

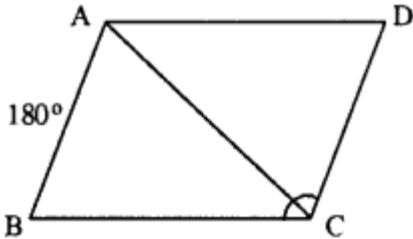
## 17. Special Types of Quadrilaterals

### EXERCISE 17

#### Question 1.

In parallelogram ABCD,  $\angle A = 3$  times  $\angle B$ . Find all the angles of the parallelogram. In the same parallelogram, if  $AB = 5x - 7$  and  $CD = 3x + 1$ ; find the length of CD.

**Solution:**



Let  $\angle B = x$

$\angle A = 3 \angle B = 3x$

$AD \parallel BC$

$\angle A + \angle B = 180^\circ$

$3x + x = 180^\circ$

$\Rightarrow 4x = 180^\circ$

$\Rightarrow x = 45^\circ$

$\angle B = 45^\circ$

$\angle A = 3x = 3 \times 45 = 135^\circ$

and  $\angle B = \angle D = 45^\circ$

opposite angles of  $\parallel$  gm are equal.

$\angle A = \angle C = 135^\circ$

opposite sides of  $\parallel$  gm are equal.

$AB = CD$

$5x - 7 = 3x + 1$

$\Rightarrow 5x - 3x = 1 + 7$

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

$CD = 3 \times 4 + 1 = 13$

Hence  $135^\circ, 45^\circ, 135^\circ$  and  $45^\circ$ ; 13

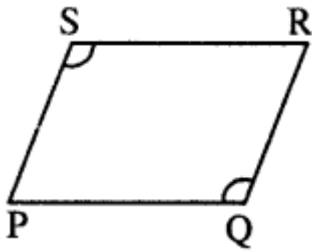
#### Question 2.

In parallelogram PQRS,  $\angle Q = (4x - 5)^\circ$  and  $\angle S = (3x + 10)^\circ$ . Calculate :  $\angle Q$  and  $\angle R$ .

**Solution:**

In parallelogram PQRS,

$\angle Q = (4x - 5)^\circ$  and  $\angle S = (3x + 10)^\circ$



opposite  $\angle$ s of //gm are equal.

$$\angle Q = \angle S$$

$$4x - 5 = 3x + 10$$

$$4x - 3x = 10 + 5$$

$$x = 15$$

$$\angle Q = 4x - 5 = 4 \times 15 - 5 = 55^\circ$$

$$\text{Also } \angle Q + \angle R = 180^\circ$$

$$55^\circ + \angle R = 180^\circ$$

$$\angle R = 180^\circ - 55^\circ = 125^\circ$$

$$\angle Q = 55^\circ ; \angle R = 125^\circ$$

### Question 3.

In rhombus ABCD ;

(i) if  $\angle A = 74^\circ$  ; find  $\angle B$  and  $\angle C$ .

(ii) if  $AD = 7.5$  cm ; find  $BC$  and  $CD$ .

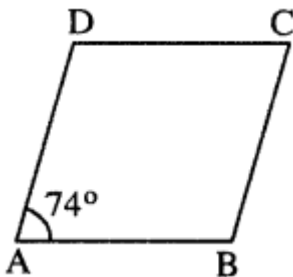
**Solution:**

$AD \parallel BC$

$$\angle A + \angle B = 180^\circ$$

$$74^\circ + \angle B = 180^\circ$$

$$\angle B = 180^\circ - 74^\circ = 106^\circ$$



opposite angles of Rhombus are equal.

$$\angle A = \angle C = 74^\circ$$

Sides of Rhombus are equal.

$$BC = CD = AD = 7.5 \text{ cm}$$

(i)  $\angle B = 106^\circ$  ;  $\angle C = 74^\circ$

(ii)  $BC = 7.5$  cm and  $CD = 7.5$  cm Ans.

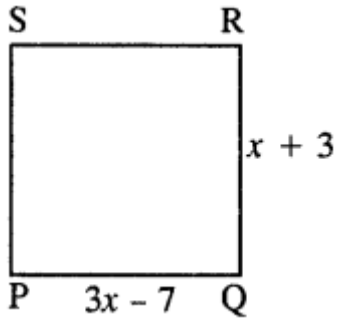
### Question 4.

