

## 2. Exponents (Powers)

### Exercise 2 (A)

#### Question 1.

Evaluate:

$$(i) (3^{-1} \times 9^{-1}) \div 3^{-2}$$

$$(ii) (3^{-1} \times 4^{-1}) \div 6^{-1}$$

$$(iii) (2^{-1} + 3^{-1})^3$$

$$(iv) (3^{-1} \div 4^{-1})^2$$

$$(v) (2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$$

$$(vi) (5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$$

$$(vii) \left[ \left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3} \right] \div \left(\frac{1}{6}\right)^{-3}$$

$$(viii) \left[ \left(-\frac{3}{4}\right)^{-2} \right]^2$$

$$(ix) \left\{ \left(\frac{3}{5}\right)^{-2} \right\}^{-2}$$

$$(x) (5^{-1} \times 3^{-1}) \div 6^{-1}$$

**Solution:**

$$(i) (3^{-1} \times 9^{-1}) \div 3^{-2}$$

$$= \left(\frac{1}{3} \times \frac{1}{9}\right) \div \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{27} \div \frac{1}{9}$$

$$= \frac{1}{27} \times \frac{9}{1} = \frac{1}{3}$$

$$(ii) (3^{-1} \times 4^{-1}) \div 6^{-1}$$

$$= \left(\frac{1}{3} \times \frac{1}{4}\right) \div \frac{1}{6}$$

$$= \frac{1}{12} \div \frac{1}{6}$$

$$= \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$$

$$(iii) (2^{-1} + 3^{-1})^3$$

$$= \left(\frac{1}{2} + \frac{1}{3}\right)^3 = \left(\frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2}\right)^3$$

$$= \left(\frac{3+2}{6}\right)^3 = \left(\frac{5}{6}\right)^3$$

$$= \frac{5 \times 5 \times 5}{6 \times 6 \times 6} = \frac{125}{216}$$

$$(iv) (3^{-1} \div 4^{-1})^2$$

$$= \left(\frac{1}{3} \div \frac{1}{4}\right)^2$$

$$= \left(\frac{1}{3} \times \frac{4}{1}\right)^2 = \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9} = 1\frac{7}{9}$$

$$(v) (2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$$

$$= (2 \times 2) + (3 \times 3) \times \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$= 4 + 9 \times \frac{1}{4} = \frac{13}{4} = 3\frac{1}{4}$$

$$(vi) (5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$$

$$= (5 \times 5) - (3 \times 3) \times \left(\frac{3}{2}\right)^3$$

$$= 25 - 9 \times \left(\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}\right)$$

$$= 16 \times \frac{27}{8} = 54$$

$$(vii) \left[ \left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3} \right] \div \left(\frac{1}{6}\right)^{-3}$$

$$= \left[ \left(\frac{4}{1}\right)^3 - \left(\frac{3}{1}\right)^3 \right] \div \left(\frac{6}{1}\right)^3$$

$$= \left( \frac{4}{1} \times \frac{4}{1} \times \frac{4}{1} - \frac{3}{1} \times \frac{3}{1} \times \frac{3}{1} \right) \div \left( \frac{6}{1} \right)^3$$

$$= 64 - 27 \times \left( \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \right)$$

$$= 37 \times \frac{1}{216} = \frac{37}{216}$$

$$(viii) \left[ \left( -\frac{3}{4} \right)^{-2} \right]^2 = \left( -\frac{3}{4} \right)^{-2 \times 2} = \left( -\frac{3}{4} \right)^{-4}$$

$$= \left( \frac{4}{3} \right)^4 = \frac{4 \times 4 \times 4 \times 4}{3 \times 3 \times 3 \times 3}$$

$$= \frac{256}{81} = 3 \frac{13}{81}$$

$$(ix) \left\{ \left( \frac{3}{5} \right)^{-2} \right\}^{-2} = \left( \frac{3}{5} \right)^{-2 \times (-2)} = \left( \frac{3}{5} \right)^4$$

$$= \frac{3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5} = \frac{81}{625}$$

$$(x) (5^{-1} \times 3^{-1}) \div 6^{-1}$$

$$= \left( \frac{1}{5} \times \frac{1}{3} \right) \div \frac{1}{6}$$

$$= \frac{1}{15} \div \frac{1}{6}$$

$$= \frac{1}{15} \times \frac{6}{1} = \frac{2}{5}$$

**Question 2.**

If  $1125 = 3^m \times 5^n$ ; find  $m$  and  $n$ .

