12. Algebraic Identities

EXERCISE 12(A)

Question 1.
Use direct method to evaluate the following products :
(i) (x + 8)(x + 3)
(ii) (y + 5)(y – 3)
(iii) (a – 8)(a + 2)
(iv) (b – 3)(b – 5)
(v) (3x – 2y)(2x + y)
(vi) (5a + 16)(3a – 7)
(vii) (8 – b) (3 + b)
Solution:

$$(i)\ (x+8)(x+3) = (x\times x) + (x\times 3) + (8\times x) + (8\times 3)$$
$$= x^2 + 3x + 8x + 24$$
$$= x^2 + 11x + 24$$

$$(ii)\quad (y+5)(y-3) = (y\times y) + (y\times -3) + (5\times y) + (5\times -3)$$
$$= y^2 + (-3y) + (5y) - 15$$
$$= y^2 - 3y + 5y - 15$$
$$= y^2 + 2y - 15$$

$$(iii)\quad (a-8)(a+2) = (a\times a) + (a\times 2) + (-8)\times a + (-8)(2)$$
$$= a^2 + 2a - 8a - 16$$
$$= a^2 - 6a - 16$$

$$(iv)\quad (b-3)(b-5) = (b\times b) + (b\times -5) + (-3)\times b + (-3)(-5)$$
$$= b^2 - 5b - 3b + 15$$
$$= b^2 - 8b + 15$$

$$(v)\quad (3x-2y)(2x+y) = (3x\times 2x) + (3x\times y) + (-2y\times 2x) + (-2y\times y)$$
$$= 6x^2 + 3xy - 4xy - 2y^2$$
$$= 6x^2 - xy - 2y^2$$

$$(vi)\quad (5a+16)(3a-7) = (5a\times 3a) + (5a\times -7) + (16\times 3a) + 16\times -7$$
$$= 15a^2 + (-35a) + 48a + (-112)$$
$$= 15a^2 - 35a + 48a - 112$$
$$= 15a^2 + 13a - 112$$

$$(vii)\quad (8-b)(3+b) = (8\times 3) + (8\times b) + (-b\times 3) + (-b\times b)$$
$$= 24 + 8b - 3b - b^2$$
$$= 24 + 5b - b^2$$

Question 2.
Use direct method to evaluate :

(i) $(x+1)(x-1)$ (ii) $(2+a)(2-a)$
(iii) $(3b-1)(3b+1)$ (iv) $(4+5x)(4-5x)$
(v) $(2a+3)(2a-3)$ (vi) $(xy+4)(xy-4)$
(vii) $(ab+x^2)(ab-x^2)$
(viii) $(3x^2+5y^2)(3x^2-5y^2)$

(ix) $\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right)$

(x) $\left(\frac{3}{5}a+\frac{1}{2}\right)\left(\frac{3}{5}a-\frac{1}{2}\right)$

(xi) $(0\cdot5-2a)(0\cdot5+2a)$

(xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right)$

Solution:

Note : $(a+b)(a-b) = a^2-b^2$

(i) $(x+1)(x-1) = (x)^2-(1)^2 = x^2-1$

(ii) $(2+a)(2-a) = (2)^2-(a)^2 = 4-a^2$

(iii) $(3b-1)(3b+1) = (3b)^2-(1)^2 = 9b^2-1$

(iv) $(4+5x)(4-5x) = (4)^2-(5x)^2 = 16-25x^2$

(v) $(2a+3)(2a-3) = (2a)^2-(3)^2 = 4a^2-9$

(vi) $(xy+4)(xy-4) = (xy)^2-(4)^2 = x^2y^2-16$

(vii) $(ab+x^2)(ab-x^2) = (ab)^2-(x^2)^2 = a^2b^2-x^4$

(viii) $(3x^2+5y^2)(3x^2-5y^2) = (3x^2)^2-(5y^2)^2 = 9x^4-25y^4$

(ix) $\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right) = (z)^2-\left(\frac{2}{3}\right)^2$

$= z^2-\frac{4}{9}$

(x) $\left(\frac{3}{5}a+\frac{1}{2}\right)\left(\frac{3}{5}a-\frac{1}{2}\right)$

$$= \left(\frac{3}{5}a\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{9}{25}a^2 - \frac{1}{4}$$

(xi) $(0\cdot5-2a)\ (0\cdot5+2a)$
$= (0\cdot5)^2 - (2a)^2$
$= 0\cdot25 - 4a^2$

(xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right) = \left(\frac{a}{2}\right)^2 - \left(\frac{b}{3}\right)^2$

$$= \frac{a^2}{4} - \frac{b^2}{9}$$

Question 3.
Evaluate :

(i) $(a+1)\ (a-1)\ (a^2+1)$
(ii) $(a+b)\ (a-b)\ (a^2+b^2)$
(iii) $(2a-b)\ (2a+b)\ (4a^2+b^2)$
(iv) $(3-2x)\ (3+2x)\ (9+4x^2)$
(v) $(3x-4y)\ (3x+4y)\ (9x^2+16y^2)$

(i) $(a+1)\ (a-1)\ (a^2+1)$
$= [(a)^2-(1)^2]\ (a^2+1)$
$= (a^2-1)\ (a^2+1)$
$= (a^2)^2 - (1)^2$
$= a^4 - 1$

(ii) $(a+b)(a-b)(a^2+b^2)$

$$= (a^2-b^2)(a^2+b^2)$$
$$= (a^2)^2 - (b^2)^2$$
$$= a^4-b^4$$

(iii) $(2a-b)(2a+b)(4a^2+b^2)$

$$= [(2a)^2-(b)^2](4a^2+b^2)$$
$$= (4a^2-b^2)(4a^2+b^2)$$
$$= (4a^2)^2 - (b^2)^2$$
$$= 16a^4 - b^4$$

(iv) $(3-2x)(3+2x)(9+4x^2)$

$$= [(3)^2-(2x)^2](9+4x^2)$$
$$= (9-4x^2)(9+4x^2)$$
$$= (9)^2 - (4x^2)^2$$
$$= 81-16x^4$$

(v) $(3x-4y)(3x+4y)(9x^2+16y^2)$

$$= [(3x)^2-(4y)^2](9x^2+16y^2)$$
$$= (9x^2-16y^2)(9x^2+16y^2)$$
$$= (9x^2)^2 - (16y^2)^2$$
$$= 81x^4 - 256y^4$$

Question 4.