In square PQRS :

- (i) if  $PQ = 3x - 7$  and  $QR = x + 3$ ; find PS  
(ii) if  $PR = 5x$  and  $QR = 9x - 8$ . Find QS

**Solution:**

- (i) sides of square are equal.



$$PQ = QR$$

$$\Rightarrow 3x - 7 = x + 3$$

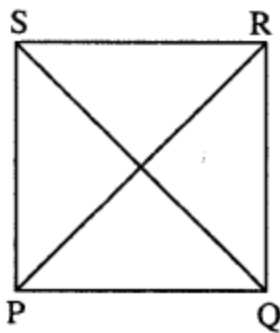
$$\Rightarrow 3x - x = 3 + 7$$

$$\Rightarrow 2x = 10$$

$$x = 5$$

$$PS = PQ = 3x - 7 = 3 \times 5 - 7 = 8$$

- (ii)  $PR = 5x$  and  $QS = 9x - 8$



As diagonals of square are equal.

$$PR = QS$$

$$5x = 9x - 8$$

$$\Rightarrow 5x - 9x = -8$$

$$\Rightarrow -4x = -8$$

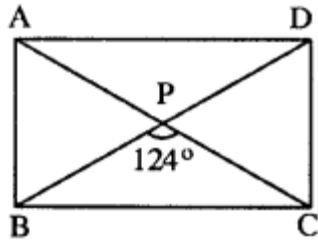
$$\Rightarrow x = 2$$

$$QS = 9x - 8 = 9 \times 2 - 8 = 10$$

**Question 5.**

ABCD is a rectangle, if  $\angle BPC = 124^\circ$

Calculate : (i)  $\angle BAP$  (ii)  $\angle ADP$



**Solution:**

Diagonals of rectangle are equal and bisect each other.

$$\angle PBC = \angle PCB = x \text{ (say)}$$

$$\text{But } \angle BPC + \angle PBC + \angle PCB = 180^\circ$$

$$124^\circ + x + x = 180^\circ$$

$$2x = 180^\circ - 124^\circ$$

$$2x = 56^\circ$$

$$\Rightarrow x = 28^\circ$$

$$\angle PBC = 28^\circ$$

But  $\angle PBC = \angle ADP$  [Alternate  $\angle$ s]

$$\angle ADP = 28^\circ$$

$$\text{Again } \angle APB = 180^\circ - 124^\circ = 56^\circ$$

Also  $PA = PB$

$$\angle BAP = \frac{1}{2} (180^\circ - \angle APB)$$

$$= \frac{1}{2} x (180^\circ - 56^\circ) = \frac{1}{2} x 124^\circ = 62^\circ$$

Hence (i)  $\angle BAP = 62^\circ$  (ii)  $\angle ADP = 28^\circ$

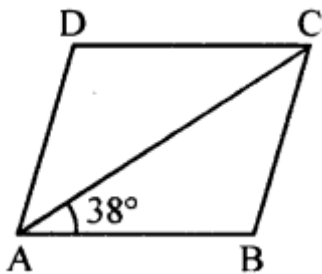
**Question 6.**

ABCD is a rhombus. If  $\angle BAC = 38^\circ$ , find :

(i)  $\angle ACB$

(ii)  $\angle DAC$

(iii)  $\angle ADC$ .



**Solution:**

ABCD is Rhombus (Given)

$$AB = BC$$

$\angle BAC = \angle ACB$  ( $\angle$ s opp. to equal sides)

But  $\angle BAC = 38^\circ$  (Given)

$$\angle ACB = 38^\circ$$

In  $\triangle ABC$ ,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\angle ABC + 38^\circ + 38^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 76^\circ = 104^\circ$$

But  $\angle ABC = \angle ADC$  (opp.  $\angle$ s of rhombus)

$$\angle ADC = 104^\circ$$

$$\angle DAC = \angle DCA \text{ ( AD = CD)}$$

$$\angle DAC = \frac{1}{2} [180^\circ - 104^\circ]$$

$$\angle DAC = \frac{1}{2} \times 76^\circ = 38^\circ$$

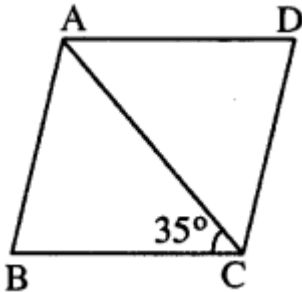
Hence (i)  $\angle ACB = 38^\circ$  (ii)  $\angle DAC = 38^\circ$  (iii)  $\angle ADC = 104^\circ$  Ans.

