11. Algebraic Expressions

EXERCISE 11(A)

Question 1.
Separate the constants and variables from the following :

$-7, 7+x, 7x+yz, \sqrt{5}, \sqrt{xy}, \frac{3yz}{8}, 4.5y-3x,$

$8-5,\ 8-5x,\ 8x-5y \times p$ and $3y^2z \div 4x$

Solution:
Clearly constants are : -7, √5, 8 – 5

Variable are : $7+x,\ 7x+yz,\ \sqrt{xy},\ \frac{3yz}{8},\ 4.5y$

$-3x$

$8-5x,\ 8x-5y \times p$ and $3y^2 z \div 4x$

Question 2.
Write the number of terms in each of the following polynomials.
(i) $5x^2 + 3 \times ax$
(ii) $ax \div 4 - 7$
(iii) $ax - by + y \times z$
(iv) $23 + a \times b \div 2$.

Solution:

$(i)\quad 5x^2 + 3 \times ax = 5x^2 + 3ax$

$\therefore$ The number of terms in this polynomial $= 2$

$(ii)\quad ax \div 4 - 7 = \frac{ax}{4} - 7$

$\therefore$ The number of terms in this polynomial $= 2$

$(iii)\quad ax - by + y \times z = ax - by + yz$

$\therefore$ The number of terms in this polynomial $= 3$

$(iv)\quad 23 + a \times b \div 2 = 23 + \frac{ab}{2}$

$\therefore$ The number of terms in this Polynomial $= 2$

Question 3.
Separate monomials, binomials, trinomials and polynomials from the following algebraic expressions :

$8 - 3x$, xy^2, $3y^2 - 5y + 8$, $9x - 3x^2 + 15x^3 - 7$, $3x \times 5y$, $3x \div 5y$, $2y \div 7 + 3x - 7$ and $4 - ax^2 + bx + y$

Solution:

Monomials are : xy^2, $3x \times 5y$, $3x \div 5y$;

Bionomials are : $8 - 3x$

Trinomials are : $3y^2 - 5y + 8$, $2y \div 7 + 3x - 7$

Polynomials are : $8 - 3x$, $3y^2 - 5y + 8$, $9x - 3x^2 + 15x^3 - 7$, $2y \div 7 + 3x - 7$, $4 - ax^2 + bx + y$

Question 4.
Write the degree of each polynomial given below :

(i) $xy + 7z$ (ii) $x^2 - 6x^3 + 8$

(iii) $y - 6y^2 + 5y^8$ (iv) $xyz - 3$

(v) $xy + yz^2 - zx^3$ (vi) $x^5y^7 - 8x^3y^8 + 10x^4y^4z^4$

Solution:
(i) degree = 2 (Polynomial is xy + 7z)
(ii) degree = 3 (Polynomial is $x^2 - 6x^3 + 8$ y)
(iii) degree = 8 (Polynomial is y – $6y^2 + 5y^8$)
(iv) degree = 3 (Polynomial is xyz – 3)
(v) degree = 4 (Polynomial is xy + $yz^2 - xz^3$)
(vi) degree = 12 (Polynomial is $x^5y^7 - 8x^3y^8 + 10x^4$, y^4z^4)

Question 5.
Write the coefficient of :
(i) ab in 7abx ,
(ii) 7a in 7abx ;
(iii) $5x^2$ in $5x^2 - 5x$;
(iv) 8 in $a^2 - 8ax + a$;
(v) 4xy in $x^2 - 4xy + y^2$.

Solution:
(i) The coefficient of ab in 7abx = 7x
(ii) The coefficient of 7a in 7abx = bx
(iii) The coefficient of $5x^2$ in $5x^2 - 5x$ = 1
(iv) The coefficient of 8 in $a^2 - 8ax + a$ = – ax
(v) The coefficient of 4xy in $x^2 - 4xy + y^2$ = -1

Question 6.

In $\frac{5}{7}$ xy^2z^3, write the coefficient of

(i) 5 (ii) $\frac{5}{7}$ (iii) $5x$

(iv) xy^2 (v) z^3 (vi) xz^3

(vii) $5xy^2$ (viii) $\frac{1}{7}yz$ (ix) z

(x) yz^2 (xi) $5xyz$

Solution:

In $\frac{5}{7}$ xy^2z^3, Co-efficient of

(i) 5 is $\frac{1}{7}xy^2z^3$ (ii) $\frac{5}{7}$ is xy^2z^3

(iii) $5x$ is $\frac{1}{7}y^2z^3$ (iv) xy^2 is $\frac{5}{7}z^3$

(v) z^3 is $\frac{5}{7}$ xy^2 (vi) xz^3 is $\frac{5}{7}$ y^2

(vii) $5xy^2$ is $\frac{1}{7}z^3$ (viii) $\frac{1}{7}yz$ is $5xyz^2$

(ix) z is $\frac{5}{7}$ xy^2 z^2 (x) yz^2 is $\frac{5}{7}$ $xy - z$

(xi) $5xyz$ is $\frac{1}{7}yz^2$

Question 7.

In each polynomial, given below, separate the like terms :

(i) $3xy$, $-4yx^2$, $2xy^2$, $2.5x^2y$, $-8yx$, $-3.2y^2x$ and x^2y

(ii) y^2z^3, xy^2z^3, $-5x^2yz$, $-4y^2z^3$, $-8xz^3y^2$, $3x^2yz$ and $2z^3y^2$

Solution:

(i) Like terms are

$3xy, -8yx$; $-4yx^2$, $2.5x^2y$ and x^2y ; $2xy^2$ and $-3.2y^2x$

(ii) y^2z^3, $-y^2z^3$ and $2z^3y^2$; xy^2z^3 and $-8xz^3y^2$; $-5x^2yz$ and $3x^2yz$

EXERCISE 11(B)

Question 1.

Evaluate :

(i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$

(ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$

(iii) $abx - 15abx - 10abx + 32abx.$

(iv) $7x - 9y + 3 - 3x - 5y + 8$

(v) $3x^2 + 5xy - 4y^2 + x^2 - 8xy - 5y^2$

Solution:

(i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$

$= 21x^2 - 12x^2$

$= 9x^2$

(ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$

$= 3b^2y - 14b^2y$

$= -11b^2y$

(iii) $abx - 15abx - 10abx + 32abx$

$= 33abx - 25abx$

$= 8abx$

(iv) $7x - 9y + 3 - 3x - 5y + 8$

$= 7x - 3x - 9y - 5y + 3 + 8$

$= 4x - 14y + 11$

(v) $3x^2 + 5xy - 4y^2 - 8xy - 5y^2$

$= 3x^2 + 5xy - 8xy - 4y^2 - 5y^2$

$= 3x^2 - 3xy - 9y^2$

Question 2.

