3. Squares and Square Roots

EXERCISE 3(A)

Question 1.
Find the square of :
(i) 59
(ii) 63
(iii) 15
Solution:
(i) Square of 59= 59 x 59 = 3481
(ii) Square of 6.3 = 6.3 x 6.3 = 39.69
(iii) Square of 15 = 15 x 15 = 225

Question 2.
By splitting into prime factors, find the square root of :
(i) 11025
(if) 396900
(iii) 194481
Solution:

(*i*) $\sqrt{11025}$

$= \sqrt{\overline{5\times5}\times\overline{7\times7}\times\overline{3\times3}}$

$= 5\times7\times3 = 105$

5	11025
5	2205
7	441
7	63
3	9
	3

(*ii*) $\sqrt{396900}$

$= \sqrt{\overline{2\times2}\times\overline{3\times3}\times\overline{3\times3}\times\overline{5\times5}\times\overline{7\times7}}$

$= 2\times3\times3\times5\times7 = 630$

2	396900
2	198450
3	99225
3	33075
3	11025
3	3675
5	1225

5	245
7	49
	7

(iii) $\sqrt{194481}$

$= \sqrt{\overline{3\times3}\times\overline{3\times3}\times\overline{7\times7}\times\overline{7\times7}}$

$= 3\times3\times7\times7 = 441$

3	194481
3	64827
3	21609
3	7203
7	2401
7	343
7	49
	7

Question 3.

(i) Find the smallest number by which 2592 be multiplied so that the product is a perfect square.
(ii) Find the smallest number by which 12748 be mutliplied so that the product is a perfect square?

Solution:

(i) $2592 = \overline{2\times2}\times\overline{2\times2}\times2\times\overline{3\times3}\times\overline{3\times3}$

On grouping the prime factors of 2592 as shown; on factor i.e. 2 is left which cannot be paired with equal factor.

2	2592
2	1296
2	648
2	324
2	162
3	81

The given number should be multiplied by 2 to make the given number a perfect square.

$12748 = \overline{2\times2}\times3187$

On grouping the prime factors of 12748 as shown; one factor i.e. 3187 is left which cannot be paired with equal factor.

2	12748
2	6374
	3187

The given number should be multiplied by 3187.

Question 4.

Find the smallest number by which 10368 be divided, so that the result is a perfect square. Also, find the square root of the resulting numbers.

Solution:

$$10368$$

$$= \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times 2\times\overline{3\times 3}\times\overline{3\times 3}$$

On grouping the prime factors of 10368 as shown; one factor *i.e.* 2 is left which cannot be paired with equal factor.

2	10368
2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
	3

$\therefore$ The given number should be divided by 2.

$$\text{Now } \sqrt{\frac{10368}{2}}$$

$$= \sqrt{\frac{\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times 2\times\overline{3\times 3}\times\overline{3\times 3}}{2}}$$

$$= 2\times 2\times 2\times 3\times 3 = 72$$

Question 5.

Find the square root of :

(i) 0.1764

(ii) $96\frac{1}{25}$
(iii) 0.0169

Solution:

$$(i)\ \sqrt{0{\cdot}1764} = \sqrt{\frac{01764}{10000}}$$

Question 6.
Evaluate

$(i)\ \sqrt{\frac{14{\cdot}4}{22{\cdot}5}}$ $(ii)\ \sqrt{\frac{0{\cdot}225}{28{\cdot}9}}$

Solution:

$(iii)\ \sqrt{\frac{25}{32}\times 2\frac{13}{18}\times 0{\cdot}25}$

$(iv)\ \sqrt{1\frac{4}{5}\times 14\frac{21}{44}\times 2\frac{7}{55}}$

$(i)\ \sqrt{\frac{14{\cdot}4}{22{\cdot}5}} = \sqrt{\frac{144}{225}}$

$= \sqrt{\frac{12\times 12}{15\times 15}}$

$= \frac{12}{15} = 0{\cdot}8$

$(ii)\ \sqrt{\frac{0{\cdot}225}{28{\cdot}9}} = \sqrt{\frac{0{\cdot}225}{28{\cdot}900}}$

