Proportion

EXERCISE 12(A)

Question 1.

In each of the following, check whether or not the given ratios form a proportion :

(i) 8 : 16 and 12 : 15

(ii) 16 : 28 and 24 : 42

(iii) 12 ÷ 3 and 8 ÷ 2

(iv) 25 : 40 and 20 : 32

(v) $\frac{15}{18} and \frac{10}{12}$

(vi) $\frac{7}{8}$ and 14 : 16

Solution:

(*i*) 8 : 16 and 12 : 15

Since $8 : 16 = \frac{8}{16} = \frac{1}{2}$

and $12 : 15 = \frac{12}{15} = \frac{4}{5}$

∴ Ratio 8 : 16 ≠ ratio 12 : 15, they are not in a proportion.

(*ii*) 16 : 28 and 24 : 42

Since $16 : 28 = \frac{16}{28} = \frac{4}{7}$

and $24 : 42 = \frac{24}{42} = \frac{4}{7}$

∴ Ratio 16 : 28 and 24 : 42 are equal, so they form a proportion.

(*iii*) 12 ÷ 3 and 8 ÷ 2

Since $\frac{12}{3} = 4$ and $\frac{8}{2} = 4$

∴ Ratio 12 ÷ 3 and 8 ÷ 2 are equal, so they form a proportion.

(*iv*) 25 : 40 and 20 : 32

Since $25 : 40 = \frac{25}{40} = \frac{5}{8}$

and $20 : 32 = \frac{20}{32} = \frac{5}{8}$

$\therefore$ Ratio 25 : 20 and 20 : 32 are equal, so they form a proportion.

(v) $\frac{15}{18}$ and $\frac{10}{12}$

Since $\frac{15}{18} = \frac{5}{6}$ and $\frac{10}{12} = \frac{5}{6}$

$\therefore$ Ratio $\frac{15}{18}$ and $\frac{10}{12}$ are equal, so they form a proportion.

(vi) $\frac{7}{8}$ and 14 : 16

Since $\frac{7}{8} = \frac{7}{8}$ and $14 : 16 = \frac{14}{16} = \frac{7}{8}$

$\therefore$ Ratio $\frac{7}{8}$ and 14 : 16 are equal, so they form a proportion.

Question 2.

Find the value of x in .each of the following proportions :

(i) x : 4 = 6 : 8

(ii) 14 : x = 7 : 9

(iii) 4 : 6 = x : 18

(iv) 8 : 10 = x : 25

(v) 5 : 15 = 4 : x

(vi) 16 : 24 = 6 : x

Solution:

$x : 4 = 6 : 8$

$\Rightarrow x \times 8 = 4 \times 6$

$\Rightarrow \quad x = \frac{4 \times 6}{8} = 3$

(ii) $14 : x = 7 : 9$

$\Rightarrow x \times 7 = 14 \times 9$

$\Rightarrow \quad x = \frac{14 \times 9}{7} = 18$

(*iii*) $4 : 6 = x : 18$

$\Rightarrow 6 \times x = 4 \times 18$

$\Rightarrow \quad x = \frac{4 \times 18}{6} = \mathbf{12}$

(*iv*) $8 : 10 = x : 25$

$\Rightarrow 10 \times x = 25 \times 8 \Rightarrow x = \frac{25 \times 8}{10} = \mathbf{20}$

(*v*) $5 : 15 = 4 : x$

$\Rightarrow 5 \times x = 15 \times 4 \quad \Rightarrow x = \frac{15 \times 4}{5} = \mathbf{12}$

(*vi*) $16 : 24 = 6 : x$

$\Rightarrow 16 \times x = 24 \times 6 \Rightarrow x = \frac{24 \times 6}{16} = \mathbf{9}$

Question 3.

Find the value of x so that the given four numbers are in proportion :

(i) x, 6, 10 and 15

(ii) x, 4, 15 and 30

(iii) 2, x, 10 and 25

(iv) 4, x, 6 and 18

(v) 9, 12, x and 8

(vi) 4, 10, 36 and x

(vii) 7, 21, x and 45

(viii) 6, 8, 12 and x.