Add :

(*i*) $5a + 3b,\ a - 2b,\ 3a + 5b$

(*ii*) $8x - 3y + 7z,\ -4x + 5y - 4z,\ -x - y - 2z$

(*iii*) $3b - 7c + 10,\ 5c - 2b - 15,\ 15 + 12c + b$

(*iv*) $a - 3b + 3\ ;\ 2a + 5 - 3c\ ;\ 6c - 15 + 6b$

(*v*) $13ab - 9cd - xy\ ;\ 5xy\ ;\ 15cd - 7ab\ ;\ 6xy - 3cd$

(*vi*) $x^3 - x^2y + 5xy^2 + y^3;\ -x^3 - 9xy^2 + y^3;\ 3x^2y + 9xy^2$

(*vii*) $a^6 - 4a^4 + 6a\ ;\ 5a^6 + 5a^4 + 6a\ ;\ 12a^6 - 10a$

(viii) $2ax - 6by + 4cz,\ 4by - 14ax,\ 9cz - 4ax - 6by$

Solution:

(*i*)
$$\begin{array}{r} 5a + 3b \\ a - 2b \\ \hline 3a + 5b \\ \hline 9a + 6b \\ \hline \end{array}$$

(*ii*)
$$\begin{array}{r} 8x - 3y + 7z \\ -4x + 5y - 4z \\ \hline -x - y - 2z \\ \hline 3x + y + z \\ \hline \end{array}$$

(*iii*)
$$\begin{array}{r} 3b - 7c + 10 \\ -2b + 5c - 15 \\ \hline +b + 12c + 15 \\ \hline 2b + 10c + 10 \\ \hline \end{array}$$

(*iv*)
$$\begin{array}{r} a - 3b + 3 \\ 2a - 3c + 5 \\ \hline + 6b + 6c - 15 \\ \hline 3a + 3b + 3c - 7 \\ \hline \end{array}$$

(*v*)
$$\begin{array}{r} 13ab - 9cd + xy \\ + 5xy \\ \hline -7ab + 15cd \\ - 3cd + 6xy \\ \hline 6ab + 3cd + 10xy \\ \hline \end{array}$$

(*vi*)
$$\begin{array}{r} x^3 - x^2y + 5xy^2 + y^3 \\ \hline -x^3 - 9xy^2 + y^3 \\ \hline + 3x^2y + 9xy^2 \\ \hline 2x^2y + 5xy^2 + 2y^3 \\ \hline \end{array}$$

Question 3.

Find the total savings of a boy who saves ₹ (4x – 6y) ; ₹ (6x + 2y) ; ₹ (4y – x) and ₹ (y – 2x) for four consecutive weeks.

Solution:

$$\begin{array}{r} 4x-6y \\ 6x+2y \\ -x+4y \\ \hline -2x+y \\ \hline 7x+y \\ \hline \end{array}$$

∴ Total savings = ₹(7x + y)

Question 4.

(*i*) $4xy^2$ from $3xy^2$;
(*ii*) $-2x^2y + 3xy^2$ from $8x^2y$;
(*iii*) $3a-5b+c+2d$ from $7a-3b+c-2d$
(*iv*) $x^3 - 4x - 1$ from $3x^3 - x^2 + 6$
(*v*) $6a+3$ from a^3-3a^2+4a+1
(*vi*) $cab - 4cad - cbd$ from $3abc + 5bcd - cda$
(*vii*) $a^2 + ab + b^2$ from $4a^2 - 3ab + 2b^2$.

Solution:

(*i*) $3xy^2 - 4xy^2 = -xy^2$

(*ii*)
$$\begin{array}{r} 8x^2y \\ -2x^2y + 3xy^2 \\ + \quad - \\ \hline 10x^2y - 3xy^2 \end{array}$$

(*iii*)
$$\begin{array}{rrrrr} 7a & - & 3b & + c & - 2d \\ 3a & - & 5b & + c & + 2d \\ - & + & & - & - \\ \hline 4a & + & 2b & & - 4d \end{array}$$

(*iv*)
$$\begin{array}{rrrrr} 3x^3 & - & x^2 & & + 6 \\ x^3 & & & - 4x & - 1 \\ - & & & + & + \\ \hline 2x^3 & - & x^2 & + 4x & + 7 \end{array}$$

(*v*)
$$\begin{array}{rrrrr} a^3 & - 3a^2 & + 4a & + 1 \\ & & + 6a & + 3 \\ & & - & - \\ \hline a^3 & - 3a^2 & - 2a & - 2 \end{array}$$

(*vi*)
$$\begin{array}{rrrrr} 3abc & + & 5bcd & - & cda \\ +cab & - & cbd & - & 4cad \\ - & + & & + & \\ \hline 2abc & + & 6bcd & + & 3cad \end{array}$$

(*vii*)
$$\begin{array}{rrrrr} 4a^2 & - & 3ab & + & 2b^2 \\ + a^2 & + & ab & + & b^2 \\ - & - & & - & \\ \hline 3a^2 & - & 4ab & + & b^2 \end{array}$$

Question 5.

(i) Take away $-3x^3 + 4x^2 - 5x + 6$ from $3x^3 - 4x^2 + 5x - 6$
(ii) Take $m^2 + m + 4$ from $-m^2 + 3m + 6$ and the result from $m^2 + m + 1$.

Solution:

$$
\begin{array}{lrrrrrrr}
(i) & 3x^3 & - & 4x^2 & + & 5x & - & 6 \\
 & -3x^3 & + & 4x^2 & - & 5x & + & 6 \\
 & + & & - & & + & & - \\
\hline
 & 6x^3 & - & 8x^2 & + & 10x & - & 12 \\
\hline
(ii) & -m^2 & + & 3m & + & 6 & & \\
 & \pm m^2 & \pm & m & \pm & 4 & & \\
\hline
 & -2m^2 & + & 2m & + & 2 & & \\
\hline
\text{A.T.Q.} & m^2 & + & m & + & 1 & & \\
 & -2m^2 & + & 2m & + & 2 & & \\
 & + & & - & & - & & \\
\hline
 & 3m^2 & - & m & - & 1 & &
\end{array}
$$

Question 6.

Subtract the sum of $5y^2 + y - 3$ and $y^2 - 3y + 7$ from $6y^2 + y - 2$.

Solution:

$$
\begin{array}{r}
5y^2 + y - 3 \\
y^2 - 3y + 7 \\
\hline
6y^2 - 2y + 4 \\
\hline
\end{array}
$$

$$
\begin{array}{rrrrr}
 & 6y^2 & + & y & - 2 \\
 & 6y^2 & - & 2y & + 4 \\
- & & + & & - \\
\hline
 & & & 3y & - 6 \\
\hline
\end{array}
$$

Question 7.

What must be added to $x^4 - x^3 + x^2 + x + 3$ to obtain $x^4 + x^2 - 1$?

Solution:

$$
\begin{array}{rrrrrrr}
 & x^4 & & + x^2 & & - & 1 \\
+ & x^4 & - x^3 & + x^2 & + x & + & 3 \\
- & + & - & & - & - & \\
\hline
 & & x^3 & & - x & - & 4 \\
\hline
\end{array}
$$

Question 8.

(i) How much more than $2x^2 + 4xy + 2y^2$ is $5x^2 + 10xy - y^2$?

(ii) How much less $2a^2 + 1$ is than $3a^2 - 6$?

Solution:

(*i*)
$$\begin{array}{rrrr} & 5x^2 & + 10xy & - y^2 \\ & +2x^2 & + 4xy & + 2y^2 \\ \hline & 3x^2 & + 6xy & - 3y^2 \end{array}$$

(*ii*)
$$\begin{array}{rrr} & 3a^2 & - 6 \\ + & 2a^2 & + 1 \\ \hline & a^2 & - 7 \end{array}$$

Question 9.