Use the product $(a + b)(a - b) = a^2 - b^2$ to evaluate:

(i) 21 x 19

(ii) 33 x 27

(iii) 103 x 97

(iv) 9.8 x 10.2

(v) 7.7 x 8.3

(vi) 4.6 x 5.4

Solution:

(i) $21 \times 19 = (20 + 1)(20 - 1)$

$= (20)^2 - (1)^2 = 400 - 1 = 399$

(ii) $33 \times 27 = (30 + 3)(30 - 3)$

$= (30)^2 - (3)^2 = 900 - 9 = 891$

(iii) $103 \times 97 = (100 + 3)(100 - 3)$

$= (100)^2 - (3)^2 = 10000 - 9 = 9991$

(iv) $9.8 \times 10.2 = (10 - .2)(10 + .2)$

$= (10)^2 - (.2)^2 = 100 - .04 = 99.96$

(v) $7.7 \times 8.3 = (8 - .3)(8 + .3)$

$= (8)^2 - (.3)^2 = 64 - .09 = 63.91$

(vi) $4.6 \times 5.4 = (5 - .4)(5 + .4)$

$= (5)^2 - (.4)^2 = 25 - .16 = 24.84$

Question 5.
Evaluate :
(i) (6 – xy) (6 + xy)

(ii) $\left(7x+\frac{2}{3}y\right)\left(7x-\frac{2}{3}y\right)$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)\left(\frac{a}{2b}-\frac{2b}{a}\right)$

(iv) $\left(3x-\frac{1}{2y}\right)\left(3x+\frac{1}{2y}\right)$

(v) $(2a + 3)(2a - 3)(4a^2 + 9)$
(vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$
(vii) $(5x + 8y)(3x + 5y)$
(viii) $(7x + 15y)(5x - 4y)$
(ix) $(2a - 3b)(3a + 4b)$
(x) $(9a - 7b)(3a - b)$

Solution:

$(6 - xy)(6 + xy) = 6(6 + xy) - xy(6 + xy)$
$= 36 + 6xy - 6xy + (xy)^2 = 36 - x^2y^2$

(ii) $\left(7x+\frac{2}{3}y\right)\left(7x-\frac{2}{3}y\right)$

$= 7x\left(7x-\frac{2}{3}y\right)+\frac{2}{3}y\left(7x-\frac{2}{3}y\right)$

$= 49x^2 - \frac{14}{3}xy+\frac{14}{3}xy-\frac{4}{9}y^2 = 49x^2-\frac{4}{9}y^2$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)\left(\frac{a}{2b}-\frac{2b}{a}\right)$

$= \frac{a}{2b}\left(\frac{a}{2b}-\frac{2b}{a}\right)+\frac{2b}{a}\left(\frac{a}{2b}-\frac{2b}{a}\right)$

$= \frac{a^2}{4b^2}-1+1-\frac{4b^2}{a^2} = \frac{a^2}{4b^2}-\frac{4b^2}{a^2}$

(iv) $\left(3x-\frac{1}{2y}\right)\left(3x+\frac{1}{2y}\right)$

$= 3x\left(3x+\frac{1}{2y}\right)-\frac{1}{2y}\left(3x+\frac{1}{2y}\right)$

$= 9x^2 + \frac{3x}{2y} - \frac{3x}{2y} - \frac{1}{4y^2} = 9x^2 - \frac{1}{4y^2}$

(v) $(2a + 3)(2a - 3)(4a^2 + 9)$

$= [(2a)^2 - (3)^2](4a^2 + 9)$

$[(a + b)(a - b) = a^2 - b^2]$

$= (4a^2 - 9)(4a^2 + 9)$

$= (4a^2)^2 - (9)^2 \quad [(a + b)(a - b) = a^2 - b^2]$

$= 16a^4 - 81$

(vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$

$= [(a)^2 - (bc)^2](a^2 + b^2c^2)$

$[(a + b)(a - b) = a^2 - b^2]$

$= (a^2 - b^2c^2)(a^2 + b^2c^2)$

$= (a^2)^2 - (b^2c^2)^2 \quad [\because (a + b)(c - b) = a^2 - b^2]$

$= a^4 - b^4c^4$

(vii) $(5x + 8y)(3x + 5y)$

$= 5x(3x + 5y) + 8y(3x + 5y)$

$= 15x^2 + 25xy + 24xy + 40y^2$

$= 15x^2 + 49xy + 40y^2$

(viii) $(7x + 15y)(5x - 4y)$

$= 7x(5x - 4y) + 15y(5x - 4y)$

$= 35x^2 - 28xy + 75xy - 60y^2$

$= 35x^2 + 47xy - 60y^2$

(ix) $(2a - 3b)(3a + 4b)$

$= 2a(3a + 4b) - 3b(3a + 4b)$

$= 6a^2 + 8ab - 9ab - 12b^2$

$= 6a^2 - ab - 12b^2$

(x) $(9a - 7b)(3a - b)$

$= 9a(3a - b) - 7b(3a - b)$

$= 27a^2 - 9ab - 21ab + 7b^2$

$= 27a^2 - 30ab + 7b^2$

EXERCISE 12(B)

Question 1.

Expand :

(i) $(2a + b)^2$

(ii) $(a - 2b)^2$

(iii) $\left(a+\frac{1}{2a}\right)^2$ (iv) $\left(2a-\frac{1}{a}\right)^2$

(v) $(a+b-c)^2$ (vi) $(a-b+c)^2$

(vii) $\left(3x+\frac{1}{3x}\right)^2$ (viii) $\left(2x-\frac{1}{2x}\right)^2$

Solution:

(i) $(2a+b)^2 = (2a)^2+(b)^2+2\times 2a\times b$

$[\because (a+b)^2=a^2+b^2+2ab]$

$= 4a^2+b^2+4ab$

(ii) $(a-2b)^2 = (a)^2+(2b)^2-2\times a\times 2b$

$[\because (a-b)^2 = a^2+b^2-2ab]$

$= a^2+4b^2-4ab$

(iii) $\left(a+\frac{1}{2a}\right)^2 = (a)^2+\left(\frac{1}{2a}\right)^2+2\times a\times \frac{1}{2a}$

$= a^2+\frac{1}{4a^2}+\frac{2a}{2a}$

$= a^2+\frac{1}{4a^2}+1$

(iv) $\left(2a-\frac{1}{a}\right)^2 = (2a)^2+\left(\frac{1}{a}\right)^2-2\times 2a\times \frac{1}{a}$

$= 4a^2+\frac{1}{a^2}-4$

(v) $(a+b-c)^2 = (a)^2+(b)^2+(-c)^2$
$+2\times a\times b+2\times b\times(-c)+2\times(-c)\times(a)$
$= a^2+b^2+c^2+2ab-2bc-2ca$

(**Note :** $(a+b+c)^2 = a^2+b^2+c^2+2ab-2bc-2ca$)

(vi) $(a-b+c)^2 = (a)^2+(-b)^2+(c)^2+2\times a\times -b$
$+2(-b)(c) + 2\times c\times a$
$= a^2+b^2+c^2-2ab-2bc+2ca$

(vii) $\left(3x+\frac{1}{3x}\right)^2 = (3x)^2+\left(\frac{1}{3x}\right)^2 +2\times 3x\times \frac{1}{3x}$

$$= 9x^2+\frac{1}{9x^2}+2$$

(viii) $\left(2x-\frac{1}{2x}\right)^2 = (2x)^2+\left(\frac{1}{2x}\right)^2 -2\times 2x\times \frac{1}{2x}$

$$=4x^2+\frac{1}{4x^2}-2$$

Question 2.

