13. Factorisation

EXERCISE 13(A)

Factorise :

Question 1.

15x + 5

Solution:

15x + 5 = 5(3x + 1)

Question 2.

$a^3 - a^2 + a$

Solution:

$a^3 - a^2 + a = a(a^2 - a + 1)$

Question 3.

$3x^2 + 6x^3$

Solution:

$3x^2 + 6x^3 = 3x^2(1 + 2x)$

Question 4.

$4a^2 - 8ab$

Solution:

$4a^2 - 8ab = 4a(a - 2b)$

Question 5.

$2x^3b^2 - 4x^5b^4$

Solution:

$2x^3b^2 - 4x^5b^4 = 2x^3b^2(1 - 2x^2b^2)$

Question 6.

$15x^4y^3 - 20x^3y$

Solution:

$15x^4y^3 - 20x^3y = 5x^3y(3xy^2 - 4)$

Question 7.

$a^3b - a^2b^2 - b^3$

Solution:

$a^3b - a^2b^2 - b^3 = b(a^3 - a^2b - b^2)$

Question 8.

$6x^2y + 9xy^2 + 4y^3$

Solution:

$6x^2y + 9xy^2 + 4y^3 = y(6x^2 + 9xy + 4y^2)$.

Question 9.

$17a^6b^8 - 34a^4b^6 + 51a^2b^4$

Solution:

$17a^6b^8 - 34a^4b^6 + 51a^2b^4$
$= 17a^2b^4 (a^4b^4 - 2a^2b^2 + 3)$

Question 10.

$3x^5y - 27x^4y^2 + 12x^3y^3$

Solution:

$3x^5y - 27x^4y^2 + 12x^3y^3$
$= 3x^3y(x^2 - 9xy + 4y^2)$

Question 11.

$x^2(a-b) - y^2(a-b) + z^2(a-b)$

Solution:

$x^2(a-b) - y^2(a-b) + z^2(a-b)$
$= (a-b)(x^2 - y^2 + z^2)$.

Question 12.

$(x+y)(a+b) + (x-y)(a+b)$

Solution:

$(x+y)(a+b) + (x-y)(a+b)$
$\qquad = (a+b)(x+y+x-y)$
$= (a+b)(2x)$
$= 2x(a+b)$.

Question 13.

$2b(2a+b) - 3c(2a+b)$

Solution:

$2b(2a+b) - 3c(2a+b)$

$= (2a+b)(2b-3c)$

Question 14.

$12abc - 6a^2b^2c^2 + 3a^3b^3c^3$

Solution:

$12abc - 6a^2b^2c^2 + 3a^3b^3c^3$

$= 3abc(4 - 2abc + a^2b^2c^2)$

Question 15.

$4x(3x - 2y) - 2y(3x - 2y)$

Solution:

$4x(3x - 2y) - 2y(3x - 2y)$

$= (3x - 2y)(4x - 2y) = (3x - 2y) \times 2(2x - y)$

$= 2(3x - 2y)(2x - y)$

Question 16.

$(a + 2b)(3a + b) - (a + b)(a + 2b) + (a + 2b)^2$

Solution:

$(a + 2b)(3a + b) - (a + b)(a + 2b) + (a + 2b)^2$

$= (a + 2b)(3a + b - a - b + a + 2b)$

$= (a + 2b)(3a + 2b)$

Question 17.

$6xy(a^2 + b^2) + 8yz(a^2 + b^2) - 10xz(a^2 + b^2)$

Solution:

$6xy(a^2 + b^2) + 8yz(a^2 + b^2) - 10xz(a^2 + b^2)$

H.C.F. of 6, 8, 10 = 2, then

$2(a^2 + b^2)(3xy + 4yz - 5xz)$

EXERCISE 13(B)

Question 1.
Factorise : $a^2 + ax + ab + bx$
Solution:

$$a^2 + ax + ab + bx$$
$$= (a^2 + ax) + (ab + bx)$$
$$= a(a+x) + b(a+x)$$
$$= (a+x)(a+b)$$

Question 2.
Factorise : $a^2 - ab - ca + bc$
Solution:

$$a^2 - ab - ca + bc$$
$$= a(a-b) - c(a-b)$$
$$= (a-b) \quad (a-c)$$

Question 3.
Factorise : $ab - 2b + a^2 - 2a$
Solution:

$$ab - 2b + a^2 - 2a$$
$$= b(a-2) + a(a-2)$$
$$= (a-2)(b+a)$$

Question 4.
Factorise : $a^3 - a^2 + a - 1$
Solution:

$$a^3 - a^2 + a - 1$$
$$= a^2(a-1) + 1(a-1)$$
$$= (a-1)(a^2+1)$$

Question 5.
Factorise : $2a - 4b - xa + 2bx$
Solution:

$$2a - 4b - xa + 2bx$$
$$= 2(a-2b) - x(a-2b)$$
$$= (a-2b)(2-x)$$

Question 6.

Factorise : $xy-ay-ax+a^2+bx-ab$

Solution:

$$xy-ay-ax+a^2+bx-ab$$
$$= y(x-a)-a(x-a)+b(x-a)$$
$$= (x-a)\ (y-a+b)$$

Question 7.

Factorise : $3x^5-6x^4-2x^3+4x^2+x-2$

Solution:

$$3x^5-6x^4-2x^3+4x^2+x-2$$
$$= 3x^4(x-2)-2x^2(x-2)+1(x-2)$$
$$= (x-2)\ (3x^4-2x^2+1)$$

Question 8.

Factorise : $-x^2y - x + 3xy + 3$

Solution:

$$-x^2y - x + 3xy + 3$$
$$= 3 - x + 3xy - x^2y \qquad \text{(By grouping)}$$
$$= 1(3 - x) + xy(3 - x)$$
$$= (3 - x)\ (1 + xy) = (xy + 1)\ (3 - x)$$

Question 9.

Factorise : $6a^2 - 3a^2b - bc^2 + 2c^2$

Solution:

$$6a^2 - 3a^2b - bc^2 + 2c^2$$
$$= 6a^2 - 3a^2b + 2c^2 - bc^2 \qquad \text{(By grouping)}$$
$$= 3a^2(2 - b) + c^2(2 - b)$$
$$= (2 - b)\ (3a^2 + c^2)$$

Question 10.

Factorise : $3a^2b - 12a^2 - 9b + 36$

Solution:

$3a^2b - 12a^2 - 9b + 36$

$= 3a^2(b - 4) - 9(b - 4)$ (By grouping)

$= (b - 4)(3a^2 - 9) = (b - 4) \, 3(a^2 - 3)$

$= 3(b - 4)(a^2 - 3)$

Question 11.