If x = 6a + 86 + 9c ; y = 2b – 3a – 6c and z = c – b + 3a ; find

(i) x + y + z

(ii) x – y + z

(iii) 2x – y – 3z

(iv) 3y – 2z – 5x

Solution:

(*i*)
$$\begin{array}{rrrrrr} & x & = & 6a & + 8b & + 9c \\ & y & = - & 3a & + 2b & - 6c \\ & z & = & +3a & - b & + c \\ \hline \text{Adding} & x+y+z & = & 6a & + 9b & +4c \end{array}$$

(*ii*) $x-y+z = (6a+8b+9c)-(2b-3a-6c) + (c-b+3a)$

$= 6a+8b+9c-2b+3a+6c+c-b+3a$

$= 6a+3a+3a+8b-2b-b+9c+6c+c$

$= 12a+5b+16c$

(*iii*) $2x-y-3z = 2(6a+8b+9c)-(2b-3a-6c) -3(c-b+3a)$

$=12a+16b+18c-2b+3a+6c-3c+3b-9a$

$=12a+3a-9a+16b+3b-2b+18c+6c-3c$

$= 6a+17b+21c$

(*iv*) $3y-2z-5x = 3(2b-3a-6c)-2(c-b+3a)- 5(6a+8b+9c)$

$=6b-9a-18c-2c+2b-6a-30a-40b-45c$

$=-9a-6a-30a+6b+2b-40b-18c-2c-45c$

$= -45a-32b-65c$

Question 10.
The sides of a triangle are $x^2 - 3xy + 8$, $4x^2 + 5xy - 3$ and $6 - 3x^2 + 4xy$. Find its perimeter.
Solution:

$$\text{Required perimeter} = \text{Sum of three sides}$$
$$= x^2 - 3xy + 8 + 4x^2 + 5xy - 3 + 6 - 3x^2 + 4xy$$
$$= x^2 + 4x^2 - 3x^2 - 3xy + 5xy + 4xy + 8 - 3 + 6$$
$$= 2x^2 + 6xy + 11$$

Question 11.
The perimeter of a triangle is $8y^2 - 9y + 4$ and its two sides are $3y^2 - 5y$ and $4y^2 + 12$. Find its third side.
Solution:

$$\text{Perimeter of the triangle} = \text{Sum of three sides} = 8y^2 - 9y + 4$$
$$\text{Sum of two sides} = 3y^2 - 5y + 4y^2 + 12 = 7y^2 - 5y + 12$$
$$\therefore (8y^2 - 9y + 4) - (7y^2 - 5y + 12)$$
$$= 8y^2 - 9y + 4 - 7y^2 + 5y - 12$$
$$= y^2 - 4y - 8$$
$$\text{Hence third side} = y^2 - 4y - 8$$

Question 12.
The two adjacent sides of a rectangle are $2x^2 - 5xy + 3z^2$ and $4xy - x^2 - z^2$. Find its perimeter.
Solution:

Adjacent sides of a rectangle are
$$2x^2 - 5xy + 3z^2 \text{ and } 4xy - x^2 - z^2$$
$$\therefore \text{Perimeter} = 2(2x^2 - 5xy + 3z^2 + 4xy - x^2 - z^2)$$
$$= 4x^2 - 10xy + 6z^2 + 8xy - 2x^2 - 2z^2$$
$$= 2x^2 - 2xy + 4z^2$$

Question 13.
What must be subtracted from $19x^4 + 2x^3 + 30x - 37$ to get $8x^4 + 22x^3 - 7x - 60$?
Solution:

The required result will be

$(19x^4 + 2x^3 + 30x - 37) - (8x^4 + 22x^3 - 7x - 60)$

$= 19x^4 + 2x^3 + 30x - 37 - 8x^4 - 22x^3 + 7x + 60$

$= 11x^4 - 20x^3 + 37x + 23$

Question 14.

How much smaller is 15x – 18y + 19z than 22x – 20y – 13z + 26 ?

Solution:

The required result is

(22x – 20y – 13z + 26) – (15x – 18y + 19z)

= 22x – 20y – 13z + 26 – 15x + 18y – 19z

= 7x – 2y – 32z + 26

Question 15.

How much bigger is $15x^2y^2 - 18xy^2 - 10x^2y$ than $-5x^2 + 6x^2y - 7xy$?

Solution:

The required result,

$(5x^2y^2 - 18xy^2 - 10x^2y) - (-5x^2 + 6x^2y - 7xy)$

$= 5x^2y^2 - 18xy^2 - 10x^2y + 5x^2 - 6x^2y + 7xy$

$= 5x^2y^2 - 18xy^2 - 16x^2y + 5x^2 + 7xy$

EXERCISE 11(C)

Question 1.

Multiply :

(*i*) $8ab^2$ by $-4a^3b^4$

(*ii*) $\frac{2}{3}ab$ by $-\frac{1}{4}a^2b$

(*iii*) $-5cd^2$ by $-5cd^2$.

(*iv*) $4a$ and $(6a + 7)$

(*v*) $-8x$ and $(4 - 2x - x^2)$

(*vi*) $2a^2 - 5a - 4$ and $-3a$.

(*vii*) $x + 4$ by $x - 5$

(*viii*) $5a - 1$ by $7a - 3$

(*ix*) $12a + 5b$ by $7a - b$

(*x*) $x^2 + x + 1$ by $1 - x$

(*xi*) $2m^2 - 3m - 1$ and $4m^2 - m - 1$

(*xii*) a^2, ab and b^2

(*xiii*) abx, $-3a^2x$ and $7b^2x^3$

(*xiv*) $-3bx$, $-5xy$ and $-7b^3y^2$

(*xv*) $\left(-\frac{3}{2}x^5y^3\right)$ and $\left(\frac{4}{9}a^2x^3y\right)$

(*xvi*) $\left(-\frac{2}{3}a^7b^2\right)$ and $\left(-\frac{9}{4}ab^5\right)$

(*xvii*) $(2a^3 - 3a^2b)$ and $\left(-\frac{1}{2}ab^2\right)$

(*xviii*) $\left(2x+\frac{1}{2}y\right)$ and $\left(2x-\frac{1}{2}y\right)$

Solution:

(*i*) $8ab^2 \times -4a^3b^4 = (8 \times -4)(ab^2 \times a^3b^4)$

$= -32a^{1+3}.\ b^{2+4}$

$= -32a^4b^6$

(*ii*) $\frac{2}{3}ab \times -\frac{1}{4}a^2b = \left(\frac{2}{3} \times \frac{-1}{4}\right)(ab \times a^2b)$

