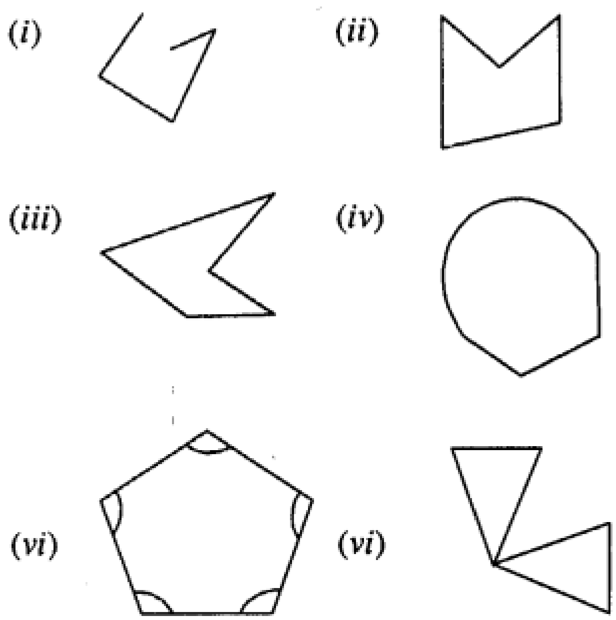
16. Understanding Shapes

(Including Polygons)

EXERCISE 16(A)

Question 1.
State which of the following are polygons :



If the given figure is a polygon, name it as convex or concave.
Solution:
Only Fig. (ii), (iii) and (v) are polygons.
Fig. (ii) and (iii) are concave polygons while
Fig. (v) is convex.

Question 2.
Calculate the sum of angles of a polygon with :
(i) 10 sides
(ii) 12 sides
(iii) 20 sides
(iv) 25 sides
Solution:
(i) No. of sides n = 10
sum of angles of polygon = $(n – 2) \times 180°$
$= (10 – 2) \times 180° = 1440°$
(ii) no. of sides n = 12
sum of angles = $(n – 2) \times 180°$
$= (12 – 2) \times 180° = 10 \times 180° = 1800°$

(iii) n = 20
Sum of angles of Polygon = (n – 2) x 180°
= (20 – 2) x 180° = 3240°
(iv) n = 25
Sum of angles of polygon = (n – 2) x 180°
= (25 – 2) x 180° = 4140°

Question 3.
Find the number of sides in a polygon if the sum of its interior angles is :
(i) 900°
(ii) 1620°
(iii) 16 right-angles
(iv) 32 right-angles.
Solution:
(i) Let no. of sides = n
Sum of angles of polygon = 900°
(n – 2) x 180° = 900°
n – 2 = $\frac{900}{180}$
n – 2 = 5
n = 5 + 2
n = 7
(ii) Let no. of sides = n
Sum of angles of polygon = 1620°
(n – 2) x 180° = 1620°
n – 2 = $\frac{1620}{180}$
n – 2 = 9
n = 9 + 2
n = 11
(iii) Let no. of sides = n
Sum of angles of polygon = 16 right angles = 16 x 90 = 1440°
(n – 2) x 180° = 1440°
n – 2 = $\frac{1440}{180}$
n – 2 = 8
n = 8 + 2
n = 10
(iv) Let no. of sides = n
Sum of angles of polygon = 32 right angles = 32 x 90 = 2880°
(n – 2) x 180° = 2880
n – 2 = $\frac{2880}{180}$
n – 2 = 16
n = 16 + 2
n = 18

Question 4.
Is it possible to have a polygon ; whose sum of interior angles is :
(i) 870°
(ii) 2340°
(iii) 7 right-angles
(iv) 4500°
Solution:
(i) Let no. of sides = n
Sum of angles = 870°
(n – 2) x 180° = 870°
n – 2 = $\frac{870}{180}$
n – 2 = $\frac{29}{6}$
n = $\frac{29}{6}$ + 2
n = $\frac{41}{6}$
Which is not a whole number.
Hence it is not possible to have a polygon, the sum of whose interior angles is 870°
(ii) Let no. of sides = n
Sum of angles = 2340°
(n – 2) x 180° = 2340°
n – 2 = $\frac{2340}{180}$
n – 2 = 13
n = 13 + 2 = 15
Which is a whole number.
Hence it is possible to have a polygon, the sum of whose interior angles is 2340°.
(iii) Let no. of sides = n
Sum of angles = 7 right angles = 7 x 90 = 630°
(n – 2) x 180° = 630°
n – 2 = $\frac{630}{180}$
n – 2 = $\frac{7}{2}$
n = $\frac{7}{2}$ + 2
n = $\frac{11}{2}$
Which is not a whole number. Hence it is not possible to have a polygon, the sum of whose interior angles is 7 right-angles.
(iv) Let no. of sides = n
(n – 2) x 180° = 4500°
n – 2 = $\frac{4500}{180}$
n – 2 = 25
n = 25 + 2
n = 27
Which is a whole number.
Hence it is possible to have a polygon, the sum of whose interior angles is 4500°.

Question 5.
(i) If all the angles of a hexagon are equal ; find the measure of each angle.
(ii) If all the angles of a 14-sided figure are equal ; find the measure of each angle.
Solution:
(i) No. of sides of hexagon, n = 6
Let each angle be = x°
Sum of angles = 6x°
(n – 2) x 180° = Sum of angles
(6 – 2) x 180° = 6x°
4 x 180 = 6x

$$x = \frac{4 \times 180}{6}$$

$$x = 120°$$

$\therefore$ Each angle of hexagon = 120° **Ans.**

(*ii*) No. of sides of polygon, $n = 14$

Let each angle = $x°$

$\therefore$ Sum of angles = $14x°$

$\therefore$ $(n-2) \times 180°$ = Sum of angles of polygon

$$\therefore \quad (14-2) \times 180° = 14x$$

$$12 \times 180° = 14x$$

$$x = \frac{12 \times 180}{14}$$

$$x = \frac{1080}{7}$$

$$x = \left(154\frac{2}{7}\right)° \quad \textbf{Ans.}$$

Question 6.
Find the sum of exterior angles obtained on producing, in order, the sides of a polygon with :
(i) 7 sides
(ii) 10 sides
(iii) 250 sides.
Solution:
(i) No. of sides n = 7
Sum of interior & exterior angles at one vertex = 180°

$$\text{Sum of all interior \& exterior angles} = 7 \times 180^\circ$$
$$= 1260^\circ$$
$$\text{Sum of interior angles} = (n-2) \times 180^\circ$$
$$= (7-2) \times 180^\circ$$
$$= 900^\circ$$
$$\therefore \text{Sum of exterior angles} = 1260^\circ - 900^\circ$$
$$= 360^\circ \ \textbf{Ans.}$$

(*ii*) No. of sides $n = 10$

$$\text{Sum of interior and exterior angles} = 10 \times 180^\circ$$
$$= 1800^\circ$$
$$\text{But sum of interior angles} = (n-2) \times 180^\circ$$
$$= (10-2) \times 180^\circ$$
$$= 1440^\circ$$
$$\therefore \text{Sum of exterior angles} = 1800 - 1440$$
$$= 360^\circ \ \textbf{Ans.}$$

(*iii*) No. of side $n = 250$

Sum of all interior and exterior angles

$$= 250 \times 180^\circ$$
$$= 45000^\circ$$
$$\text{But sum of interior angles} = (n-2) \times 180^\circ$$
$$= (250-2) \times 180^\circ$$
$$= 248 \times 180^\circ$$
$$= 44640^\circ$$
$$\therefore \text{Sum of exterior angles} = 45000 - 44640$$
$$= 360^\circ$$

Question 7.