$= \sqrt{\frac{225}{28900}} = \sqrt{\frac{15\times 15}{17\times 17\times 10\times 10}}$

$= \frac{15}{17\times 10} = \frac{15}{170} = \frac{3\times 5}{5\times 34} = \frac{3}{34}$

$$\begin{array}{r|l} 17 & 28900 \\ \hline 17 & 1700 \\ \hline 10 & 100 \\ \hline & 10 \end{array}$$

$$\begin{array}{r} 0{\cdot}8 \\ 15\overline{)120(} \\ 120 \\ \hline \times \end{array}$$

(*iii*) $\sqrt{\frac{25}{32}\times 2\frac{13}{18}\times 0{\cdot}25}$

$= \sqrt{\frac{25}{32}\times\frac{49}{18}\times 0{\cdot}25} = \sqrt{\frac{25}{32}\times\frac{49}{18}\times\frac{25}{100}}$

$= \sqrt{\frac{25\times 49\times \cancel{25}^{1}}{32\times 18\times \cancel{100}^{4}}} = \sqrt{\frac{25\times 49}{32\times 18\times 4}}$

$= \sqrt{\frac{5\times 5\times 7\times 7}{(2\times 2\times 2\times 2\times 2)\times(2\times 3\times 3)\times(2\times 2)}}$

$= \sqrt{\frac{\overline{5\times 5}\times\overline{7\times 7}}{\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}\times\overline{2\times 2}}}$

$= \frac{5\times 7}{2\times 2\times 2\times 3\times 2} = \frac{35}{48}$

(*iv*) $\sqrt{1\frac{4}{5}\times 14\frac{21}{44}\times 2\frac{7}{55}}$

$= \sqrt{\frac{9}{5}\times\frac{637}{44}\times\frac{117}{55}} = \sqrt{\frac{9\times 637\times 117}{5\times 44\times 55}}$

$= \sqrt{\frac{9\times 7\times 7\times 13\times 13\times 9}{5\times 11\times 2\times 2\times 11\times 5}}$

7	637
7	91
	13

9	117
	13

$= \frac{9\times 7\times 13}{5\times 11\times 2} = \frac{819}{110} = 7\frac{49}{110}$

Question 7.

Evaluate : (*i*) $\sqrt{3^2 \times 6^3 \times 24}$

(*ii*) $\sqrt{(0{\cdot}5)^3 \times 6 \times 3^5}$ (*iii*) $\sqrt{\left(5+2\frac{21}{25}\right) \times \frac{0{\cdot}169}{1{\cdot}6}}$

(*iv*) $\sqrt{5\left(2\frac{3}{4}-\frac{3}{10}\right)}$ (*v*) $\sqrt{248+\sqrt{52+\sqrt{144}}}$

Solution:

(*i*) $\sqrt{3^2 \times 6^3 \times 24}$

$= \sqrt{3^2 \times 6^3 \times 2 \times 2 \times 6} = \sqrt{3^2 \times 6^4 \times 2^2}$

$= 3 \times 6^2 \times 2 = 3 \times 36 \times 2 = 216$

(*ii*) $\sqrt{(0{\cdot}5)^3 \times 6 \times 3^5}$

$= \sqrt{(0{\cdot}5)^2 \times 0{\cdot}5 \times 3 \times 2 \times 3^5}$

$= \sqrt{(0{\cdot}5)^2 \times 0{\cdot}5 \times 2 \times 3 \times 3^5}$

$= \sqrt{(0{\cdot}5)^2 \times 1{\cdot}0 \times 3^6}$ $[0{\cdot}5 \times 2 = 1{\cdot}0]$