**Solution:**

$$1125 = 3^2 \times 5^3$$

The factors of 1125 are  $3 \times 3 \times 5 \times 5 \times 5$

$$\begin{array}{r|l} 3 & 1125 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\therefore 1125 = 3 \times 3 \times 5 \times 5 \times 5$$

$$\text{Now comparing, } 3^2 \times 5^3 = 3^m \times 5^n$$

$$\therefore m = 2 \quad n = 3$$

**Question 3.**

Find  $x$ , if  $9 \times 3^x = (27)^{2x-3}$

**Solution:**

$$9 \times 3^x = (27)^{2x-3}$$

$$3^2 \times 3^x = (3 \times 3 \times 3)^{2x-3}$$

$$\Rightarrow 3^{x+2} = (3)^{3(2x-3)}$$

$$\Rightarrow 3^{x+2} = (3)^{6x-9}$$

Since, bases are same, compare them,

$$x + 2 = 6x - 9$$

$$6x - x = 9 + 2$$

$$\Rightarrow 5x = 11$$

$$\Rightarrow x = \frac{11}{5}$$

$$\Rightarrow x = 2\frac{1}{5}$$

## Exercise 2 (B)

### Question 1.

Compute:

(i)  $1^8 \times 3^0 \times 5^3 \times 2^2$

(ii)  $(4^7)^2 \times (4^{-3})^4$

(iii)  $(2^{-9} \div 2^{-11})^3$

(iv)  $\left(\frac{2}{3}\right)^{-4} \times \left(\frac{27}{8}\right)^{-2}$

(v)  $\left(\frac{56}{28}\right)^0 \div \left(\frac{2}{5}\right)^3 \times \frac{16}{25}$

(vi)  $(12)^{-2} \times 3^3$

(vii)  $(-5)^4 \times (-5)^6 \div (-5)^9$

(viii)  $\left(-\frac{1}{3}\right)^4 \div \left(-\frac{1}{3}\right)^8 \times \left(-\frac{1}{3}\right)^5$

(ix)  $9^0 \times 4^{-1} \div 2^{-4}$

(x)  $(625)^{-\frac{3}{4}}$

**Solution:**

$$(xi) \left(\frac{27}{64}\right)^{-\frac{2}{3}}$$

$$(xii) \left(\frac{1}{32}\right)^{-\frac{2}{5}}$$

$$(xiii) (125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}}$$

$$(xiv) (243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$$

$$(xv) (-3)^4 - (\sqrt[4]{3})^0 \times (-2)^5 \div (64)^{\frac{2}{3}}$$

$$(xvi) (27)^{\frac{2}{3}} \div \left(\frac{81}{16}\right)^{-\frac{1}{4}}$$

$$\begin{aligned}(i) & 1^8 \times 3^0 \times 5^3 \times 2^2 \\ & = 1 \times 1 \times 5 \times 5 \times 5 \times 2 \times 2 \\ & = 125 \times 4 \\ & = 500\end{aligned}$$

$$\begin{aligned}(ii) & (4^7)^2 \times (4^{-3})^4 \\ & = 4^{14} \times 4^{-12} \\ & = 4^{14-12} = 4^2 \\ & = 4 \times 4 \\ & = 16\end{aligned}$$

$$\begin{aligned}(iii) & (2^{-9} \div 2^{-11})^3 = \left(\frac{2^{-9}}{2^{-11}}\right)^3 \\ & = (2^{-9+11})^3 \\ & = (2^2)^3 = 2^6\end{aligned}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 64$$

$$(iv) \left(\frac{2}{3}\right)^{-4} \times \left(\frac{27}{8}\right)^{-2} = \left(\frac{2}{3}\right)^{-4} \times \left(\frac{3^3}{2^3}\right)^{-2}$$

$$= \frac{2^{-4}}{3^{-4}} \times \frac{3^{-6}}{2^{-6}} = \frac{2^{-4}}{2^{-6}} \times \frac{3^{-6}}{3^{-4}}$$

$$= 2^{-4+6} \times \frac{1}{3^{-4+6}} = \frac{2^2}{3^2}$$

$$= \frac{4}{9}$$

$$(v) \left(\frac{56}{28}\right)^0 \div \left(\frac{2}{5}\right)^3 \times \frac{16}{25}$$

$$= 1 \div \frac{2^3}{5^3} \times \frac{2 \times 2 \times 2 \times 2}{5 \times 5}$$

