

Chapter 5. Linear Inequations

Ex 5.1

Answer 1.

(i) $x + 5 > 11$

$$x + 5 > 11$$

$$\Rightarrow x > 11 - 5$$

$$\Rightarrow x > 6$$

$$\text{But}\{x : x \in \mathbb{N}; N < 10\}$$

Therefore, solution set $x = \{7, 8, 9\}$

(ii) $2x + 1 < 17$

$$2x + 1 < 17$$

$$\Rightarrow 2x < 17 - 1$$

$$\Rightarrow x < \frac{16}{2}$$

$$\Rightarrow x < 8$$

$$\text{But}\{x : x \in \mathbb{N}; N < 10\}$$

Therefore, solution set $x = \{1, 2, 3, 4, 5, 6, 7\}$

(iii) $3x - 5 \leq 7$

$$3x - 5 \leq 7$$

$$\Rightarrow 3x \leq 7 + 5$$

$$\Rightarrow 3x \leq 12$$

$$\Rightarrow x \leq 4$$

$$\text{But}\{x : x \in \mathbb{N}; N < 10\}$$

Therefore, solution set $x = \{1, 2, 3, 4\}$

(iv) $8 - 3x \geq 2$

$$8 - 3x \geq 2$$

$$\Rightarrow -3x \geq -6$$

$$\Rightarrow x \geq 2$$

$$\text{But}\{x : x \in \mathbb{N}; N < 10\}$$

Therefore, solution set $x = \{1, 2\}$

(v) $5 - 2x < 11$

$$5 - 2x < 11$$

$$\Rightarrow -2x < 11 - 5$$

$$\Rightarrow -2x < 6$$

$$\Rightarrow x > -3$$

$$\text{But}\{x : x \in \mathbb{N}; N < 10\}$$

Therefore, solution set $x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Answer 2.

(i) $3x > 12$

$$3x > 12$$

$$x > \frac{12}{3}$$

$$x > 4$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x > 4\}$$

(ii) $2x - 3 < 7$

$$2x - 3 < 7$$

$$2x < 7 + 3$$

$$2x < 10$$

$$x < \frac{10}{2}$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x < 5\}$$

(iii) $3x + 2 \leq 11$

$$3x + 2 \leq 11$$

$$3x \leq 11 - 2$$

$$3x \leq 9$$

$$x \leq \frac{9}{3}$$

$$x \leq 3$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x \leq 3\}$$

(iv) $14 - 3x \geq 5$

$$14 - 3x \geq 5$$

$$-3x \geq 5 - 14$$

$$3x \leq 9 \quad (\text{multiplying by } -1 \text{ changes the sign})$$

$$x \leq \frac{9}{3}$$

$$x \leq 3$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x \leq 3\}$$

$$(v) 7x + 11 > 16 - 3x$$

$$7x + 11 > 16 - 3x$$

$$7x + 3x > 16 - 11$$

$$10x > 5$$

$$x > \frac{5}{10} = 0.5$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x > 0.5\}$$