The sides of a hexagon are produced in order. If the measures of exterior angles so obtained are $(6x - 1)^\circ$, $(10x + 2)^\circ$, $(8x + 2)^\circ$ $(9x - 3)^\circ$, $(5x + 4)^\circ$ and $(12x + 6)^\circ$; find each exterior angle.

Solution:

Sum of exterior angles of hexagon formed by producing sides of order = 360°

$$\therefore\ (6x-1)^\circ+(10x+2)^\circ+(8x+2)^\circ+(9x-3)^\circ$$
$$+\ (5x+4)^\circ+(12x+6)^\circ = 360^\circ$$
$$50x+10^\circ = 360^\circ$$
$$50x = 360^\circ-10^\circ$$
$$50x = 350^\circ$$
$$x = \frac{350}{50}$$
$$x = 7$$

$\therefore$ Angles are

$(6x-1)^\circ$; $(10x+2)^\circ$; $(8x+2)^\circ$; $(9x-3)^\circ$; $(5x+4)^\circ$ and $(12x+6)^\circ$

i.e. $(6\times7-1)^\circ$; $(10\times7+2)^\circ$; $(8\times7+2)^\circ$; $(9\times7-3)^\circ$; $(5\times7+4)^\circ$; $(12\times7+6)^\circ$

i.e. 41° ; 72°, 58° ; 60° ; 39° and 90°

Question 8.

The interior angles of a pentagon are in the ratio 4 : 5 : 6 : 7 : 5. Find each angle of the pentagon.

Solution:

Let the interior angles of the pentagon be 4x, 5x, 6x, 7x, 5x.

Their sum = 4x + 5x + 6x + 7x + 5x = 21x

Sum of interior angles of a polygon = $(n-2)\times 180^\circ = (5-2)\times 180^\circ = 540^\circ$

$$\therefore\ 27x = 540 \quad \Rightarrow\ x = \frac{540}{27} \quad \Rightarrow\ x = 20^\circ$$

$\therefore$ Angles are $4\times 20^\circ = 80^\circ$

$5\times 20^\circ = 100^\circ$

$6\times 20^\circ = 120^\circ$

$7\times 20^\circ = 140^\circ$

$5\times 20 = 100^\circ$

Question 9.

Two angles of a hexagon are 120° and 160°. If the remaining four angles are equal, find each equal angle.

Solution:

Two angles of a hexagon are 120°, 160°

Let remaining four angles be x, x, x and x.
Their sum = 4x + 280°
But sum of all the interior angles of a hexagon

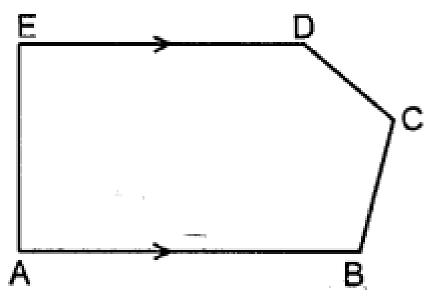
$$= (6 - 2) \times 180°$$
$$= 4 \times 180° = 720°$$

$\therefore \quad 4x + 280° = 720°$

$\Rightarrow \quad 4x = 720° - 280° = 440° \quad \Rightarrow \quad x = 110°$

$\therefore$ Equal angles are 110° (each)

Question 10.
The figure, given below, shows a pentagon ABCDE with sides AB and ED parallel to each other, and ∠B : ∠C : ∠D = 5 : 6 : 7.



(i) Using formula, find the sum of interior angles of the pentagon.
(ii) Write the value of ∠A + ∠E
(iii) Find angles B, C and D.
Solution:
(i) Sum of interior angles of the pentagon

$= (5 - 2) \times 180°$

$= 3 \times 180° = 540°$ [$\because$ sum for a polygon of x sides $= (x - 2) \times 180°$]

(ii) Since AB || ED

$\therefore \angle A + \angle E = 180°$

(iii) Let $\angle B = 5x \quad \angle C = 6x \quad \angle D = 7x$

$\therefore 5x + 6x + 7x + 180° = 540°$

($\angle A + \angle E = 180°$)

Proved in (ii)

$18x = 540° - 180°$

$\Rightarrow \quad 18x = 360° \quad \Rightarrow \quad x = 20°$

$\therefore \angle B = 5 \times 20° = 100°, \angle C = 6 \times 20 = 120°$

$\angle D = 7 \times 20 = 140°$

Question 11.
Two angles of a polygon are right angles and the remaining are 120° each. Find the number of sides in it.
Solution:
Let number of sides = n

$$\begin{aligned}
\text{Sum of interior angles} &= (n-2)\times 180^\circ \\
&= 180n - 360^\circ \\
\text{Sum of 2 right angles} &= 2\times 90^\circ \\
&= 180^\circ \\
\therefore \quad \text{Sum of other angles} &= 180n - 360^\circ - 180^\circ \\
&= 180n - 540
\end{aligned}$$

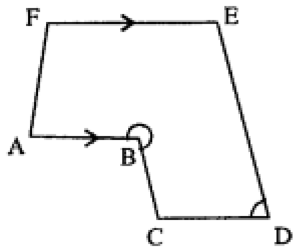
No. of vertices at which these angles are formed

$$= n - 2$$

$$\begin{aligned}
\therefore \quad \text{Each interior angle} &= \frac{180n - 540}{n-2} \\
\therefore \quad \frac{180n - 540}{n-2} &= 120^\circ \\
180n - 540 &= 120n - 240 \\
180n - 120n &= -240 + 540 \\
60n &= 300
\end{aligned}$$

n = $\frac{300}{60}$
n = 5

Question 12.
In a hexagon ABCDEF, side AB is parallel to side FE and ∠B : ∠C : ∠D : ∠E = 6 : 4 : 2 : 3. Find ∠B and ∠D.
Solution:



Given : Hexagon ABCDEF in which AB || EF and $\angle B : \angle C : \angle D : \angle E = 6 : 4 : 2 : 3$.
To find : ∠B and ∠D
Proof : No. of sides $n = 6$
$\therefore$ Sum of interior angles $= (n-2)\times 180^\circ$
$= (6-2)\times 180^\circ = 720^\circ$
$\because$ AB || EF (Given)
$\therefore \angle A + \angle F = 180^\circ$
But $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F = 720^\circ$ (Proved)
$\angle B+\angle C+\angle D+\angle E+\angle 180^\circ = 720^\circ$
$\therefore \angle B+\angle C+\angle D+\angle E = 720^\circ - 180^\circ = 540^\circ$
Ratio $= 6 : 4 : 2 : 3$
Sum of parts $= 6 + 4 + 2 + 3 = 15$

$$\therefore \angle B = \frac{6}{15}\times 540 = 216^\circ$$

$$\angle D = \frac{2}{15}\times 540^\circ = 72^\circ$$

Hence $\angle B = 216^\circ$; $\angle D = 72^\circ$ **Ans.**

Question 13.
the angles of a hexagon are x + 10°, 2x + 20°, 2x – 20°, 3x – 50°, x + 40° and x + 20°. Find x.
Solution:

Sol. Angles of a hexagon are $x + 10°$, $2x + 20°$, $2x - 20°$, $3x - 50°$, $x + 40°$ and $x + 20°$

$\therefore$ But sum of angles of a hexagon $= (x - 2) \times 180°$
$= (6 - 2) \times 180° = 4 \times 180° = 720°$
But sum $= x + 10 + 2x + 20° + 2x - 20° + 3x - 50° + x + 40 + x + 20$
$= 10x + 90 - 70 = 10x + 20$

$\therefore\ 10x + 20 = 720° \Rightarrow 10x = 720 - 20 = 700$

$$\Rightarrow x = \frac{700°}{10} = 70°$$

$\therefore\ x = 70°$

Question 14.

In a pentagon, two angles are 40° and 60°, and the rest are in the ratio 1 : 3 : 7. Find the biggest angle of the pentagon.

Solution:

In a pentagon, two angles are 40° and 60° Sum of remaining 3 angles = 3 x 180°
= 540° – 40° – 60° = 540° – 100° = 440°
Ratio in these 3 angles =1 : 3 : 7
Sum of ratios =1 + 3 + 7 = 11
Biggest angle = $\frac{440 \times 7}{11}$ = 280°

EXERCISE 16(B)

Question 1.

Fill in the blanks :
In case of regular polygon, with :

no. of sides	each exterior angle	each interior angle
(i) ....8....	....	....
(ii) ....12....	....	....
(iii)	.....72°.....	....
(iv)	....45°.....	....
(v)	....	....150°.....
(vi)	....	....140°.....

Solution:

no. of sides	each exterior angle	each interior angle
(i) 8	$45°$	$135°$
(ii) 12	$30°$	$150°$
(iii) 5	$72°$	$108°$
(iv) 8	$45°$	$135°$
(v) 12	$30°$	$150°$
(vi) 9	$40°$	$140°$

Explanation

(i) Each exterior angle $= \dfrac{360°}{8} = 45°$

Each interior angle $= 180° - 45° = 135°$

(ii) Each exterior angle $= \dfrac{360°}{12} = 30°$

Each interior angle $= 180° - 30° = 150°$

(iii) Since each exterior $= 72°$

$\therefore$ Number of sides $= \dfrac{360°}{72°} = 5$

Also interior angle $= 180° - 72° = 108°$

(iv) Since each exterior angle $= 45°$

$\therefore$ Number of sides $= \dfrac{360°}{45°} = 8$

Interior angle $= 180° - 45° = 135°$

(v) Since interior angle $= 150°$

$\therefore$ Exterior angle $= 180° - 150° = 30°$

$\therefore$ Number of sides $= \dfrac{360°}{30°} = 12$

(vi) Since interior angle $= 140°$

$\therefore$ Exterior angle $= 180° - 140° = 40°$

$\therefore$ Number of sides $= \dfrac{360°}{40°} = 9$

Question 2.
Find the number of sides in a regular polygon, if its each interior angle is :
(i) 160°
(ii) 135°
(iii) $1\frac{1}{5}$ of a right-angle
Solution:

(*i*) Let no. of sides of regular polygon be n.

Each interior angle $= 160°$

$$\therefore \quad \frac{(n-2)}{n} \times 180° = 160°$$

$$180n - 360° = 160n$$

$$180n - 160n = 360°$$

$$20n = 360°$$

$$n = 18 \text{ Ans.}$$

(*ii*) No. of sides $= n$

Each interior angle $= 135°$

$$\frac{(n-2)}{n} \times 180° = 135°$$

$$180n - 360° = 135n$$

$$180n - 135n = 360°$$

$$45n = 360°$$

$$n = 8 \text{ Ans.}$$

(*iii*) No. of sides $= n$

Each interior angle $= 1\frac{1}{5}$ right angles

$$= \frac{6}{5} \times 90$$

$$= 108°$$

$$\therefore \quad \frac{(n-2)}{n} \times 180° = 108°$$

$$180n - 360° = 108n$$

$$180n - 108n = 360°$$

$$72n = 360°$$

$$n = 5 \quad \text{Ans.}$$

Question 3.
Find the number of sides in a regular polygon, if its each exterior angle is :
(i) $\frac{1}{3}$ of a right angle
(ii) two-fifth of a right-angle.
Solution:
(i) Each exterior angle = $\frac{1}{3}$ of a right angle
= $\frac{1}{3}$ x 90

$$\therefore \quad \frac{360°}{n} = 30°$$

$$\therefore \quad n = \frac{360°}{30°}$$

$$n = 12 \textbf{ Ans.}$$

(ii) Each exterior angle $= \frac{2}{5}$ of a right-angle

$$= \frac{2}{5} \times 90°$$

$$= 36°$$

Let number of sides $= n$

$$\therefore \quad \frac{360°}{n} = 36°$$

$$n = \frac{360°}{36°}$$

$$n = 10 \quad \textbf{Ans.}$$

Question 4.
Is it possible to have a regular polygon whose each interior angle is :
(i) 170°
(ii) 138°
Solution:
(i) No. of sides = n
each interior angle = 170°

$$\therefore \quad \frac{(n-2)}{n} \times 180° = 170°$$

$$180n - 360° = 170n$$

$$180n - 170n = 360°$$

$$10n = 360°$$

$$n = \frac{360°}{10}$$

$$n = 36$$

which is a whole number.