### Question 7.

ABCD is a rhombus. If  $\angle BCA = 35^\circ$ . find  $\angle ADC$ .

**Solution:**

**Given :** Rhombus ABCD in which  $\angle BCA = 35^\circ$



**To find :**  $\angle ADC$

**Proof :**  $AD \parallel BC$

$$\angle DAC = \angle BCA \text{ (Alternate } \angle\text{s)}$$

But  $\angle BCA = 35^\circ$  (Given)

$$\angle DAC = 35^\circ$$

But  $\angle DAC = \angle ACD$  (  $AD = CD$ ) &  $\angle DAC + \angle ACD + \angle ADC = 180^\circ$

$$35^\circ + 35^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 70^\circ = 110^\circ$$

Hence  $\angle ADC = 110^\circ$

### Question 8.

PQRS is a parallelogram whose diagonals intersect at M.

If  $\angle PMS = 54^\circ$ ,  $\angle QSR = 25^\circ$  and  $\angle SQR = 30^\circ$ ; find :

(i)  $\angle RPS$

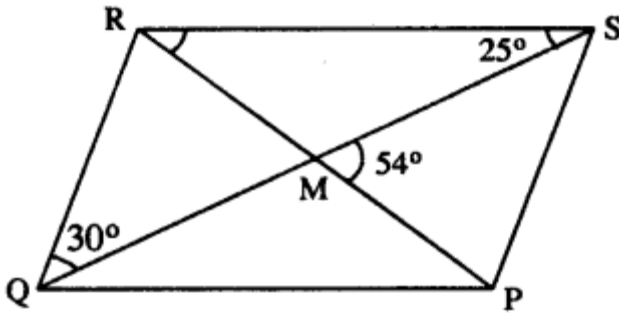
(ii)  $\angle PRS$

(iii)  $\angle PSR$ .

**Solution:**

**Given :**  $\parallel$ gm PQRS in which diagonals PR & QS intersect at M.

$\angle PMS = 54^\circ$ ;  $\angle QSR = 25^\circ$  and  $\angle SQR = 30^\circ$



**To find :** (i)  $\angle RPS$  (ii)  $\angle PRS$  (iii)  $\angle PSR$

**Proof :**  $QR \parallel PS$

$\Rightarrow \angle PSQ = \angle SQR$  (Alternate  $\angle$ s)

But  $\angle SQR = 30^\circ$  (Given)

$\angle PSQ = 30^\circ$

In  $\triangle SMP$ ,

$\angle PMS + \angle PSM + \angle MPS = 180^\circ$  or  $54^\circ + 30^\circ + \angle RPS = 180^\circ$

$\angle RPS = 180^\circ - 84^\circ = 96^\circ$

Now  $\angle PRS + \angle RSQ = \angle PMS$

$\angle PRS + 25^\circ = 54^\circ$

$\angle PRS = 54^\circ - 25^\circ = 29^\circ$

$\angle PSR = \angle PSQ + \angle RSQ = 30^\circ + 25^\circ = 55^\circ$

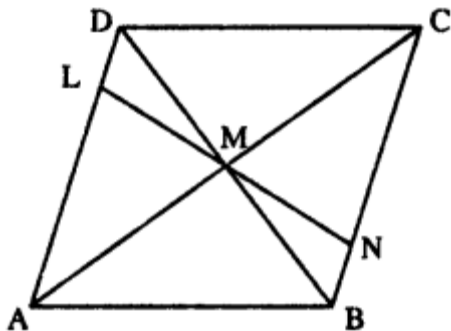
Hence (i)  $\angle RPS = 96^\circ$  (ii)  $\angle PRS = 29^\circ$  (iii)  $\angle PSR = 55^\circ$