$= -\frac{1}{6}a^{1+2}.b^{1+1}$

$= -\frac{1}{6}a^3b^2$

(*iii*) $-5cd^2 \times -5cd^2 = (-5 \times -5)\ (cd^2 \times cd^2)$

$= 25c^{1+1}d^{2+2}$

$= 25c^2d^4$

(*iv*) $4a\ (6a + 7)$

$= 4a \times 6a + 4a \times 7$

$= 24a^2 + 28a$

(*v*) $-8x\ (4 - 2x - x^2)$

$= -8x \times 4 - 8x \times -2x - 8x \times -x^2$

$= -32x + 16x^2 + 8x^3$

(*vi*) $-3a\ (2a^2 - 5a - 4)$

$= -3a \times 2a^2 - 5a \times -3a - 4 \times -3a$

$= -6a^3 + 15a^2 + 12a$

(*vii*) $(x+4)(x-5) = x(x-5) + 4\ (x-5)$

$= x^2 - 5x + 4x - 20$

$= x^2 - x - 20$

(*viii*) $(5a - 1)(7a - 3)$

$= 5a\ (7a - 3) - 1\ (7a - 3)$

$= 35a^2 - 15a - 7a + 3$

$= 35a^2 - 22a + 3$

(*ix*) $(12a + 5b)\ (7a - b) = 12a\ (7a - b) + 5b\ (7a - b)$

$= 84a^2 - 12ab + 35ab - 5b^2$

$= 84a^2 + 23ab - 5b^2$

(*x*) $(x^2+x+1)\ (1-x) = 1\ (x^2+x+1) - x(x^2+x+1)$

$= x^2+x+1 - x^3-x^2-x.$

$= 1-x^3$

(xi) $(2m^2 - 3m - 1)(4m^2 - m - 1)$

$= 2m^2(4m^2-m-1)-3m(4m^2-m-1)-1(4m^2-m-1)$

$= 8m^4-2m^3-2m^2-12m^3+3m^2+3m-4m^2+m+1$

$= 8m^4-14m^3-6m^2+3m^2+4m+1$

$= 8m^4-14m^3-3m^2+4m+1$

(xii) $a^2 \times ab \times b^2 = a^{2+1}.b^{1+2}$

$= a^3b^3$

(xiii) $abx \times -3a^2x \times 7b^2x^3$

$= (-3\times7)(a\times a^2)(b\times b^2)(x\times x\times x^3)$

$= -21a^3b^3x^5$

(xiv) $-3bx \times -5xy \times -7b^3y^2$

$= (-3\times-5\times-7)(b\times b^3)(x\times x)(y\times y^2)$

$= -105\ b^4x^2y^3$

(xv) $\left(-\frac{3}{2}x^5y^3\right)\left(\frac{4}{9}a^2x^3y\right)$

$= \left(-\frac{3}{2}\times\frac{4}{9}\right)(a^2)(x^5\times x^3)(y^3\times y)$

(xvi) $\left(-\frac{2}{3}a^7b^2\right)\left(-\frac{9}{4}ab^5\right)$

$= \left(-\frac{2}{3}\times\frac{-9}{4}\right)(a^7\times a)(b^2\times b^5)$

$= \frac{3}{2}a^8b^7$

(xvii) $(2a^3 - 3a^2b)\left(-\frac{1}{2}ab^2\right)$

$= -\frac{1}{2}ab^2\ (2a^3-3a^2b)$

$= 2a^3\times-\frac{1}{2}ab^2-3a^2b\times-\frac{1}{2}ab^2$

$= -a^4b^2 + \frac{3}{2}a^3b^3$

(xviii) $\left(2x+\frac{1}{2}y\right)\left(2x-\frac{1}{2}y\right)$

$= 2x\left(2x-\frac{1}{2}y\right)+\frac{1}{2}y\left(2x-\frac{1}{2}y\right)$

$= 4x^2 - xy + xy - \frac{1}{4}y^2$

$= 4x^2 - \frac{1}{4}y^2$

Question 2.

Multiply :

(i) $5x^2 - 8xy + 6y^2 - 3$ by $-3xy$

(ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$ by $-21x^2y^2$

(iii) $6x^3 - 5x + 10$ by $4 - 3x^2$

(iv) $2y - 4y^3 + 6y^5$ by $y^2 + y - 3$

(v) $5p^2 + 25pq + 4q^2$ by $2p^2 - 2pq + 3q^2$

Solution:

(i) $5x^2 - 8xy + 6y^2 - 3 \times -3xy$

$= 15x^3y^3 + 24x^2y^2 - 18xy^3 + 9xy$

(ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$

$\times \quad -21x^2y^2$

$-63x^2y^2 + 14x^3y^3 - 15x^3y^4 + 16x^4y^3$

(iii)
$$\begin{array}{r} 6x^3 - 5x + 10 \\ \times \quad 4 - 3x^2 \\ \hline 24x^3 - 20x + 40 \\ -18x^5 + 15x^3 - 30x^2 \\ \hline -18x^5 + 39x^3 - 30x^2 - 20x + 40 \end{array}$$

(iv)
$$\begin{array}{r} 2y - 4y^3 + 6y^5 \\ \times \quad y^2 + y - 3 \\ \hline 2y^3 - 4y^5 + 6y^7 \\ +2y^2 - 4y^4 + 6y^6 \\ \hline -6y + 12y^3 - 18y^5 \end{array}$$

$6y^7 + 6y^6 - (4 + 18)y^5 - 4y^4 + (2 + 12)y^3 + 2y^2 - 6y$

$= 6y^7 + 6y^6 - 22y^5 - 4y^4 + 14y^3 + 2y^2 - 6y$

(v)
$$\begin{array}{r} 5p^2 + 25pq + 4q^2 \\ \times \quad 2p^2 - 2pq + 3q^2 \\ \hline 10p^4 + 50p^3q + 8p^2q^2 \\ -10p^3q - 50p^2q^2 - 8pq^3 \\ +15p^2q^2 + 75pq^3 + 12q^4 \\ \hline 10p^4 + 40p^3q - 27p^2q^2 + 67pq^3 + 12q^4 \end{array}$$

Question 3.

Simplify :

(i) (7x – 8) (3x + 2)

(ii) (px – q) (px + q)

(iii) (5a + 5b – c) (2b – 3c)

(iv) (4x – 5y) (5x – 4y)

(v) (3y + 4z) (3y – 4z) + (2y + 7z) (y + z)

Solution:

(i) $(7x - 8)(3x + 2) = 7x(3x + 2) - 8(3x + 2)$

$= 21x^2 + 14x - 24x - 16 = 21x^2 - 10x - 16$

(ii) $(px - q)(px + q) = px(px + q) - q(px + q)$

$= p^2x^2 + pxq - pqx - q^2 = p^2x^2 - q^2$

(iii) $(5a + 5b - c)(2b - 3c)$

$= 5a(2b - 3c) + 5b(2b - 3c) - c(2b - 3c)$

$= 10ab - 15ac + 10b^2 - 15bc - 2bc + 3c^2$

$= 10ab + 10b^2 - 17bc - 15ac + 3c^2$

(iv) $(4x - 5y)(5x - 4y)$

$= 4x(5x - 4y) - 5y(5x - 4y)$

$= 20x^2 - 16xy - 25xy + 20y^2$

$= 20x^2 - 41xy + 20y^2$

(v) $(3y + 4z)(3y - 4z) + (2y + 7z)(y + z)$

$= 3y(3y - 4z) + 4z(3y - 4z) + 2y(y + z) + 7z(y + z)$

$= 9y^2 - 12yz + 12yz - 16z^2 + 2y^2 + 2yz + 7yz + 7z^2$

$= (9 + 2)y^2 + (-12 + 12 + 2 + 7)yz + (-16 + 7)z^2$

$= 11y^2 + 9yz - 9z^2$

Question 4.
The adjacent sides of a rectangle are $x^2 - 4xy + 7y^2$ and $x^3 - 5xy^2$. Find its area.
Solution:

Reqd. area $= (x^2 - 4xy + 7y^2)(x^3 - 5xy^2)$

$= x^2(x^3 - 5xy^2) - 4xy(x^3 - 5xy^2) + 7y^2(x^3 - 5xy^2)$

$= x^5 - 5x^3y^2 - 4x^4y + 20x^2y^3 + 7x^3y^2 - 35xy^4$

$= x^5 + (7 - 5)x^3y^2 - 4x^4y + 20x^2y^3 - 35xy^4$

$= x^5 + 2x^3y^2 - 4x^4y + 20x^2y^3 - 35xy^4$

$= (x^5 - 4x^4y + 2x^3y^2 + 20x^2y^3 - 35xy^4)$ sq. unit.

Question 5.
The base and the altitude of a triangle are $(3x - 4y)$ and $(6x + 5y)$ respectively. Find its area.
Solution:

Reqd. Area $= \frac{1}{2}$ (base) x (altitude)

$$= \frac{1}{2}(3x - 4y)(6x + 5y)$$

$$= \frac{1}{2}(18x^2 + 15xy - 24xy - 20y^2)$$

$$= \frac{1}{2}(18x^2 - 9xy - 20y^2) \text{ sq. unit.}$$

Question 6.
Multiply $-4xy^3$ and $6x^2y$ and verify your result for $x = 2$ and $y = 1$.
Solution:

$(-4xy^3) \times (6x^2y) = (-4 \times 6)(x \times x^2)(y^3 \times y)$

$= -24x^3y^4$

For $x = 2$ and $y = 1$

$(-4xy^3) \times (6x^2y) = (-4 \times 2 \times 1^3) \times (6 \times 2^2 \times 1)$

$= (-8) \times 24 = -192$

And, $-24x^3y^4 = -24 \times 2^3 \times 1^4$

$= -24 \times 8 \times 1 = -192$

$\therefore$ For $x = 2$ and $y = 1$, it is verified that

$(-4xy^3) \times (6x^2y) = -24x^3y^4$

Question 7.
Find the value of $(3x^3) \times (-5xy^2) \times (2x^2yz^3)$ for x = 1, y = 2 and z = 3.
Solution:

For $x = 1, y = 2$ and $z = 3$

$(3x^3) \times (-5xy^2) \times (2x^2yz^3)$

$(3 \times 1^3) \times (-5 \times 1 \times 2^2) \times (2 \times 1^2 \times 2 \times 3^3)$

$3 \times (-5 \times 4) \times (2 \times 1 \times 2 \times 27)$

$3 \times (-20) \times 108 = -6480$

Question 8.
Evaluate $(3x^4y^2)\ (2x^2y^3)$ for x = 1 and y = 2.

Solution:

$(3x^4y^2)\ (2x^2y^3)$

$(3 \times 1^4 \times 2^2) \times (2 \times 1^2 \times 2^3)$

$(3 \times 1 \times 4) \times (2 \times 1 \times 8)$

$= 12 \times 16 = 192$

Question 9.
Evaluate $(x^5) \times (3x^2) \times (-2x)$ for x = 1.
Solution:

For $x = 1$

$(x^5) \times (3x^2) \times (-2x)$

$(1^5) \times (3 \times 1^2) \times (-2 \times 1)$

$1 \times 3 \times (-2) = -6$

Question 10.
If x = 2 and y = 1; find the value of $(-4x^2y^3) \times (-5x^2y^5)$.
Solution:

For $x = 2$ and $y = 1$

$(-4x^2y^3) \times (-5x^2y^5)$

$(-4 \times 2^2 \times 1^3) \times (-5 \times 2^2 \times 1^5)$

$(-4 \times 4 \times 1) \times (-5 \times 4 \times 1)$

$-16 \times -20 = 320$

Question 11.
Evaluate:
(i) $(3x - 2)(x + 5)$ for x = 2.
(ii) $(2x - 5y)(2x + 3y)$ for x = 2 and y = 3.
(iii) $xz\ (x^2 + y^2)$ for x = 2, y = 1 and z= 1.

Solution:

(*i*) For $x = 2$

$(3x - 2)(x + 5)$

$(3 \times 2 - 2)(2 + 5)$

$(6 - 2) \times 7$

$4 \times 7 = 28$

(*ii*) For $x = 2$ and $y = 1$

$xy^2(x - 5y) + 1$

$2 \times 1^2(2 - 5 \times 1) + 1$

$2 \times (2 - 5) + 1$

$2 \times (-3) + 1$

$-6 + 1 = -5$

(*iii*) For $x = 2$, $y = 1$ and $z = 1$

$xz(x^2 + y^2)$

$2 \times 1(2^2 + 1^2)$

$2(2 + 1)$

$= 2 \times 3 = 6$

Question 12.

Evaluate:

(i) $x(x - 5) + 2$ for $x = 1$.

(ii) $xy^2(x - 5y) + 1$ for $x = 2$ and $y = 1$.

(iii) $2x(3x - 5) - 5(x - 2) - 18$ for $x = 2$.

Solution:

(*i*) For $x = 1$

$x(x - 5) + 2$

$1(1 - 5) + 2$

$-4 + 2 = -2$

(*ii*) For $x = 2$ and $y = 1$

$xy^2(x - 5y)$

$2 \times 1^2(2 - 5 \times 1)$

$2 \times (2 - 5)$

$2 \times (-3) = -6$

(*iii*) For $x = 2$

$2x(3x - 5) - 5(x - 2) - 18$

$2 \times 2(3 \times 2 - 5) - 5(2 - 2) - 18$

$4(6 - 5) - 5 \times 0 - 18$

$4 - 18 = -14$

Question 13.

Multiply and then verify :

$-3x^2y^2$ and $(x - 2y)$ for $x = 1$ and $y = 2$.

Solution:

$(-3x^2y^2) \times (x - 2y)$

$= (-3x^2y^2) \times (x) - (-3x^2y^2)(2y)$

$= -3x^3y^2 + 6x^2y^3$

$= 6x^2y^3 - 3x^3y^2$

For $x = 1$ and $y = 2$

$(-3x^2y^2) \times (x - 2y)$

$= (-3 \times 1^2 \times 2^2) \times (1 - 2 \times 2)$

$= (6 \times 1 \times 8) - (3 \times 1 \times 4)$

$= 48 - 12 = 36$

$\therefore$ For $x = 1$ and $y = 2$, it is verified that,

$(-3x^2y^2) \times (x - 2y) = 6x^2y^3 - 3x^3y^2$

Question 14.

