HCF and LCM

IMPORTANT POINTS

Factor of a given number is that number by which the given number can be divided completely.

1. Prime Numbers :

A Natural number, which is divisible by 1 (one) and itself only is called a prime number.

2. Highest Common Factor :

H.C.F. stands for Highest Common Factor and H.C.F. of two or more given numbers is the greatest number (factor) which divides each given number completely.

3. Lowest Common Factor :

L.C.M. stands for Lowest Common Multiple. The L.C.M. of two or more given numbers is the lowest (smallest) number which is exactly divisible by each of the given numbers.

EXERCISE 8(A)

Question 1.

Write all the factors of :

- (i) 15
- (ii) 55 (iii) 48
- (III) 40 (iv) 26

(iv) 36

(v) 84

Solution:

(i) Factors of $15 = F_{15} = 1$, 3, 5 and 15 (ii) Factors of $55 = F_{55} = 1$, 5, 11 and 55 (iii) Factors of $48 = F_{48} = 1$, 2, 3, 4, 6, 8, 12, 16, 24 and 48 (iv) Factors of $36 = F_{56} = 1$, 2, 3, 4, 6, 9, 12, 18 and 36. (v) Factors of $84 = F_{84} = 1$, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84.

Question 2.

Write all prime numbers :
(i) less than 25
(ii) between 15 and 35
(iii) between 8 and 76
Solution:
(i) 2, 3, 5, 7, 11, 13, 17, 19 and 23
(ii) 17, 19, 23, 29 and 31
(iii) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71 and 73.

Question 3.

Write the prime-numbers from : (i) 5 to 45 (ii) 2 to 32 (iii) 8 to 48 (iv) 9 to 59 **Solution:** (i) 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 and 43. (ii) 3, 5, 7, 11, 13, 17, 19, 23 29 and 31. (iii) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

(iv) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 and 53.

Question 4.

Write the prime factors of: (i) 16 (ii) 27 (iii) 35 (iv) 49 **Solution:** (i) Prime factors of 16 = 22 16

(ii) Prime factors of 27 = 3

 $\begin{array}{c|c}
 3 & 27 \\
 3 & 9 \\
 3 & 3
 \end{array}$

1

(iii) Prime factors of 35 = 5, 7

(iv) Prime factors of 49 = 7

Question 5.

If P_n means prime factors of n, find: (i) p_6 (ii) P_{24} (iii) p_{50} (iv) P_{42} **Solution:** (i) $F_6 = 1, 2, 3, 6$ P.F₆ (Prime factor of 6) = 2 and 3. (ii) $F_{24} = 1, 2, 3, 4, 6, 8, 12, 24$ P.F₂₄ = 2 and 3. (iii) $F_{50} = 2, 5, 5$ P.F₅₀ = 2 and 5. (iv) $F_{42} = 1, 2, 3, 6, 7, 14, 21, 42$ P.F₄₂ = 2, 3 and 7.

EXERCISE 8(B)

Question 1.

Using the common factor method, find the H.C.F. of : (i) 16 and 35 (ii) 25 and 20 (iii) 27 and 75 (iv) 8, 12 and 18 (v) 24, 36, 45 and 60 Solution: (i) $F_{16} = 1, 2, 4, 8, 16$ $F_{35} = 1, 5, 7, 35$ Common factors between 16 and 35 = 1 H.C.F. of 16 and 35 = 1 (ii) $F_{25} = 1, 5, 25$ $F_{20} = 1, 2, 4, 5, 10, 20$ Common factors between 25 and 20 = 1, 5 H.C.F. of 25 and 20 = 5 (iii) $F_{27} = 1, 3, 9, 27$ F₇₅ = 1, 3, 5, 15, 25, 75 Common factors between 27 and 75 = 1, 3 H.C.F. of 27 and 75 = 3 (iv) $F_8 = 1, 2, 4, 8$ $F_{12} = 1, 2, 3, 4, 6, 12$ $F_{18} = 1, 2, 3, 6, 9, 18$ Common factors between 8, 12 and 18 = 1, 2H.C.F. of 8, 12 and 18 = 2 (v) $F_{24} = 1, 2, 3, 4, 6, 8, 12, 24$ $F_{36} = 1, 2, 3, 4, 6, 12, 18, 36$ F₄₅ = 1, 3, 5, 9, 15, 45

 $F_{60} = 1, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$ Common factor between 24, 36, 45 and 60 = 1, 3 H.C.F. of 24, 36, 45 and 60 = 3

Question 2.