### Question 9.

**Given :** Parallelogram ABCD in which diagonals AC and BD intersect at M.

**Prove :** M is mid-point of LN.

**Solution:**



**Proof :** Diagonals of //gm bisect each other.

$MD = MB$

Also  $\angle ADB = \angle DBN$  (Alternate  $\angle$ s)

&  $\angle DML = \angle BMN$  (Vert. opp.  $\angle$ s)

$\triangle DML = \triangle BMN$

$LM = MN$

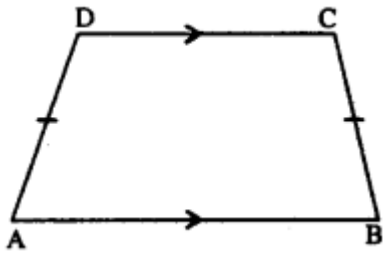
M is mid-point of LN.

Hence proved.

**Question 10.**

In an Isosceles-trapezium, show that the opposite angles are supplementary.

**Solution:**



**Given :** ABCD is isosceles trapezium in which  $AD = BC$

**To Prove :** (i)  $\angle A + \angle C = 180^\circ$

(ii)  $\angle B + \angle D = 180^\circ$

**Proof :**  $AB \parallel CD$ .

$\Rightarrow \angle A + \angle D = 180^\circ$

But  $\angle A = \angle B$  [Trapezium is isosceles]

$\angle B + \angle D = 180^\circ$

Similarly  $\angle A + \angle C = 180^\circ$

Hence the result.

**Question 11.**

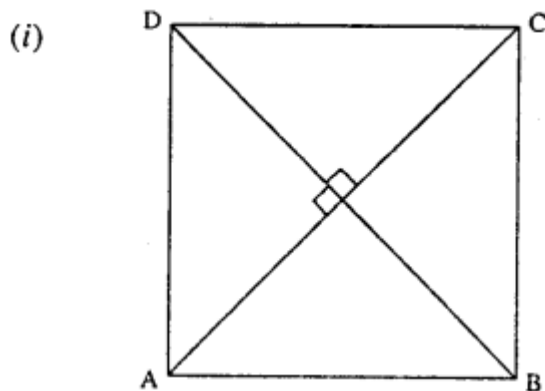
ABCD is a parallelogram. What kind of quadrilateral is it if :

(i)  $AC = BD$  and  $AC$  is perpendicular to  $BD$ ?

(ii)  $AC$  is perpendicular to  $BD$  but is not equal to it ?

(iii)  $AC = BD$  but  $AC$  is not perpendicular to  $BD$  ?

**Solution:**



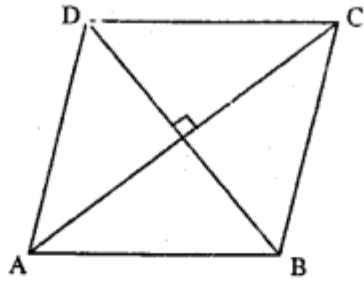
$$AC = BD \text{ (Given)}$$

$$\& AC \perp BD \text{ (Given)}$$

*i.e.* Diagonals of quadrilateral are equal and they are  $\perp$  to each other.

$\therefore$  ABCD is square

(ii)

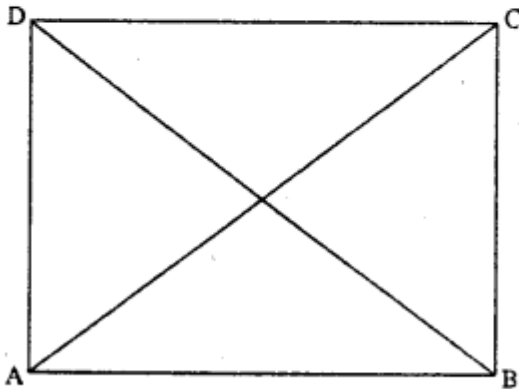


$AC \perp BD$  (Given)

But  $AC$  &  $BD$  are not equal

$\therefore$   $ABCD$  is a Rhombus.

