Chapter 11. Matrices

Ex 11.1

Answer 2.

[2a+3b a-b] = [19 2]

[2a+3b a-b] is 1×2 matrix and [19 2] is 1×2 matrix

2a+3b = 19 ____ (1)

a-b = 2 _____ (2)

=>a= 2+b

Putting the value of a in equation (1)

4+2b+3b = 19 .... (1)

=>5b = 15

=> b = 3

From (2)

a = 2+3 = 5

Answer 3.

$$\begin{bmatrix} 3x - y \\ 5 \end{bmatrix}_{2\times1} = \begin{bmatrix} 7 \\ x + y \end{bmatrix}_{2\times1}$$

$3x - y = 7 \quad -(1)$

$x + y = 5 \quad -(2)$

$\Rightarrow x = 5 - y$

putting the value of x in(1)

$3(5 - y) - y = 7$

$\Rightarrow 15 - 3y - y = 7$

$\Rightarrow -4y = -8$

$\Rightarrow \quad y = 2$

from(2)

$x + 2 = 5$

$\Rightarrow x = 3$

Answer 6.

$A = [4\ 7]_{1\times2}\quad B = [3\ \ 1]_{1\times2}$

(i) A + 2B

$$A + 2B = [4\ \ 7] + [6\ \ 2]$$
$$= [4+6\ \ \ 7+2]$$
$$= [10\ \ \ 9]_{1\times2}$$

(ii) A - B

$$A\text{-}B = [4\text{-}3\ \ 7\text{-}1]$$
$$= [1\ \ \ 6]_{1\times2}$$

(iii) 2A - 3B

$$2A\text{-}3B = [8\ \ \ 14] - [9\ \ \ 3]$$
$$= [-1\ \ \ 11]_{1\times2}$$

Answer 7.

$P = \begin{bmatrix} 2 & 9 \\ 5 & 7 \end{bmatrix}_{2\times2},\quad Q = \begin{bmatrix} 7 & 3 \\ 4 & 1 \end{bmatrix}_{2\times2}$

(i) $$2P+3Q = \begin{bmatrix} 4 & 18 \\ 10 & 14 \end{bmatrix} + \begin{bmatrix} 21 & 9 \\ 12 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+21 & 18+9 \\ 10+12 & 14+3 \end{bmatrix} = \begin{bmatrix} 25 & 27 \\ 22 & 17 \end{bmatrix}_{2\times2}$$

(ii) $$2Q\text{-}P = \begin{bmatrix} 14 & 6 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 9 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 14-2 & 6-9 \\ 8-5 & 2-7 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ 3 & -5 \end{bmatrix}_{2\times2}$$

(iii) $$3P\text{-}2Q = \begin{bmatrix} 6 & 27 \\ 15 & 21 \end{bmatrix} - \begin{bmatrix} 14 & 6 \\ 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-14 & 27-6 \\ 15-8 & 21-2 \end{bmatrix} = \begin{bmatrix} -8 & 21 \\ 7 & 19 \end{bmatrix}_{2\times2}$$

Answer 8.

$$A=\begin{bmatrix}17 & 5 & 19\\ 11 & 8 & 13\end{bmatrix}_{2\times3},\ B=\begin{bmatrix}9 & 3 & 7\\ 1 & 6 & 5\end{bmatrix}_{2\times3}$$

$$2A-3B=\begin{bmatrix}34 & 10 & 38\\ 22 & 16 & 26\end{bmatrix}-\begin{bmatrix}27 & 9 & 21\\ 3 & 18 & 15\end{bmatrix}$$

$$=\begin{bmatrix}7 & 1 & 17\\ 19 & -2 & 11\end{bmatrix}_{2\times3}$$

Answer 9.