Find the square of :

(i) $x+3y$ (ii) $2x-5y$

(iii) $a+\frac{1}{5a}$ (iv) $2a-\frac{1}{a}$

(v) $x-2y+1$ (vi) $3a-2b-5c$

(vii) $2x+\frac{1}{x}+1$ (viii) $5-x+\frac{2}{x}$

(ix) $2x-3y+z$ (x) $x+\frac{1}{x}-1$

Solution:

$(i)\quad (x+3y)^2 = (x)^2 + (3y)^2+2\times x\times 3y$

$= x^2+9y^2+6xy$

$(ii)\quad (2x-5y)^2 = (2x)^2+(5y)^2-2\times 2x\times 5y$

$= 4x^2+25y^2-20xy$

$$(iii)\quad \left(a+\frac{1}{5a}\right)^2 = (a)^2+\left(\frac{1}{5a}\right)^2+2\times a\times\frac{1}{5a}$$

$$= a^2+\frac{1}{25a^2}+\frac{2}{5}$$

$$(iv)\quad \left(2a-\frac{1}{a}\right)^2 = (2a)^2+\left(\frac{1}{a}\right)^2-2\times 2a\times\frac{1}{a}$$

$$= 4a^2+\frac{1}{a^2}-4$$

$(v)\quad (x-2y+1)^2 = (x)^2+(-2y)^2+(1)^2+2\times x$

$\times -2y+2\times(-2y)\times 1+2\times 1\times x$

$= x^2+4y^2+1-4xy-4y+2x$

$(vi)\quad (3a-2b-5c)^2 = (3a)^2+(-2b)^2+(-5c)^2$

$+2\times 3a\times -2b+2\times(-2b)(-5c)$

$+2\times -5c\times 3a$

$= 9a^2+4b^2+25c^2-12ab$

$+20bc-30ca$

$$(vii)\quad \left(2x+\frac{1}{x}+1\right) = (2x)^2+\left(\frac{1}{x}\right)^2+(1)^2+2\times$$

$$2x\times\frac{1}{x}+2\times\frac{1}{x}\times1+2\times1\times2x$$

$$=4x^2+\frac{1}{x^2}+1+4+\frac{2}{x}+4x$$

$$=4x^2+\frac{1}{x^2}+5+\frac{2}{x}+4x$$

(viii) $\left(5-x+\frac{2}{x}\right)^2=(5)^2+(-x)^2+\left(\frac{2}{x}\right)^2$

$$+2\times5\times(-x)+2(-x)\times\frac{2}{x}+2\times\frac{2}{x}\times5$$

$$=25+x^2+\frac{4}{x^2}-10x-4+\frac{20}{x}$$

$$=21+x^2+\frac{4}{x^2}-10x+\frac{20}{x}$$

(ix) $(2x-3y+z)^2=(2x)^2+(-3y)^2+(z)^2+2\times2x\times$

$$-3y+2(-3y)\times z+2\times z\times2x$$

$$=4x^2+9y^2+z^2-12xy-6yz+4zx$$

(x) $\left(x+\frac{1}{x}-1\right)^2=(x)^2+\left(\frac{1}{x}\right)^2+(-1)^2$

$$+2\times x\times\frac{1}{x}+2\times\frac{1}{x}\times(-1)+2(-1)\times x$$

$$=x^2+\frac{1}{x^2}+1+2-\frac{2}{x}-2x$$

$$=x^2+\frac{1}{x^2}+3-\frac{2}{x}-2x$$

Question 3.
Evaluate:
Using expansion of $(a+b)^2$ or $(a-b)^2$
(i) $(208)^2$
(ii) $(92)^2$
(iii) $(415)^2$
(iv) $(188)^2$
(v) $(9.4)^2$
(vi) $(20.7)^2$

Solution:

(i) $(208)^2 = (200 + 8)^2$

$= (200)^2 + (8)^2 + 2(200)(8) = 40000 + 64 + 3200 = 43264$

(ii) $(92)^2 = (100 - 8)^2 = (100)^2 + (8)^2 - 2(100)(8)$

$= 10000 + 64 - 1600 = 10064 - 1600 = 8464$

(iii) $(415)^2 = (400 + 15)^2$

$= (400)^2 + (15)^2 + 2(400)(15) = 160000 + 225 + 12000 = 172225$

(iv) $(188)^2 = (200 - 12)^2$

$= (200)^2 + (12)^2 - 2(200)(12) = 40000 + 144 - 4800$

$= 40144 - 4800 = 35344$

(v) $(9.4)^2 = (10 - .6)^2$

$= (10)^2 + (.6)^2 - 2(10)(.6) = 100 + .36 - 12$

$= 88 + .36 = 88.36$

(vi) $(20.7)^2 = (20 + .7)^2 = (20)^2 + (.7)^2 + 2(20)(.7)$

$= 400 + .49 + 28 = 428 + .49 = 428.49$

Question 4.

Expand :

(i) $(2a+b)^3$ (ii) $(a-2b)^3$

(iii) $(3x-2y)^3$ (iv) $(x+5y)^3$

(v) $\left(a+\frac{1}{a}\right)^3$ (vi) $\left(2a-\frac{1}{2a}\right)^3$

Solution:

(i) $(2a+b)^3 = (2a)^3+(b)^3+3\times 2a\times b(2a+b)$

$[(a+b)^3 = a^3+b^3+3ab\,(a+b)]$

$= 8a^3+b^3+6ab\,(2a+b)$

$= 8a^3+b^3+12a^2b+6ab^2$

(ii) $(a-2b)^3 = (a)^3-(2b)^3-3\times a\times 2b\,(a-2b)$

$[(a-b)^3=a^3-b^3-3ab(a-b)]$

$= a^3-8b^3-6ab\,(a-2b)$

$= a^3-8b^3-6a^2b+12ab^2$

(iii) $(3x-2y)^3= (3x)^3-(2y)^3-3\times 3x\times 2y\,(3x-2y)$

$= 27x^3-8y^3-18xy\,(3x-2y)$

$= 27x^3-8y^3-54x^2y+36xy^2$

(iv) $(x+5y)^3 = (x)^3+(5y)^3+3\times x\times 5y\,(x+5y)$

$= x^3+125y^3+15xy\,(x+5y)$

$= x^3+125y^3+15x^2y+75xy^2$

(v) $\left(a+\frac{1}{a}\right)^3$

$$= a^3+\left(\frac{1}{a}\right)^3+3\times a\times\frac{1}{a}\times\left(a+\frac{1}{a}\right)$$

$$= a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$$

$$= a^3+\frac{1}{a^3}+3a+\frac{3}{a}$$

(vi) $\left(2a-\frac{1}{2a}\right)^3 = (2a)^3-\left(\frac{1}{2a}\right)^3-3\times 2a$

$$\times\frac{1}{2a}\left(2a-\frac{1}{2a}\right)$$

$$= 8a^3-\frac{1}{8a^3}-3\left(2a-\frac{1}{2a}\right)$$

$$= 8a^3-\frac{1}{8a^3}-6a+\frac{3}{2a}$$

Question 5.
Find the cube of :