(iii)



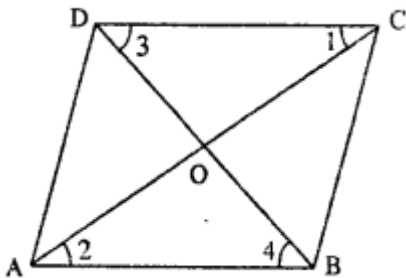
$AC = BD$  but  $AC$  &  $BD$  are not  $\perp$  to each other.

$\therefore$   $ABCD$  is a Rectangle.

### Question 12.

Prove that the diagonals of a parallelogram bisect each other.

**Solution:**



**Given :**  $\parallel$ gm  $ABCD$  in which diagonals  $AC$  and  $BD$  bisect each other.

**To Prove :**  $OA = OC$  and  $OB = OD$

**Proof :**  $AB \parallel CD$  (Given)

$\angle 1 = \angle 2$  (alternate  $\angle$ s)

$\angle 3 = \angle 4$  (alternate  $\angle$ s)

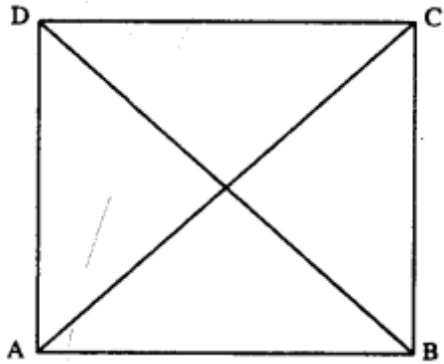


and  $AB = CD$  (opposite sides of //gm)  
 $\triangle COD = \triangle AOB$  (A.S.A. rule)  
 $OA = OC$  and  $OB = OD$   
Hence the result.

**Question 13.**

If the diagonals of a parallelogram are of equal lengths, the parallelogram is a rectangle.  
Prove it.

**Solution:**



**Given :** //gm ABCD in which  $AC = BD$

**To Prove :** ABCD is rectangle.

**Proof :** In  $\triangle ABC$  and  $\triangle ABD$

$AB = AB$  (Common)

$AC = BD$  (Given)

$BC = AD$  (opposite sides of //gm)

$\triangle ABC = \triangle ABD$  (S.S.S. Rule)

$\angle A = \angle B$

But  $AD \parallel BC$  (opp. sides of //gm are ||)

$\angle A + \angle B = 180^\circ$

$\angle A = \angle B = 90^\circ$

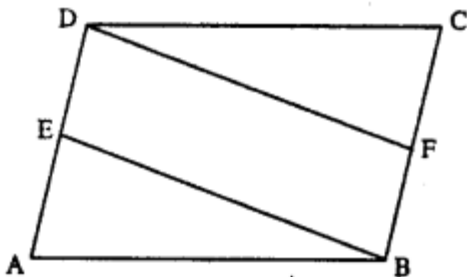
Similarly  $\angle D = \angle C = 90^\circ$

Hence ABCD is a rectangle.

**Question 14.**

In parallelogram ABCD, E is the mid-point of AD and F is the mid-point of BC. Prove that BFDE is a parallelogram.

**Solution:**



**Given :** //gm ABCD in which E and F are mid-points of AD and BC respectively.

**To Prove :** BFDE is a //gm.

**Proof :** E is mid-point of AD. (Given)

$$DE = \frac{1}{2} AD$$

Also F is mid-point of BC (Given)

$$BF = \frac{1}{2} BC$$

But  $AD = BC$  (opp. sides of //gm)

$$BF = DE$$

Again  $AD \parallel BC$

$$\Rightarrow DE \parallel BF$$

Now  $DE \parallel BF$  and  $DE = BF$

Hence BFDE is a //gm.

### Question 15.

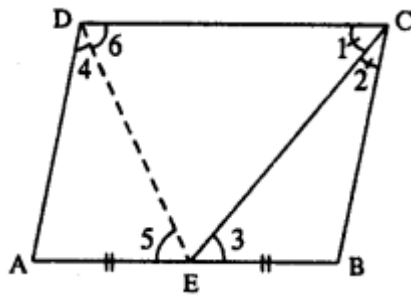
In parallelogram ABCD, E is the mid-point of side AB and CE bisects angle BCD. Prove that :

(i)  $AE = AD$ ,

(ii) DE bisects  $\angle ADC$  and

(iii) Angle DEC is a right angle.