Factorise : $x^2 - (a-3)x - 3a$

Solution:

$x^2 - (a-3)x - 3a$

$= x^2 - ax + 3x - 3a$

$= x(x-a) + 3(x-a)$

$= (x-a)(x+3)$

Question 12.

Factorise : $x^2 - (b-2)x - 2b$

Solution:

$x^2 - (b-2)x - 2b$

$= x^2 - bx + 2x - 2b$

$= x(x-b) + 2(x-b)$

$= (x-b)(x+2)$

Question 13.

Factorise : $a(b-c) - d(c-b)$

Solution:

$a(b-c) - d(c-b)$

$= a(b-c) + d(b-c)$

$= (b-c)(a+d)$

Question 14.

Factorise : $ab^2 - (a - c)\, b - c$

Solution:

$ab^2 - (a - c)\, b - c$

$= ab^2 - ab + bc - c$

$= ab\, (b - 1) + c\, (b - 1)$

$= (b - 1)\, (ab + c)$

Question 15.

Factorise : $(a^2 - b^2)\, c + (b^2 - c^2)\, a$

Solution:

$(a^2 - b^2)\, c + (b^2 - c^2)\, a$

$= a^2 c - b^2 c + ab^2 - ac^2$

$= a^2 c - ac^2 + ab^2 - b^2 c$

$= ac\, (a - c) + b^2\, (a - c)$

$= (a - c)\, (ac + b^2)$

Question 16.

Factorise : $a^3 - a^2 - ab + a + b - 1$

Solution:

$a^3 - a^2 - ab + a + b - 1$

$= a^3 - a^2 - ab + b + a - 1$

$= a^2\, (a - 1) - b\, (a - 1) + 1\, (a - 1)$

$= (a - 1)\, (a^2 - b + 1)$

Question 17.

Factorise : $ab\, (c^2 + d^2) - a^2 cd - b^2 cd$

Solution:

$ab\, (c^2 + d^2) - a^2 cd - b^2 cd$

$= abc^2 + abd^2 - a^2 cd - b^2 cd$

$= abc^2 - a^2 cd - b^2 cd + abd^2$

$= ac\, (bc - ad) - bd\, (bc - ad)$

$= (bc - ad)\, (ac - bd)$

Question 18.

Factorise : $2ab^2 - aby + 2cby - cy^2$

Solution:

$$= 2ab^2 + 2cby - aby - cy^2$$
$$= 2b\,(ab + cy) - y\,(ab + cy)$$
$$= (ab + cy)\,(2b - y)$$

Question 19.

Factorise : $ax + 2bx + 3cx - 3a - 6b - 9c$

Solution:

$$ax + 2bx + 3cx - 3a - 6b - 9c$$
$$= x(a + 2b + 3c) - 3(a + 2b + 3c)$$
$$\qquad\qquad\qquad\qquad \text{(By grouping)}$$
$$= (a + 2b + 3c)\,(x - 3)$$

Question 20.

Factorise : $2ab^2c - 2a + 3b^3c - 3b - 4b^2c^2 + 4c$

Solution:

$$2ab^2c - 2a + 3b^3c - 3b - 4b^2c^2 + 4c$$
$$= 2a(b^2c - 1) + 3b(b^2c - 1) - 4c(b^2c - 1)$$
$$= (b^2c - 1)\,(2a + 3b - 4c)$$

EXERCISE 13(C)

Note : $a^2 - b^2 = (a + b)\,(a - b)$

Question 1.
Factorise : $16 - 9x^2$
Solution:
$16 - 9x^2 = (4)^2 - (3x)^2 = (4 + 3x)\,(4 - 3x)$

Question 2.
Factorise : $1 - 100a^2$
Solution:
$1 - 100a^2 = (1)^2 - (10a)^2 = (1 + 10a)\,(1 - 10a)$

Question 3.
Factorise : $4x^2 - 81y^2$
Solution:
$4x^2 - 81y^2 = (2x)^2 - (9y)^2 = (2x + 9y)\,(2x - 9y)$

Question 4.

Factorise : $\dfrac{4}{25} - 25b^2$

Solution:

$$\dfrac{4}{25} - 25b^2 = \left(\dfrac{2}{5}\right)^2 - (5b)^2$$

$$= \left(\dfrac{2}{5} + 5b\right)\left(\dfrac{2}{5} - 5b\right)$$

Question 5.

Factorise : $(a+2b)^2 - a^2$

Solution:

$$\begin{aligned}
(a+2b)^2 - a^2 &= (a+2b)^2 - (a)^2 \\
&= (a+2b+a)(a+2b-a) \\
&= (2a+2b)(2b) \\
&= 2(a+b)(2b) \\
&= 2 \times 2b(a+b) \\
&= 4b(a+b)
\end{aligned}$$

Question 6.

Factorise : $(5a-3b)^2 - 16b^2$

Solution:

$$\begin{aligned}
(5a-3b)^2 - 16b^2 \\
&= (5a-3b)^2 - (4b)^2 \\
&= (5a-3b+4b)(5a-3b-4b) \\
&= (5a+b)(5a-7b)
\end{aligned}$$

Question 7.

Factorise : $a^4 - (a^2 - 3b^2)^2$

Solution:

$a^4-(a^2-3b^2)^2$

$$= (a^2)^2-(a^2-3b^2)^2$$
$$= (a^2+a^2-3b^2)\ (a^2-a^2+3b^2)$$
$$= (2a^2-3b^2)\ (3b)^2$$
$$= 3b^2\ (2a^2-3b^2)$$

Question 8.

Factorise : $(5a-2b)^2 - (2a-b)^2$

Solution:

$(5a-2b)^2 - (2a-b)^2$
$$= (5a-2b+2a-b)\ (5a-2b-2a+b)$$
$$= (7a-3b)\ (3a-b)$$

Question 9.

Factorise : $1-25\ (a+b)^2$

Solution:

$1-25(a+b)^2$

$$= (1)^2-[5(a+b)]^2$$
$$= [1+5(a+b)]\ [1-5(a+b)]$$
$$= (1+5a+5b)\ (1-5a-5b)$$

Question 10.

Factorise : $4\ (2a+b)^2 - (a-b)^2$

Solution:

$4\ (2a+b)^2 - (a-b)^2$

$$= [2(2a+b)]^2 - (a-b)^2$$
$$= [2(2a+b)+a-b][2(2a+b)$$
$$\qquad\qquad -a+b]$$
$$= (4a+2b+a-b)\ (4a+2b-a+b)$$
$$= (5a+b)\ (3a+3b)$$
$$= (5a+b)\ 3(a+b)$$
$$= 3(5a+b)\ (a+b)$$

Question 11.