(i) $a+2$ (ii) $2a-1$
(iii) $2a+3b$ (iv) $3b-2a$
(v) $2x+\frac{1}{x}$ (v) $x-\frac{1}{2}$

Solution:

(i) $(a+2)^3 = (a)^3+(2)^3+3\times a\times 2(a+2)$
$= a^3+8+6a(a+2)$
$= a^3+8+6a^2+12a$
$= a^3+6a^2+12a+8$

(ii) $(2a-1)^3 = (2a)^3-(1)^3-3\times 2a\times 1(2a-1)$
$= 8a^3-1-6a\ (2a-1)$
$= 8a^3-1-12a^2+6a$
$= 8a^3-12a^2+6a-1$

(iii) $(2a+3b)^3 = (2a)^3+(3b)^3+3\times 2a\times 3b\ (2a+3b)$
$= 8a^3+27b^3+18ab\ (2a+3b)$
$= 8a^3+27b^3+36a^2b+54ab^2$
$= 8a^3+36a^2b+54ab^2+27b^3$

(iv) $(3b-2a)^3 = (3b)^3-(2a)^3-3\times 3b\times 2a(3b-2a)$
$= 27b^3-8a^3-18ab\ (3b-2a)$

$$= 27b^3 - 8a^3 - 54ab^2 + 36a^2b$$
$$= 27b^3 - 54b^2a + 36ba^2 - 8a^3$$

(v) $\left(2x + \frac{1}{x}\right)^3$

$$= (2x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times 2x \times \frac{1}{x}\left(2x + \frac{1}{x}\right)$$

$$= 8x^3 + \frac{1}{x^3} + 6\left(2x + \frac{1}{x}\right)$$

$$= 8x^3 + \frac{1}{x^3} + 12x + \frac{6}{x}$$

$$= 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$$

(vi) $\left(x - \frac{1}{2}\right)^3$

$$= (x)^3 - \left(\frac{1}{2}\right)^3 - 3 \times x \times \frac{1}{2}\left(x - \frac{1}{2}\right)$$

$$= x^3 - \frac{1}{8} - \frac{3x}{2}\left(x - \frac{1}{2}\right)$$

$$= x^3 - \frac{1}{8} - \frac{3x^2}{2} + \frac{3x}{4}$$

$$= x^3 - \frac{3x^2}{2} + \frac{3x}{4} - \frac{1}{8}$$

EXERCISE 12(C)

Question 1.

If a+b=5 and ab = 6; find $a^2 + b^2$

Solution:

$$(a+b)^2 = a^2 + b^2 + 2ab$$
$$\Rightarrow (5)^2 = a^2 + b^2 + 2 \times 6$$
$$\Rightarrow 25 = a^2 + b^2 + 12$$
$$\Rightarrow 25 - 12 = a^2 + b^2$$
$$\Rightarrow 13 = a^2 + b^2$$
$$\therefore a^2 + b^2 = 13$$

Question 2.
If a – b = 6 and ab = 16; find $a^2 + b^2$
Solution:

$$\begin{aligned} (a-b)^2 &= a^2+b^2-2ab \\ \Rightarrow \quad (6)^2 &= a^2+b^2-2\times16 \\ \Rightarrow \quad 36 &= a^2+b^2-32 \\ \Rightarrow \quad 36+32 &= a^2+b^2 \\ \Rightarrow \quad 68 &= a^2+b^2 \\ \therefore \quad a^2+b^2 &= 68 \end{aligned}$$

Question 3.
If $a^2 + b^2 = 29$ and ab = 10 ; find :
(i) a + b
(ii) a – b
Solution:

$$\begin{aligned} (i) \quad (a+b)^2 &= a^2+b^2+2ab \\ \Rightarrow \quad (a+b)^2 &= 29+2\times10 \\ \Rightarrow \quad (a+b)^2 &= 29+20 \\ \Rightarrow \quad (a+b)^2 &= 49 \\ \Rightarrow \quad a+b &= \sqrt{49} \\ \Rightarrow \quad a+b &= 7 \\ (ii) \quad (a-b)^2 &= a^2+b^2-2ab \\ \Rightarrow \quad (a-b)^2 &= 29-2\times10 \\ \Rightarrow \quad (a-b)^2 &= 29-20 \\ \Rightarrow \quad (a-b)^2 &= 9 \\ \Rightarrow \quad a-b &= \sqrt{9} \\ \Rightarrow \quad a-b &= 3 \end{aligned}$$

Question 4.
If $a^2 + b^2 = 10$ and ab = 3; find :
(i) a – b
(ii) a + b

Solution:

$$\begin{aligned} (i) \quad (a-b)^2 &= a^2+b^2-2ab \\ \Rightarrow \quad (a-b)^2 &= 10-2\times3 \\ \Rightarrow \quad (a-b)^2 &= 10-6 \\ \Rightarrow \quad (a-b)^2 &= 4 \\ \Rightarrow \quad (a-b) &= \sqrt{4} \\ \Rightarrow \quad a-b &= 2 \\ (ii) \quad (a+b)^2 &= a^2+b^2+2ab \\ \Rightarrow \quad (a+b)^2 &= 10+2\times3 \\ \Rightarrow \quad (a+b)^2 &= 10+6 \\ \Rightarrow \quad (a+b)^2 &= 16 \\ \Rightarrow \quad (a+b) &= \sqrt{16} \\ \Rightarrow \quad (a+b) &= 4 \end{aligned}$$

Question 5.

If $a+\frac{1}{a}=3$; find $a^2+\frac{1}{a^2}$

Solution:

$$\left(a+\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}+2$$

$$\Rightarrow \quad (3)^2 = a^2+\frac{1}{a^2}+2$$

$$\Rightarrow \quad 9 = a^2+\frac{1}{a^2}+2$$

$$\Rightarrow \quad 9-2 = a^2+\frac{1}{a^2}$$

$$\Rightarrow \quad 7 = a^2+\frac{1}{a^2}$$

$$\therefore \quad a^2+\frac{1}{a^2} = 7$$

Alternative Method :

$$a+\frac{1}{a} = 3$$

$$\Rightarrow \quad \left(a+\frac{1}{a}\right)^2 = (3)^2$$

$$\Rightarrow \quad a^2+\frac{1}{a^2}+2 = 9$$

$$\Rightarrow \quad a^2+\frac{1}{a^2} = 9-2$$

$$\Rightarrow \quad a^2+\frac{1}{a^2} = 7$$

Question 6.