$$\left[ \because \left(\frac{56}{28}\right)^0 = 1 \right]$$

$$= 1 \times \frac{5^3}{2^3} \times \frac{2^4}{5^2} = 5^{3-2} \times 2^{4-3}$$

$$= 5^1 \times 2^1 = 10$$

$$(vi) (12)^{-2} \times 3^3 = (2 \times 2 \times 3)^{-2} \times 3^3$$

$$= (2^2 \times 3)^{-2} \times 3^3$$

$$= 2^{-2 \times 2} \times 3^{-2} \times 3^3$$

$$= 2^{-4} \times 3^{-2+3}$$

$$= 2^{-4} \times 3^1$$

$$= \frac{3}{2^4} = \frac{3}{2 \times 2 \times 2 \times 2}$$

$$= \frac{3}{16}$$

$$(vii) (-5)^4 \times (-5)^6 \div (-5)^9$$

$$= (-5)^4 \times (-5)^6 \times \frac{1}{(-5)^9}$$

$$= (-5)^{4+6-9}$$



$$= (-5)^{10-9} = -5$$

$$\begin{aligned} \text{(viii)} \quad \left(-\frac{1}{3}\right)^4 \div \left(-\frac{1}{3}\right)^8 \times \left(-\frac{1}{3}\right)^5 \\ &= \left(-\frac{1}{3}\right)^4 \times \frac{1}{\left(-\frac{1}{3}\right)^8} \times \left(-\frac{1}{3}\right)^5 \\ &= \left(-\frac{1}{3}\right)^{4+5-8} = \left(-\frac{1}{3}\right)^{9-8} \\ &= -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad 9^0 \times 4^{-1} \div 2^{-4} &= 1 \times \frac{1}{4^1} \times \frac{1}{2^{-4}} \\ &= 1 \times \frac{1}{4} \times 2^4 = 1 \times \frac{1}{2^2} \times 2^4 \\ &= 2^{4-2} = 2^2 = 4 \end{aligned}$$

$$\begin{aligned} \text{(x)} \quad (625)^{-\frac{3}{4}} &= (5 \times 5 \times 5 \times 5)^{-\frac{3}{4}} \\ &= (5^4)^{-\frac{3}{4}} = 5^{4 \times -\frac{3}{4}} \\ &= 5^{-3} = \frac{1}{5^3} \\ &= \frac{1}{5 \times 5 \times 5} \\ &= \frac{1}{125} \end{aligned}$$

$$\begin{aligned}
 \text{(xi)} \quad \left(\frac{27}{64}\right)^{-\frac{2}{3}} &= \left[\frac{(3^3)}{(4^3)}\right]^{-\frac{2}{3}} \\
 &= \frac{3^{3 \times -\frac{2}{3}}}{4^{3 \times -\frac{2}{3}}} = \frac{3^{-2}}{4^{-2}} \\
 &= \frac{4^2}{3^2} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9} \\
 &= 1\frac{7}{9}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xii)} \quad \left(\frac{1}{32}\right)^{-\frac{2}{5}} &= \left(\frac{1}{2 \times 2 \times 2 \times 2 \times 2}\right)^{-\frac{2}{5}} \\
 &= \left(\frac{1}{2^5}\right)^{-\frac{2}{5}} = \frac{1}{2^{5 \times -\frac{2}{5}}} \\
 &= \frac{1}{2^{-2}} = 2^2 = 4
 \end{aligned}$$

$$\begin{aligned}
 \text{(xiii)} \quad (125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}} &= (5^3)^{-\frac{2}{3}} \div (2^3)^{\frac{2}{3}} \\
 &= 5^{-\frac{2}{3} \times 3} \div 2^{3 \times \frac{2}{3}} \\
 &= 5^{-2} \div 2^2 = \frac{1}{5^2} \times \frac{1}{2^2} \\
 &= \frac{1}{25} \times \frac{1}{4} = \frac{1}{100}
 \end{aligned}$$

$$\text{(xiv)} \quad (243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$$

$$\begin{aligned}
&= (3 \times 3 \times 3 \times 3 \times 3)^{\frac{2}{5}} + (2 \times 2 \times 2 \times 2 \times 2)^{-\frac{2}{5}} \\
&= (3^5)^{\frac{2}{5}} + (2^5)^{-\frac{2}{5}} \\
&= 3^{5 \times \frac{2}{5}} + 2^{-\frac{2}{5} \times 5} = 3^2 + 2^{-2} \\
&= 3^2 \times \frac{1}{2^{-2}} = 3^2 \times 2^{+2} \\
&= 3 \times 3 \times 2 \times 2 = 36
\end{aligned}$$

$$\begin{aligned}
\text{(xv)} \quad &(-3)^4 - (\sqrt[4]{3})^0 \times (-2)^5 \div (64)^{\frac{2}{3}} \\
&= (-3 \times -3 \times -3 \times -3) \\
&\quad - 1 \times -2 \times -2 \times -2 \times -2 \times -2 \div (2^6)^{\frac{2}{3}}
\end{aligned}$$