$$(vi) 3x + 25 < 8x - 10$$

$$3x + 25 < 8x - 10$$

$$25 + 10 < 8x - 3x$$

$$35 < 5x$$

$$x > 7$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x > 7\}$$

$$(vii) 2(3x - 5) \leq 8$$

$$2(3x - 5) \leq 8$$

$$6x - 10 \leq 8$$

$$6x \leq 8 + 10$$

$$6x \leq 18$$

$$x \leq 3$$

Since, replacement set is R

$$\text{Solution set} = \{x : x \in \mathbb{R} \text{ and } x \leq 3\}$$

$$(viii) x + 7 \geq 15 + 3x$$

$$x + 7 \geq 15 + 3x$$

$$x - 3x \geq 15 - 7$$

$$-2x \geq 8$$

$$2x \leq -8$$

$$x \leq -4$$

Since, replacement set is R

Solution set = $\{x : x \in \mathbb{R} \text{ and } x \leq -4\}$

$$(ix) 2x - 7 \geq 5x + 8$$

$$2x - 7 \geq 5x + 8$$

$$2x - 5x \geq 8 + 7$$

$$-3x \geq 15$$

$$3x \leq -15$$

$$x \leq -5$$

Since, replacement set is \mathbb{R}

Solution set = $\{x : x \in \mathbb{R} \text{ and } x \leq -5\}$

$$(x) 9 - 4x \leq 15 - 7x$$

$$9 - 4x \leq 15 - 7x$$

$$9 - 15 \leq 4x - 7x$$

$$-6 \leq -3x$$

$$6 \geq 3x \quad (\text{multiplying by } -1 \text{ changes the sign})$$

$$x \leq 2$$

Since, replacement set is \mathbb{R}

Solution set = $\{x : x \in \mathbb{R} \text{ and } x \leq 2\}$

Answer 3.

$$6 - 10x < 36$$

$$-10x < 36 - 6$$

$$-10x < 30$$

$$10x > -30$$

$$x > -3$$

Solution set = $\{-2, -1, 0, 1, 2\}$

Answer 4.

$$3 - 2x \geq x - 12$$

$$3 + 12 \geq x + 2x$$

$$15 \geq 3x$$

$$x \leq 5$$

$$\text{Solution set} = \{1, 2, 3, 4, 5\}$$

Answer 5.

$$5x - 9 \leq 15 - 7x$$

$$5x + 7x \leq 15 + 9$$

$$12x \leq 24$$

$$x \leq 2$$

$$\text{Solution set} = \{0, 1, 2\}$$

Answer 6.

$$7 + 5x > x - 13$$

$$-x + 5x > -13 - 7$$

$$4x > -20$$

$$x > -5$$

$$\text{Solution set} = \{-4, -3, -2, -1\}$$

Answer 7

$$5x - 14 < 18 - 3x$$

$$5x + 3x < 18 + 14$$

$$8x < 32$$

$$x < 4$$

$$\text{Solution set} = \{0, 1, 2, 3\}$$

Answer 8.

$$2x + 7 \geq 5x - 14$$

$$2x - 5x \geq -14 - 7$$

$$-3x \geq -21$$

$$3x \leq 21$$

$$x \leq 7$$

$$\text{Solution set} = \{2,3,5,7\}$$

Answer 9.

$$\frac{x}{4} + 3 \leq \frac{x}{3} + 4$$

$$\frac{x+12}{4} \leq \frac{x+12}{3}$$

$$3x + 36 \leq 4x + 48$$

$$3x - 4x \leq 48 - 36$$

$$-x \leq 12$$

$$x \geq -12$$

$$\text{Solution set} = \{-11, -9, -7, -5, -3, -1\}$$

Answer 10.

$$\frac{x+3}{3} \leq \frac{x+8}{4}$$

$$4x + 12 \leq 3x + 24$$

$$4x - 3x \leq 24 - 12$$

$$x \leq 12$$

$$\text{Solution set} = \{2,4,6,8,10,12\}$$

Answer 11.

(i) $x \in \mathbb{Z}$

$$x + 17 \leq 4x + 9$$

$$x - 4x \leq 9 - 17$$

$$-3x \leq -8$$

$$3x \geq 8$$

$$x \geq \frac{8}{3}$$

Since $x \in \mathbb{Z}$

Smallest value of $x = [3]$

(ii) $x \in \mathbb{R}$

$$x + 17 \leq 4x + 9$$

$$x - 4x \leq 9 - 17$$

$$-3x \leq -8$$

$$3x \geq 8$$

$$x \geq \frac{8}{3}$$

Since $x \in \mathbb{R}$

Smallest value of $x = \left[2\frac{2}{3}\right]$

Answer 12.

$$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$$

$$2 - 5x + 2x^2 = 0$$

$$2x^2 - 5x + 2 = 0$$

$$x^2 - \frac{5}{2}x + 1 = 0$$

$$x^2 - 2x - \frac{1}{2}x + 1 = 0$$

$$x(x - 2) - \frac{1}{2}(x - 2) = 0$$

$$(x - 2)\left(x - \frac{1}{2}\right) = 0$$

$$(x - 2) = 0, \left(x - \frac{1}{2}\right) = 0$$

$$x = 2, x = \frac{1}{2}$$

Answer 13.