Solution:

(*i*) $x : 6 : 10 : 15$

$\Rightarrow x \times 15 = 6 \times 10 \Rightarrow x = \frac{6\times10}{15} = 4.$

(*ii*) $x : 4 : 15 : 30$

$\Rightarrow x \times 30 = 4 \times 15 \Rightarrow x = \frac{4\times15}{30} = 2.$

(*iii*) $2 : x : 10 : 25$

$\Rightarrow x \times 10 = 2 \times 25 \Rightarrow x = \frac{2\times25}{10} = \frac{25}{5} = 5.$

(*iv*) $4 : x : 6 : 18$

$\Rightarrow x \times 6 = 18 \times 4 \Rightarrow x = \frac{18\times4}{6} = 12.$

(*v*) $9 : 12 : x : 8$

$\Rightarrow 12 \times x = 9 \times 8 \Rightarrow x = \frac{9\times8}{12} = 6.$

(*vi*) $4 : 10 : 36 : x$

$\Rightarrow 4 \times x = 10 \times 36 \Rightarrow x = \frac{10\times36}{4} = 90.$

(*vii*) $7 : 21 : x : 45$

$\Rightarrow 21 \times x = 7 \times 45$

$\Rightarrow \quad x = \frac{7\times45}{21} = \frac{45}{3} = 15.$

(*viii*) $6 : 8 : 12 : x$

$\Rightarrow 6 \times x = 12 \times 8 \Rightarrow x = \frac{12\times8}{6} = 16.$

Question 4.

The first, second and the fourth terms of a proportion are 6, 18 and 75, respectively. Find its third term.

Solution:

Let the third term = x

6 : 18 : : x : 75

= 18 x x = 6 x 75

x = $\frac{6\times75}{18} = \frac{75}{3}$ = 25

The third term of proportion is 25

Question 5.
Find the second term of the proportion whose first, third and fourth terms are 9, 8 and 24 respectively.
Solution:
Let the second term = x
9 : x : : 8 : 24
=> x x 8 = 24 x 9
x = $\frac{24 \times 9}{8}$ = 3 x 9 = 27
The second term of proportion = 27

Question 6.
Find the fourth term of the proportion whose first, second and third terms are 18, 27, and 32 respectively.
Solution:
Let the fourth term = x
18 : 27 : : 32 : x
=> 18 x x = 27 x 32
=> x = $\frac{27 \times 32}{18}$ = 3 x 16 = 48
Fourth term = 48

Question 7.
The ratio of the length and the width of a school ground is 5 : 2. Find the length, if the width is 40 metres.
Solution:
Let the length = x m,
width = 40 m
The ratio of length to width = x : 40
as per given statement 5 : 2 = x : 40
=> 2 x x = 40 x 5
x = $\frac{40 \times 5}{2}$ = 20 x 5 = 100 m

Question 8.
The ratio of the sale of eggs on a Sunday and that of the whole week at a grocery shop was 2 : 9. If the total value of the sale of eggs in the same week was Rs 360, find the value of the sale of eggs that Sunday.
Solution:
Let, the sale of eggs on Sunday = x
Sale in week = Rs 360
According to question, 2 : 9 = x : 360
=> 9 x x = 360 x 2
x = $\frac{360 \times 2}{9}$ = Rs 80
Sale on Sunday = Rs 80

Question 9.
The ratio of copper and zinc in an alloy is 9 : 8. If the weight of zinc, in the alloy, is 9.6 kg ; find the weight of copper in the alloy.
Solution:
Let the weight of copper = x kg
Weight of zinc = 9.6 kg.
According to question,
9 : 8 = x : 9.6
=> 8 x x = 9 x 9.6
=> x = $\frac{9\times9.6}{8}$ = 9 x 1.2 = 10.8 kg.
Weight of cooper in alloy = 10.8

Question 10.
The ratio of the number of girls to the number of boys in a school is 2 : 5. If the number of boys is 225 ; find:
(i) the number of girls in the school.
(ii) the number of students in the school.
Solution:
Let, the number of girls in school = x
Number of boys in school = 225
According to question 2 : 5 = x : 225
=> 5 x x = 2 x 225
x = $\frac{2\times225}{5}$ = 2 x 45 = 90
Number of girls in school = 90
Total number of student in the school = (number of boys + number of girls) = (225 + 90) = 315

Question 11.
In a class, one out of every 5 students pass. If there are 225 students in all the sections of a class, find how many pass ?
Solution:
Total number of students in all sections = 225
Given, One of every five students pass
Total students pass = 225 x $\frac{1}{5}$ = 45 studetns