Using the prime factor method, find the H.C.F. of: (i) 5 and 8 (ii) 24 and 49 (iii) 40, 60 and 80 (iv) 48, 84 and 88 (v) 12, 16 and 28 Solution: (i) Prime factor of 5 = 5Prime factor of $8 = 2 \times 2 \times 2$ No common prime factor H.C.F. of 5 and 8 = 1 (as both the number are co-prime) (ii) Prime factor of $24 = 2 \times 2 \times 2 \times 3$ Prime factor of $49 = 7 \times 7$ No common prime factor, number are co-prime. H.C.F. of 24 and 49 = 1. (iii) Prime factor of $40 = 2 \times 2 \times 2 \times 5$ Prime factor of $60 = 2 \times 2 \times 3 \times 5$ Prime factor of $80 = 2 \times 2 \times 2 \times 2 \times 5$ Common prime factor = $2 \times 2 \times 5$ H.C.F. of 40, 60 and 80 = 2 x 2 x 5 = 20 (iv) Prime factor of $48 = 2 \times 2 \times 2 \times 2 \times 3$ Prime factor of $84 = 2 \times 2 \times 3 \times 7$ Prime factor of $88 = 2 \times 2 \times 2 \times 11$ Common prime factor of 48, 84 and 88 = 2×2 H.C.F. of 48. 84 and $88 = 2 \times 2 = 4$ (v) Prime factor of $12 = 2 \times 2 \times 3$ Prime factor of $16 = 2 \times 2 \times 2 \times 2$ Prime factor of $28 = 2 \times 2 \times 7$ Common prime factor between 12, 16 and $28 = 2 \times 2$ H.C.F. of 12, 16 and 28 = 2 x 2 = 4

Question 3.

Using the division method, find the H.C.F. of the following : (i) 16 and 24 (ii) 18 and 30 (iii) 7, 14 and 24 (iv) 70,80,120 and 150 (v) 32, 56 and 46 **Solution:**

(i) 16 and 24
16) 24 (1
16
8) 16 (2
16
8) 16 (2
16
8) 16 (2
16
8) 16 (2
16
8) 16 (2
16
17
18 and 24 = 8
(ii) 18 and 30
18) 30 (1
18
12) 18 (1
12
6) 12 (2
12
×
Since last division is 6

$$\therefore$$
 H.C.F. of 18 and 30 = 6
(iii) 7, 14 and 24
7) 14 (2
14
×
7) 24 (3
21
3) 7 (2
6
1) 3 (3
×
Since the last division is 1
 \therefore H.C.F. of 7, 14 and 24 = 1

(*iv*) 70, 80, 120 and 150
70) 80 (1
70
10) 70 (7
70
10) 70 (7
70

$$\times$$

10) 120 (12
 120
 \times
10) 150 (15
 150
 \times
Since the last division = 10

: H.C.F. of 70, 80, 120 and 150 = 10

(v) 32, 56 and 46

$$32\overline{)56(1)} \\ 32\overline{)24(3)} \\ 24\overline{)32(1)} \\ 24\overline{)8(3)} \\ 24\overline{)32(3)} \\ 32\overline{)56(1)} \\ 32\overline{)32(1)} \\ 3$$

∴ H.C.F. of 32, 56 and 46 = 2

Question 4.

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Use a method of your own choice to find the H.C.F. of : (i) 45, 75 and 135 (ii) 48, 36 and 96 (iii) 66, 33 and 132 (iv) 24, 36, 60 and 132 (v) 30, 60, 90 and 105 Solution:

(i) Factor of $45 = F_{45} = 3 \times 3 \times 5$ Factor of $75 = F_{75} = 3 \times 5 \times 5$ and Factor of $135 = F_{135} = 3 \times 3 \times 3 \times 5$ Now the common factors of 45, 75 and 135 = 3 and 5 H.C.F. = 3 x 5 = 15 (ii) Factor of $48 = F_{48} = 2 \times 2 \times 2 \times 2 \times 3$ Factor of $36 = F_{36} = 2 \times 2 \times 3 \times 3$ and factor of $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ Now the common factor of 48, 36 and 96 = 2, 2 and 3 $H.C.F. = 2 \times 2 \times 3 = 12$ (iii) Factor of $66 = F_{66} = 2 \times 3 \times 11$ Factor of $33 = F_{33} = 3 \times 11$ and factor of $132 = F_{132} = 2 \times 2 \times 3 \times 11$ Now the common factor of 66, 33 and 132 = 3 and 11 H.C.F. = 3 x 11 = 33 (iv) Factor of $24 = F_{24} = 2 \times 2 \times 2 \times 3$ Factor of $36 = F_{36} = 2 \times 2 \times 3 \times 3$ Factor of $60 = F_{60} = 2 \times 2 \times 3 \times 5$ and Factor of $132 = F_{132} = 2 \times 2 \times 3 \times 11$ Now the common factors of 24, 36, 60 and 132 = 2, 2 and 3 $H.C.F. = 2 \times 2 \times 3 = 12$ (v) Factor of $30 = F_{30} = 2 \times 3 \times 5$ Factor of $60 = F_{60} = 2 \times 2 \times 3 \times 5$ Factor of $90 = F_{90} = 2 \times 3 \times 3 \times 5$ and factor of $105 = F_{105} = 3 \times 5 \times 7$ Now the common factor of 30, 60, 90 and 105 = 3 and 5 $H.C.F. = 3 \times 5 = 15$

Question 5.

Find the greatest number that divides each of 180, 225 and 315 completely. **Solution:**

The greatest number that divides 180, 225 and 315 will be HCF of 180, 225, 315

Let us first find HCF of 180 and 225

180)225(1)
 180
 45)180(4)
 180
 <u>x</u>

Since third number is 315, and HCF obtained above is 45, find the HCF of 315 and 45.

:. HCF of given number 80, 225 and 315

= 45

Question 6.

Show that 45 and 56 are co-prime numbers.

Solution:

The HCF of two co-prime numbers is always HCF of 45 and 56

From above it is proved that HCF of 45 and 56 is 1

Hence 45 and 56 are co-prime numbers.

Question 7.

Out of 15, 16, 21 and 28, find out all the pairs of co-prime numbers. **Solution:**



From above it is clear that 15 and 16 are co-prime because common factor is 1 Hence pairs 15 and 16, 16, 21, 15, 28 are co-prime number.

Question 8.

Find the greatest no. that will divide 93, 111 and 129, leaving remainder 3 in each case. **Solution:**

Since Remainder is 3 in each case numbers are



EXERCISE 8(C)

Question 1.

Using the common multiple method, find the L.C.M. of the following : (i) 8, 12 and 24 (ii) 10, 15 and 20 (iii) 3, 6, 9 and 12 Solution:

(i) 8, 12 and 24 8, 12, 24 6 $\frac{\overline{3}}{2}$ 3, 2, 2 2, 1, 1 1, 1. \therefore L.C.M. = 4 × 3 × 2 = 24 (ii) 10, 15 and 20 2 10,15,20 25 5,15,10 5,15,5 \therefore L.C.M. = 2 × 2 × 5 × 3 = 60 (iii) 3, 6, 9 and 12

∴ L.C.M. =
$$3 \times 2 \times 3 \times 2 = 36$$

Question 2.

Find the L.C.M. of each the following groups of numbers, using (i) the prime factor method and (ii) the common division method : (i) 18, 24 and 96 (ii) 100, 150 and 200 (iii) 14, 21 and 98 (iv) 22, 121 and 33 (v) 34, 85 and 51 Solution: (i) L.C.M. of 18, 24 and 96 (i) By prime factors Prime factors of $18 = 2 \times 3 \times 3$ Prime factors of $24 = 2 \times 2 \times 2 \times 3$ Prime factors of $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ $L.C.M. = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$ By common division method L.C.M. of 18, 24 and 96 = 2 x 2 x 2 x 3 x 3 x 4 = 288