If $a - \frac{1}{a} = 4$; find $a^2 + \frac{1}{a^2}$

Solution:

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \quad (4)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \quad 16 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \quad 16 + 2 = a^2 + \frac{1}{a^2}$$

$$\Rightarrow \quad 18 = a^2 + \frac{1}{a^2}$$

$$\therefore \quad a^2 + \frac{1}{a^2} = 18$$

Alternative Method :

$$a - \frac{1}{a} = 4$$

$$\Rightarrow \quad \left(a - \frac{1}{a}\right)^2 = (4)^2$$

$$\Rightarrow \quad a^2 + \frac{1}{a^2} - 2 = 16$$

$$\Rightarrow \quad a^2 + \frac{1}{a^2} = 16 + 2$$

$$\Rightarrow \quad a^2 + \frac{1}{a^2} = 18$$

Question 7.

If $a^2 + \frac{1}{a^2} = 23$; find $a + \frac{1}{a}$

Solution:

$$\left(a+\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}+2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 23+2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 25$$

$$\Rightarrow a+\frac{1}{a} = \sqrt{25}$$

$$\Rightarrow a+\frac{1}{a} = 5$$

Question 8.

If $a^2+\frac{1}{a^2} = 11$; find $a-\frac{1}{a}$

Solution:

$$\left(a-\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}-2$$

$$\Rightarrow \left(a-\frac{1}{a}\right)^2 = 11-2$$

$$\Rightarrow \left(a-\frac{1}{a}\right)^2 = 9$$

$$\Rightarrow a-\frac{1}{a} = \sqrt{9}$$

$$\Rightarrow a-\frac{1}{a} = 3$$

Question 9.

If $a + b + c = 10$ and $a^2 + b^2 + c^2 = 38$; find $ab + bc + ca$

Solution:

$$a+b+c = 10$$
$$\Rightarrow (a+b+c)^2 = (10)^2$$
$$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca = 100$$
$$\Rightarrow 38+2(ab+bc+ca) = 100$$
$$\Rightarrow 2(ab+bc+ca) = 100-38$$
$$\Rightarrow 2(ab+bc+ca) = 62$$
$$\Rightarrow (ab+bc+ca) = \frac{62}{2}$$
$$\Rightarrow ab+bc+ca = 31$$

Alternative Method :

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$
$$\Rightarrow (10)^2 = 38+2(ab+bc+ca)$$
$$\Rightarrow 100 = 38+2(ab+bc+ca)$$
$$\Rightarrow 100-38 = 2(ab+bc+ca)$$
$$\Rightarrow 62 = 2(ab+bc+ca)$$
$$\Rightarrow \frac{62}{2} = ab+bc+ca$$
$$\Rightarrow 31 = ab+bc+ca$$
$$\therefore ab+bc+ca = 31$$

Question 10.
Find $a^2 + b^2 + c^2$; if $a + b + c = 9$ and $ab + bc + ca = 24$
Solution:

$$a+b+c = 9$$
$$\Rightarrow (a+b+c)^2 = (9)^2$$
$$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca = 81$$
$$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 81$$
$$\Rightarrow a^2+b^2+c^2+2\times 24 = 81$$
$$\Rightarrow a^2+b^2+c^2+48 = 81$$
$$\Rightarrow a^2+b^2+c^2 = 81-48$$
$$\Rightarrow a^2+b^2+c^2 = 33$$

Question 11.
Find $a + b + c$; if $a^2 + b^2 + c^2 = 83$ and $ab + bc + ca = 71$

Solution:

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$
$$\Rightarrow \quad (a+b+c)^2 = 83+2(ab+bc+ca)$$
$$\Rightarrow \quad (a+b+c)^2 = 83+2\times71$$
$$\Rightarrow \quad (a+b+c)^2 = 83+142$$
$$\Rightarrow \quad (a+b+c)^2 = 225$$
$$\Rightarrow \quad a+b+c = \sqrt{225}$$
$$\Rightarrow \quad a+b+c = 15$$

Question 12.

If a + b = 6 and ab=8; find $a^3 + b^3$

Solution:

$$a+b = 6$$
$$\Rightarrow \quad (a+b)^3 = (6)^3$$
$$\Rightarrow \quad a^3+b^3+3ab\ (a+b) = 216$$
$$\Rightarrow \quad a^3+b^3+3\times8\ (6) = 216$$
$$\Rightarrow \quad a^3+b^3+144 = 216$$
$$\Rightarrow \quad a^3+b^3 = 216-144$$
$$\Rightarrow \quad a^3+b^3 = 72$$

Alternative Method :

$$(a+b)^3 = a^3+b^3+3ab\ (a+b)$$
$$\Rightarrow \quad (6)^3 = a^3+b^3+3\times8\ (6)$$
$$\Rightarrow \quad 216 = a^3+b^3+144$$
$$\Rightarrow \quad 216-144 = a^3+b^3$$
$$\Rightarrow \quad 72 = a^3+b^3$$
$$\Rightarrow \quad a^3+b^3 = 72$$

Question 13.

If a – b=3 and ab = 10; find $a^3 - b^3$

Solution:

$$a-b = 3$$

$$\Rightarrow \quad (a-b)^3 = (3)^3$$

$$\Rightarrow \quad a^3-b^3-3ab(a-b) = 27$$

$$\Rightarrow \quad a^3-b^3-3\times 10\ (3) = 27$$

$$\Rightarrow \quad a^3-b^3-90 = 27$$

$$\Rightarrow \quad a^3-b^3 = 27+90$$

$$\Rightarrow \quad a^3-b^3 = 117$$

Alternative Method :

$$(a-b)^3 = a^3-b^3-3ab\ (a-b)$$

$$\Rightarrow \quad (3)^3 = a^3-b^3-3\times 10\ (3)$$

$$\Rightarrow \quad 27 = a^3-b^3-90$$

$$\Rightarrow \quad 27+90 = a^3-b^3$$

$$\Rightarrow \quad 117 = a^3-b^3$$

$$\Rightarrow \quad a^3-b^3 = 117$$

Question 14.

Find $a^3+\frac{1}{a^3}$ if $a+\frac{1}{a} = 5$

Solution:

$$a+\frac{1}{a} = 5$$

$$\Rightarrow \quad \left(a+\frac{1}{a}\right)^3 = (5)^3$$

$$\Rightarrow \quad a^3+\frac{1}{a^3}+3a\times\frac{1}{a}\left(a+\frac{1}{a}\right) = 125$$

$$\Rightarrow \quad a^3+\frac{1}{a^3}+3(5) = 125 \qquad \left[a+\frac{1}{a}=5\right]$$

$$\Rightarrow \quad a^3+\frac{1}{a^3}+15 = 125$$

$$\Rightarrow \quad a^3+\frac{1}{a^3} = 125-15$$

$$\Rightarrow \quad a^3+\frac{1}{a^3} = 110$$

Question 15.

Find $a^3-\frac{1}{a^3}$ if $a-\frac{1}{a}=4$

Solution:

$$a-\frac{1}{a} = 4$$

$$\Rightarrow \left(a-\frac{1}{a}\right)^3 = (4)^3$$

$$\Rightarrow a^3-\frac{1}{a^3}-3a\times\frac{1}{a}\left(a-\frac{1}{a}\right) = 64$$

$$\Rightarrow a^3-\frac{1}{a^3}-3(4) = 64 \qquad \left[\because a-\frac{1}{a} = 4\right]$$

$$\Rightarrow a^3-\frac{1}{a^3}-12 = 64$$

$$\Rightarrow a^3-\frac{1}{a^3} = 64+12$$

$$\Rightarrow a^3-\frac{1}{a^3} = 76$$

Question 16.