(i) $2x - 11 \leq 7 - 3x, x \in \mathbb{N}$

$$2x - 11 \leq 7 - 3x$$

$$2x + 3x \leq 7 + 11$$

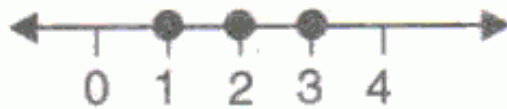
$$5x \leq 18$$

$$x \leq \frac{18}{5}$$

$$x \leq 3.6$$

Since $x \in \mathbb{N}$

$$\text{Solution set} = \{1, 2, 3\}$$



(ii) $3(5x + 3) \geq 2(9x - 17), x \in \mathbb{W}$

$$3(5x + 3) \geq 2(9x - 17)$$

$$15x + 9 \geq 18x - 34$$

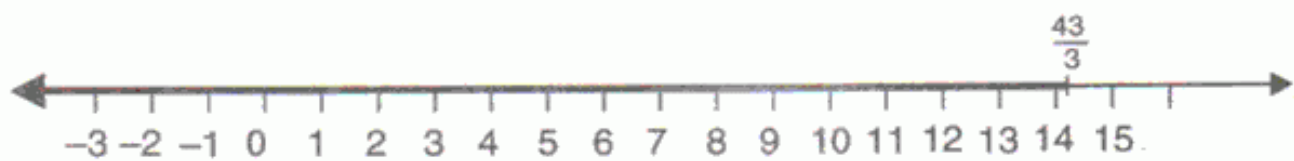
$$15x - 18x \geq -34 - 9$$

$$-3x \geq -43$$

$$3x \leq 43$$

$$x \leq \frac{43}{3}$$

$$\text{Solution set} = \left[x \leq \frac{43}{3} \right]$$



(iii) $2(3x - 5) > 5(13 - 2x), x \in \mathbb{W}$

$$2(3x - 5) > 5(13 - 2x)$$

$$6x - 10 > 65 - 10x$$

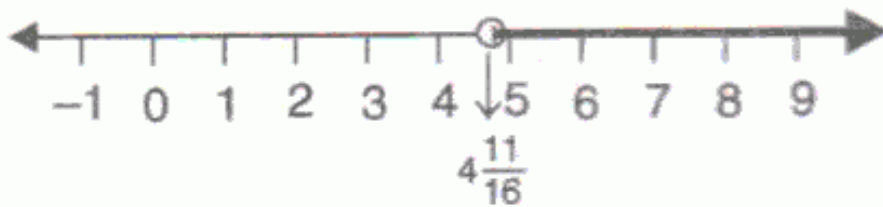
$$6x + 10x > 65 + 10$$

$$16x > 75$$

$$x > \frac{75}{16}$$

$$x > 4\frac{11}{16}$$

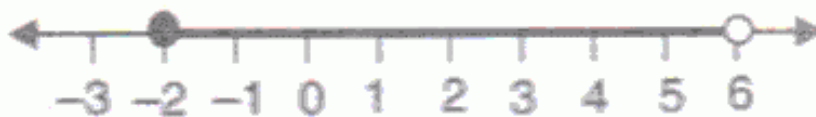
$$\text{Solution set} = \left[x > 4\frac{11}{16} \right]$$



$$(iv) 3x - 9 \leq 4x - 7 < 2x + 5, x \in \mathbb{R}$$

$$\begin{array}{ll} 3x - 9 \leq 4x - 7 & 4x - 7 < 2x + 5 \\ 3x - 4x \leq -7 + 9 & 4x - 2x < 5 + 7 \\ -x \leq 2 & 2x < 12 \\ x \geq -2 & x < 6 \end{array} \quad \text{and}$$

$$\text{Solution set} = [-2 \leq x < 6]$$



$$(v) 2x - 7 < 5x + 2 \leq 3x + 14, x \in \mathbb{R}$$

$$\begin{array}{ll} 2x - 7 < 5x + 2 & 5x + 2 \leq 3x + 14 \\ 2x - 5x < 2 + 7 & 5x - 3x \leq 14 - 2 \\ -3x < 9 & 2x \leq 12 \\ x > -3 & x \leq 6 \end{array} \quad \text{and}$$