Hence it is possible to have a regular polygon whose interior angle is 170°.

(*ii*) Let no. of sides $= n$

each interior angle $= 138°$

$$\therefore \quad \frac{(n-2)}{n} \times 180° = 138°$$

$$180n - 360° = 138n$$

$$180n - 138n = 360°$$

$$42n = 360°$$

$$n = \frac{360°}{42}$$

$$n = \frac{60°}{7}$$

Which is not a whole number.

Hence it is not possible to have a regular polygon having interior angle of 138°.

Question 5.

Is it possible to have a regular polygon whose each exterior angle is :

(i) 80°

(ii) 40% of a right angle.

Solution:

(i) Let no. of sides = n each exterior angle = 80°

$$\frac{360°}{n} = 80°$$

$$n = \frac{360°}{80°}$$

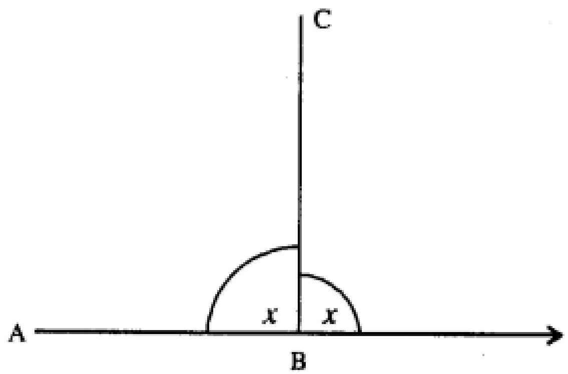
$$n = \frac{9}{2}$$

Which is not a whole number.
Hence it is not possible to have a regular polygon whose each exterior angle is of 80°.
(ii) Let number of sides = n
Each exterior angle = 40% of a right angle

$$= \frac{40}{100} \times 90$$
$$= 36°$$
$$n = \frac{360°}{36°}$$
$$n = 10$$

Which is a whole number.
Hence it is possible to have a regular polygon whose each exterior angle is 40% of a right angle.

Question 6.
Find the number of sides in a regular polygon, if its interior angle is equal to its exterior angle.
Solution:
Let each exterior angle or interior angle be = x°

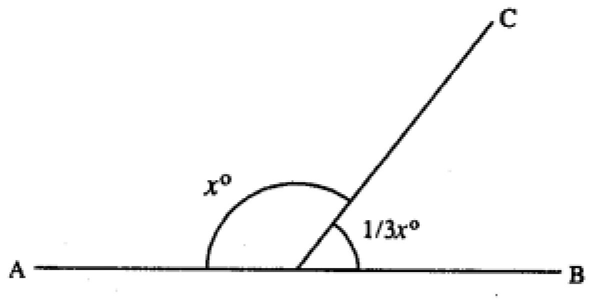


$$\therefore \quad x + x = 180°$$
$$2x = 180°$$
$$x = 90°$$

Now, let no. of sides = n

$$\because \quad \text{each exterior angle} = \frac{360°}{n}$$
$$\therefore \quad 90° = \frac{360°}{n}$$
$$n = \frac{360°}{90°}$$
$$n = 4 \quad \textbf{Ans.}$$

Question 7.
The exterior angle of a regular polygon is one-third of its interior angle. Find the number of sides in the polygon.
Solution:
Let interior angle = x°
Exterior angle = $\frac{1}{3}$ x°



$$\therefore \quad x+\frac{1}{3}x = 180°$$
$$3x+x = 540$$
$$4x = 540$$
$$x = \frac{540}{4}$$
$$x = 135°$$
$$\therefore \quad \text{Exterior angle} = \frac{1}{3}\times 135°$$
$$= 45°$$
Let no. of sides = n
$$\because \quad \text{each exterior angle} = \frac{360°}{n}$$
$$\therefore \quad 45° = \frac{360°}{n}$$
$$\therefore \quad n = \frac{360°}{45°}$$
$$n = 8 \text{ Ans.}$$

Question 8.
The measure of each interior angle of a regular polygon is five times the measure of its exterior angle. Find :
(i) measure of each interior angle ;
(ii) measure of each exterior angle and
(iii) number of sides in the polygon.
Solution:
Let exterior angle = x°

Interior angle = 5x°
x + 5x = 180°
6x = 180°
x = 30°
Each exterior angle = 30°
Each interior angle = 5 x 30° = 150°
Let no. of sides = n

$$\because \quad \text{each exterior angle} = \frac{360°}{n}$$

$$30° = \frac{360°}{n}$$

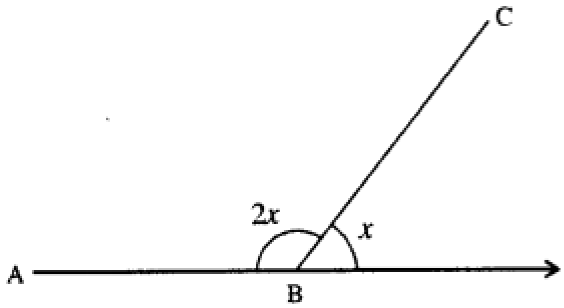
$$n = \frac{360°}{30°}$$

$$n = 12$$

Hence (*i*) 150° (*ii*) 30° (*iii*) 12

Question 9.
The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find :
(i) each exterior angle of the polygon ;
(ii) number of sides in the polygon
Solution:
Interior angle : exterior angle = 2 : 1

Let interior angle = 2x° & exterior angle = x°



$\therefore \quad 2x° + x° = 180°$

$3x = 180°$

(*i*) $\quad x = 60°$

$\therefore$ Each exterior angle = $60°$

Let no. of sides = n

$$\therefore \quad \frac{360°}{n} = 60°$$

$$n = \frac{360°}{60°}$$

(*ii*) $\quad n = 6$

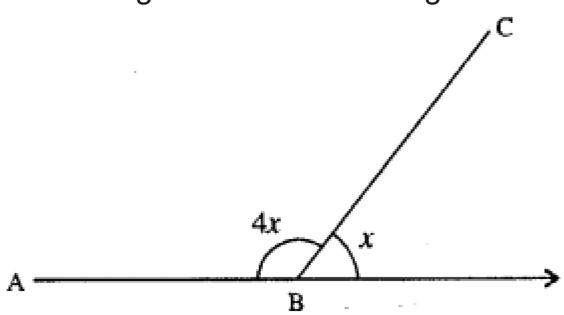
$\therefore$ (*i*) $60°$ (*ii*) 6 **Ans.**

Question 10.

The ratio between the exterior angle and the interior angle of a regular polygon is 1 : 4. Find the number of sides in the polygon.