If $2x-\frac{1}{2x}=4$; find : (*i*) $4x^2+\frac{1}{4x^2}$

(*ii*) $8x^3-\frac{1}{8x^3}$

Solution:

(i) $2x - \frac{1}{2x} = 4$

$$\Rightarrow \left(2x - \frac{1}{2x}\right)^2 = (4)^2$$

$$\Rightarrow (2x)^2 + \left(\frac{1}{2x}\right)^2 - 2 \times 2x \times \frac{1}{2x} = 16$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} - 2 = 16$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} = 16 + 2$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} = 18$$

(ii) $2x - \frac{1}{2x} = 4$

$$\Rightarrow \left(2x - \frac{1}{2x}\right)^3 = (4)^3$$

$$\Rightarrow (2x)^3 - \left(\frac{1}{2x}\right)^3 - 3 \times 2x \times \frac{1}{2x}\left(2x - \frac{1}{2x}\right) = 64$$

$$\Rightarrow 8x^3 - \frac{1}{8x^3} - 3(4) = 64$$

$$\Rightarrow 8x^3 - \frac{1}{8x^3} - 12 = 64$$

$$\Rightarrow 8x^3 - \frac{1}{8x^3} = 64 + 12 \Rightarrow 8x^3 - \frac{1}{8x^3} = 76$$

Question 17.

If $3x+\frac{1}{3x}=3$; find : (*i*) $9x^2+\frac{1}{9x^2}$

(*ii*) $27x^3+\frac{1}{27x^3}$

Solution:

(*i*) $3x+\frac{1}{3x}=3$

$$\Rightarrow \left(3x+\frac{1}{3x}\right)^2 = (3)^2$$

$$\Rightarrow (3x)^2+\left(\frac{1}{3x}\right)^2+2\times 3x\times\frac{1}{3x} = 9$$

$$\Rightarrow 9x^2+\frac{1}{9x^2}+2 = 9 \Rightarrow 9x^2+\frac{1}{9x^2} = 9-2$$

$$\Rightarrow 9x^2+\frac{1}{9x^2} = 7$$

(*ii*) $3x+\frac{1}{3x} = 3$

$$\Rightarrow \left(3x+\frac{1}{3x}\right)^3 = (3)^3$$

$$\Rightarrow (3x)^3+\left(\frac{1}{3x}\right)^3+3\times 3x\times\frac{1}{3x}\left(3x+\frac{1}{3x}\right) = 27$$

$$\Rightarrow 27x^3+\frac{1}{27x^3}+3\left(3x+\frac{1}{3x}\right) = 27$$

$$\Rightarrow 27x^3+\frac{1}{27x^3}+3(3) = 27$$

$$\Rightarrow 27x^3+\frac{1}{27x^3}+9 = 27$$

$$\Rightarrow 27x^3+\frac{1}{27x^3} = 27-9$$

$$\Rightarrow 27x^3+\frac{1}{27x^3} = 18$$

Question 18.

The sum of the squares of two numbers is 13 and their product is 6. Find:
(i) the sum of the two numbers.
(ii) the difference between them.

Solution:

Let x and y be the two numbers, then

$x^2 + y^2 = 13$ and $xy = 6$

(i) $(x+y)^2 = x^2 + y^2 + 2xy$
$= 13 + 2 \times 6 = 13 + 12 = 25$

$\therefore\ x + y = \pm\sqrt{25} = \pm 5$

(ii) $(x-y)^2 = x^2 + y^2 - 2xy = 13 - 12 = 1$
$\therefore\ x - y = \pm 1$

EXERCISE 12(D)

Question 1.

(i) $\left(3x+\frac{1}{2}\right)\left(2x+\frac{1}{3}\right)$

(ii) $(2a + 0.5)(7a - 0.3)$

(iii) $(9 - y)(7 + y)$ (iv) $(2 - z)(15 - z)$

(v) $(a^2 + 5)(a^2 - 3)$ (vi) $(4 - ab)(8 + ab)$

(vii) $(5xy - 7)(7xy + 9)$(viii) $(3a^2 - 4b^2)(8a^2 - 3b^2)$

Solution:

(i) $\left(3x+\frac{1}{2}\right)\left(2x+\frac{1}{3}\right)$

$= 3x\left(2x+\frac{1}{3}\right)+\frac{1}{2}\left(2x+\frac{1}{3}\right)$

$= 6x^2 + x + x + \frac{1}{6} = 6x^2 + 2x + \frac{1}{6}$

(ii) $(2a + 0.5)(7a - 0.3)$
$= 2a(7a - 0.3) + 0.5(7a - 0.3)$
$= 14a^2 - 0.6a + 3.5a - 0.15$
$= 14a^2 + 2.9a - 0.15$

(iii) $(9 - y)(7 + y) = 9(7 + y) - y(7 + y)$
$= 63 + 9y - 7y - y^2 = 63 + 2y - y^2$

(iv) $(2 - z)(15 - z) = 2(15 - z) - z(15 - z)$
$= 30 - 2z - 15z + z^2 = 30 - 17z + z^2$

(v) $(a^2 + 5)(a^2 - 3) = a^2(a^2 - 3) + 5(a^2 - 3)$

$= a^4 - 3a^2 + 5a^2 - 15 = a^4 + 2a^2 - 15$

(vi) $(4 - ab)(8 + ab) = 4(8 + ab) - ab(8 + ab)$

$= 32 + 4ab - 8ab - a^2b^2 = 32 - 4ab - a^2b^2$

(vii) $(5xy - 7)(7xy + 9) = 5xy(7xy + 9) - 7(7xy + 9)$

$= 35x^2y^2 + 45xy - 49xy - 63$

$= 35x^2y^2 - 4xy - 63$

(viii) $(3a^2 - 4b^2)(8a^2 - 3b^2)$

$= 3a^2(8a^2 - 3b^2) - 4b^2(8a^2 - 3b^2)$

$= 24a^4 - 9a^2b^2 - 32a^2b^2 + 12b^4$

$= 24a^4 - 41a^2b^2 + 12b^4$

Question 2.
Evaluate:

(i) $\left(2x-\frac{3}{5}\right)\left(2x+\frac{3}{5}\right)$ (ii) $\left(\frac{4}{7}a+\frac{3}{4}b\right)\left(\frac{4}{7}a-\frac{3}{4}b\right)$

(iii) $(6 - 5xy)(6 + 5xy)$

(iv) $\left(2a+\frac{1}{2a}\right)\left(2a-\frac{1}{2a}\right)$

(v) $(4x^2 - 5y^2)(4x^2 + 5y^2)$

(vi) $(1.6x + 0.7y)(1.6x - 0.7y)$

(vii) $(m + 3)(m - 3)(m^2 + 9)$

(viii) $(3x + 4y)(3x - 4y)(9x^2 + 16y^2)$

(ix) $(a + bc)(a - bc)(a^2 + b^2c^2)$

(x) 203×197 (xi) 20.8×19.2

Solution:

(i) $\left(2x-\frac{3}{5}\right)\left(2x+\frac{3}{5}\right)$

$= (2x)^2 - \left(\frac{3}{5}\right)^2$ [$\because (a - b)(a + b) = a^2b^2$]

$= 4x^2 - \frac{9}{25}$

(ii) $\left(\frac{4}{7}a+\frac{3}{4}b\right)\left(\frac{4}{7}a-\frac{3}{4}b\right)$

$= \left(\frac{4}{7}a\right)^2 - \left(\frac{3}{4}b\right)^2$ [$\because (a - b)(a + b) = a^2b^2$]

$= \frac{16}{49}a^2 - \frac{9}{16}b^2$

(iii) $(6 - 5xy)(6 + 5xy)$

$= (6)^2 - (5xy)^2$

$= 36 - 25x^2y^2$ $\quad [\because (a - b)(a + b) = a^2b^2]$

(iv) $\left(2a + \frac{1}{2a}\right)\left(2a - \frac{1}{2a}\right)$

$= (2a)^2 - \left(\frac{1}{2a}\right)^2$ $\quad [\because (a - b)(a + b) = a^2b^2]$

$= 4a^2 - \frac{1}{4a^2}$

(v) $(4x^2 - 5y^2)(4x^2 + 5y^2)$

$= (4x^2)^2 - (5y^2)^2$

$= 16x^4 - 25y^4$ $\quad [\because (a - b)(a + b) = a^2 - b^2]$

(vi) $(1.6x + 0.7y)(1.6x - 0.7y)$

$= (1.6x)^2 - (0.7y)^2$ $\quad [\because (a - b)(a + b) = a^2 - b^2]$

$= 2.56x^2 - 0.49y^2$

(vii) $(m + 3)(m - 3)(m^2 + 9)$

$= (m)^2 - (3)^2 (m^2 + 9)$ $\quad [\because (a - b)(a + b) = a^2 -$
$b^2]$

$= (m^2 - 9)(m^2 + 9)$

$= (m^2)^2 - 9^2 = m^4 - 81$

(viii) $(3x + 4y)(3x - 4y)(9x^2 + 16y^2)$

$= [(3x)^2 - (4y^2)](9x^2 + 16y^2)$

$[\because (a - b)(a + b) = a^2 - b^2]$

$= (9x^2 - 16y^2)(9x^2 + 16y^2)$

$= (9x^2)^2 - (16y^2)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$

$= 81x^4 - 256y^4$

(ix) $(a + bc)(a - bc)(a^2 + b^2c^2)$

$= [a^2 - (bc)^2](a^2 + b^2c^2)$

$[\because (a - b)(a + b) = a^2 - b^2]$

$= (a^2 - b^2c^2)(a^2 + b^2c^2)$

$= (a^2)^2 - (b^2c^2)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$

$= a^4 - b^4c^4$

(x) 203×197

$= (200 + 3)(200 - 3)$

$= (200)^2 - (3)^2 = 40000 - 9$

$[\because (a - b)(a + b) = a^2 - b^2]$

$= 39991$

(xi) $20.8 \times 19.2 = (20 + .8)(20 - .8)$

$= (20)^2 - (.8)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$

$= 400 - .64 = 399.36$

Question 3.

Find the square of :

(i) $3x + \frac{2}{y}$ (ii) $\frac{5a}{6b} - \frac{6b}{5a}$

(iii) $2m^2 - \frac{2}{3}n^2$ (iv) $5x + \frac{1}{5x}$

(v) $8x + \frac{3}{2}y$ (vi) 607

(vii) 391 (viii) 9.7

Solution:

(i) $3x + \frac{2}{y}$

$$\left(3x+\frac{2}{y}\right)^2 = (3x)^2 + \left(\frac{2}{y}\right)^2 + 2(3x)\left(\frac{2}{y}\right)$$

$$= 9x^2 + \frac{4}{y^2} + \frac{12x}{y}$$

(ii) $$\left(\frac{5a}{6b}-\frac{6b}{5a}\right)^2 = \left(\frac{5a}{6b}\right)^2 + \left(\frac{6b}{5a}\right)^2 - 2\times\frac{5a}{6b}\times\frac{6b}{5a}$$

$$= \frac{25a^2}{36b^2} - 2 + \frac{36b^2}{25a^2}$$

(iii) $2m^2 - \frac{2}{3}n^2$

$$\left(2m^2 - \frac{2}{3}n^2\right)^2 = (2m^2)^2 + \left(\frac{2}{3}n^2\right)^2 - 2\times 2m^2 \times \frac{2}{3}n^2$$

$$= 4m^4 + \frac{4}{9}n^4 - \frac{8}{3}m^2n^2$$

$$= 4m^4 - \frac{8}{3}m^2n^2 + \frac{4}{9}n^2$$

(iv) $$\left(5x+\frac{1}{5x}\right)^2 = (5x)^2 + \frac{1}{(5x)^2} + 2\times 5x \times \frac{1}{5x}$$

$$= 25x^2 + \frac{1}{25x^2} + 2 = 25x^2 + 2 + \frac{1}{25x^2}$$

(v) $$\left(8x+\frac{3}{2}y\right)^2 = (8x)^2 + \left(\frac{3}{2}y\right)^2 + 2\times 8x\times\frac{3}{2}y$$

$$= 64x^2 + \frac{9}{4}y^2 + 24xy = 64x^2 + 24xy + \frac{9}{4}y^2$$

(vi) $(607)^2 = (600 + 7)^2 = (600)^2 + (7)^2 + 2\,(600)\,(7)$

$= 360000 + 49 + 8400 = 368449$

(vii) $(391)^2 = (400 - 9)^2 = (400)^2 + 9^2 - 2\,(400)\,(9)$

$= 160000 + 81 - 7200 = 152881$

(viii) $(9.7)^2 = (10 - .3)^2 = (10)^2 + (.3)^2 - 2\,(10)\,(.3)$

$= 100 + .09 - 6 = 100.09 - 6.00 = 94.09$

Question 4.

. If $a + \frac{1}{a} = 2$, find :

(i) $a^2 + \frac{1}{a^2}$ (ii) $a^4 + \frac{1}{a^4}$

Solution:

(i) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$

$= (2)^2 - 2 = 4 - 2 = 2$

(ii) $a^4 + \frac{1}{a^4} = \left(a^2 + \frac{1}{a^2}\right)^2 - 2$

$= (2)^2 - 2 = 4 - 2 = 2$

Question 5.