**Solution:**



**Given :** //gm ABCD in which E is mid-point of AB and CE bisects ZBCD.

**To Prove :** (i)  $AE = AD$

(ii) DE bisects  $\angle ADC$

(iii)  $\angle DEC = 90^\circ$

**Const.** Join DE

**Proof :** (i)  $AB \parallel CD$  (Given)

and CE bisects it.

$$\angle 1 = \angle 3 \text{ (alternate } \angle\text{s) } \dots\dots\dots \text{(i)}$$

$$\text{But } \angle 1 = \angle 2 \text{ (Given) } \dots\dots\dots \text{(ii)}$$

From (i) & (ii)

$$\angle 2 = \angle 3$$

$BC = BE$  (sides opp. to equal angles)

But  $BC = AD$  (opp. sides of //gm)

and  $BE = AE$  (Given)

$$AD = AE$$

$$\angle 4 = \angle 5 \text{ (} \angle\text{s opp. to equal sides)}$$

$$\text{But } \angle 5 = \angle 6 \text{ (alternate } \angle\text{s)}$$

$$\Rightarrow \angle 4 = \angle 6$$

DE bisects  $\angle ADC$ .

Now  $AD \parallel BC$

$$\Rightarrow \angle D + \angle C = 180^\circ$$

$$2\angle 6 + 2\angle 1 = 180^\circ$$

DE and CE are bisectors.

$$\angle 6 + \angle 1 = \frac{180^\circ}{2}$$

$$\angle 6 + \angle 1 = 90^\circ$$

$$\text{But } \angle DEC + \angle 6 + \angle 1 = 180^\circ$$

$$\angle DEC + 90^\circ = 180^\circ$$

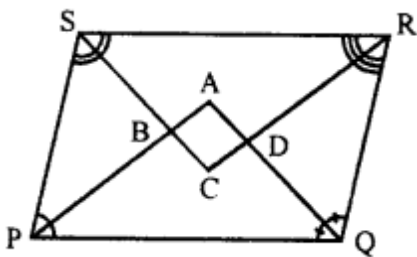
$$\angle DEC = 180^\circ - 90^\circ$$

$$\angle DEC = 90^\circ$$

Hence the result.

### Question 16.

In the following diagram, the bisectors of interior angles of the parallelogram PQRS enclose a quadrilateral ABCD.



Show that:

(i)  $\angle PSB + \angle SPB = 90^\circ$

(ii)  $\angle PBS = 90^\circ$

(iii)  $\angle ABC = 90^\circ$

(iv)  $\angle ADC = 90^\circ$

(v)  $\angle A = 90^\circ$

(vi) ABCD is a rectangle

Thus, the bisectors of the angles of a parallelogram enclose a rectangle.

#### Solution:

**Given :** In parallelogram PQRS bisector of angles P and Q, meet at A, bisectors of  $\angle R$  and  $\angle S$  meet at C. Forming a quadrilateral ABCD as shown in the figure.

**To prove :**

(i)  $\angle PSB + \angle SPB = 90^\circ$

(ii)  $\angle PBS = 90^\circ$

(iii)  $\angle ABC = 90^\circ$

(iv)  $\angle ADC = 90^\circ$

(v)  $\angle A = 9^\circ$

(vi) ABCD is a rectangle

**Proof :** In parallelogram PQRS,

$PS \parallel QR$  (opposite sides)

$$\angle P + \angle Q = 180^\circ$$

and AP and AQ are the bisectors of consecutive angles  $\angle P$  and  $\angle Q$  of the parallelogram

$$\angle APQ + \angle AQP = \frac{1}{2} \times 180^\circ = 90^\circ$$

But in  $\triangle APQ$ ,

$$\angle A + \angle APQ + \angle AQP = 180^\circ \text{ (Angles of a triangle)}$$

$$\angle A + 90^\circ = 180^\circ$$

$$\angle A = 180^\circ - 90^\circ$$

$$(v) \angle A = 90^\circ$$

Similarly  $PQ \parallel SR$

$$\angle PSB + \angle SPB = 90^\circ$$

$$(ii) \text{ and } \angle PBS = 90^\circ$$

But,  $\angle ABC = \angle PBS$  (Vertically opposite angles)

$$(iii) \angle ABC = 90^\circ$$

Similarly we can prove that

$$(iv) \angle ADC = 90^\circ \text{ and } \angle C = 90^\circ$$

(vi) ABCD is a rectangle (Each angle of a quadrilateral is  $90^\circ$ )

Hence proved.