$= \sqrt{(0{\cdot}5)^2 \times 1 \times 3^6} = 0{\cdot}5 \times 3^3$

$= 0{\cdot}5 \times 27 = 13{\cdot}5$

(*iii*) $\sqrt{\left(5+2\frac{21}{25}\right) \times \frac{0{\cdot}169}{1{\cdot}6}}$

$$= \sqrt{\left(5+\frac{71}{25}\right)\times\frac{0{\cdot}169}{1{\cdot}600}} = \sqrt{\frac{196}{25}\times\frac{169}{1600}}$$

$$= \sqrt{\frac{14\times14}{5\times5}\times\frac{13\times13}{40\times40}} = \frac{14\times13}{5\times40}$$

$$= \frac{7\times13}{5\times20} = \frac{91}{100} = 0{\cdot}91$$

(iv) $\sqrt{5\left(2\frac{3}{4}-\frac{3}{10}\right)} = \sqrt{5\left(\frac{11}{4}-\frac{3}{10}\right)}$

$$= \sqrt{5\left(\frac{55-6}{20}\right)} = \sqrt{5\left(\frac{49}{20}\right)}$$

$$= \sqrt{\frac{5\times49}{20}} = \sqrt{\frac{49}{4}} = \sqrt{\frac{7\times7}{2\times2}}$$

$$= \frac{7}{2} = 3\frac{1}{2}$$

(v) $\sqrt{248+\sqrt{52+\sqrt{144}}}$

$= \sqrt{248+\sqrt{52+12}}$ $(\because \sqrt{144} = 12)$

$= \sqrt{248+\sqrt{64}} = \sqrt{248+8}$ $(\because \sqrt{64} = 8)$

$= \sqrt{256} = 16$ $(\because \sqrt{256} = \sqrt{16\times16} = 16)$

Question 8.

A man, after a tour, finds that he had spent every day as many rupees as the number of days he had been on tour. How long did his tour last, if he had spent in all ₹ 1,296

Solution:

Let the number of days he had spent = x

Number of rupees spent in each day = x

Total money spent = x x x = x^2 = 1,296 (given)

$\therefore x = \sqrt{1296}$

$\Rightarrow x = \sqrt{4 \times 4 \times 9 \times 9}$

$x = 4 \times 9$

$\Rightarrow x = 36$

4	1296
4	324
9	81
	9

Hence required number of days = 36

Question 9.

Out of 745 students, maximum are to be arranged in the school field for a P.T. display, such that the number of rows is equal to the number of columns. Find the number of rows if 16 students were left out after the arrangement.

Solution:

Total number of students = 745

Students left after standing in arrangement = 16

No. of students who were to be arranged = 745 – 16 = 729

The number of rows = no. of students in each row

No. of rows = √729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$= \sqrt{3 \times 3 \times 3 \times 3 \times 3 \times 3} = 3 \times 3 \times 3 = 27$

Question 10.

13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Find two more such pairs.

Solution:

$(13)^2 = 169$ and $(31)^2 = 961$

Similarly, two such number can be 12 and 21

$\therefore$ $(12)^2 = 144$ and $(21)^2 = 441$

and 102, 201

$(102)^2 = 102 \times 102 = 10404$

and $(201)^2 = 201 \times 201 = 40401$

$$\begin{array}{r} 102 \\ \times 102 \\ \hline 204 \\ 1020 \\ \hline 10404 \end{array} \qquad \begin{array}{r} 201 \\ \times 201 \\ \hline 201 \\ 4020 \\ \hline 40401 \end{array}$$

Question 11.

Find the smallest perfect square divisible by 3, 4, 5 and 6.

Solution:

L.C.M. of 3, 4, 5, 6 = 2 x 2 x 3 x 5 = 60

2	3, 4, 5, 6
3	3, 2, 5; 3
	1, 2, 5, 1

in which 3 and 5 are not in pairs L.C.M. = 2 x 3 x 2 x 5 = 60

We should multiple it by 3 x 5 i.e. by 15

Required perfect square = 60 x 15 = 900

Question 12.