Solution:

Let exterior angle = x° & interior angle = 4x°



$\therefore \quad 4x + x = 180°$

$5x = 180°$

$x = 36°$

$\therefore$ Each exterior angle = $36°$

Let no. of sides = n

$$\therefore \quad \frac{360°}{n} = 36°$$

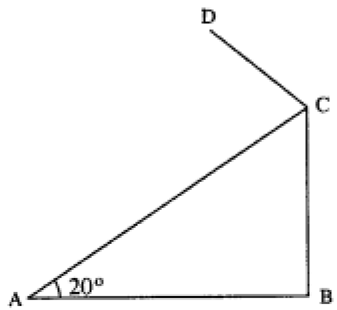
$$n = \frac{360°}{36}$$

$n = 10$ **Ans.**

Question 11.
The sum of interior angles of a regular polygon is twice the sum of its exterior angles. Find the number of sides of the polygon.
Solution:
Let number of sides = n
Sum of exterior angles = 360°
Sum of interior angles = 360° x 2 = 720°
Sum of interior angles = (n – 2) x 180°
720° = (n – 2) x 180°
n – 2 = $\frac{720}{180}$
n – 2 = 4
n = 4 + 2
n = 6

Question 12.
AB, BC and CD are three consecutive sides of a regular polygon. If angle BAC = 20° ; find :
(i) its each interior angle,
(ii) its each exterior angle
(iii) the number of sides in the polygon.

Solution:



$\because$ Polygon is regular (Given)

$\therefore$ $AB = BC$

$\Rightarrow$ $\angle BAC = \angle BCA$

[$\angle s$ opp. to equal sides]

But $\angle BAC = 20°$

$\therefore$ $\angle BCA = 20°$

i.e. In ΔABC,

$$\angle B + \angle BAC + \angle BCA = 180°$$
$$\angle B + 20° + 20° = 180°$$
$$\angle B = 180° - 40°$$
$$\angle B = 140°$$

(*i*) each interior angle $= 140°$

(*ii*) each exterior angle $= 180° - 140°$

$= 40°$

(*iii*) Let no. of sides $= n$

$\therefore$ $\dfrac{360°}{n} = 40°$

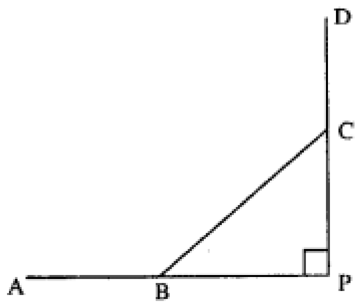
$$n = \frac{360°}{40°} = 9,$$
$$n = 9$$

$\therefore$ (*i*) 140° (*ii*) 9 **Ans.**

Question 13.

Two alternate sides of a regular polygon, when produced, meet at the right angle. Calculate the number of sides in the polygon.

Solution:



Let number of sides of regular polygon $= n$

AB & DC when produced meet at P such that $\angle P = 90°$

$\because$ Interior angles are equal.

$$\therefore \quad \angle ABC = \angle BCD$$

$$\therefore \quad 180° - \angle ABC = 180° - \angle BCD$$

$$\therefore \quad \angle PBC = \angle BCP$$

$$\text{But} \quad \angle P = 90° \quad \text{(Given)}$$

$$\therefore \quad \angle PBC + \angle BCP = 180° - 90° = 90°$$

$$\therefore \quad \angle PBC = \angle BCP$$

$$= \frac{1}{2} \times 90° = 45°$$

$\therefore$ Each exterior angle $= 45°$

$$\therefore \quad 45° = \frac{360°}{n}$$

$$n = \frac{360°}{45°}$$

$$n = 8 \quad \textbf{Ans.}$$

Question 14.

In a regular pentagon ABCDE, draw a diagonal BE and then find the measure of:

(i) ∠BAE

(ii) ∠ABE

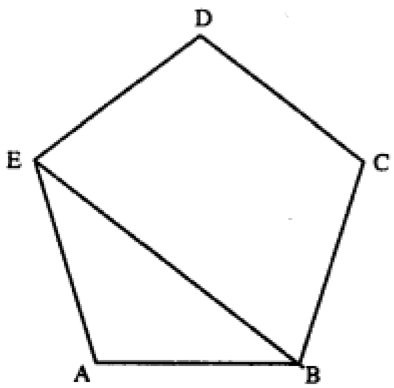
(iii) ∠BED

Solution:

(i) Since number of sides in the pentagon = 5

Each exterior angle = $\frac{360}{5}$ = 72°

$\angle BAE = 180° – 72°= 108°$



(*ii*) In ΔABE, $AB = AE$

$\because \therefore \angle ABE = \angle AEB$

But $\angle BAE + \angle ABE + \angle AEB = 180°$

$\therefore 108° + 2\angle ABE = 180° - 108° = 72°$

$\Rightarrow \angle ABE = 36°$

(iii) Since $\angle AED = 108°$

[$\because$ each interior angle $= 108°$]

$\Rightarrow \angle AEB = 36°$

$\Rightarrow \angle BED = 108° - 36° = 72°$

Question 15.

The difference between the exterior angles of two regular polygons, having the sides equal to (n – 1) and (n + 1) is 9°. Find the value of n.

Solution:

We know that sum of exterior angles of a polynomial is 360°

(i) If sides of a regular polygon = n – 1

Then each angle $= \dfrac{360°}{n-1}$

and if sides are $n + 1$, then

each angle $= \dfrac{360°}{n+1}$

According to the condition,

$$\frac{360°}{n-1} - \frac{360°}{n+1} = 9$$

$$\Rightarrow 360\left[\frac{1}{x-1} - \frac{1}{x+1}\right] = 9$$

$$\Rightarrow 360\left[\frac{n+1-n+1}{(n-1)(n+1)}\right]=9$$

$$\Rightarrow \frac{2\times 360}{n^2-1}=9 \Rightarrow n^2-1=\frac{2\times 360}{9}=80$$

$\Rightarrow n^2 - 1 = 80 \Rightarrow n^2 = 1 - 80 = 0$

$\Rightarrow n^2 - 81 = 0$

$\Rightarrow (n)^2 - (9)^2 = 0$

$\Rightarrow (n + 9)(n - 9) = 0$

Either $n + 9 = 0$, then $n = -9$ which is not possible being negative,

or $n - 9 = 0$, then $n = 9$

$\therefore n = 9$

$\therefore$ No. of sides of a regular polygon = 9

Question 16.

If the difference between the exterior angle of a n sided regular polygon and an (n + 1) sided regular polygon is 12°, find the value of n.