### Question 17.

In parallelogram ABCD, X and Y are midpoints of opposite sides AB and DC respectively. Prove that:

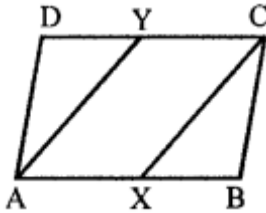
$$(i) AX = YC$$

(ii) AX is parallel to YC

(iii) AXCY is a parallelogram.

#### Solution:

**Given :** In parallelogram ABCD, X and Y are the mid-points of sides AB and DC respectively AY and CX are joined



**To prove :**

$$(i) AX = YC$$

(ii) AX is parallel to YC

(iii) AXCY is a parallelogram

**Proof :**  $AB \parallel DC$  and X and Y are the mid-points of the sides AB and DC respectively

$$(i) AX = YC \left( \frac{1}{2} \text{ of opposite sides of a parallelogram} \right)$$

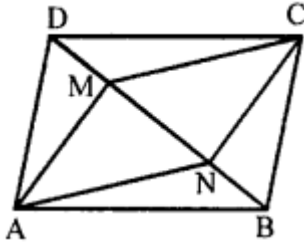
(ii) and  $AX \parallel YC$

(iii) AXCY is a parallelogram (A pair of opposite sides are equal and parallel)

Hence proved.

### Question 18.

The given figure shows parallelogram ABCD. Points M and N lie in diagonal BD such that  $DM = BN$ .



Prove that:

- (i)  $\triangle DMC = \triangle BNA$  and so  $CM = AN$
- (ii)  $\triangle AMD = \triangle CNB$  and so  $AM = CN$
- (iii) ANCM is a parallelogram.

**Solution:**

**Given :** In parallelogram ABCD, points M and N lie on the diagonal BD such that  $DM = BN$

AN, NC, CM and MA are joined

**To prove :**

- (i)  $\triangle DMC = \triangle BNA$  and so  $CM = AN$
- (ii)  $\triangle AMD = \triangle CNB$  and so  $AM = CN$
- (iii) ANCM is a parallelogram

**Proof :**

(i) In  $\triangle DMC$  and  $\triangle BNA$ .

$CD = AB$  (opposite sides of ||gm ABCD)

$DM = BN$  (given)

$\angle CDM = \angle ABN$  (alternate angles)

$\triangle DMC = \triangle BNA$  (SAS axiom)

$CM = AN$  (c.p.c.t.)

Similarly, in  $\triangle AMD$  and  $\triangle CNB$

$AD = BC$  (opposite sides of ||gm)

$DM = BN$  (given)

$\angle ADM = \angle CBN$  – (alternate angles)

$\triangle AMD = \triangle CNB$  (SAS axiom)

$AM = CN$  (c.p.c.t.)

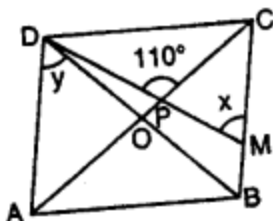
(iii)  $CM = AN$  and  $AM = CN$  (proved)

ANCM is a parallelogram (opposite sides are equal)

Hence proved.

### Question 19.

The given figure shows a rhombus ABCD in which angle  $BCD = 80^\circ$ . Find angles x and y.