If $\sqrt{784} = 28$, find the value of:

(i) $\sqrt{7.84} + \sqrt{78400}$

(ii) $\sqrt{0.0784} + \sqrt{0.000784}$

Solution:

$$\sqrt{784} = 28$$

$$\therefore \sqrt{7.84} = \sqrt{\frac{784}{100}} = \frac{28}{10} = 2.8$$

$$\sqrt{78400} = \sqrt{28\times28\times10\times10}$$
$$= 28 \times 10 = 280$$

$$\sqrt{0.0784} = \sqrt{\frac{784}{10000}} = \frac{28}{100} = 0.28$$

$$\text{and } \sqrt{0.000784} = \sqrt{\frac{784}{1000000}}$$

$$= \sqrt{\frac{28\times28}{10\times10\times10\times10\times10\times10}}$$

$$= \frac{28}{10\times10\times10} = \frac{28}{1000} = 0.028$$

Now,

(*i*) $\sqrt{7.84} + \sqrt{78400} = 2.8 + 280 = 282.8$

(*ii*) $\sqrt{0.0784} + \sqrt{0.000784}$
$= 0.28 + 0.028 = 0.308$

EXERCISE 3(B)

Question 1.

Find the square root of:

(i) 4761
(ii) 7744
(iii) 15129
(iv) 0.2916
(v) 0.001225
(vi) 0.023104
(vii) 27.3529

Solution:

Sol. (*i*) 4761

	69
6	4761
	36
129	1161
	1161
	×

Required square root = 69

(*ii*) 7744

	88
8	7744
	64
168	1344
	1344
	×

Required square root = 88

(*iii*) 15129

	123
1	15129
	1
22	51
	44
243	729
	729
	×

Required square root = 123

(*iv*) 0.2916

	0.54
0.5	0.2916
	0.25
0.104	416
	416
	×

Required square root = 0.54

(*v*) 0.001225

	0.035
0.03	0.001225
	9
0.065	325
	325
	×

Required square root = 0.035

(*vi*) 0.023104

	0.152
0.1	0.023104
	0.01
.25	131
	125
.302	604
	604
	×

Required square root of = 0.152

(*vii*) 27.3529

	5.23
5	27.3529
	25
102	2.35
	2.04
1043	3129
	3129
	×

Required square root = 5.23

Question 2.
Find the square root of:
(i) 4.2025
(ii) 531.7636
(iii) 0.007225
Solution:
Sol. (*i*) 4.2025

	2.05
2	4.2025
	4
405	.2025
	.2025
	×

Required square root = 2.05

(*ii*) 531.7636

	23.06
2	531.7636
	4
43	131
	129
4606	2.7636
	2.7636
	×

Required square root = 23.06

(*iii*) 0.007225

	0.085
.8	.007225
	64
0.165	825
	825
	×

Required square root = 0.085

Question 3.
Find the square root of:
(i) 245 correct to two places of decimal.
(ii) 496 correct to three places of decimal.
(iii) 82.6 correct to two places of decimal.
(iv) 0.065 correct to three places of decimal.

(v) 5.2005 correct to two places of decimal.
(vi) 0.602 correct to two places of decimal

Solution:

Sol. (*i*) 245

	15.65
1	245
	1
25	145
	125
306	2000
	1836
3125	16400
	15625
	775

Required square root = 15.65 upto two places of decimal.

(*ii*) 496

	22.271
2	496
	4
42	96
	84
442	1200
	884
4447	31600
	31129
44541	47100
	44541

Required square root = 22.2708 = 22.271 upto three places of decimals.

(*iii*) 82.6

	9.088
9	82.60
	81
1808	16000
	14464
18168	153600
	145324

Required square root = 9.088 = 9.09 upto two places of decimal.

(*iv*) 0.065

	.2549
.2	0.0650
	.04
.45	250
	225
.504	2500
	2016
.5089	48400
	45801

Required square root = .255 upto three places of decimal.

(v) 5.2005

	2.28
2	5.2005
	4
42	120
	84
448	3605
	3584
456	2100

Required square root = 2.28 upto two places of decimal.