**Note :**  $(\sqrt[4]{3})^0 = 1$

$$\begin{aligned}
&= 3^4 + 2^5 \div 2^{6 \times \frac{2}{3}} \\
&= 3^4 + 2^5 \div 2^4 = 3^4 + \frac{2^5}{2^4} \\
&= 3^4 + 2^{5-4} = 3^4 + 2 = 3 \times 3 \times 3 \times 3 + 2 \\
&= 81 + 2 = 83
\end{aligned}$$

$$\begin{aligned}
\text{(xvi)} \quad &(27)^{\frac{2}{3}} \div \left(\frac{81}{16}\right)^{-\frac{1}{4}} = (3^3)^{\frac{2}{3}} \div \left(\frac{3^4}{2^4}\right)^{-\frac{1}{4}} \\
&= 3^{3 \times \frac{2}{3}} \div \frac{3^{\frac{1}{4} \times 4}}{2^{\frac{1}{4} \times 4}} = 3^2 \div \frac{3^{-1}}{2^{-1}} \\
&= 3^2 \times \frac{2^{-1}}{3^{-1}} \\
&= 3^{2+1} \times 2^{-1} = 3^3 \times \frac{1}{2^{+1}} \\
&= \frac{3 \times 3 \times 3}{2} = \frac{27}{2} = 13\frac{1}{2}
\end{aligned}$$

**Question 2.**

Simplify:

$$(i) 8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$$

$$(ii) [(64)^{-2}]^{-3} \div [((-8)^2)^3]^2$$

$$(iii) (2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$$

**Solution:**

$$(i) 8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$$

$$= (2^3)^{\frac{4}{3}} + (5^2)^{\frac{3}{2}} - \left(\frac{1}{3^3}\right)^{-\frac{2}{3}}$$

$$= 2^{3 \times \frac{4}{3}} + 5^{2 \times \frac{3}{2}} - \frac{1}{3^{3 \times \left(\frac{-2}{3}\right)}}$$

$$= 2^4 + 5^3 - \frac{1}{3^{-2}}$$

$$= 16 + 125 - 3^2$$

$$= 141 - 9 = 132$$

$$= 2^{3 \times \frac{4}{3}} + 5^{2 \times \frac{3}{2}} - \frac{1}{3^{3 \times \left(\frac{-2}{3}\right)}}$$

$$= 2^4 + 5^3 - \frac{1}{3^{-2}}$$

$$= 16 + 125 - 3^2$$

$$= 141 - 9 = 132$$

$$(ii) [(64)^{-2}]^{-3} \div [ \{(-8)^2\}^3 ]^2$$

$$= (2^6)^{-2 \times -3} \div (-8)^{2 \times 3 \times 2}$$

$$= 2^{6 \times (6)} \div (-8)^{12}$$

$$= 2^{+36} \div (-8)^{12}$$

$$= 2^{+36} \div [(-2)^3]^{12} = 2^{36} \div (-2)^{36}$$

$$= \frac{2^{36}}{(-2)^{36}} = \frac{2^{36}}{2^{36}} \quad (\because 36 \text{ is even})$$

$$= 2^{36-36} = 2^0 = 1 \quad (\because a^0 = 1)$$

$$(iii) (2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$$

$$= (2^{-3})^2 - (2^{-4})^2$$

$$\{\because (a - b)(a + b) = a^2 - b^2\}$$

$$= 2^{-6} - 2^{-8} = \frac{1}{2^6} - \frac{1}{2^8}$$

$$= \frac{1}{64} - \frac{1}{256}$$

$$= \frac{4-1}{256} = \frac{3}{256}$$

### Question 3.

Evaluate:

- (i)  $(-5)^0$                       (ii)  $8^0 + 4^0 + 2^0$   
 (iii)  $(8 + 4 + 2)^0$             (iv)  $4x^0$   
 (v)  $(4x)^0$                       (vi)  $[(10^3)^0]^5$   
 (vii)  $(7x^0)^2$   
 (viii)  $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