$$\text{Solution set} = [-3 < x \leq 6]$$



$$(vi) -3 \leq \frac{1}{2} - \frac{2x}{3} \leq 2\frac{2}{3}, x \in \mathbb{N}$$

$$\begin{array}{ll} -3 \leq \frac{1}{2} - \frac{2x}{3} & \frac{1}{2} - \frac{2x}{3} \leq 2\frac{2}{3} \\ -3 \leq \frac{3 - 4x}{6} & \frac{3 - 4x}{6} \leq \frac{8}{3} \\ -18 \leq 3 - 4x & 3 - 4x \leq \frac{8}{3} \\ -18 - 3 \leq -4x & \text{and } 9 - 12x \leq 48 \\ -21 \leq -4x & -12x \leq 39 \\ x \leq \frac{21}{4} & 12x \geq -39 \\ x \leq 5\frac{1}{4} & x \geq -3\frac{1}{4} \end{array}$$

$$\text{Solution set} = \left[-3\frac{1}{4} \leq x \leq 5\frac{1}{4} \right]$$



$$(vii) \quad 4\frac{3}{4} \geq x + \frac{5}{6} > \frac{1}{3}, x \in \mathbb{R}$$

$$\begin{array}{ll} 4\frac{3}{4} \geq x + \frac{5}{6} & x + \frac{5}{6} > \frac{1}{3} \\ \frac{19}{4} \geq \frac{6x+5}{6} & \frac{6x+5}{6} > \frac{1}{3} \\ 114 \geq 24x + 20 & \text{and} \quad 18x + 15 > 6 \\ 114 - 20 \geq 24x & 18x > 6 - 15 \\ 94 \geq 24x & 18x > -9 \\ x \leq 3\frac{11}{12} & x > -\frac{1}{2} \end{array}$$

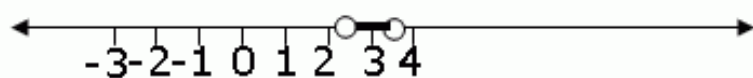
$$\text{Solution set} = \left[-\frac{1}{2} < x \leq 3\frac{11}{12} \right]$$



$$(viii) \quad \frac{1}{3}(2x - 1) < \frac{1}{4}(x + 5) < \frac{1}{6}(3x + 4), x \in \mathbb{R}$$

$$\begin{array}{ll} \frac{1}{3}(2x - 1) < \frac{1}{4}(x + 5) & \frac{1}{4}(x + 5) < \frac{1}{6}(3x + 4) \\ 4(2x - 1) < 3(x + 5) & 6(x + 5) < 4(3x + 4) \\ 8x - 4 < 3x + 15 & 6x + 30 < 12x + 16 \\ 8x - 3x < 15 + 4 & \text{and} \quad 6x - 12x < 16 - 30 \\ 5x < 19 & -6x < -14 \\ x < 3\frac{4}{5} & x > 2\frac{1}{3} \end{array}$$

$$\text{Solution set} = \left[2\frac{1}{3} < x < 3\frac{4}{5} \right]$$



$$(ix) \quad \frac{1}{3}(5x - 8) \geq \frac{1}{2}(4x - 7), x \in \mathbb{R}$$

$$\frac{1}{3}(5x - 8) \geq \frac{1}{2}(4x - 7)$$

$$2(5x - 8) \geq 3(4x - 7)$$

$$10x - 16 \geq 12x - 21$$

$$10x - 12x \geq -21 + 16$$

$$-2x \geq -5$$

$$x \leq \frac{5}{2}$$

$$x \leq 2\frac{1}{2}$$

$$\text{Solution set} = \left[x \leq 2\frac{1}{2} \right]$$



$$(x) \frac{5}{4}x > 1 + \frac{1}{3}(4x - 1), x \in \mathbb{R}$$

$$\frac{5}{4}x > 1 + \frac{1}{3}(4x - 1)$$

$$\frac{5}{4}x > \frac{3 + (4x - 1)}{3}$$

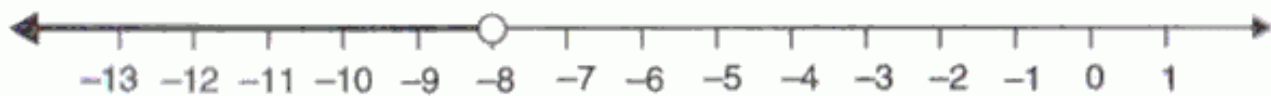
$$15x > 12 + 16x - 4$$

$$15x - 16x > 8$$

$$-x > 8$$

$$x < -8$$

$$\text{Solution set} = [x < -8]$$



Answer 14.