Question 12.
Make set of all possible proportions from the numbers 15, 18, 35 and 42.
Solution:
The possible proportions that can be made from the numbers 15, 18, 35 and 42 are
(i) 15 : 35 :: 18 : 42
(ii) 42 : 18 :: 35 : 15
(iii) 42 : 35 :: 18 : 15
(iv) 15 : 18 :: 35 : 42

EXERCISE 12(B)

Question 1.
If x, y and z are in continued proportion, then which of the following is true :
(i) x : y = x : z
(ii) x : x = z : y
(iii) x : y = y : z
(iv) y : x = y : z
Solution:
(iii) x : y = y : z

Question 2.
Which of the following numbers are in continued proportion :
(i) 3, 6 and 15
(ii) 15, 45 and 48
(iii) 6, 12 and 24
(iv) 12, 18 and 27
Solution:
(iii) and (iv)

Question 3.
Find the mean proportion between
(i) 3 and 27
(ii) 0.06 and 0.96
Solution:

(*i*) Mean proportional between 3 and 27

$$= \sqrt{3 \times 27} = \sqrt{81} = 9$$

(*ii*) Mean proportional between 0·6 and 9·6

$$= \sqrt{0{\cdot}6 \times 9{\cdot}6} = \sqrt{\frac{6}{10} \times \frac{96}{10}}$$

$$= \sqrt{\frac{576}{100}} = \frac{24}{10} = 2{\cdot}4$$

Question 4.
Find the third proportional to :
(i) 36, 18
(ii) 5.25, 7
(iii) ₹ 1.60, ₹ 0.40
Solution:

(i) Let the required third proportional be x

$\therefore$ 36, 18, x are in continued proportion

$\Rightarrow 36 : 18 = 18 : x$

$\Rightarrow 36x = 18 \times 18$

$$\Rightarrow x = \frac{18 \times 18}{36}$$

$\Rightarrow x = 9$

$\therefore$ Required proportional = 9

(ii) Let the required third proportional be x

$\therefore$ 5.25, 7, x are in continued proportion

$\Rightarrow 5.25 : 7 = 7 : x$

$\Rightarrow 5x = 7 \times 7$

$$\Rightarrow x = \frac{7 \times 7}{5.25}$$

$$\Rightarrow x = \frac{49}{5.25} = \frac{28}{3}$$

$$\Rightarrow x = 9\frac{1}{3}$$

(iii) Let the required third proportional be x

$\therefore$ ₹1.60, ₹0.40, ₹x are in continued proportion.

$\Rightarrow 1.60 \times x = 0.40 \times 0.40$

$$\Rightarrow x = \frac{0.40 \times 0.40}{1.60}$$

=> x = 0.1

Question 5.

The ratio between 7 and 5 is same as the ratio between ₹ x and ₹ 20.50 ; find the value of x.

Solution:

Since, It is given that the ratio between 7 and 5 is same as the ratio between ₹ x and ₹

20.50

$\therefore \quad 7 : 5 = x = 20.50$

$\Rightarrow \quad 5x = 7 \times 20.50$

$$\Rightarrow \quad x = \frac{7 \times 20.50}{5}$$

$\Rightarrow \quad x = 82.7$

Question 6.

If (4x + 3y) : (3x + 5y) = 6 : 7, find :

(i) x : y

(ii) x, if y = 10

(iii) y, if x = 27

Solution:

(i) $7x\,(4x + 3y) = 6x\,(3x + 5y)$

$28x + 21y = 18x + 30y$

$28x - 18x = 30y - 21y$

$10x = 9y$

$$\frac{x}{y} = \frac{9}{10}$$

$\therefore \quad x : y = 9 : 10$

(ii) $(4x + 3y) : (3x + 5y) = 6 : 7$

Given, $y = 10$

$\therefore \quad (4x + 3 \times 10) : (3x + 5 \times 10) = 6 : 7$

$(4x + 30) : (3x + 50) = 6 : 7$

$7 \times (4x + 30) = 6 \times (3x + 50)$

$28x + 210 = 18x + 300$

$28x - 18x = 300 - 210$

$10x = 90$

$$\Rightarrow \quad x = \frac{90}{10} = 9$$

(iii) $(4x + 3y) : (3x + 5y) = 6 : 7$

Given, $x = 27$

$\therefore \quad (4 \times 27 + 3y) : (3 \times 27 + 5y) = 6 : 7$

$(108 + 3y) : (81 + 5y) = 6 : 7$

$7 \times (108 + 3y) = 6 \times (81 + 5y)$

$756 + 21y = 486 + 30y$

$9y = 270$

$\Rightarrow \quad y = \frac{270}{9} = 30$

Question 7.