Factorise : $25(2x+y)^2 - 16(x-y)^2$

Solution:

$25(2x+y)^2 - 16(x-y)^2$

$$= [5(2x+y)]^2 - [4(x-y)]^2$$
$$= (10x+5y)^2 - (4x-4y)^2$$
$$= (10x+5y+4x-4y)\,(10x+5y$$
$$\qquad\qquad\qquad - 4x+4y)$$
$$= (14x+y)\,(6x+9y)$$
$$= (14x+y)\,3(2x+3y)$$
$$= 3\,(14x+y)\,(2x+3y)$$

Question 12.

Factorise : $49\,(x-y)^2 - 9\,(2x+y)^2$

Solution:

$$= [7(x-y)]^2 - [3(2x+y)]^2$$
$$= (7x-7y)^2 - (6x+3y)^2$$
$$= (7x-7y+6x+3y)\,(7x - 7y - 6x - 3y)$$
$$= (13x - 4y)\,(x-10y)$$

Question 13.

Evaluate : $\left(6\dfrac{2}{3}\right)^2 - \left(2\dfrac{1}{3}\right)^2$

Solution:

$$\left(6\dfrac{2}{3}\right)^2 - \left(2\dfrac{1}{3}\right)^2 = \left(\dfrac{20}{3}\right)^2 - \left(\dfrac{7}{3}\right)^2$$

$$= \left(\dfrac{20}{3} + \dfrac{7}{3}\right)\left(\dfrac{20}{3} - \dfrac{7}{3}\right)$$

$$= \left(\dfrac{27}{3}\right)\left(\dfrac{13}{3}\right) = 9 \times \dfrac{13}{3} = 39$$

Question 14.
Evaluate :

$$\left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2$$

Solution:

$$\left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2$$

$$= \left(\frac{73}{10}\right)^2 - \left(\frac{21}{10}\right)^2$$

$$= \left(\frac{73}{10} + \frac{21}{10}\right)\left(\frac{73}{10} - \frac{21}{10}\right)$$

$$= \left(\frac{94}{10}\right)\left(\frac{52}{10}\right)$$

$$= \left(\frac{47}{5}\right)\left(\frac{26}{5}\right) = \frac{1222}{25} = 48\frac{22}{25}$$

Question 15.
Evaluate :
$(0.7)^2 - (0.3)^2$
Solution:

$(0.7)^2 - (0.3)^2$

$= (0.7+0.3)\ (0.7-0.3)$

$= 1 \times 0.4 = 0.4$

Question 16.
Evaluate :
$(4.5)^2 - (1.5)^2$
Solution:

$(4.5)^2 - (1.5)^2$

$= (4.5+1.5)\ (4.5-1.5)$

$= 6 \times 3 = 18$

Question 17.

Factorise : $75(x+y)^2 - 48(x-y)^2$

Solution:

$75(x + y)^2 - 48(x - y)^2$

$= 3[25(x + y)^2 - 16(x - y)^2]$

$= 3[\{5(x + y)^2\} - \{4(x - y)\}^2]$

Using $a^2 - b^2 = (a + b)(a - b)$

$= 3[5(x + y) + 4(x - y)][5(x + y) - 4(x - y)]$

$= 3[5x + 5y + 4x - 4y][5x + 5y - 4x + 4y]$

$= 3(9x + y)(x + 9y)$

Question 18.

Factorise : $a^2 + 4a + 4 - b^2$

Solution:

$a^2 + 4a + 4 - b^2$

$$\left\{ \begin{array}{l} \because (a + b)^2 = a^2 + 2ab + b^2 \\ a^2 - b^2 = (a + b)(a - b) \end{array} \right\}$$

$= (a)^2 + 2 \times a \times 2 + (2)^2 - (b)^2$

$= (a + 2)^2 - (b)^2$

$= (a + 2 + b)(a + 2 - b)$

$= (a + b + 2)(a - b + 2)$

Question 19.

Factorise : $a^2 - b^2 - 2b - 1$

Solution:

$a^2 - b^2 - 2b - 1$

$= a^2 - (b^2 + 2b + 1)$

$$\left\{ \begin{array}{l} \because (a + b)^2 = a^2 + b^2 + 2ab \\ a^2 - b^2 = (a + b)(a - b) \end{array} \right\}$$

$= (a)^2 - (b + 1)^2$

$= (a + b + 1)(a - b - 1)$

Question 20.

Factorise : $x^2 + 6x + 9 - 4y^2$

Solution:

$x^2 + 6x + 9 - 4y^2$

$= (x)^2 + 2 \times x \times 3 + (3)^2 - (2y)^2$

$$\left. \begin{cases} \because (a+b)^2 = a^2 + 2ab + b^2 \\ \quad a^2 - b^2 = (a+b)(a-b) \end{cases} \right\}$$

$= (x+3)^2 - (2y)^2$

$= (x + 3 + 2y)(x + 3 - 2y)$

$= (x + 2y + 3)(x - 2y + 3)$

EXERCISE 13(D)

Question 1.

Factorise : $x^2 + 6x + 8$

Solution:

$x^2 + 6x + 8 = x^2 + 4x + 2x + 8$

$\qquad\qquad = x(x+4) + 2(x+4)$

$= (x+4)(x+2)$

Question 2.

Factorise : $x^2 + 4x + 3$

Solution:

$x^2 + 4x + 3 = x^2 + 3x + x + 3$

$\qquad\qquad = x(x+3) + 1(x+3)$

$\qquad\qquad = (x+3)(x+1)$

Question 3.

Factorise : $a^2 + 5a + 6$

Solution:

$a^2 + 5a + 6 = a^2 + 3a + 2a + 6$

$\qquad\qquad = a(a+3) + 2(a+3)$

$\qquad\qquad = (a+3)(a+2)$

Question 4.

Factorise : $a^2 - 5a + 6$

Solution:

$$a^2-5a+6 = a^2-3a-2a+6$$
$$= a(a-3)-2(a-3)$$
$$= (a-3)\ (a-2)$$

Question 5.

Factorise : a^2+5a-6

Solution:

$$a^2+5a-6 = a^2+6a-a-6$$
$$= a(a+6)-1(a+6)$$
$$= (a+6)\ (a-1)$$

Question 6.

Factorise : $x^2+5xy+4y^2$

Solution:

$$x^2+5xy+4y^2 = x^2+4xy+xy+4y^2$$
$$= x(x+4y)+y(x+4y)$$
$$= (x+4y)\ (x+y)$$

Question 7.