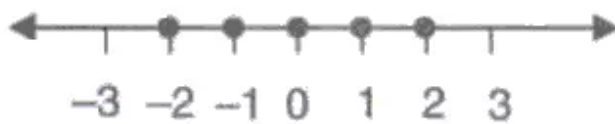
$$P = \{x : -3 < x \leq 7, x \in \mathbb{R}\}$$

$$Q = \{x : -7 \leq x < 3, x \in \mathbb{R}\}$$

$$P = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\} \text{ and } Q = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}$$

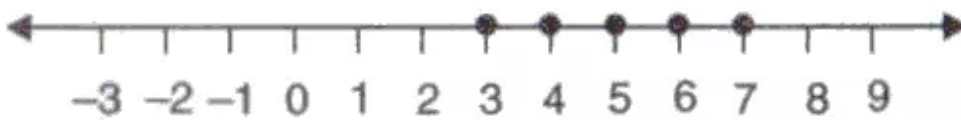
(i) $P \cap Q$

$$P \cap Q = \{-2, -1, 0, 1, 2\}$$



(ii) $Q' \cap P$

$$Q' \cap P = \{3, 4, 5, 6, 7\}$$



(iii) $P - Q$

$$P - Q = \{3, 4, 5, 6, 7\}$$



Answer 15.

$$P = \{x : 7x - 2 > 4x + 1, x \in \mathbb{R}\}$$

$$7x - 2 > 4x + 1$$

$$7x - 4x > 1 + 2$$

$$3x > 3$$

$$x > 1$$

$$P = \{2, 3, 4, 5, \dots\}$$

$$Q = \{x : 9x - 45 \geq 5(x - 5), x \in \mathbb{R}\}$$

$$9x - 45 \geq 5x - 25$$

$$9x - 5x \geq -25 + 45$$

$$4x \geq 20$$

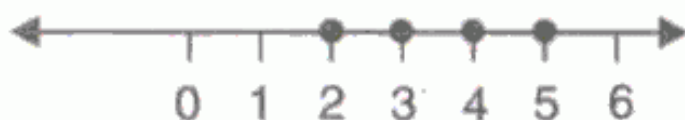
$$x \geq 5$$

$$Q = \{5, 6, 7, 8, 9, \dots\}$$

and

(i) $P \cap Q$

$$P \cap Q = \{2, 3, 4, 5\}$$



(ii) $P - Q$

$$P - Q = \{6, 7, 8, 9, \dots\}$$



(iii) $P \cap Q'$

$$P \cap Q' = \{6, 7, 8, 9, \dots\}$$



Answer 16.

$$P = \{x : 7x - 4 > 5x + 2, x \in \mathbb{R}\}$$

$$7x - 4 > 5x + 2$$

$$7x - 5x > 2 + 4$$

$$2x > 6$$

$$x > 3$$

$$P = \{4, 5, 6, 7, \dots\}$$

$$Q = \{x : x - 19 \geq 1 - 3x, x \in \mathbb{R}\}$$

$$x - 19 \geq 1 - 3x$$

$$x + 3x \geq 1 + 19$$

$$4x \geq 20$$

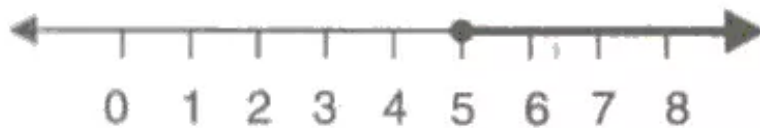
$$x \geq 5$$

$$Q = \{5, 6, 7, 8, 9, \dots\}$$

and

(i) $P \cap Q$

$$P \cap Q = \{5, 6, 7, 8, \dots\}$$



(ii) $P' \cap Q$

$$P' \cap Q = \{\emptyset\}$$