(vi) 0.602

	0.775
0.7	0.6020
	.49
0.147	1120
	1029
1545	9100
	7725
	1375

Required square root = 0.78 upto two places of decimals.

Question 4.

Find the square root of each of the following correct to two decimal places:

(i) $3\frac{4}{5}$

(ii) $6\frac{7}{8}$

Solution:

Sol. (*i*) $3\frac{4}{5} = 3.80$

	1.949
1	3.80
	1
29	280
	261
384	1900
	1536
3889	36400
	35001
	1399

Reqd. square root = 1.949 = 1.95 upto two places of decimal

(*ii*) $6\frac{7}{8} = 6.875$

	2.62
2	6.8750
	4
46	287
	276
522	1150
	1044
	106

Reqd. sqare root =2.62

Question 5.
For each of the following, find the least number that must be subtracted so that the resulting number is a perfect square.
(i) 796
(ii) 1886
(iii) 23497
Solution:

Sol. (*i*) 796

Taking square root of 796, we find that 12 has been left

$$\begin{array}{r|l} & 28 \\ \hline 2 & \underline{7}\,\underline{96} \\ & 4 \\ \hline 48 & 396 \\ & 384 \\ \hline & 12 \end{array}$$

$\therefore$ Least number to be subtracted = 12

(*ii*) 1886

Taking square root of 1886, we find that 37 has been left

$$\begin{array}{r|l} & 43 \\ \hline 4 & \underline{18}\,\underline{86} \\ & 16 \\ \hline 83 & 286 \\ & 249 \\ \hline & 37 \end{array}$$

$\therefore$ Least number to be subtracted = 37

(*iii*) 23497

Taking square root of 23497, we find that 88 has been left

$$\begin{array}{r|l} & 153 \\ \hline 1 & 2\,\underline{34}\,\underline{97} \\ & 1 \\ \hline 25 & 134 \\ & 125 \\ \hline 303 & 997 \\ & 909 \\ \hline & 88 \end{array}$$

$\therefore$ Least number to be subtracted = 88

Question 6.

For each of the following, find the least number that must be added so that the resulting number is a perfect square.

(i) 511
(ii) 7172
(iii) 55078

Solution:

(i) 511

Taking square root of 511, we find that 27 has been left We see that 511 is greater than $(22)^2$

```
   22
 2 | 5 11
   | 4
42 | 111
   |  84
   |  27
```

On adding the required number to 511, we get $(23)^2$ i.e., 529

So, the required number = 529 – 511 = 18

(ii) 7172

Taking square root of 7172, we find that 116 has been left

We see that 7172 is greater than $(84)^2$

```
   84
 8 | 71 72
   | 64
164 | 772
    | 656
    | 116
```

∴ On adding the required number to 7172, we get $(85)^2$ i.e., 7225

Required number = 7225 – 7172 = 53

(iii) 55078

```
    234
 2 | 5 50 78
   | 4
43 | 150
   | 129
464 | 2178
    | 1856
    |  322
```

Taking square root of 55078, we find that 322 has been left

We see that 55078 is greater than $(234)^2$

On adding the required number to 55078, we get $(235)^2$ i.e., 55225

Required number = 55225 – 55078 = 147

Question 7.
Find the square root of 7 correct to two decimal places; then use it to find the value of $\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}}$ correct to three significant digits.
Solution:
$\sqrt{7} = 2.645 = 2.65$

$$\begin{array}{r|l} & 2.645 \\ 2 & \overline{7.00\ 00\ 00} \\ & 4 \\ \hline 46 & 300 \\ & 276 \\ \hline 524 & 2400 \\ & 2096 \\ \hline 5285 & 30400 \\ & 26425 \\ \hline & 3975 \end{array}$$

$$\text{Now, } \sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}}$$

$$= \sqrt{\frac{(4+\sqrt{7})(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})}}$$

$$= \sqrt{\frac{(4+\sqrt{7})^2}{(4)^2-(\sqrt{7})^2}}$$

$$= \sqrt{\frac{(4+\sqrt{7})^2}{16-7}} = \sqrt{\frac{(4+\sqrt{7})^2}{9}}$$

$$= \frac{4+\sqrt{7}}{3} = \frac{4+2.65}{3} = \frac{6.65}{3} = 2.22$$

Question 8.
Find the value of $\sqrt{5}$ correct to 2 decimal places; then use it to find the square root of $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$ correct to 2 significant digits.
Solution:

Sol. $\sqrt{5} = 2.236 = 2.24$

$$
\begin{array}{r|l}
 & 2.236 \\
\hline
2 & \overline{5.00\ 00\ 00} \\
 & 4 \\
\hline
42 & 100 \\
 & 84 \\
\hline
443 & 1600 \\
 & 1329 \\
\hline
4466 & 27100 \\
 & 26796 \\
\hline
 & 304
\end{array}
$$

$$\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}}$$

$$= \sqrt{\frac{(3-\sqrt{5})^2}{(3)^2-(\sqrt{5})^2}} = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}}$$

$$= \sqrt{\frac{(3-\sqrt{5})^2}{4}} = \frac{(3-2.24)}{2}$$

$$= \frac{(0.76)}{2} = 0.38$$

Question 9.

Find the square root of:

(i) $\frac{1764}{2809}$

(ii) $\frac{507}{4107}$

(iii) $\sqrt{108 \times 2028}$

(iv) 0.01 + √0.0064

Solution:

$$\begin{array}{r|l} & 42 \\ \hline 4 & 1764 \\ & 16 \\ \hline 82 & 164 \\ & 164 \\ \hline & \times \end{array} \qquad \begin{array}{r|l} & 53 \\ \hline 5 & 2809 \\ & 25 \\ \hline 103 & 309 \\ & 309 \\ \hline & \times \end{array}$$

Hence, square root of $\sqrt{\frac{1764}{2809}} = \frac{42}{53}$

(*ii*) $\frac{507}{4107} = \frac{507 \div 3}{4107 \div 3} = \frac{169}{1369}$

$$\begin{array}{r|l} & 13 \\ \hline 1 & 169 \\ & 1 \\ \hline 23 & 69 \\ & 69 \\ \hline & \times \end{array} \qquad \begin{array}{r|l} & 37 \\ \hline 3 & 1369 \\ & 9 \\ \hline 67 & 469 \\ & 469 \\ \hline & \times \end{array}$$

Hence, square root of $\sqrt{\frac{169}{1369}} = \frac{13}{37}$

(*iii*) $\sqrt{108 \times 2028} = \sqrt{219024}$

```
      468
   4 | 219024
     | 16
  86 |  590
     |  516
 928 |   7424
     |   7424
     |    ×
```

Hence, $\sqrt{108 \times 2028} = 468$

OR

$\sqrt{108 \times 2028}$

2	108
2	54
3	27
3	9
3	3
	1

2	2028
2	1014
3	507
13	169
13	13
	1

$= \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 2 \times 3 \times 13 \times 13}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 13 \times 13}$

$= 2 \times 2 \times 3 \times 13 = 468$

(*iv*) $0{\cdot}01 + \sqrt{0.0064}$

$= 0.01 + 0.08 = 0.09$

```
   0·08
 8 | 0·0064
   |   64
   |    ×
```

Question 10.

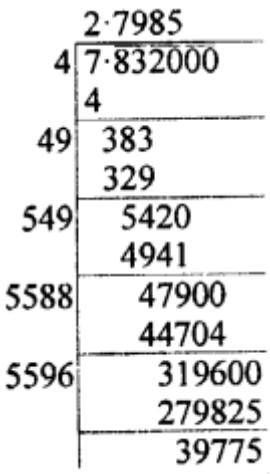
Find the square root of 7.832 correct to :

(i) 2 decimal places

(ii) 2 significant digits.