If $\frac{2y+5x}{3y-5x} = 2\frac{1}{2}$, find:

(i) x : y

(ii) x, if y = 70

(iii) y, if x = 33

Solution:

(i) $\frac{2y+5x}{3y-5x} = \frac{2\times2+1}{2}$

$\frac{2y+5x}{3y-5x} = \frac{5}{2}$

$\Rightarrow \quad 2(2y + 5x) = 5 \times (3y - 5x)$

$\Rightarrow \quad 4y + 10x = 15y - 25x$

$\Rightarrow \quad 35x = 11y$

$\Rightarrow \quad \frac{x}{y} = \frac{11}{35}$ *i.e.* $x : y = 11 : 35$

(ii) $\frac{2y+5x}{3y-5x} = \frac{5}{2}$

Given $y = 70$

$$\frac{2\times 70+5x}{3\times 70-5x}=\frac{5}{2} \Rightarrow \frac{140+5x}{210-5x}=\frac{5}{2}$$

$\Rightarrow 2 \times (140 + 5x) = 5 \times (210 - 5x)$

$\Rightarrow 280 + 10x = 1050 - 25x$

$\Rightarrow 35x = 1050 - 280$

$\Rightarrow 35x = 770 \qquad \Rightarrow x = \frac{770}{35} = 22$

(iii) $\frac{2y+5x}{3y-5x} = \frac{5}{2}$

Given $x = 33$

$$\frac{2y+5\times 33}{3y-5\times 33} = \frac{5}{2} \Rightarrow \frac{2y+165}{3y-165}=\frac{5}{2}$$

$\Rightarrow 2 \times (2y + 165) = 5 \times (3y - 165)$

$\Rightarrow 4y + 330 = 15y - 825$

$\Rightarrow 11y = 1155$

$\Rightarrow y = \frac{1155}{11} = 105$

EXERCISE 12(C)

Question 1.

Are the following numbers in proportion:

(i) 32, 40, 48 and 60 ?

(ii) 12,15,18 and 20 ?

Solution:

(i) 32, 40, 48 and 60 are in proportion

if 32 : 40 = 48 : 60

if 32 x 60 = 40 x 48

$\left\{\frac{a}{b} = \frac{c}{d} \Longrightarrow \quad ad = bc\right\}$

if 1920 = 1920

Which is true.

32, 40, 48 and 60 are in proportion

(ii) 12, 15, 18 and 20 are in proportion

if 12 : 15 = 18 : 20

if 12 x 20 = 15 x 18 {ad = bc}

if 240 = 270

which is not true.
12, 15, 18 and 20 are not in proportion.

Question 2.
Find the value of x in each of the following such that the given numbers are in proportion.
(i) 14, 42, x and 75
(ii) 45, 135, 90 and x
Solution:
14, 42, x and 75 are in proportion
$\frac{14}{42} = \frac{x}{75}$
=> 14 x 75 =x x 42

$$\Rightarrow x = \frac{14 \times 75}{42} = 25$$

$$\therefore x = 25$$

(ii) $\because$ 45, 135, 90 and x are in proportion

$$\therefore \frac{45}{135} = \frac{90}{x} \Rightarrow 45 \times x = 90 \times 135$$

$$\Rightarrow x = \frac{90 \times 135}{45} = 270$$

$$\therefore x = 270$$

Question 3.
The costs of two articles are in the ratio 7 : 4. If the cost of the first article is Rs. 2,800 ; find the cost of the second article.
Solution:
Ratio in the cost of two articles = 7 : 4
Cost of first article = Rs. 2800
Let cost of the second article = x
7 : 4 = 2800 : x

$$\Rightarrow \frac{7}{4} = \frac{2800}{x} \Rightarrow 7 \times x = 2800 \times 4$$

$$\Rightarrow x = \frac{2800 \times 4}{7} = 1600$$

$$\therefore \text{Cost of second article} = \text{Rs. } 1600$$

Question 4.
The ratio of the length and the width of a rectangular sheet of paper is 8 : 5. If the width

of the sheet is 17.5 cm; find the length.