$$M=\begin{bmatrix}8 & 3\\ 9 & 7\\ 4 & 3\end{bmatrix}_{3\times2},\quad N=\begin{bmatrix}4 & 7\\ 5 & 3\\ 10 & 1\end{bmatrix}_{3\times2}$$

$$\text{(i)}\quad M+N=\begin{bmatrix}8+4 & 3+7\\ 9+5 & 7+3\\ 4+10 & 3+1\end{bmatrix}=\begin{bmatrix}12 & 10\\ 14 & 10\\ 14 & 4\end{bmatrix}_{3\times2}$$

$$\text{(ii)}\quad M-N=\begin{bmatrix}8-4 & 3-7\\ 9-5 & 7-3\\ 4-10 & 3-1\end{bmatrix}=\begin{bmatrix}4 & -4\\ 4 & 4\\ -6 & 2\end{bmatrix}$$

Answer 10.

$$A=\begin{bmatrix}1 & 9 & 4\\ 5 & 0 & 3\end{bmatrix}_{2\times3}$$

$$\text{(i)}\ \text{Negative}\,A=\begin{bmatrix}-1 & -9 & -4\\ -5 & 0 & -3\end{bmatrix}_{2\times3}$$

$$\text{(ii)}\ A^t=\begin{bmatrix}1 & 5\\ 9 & 0\\ 4 & 3\end{bmatrix}_{3\times2}$$

Answer 11.

$$P = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix}_{2\times2}$$

$$\text{(i)}\ \ P^t = \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}_{2\times2}$$

$$\text{(ii)}\ \ P + P^t = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 12 \\ 12 & 4 \end{bmatrix}_{2\times2}$$

$$\text{(iii)}\ \ P + p^t = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}_{2\times2}$$

Answer 12.

$$B = \begin{bmatrix} 15 & 13 \\ 11 & 12 \\ 10 & 17 \end{bmatrix}_{3\times2}$$

$$B^t = \begin{bmatrix} 15 & 11 & 10 \\ 13 & 12 & 17 \end{bmatrix}_{2\times3}$$

To add two matrixes their order i.e. their corresponding number of rows and number of columns should be same, whereas in this case order of B and B^t are not same hence, we cannot add them.

Answer 13.

$$A=\begin{bmatrix}5 & r\\ p & 7\end{bmatrix}, B=\begin{bmatrix}q & 4\\ 3 & s\end{bmatrix}$$

$$A+B=\begin{bmatrix}s & r\\ p & 7\end{bmatrix}+\begin{bmatrix}q & 4\\ 3 & s\end{bmatrix}$$

$$A+B=\begin{bmatrix}s+q & r+4\\ p+3 & 7+s\end{bmatrix}_{2\times 2} \quad -(1)$$

But, given

$$A+B=\begin{bmatrix}9 & 7\\ 5 & 8\end{bmatrix}_{2\times 2} \quad -(2)$$

from (1) & (2)

$$\begin{bmatrix}5+q & r+4\\ p+3 & 7+s\end{bmatrix}=\begin{bmatrix}9 & 7\\ 5 & 8\end{bmatrix}$$

$5+q=9$

$\Rightarrow q=4$

$r+4=7$

$\Rightarrow r=3$

$p+3=5$

$\Rightarrow p=2$

$7+s=8$

$\Rightarrow s=1$

Answer 14.

$$A=\begin{bmatrix} p & q \\ 8 & 5 \end{bmatrix}_{2\times2},\ B=\begin{bmatrix} 3p & 5q \\ 2q & 7 \end{bmatrix}_{2\times2}$$

$$A+B=\begin{bmatrix} p+3q & q+5q \\ 8+2q & 5+7 \end{bmatrix}$$

$$A+B=\begin{bmatrix} 4p & 6q \\ 8+2q & 12 \end{bmatrix}_{2\times2} \quad - \quad (1)$$

$$\text{But given, } A+B=\begin{bmatrix} 12 & 6 \\ 2r & 3s \end{bmatrix}_{2\times2} \quad -(2)$$

from (1) and (2)

$$\begin{bmatrix} 4p & 6q \\ 8+2q & 12 \end{bmatrix}=\begin{bmatrix} 12 & 6 \\ 2r & 3s \end{bmatrix}$$

$4p=12$

$\Rightarrow p=3$

$6q=6$

$\Rightarrow q=1$

$8+2q=2r$

$8+2=2r$

$\Rightarrow r=5$

$12=3s$

$\Rightarrow s=4$

Answer 15.