Factorise : $a^2-3a-40$

Solution:

$$a^2-3a-40 = a^2-8a+5a-40$$
$$= a(a-8)+5(a-8)$$
$$= (a-8)\ (a+5)$$

Question 8.

Factorise : x^2-x-72

Solution:

$$x^2-x-72 = x^2-9x+8x-72$$
$$= x(x-9)\ +8(x-9)$$
$$= (x-9)\ (x+8)$$

Question 9.

Factorise : $x^2-10xy+24y^2$

Solution:

$$x^2-10xy+24y^2 = x^2-6xy-4xy+24y^2$$
$$= x(x-6y) - 4y(x-6y)$$
$$= (x-6y)(x-4y)$$

Question 10.

Factorise : $2a^2+7a+6$

Solution:

$$2a^2+7a+6 = 2a^2+4a+3a+6$$
$$= 2a(a+2)+3(a+2)$$
$$= (a+2)(2a+3)$$

Question 11.

Factorise : $3a^2-5a+2$

Solution:

$$3a^2-5a+2 = 3a^2-3a-2a+2$$
$$= 3a(a-1) -2(a-1)$$
$$= (a – 1)(3a – 2)$$

Question 12.

Factorise : $7b^2-8b+1$

Solution:

$$7b^2-8b+1 = 7b^2-7b-b+1$$
$$= 7b(b-1)-1(b-1)$$
$$= (b-1)(7b-1)$$

Question 13.

Factorise : $2a^2-17ab+26b^2$

Solution:

$$2a^2-17ab+26b^2$$
$$= 2a^2-13ab-4ab+26b^2$$
$$= a(2a-13b)-2b(2a-13b)$$
$$= (2a-13b)(a-2b)$$

Question 14.

Factorise : $2x^2+xy-6y^2$

Solution:

$$2x^2 + xy - 6y^2 = 2x^2 + 4xy - 3xy - 6y^2$$
$$= 2x(x + 2y) - 3y(x + 2y)$$
$$= (x + 2y)(2x - 3y)$$

Question 15.

Factorise : $4c^2 + 3c - 10$

Solution:

$$4c^2 + 3c - 10 = 4c^2 + 8c - 5c - 10$$
$$= 4c(c + 2) - 5(c + 2)$$
$$= (c + 2)(4c - 5)$$

Question 16.

Factorise : $14x^2 + x - 3$

Solution:

$$14x^2 + x - 3 = 14x^2 + 7x - 6x - 3$$
$$= 7x(2x + 1) - 3(2x + 1)$$
$$= (2x + 1)(7x - 3)$$

Question 17.

Factorise : $6 + 7b - 3b^2$

Solution:

$$6 + 7b - 3b^2 = 6 + 9b - 2b - 3b^2$$
$$= 3(2 + 3b) - b(2 + 3b)$$
$$= (2 + 3b)(3 - b)$$

Question 18.

Factorise : $5 + 7x - 6x^2$

Solution:

$$5 + 7x - 6x^2 = 5 + 10x - 3x - 6x^2$$
$$= 5(1 + 2x) - 3x(1 + 2x)$$
$$= (1 + 2x)(5 - 3x)$$

Question 19.

Factorise : $4 + y - 14y^2$

Solution:

$$4+y-14y^2 = 4+8y-7y-14y^2$$
$$= 4(1+2y)-7y(1+2y)$$
$$= (1+2y)\ (4-7y)$$

Question 20.

Factorise : $5+3a-14a^2$

Solution:

$$5+3a-14a^2 = 5+10a-7a-14a^2$$
$$= 5(1+2a)\ -7a(1+2a)$$
$$= (1+2a)\ (5-7a)$$

Question 21.

Factorise : $(2a+b)^2+5(2a+b)+6$

Solution:

$$\text{Let}\quad (2a+b) = x$$
$$(2a+b)^2 = x^2$$
$$(2a+b)^2+5(2a+b)+6$$
$$= x^2+5x+6$$
$$= x^2+3x+2x+6$$
$$= x(x+3)+2(x+3)$$
$$= (x+3)(x+2)$$
$$= (2a+b+3)(2a+b+2)$$
$$\text{(Substituting the value of } x)$$

Question 22.

Factorise : $1-(2x+3y)-6(2x+3y)^2$

Solution:

Let $(2x+3y) = a$

$\therefore \quad (2x+3y)^2 = a^2$

$\therefore \quad 1-(2x+3y)-6(2x+3y)^2$

$$= 1-a-6a^2$$
$$= 1-3a+2a-6a^2$$
$$= 1(1-3a)+2a(1-3a)$$
$$= (1-3a)\ (1+2a)$$
$$= [1-3(2x+3y)][1+2(2x+3y)]$$

(Substituting the value of a)

$$= (1-6x-9y)\ (1+4x+6y)$$

Question 23.

Factorise : $(x-2y)^2-12(x-2y)+32$

Solution:

Let $\quad (x-2y) = a$

$\therefore \qquad (x-2y)^2 = a^2$

$\therefore \quad (x-2y)^2-12(x-2y)+32$

$$= a^2-12a+32$$
$$= a^2-8a-4a+32$$
$$= a(a-8)-4(a-8)$$
$$= (a-8)(a-4)$$
$$= (x-2y-8)\ (x-2y-4)$$

(Substituting the value of a)

Question 24.

Factorise : $8 + 6\,(a+b) - 5\,(a+b)^2$

Solution:

Let $a+b = x$

$(a+b)^2 = x^2$

$8+6(a+b)-5(a+b)^2$

$= 8+6x-5x^2$

$= 8+10x-4x-5x^2$

$= 2(4+5x)-x(4+5x)$

$= (4+5x)(2-x)$

$= [4+5(a+b)][2-(a+b)]$

(Substituting the value of x)

$=[4+5a+5b][2-a-b]$

Question 25.

Factorise : $2(x + 2y)^2 - 5(x + 2y) + 2$

Solution:

$2(x + 2y)^2 - 5(x + 2y) + 2$

Let $x + 2y = a$, then

$2a^2 - 5a + 2$

$\Rightarrow 2a^2 - a - 4a + 2$

$= a(2a - 1) - 2(2a - 1)$

$= (2a - 1)(a - 2)$

$= \{2(x + 2y - 1)\}\{(x + 2y) - 2\}$

$= (2x + 4y - 2)(x + 2y - 2)$

$$\begin{bmatrix} \because 2 \times 2 = 4 \\ -5 = -1-4 \\ 4 = (-1)\times(-4) \end{bmatrix}$$

EXERCISE 13(E)

Question 1.