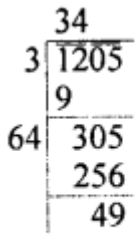
Solution:

Square root of 7.832



$\sqrt{7.832}$ = 2.80 upto two decimal places
= 2.8 upto two significant places

Question 11.
Find the least number which must be subtracted from 1205 so that the resulting number is a perfect square.
Solution:
Clearly, if 49 is subtracted from 1205, the number will be a perfect square.



$\therefore$ 1205 – 49 = 1156 and $\sqrt{1156}$ = 34

Question 12.
Find the least number which must be added to 1205 so that the resulting number is a perfect square.

Solution:

Sol. Clearly, 1205 is greater than 34^2

$$\begin{array}{r|l} & 34 \\ \hline 3 & 1205 \\ & 9 \\ \hline 64 & 305 \\ & 256 \\ \hline & 49 \end{array}$$

$\therefore$ On adding the required number to 1205, we shall be getting 35^2 *i.e.*, 1225

$\therefore$ The required number = 1225 – 1205 = 20

Question 13.

Find the least number which must be subtracted from 2037 so that the resulting number is a perfect square.

Solution:

Clearly; if 12 is subtracted from 2037, the remainder will be a perfect square.

$\therefore$ 2037 – 12 = 2035 and $\sqrt{2035} = 45$

$$\begin{array}{r|l} & 45 \\ \hline 4 & 2037 \\ & 49 \\ \hline 85 & 437 \\ & 425 \\ \hline & 12 \end{array}$$

Question 14.

Find the least number which must be added to 5483 so that the resulting number is a perfect square.

Solution:

Sol. Clearly, 5483 is greater than 74^2.

$$\begin{array}{r|l} & 74 \\ 9 & 5483 \\ & 49 \\ \hline 144 & 583 \\ & 576 \\ \hline & 7 \end{array}$$

$\therefore$ On adding the required number to 5483, we shall be getting 75^2 *i.e.* 5625.

Hence, the required number = 5625 – 5483

= 142

EXERCISE 3(C)

Question 1.

Seeing the value of the digit at unit's place, state which of the following can be square of a number :

(i) 3051

(ii) 2332

(iii) 5684

(iv) 6908

(v) 50699

Solution:

We know that the ending digit (the digit at units place) of the square of a number is 0, 1, 4, 5, 6, or 9

So, the following numbers can be squares : 3051, 5684, and 50699 i.e., (i), (iii), and (v)

Question 2.

Squares of which of the following numbers will have 1 (one) at their unit's place :

(i) 57

(ii) 81

(iii) 139

(iv) 73

(v) 64

Solution:

The square of the following numbers will have 1 at their units place as $(1)^2 = 1$, $(9)^2 = 81$

81 and 139 i.e., (i) and (iii)

Question 3.

Which of the following numbers will not have 1 (one) at their unit's place :

(i) 322
(ii) 572
(iii) 692
(iv) 3212
(v) 2652

Solution:

The square of the following numbers will not have 1 at their units place : as only $(1)^2 = 1$, $(9)^2 = 81$ have 1 at then units place
322, 572, 2652 i.e., (i), (ii) and (v)

Question 4.

Square of which of the following numbers will not have 6 at their unit's place :
(i) 35
(ii) 23
(iii) 64
(iv) 76
(v) 98

Solution:

The squares of the following numbers, Will not have 6 at their units place as only $(4)^2 = 16$, $(6)^2 = 36$ has but its units place 35, 23 and 98 i.e., (i), (ii), and (v)

Question 5.

Which of the following numbers will have 6 at their unit's place :
(i) 262
(ii) 492
(iii) 342
(iv) 432
(v) 2442

Solution:

The following numbers have 6 at their units place as $(4)^2 = 16$, $(6)^2 = 36$ has 6 at their units place 262, 342, 2442 i.e., (i), (iii) and (v)

Question 6.

If a number ends with 3 zeroes, how many zeroes will its square have ?