**Solution:**

- (i)  $(-5)^0 = 1$                       ( $\because a^0 = 1$ )  
 (ii)  $8^0 + 4^0 + 2^0$   
 $= 1 + 1 + 1 = 3$                       ( $\because a^0 = 1$ )  
 (iii)  $(8 + 4 + 2)^0 = (14)^0 = 1$                       ( $\because a^0 = 1$ )  
 (iv)  $4x^0 = 4 \times 1 = 4$   
 (v)  $(4x)^0 = 1$   
 (vi)  $[(10^3)^0]^5 = 10^{3 \times 0 \times 5} = 10^0 = 1$   
 (vii)  $(7x^0)^2 = 7^2 \times x^{0 \times 2} = 49 \times 1 = 49$   
 (viii)  $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

$$= 1 + \frac{1}{9} - \frac{1}{9^2} + (3^2)^{\frac{1}{2}} - (3^2)^{-\frac{1}{2}}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + 3^{2 \times \frac{1}{2}} - 3^{2 \times (-\frac{1}{2})}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + 3 - 3^{-1}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + \frac{3}{1} - \frac{1}{3}$$

$$= \frac{81 + 9 - 1 + 243 - 27}{81} = \frac{333 - 28}{81}$$

$$= \frac{305}{81} = 3 \frac{62}{81}$$

**Question 4.**

Simplify:

(i)  $\frac{a^5b^2}{a^2b^{-3}}$

(ii)  $15y^8 \div 3y^3$

(iii)  $x^{10}y^6 \div x^3y^{-2}$

(iv)  $5z^{16} \div 15z^{-11}$

(v)  $(36x^2)^{\frac{1}{2}}$

(vi)  $(125x^{-3})^{\frac{1}{3}}$

(vii)  $(2x^2y^{-3})^{-2}$

(viii)  $(27x^{-3}y^6)^{\frac{2}{3}}$

(ix)  $(-2x^{\frac{2}{3}}y^{-\frac{3}{2}})^6$

**Solution:**

$$(i) \quad \frac{a^5 b^2}{a^2 b^{-3}} = a^{5-2} \cdot b^{2+3} \\ = a^3 b^5$$

$$(ii) \quad 15y^8 \div 3y^3 = \frac{15y^8}{3y^3} \\ = 5y^{8-3} \\ = 5y^5$$

$$(iii) \quad x^{10}y^6 \div x^3y^{-2} = \frac{x^{10}y^6}{x^3y^{-2}} \\ = x^{10-3} \cdot y^{6+2} \\ = x^7y^8$$

$$(iv) \quad 5z^{16} + 15z^{-11} = \frac{5z^{16}}{15z^{-11}} \\ = \frac{5}{15} z^{16+11} \\ = \frac{1}{3} z^{27}$$

$$(v) \quad (36x^2)^{1/2} = (36)^{1/2} \cdot x^{2 \times \frac{1}{2}} \\ = (6 \times 6)^{1/2} \cdot x = (6^2)^{1/2} \cdot x \\ = 6x$$

$$(vi) \quad (125x^{-3})^{1/3} = (125)^{1/3} \cdot x^{-3 \times 1/3} \\ = (5 \times 5 \times 5)^{1/3} \cdot x^{-1}$$



$$(5^3)^{\frac{1}{3}} \cdot x^{-1} = 5x^{-1}$$

$$= \frac{5}{x} = 5x^{-1}$$

$$(vii) \quad (2x^2y^{-3})^{-2} = 2^{-2} \cdot x^{2 \times -2} \cdot y^{-3 \times -2}$$

$$= \frac{1}{2^2} x^{-4} \cdot y^6$$

$$= \frac{1}{4} \times \frac{y^6}{x^4}$$

$$= \frac{y^6}{4x^4} = \frac{1}{4} \cdot y^6 \cdot x^{-4}$$

$$(viii) \quad (27x^{-3}y^6)^{2/3} = (27)^{2/3} \cdot x^{-3 \times \frac{2}{3}} \cdot y^{6 \times \frac{2}{3}}$$

$$= (3 \times 3 \times 3)^{2/3} \cdot x^{-2} \cdot y^4$$

$$= [(3 \times 3 \times 3)^{1/3}]^2 \cdot x^{-2} \cdot y^4$$

$$= 3^2 \cdot x^{-2} \cdot y^4$$

$$= 9x^{-2}y^4$$

$$= \frac{9y^4}{x^2} = 9x^{-2}y^4$$

$$(ix) \quad (-2x^{2/3}y^{-3/2})^6$$

$$= (-2)^6 \cdot x^{2/3 \times 6} \cdot y^{-3/2 \times 6}$$

$$= 64x^4y^{-9}$$

$$= \frac{64x^4}{y^9}$$

$$= 64x^4y^{-9}$$

### Question 5.

Simplify:

$$(x^{a+b})^{a-b} \cdot (x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a}$$