If $m - \frac{1}{m} = 5$, find :

(i) $m^2 + \frac{1}{m^2}$ (ii) $m^4 + \frac{1}{m^4}$

(iii) $m^2 - \frac{1}{m^2}$

Solution:

$$\text{(i) } m^2 + \frac{1}{m^2} = \left(m - \frac{1}{m}\right)^2 + 2$$

$= (5)^2 + 2 = 25 + 2 = 27$

$$\text{(ii) } m^4 + \frac{1}{m^4} = \left(m^2 + \frac{1}{m^2}\right)^2 - 2$$

$= (27)^2 - 2 = 729 - 2 = 727$

$$\text{(iii) } m^2 - \frac{1}{m^2} = \left(m + \frac{1}{m}\right)\left(m - \frac{1}{m}\right)$$

$$= 5\left(m + \frac{1}{m}\right)$$

$$\text{Now } \left(m + \frac{1}{m}\right)^2 = \left(m - \frac{1}{m}\right)^2 + 4$$

$$= (5)^2 + 4 = 25 + 4 = 29$$

$$\therefore m + \frac{1}{m} = \sqrt{29}$$

$$\therefore m^2 - \frac{1}{m^2} = (5)\,(\sqrt{29})\; 5\sqrt{29}$$

Question 6.
If $a^2 + b^2 = 41$ and $ab = 4$, find :
(i) $a - b$
(ii) $a + b$

Solution:

(i) $(a - b)^2 = a^2 + b^2 - 2ab = 41 - 2(4) = 41 - 8 = 33$

$\therefore a - b = \sqrt{33}$

(ii) $(a + b)^2 = a^2 + b^2 + 2ab = 41 + 2(4) = 41 + 8 = 49 \Rightarrow (a + b)^2 = 49$

$\therefore a + b = 7$

Question 7.

If $2a + \frac{1}{2a} = 8$, find :

(i) $4a^2 + \frac{1}{4a^2}$ (ii) $16a^4 + \frac{1}{16a^4}$

Solution:

(i) $4a^2 + \frac{1}{4a^2} = \left(2a+\frac{1}{2a}\right)^2 - 2.2a\frac{1}{2a}$

$\left(2a+\frac{1}{2a}\right)^2 - 2 = (8)^2 - 2 = 64 - 2 = 62$

(ii) $16a^4 + \frac{1}{16a^4} = \left(4a^2+\frac{1}{4a^2}\right)^2 - 2.4a^2.\frac{1}{4a^2}$

$= (62)^2 - 2 = 3844 - 2 = 3842$

Question 8.

If $3x - \frac{1}{3x} = 5$, find :

(i) $9x^2 + \frac{1}{9x^2}$ (ii) $81x^4 + \frac{1}{81x^4}$

Solution:

(i) $9x^2 + \frac{1}{9x^2} = \left(3x-\frac{1}{3x}\right)^2 + 2$

$= (5)^2 + 2 = 25 + 2 = 27$

(ii) $81x^4 + \frac{1}{81x^4} = \left(9x^2+\frac{1}{9x^2}\right)^2 - 2$

$= (27)^2 - 2 = 729 - 2 = 727$

Question 9.

Expand :

(i) $(3x - 4y + 5z)^2$

(ii) $(2a - 5b - 4c)^2$

(iii) $(5x + 3y)^3$
(iv) $(6a – 7b)^3$
Solution:

(i) $(3x - 4y + 5z)^2$

$(3x)^2 + (-4y)^2 + (5z)^2 + 2(3x)(-4y) + 2(-4y)(5z) + 2(5z)(3x)$

$= 9x^2 + 16y^2 + 25z^2 - 24xy - 40yz + 30zx$

(ii) $(2a - 5b - 4c)^2 = (2a)^2 + (-5b)^2 + (-4c)^2 + 2(2a)(-5b) + 2(-5b)(-4c) + 2(-4c)(2a)$

$= 4a^2 + 25b^2 + 16c^2 - 20ab + 40bc - 16ca$

(iii) $(5x + 3y)^3 = (5x)^3 + (3y)^3 + 3(5x)(3y)(5x + 3y)$

$= 125x^3 + 27y^3 + 45xy(5x + 3y)$

$= 125x^3 + 27y^3 + 225x^2y + 135xy^2$

(iv) $(6a - 7b)^3 = (6a)^3 - (7b)^3 - 3(6a)(7b)(6a - 7b)$

$= 216a^3 - 343b^3 - 126ab(6a - 7b)$

$= 216a^3 - 343b^3 - 756a^2b + 882ab^2$

$= 216a^3 - 756a^2b + 882ab^2 - 343b^3$

Question 10.

If a + b + c = 9 and ab + bc + ca = 15, find: $a^2 + b^2 + c^2$.
Solution:

Since $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\therefore (9)^2 = a^2 + b^2 + c^2 + 2(15)$

$81 = a^2 + b^2 + c^2 + 30$

$\therefore a^2 + b^2 + c^2 = 81 - 30 = 51$

Question 11.

If a + b + c = 11 and $a^2 + b^2 + c^2 = 81$, find ab + bc + ca.

Solution:

Since $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\therefore \ (11)^2 = 81 + 2(ab + bc + ca)$

$\therefore 2(ab + bc + ca) = 121 - 81 = 40$

$ab + bc + ca = \frac{40}{2}$

$\Rightarrow ab + bc + ca = 20$

Question 12.
If 3x – 4y = 5 and xy = 3, find : $27x^3 - 64y^3$.
Solution:
$27x^3 - 64x^3 = (3x)^3 - (4y)^3$

$= (3x - 4y)^3 (3x - 4y)^3 + 3(3x)(4y)(3x - 4y)$

$[\because a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$= (5)^3 + 36(xy)(3x - 4y) = 125 + 36(3)(5)$
$= 125 + 540 = 665$

Question 13.
If a + b = 8 and ab = 15, find : $a^3 + b^3$.
Solution:

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (8)^3 - 3(15)(8) = 512 - 360 = 152$

Question 14.
If 3x + 2y = 9 and xy = 3, find : $27x^3 + 8y^3$
Solution:

$27x^3 + 8y^3 = (3x)^3 + (2y)^3 = (3x + 2y)^3 - 3.3x \,.\, 2y\,(3x + 2y)$

$= (3x - 2y)^3 - 18xy\,(3x + 2y)$

$= (9)^3 - 18(3)(9) = 729 - 486 = 243$

Question 15.
If $5x - 4y = 7$ and $xy = 8$, find : $125x^3 - 64y^3$

Solution:
$125x^3 - 64y^3 = (5x)^3 - (4y)^3 = (5x - 4y)^3 + 3(5x)$
$(4y)(5x - 4y)$
$= (5x - 4y)^3 + 60xy(5x - 4y)$
$= (7)^3 + 60(8)(7) = 343 + 3360 = 3703$

Question 16.
The difference between two numbers is 5 and their products is 14. Find the difference between their cubes.
Solution:
Let x and y be two numbers, then $x - y = 5$ and $xy = 14$
$\therefore x^3 - y^3 = (x - y)^3 + 3xy(x - y)$
$= (5)^3 + 3 \times 14 \times 5$
$= 125 + 210 = 335$