Solution:

We know that sum of exterior angles of a polygon = 360°

Each exterior angle of a regular polygon of 360°

n sides $= \frac{360°}{n}$

and exterior angle of the regular polygon of

$(n + 1)$ sides $= \frac{360°}{n+1}$

$$\therefore \frac{360°}{n}-\frac{360°}{n+1}=12$$

$$\Rightarrow 360\left[\frac{1}{n}-\frac{1}{n+1}\right]=12 \Rightarrow 360\left[\frac{n+1-n}{n(n+1)}\right]=12$$

$$\Rightarrow \frac{30\times 1}{n^2+n}=12 \Rightarrow 12(n^2+n)=360°$$

$\Rightarrow n^2 + n = 36°$ (Dividing by 12)

$\Rightarrow n^2 + n - 30 = 0$

$$\Rightarrow n^2+6n-5n-30=0\left\{\begin{array}{c}\because -30=6\times(-5) \\ 1=6-5\end{array}\right\}$$

$\Rightarrow n(n + 6) - 5(n + 6) = 0$

$\Rightarrow (n + 6)(n - 5) = 0$

Either $n + 6 = 0$, then $n = -6$ which is not possible being negative

or $n - 5 = 0$, then $n = 5$

Hence $n = 5$.

Question 17.
The ratio between the number of sides of two regular polygons is 3 : 4 and the ratio between the sum of their interior angles is 2 : 3. Find the number of sides in each polygon.

Solution:
Ratio of sides of two regular polygons = 3 : 4
Let sides of first polygon = 3n
and sides of second polygon = 4n

Sum of interior angles of first polygon

$= (2 \times 3n - 4) \times 90^\circ = (6n - 4) \times 90^\circ$

and sum of interior angle of second polygon

$= (2 \times 4n - 4) \times 90^\circ = (8n - 4) \times 90^\circ$

$$\therefore \frac{(6n-4)\times 90^\circ}{(8n-4)\times 90^\circ} = \frac{2}{3}$$

$$\Rightarrow \frac{6n-4}{8n-4} = \frac{2}{3}$$

$\Rightarrow 18n - 12 = 16n - 8$

$\Rightarrow 18n - 16n = -8 + 12$

$\Rightarrow 2n = 4$

$\Rightarrow n = 2$

$\therefore$ No. of sides of first polygon

$= 3n = 3 \times 2 = 6$

and no. of sides of second polygon

$= 4n = 4 \times 2 = 8$

Question 18.
Three of the exterior angles of a hexagon are 40°, 51 ° and 86°. If each of the remaining exterior angles is x°, find the value of x.
Solution:
Sum of exterior angles of a hexagon = 4 x 90° = 360°
Three angles are 40°, 51° and 86°
Sum of three angle = 40° + 51° + 86° = 177°
Sum of other three angles = 360° – 177° = 183°
Each angle is x°
3x = 183°
x = $\frac{183}{3}$
Hence x = 61

Question 19.
Calculate the number of sides of a regular polygon, if:
(i) its interior angle is five times its exterior angle.
(ii) the ratio between its exterior angle and interior angle is 2 : 7.
(iii) its exterior angle exceeds its interior angle by 60°.
Solution:
Let number of sides of a regular polygon = n
(i) Let exterior angle = x
Then interior angle = 5x
x + 5x = 180°
=> 6x = 180°

$$\Rightarrow x = \frac{180^\circ}{6} = 30^\circ$$

$$\therefore \text{ Number of sides } (n) = \frac{360^\circ}{30} = 12$$

(ii) Ratio between exterior angle and interior angle = 2 : 7

Let exterior angle = $2x$

Then interior angle = $7x$

$$\therefore 2x + 7x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{9} = 20^\circ$$

$$\therefore \text{ Ext. angle} = 2x = 2 \times 20^\circ = 40^\circ$$

$$\therefore \text{ No. of sides} = \frac{360^\circ}{40} = 9$$

(*iii*) Let interior angle = x

Then exterior angle = $x + 60$

$\therefore\ x + x + 60° = 180°$

$\Rightarrow 2x = 180° - 60° = 120° \quad \Rightarrow x = \frac{120°}{2} = 60°$

$\therefore$ Exterior angle = $60° + 60° = 120°$

$\therefore$ Number of sides = $\frac{360°}{120°} = 3$

Question 20.

The sum of interior angles of a regular polygon is thrice the sum of its exterior angles. Find the number of sides in the polygon.

Solution:

Sum of interior angles = 3 x Sum of exterior angles
Let exterior angle = x
The interior angle = 3x
x + 3x=180°
=> 4x = 180°
=> x = $\frac{180}{4}$
=> x = 45°
Number of sides = $\frac{360}{45}$ = 8

EXERCISE 16(C)

Question 1.
Two angles of a quadrilateral are 89° and 113°. If the other two angles are equal; find the equal angles.

Solution:
Let the other angle = x°
According to given,
89° + 113° + x° + x° = 360°
2x° = 360° – 202°
2x° = 158°
x° = $\frac{158}{2}$
other two angles = 79° each

Question 2.
Two angles of a quadrilateral are 68° and 76°. If the other two angles are in the ratio 5 : 7; find the measure of each of them.

Solution:
Two angles are 68° and 76°
Let other two angles be 5x and 7x
68° + 76°+ 5x + 7x = 360°
12x + 144° = 360°
12x = 360° – 144°
12x = 216°
x = 18°
angles are 5x and 7x
i.e. 5 x 18° and 7 x 18° i.e. 90° and 126°

Question 3.
Angles of a quadrilateral are (4x)°, 5(x+2)°, (7x – 20)° and 6(x+3)°. Find :
(i) the value of x.
(ii) each angle of the quadrilateral.

Solution:
Angles of quadrilateral are,

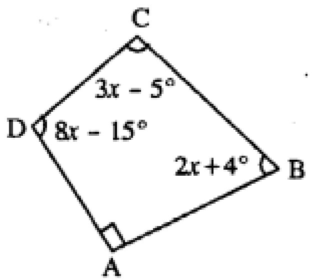
$(4x)°,\ 5(x+2)°,\ (7x-20)°$ and $6(x+3)°$.

$$\therefore\ 4x+5(x+2)+(7x-20)+6(x+3) = 360°$$
$$4x+5x+10+7x-20+6x+18 = 360°$$
$$22x+8 = 360°$$
$$22x = 360°-8°$$
$$22x = 352°$$
$$x = 16° \text{ Ans.}$$

Hence angles are,

$$(4x)° = (4\times16)° = 64°,$$
$$5(x+2)° = 5(16+2)° = 90°,$$
$$(7x-20)° = (7\times16-20)° = 92°$$
$$6(x+3)° = 6(16+3) = 114° \text{ Ans.}$$