**Solution:**

$$\begin{aligned} & (x^{a+b})^{a-b} \cdot (x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a} \\ &= x^{(a+b)(a-b)} \cdot x^{(b+c)(b-c)} \cdot x^{(c+a)(c-a)} \\ &= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2} \\ &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\ &= x^0 \\ &= 1 \end{aligned}$$

**Question 6.**

Simplify:

$$(i) \quad \sqrt[5]{x^{20}y^{-10}z^5} + \frac{x^3}{y^3}$$

**Solution:**

$$(ii) \quad \left( \frac{256a^{16}}{81b^4} \right)^{\frac{-3}{4}}$$

$$\begin{aligned}
 (i) \quad & \sqrt[5]{x^{20}y^{-10}z^5} + \frac{x^3}{y^3} \\
 &= (x^{20}y^{-10}z^5)^{1/5} + \frac{x^3}{y^3} \\
 &= x^{20 \times \frac{1}{5}} \cdot y^{-10 \times \frac{1}{5}} \cdot z^{5 \times \frac{1}{5}} + \frac{x^3}{y^3} \\
 &= x^4 \cdot y^{-2} \cdot z^1 \times \frac{y^3}{x^3} \\
 &= x^{4-3} \cdot y^{-2+3} \cdot z^1 \\
 &= xyz
 \end{aligned}$$

$$(ii) \quad \left[ \frac{256a^{16}}{81b^4} \right]^{-3/4} = \left[ \frac{4^4 a^{16}}{3^4 b^4} \right]^{-3/4}$$

$  \begin{aligned}  256 &= 4 \times 4 \times 4 \times 4 = 4^4 \\  81 &= 3 \times 3 \times 3 \times 3 = 3^4  \end{aligned}  $
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$$\begin{aligned}
 &= \frac{4^{4 \times \frac{-3}{4}} \cdot a^{16 \times \frac{-3}{4}}}{3^{4 \times \frac{-3}{4}} \cdot b^{4 \times \frac{-3}{4}}} \\
 &= \frac{4^{-3} \cdot a^{-12}}{3^{-3} \cdot b^{-3}} \\
 &= \frac{3^3 b^3}{4^3 a^{12}} \\
 &= \frac{27b^3}{64a^{12}} \\
 &= \frac{27}{64} \cdot a^{-12} b^3
 \end{aligned}$$

<p><b>Note :</b></p> $4^{-3} = \frac{1}{4^3}$ $\frac{1}{3^{-3}} = 3^3$ $a^{-12} = \frac{1}{a^{12}}$ $\frac{1}{b^{-3}} = b^3$
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**Question 7.**

(i)  $(a^{-2})^2 \cdot (ab)^{-3}$

$$(ii) (x^ny^{-m})^4 \times (x^3y^{-2})^{-n}$$

$$(iii) \left( \frac{125a^{-3}}{y^6} \right)^{-1/3}$$

$$(iv) \left( \frac{32x^{-5}}{243y^{-5}} \right)^{-1/5}$$

$$(v) (a^{-2}b)^{1/2} \times (ab^{-3})^{1/3}$$

$$(vi) (xy)^{m-n} \cdot (yz)^{n-l} \cdot (zx)^{l-m}$$

**Solution:**

$$(i) (a^{-2}b)^{-2} \cdot (ab)^{-3}$$

$$= (a^{-2 \times -2} \cdot b^{-2}) \cdot (a^{-3} \cdot b^{-3})$$

$$= a^{+4} \cdot b^{-2} \cdot a^{-3} \cdot b^{-3}$$

$$= a^{4-3} \cdot b^{-2-3}$$

$$= ab^{-5}$$

$$= \frac{a}{b^5} \text{ Ans.}$$

$$(ii) (x^ny^{-m})^4 \times (x^3y^{-2})^{-n}$$

$$= x^{4n} y^{-4m} \times x^{-3n} y^{2n}$$

$$= x^{4n-3n} \cdot y^{-4m+2n}$$

$$= x^ny^{-4m+2n}$$

$$(iii) \left[ \frac{125a^{-3}}{y^6} \right]^{-1/3} = \left[ \frac{5^3 a^{-3}}{y^6} \right]^{-1/3}$$

$$\boxed{125 = 5 \times 5 \times 5 = 5^3}$$

$$= \frac{5^{3 \times \frac{-1}{3}} \cdot a^{-3 \times \frac{-1}{3}}}{y^{6 \times \frac{-1}{3}}}$$

$$= \frac{5^{-1} \cdot a^1}{y^{-2}}$$

$$(iv) \left[ \frac{32x^{-5}}{243y^{-5}} \right]^{\frac{-1}{5}} = \left[ \frac{2^5 x^{-5}}{3^5 y^{-5}} \right]^{\frac{-1}{5}}$$