In each case find whether the trinomial is a perfect square or not:

(i) $x^2+14x+49$ (ii) $a^2-10a+25$

(iii) $4x^2+4x+1$ (iv) $9b^2+12b+16$

(v) $16x^2-16xy+y^2$ (vi) $x^2- 4x+16$

Solution:

(i) $x^2+14x+49$

$\quad = (x)^2+2\times x\times 7+(7)^2$

$\quad = (x+7)^2$

$\qquad [\because a^2+2ab+b^2 = (a+b)^2]$

$\therefore$ The given trinomial $x^2+14x+49$ is a perfect square.

(ii) $\quad a^2-10a+25 = (a)^2-2\times a\times 5+(5)^2$

$\qquad\qquad\qquad = (a-5)^2$

$\qquad\qquad\quad [\because a^2-2ab+b^2 = (a-b)^2]$

$\therefore$ The given trinomial $a^2-10a+25$ is a perfect square.

(iii) $\quad 4x^2+4x+1 = (2x)^2+2\times 2x\times 1+(1)^2$

$\qquad\qquad\qquad = (2x+1)^2$

$\qquad\qquad\quad [\because a^2+2ab+b^2 = (a+b)^2]$

$\therefore$ The given trinomial $4x^2 + 4x = 1$ is a perfect square.

(iv) $\quad 9b^2+12b+16 = (3b)^2+3b\times 4+(4)^2$

$\qquad\qquad\qquad = x^2+xy+y^2$

$\qquad\qquad$ [Taking $3b=x$, and $4=y$]

$\therefore$ The given trinomial cannot be expressed as $x^2+2xy+y^2$. Hence, it is not a perfect square.

(v) $\quad 16x^2-16xy+y^2 = (4x)^2-4\times 4x\times y+(y)^2$

$\qquad\qquad\qquad = a^2-4ab+b^2$

$\qquad\qquad$ [Taking $4x=a$, and $y=b$]

$\therefore$ The given trinomial cannot be expressed as $a^2-2ab+b^2$.

$\therefore$ It is not a perfect square.

(vi) $\quad x^2-4x+16 \qquad = (x)^2-x\times 4+(4)^2$

$\qquad\qquad\qquad = a-ab+b^2$

$\qquad\qquad$ [Taking $x=a$, and $4=b$]

$\therefore$ The given trinomial cannot be expressed as $a^2-2ab+b^2$.

Hence, it is not a perfect square.

Question 2.
Factorise completely $2 - 8x^2$.
Solution:

$$2 - 8x^2 = 2(1 - 4x^2)$$
$$= 2[(1)^2 - (2x)^2]$$
$$= 2(1 + 2x)(1 - 2x)$$

Note : $a^2 - b^2 = (a+b)(a-b)$

Question 3.
Factorise completely : $8x^2y - 18y^3$
Solution:

$$8x^2y - 18y^3 = 2y(4x^2 - 9y^2)$$
$$= 2y[(2x)^2 - (3y)^2]$$
$$= 2y(2x + 3y)(2x - 3y)$$

Question 4.
Factorise completely : $ax^2 - ay^2$
Solution:

$$ax^2 - ay^2 = a(x^2 - y^2)$$
$$= a(x + y)(x - y)$$

Question 5.
Factorise completely : $25x^3 - x$
Solution:

$$25x^3 - x = x(25x^2 - 1)$$
$$= x[(5x)^2 - (1)^2]$$
$$= x(5x + 1)(5x - 1)$$

Question 6.
Factorise completely : $a^4 - b^4$
Solution:

$$a^4 - b^4 = (a^2)^2 - (b^2)^2$$
$$= (a^2 + b^2)(a^2 - b^2)$$
$$= (a^2 + b^2)(a + b)(a - b)$$

Question 7.
Factorise completely : $16x^4 - 81y^4$

Solution:

$$16x^4 - 81y^4 = (4x^2)^2 - (9y^2)^2$$
$$= (4x^2 + 9y^2)(4x^2 - 9y^2)$$
$$= (4x^2 + 9y^2)[(2x)^2 - (3y)^2]$$
$$= (4x^2 + 9y^2)(2x + 3y)(2x - 3y)$$

Question 8.
Factorise completely : $625 - x^4$
Solution:

$$625 - x^4 = (25)^2 - (x^2)^2$$
$$= (25 + x^2)(25 - x^2)$$
$$= (25 + x^2)[(5)^2 - (x)^2]$$
$$= (25 + x^2)(5 + x)(5 - x)$$

Question 9.
Factorise completely : $x^2 - y^2 - 3x - 3y$
Solution:

$$x^2 - y^2 - 3x - 3y = (x^2 - y^2) - 3(x + y)$$
$$= (x + y)(x - y) - 3(x + y)$$
$$= (x + y)(x - y - 3)$$

Question 10.
Factorise completely : $x^2 - y^2 - 2x + 2y$
Solution:

$$x^2 - y^2 - 2x + 2y = (x^2 - y^2) - 2(x - y)$$
$$= (x + y)(x - y) - 2(x - y)$$
$$= (x - y)(x + y - 2)$$

Question 11.
Factorise completely : $3x^2 + 15x - 72$
Solution:

$$3x^2 + 15x - 72 = 3(x^2 + 5x - 24)$$
$$= 3[x^2 + 8x - 3x - 24]$$
$$= 3[x(x + 8) - 3(x + 8)]$$
$$= 3[(x + 8)(x - 3)]$$
$$= 3(x + 8)(x - 3)$$

Question 12.

Factorise completely : $2a^2 - 8a - 64$

Solution:

$$2a^2 - 8a - 64 = 2[a^2 - 4a - 32]$$
$$= 2[a^2 - 8a + 4a - 32]$$
$$= 2[a(a-8) + 4(a-8)]$$
$$= 2[(a-8)(a+4)]$$
$$= 2(a-8)(a+4)$$

Question 13.

Factorise completely : $5b^2 + 45b + 90$

Solution:

$$5b^2 + 45b + 90 = 5[b^2 + 9b + 18]$$
$$= 5[b^2 + 6b + 3b + 18]$$
$$= 5[b(b+6) + 3(b+6)]$$
$$= 5[(b+6)(b+3)]$$
$$= 5(b+6)(b+3)$$

Question 14.