Solution:

Let length of sheet = x cm

Ratio in length and breadth = 8 : 5

and width = 17.5 cm

8 : 5 = x : 17.5

$$\Rightarrow \frac{8}{5} = \frac{x}{17.5} \Rightarrow 8 \times 17.5 = x \times 5$$

$$\Rightarrow x = \frac{8 \times 17.5}{5} = 8 \times 3.5 = 28$$

Length of sheet = 28 cm

Question 5.

The ages of A and B are in the ratio 6 : 5. If A's age is 18 years, find the age of B.

Solution:

Ratio in the ages of A and B = 6 : 5

A's age = 18 years

Let B's age = x years

6 : 5 = 18 : x

$$\Rightarrow \frac{6}{5} = \frac{18}{x} \Rightarrow 6 \times x = 18 \times 5$$

$$\Rightarrow x = \frac{18 \times 5}{6} = 15$$

$\therefore$ B's age = 15 years.

Question 6.

A sum of Rs. 10, 500 is divided among A, B and C in the ratio 5 : 6 : 4. Find the share of each.

Solution:

Total amount = Rs. 10, 500

Ratio in A, B, and C = 5 : 6 : 4

Sum of ratio = 5 + 6 + 4 = 15

$$\therefore \text{ A's share} = \text{Rs. } \frac{10500}{15} \times 5$$

$$= \text{Rs. } 700 \times 5 = \text{Rs. } 3500$$

$$\text{B's share} = \text{Rs. } \frac{10500 \times 6}{15}$$

$$= \text{Rs. } 700 \times 6$$

$$= \text{Rs. } 4200$$

$$\text{and C's share} = \text{Rs. } \frac{10500 \times 4}{15}$$

$$= \text{Rs. } 700 \times 4 = \text{Rs. } 2800$$

Question 7.
Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?
Solution:
15 cm : 2 m : : 10 sec : 3 min
15 cm : 2 x 100 cm :: 10 sec : 30 x 60 sec
15 : 200 :: 10 : 1800
3 : 40 :: 1 : 180
No, they donot form a proportion

Question 8.
Do the ratios 2 kg : 80 kg and 25 g : 625 g form a proportion ?
Solution:
2 kg : 80 kg : : 25 g : 625 g
2 : 80 :: 25 : 625
1 : 40 :: 1 : 25
No, they do not form a proportion.

Question 9.
10 kg sugar cost ₹ 350. If x kg sugar of the same kind costs ₹ 175, find the value of x
Solution:
10 kg of sugar costs = ₹ 350
and x kg of sugar cost = ₹ 175
A.T.Q.
10 kg : x kg :: 350 : 175
=> 10 x 175 = 350 x x
=> 350x= 1750
=> x = $\frac{1750}{350}$ = 5
Hence, 5 kg of sugar costs ₹ 175

Question 10.
The length of two ropes are in the ratio 7 : 5. Find the length of:
(i) shorter rope, if the longer one is 22.5 ni
(ii) longer rope, if the shorter is 9.8 m.
Solution:
Length of the ropes are in the ratio = 7 : 5
(i) Let the length of shorter rope = x
Length of longer rope = 22.5 m
A.T.Q.

$$7 : 5 = 22.5 : x$$

$$\Rightarrow 7x = 22.5 \times 5$$

$$\Rightarrow x = \frac{22.5 \times 5}{7}$$

$$\Rightarrow x = 16.07 \text{ m}$$

(ii) Let length of the longer side = x
length of shorter rope = 9.8 m
A.T.Q.

$$7 : 5 = x : 9.8$$

$$\Rightarrow 5 \times x = 9.8 \times 7$$

$$\Rightarrow x = \frac{9.8 \times 7}{5}$$

$$\Rightarrow x = 13.72 \text{ m}$$

Question 11.
If 4, x and 9 are in continued proportion, find the value of x.
Solution:
4, x and 9 are in continued proportion
=> 4 : x = x : 9
=> $x^2 = 9 \times 4$
=> $x = \sqrt{36}$
x = 6

Question 12.
If 25, 35 and x are in continued proportion, find the value of x.
Solution:
25, 35 and x are in continued proportion
=> 25 : 35 = 35 : x
=> 25 x x = 35 x 35

=> $x = \frac{35 \times 35}{25}$