Multiply:

(i) $2x^2 - 4x + 5$ by $x^2 + 3x - 7$

(ii) $(ab - 1)(3 - 2ab)$

(*i*) $2x^2 - 4x + 5$ by $x^2 + 3x - 7$

$(2x^2 - 4x + 5) \times (x^2 + 3x - 7)$

$2x^2(x^2 + 3x - 7) - 4x(x^2 + 3x - 7) + 5(x^2 + 3x - 7)$

$2x^4 + 6x^3 - 14x^2 - 4x^3 - 12x^2 + 28x + 5x^2 + 15x - 35$

$2x^4 + 6x^3 - 4x^3 - 14x^2 - 12x^2 + 5x^2 + 28x + 15x - 35$

$2x^4 + 2x^3 - 21x^2 + 43x - 35$

(*ii*) $(ab - 1)(3 - 2ab)$

$ab(3 - 2ab) - 1(3 - 2ab)$

$3ab - 2a^2b^2 - 3 + 2ab$

$-2a^2b^2 + 5ab - 3$

$2a^2b^2 - 5ab + 3$

Question 15.

Simplify : (5 – x)(6 – 5x)(2 -x).

Solution:

$(5 - x)\,(6 - 5x)\,(2 - x)$

$[(5 - x)\,(6 - 5x)]\,(2 - x)$

$[5(6 - 5x) - x(6 - 5x)]\,(2 - x)$

$[30 - 25x - 6x + 5x^2]\,(2 - x)$

$(5x^2 - 31x + 30)\,(2 - x)$

$2(5x^2 - 31x + 30) - x\,(5x^2 - 31x + 30)$

$10x^2 - 62x + 60 - 5x^3 + 31x^2 - 30x$

$-5x^3 + 10x^2 + 31x^2 - 62x - 30x + 60$

$-5x^3 + 41x^2 - 92x + 60$

EXERCISE 11(D)

Question 1.

Divide :

(*i*) $-70a^3$ by $14a^2$

(*ii*) $24x^3y^3$ by $-8y^2$

(*iii*) $15a^4b$ by $-5a^3b$

(*iv*) $-24x^4d^3$ by $-2x^2d^5$

(*v*) $63a^4b^5c^6$ by $-9a^2b^4c^3$

(*vi*) $8x - 10y + 6c$ by 2.

(*vii*) $15a^3b^4 - 10a^4b^3 - 25a^3b^6$ by $-5a^3b^2$

(*viii*) $-14x^6y^3 - 21x^4y^5 + 7x^5y^4$ by $7x^2y^2$

(*ix*) $a^2 + 7a + 12$ by $a + 4$

(*x*) $x^2 + 3x - 54$ by $x - 6$

(*xi*) $12x^2 + 7xy - 12y^2$ by $3x + 4y$

(*xii*) $x^6 - 8$ by $x^2 - 2$

(*xiii*) $6x^3 - 13x^2 - 13x + 30$ by $2x^2 - x - 6$

(*xiv*) $4a^2 + 12ab + 9b^2 - 25c^2$ by $2a + 3b + 5c$.

(*xv*) $16 + 8x + x^6 - 8x^3 - 2x^4 + x^2$ by $x + 4 - x^3$

Solution:

(*i*) $$\frac{-70a^3}{14a^2} = \left(\frac{-70}{14}\right)\left(\frac{a^3}{a^2}\right)$$

$$= -5a^{3-2}$$

$$= -5a$$

(ii) $\frac{24x^3y^3}{-8y^2} = \left(\frac{24}{-8}\right)(x^3)\left(\frac{y^3}{y^2}\right)$

$= -3x^3y^{3-2}$

$= -3x^3y$

(iii) $\frac{15a^4b}{-5a^3b} = \left(\frac{15}{-5}\right)\left(\frac{a^4}{a^3}\right)\left(\frac{b}{b}\right)$

$= -3a^{4-3}b^{1-1}$

$= -3a\ b^0$

$= -3a \times 1 \quad (\because b^0 = 1)$

$= -3a$

(iv) $\frac{-24x^4d^3}{-2x^2d^5} = \left(\frac{-24}{-2}\right)\left(\frac{x^4}{x^2}\right)\left(\frac{d^3}{d^5}\right)$

$= 12x^{4-2}d^{3-5} = 12x^2d^{-2}$

$= \frac{12x^2}{d^2}$

(v) $\frac{63a^4b^5c^6}{-9a^2b^4c^3} = \left(\frac{63}{-9}\right)\left(\frac{a^4}{a^2}\right)\left(\frac{b^5}{b^4}\right)\left(\frac{c^6}{c^3}\right)$

$= -7a^{4-2}.\ b^{5-4}.\ c^{6-3}$

$= -7a^2bc^3$

(vi) $\frac{8x-10y+6c}{2}$

$= \frac{8x}{2} - \frac{10y}{2} + \frac{6c}{2}$

$= 4x-5y+3c$

(vii) $\frac{15a^3b^4 - 10a^4b^3 - 25a^3b^6}{-5a^3b^2}$

$$= \frac{15a^3b^4}{-5a^3b^2} - \frac{10a^4b^3}{-5a^3b^2} - \frac{25a^3b^6}{-5a^3b^2}$$

$$= -3b^{4-2} + 2a^{4-3}b^{3-2} + 5b^{6-2}$$

$$= -3b^2 + 2ab + 5b^4$$

(*viii*) $\dfrac{-14x^6y^3 - 21x^4y^5 + 7x^5y^4}{7x^2y^2}$

$$= \frac{-14x^6y^3}{7x^2y^2} - \frac{21x^4y^5}{7x^2y^2} + \frac{7x^5y^4}{7x^2y^2}$$

$$= -2x^{6-2}y^{3-2} - 3x^{4-2}y^{5-2} + x^{5-2}y^{4-2}$$

$$= -2x^4y - 3x^2y^3 + x^3y^2$$

(*ix*)

$$\begin{array}{l} a+4\,\overline{)\,a^2+7a+12\,}\,(a+3 \\ \quad a^2+4a \\ \quad \underline{-\ \ -\qquad\quad} \\ \qquad\quad 3a+12 \\ \qquad\quad 3a+12 \\ \qquad\quad \underline{-\ \ -\qquad\quad} \\ \qquad\qquad\quad \times \end{array}$$

$\therefore$ Answer $= a + 3$

(*x*)

$$\begin{array}{l} x-6\,\overline{)\,x^2+3x-54\,}\,(x+9 \\ \quad x^2-6x \\ \quad \underline{-\ \ +\qquad\quad} \\ \qquad\quad 9x-54 \\ \qquad +\ 9x-54 \\ \qquad\quad \underline{-\ \ +\qquad\quad} \\ \qquad\qquad\quad \times \end{array}$$

$\therefore$ Answer $= x + 9$

(*xi*)
$$
\begin{array}{r}
4x-3y \\
3x+4y \;\overline{)\,12x^2+7xy-12y^2} \\
12x^2+16xy \\
\hline
-9xy-12y^2 \\
-9xy-12y^2 \\
\hline
\times
\end{array}
$$

$\therefore$ Answer $= 4x - 3y$

(*xii*)
$$
\begin{array}{r}
x^4+2x^2+4 \\
x^2-2 \;\overline{)\,x^6-8} \\
x^6 \quad -2x^4 \\
\hline
2x^4 - 8 \\
2x^4 \quad - 4x^2 \\
\hline
4x^2 - 8 \\
4x^2 - 8 \\
\hline
\times
\end{array}
$$