Factorise completely : $3x^2y + 11xy + 6y$

Solution:

$$3x^2y + 11xy + 6y = y(3x^2 + 11x + 6)$$
$$= y[(3x^2 + 9x + 2x + 6)]$$
$$= y[3x(x+3) + 2(x+3)]$$
$$= y[(x+3)(3x+2)]$$
$$= y(x+3)(3x+2)$$

Question 15.

Factorise completely : $5ap^2 + 11ap + 2a$

Solution:

$$5ap^2 + 11ap + 2a = a[5p^2 + 11p + 2]$$
$$= a[5p^2 + 10p + p + 2]$$
$$= a[5p(p+2) + 1(p+2)]$$
$$= a[(p+2)(5p+1)]$$
$$= a(p+2)(5p+1)$$

Question 16.
Factorise completely : $a^2 + 2ab + b^2 - c^2$
Solution:

$$a^2 + 2ab + b^2 - c^2 = (a^2 + 2ab + b^2) - c^2$$
$$= (a+b)^2 - (c)^2$$
$$= (a+b+c)(a+b-c)$$

Question 17.
Factorise completely : $x^2 + 6xy + 9y^2 + x + 3y$
Solution:

$$x^2 + 6xy + 9y^2 + x + 3y$$
$$= [(x)^2 + 2 \times x \times 3y + (3y)^2] + (x+3y)$$
$$= [x+3y]^2 + (x+3y)$$
$$= (x+3y)(x+3y) + (x+3y)$$
$$= (x+3y)(x+3y+1)$$

Question 18.
Factorise completely : $4a^2 - 12ab + 9b^2 + 4a - 6b$
Solution:

$$[4a^2 - 12ab + 9b^2] + (4a - 6b)$$
$$= [(2a)^2 - 2 \times 2a \times 3b + (3b)^2] + 2(2a - 3b)$$
$$= (2a - 3b)^2 + 2(2a - 3b)$$
$$= (2a - 3b)(2a - 3b + 2)$$

Question 19.
Factorise completely : $2a^2b^2 - 98b^4$
Solution:

$$2a^2b^2 - 98b^4 = 2b^2(a^2 - 49b^2)$$
$$= 2b^2[(a)^2 - (7b)^2]$$
$$= 2b^2(a + 7b)(a - 7b)$$

Question 20.
Factorise completely : $a^2 - 16b^2 - 2a - 8b$
Solution:

$$(a^2 - 16b)^2 - 2a - 8b = [(a)^4 - (4b)^2] - 2(a + 4b)$$
$$= (a + 4b)(a - 4b) - 2(a + 4b)$$
$$= (a + 4b)(a - 4b - 2)$$

EXERCISE 13(F)

Question 1.

Factorise :

(i) $6x^3 - 8x^2$ (ii) $35a^3b^2c + 42ab^2c^2$

(iii) $36x^2y^2 - 30x^3y^3 + 48x^3y^2$

(iv) $8(2a + 3b)^3 - 12(2a + 3b)^2$

(v) $9a(x - 2y)^4 - 12a(x - 2y)^3$

Solution:

(i) $6x^3 - 8x^2 = 2x^2(3x - 4)$

(ii) $35a^3b^2c + 42ab^2c^2 = 7ab^2c(5a^2 + 6c)$

(iii) $36x^2y^2 - 30x^3y^3 + 48x^3y^3 = 6x^2y^2(6 - 5xy + 8xy)$

(iv) $8(2a + 3b)^3 - 12(2a + 3b)^2$

$= 4(2a + 3b)^2[2(2a + 3b) - 3]$

$= 4(2a + 3b)^2[4a + 6b - 3]$

(v) $9a(x - 2y)^4 - 12a(x - 2y)^3$

$= 3a(x - 2y)^3(3(x - 2y) - 4$

$= 3a(x - 2y)^3(3x - 6y - 4)$

Question 2.

Factorise :

(i) $a^2 - ab - 3a + 3b$ (ii) $x^2y - xy^2 + 5x - 5y$

(iii) $a^2 - ab(1 - b) - b^3$

(iv) $xy^2 + (x - 1)y - 1$

(v) $(ax + by)^2 + (bx - ay)^2$

(vi) $ab(x^2 + y^2) - xy(a^2 + b^2)$

(vii) $m - 1 - (m - 1)^2 + am - a$

Solution:

(i) $a^2 - ab - 3a + 3b = a(a - b) - 3(a - b)$
      $= (a - b)(a - 3)$

(ii) $x^2 y - xy^2 + 5x - 5y = xy(x - y) + 5(x - y)$
      $= (x - y)(xy + 5)$

(iii) $a^2 - ab(1 - b) - b^3 = a^2 - ab + ab^2 - b^3$
$= a(a - b) + b^2(a - b) = (a - b)(q + b)^2$

(iv) $xy^2 + (x - 1)y - 1 = xy^2 + xy - y - 1$
$= xy(y + 1) - 1(y + 1) = (xy - 1)(y + 1)$

(v) $(ax + by)^2 + (bx - ay)^2$
$= a^2 x^2 + b^2 y^2 + 2abxy + b^2 x^2 + a^2 y^2 - 2abxy$
$= a^2 x^2 + b^2 y^2 + b^2 x^2 + y^2 = a^2 x^2 + a^2 y^2 + b^2 x^2 + b^2 y^2$
$= a^2(x^2 + y^2) + b^2(x^2 + y^2) = (x^2 + y^2)(a^2 + b^2)$

(vi) $ab(x^2 + y^2) - xy(a^2 + b^2)$
$= abx^2 + aby^2 - a^2 xy - b^2 xy = abx^2 - a^2 xy + aby^2 - b^2 xy$
$= abx^2 - a^2 xy - b^2 xy + aby^2$
$= ax(bx - ay) - by(bx - ay)$
$= (bx - ay)(ax - by)$

(vii) $m - 1 - (m - 1)^2 + am - a$
$= (m - 1) - (m - 1)^2 + a(m - 1)$
$= (m - 1)(1 - (m - 1) + a)$
$= (m - 1)(1 - m + 1 + a)$
$= (m - 1)(2 - m + a)$

Question 3.
Factorise :
(i) $a^2 - (b - c)^2$ (ii) $25(2x - y)^2 - 16(x - 2y)^2$

(iii) $16(5x + 4)^2 - 9(3x - 2)^2$ (iv) $9x^2 - \dfrac{1}{16}$

(v) $25(x - 2y)^2 - 4$
Solution:

(i) $a^2 - (b - c)^2$

$= (a - (b - c)) (a + b - c)$

$\qquad [a^2 - b^2 = (a - b) (a + b)]$

$= (a - b + c) (a + b - c)$

(ii) $25 (2x - y)^2 - 16 (x - 2y)^2$

$= (5 (2x - y))^2 - (4 (x - 2y))^2$

$= [5(2x - y) - 4 (x - 2y)] [5 (2x - y) + 4 (x - 2y)$

$= [10x - 5y - 4x + 8y] [10x - 5y + 4x - 8y]$

$\qquad [a^2 - b^2 = (a - b) (a + b)]$

$= (6x + 3y) (14x - 13y) = 3 (2x + y) (14x - 13y)$

(iii) $16 (5x + 4)^2 - 9 (3x - 2)^2$

$= (4 (5x + y))^2 - (3 (3x - 2))^2$

$= [4 (5x + 4) - 3 (3x - 2)] [4 (5x + 4) + 3 (3x - 2)]$

$\qquad [a^2 - b^2 = (a - b) (a + b)]$

$= (20x + 16 - 9x + 6] [20x + 16 + 9x - 6]$

$= [11x + 22] (29x + 10) = 11 (x + 2) (29x + 10)$

(iv) $9x^2 - \dfrac{1}{16} = (3x)^2 - \left(\dfrac{1}{4}\right)^2$

$= \left(3x - \dfrac{1}{4}\right)\left(3x + \dfrac{1}{4}\right) \quad [(a^2 - b^2) = (a - b) (a + b)]$

(v) $25(x - 2y)^2 - 4 = (5 (x - 2y))^2 - 2^2$

$= [5 (x - 2y) - 2] [5 (x - 2y) + 2]$

$\qquad [a^2 - b^2 = (a - b) (a + b)]$

$= (5x - 10y - 2) (5x - 10y + 2)$

Question 4.

Factorise :

(i) $a^2 - 23a + 42$ (ii) $a^2 - 23a - 108$

(iii) $1 - 18x - 63x^2$ (iv) $5x^2 - 4xy - 12y^2$

(v) $x (3x + 14) + 8$ (vi) $5 - 4x (1 + 3x)$

(vii) $x^2 y^2 - 3xy - 40$

(viii) $(3x - 2y)^2 - 5 (3x - 2y) - 24$

(ix) $12 (a + b)^2 - (a + b) - 35$

Solution:

(i) $a^2 - 23a + 42$

                 [42 = 21 × 2 and 21 + 2 = 23]

$= a^2 - 21a - 2a + 42 = a(a - 21) - 2(a - 21)$

$= (a - 21)(a - 2)$

(ii) $a^2 - 23a - 108$

$= a^2 - 27a + 4a - 108$

                 [27 × 4 = 108 and 27 − 4 = 23]

$= a(a - 27) + 4(a - 27)$

$= (a - 27)(a + 4)$

(iii) $1 - 18x - 63x^2 = 1 - 21x + 3x - 63x^2$

$= 1(1 - 21x) + 3x(1 - 21x)$

$= (1 - 21x)(1 + 3x)$

(iv) $5x^2 - 4xy - 12y^2 = 5x^2 - 10xy + 6xy - 12y^2$

$= 5x(x - 2y) + 6y(x - 2y) = (x - 2y)(5x + 6y)$

(v) $x(3x + 14) + 8 = 3x^2 + 14x + 8$

$= 3x^2 + 12x + 2x + 8 = 3x(x + 4) + 2(x + 4)$

$= (x + 4)(3x + 2)$

(vi) $5 - 4x(1 + 3x) = 5 - 4x - 12x^2$

$= 5 - 10x + 6x - 12x^2 = 5(1 - 2x) + 6x(1 - 2x)$

$= (1 - 2x)(5 + 6x)$

(vii) $x^2 y^2 - 3xy - 40 = x^2 y^2 - 8xy + 5xy - 40$

 $= xy(xy - 8) + 5(xy - 8) = (xy - 8)(xy + 5)$

(viii) $(3x - 2y)^2 - 5(3x - 2y) - 24$

$= (3x - 2y)^2 - 8(3x - 2y) + 3(3x - 2y) - 24$

$= (3x - 2y)(3x - 2y - 8) + 3(3x - 2y - 8)$

$= (3x - 2y - 8)(3x - 2y + 3)$

(ix) $12(a + b)^2 - (a + b) - 35$

$= 12(a + b)^2 - 21(a + b) + 20(a + b) - 35$

$= 3(a + b)[4(a + b) - 7] + 5[4(a + b) - 7]$

$= (4a + 4b - 7)(3a + 3b + 5)$

Question 5.
Factorise :

(i) $15 (5x - 4)^3 - 10 (5x - 4)$

(ii) $3a^2 x - bx + 3a^2 - b$

(iii) $b (c - d)^2 + a (d - c) + 3 (c - d)$

(iv) $ax^2 + b^2y - ab^2 - x^2y$

(v) $1 - 3x - 3y - 4 (x + y)^2$

Solution:

(i) $15 (5x - 4)^3 - 10 (5x - 4) = 5 (5x - 4) [3 (5x - 4)^2 - 2]$

$= 5 (5x - 4) [3 (25x^2 - 40x + 16) - 2]$

$= 5(5x - 4) (75x^2 - 120x + 46)$

(ii) $3a^2 x - bx + 3a^2 - b$

$= x (3a^2 - b) + 1 (3a^2 - b) = (x + 1) (3a^2 - b)$

(iii) $b (c - d)^2 + a (d - c) + 3 (c - d)$

$= b (c - d)^2 - a (c - d) + 3 (c - d)$

$= (c - d) [b (c - d) - a + 3]$

$= (c - d) (bc - bd - a + 3)$

(iv) $ax^2 + b^2y - ab^2 - x^2y$

$= ax^2 - ab^2 + b^2y - x^2 y = a (x^2 - b^2) + y (b^2 - x^2)$

$= a (x^2 - b^2) - y (x^2 - b^2) = (x^2 - b^2) (a - y)$

$= (x - b) (x + b) (a - y)$

(v) $1 - 3x - 3y - 4 (x + y)^2$

$= 1 - 3 (x + y) - 4 (x + y)^2$

$= 1 - 4 (x + y) + (x + y) - 4 (x + y)^2$

$= 1[1 - 4 (x + y)] + (x + y) [1 - 4 (x + y)]$

$= [1 - 4x - 4y] (1 + x + y)$

Question 6.
Factorise :

(i) $2a^3 - 50a$ (ii) $54a^2 b^2 - 6$

(iii) $64a^2 b - 144b^3$ (iv) $(2x - y)^3 - (2x - y)$

(v) $x^2 - 2xy + y^2 - z^2$ (vi) $x^2 - y^2 - 2yz - z^2$

(vii) $7a^5 - 567a$ (viii) $5x^2 - \dfrac{20x^4}{9}$

Solution:

(i) $2a^3 - 50a = 2a\,(a^2 - 25) = 2a\,(a^2 - 5^2)$

$= 2a\,(a - 5)\,(a + 5)$

(ii) $54a^2\,b^2 - 6 = 6\,(9a^2\,b^2 - 1) = 6\,[(3ab)^2 - (1)^2]$

$= 6\,(3ab - 1)\,(3ab + 1)$

(iii) $64a^2\,b - 144b^3$

$= 16b\,(4a^2 - 9b^2) = 16b\,[(2a)^2 - (3b)^2]$

$= 16b\,(2a + 3b)\,(2a - 3b)$

(iv) $(2x - y)^3 - (2x - y) = (2x - y)\,[(2x - y)^2 - 1]$

$= (2x - y)\,(2x - y - 1)\,(2x - y + 1)$

(v) $x^2 - 2xy + y^2 - z^2$

$= (x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2$

$= (x - y - z)\,(x - y + z)$

(vi) $x^2 - y^2 - 2yz - z^2 = x^2 - (y^2 + 2yz + z^2)$

$= x^2 - (y + z)^2 = (x - y - z)\,(x + y + z)$

(vii) $7a^5 - 567a = 7a\,(a^4 - 81)$

$= 7a\,(a^2)^2 - (9)^2$

$= 7a\,(a^2 + 9)\,(a^2 - 9)$

$= 7a\,(a^2 + 9)\,((a)^2 - (3)^2)$

$= 7a\,(a^2 + 9)\,(a + 3)\,(a - 3)$

(viii) $5x^2 - \dfrac{20x^4}{9} = 5x^2\left[1 - \dfrac{4x^2}{9}\right]$

$= 5x^2\left[1 - \dfrac{2x}{3}\right]\left[1 + \dfrac{2x}{3}\right]$

Question 7.

Factorise $xy^2 - xz^2$, Hence, find the value of:

(i) $9 \times 8^2 - 9 \times 2^2$ (ii) $40 \times 5.5^2 - 40 \times 4.5^2$

Solution:

$$xy^2 - xz^2 = x(y^2 - z^2) = x(y - z)(y + z)$$

(i) $9 \times 8^2 - 9 \times 2^2 = 9(8^2 - 2^2)$

$= 9(8 - 2)(8 + 2) = 9(6)(10) = 540$

(ii) $40 \times 5.5^2 - 40 \times 4.5^2 = 40(5.5)^2 - (4.5^2)$

$= 40(5.5 - 4.5)(5.5 + 4.5)$

$= 40(1)(10) = 400$

Question 8.
Factorise :

(i) $(a - 3b)^2 - 36 b^2$

(ii) $25(a - 5b)^2 - 4(a - 3b)^2$

(iii) $a^2 - 0.36 b^2$

(iv) $a^4 - 625$

(v) $x^4 - 5x^2 - 36$

(vi) $15(2x - y)^2 - 16(2x - y) - 15$

Solution:

(i) $(a - 3b)^2 - 36 b^2$

$= (a - 3b)^2 - (6b)^2$

$= (a - 3b + 6b)(a - 3b - 6b)$

$= (a + 3b)(a - 9b)$
$\qquad \left\{ \begin{array}{l} a^2 - b^2 \\ = (a + b)(a - b) \end{array} \right\}$

(ii) $25(a - 5b)^2 - 4(a - 3b)^2$

$= [5(a - 5b)]^2 - [2(a - 3b)]^2$

$= (5a - 25b)^2 - (2a - 6b)^2$

$= (5a - 25b + 2a - 6b)(5a - 25b - 2a + 6b)$

$\qquad \{a^2 - b^2 = (a + b)(a - b)\}$

$= (7a = 31b)(3a - 19b)$

(iii) $a^2 - 0.36\ b^2$
$= (a)^2 - (0.6b)^2$
$= (a + 0.6b)\ (a - 0.6b)$
$$\{a^2 - b^2 = (a + b)\ (a - b)\}$$

(iv) $a^4 - 625 = (a^2)^2 - (25)^2$
$= (a^2 + 25)\ (a^2 - 25)$
$$\{a^2 - b^2 = (a + b)\ (a - b)\}$$
$= (a^2 + 25)\ \{(a)^2 - (5)^2\}$
$= (a^2 + 25)\ (a + 5)\ (a - 5)$

(v) $x^4 - 5x^2 - 36$
$= (x^2)^2 - 5x^2 - 36$
$= (x^2)^2 - 9x^2 + 4x^2 - 36 \quad \begin{bmatrix} \because -36 = -9 \times 4 \\ -5 = -9 + 4 \end{bmatrix}$
$= x^2\ (x^2 - 9) + 4\ (x^2 - 9)$
$= (x^2 - 9)\ (x^2 + 4)$
$= \{x^2 - (3)^2\}\ \{x^2 + 4\}$
$= (x + 3)\ (x - 3)\ (x^2 + 4)$
$= (x^2 + 4)\ (x + 3)\ (x - 3)$

(vi) $15\ (2x - y)^2 - 16\ (2x - y) - 15$
Let $2x - y = a$, then
$15a^2 - 16a - 15$
$= 15a^2 - 25a + 9a - 15$
$$\begin{bmatrix} \because 15 \times (-15) = -225 \\ \therefore -225 = -25 \times 9 \\ -16 = -25 + 9 \end{bmatrix}$$
$= 5a\ (3a - 5) + 3\ (3a - 5)$
$= (3a - 5)\ (5a + 3)$
$= [3\ (2x - y) - 5]\ [5\ (2x - y) + 3]$
$= (6x - 3y - 5)\ (10x - 5y + 3)$

Question 9.
Factorise $a^2b - b^3$ Using this result, find the value of $101^2 \times 100 - 100^3$.

Solution:

$$a^2b - b^3$$
$$b\,(a^2 - b^2)$$
$$b\,(a + b)\,(a - b)$$

Now,

$$101^2 \times 100 - 100^3$$
$$= 100\,(101^2 - 100^2)$$
$$= 100\,(101 + 100)\,(101 - 100)$$
$$= 100(201)\,(1)$$
$$= 20100$$

Question 10.
Evaluate (using factors): $301^2 \times 300 - 300^3$.
Solution:

$$301^2 \times 300 - 300^3$$
$$= 300(301^2 - 300^2)$$
$$= 300(301 + 300)\,(301 - 300)$$
$$= 300\,(601)\,(1)$$
$$= 180300$$