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$
---

$$= \frac{2^{5 \times \frac{-1}{5}} \cdot x^{-5 \times \frac{-1}{5}}}{3^{5 \times \frac{-1}{5}} y^{-5 \times \frac{-1}{5}}}$$

$$= \frac{2^{-1} x^1}{3^{-1} y^1}$$

$$= \frac{3x}{2y}$$

$$(v) (a^{-2}b)^{\frac{1}{2}} \times (ab^{-3})^{\frac{1}{3}}$$

$$= (a^{-2 \times \frac{1}{2}} b^{1/2}) \times (a^{1/3} b^{-3 \times \frac{1}{3}})$$

$$= a^{-1} b^{1/2} \times a^{1/3} b^{-1}$$

$$= a^{-1 + \frac{1}{3}} b^{\frac{1}{2} - 1}$$

$$= a^{-2/3} b^{-1/2}$$

$$= \frac{1}{a^{2/3} b^{1/2}}$$

$$(vi) (xy)^{m-n} \cdot (yz)^{n-l} \cdot (xz)^{l-m}$$

$$= x^{m-n} \cdot y^{m-n} \cdot y^{n-l} \cdot z^{n-l} \cdot x^{l-m} \cdot z^{l-m}$$

$$= x^{m-n+l-m} \cdot y^{m-n+n-l} \cdot z^{n-l+l-m}$$

$$= x^{l-n} \cdot y^{m-l} \cdot z^{n-m}$$

### Question 8.

Show that:

$$\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a} = 1$$

**Solution:**

**L.H.S.**

$$\begin{aligned} & \left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a} \\ &= (x^{a+b})^{a-b} \cdot (x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a} \\ &= x^{(a+b)(a-b)} \cdot x^{(b+c)(b-c)} \cdot x^{(c+a)(c-a)} \\ &= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2} \\ &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\ &= x^0 \\ &= 1 = \text{R.H.S.} \end{aligned}$$

**Question 9.**

Evaluate:

$$\frac{x^{5+n}(x^2)^{3n+1}}{x^{7n-2}}$$

**Solution:**

$$\begin{aligned} & \frac{x^{5+n} \times (x^2)^{3n+1}}{x^{7n-2}} \\ &= \frac{x^{5+n} \times x^{2(3n+1)}}{x^{7n-2}} \\ &= \frac{x^{5+n} \times x^{6n+2}}{x^{7n-2}} \\ &= x^{5+n+6n+2-7n+2} \\ &= x^9 \end{aligned}$$

**Question 10.**

Evaluate:

$$\frac{a^{2n+1} \times a^{(2n+1)(2n-1)}}{a^{n(4n-1)} \times (a^2)^{2n+3}}$$

**Solution:**

$$\begin{aligned} & \frac{a^{2n+1} \times a^{(2n+1)(2n-1)}}{a^{n(4n-1)} \times (a^2)^{2n+3}} \\ &= \frac{a^{2n+1} \times a^{(2n)^2 - (1)^2}}{a^{4n^2 - n} \times a^{2(2n+3)}} \\ &= \frac{a^{2n+1} \times a^{4n^2 - 1}}{a^{4n^2 - n} \times a^{4n+6}} \\ &= a^{2n+1+4n^2-1-4n^2+n-4n-6} \\ &= a^{-n-6} \\ &= a^{-(n+6)} \\ &= \frac{1}{a^{n+6}} \end{aligned}$$

**Question 11.**

$$(m+n)^{-1} (m^{-1} + n^{-1}) = (mn)^{-1}$$

**Solution:**

$$\text{L.H.S. } (m+n)^{-1} (m^{-1} + n^{-1})$$

$$\begin{aligned} &= \frac{1}{m+n} \left( \frac{1}{m} + \frac{1}{n} \right) = \frac{1}{m+n} \cdot \frac{n+m}{mn} = \frac{1}{mn} \\ &= (mn)^{-1} \\ &= \text{R.H.S.} \end{aligned}$$

Hence proved.