**Solution:**

In rhombus ABCD, diagonals AC and BD bisect each other at  $90^\circ$

$$\angle BCD = 80^\circ$$

Diagonals bisect the opposite angles also  $\angle BCD = \angle BAD$  (Opposite angles of rhombus)

$$\angle BAD = 80^\circ \text{ and } \angle ABC = \angle ADC = 180^\circ - 80^\circ = 100^\circ$$

Diagonals bisect opposite angles

$$\angle OCB \text{ or } \angle PCB = \frac{80^\circ}{2} = 40^\circ$$

In  $\triangle PCM$ ,

$$\text{Ext. } \angle CPD = \angle OCB + \angle PMC$$

$$110^\circ = 40^\circ + x$$

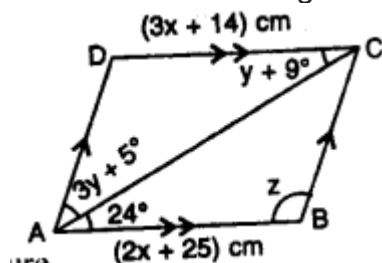
$$\Rightarrow x = 110^\circ - 40^\circ = 70^\circ$$

$$\text{and } \angle ADO = \frac{1}{2} \angle ADC = \frac{1}{2} \times 100^\circ = 50^\circ$$

$$\text{Hence } x = 70^\circ \text{ and } y = 50^\circ$$

### Question 20.

Use the information given in the alongside diagram to find the value of x, y and z.



### Solution:

ABCD is a parallelogram and AC is its diagonal which bisects the opposite angle

Opposite sides of a parallelogram are equal

$$3x + 14 = 2x + 25$$

$$\Rightarrow 3x - 2x = 25 - 14$$

$$\Rightarrow x = 11$$

$$\therefore x = 11 \text{ cm}$$

$$\angle DCA = \angle CAB \text{ (Alternate angles)}$$

$$y + 9^\circ = 24$$

$$y = 24^\circ - 9^\circ = 15^\circ$$

$$\angle DAB = 3y^\circ + 5^\circ + 24^\circ = 3 \times 15 + 5 + 24^\circ = 50^\circ + 24^\circ = 74^\circ$$

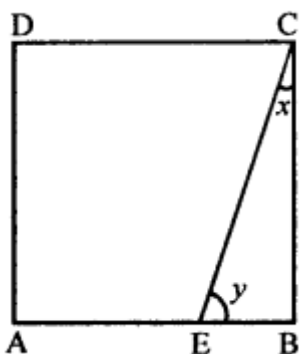
$$\angle ABC = 180^\circ - \angle DAB = 180^\circ - 74^\circ = 106^\circ$$

$$z = 106^\circ$$

$$\text{Hence } x = 11 \text{ cm, } y = 15^\circ, z = 106^\circ$$

### Question 21.

The following figure is a rectangle in which  $x : y = 3 : 7$ ; find the values of x and y.



**Solution:**

ABCD is a rectangle,

$$x : y = 3 : 1$$

In  $\triangle BCE$ ,  $\angle B = 90^\circ$

$$x + y = 90^\circ$$

But  $x : y = 3 : 7$

Sum of ratios =  $3 + 7 = 10$

$$\therefore x = \frac{90^\circ \times 3}{10} = 27^\circ$$

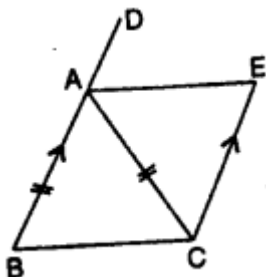
$$\text{and } y = \frac{90^\circ \times 7}{10} = 63^\circ$$

Hence  $x = 27^\circ, y = 63^\circ$

Hence  $x = 27^\circ, y = 63^\circ$

**Question 22.**

In the given figure,  $AB \parallel EC$ ,  $AB = AC$  and  $AE$  bisects  $\angle DAC$ . Prove that:



(i)  $\angle EAC = \angle ACB$

(ii) ABCE is a parallelogram.

**Solution:**

ABCE is a quadrilateral in which AC is its diagonal and  $AB \parallel EC$ ,  $AB = AC$

BA is produced to D

AE bisects  $\angle DAC$

**To prove:**

(i)  $\angle EAC = \angle ACB$

(ii) ABCE is a parallelogram

**Proof:**

(i) In  $\triangle ABC$  and  $\triangle ZAEC$

$AC=AC$  (common)

$AB = CE$  (given)

$\angle BAC = \angle ACE$  (Alternate angle)

$\triangle ABC = \triangle AEC$  (SAS Axiom)

(ii)  $\angle BCA = \angle CAE$  (c.p.c.t.)

But these are alternate angles

$AE \parallel BC$

But  $AB \parallel EC$  (given)

$\therefore ABCD$  is a parallelogram