$\therefore$ Answer $= x^4+2x^2+4$

(*xiii*)
$$
\begin{array}{r}
3x-5 \\
2x^2-x-6 \;\overline{)\,6x^3-13x^2-13x+30} \\
6x^3-3x^2-18x \\
\hline
-10x^2+5x+30 \\
-10x^2+5x+30 \\
\hline
\times
\end{array}
$$

$\therefore$ Answer $= 3x - 5$

(*xiv*)

$$2a+3b+5c \;)\; 4a^2+12ab+9b^2-25c^2 \;(2a+3b-5c$$

$$4a^2+6ab \qquad +10ca$$

$$- \quad - \qquad\qquad -$$

$$6ab+9b^2-25c^2 \; -10ca$$

$$6ab+9b^2 \qquad +15bc$$

$$- \quad - \qquad\qquad -$$

$$-10ca-25c^2-15bc$$

$$-10ca-25c^2-15bc$$

$$+ \quad + \quad +$$

$$\times$$

$\therefore$ Answer $= 2a+3b-5c$

(*xv*)

$$-x^3+x+4) +x^6-2x^4-8x^3+x^2+8x+16 \;(-x^3+x+4$$

$$+x^6- x^4- 4x^3$$

$$- \quad + \quad +$$

$$-x^4-4x^3+x^2+8x+16$$

$$-x^4 \qquad +x^2+4x$$

$$+ \qquad - \quad -$$

$$-4x^3+4x+16$$

$$-4x^3+4x+16$$

$$+ \quad - \quad -$$

$$\times$$

$\therefore$ Answer $= -x^3 + x + 4$

Question 2.

Find the quotient and the remainder (if any) when :

(i) $a^3 - 5a^2 + 8a + 15$ is divided by $a + 1$.

(ii) $3x^4 + 6x^3 - 6x^2 + 2x - 7$ is divided by $x - 3$.

(iii) $6x^2 + x - 15$ is divided by $3x + 5$. In each case, verify your answer.

(iv) $6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ is divided by $2y^3 + 1$.

Solution:

(i) $a + 1) a^3 - 5a^2 + 8a + 15 (a^2 - 6a + 14$

$a^3 + a^2$

$- \quad -$

$-6a^2 + 8a + 15$

$-6a^2 - 6a$

$+ \quad +$

$14a + 15$

$14a + 14$

1

$\therefore$ Quotient $= a^2 - 6a + 14$ and remainder $= 1$

(ii) $x - 3) 3x^4 + 6x^3 - 6x^2 + 2x - 7 (3x^3 + 15x^2 + 39x + 119$

$3x^4 - 9x^3$

$- \quad +$

$15x^3 - 6x^2 + 2x - 7$

$15x^3 - 45x^2$

$- \quad +$

$39x^2 + 2x - 7$

$39x^2 - 117x$

$- \quad +$

$119x - 7$

$119x - 357$

$- \quad +$

350

$\therefore$ Quotient $= 3x^3 + 15x^2 + 39x + 119$ and remainder $= 350$

(iii) $3x + 5\overline{)6x^2 + x - 15}(2x - 3$

$6x^2 + 10x$

$- \quad -$

$-9x - 15$

$-9x - 15$

$+ \quad +$

$\times$

$\therefore$ Quotient $= 2x - 3$ and remainder $= 0$

(iv) $2y^3 + 1\overline{)6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3}(3y^2 + 15y + 9$

$6y^5 + 3y^2$

$- \quad - \quad -$

$30y^4 + 18y^3 + 3y^2 + 15y + 3$

$30y^4 \qquad + 15y$

$- \quad - \quad -$

$18y^3 + 3y^2 + 3$

$18y^3 \qquad + 9$

$- \quad -$

$3y^2 - 6$

$\therefore$ Quotient $= 3y^2 + 15y + 9$ and remainder $= 3y^2 - 6$

(*i*) Verification.

Dividend = Quotient × Divisor + Remainder

$= (a^2 - 6a + 14) \times (a + 1) + 1$

$= a^3 - 6a^2 + 14a + a^2 - 6a + 14 + 1$

$= a^3 - 5a^2 + 8a + 15$ which is given

(*ii*) Verification:

Dividend = Quotient × Divisor + Remainder

$= (3x^3 + 15x^2 + 39x + 119)(x - 3) + 350$

$= 3x^4 + 15x^3 + 39x^2 + 119x - 9x^3 - 45x^2 - 117x - 357 + 350$

$= 3x^4 + 6x^3 - 6x^2 + 2x - 7$ which is given

(*iii*) Verification:

Dividend = Quotient × Divisor + Remainder

$= (2x - 3)(3x + 5) + 0$

$= 6x^2 + 10x - 9x - 15 + 0$

$= 6x^2 - x - 15$ which is given

(*iv*) Verification:

Dividend = Quotient × Divisor + Remainder

$= (3y^2 + 15y + 9)(2y^3 + 1) + 3y^2 - 6$

$= 6y^5 + 30y^4 + 18y^3 + 3y^2 + 15y + 9 + 3y^2 - 6$

$= 6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ which is given

Question 3.

The area of a rectangle is $x^3 - 8x^2 + 7$ and one of its sides is $x - 1$. Find the length of the adjacent side.

Solution:

$$\text{Area} = x^3 - 8x^2 + 7$$

$$\text{One side} = x - 1$$

$$\therefore \text{Adjacent side} = (x^3 - 8x^2 + 7) \div (x - 1)$$

$$x - 1 \overline{)x^3 - 8x^2 + 7} (x^2 - 7x - 7$$

$$x^3 - x^2$$

$$- \quad +$$

$$-7x^2 + 7$$

$$-7x^2 + 7x$$

$$+ \quad -$$

$$-7x + 7$$

$$-7x + 7$$

$$+ \quad -$$

$$\times$$

$$\therefore \text{Other side} = x^2 - 7x - 7$$

Question 4.

The product of two numbers-is $16x^4 - 1$. If one number is $2x - 1$, find the other.

Solution:

$$\text{Product of two numbers} = 16x^4 - 1$$

$$\text{One number} = 2x - 1$$

$$\text{Then second number} = \frac{16x^4 - 1}{2x - 1}$$

$$= 8x^3 + 4x^2 + 2x + 1$$

$$8x^3 + 4x^2 + 2x + 1$$

$$2x - 1 \overline{)16x^4} \qquad - 1($$

$$16x^4 - 8x^3$$

$$- \quad +$$

$$8x^3$$

$$8x^3 - 4x^2$$

$$- \quad +$$

$$4x^2$$

$$4x^2 - 2x$$

$$- \quad +$$

$$2x - 1$$

$$2x - 1$$

$$- \quad +$$

$$\times$$

Question 5.

Divide $x^6 - y^6$ by the product of $x^2 + xy + y^2$ and $x - y$.