Solution:

We know that if a number ends with n zeros, then its square will have 2n zeroes at their ends
A number ends with 3 zeroes, then its square will have 3 x 2 = 6 zeroes

Question 7.

If the square of a number ends with 10 zeroes, how many zeroes will the number have ?

Solution:

We know that if a number ends with n zeros Then its square will have 2n zeroes
Conversely, if square of a number have 2n zeros at their ends then the number will

have n zeroes
The square of a number ends 10 zeroes, then the number will have $\frac{10}{2}$ = 5 zeroes

Question 8.
Is it possible for the square of a number to end with 5 zeroes ? Give reason.
Solution:
No, it is not possible for the square of a number, to have 5 zeroes which is odd because the number of zeros of the square must be 2n zeroes i.e., even number of zeroes.

Question 9.
Give reason to show that none of the numbers, given below, is a perfect square.
(i) 2162
(ii) 6843
(iii) 9637
(iv) 6598
Solution:
A number having 2,3,7 or 8 at the unit place is never a perfect square.

Question 10.
State, whether the square of the following numbers is even or odd?
(i) 23
(ii) 54
(iii) 76
(iv) 75
Solution:
(i) 23 – odd
(ii) 54 – even
(iii) 76 – odd
(iv) 75 – even

Question 11.
Give reason to show that none of the numbers 640, 81000 and 3600000 is a perfect square.
Solution:
No, number has an even number of zeroes.

Question 12.
Evaluate:
(i) $37^2 - 36^2$
(ii) $85^2 - 84^2$
(iii) $101^2 - 100^2$
Solution:

Sol. (*i*) $37^2 - 36^2$

Using property, for any natural number n,

$(n + 1)^2 - n^2 = (n + 1) + n$

$\Rightarrow (36 + 1)^2 - 36^2 = (36 + 1) + 36$

$\Rightarrow 37^2 - 36^2 = 37 + 36$

$\Rightarrow 37^2 - 36^2 = 73$

(*ii*) $85^2 - 84^2$

Using property, for any natural number n,

$(n + 1)^2 - n^2 = (n + 1) + n$

$\Rightarrow (84 + 1)^2 - 84^2 = (84 + 1) + 84$

$\Rightarrow 85^2 - 84^2 = 85 + 84$

$\Rightarrow 85^2 - 84^2 = 169$

(*iii*) $101^2 - 100^2$

Using property, for any natural number n,

$(n + 1)^2 - n^2 = (n + 1) + n$

$\Rightarrow (100 + 1)^2 - 100^2 = (100 + 1) + 100$

$\Rightarrow 101^2 - 100^2 = 101 + 100$

$\Rightarrow 101^2 - 100^2 = 201$

Question 13.

Without doing the actual addition, find the sum of:

(i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

(ii) 1 + 3 + 5 + 7 + 9 + ... + 39 + 41

(iii) 1 + 3 + 5 + 7 + 9 + ... + 51 + 53

Solution:

(i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 +23

= Sum of first 12 odd natural numbers = 122 = 144

(ii) 1+3 + 5 + 7 + 9 + ... + 39 + 41

= Sum of first 21 odd natural numbers = 212 = 441

(iii) 1 + 3 + 5 + 7 + 9 + ... + 51 + 53

= Sum of first 27 odd natural number = 272 = 729

Question 14.

Write three sets of Pythagorean triplets such that each set has numbers less than 30.

Solution:

The three sets of Pythagorean triplets such that each set has numbers less than 30 are 3, 4 and 5; 6, 8 and 10; 5, 12 and 13

Proof:

In 3, 4 and 5

$3^2 + 4^2 = 5^2$

$\Rightarrow 9 + 16 = 25$

$\Rightarrow 25 = 25$

In 6, 8 and 10

$6^2 + 8^2 = 10^2$

$\Rightarrow 36 + 64 = 100$

$\Rightarrow 100 = 100$

In 5, 12, and 13

$5^2 + 12^2 = 13^2$

$\Rightarrow 25 + 144 = 169$

$\Rightarrow 169 = 169$