(<i>ii</i>) 100, 150 and 200 Factor of 100 = $2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$ Factor of 150 = $2 \times 3 \times 5 \times 5$ = $2^1 \times 3^1 \times 5^2$ Factor of 200 = $2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$ $\therefore L \in M_{1} = 5100 = 150 \text{ and } 200$
Factor of $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$ Factor of $150 = 2 \times 3 \times 5 \times 5$ $= 2^1 \times 3^1 \times 5^2$ Factor of $200 = 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$ $= 1.00 \pm 50 \text{ and } 200$
Factor of $150 = 2 \times 3 \times 5 \times 5$ = $2^1 \times 3^1 \times 5^2$ Factor of $200 = 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$
$= 2^{1} \times 3^{1} \times 5^{2}$ Factor of 200 = 2 × 2 × 2 × 5 × 5 = 2 ³ × 5 ²
Factor of $200 = 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$
. I C M of 100 150 and 200
. L.C.M. of 100, 150 and 200
$= 2^3 \times 3^1 \times 5^2 = 600$
Common Division Method :
2 100 150 200
$\frac{2}{2}$ 50 75 100
$\frac{2}{5}$ 25 75 50
$\frac{5}{5}$ $\frac{25}{5}$ $\frac{15}{15}$ $\frac{10}{10}$
$\frac{1}{1}$ $\frac{1}{3}$ $\frac{2}{2}$

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: L.C.M. of 100, 150 and 200

 $= 2 \times 2 \times 5 \times 5 \times 3 \times 2 = 600$

(iii) 14, 21, 98 Factor of $14 = 2 \times 7 = 2^1 \times 7^1$ Factor of $21 = 3 \times 7 = 3^1 \times 7^1$ Factor of $98 = 2 \times 7 \times 7 = 2^1 \times 7^2$: L.C.M. of 14, 21 and 98 $= 2^1 \times 3^1 \times 7^2 = 294$ ---Common Division Method : : L.C.M. of 14, 21, 98 = $2 \times 7 \times 3 \times 7 = 294$ (iv) 22, 121 and 33 Factor of $22 = 2 \times 11 = 2^1 \times 11^1$ Factor of $121 = 11 \times 11 = 11^2$ Factor of $33 = 3 \times 11 = 3^1 \times 11^1$: L.C.M. of 22, 121 and 33 $= 2^1 \times 3^1 \times 11^2 = 726$ Common Division Method :

2	22,	121,	33	
11	11,	121,	33	
	1,	11,	3	
:. I	L.C.M	l. of 22	2, 12	and 33
= 2	× 11	× 11 >	< 3 =	726
34, 85 and 51				

Factor of $34 = 2 \times 17 = 2^{1} \times 17^{1}$ Factor of $85 = 5 \times 17 = 5^{1} \times 17^{1}$ Factor of $51 = 3 \times 17 = 3^{1} \times 17^{1}$ \therefore L.C.M. of 34, 85 and 51 $= 2^{1} \times 5^{1} \times 3^{1} \times 17 = 510$

Common Division Method :

Question 3.

(v)

The H.C.F. and the L.C.M. of two numbers are 50 and 300 respectively. If one of the numbers is 150, find the other one.

Solution:

H.C.F. = 50L.C.M. = 300 Product of L.C.M. and $H.C.F. = 300 \times 50 = 15000$ One number = 150 The other number

 $= \frac{\text{Product of L.C.M. and H.C.F.}}{\text{One number}} = \frac{15000}{150} = 100$

Question 4.

The product of two numbers is 432 and their L.C.M. is 72. Find their H.C.F. **Solution:** Product of two numbers = Product of their L.C.M. and H.C.F.

Here, product of two number = 432 L.C.M. = 72 H.C.F. = $\frac{432}{72}$ = 6

Question 5.

The product of two numbers is 19,200 and their H.C.F. is 40. Find their L.C.M. **Solution:**

L.C.M. = $\frac{\text{Product of number}}{\text{H.C.F.}}$ Product of number = 19,200 H.C.F. = 40 \therefore L.C.M. = $\frac{19,200}{40}$ = 480

Question 6.

Find the smallest number which, when divided by 12, 15, 18, 24 and 36 leaves no remainder

Solution:

The least number which is exactly divisible by each given number is their L.C.M. Required number L.C.M. of 12, 15, 18, 24 and 36.

2	12,	15,	18,	24,	36
2	6,	15,	9,	12,	18
3	3,	15,	9,	6,	9
3	1,	5,	3,	2,	3
_	1,	5,	1,	2,	1

: L.C.M. = least required number

 $= 2 \times 2 \times 3 \times 3 \times 5 \times 2 = 360$

Hence, the least required number = 360

Question 7.