Question 4.
Use the information given in the following figure to find :
(i) x
(ii) ∠B and ∠C



Solution:

$\because \angle A = 90°$ (Given)

$\angle B = (2x+4°)$

$\angle C = (3x-5°)$

$\angle D = (8x - 15°)$

$\angle A+\angle B+\angle C+\angle D = 360°$

$90°+(2x+4°) + (3x-5°) + (8x - 15°) = 360°$

$90°+2x+4° + 3x - 5° + 8x - 15° = 360°$

$\Rightarrow 74° + 13x = 360°$

$\Rightarrow 13x = 360° - 74°$

$\Rightarrow 13x = 286°$

$\Rightarrow x = 22°$

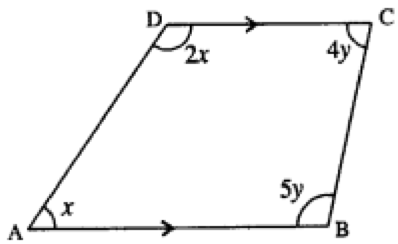
$\because \angle B = 2x + 4 = 2 \times 22° + 4 = 48°$

$\angle C = 3x - 5 = 3\times22° - 5 = 61°$

Hence (*i*) 22° (*ii*) ∠B = 48°, ∠C = 61° **Ans.**

Question 5.
In quadrilateral ABCD, side AB is parallel to side DC. If ∠A : ∠D = 1 : 2 and ∠C : ∠B = 4 : 5
(i) Calculate each angle of the quadrilateral.
(ii) Assign a special name to quadrilateral ABCD

Solution:



$\because$ $\angle A : \angle D = 1 : 2$

Let $\angle A = x$ and $\angle B = 2x$

$\because$ $\angle C : \angle B = 4 : 5$

Let $\angle C = 4y$ and $\angle B = 5y$

$\because$ $AB \parallel DC$

$\therefore$ $\angle A + \angle D = 180°$

$x + 2x = 180°$

$3x = 180°$

$x = 60°$

$\therefore$ $A = 60°$

$\angle D = 2x = 2 \times 60 = 120°$

Again $\angle B + \angle C = 180°$

$5y + 4y = 180°$

$9y = 180°$

$y = 20°$

$\therefore$ $\angle B = 5y = 5 \times 20 = 100°$

$\angle C = 4y = 4 \times 20 = 80°$

Hence $\angle A = 60°$; $\angle B = 100°$; $\angle C = 80°$ and $\angle D = 120°$ **Ans.**

(ii) Quadrilateral ABCD is a trapezium because one pair of opposite side is parallel

Question 6.

From the following figure find ;

(i) x

(ii) ∠ABC

(iii) ∠ACD

Solution:

$$x = \frac{312}{12} = 26^\circ$$

(ii) $\angle ABC = 4x$

$4 \times 26 = 104^\circ$

(iii) $\angle ACD = 180^\circ - 4x - 48^\circ$

$= 180^\circ - 4 \times 26^\circ - 48^\circ$

$= 180^\circ - 104^\circ - 48^\circ$

$= 180^\circ - 152^\circ = 28^\circ$

(i) In Quadrilateral ABCD,
x + 4x + 3x + 4x + 48° = 360°
12x = 360° – 48°
12x = 312

Question 7.

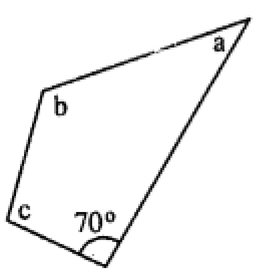
Given : In quadrilateral ABCD ; ∠C = 64°, ∠D = ∠C – 8° ; ∠A = 5(a+2)° and ∠B = 2(2a+7)°.
Calculate ∠A.

Solution:

∠C = 64° (Given)
∠D = ∠C – 8° = 64°- 8° = 56°
∠A = 5(a+2)°
∠B = 2(2a+7)°
Now ∠A + ∠B + ∠C + ∠D = 360°
5(a+2)° + 2(2a+7)° + 64° + 56° = 360°
5a + 10 + 4a + 14° + 64° + 56° = 360°
9a + 144° = 360°
9a = 360° – 144°
9a = 216°
a = 24°
∠A = 5 (a + 2) = 5(24+2) = 130°

Question 8.

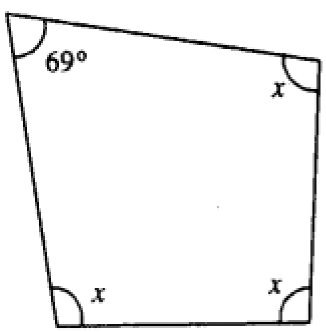
In the given figure : ∠b = 2a + 15 and ∠c = 3a + 5; find the values of b and c.



Solution:

Stun of angles of quadrilateral = 360°
70° + a + 2a + 15 + 3a + 5 = 360°
6a + 90° = 360°
6a = 270°
a = 45°
b = 2a + 15 = 2 x 45 + 15 = 105°
c = 3a + 5 = 3 x 45 + 5 = 140°
Hence ∠b and ∠c are 105° and 140°

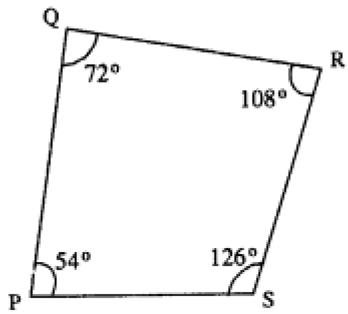
Question 9.
Three angles of a quadrilateral are equal. If the fourth angle is 69°; find the measure of equal angles.
Solution:
Let each equal angle be x°
x + x + x + 69° = 360°



3x = 360°- 69
3x = 291
x = 97°
Each, equal angle = 97°

Question 10.
In quadrilateral PQRS, ∠P : ∠Q : ∠R : ∠S = 3 : 4 : 6 : 7.
Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other
(i) Is PS also parallel to QR ?
(ii) Assign a special name to quadrilateral PQRS.

Solution:



$\because \quad \angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$

Let $\angle P = 3x$

$\angle Q = 4x$

$\angle R = 6x$

& $\angle S = 7x$

$\therefore \angle P+\angle Q+\angle R+\angle S = 360°$

$3x+4x+6x+7x = 360°$

$20x = 360°$

$x = 18°$

$\therefore \quad \angle P = 3x = 3\times18 = 54°$

$\angle Q = 4x = 4\times18 = 72°$

$\angle R = 6x = 6\times18 = 108°$

$\angle S = 7x = 7\times18 = 126°$

$\angle Q+\angle R = 72°+108° = 180°$

or $\angle P+\angle S = 54°+126° = 180°$

Hence PQ || SR

As $\angle P + \angle Q = 72°+54° = 126°$

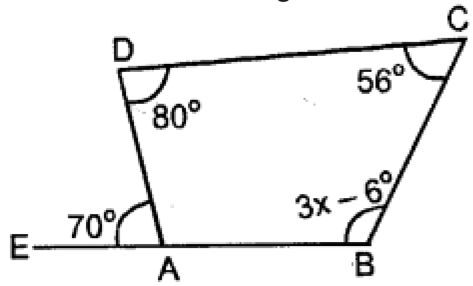
Which is $\neq 180°$.