Given,

$$\begin{bmatrix} 2a+b & c \\ d & 3a-b \end{bmatrix}_{2\times2} = \begin{bmatrix} 4 & 3a \\ 7 & 6 \end{bmatrix}_{2\times2}$$

$2a+b=4 \quad -(1)$

$3a-b=6 \quad -(2)$

Adding (1) and (2), we get

$5a=10$

$\Rightarrow a=2$

from(1)

$2(2)+b=4$

$\Rightarrow b=0$

$c=3a$

$\Rightarrow c=3\times2$

$\Rightarrow c=6$

$\Rightarrow d=7$

Answer 17.

$$A=\begin{bmatrix} 15 & 7 \\ 13 & 8 \end{bmatrix}_{2\times2}, B=\begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}_{2\times2}$$

$(1) \quad A+X=B$

$X=B-A$

$$\therefore X=\begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}-\begin{bmatrix} 15 & 7 \\ 13 & 8 \end{bmatrix}=\begin{bmatrix} 1 & 5 \\ 14 & 3 \end{bmatrix}_{2\times2}$$

$(ii) \quad 2A-X=B$

$X=2A-B$

$$X=\begin{bmatrix} 30 & 14 \\ 26 & 16 \end{bmatrix}-\begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}$$

$$=\begin{bmatrix} 14 & 2 \\ -1 & 5 \end{bmatrix}_{2\times2}$$

Answer 18.

$$P = \begin{bmatrix} 14 & 17 \\ 13 & 1 \end{bmatrix}_{2\times2}, \quad Q = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}_{2\times2}$$

$$P - M = 3Q$$

$$M = P - 3Q$$

$$M = \begin{bmatrix} 14 & 17 \\ 13 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ 9 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 14-6 & 17-3 \\ 13-9 & 1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 14 \\ 4 & 10 \end{bmatrix}_{2\times2}$$

Ex 11.2

Answer 2.

$$A=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}_{2\times2},\quad B=\begin{bmatrix}-2 & 3\\ -4 & 1\end{bmatrix}_{2\times2}$$

(a) $$AB=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}\begin{bmatrix}-2 & 3\\ -4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2-12 & 3+3\\ -6-8 & 9+2\end{bmatrix}=\begin{bmatrix}-14 & 6\\ -14 & 11\end{bmatrix}_{2\times2}$$

(b) $$BA=\begin{bmatrix}-2 & 3\\ -4 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-2+9 & -6+6\\ -4+3 & -12+2\end{bmatrix}$$

$$=\begin{bmatrix}7 & 0\\ -1 & -10\end{bmatrix}_{2\times2}$$

Answer 3.

Sol: Let $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}_{2\times2}$

$$AI=\begin{bmatrix}p & q\\ r & s\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}p+0 & 0+q\\ r+0 & 0+s\end{bmatrix}$$

$$=\begin{bmatrix}p & q\\ r & s\end{bmatrix}=A$$

$$IA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}p & q\\ r & s\end{bmatrix}$$

$$=\begin{bmatrix}p+0 & q+0\\ 0+r & 0+s\end{bmatrix}$$

$$=\begin{bmatrix}p & q\\ r & s\end{bmatrix}=a$$

Hence proved $AI=IA=A$.

Answer 4.

$$P=\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}_{2\times2},\quad Q=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}_{2\times2}$$

$$QP=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & 4+1\\1+4 & 2+2\end{bmatrix}$$

$$=\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}_{2\times2}$$

$$P(QP)=\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$$

$$=\begin{bmatrix}4+10 & 5+8\\8+5 & 10+4\end{bmatrix}$$

$$=\begin{bmatrix}14 & 13\\13 & 14\end{bmatrix}_{2\times2}$$