Find the smallest number which, when increased by one is exactly divisible by 12, 18, 24, 32 and 40

Solution:

L.C.M. of given numbers

2	12,	18,	24,	32,	40
2	6,	9,	12,	16,	20
2	3,	9,	6,	8,	10
3	3,	9,	3,	4,	5
	1,	3,	1,	4 ,	5

 $\therefore \text{ L.C.M.} = 2 \times 2 \times 2 \times 3 \times 3 \times 4 \times 5$

= 1440 = One increasing

 \therefore The required number = 1440 - 1 = 1439

Question 8.

Find the smallest number which, on being decreased by 3, is completely divisible by 18, 36, 32 and 27.

Solution:

LCM of 18, 36, 32 and 27

 $= 2 \times 2 \times 3 \times 3 \times 3 \times 8 = 864$

 \therefore Required number = 864 + 3 = 867

REVISION EXERCISE

Question 1.

Find the H.C.F. of : (i) 108, 288 and 420 (ii) 36, 54 and 138 **Solution:** (i) H.C.F. of 108, 288, 420 = 12

108)288(2	36)420(11
216	396
72)108(1	24)36(1
72	24
36)72(2	12)24(2
72	24
×	×

(ii) H.C.F. of 36, 54 and 138 = 6

36)54(1	18)138(7
36	126
18)36(2	12)18(1
36	12
×	6)12(2
	12
	X

Question 2.

Find the L.C.M. of: (i) 72, 80 and 252 (ii) 48, 66 and 120 **Solution:** L.C.M. 72, 80, 252

2	72, 80, 252
2	36, 40, 126
2	18, 20, 63
3	9, 10, 63
3	3, 10, 21
	1, 10, 7

= $2 \times 2 \times 2 \times 3 \times 3 \times 10 \times 7 = 5040$ (*ii*) L.C.M. of 48, 66 and 120

2	48, 66, 120
2	24, 33, 60
2	12, 33, 30
3	6, 33, 15
	2, 11, 5

 $= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 11 = 2640$

Question 3.

State true or false : Give an example.

- (i) H.C.F. of two prime numbers is 1.
- (ii) H.C.F. of two co-prime numbers is 1.
- (iii) L.C.M. of two prime numbers is equal to their product.
- (iv) L.C.M. of two co-prime numbers is equal to their product.

Solution:

(i) True : Because the prime numbers have no common factor except 1.

- (ii) True : Because co-prime numbers have no common factor except 1.
- (iii) True : Because the prime number have no common factor except 1.
- (iv) True : Because co-prime numbers have no common factor except 1.

Question 4.

The product of two numbers is 12096 and their H.C.F. is 36. Find their L.C.M. **Solution:**

We know that Product of two numbers = Product of their H.C.F. and L.C.M. => 12096 = 36 x L.C.M. => L.C.M. = $\frac{12096}{36}$ = 336

Question 5.

The product of the H.C.F. and the L.C.M. of two numbers is 1152. If one number is 48, find the other one.

Solution:

We know that: Product of two numbers = Product of their H.C.F. and L.C.M. => 1st number x 2nd number = Product of their H.C.F. and L.C.M. => 48 x 2nd number = 1152 => 2nd number = $\frac{1152}{48}$ = 24

Question 6.

(i) Find the smallest number that is completely divisible by 28 and 42.

(ii) Find the largest number that can divide 28 and 42 completely.

Solution:

(i) We know that the least number which is divisible by 28 and 42 is their L.C.M.

 $\begin{array}{r}
 2 & 28, 42 \\
 \overline{7} & 14, 21 \\
 \hline
 2, 3
 \end{array}$

L.C.M. of 28 and 42 = 2 x 2 x 3 x 7 = 84

(ii) We know that the largest number which can divide 28 and 42 completely will be their H.C.F.

$$28)\overline{42(1)}$$

$$28\overline{14)28(2)}$$

$$28\overline{14)28(2)}$$

$$28\overline{\times}$$

H.C.F. of 28 and 42 = 14

Question 7.

Find the L.C.M. of 140 and 168. Use the L.C.M. obtained to find the H.C.F. of the given numbers.

Solution:

Numbers are 140 and 168 L.C.M. of 140 and 168

$$= 2 \times 2 \times 7 \times 5 \times 6 = 840$$

 $H.C.F. = \frac{1st \text{ number} \times 2nd \text{ number}}{L.C.M.}$

$$=\frac{140 \times 168}{840}=28$$

Question 8.

Find the H.C.F. of 108 and 450 and use the H.C.F. obtained to find the L.C.M. of the given numbers.

Solution:

Numbers are given : 108 and 450