$\therefore$ PS and QR are not parallel.

(ii) PQRS is a Trapezium as its one pair of opposite side is parallel.

Question 11.

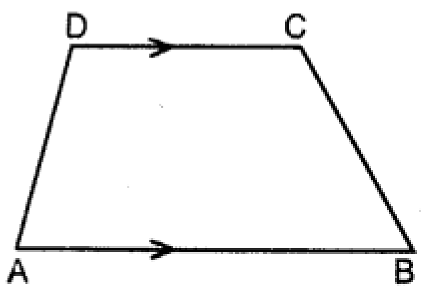
Use the informations given in the following figure to find the value of x.



Solution:

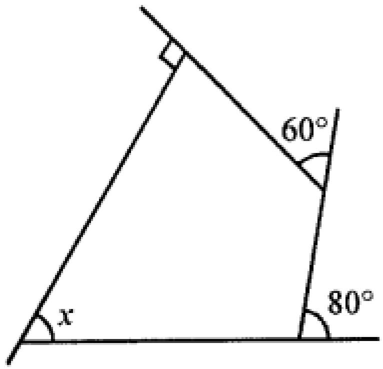
Take A, B, C, D as the vertices of Quadrilateral and BA is produced to E (say).
Since ∠EAD = 70°
∠DAB = 180° – 70°= 110°
[EAB is a straight line and AD stands on it ∠EAD+ ∠DAB = 180°]
110° + 80° + 56° + 3x – 6° = 360°
[sum of interior angles of a quadrilateral = 360°]
3x = 360° – 110° – 80° – 56° + 6°
3x = 360° – 240° = 120°
x = 40°

Question 12.
The following figure shows a quadrilateral in which sides AB and DC are parallel. If ∠A : ∠D = 4 : 5, ∠B = (3x – 15)° and ∠C = (4x + 20)°, find each angle of the quadrilateral ABCD.

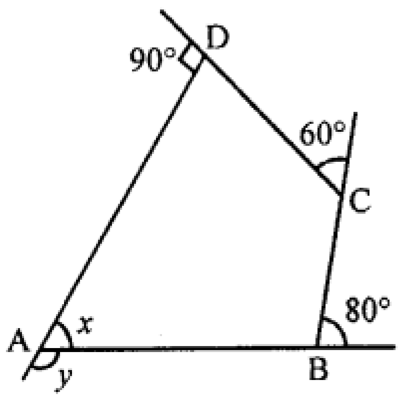


Solution:
Let ∠A = 4x
∠D = 5x
Since ∠A + ∠D = 180° [AB||DC]
4x + 5x = 180°
=> 9x = 180°
=> x = 20°
∠A = 4 (20) = 80°,
∠D = 5 (20) = 100°
Again ∠B + ∠C = 180° [AB||DC]
3x – 15° + 4x + 20° = 180°
7x = 180° – 5°
=> 7x = 175°
=> x = 25°
∠B = 75° – 15° = 60°
and ∠C = 4 (25) + 20 = 100°+ 20°= 120°

Question 13.
Use the following figure to find the value of x

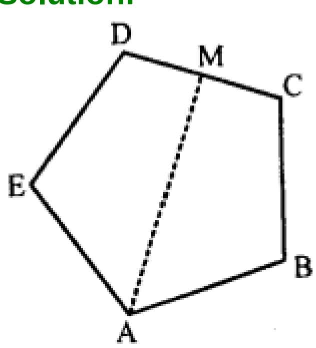


Solution:
The sum of exterior angles of a quadrilateral



=> y + 80° + 60° + 90° = 360°
=> y + 230° = 360°
=> y = 360° – 230° = 130°
At vertex A,
∠y + ∠x = 180° (Linear pair)
x = 180° – 130°
=> x = 50°

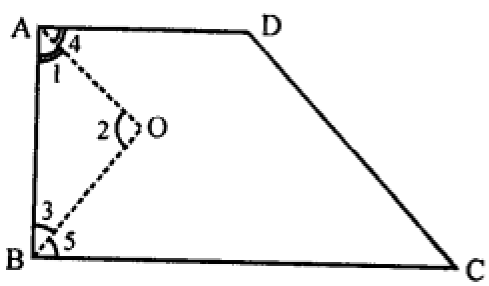
Question 14.
ABCDE is a regular pentagon. The bisector of angle A of the pentagon meets the side CD in point M. Show that ∠AMC = 90°.
Solution:



Given : ABCDE is a regular pentagon.

The bisector ∠A of the pentagon meets the side CD at point M.
To prove : ∠AMC = 90°
Proof: We know that, the measure of each interior angle of a regular pentagon is 108°.
∠BAM = $\frac{1}{2}$ x 108° = 54°
Since, we know that the sum of a quadrilateral is 360°
In quadrilateral ABCM, we have
∠BAM + ∠ABC + ∠BCM + ∠AMC = 360°
54° + 108° + 108° + ∠AMC = 360°
∠AMC = 360° – 270°
∠AMC = 90°

Question 15.
In a quadrilateral ABCD, AO and BO are bisectors of angle A and angle B respectively.
Show that:
∠AOB = $\frac{1}{2}$ (∠C + ∠D)
Solution:
Given : AO and BO are the bisectors of ∠A and ∠B respectively.
∠1 = ∠4 and ∠3 = ∠5 ……..(i)



To prove : ∠AOB = $\frac{1}{2}$ (∠C + ∠D)
Proof: In quadrilateral ABCD
∠A + ∠B + ∠C + ∠D = 360°
$\frac{1}{2}$ (∠A + ∠B + ∠C + ∠D) = 180° …………(ii)
Now in ΔAOB
∠1 + ∠2 + ∠3 = 180° ………(iii)
Equating equation (ii) and equation (iii), we get
∠1 + ∠2 + ∠3 = ∠A + ∠B + $\frac{1}{2}$ (∠C + ∠D)
∠1 + ∠2 + ∠3 = ∠1 + ∠3 + $\frac{1}{2}$ (∠C + ∠D)
∠2 = $\frac{1}{2}$ (∠C + ∠D)
∠AOB = $\frac{1}{2}$ (∠C + ∠D)
Hence proved.