Solution:

Product of $(x^2 + xy + y^2)$ and $(x - y)$

$= (x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3$

Now, $(x^6 - y^6) \div (x^3 - y^3)$

$= x^3 + y^3$

$$
\begin{array}{l}
x^3 - y^3 \overline{)\,x^6 \quad\quad - y^6\,(x^3 + y^3} \\
\quad x^6 - x^3y^3 \\
\quad - \quad + \\
\hline
\quad\quad x^3y^3 - y^6 \\
\quad\quad x^3y^3 - y^6 \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad\; \times \\
\hline
\end{array}
$$

Simplification
(Using removal of brackets)

The signs for different types of brackets are :

1. ____ ; Vinculum or bar brackets,
2. ( ); Parenthesis or small brackets,
3. { }; Curly brackets or middle brackets,
4. [ ]; Square brackets or big brackets.
   In a combined operation, the brackets must be removed in the same order as written above:

EXERCISE 11(E)

Simplify :

Question 1.

$a^2 - 2a + \{5a^2 - (3a - 4a^2)\}$

Solution:

$= a^2 - 2a + \{5a^2 - 3a + 4a^2\}$

$= a^2 - 2a + \{9a - 3a\}$

$= a^2 - 2a + 9a^2 - 3a = 10a^2 - 5a$

Question 2.

$x - y - \{x - y - (x + y) - \overline{x - y}\}$

Solution:

$x - y - \{x - y - (x + y) - \overline{x - y}\}$

$= x - y - \{x - y - (x + y) - x + y\}$

$= x - y - \{x - y - x - y - x + y\}$

$= x - y - x + y + x + y + x - y = 2x$

Question 3.

$-3\ (1 - x^2) - 2\ \{x^2 - (3 - 2x^2)\}$

Solution:

$-3\ (1 - x^2) - 2\ \{x^2 - (3 - 2x^2)\}$

$= -3 + 3x^2 - 2\ \{x^2 - 3 + 2x^2\}$

$= -3 + 3x^2 - 2\{3x^2 - 3\}$

$= -3 + 3x^2 - 6x^2 + 6 = 3 - 3x^2$

Question 4.

$2\{m - 3\ (n + \overline{m - 2n})\}$

Solution:

$2\{m - 3\ (n + \overline{m - 2n})\}$

$= 2\{m - 3\ (n + m - 2n)\} = 2\{m - 3\ (m - n)\}$

$= 2\{m - 3m + 3n\} = 2\{3n - 2m\} = 6n - 4m$

Question 5.

$3x - [3x - \{3x - (3x - \overline{3x - y})\}]$

Solution:

$3x - [3x - \{3x - (3x - \overline{3x - y})\}]$

$= 3x - [3x - \{3x - (3x - 3x + y)\}]$

$= 3x - [3x - \{3x - y\}] = 3x - [3x - 3x + y]$

$= 3x - y$

Question 6.

$p^2x - 2\{px - 3x\ (x^2 - \overline{3a - x^2}\ )\}$

Solution:

$p^2x - 2\{px - 3x\ (x^2 - \overline{3a - x^2}\ )\}$

$= p^2x - 2\{px - 3x\ (x^2 - 3a + x^2)\}$

$= p^2x - 2\{px - 3x\ (2x^2 - 3a)\}$

$= p^2x - 2\{px - 6x^3 + 9ax\}$

$= p^2x - 2px + 12x^3 - 18ax$

Question 7.

$2[6 + 4\{m - 6(7 - \overline{n + p}\ ) + q\}]$

Solution:

$2[6 + 4\{m - 6(7 - \overline{n + p}\ ) + q\}]$

$= 2[6 + 4\{m - 6\ (7 - n - p) + q\}]$

$= 2[6 + 4\{m - 42 + 6n + 6p + q\}]$

$= 2[6 + 4m - 168 + 24n + 24p + 4q]$

$= 2[4m + 24n + 24p + 4q - 162]$

$= 8m + 48n + 48p + 8q - 324$

Question 8.

$a - [a - \overline{b + a}\ - \{a - (a - \overline{b - a}\ )\}]$

Solution:

$a - [a - \overline{b + a}\ - \{a - (a - \overline{b - a}\ )\}]$

$= a - [a - b - a - \{a - (a - b + a)\}]$

$= a - [- b - \{a - a + b - a\}]$

$= a - [- b - b + a]$

$= a + b + b - a = 2b$

Question 9.

$$3x - [4x - \overline{3x-5y} - 3\{2x - (3x - \overline{2x-3y})\}]$$

Solution:

$$3x - [4x - \overline{3x-5y} - 3\{2x - (3x - \overline{2x-3y})\}]$$

$$= 3x - [4x - 3x + 5y - 3\{2x - (3x - 2x + 3y)\}]$$

$$= 3x - [4x - 3x + 5y - 3\{2x - (x + 3y)\}]$$

$$= 3x - [4x - 3x + 5y - 3\{2x - x - 3y\}]$$

$$= 3x - [x + 5y - 6x + 3x + 9y]$$

$$= 3x - [-2x + 14y]$$

$$= 3x + 2x - 14y$$

$$= 5x - 14y$$

Question 10.

$a^5 \div a^3 + 3a \times 2a$

Solution:

$a^5 \div a^3 + 3a \times 2a = a^{5-3} + 3a \times 2a = a^2 + 6a^2 = 7a^2$

Question 11.

$x^5 \div (x^2 \times y^2) \times y^3$

Solution:

$$x^5 \div (x^2 \times y^2) \times y^3 = \frac{x^5}{x^2y^2} \times y^3 = x^{5-2} - y^{3-2} = x^3y$$

Question 12.

$(x^5 \div x^2) \times y^2 \times y^3$

Solution:

$(x^5 \div x^2) \times y^2 \times y^3 = x^{5-2} \times y^{2+3} = x^3y^5$

Question 13.

$(y^3 - 5y^2) \div y \times (y - 1)$

Solution:

$$(y^3 - 5y^2) \div y \times (y - 1) = \frac{y^3 - 5y^2}{y} \times y - 1$$

$$= (y^2 - 5y) \times (y - 1) = y^2(y - 1) - 5y(y - 1) = y^3 - y^2 - 5y^2 + 5y = y^3 - 6y^2 + 5y$$

Question 14.

$3a \times [8b \div 4 - 6\{a - (5a - \overline{3b - 2a})\}]$

Solution:

$3a \times [8b \div 4 - 6\{a - (5a - \overline{3b - 2a})\}]$

$= 3a \times \left[\frac{8b}{4} - 6\{a - (5a - 3b + 2a)\}\right] = 3a \times [2b - 6\{a - 7a + 3b\}] = 3a \times [2b - 6\{-6a + 3b\}]$

$= 3a \times [2b + 36a - 18b] = 3a \times [36a - 16b] = 108a^2 - 48ab$

Question 15.

$7x + 4\{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$

Solution:

$7x + 4\{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$

$= 7x + 4\left\{x^2 \div \left(\frac{5x}{10}\right)\right\} - 3\left\{2 - x^3 \div \left(\frac{3x^2}{x}\right)\right\}$

$= 7x + 4\left\{x^2 \div \frac{x}{2}\right\} - 3\left\{2 - x^3 \div 3x\right\} = 7x + 4\left\{x^2 \times \frac{2}{x}\right\} - 3\left\{2 - \frac{x^3}{3x}\right\}$

$= 7x + 8x - 6 + x^2 = x^2 + 15x - 6$