**Question 12.**

Prove that:

$$(i) \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$$

$$(ii) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

**Solution:**

$$(i) \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$$

$$\text{L.H.S.} = \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$$

$$(x^{a-b})^{\frac{1}{ab}} (x^{b-c})^{\frac{1}{bc}} (x^{c-a})^{\frac{1}{ca}}$$



$$= x^{\frac{a-b}{ab}} x^{\frac{b-c}{bc}} x^{\frac{c-a}{ca}} \quad \left\{ (x^a)^b = x^{ab} \right\}$$

$$= x^{\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ca}}$$

$$= x^{\frac{ac-bc+ab-ac+bc-ab}{abc}}$$

$$= x^0 = 1 = \text{R.H.S.} \quad (\because x^0 = 1)$$

$$(ii) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

$$\text{L.H.S.} = \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$$

$$= \frac{1}{x^{a-a} + x^{a-b}} + \frac{1}{x^{b-b} + x^{b-a}}$$

$$= \frac{1}{x^a \cdot x^{-a} + x^a \cdot x^{-b}} + \frac{1}{x^b \cdot x^{-b} + x^b \cdot x^{-a}}$$

$$= \frac{1}{x^a (x^{-a} + x^{-b})} + \frac{1}{x^b (x^{-b} + x^{-a})}$$

$$= \frac{1}{(x^{-a} + x^{-b})} \left[ \frac{1}{x^a} + \frac{1}{x^b} \right]$$

$$= \frac{1}{x^{-a} + x^{-b}} [x^a + x^b] = 1 = \text{R.H.S.}$$

### Question 13.

Find the values of n, when:

$$(i) 12^{-5} \times 12^{2n+1} = 12^{13} \div 12^7$$

$$(ii) \frac{a^{2n-3} \times (a^2)^{n+1}}{(a^4)^{-3}} = (a^3)^3 \div (a^6)^{-3}$$

**Solution:**

$$(i) 12^{-5} \times 12^{2n+1} = 12^{13} \div 12^7$$

$$12^{-5+2n+1} = \frac{12^{13}}{12^7}$$

$$12^{2n-4} = 12^{13-7}$$

$$12^{2n-4} = 12^6$$

Comparing both sides, we get

$$2n - 4 = 6$$

$$\Rightarrow 2n = 6 + 4$$

$$\Rightarrow 2n = 10$$

$$\Rightarrow n = 5$$

$$(ii) \frac{a^{2n-3} \times (a^2)^{n+1}}{(a^4)^{-3}} = (a^3)^3 \div (a^6)^{-3}$$

$$\frac{a^{2n-3} \times 2^{2n+2}}{a^{-12}} = a^9 \div a^{-18}$$

$$\frac{a^{2n-3} \times 2^{2n+2}}{a^{-12}} = \frac{a^9}{a^{-18}}$$

$$a^{2n-3+2n+2-(-12)} = a^9 - (-18)$$

$$a^{4n+11} = a^{27}$$

Comparing both sides, we get

$$4n + 11 = 27$$

$$\Rightarrow 4n = 27 - 11$$

$$\Rightarrow n = \frac{16}{4} = 4$$

#### Question 14.

Simplify:

$$(i) \frac{a^{2n-3} \cdot a^{(2n+1)(n+2)}}{(a^3)^{2n+1} \cdot a^{n(2n+1)}}$$

$$(ii) \frac{x^{2n+7} \cdot (x^2)^{3n+2}}{x^{4(2n+3)}}$$

**Solution:**

$$(i) \frac{a^{2n-3} \cdot a^{(2n+1)(n+2)}}{(a^3)^{2n+1} \cdot a^{n(2n+1)}}$$

$$\text{Given expression} = \frac{a^{2n+3} \cdot a^{(2n^2+4n+n+2)}}{a^{6n+3} \cdot a^{2n^2+n}}$$

$$= \frac{a^{2n+3+2n^2+5n+2}}{a^{6n+3+2n^2+n}} = \frac{a^{2n^2+7n+5}}{a^{2n^2+7n+3}}$$

$$= \frac{a^{(2n^2+7n+3)+2}}{a^{2n^2+7n+3}} = a^2$$

$$(ii) \frac{x^{2n+7} \cdot (x^2)^{3n+2}}{x^{4(2n+3)}}$$

$$\text{Given expression} = \frac{x^{2n+7} \cdot x^{6n+4}}{x^{8n+12}}$$

$$= \frac{x^{2n+7+6n+4}}{x^{8n+12}} = \frac{x^{8n+11}}{x^{8n+12}}$$

$$= x^{8n+11-8n-12} = x^{-1}$$

$$= \frac{1}{x}$$