4. Cubes and Cube-Roots

(Including use of tables for natural numbers)

EXERCISE 4(A)

Question 1.
Find the cube of :
(i) 7
(ii) 11
(iii) 16
(iv) 23
(v) 31
(vi) 42
(vii) 54
Solution:
(i) $(7)^3 = 7 \times 7 \times 7 = 343$
(ii) $(11)^3 = 11 \times 11 \times 11 = 1331$
(iii) $(16)^3 = 16 \times 16 \times 16 = 4096$
(iv) $(23)^3 = 23 \times 23 \times 23 = 12167$
(v) $(31)^3 = 31 \times 31 \times 31 = 29791$
(vi) $(42)^3 = 42 \times 42 \times 42 = 74088$
(vii) $(54)^3 = 54 \times 54 \times 54 = 157464$

Question 2.
Find which of the following are perfect cubes :
(i) 243
(ii) 588
(iii) 1331
(iv) 24000
(v) 1728
(vi) 1938
Solution:

(*i*) 243

3	243
3	81
3	27
3	9
3	3
	1

$\because$ $243 = 3 \times 3 \times 3 \times 3$

$= (3 \times 3 \times 3) \times 3$

$= 3^3 \times 3$

$\therefore$ 297 is not a perfect cube.

(*ii*) 588

2	588
2	294
7	147
7	21
3	3
	1

$588 = 2 \times 2 \times 7 \times 7 \times 3$

$\therefore$ 588 is not a perfect cube.

(*iii*) 1331

11	1331
11	121
11	11
	1

$\because$ $1331 = 11 \times 11 \times 11 = (11)^3$

$\therefore$ 1331 is a perfect cube.

(*iv*) 24000

$\because$ $24000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

$= (2)^3 \times (2)^3 \times (5)^3 \times 3$

$\therefore$ 24000 is not a perfect cube.

(v) 1728

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$\because$ $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2)^3 \times (2)^3 \times (3)^3$

$\therefore$ 1728 is a perfect cube.

(vi) 1938

2	1938
3	936
17	323
19	19
	1

1938 = 2 x 3 x 17 x 19

1938 is not a perfect cube.

Question 3.

Find the cubes of :

(i) 2.1

(ii) 0.4

(iii) 1.6

(iv) 2.5

(v) 0.12

(vi) 0.02

(vii) 0.8

Solution:

(*i*) $2.1 = (2.1)^3 = \left(\frac{21}{10}\right)^3 = \frac{21\times21\times21}{10\times10\times10}$

$= \frac{9261}{1000} = 9.261$

(*ii*) $0.4 = (0.4)^3 = \left(\frac{4}{10}\right)^3 = \frac{4\times4\times4}{10\times10\times10}$

$= \frac{64}{1000} = 0.064$

(*iii*) $1.6 = (1.6)^3 = \left(\frac{16}{10}\right)^3 = \frac{16\times16\times16}{10\times10\times10}$

$= \frac{4096}{1000} = 4.096$

(*iv*) $2.5 = (2.5)^3 = \left(\frac{25}{10}\right)^3 = \frac{25\times25\times25}{10\times10\times10}$

$= \frac{15625}{1000} = 15.625$

(*v*) $0.12 = (0.12)^3 = \left(\frac{12}{100}\right)^3 = \frac{12\times12\times12}{100\times100\times100}$

$= \frac{1728}{1000000} = 0.001728$

(*vi*) $0.02 = (0.02)^3 = \left(\frac{2}{100}\right)^3 = \frac{2\times2\times2}{100\times100\times100}$

$= \frac{8}{1000000} = 0.000008$

$$(vii)\ 0.8 = (0.8)^3 = \left(\frac{8}{10}\right)^3 = \frac{8\times8\times8}{10\times10\times10}$$

$$= \frac{512}{1000} = 0.512$$

Question 4.

Find the cubes of :

(i) $\frac{3}{7}$

(ii) $\frac{8}{9}$

(iii) $\frac{10}{13}$

(iv) $1\frac{2}{7}$

(v) $2\frac{1}{2}$

Solution:

$$(i)\ \frac{3}{7} = \left(\frac{3}{7}\right)^3 = \frac{3\times3\times3}{7\times7\times7} = \frac{27}{343}$$

$$(ii)\ \frac{8}{9} = \left(\frac{8}{9}\right)^3 = \frac{8\times8\times8}{9\times9\times9} = \frac{512}{729}$$

$$(iii)\ \frac{10}{13} = \left(\frac{10}{13}\right)^3 = \frac{10\times10\times10}{13\times13\times13} = \frac{1000}{2197}$$

$$(iv)\ 1\frac{2}{7} = \left(1\frac{2}{7}\right)^3 = \left(\frac{1\times7+2}{7}\right)^3 = \left(\frac{9}{7}\right)^3$$

$$= \frac{9\times9\times9}{7\times7\times7} = \frac{729}{343} = 2\frac{43}{343}$$

$$(v)\ 2\frac{1}{2} = \left(2\frac{1}{2}\right)^3 = \left(\frac{5}{2}\right)^3$$

$$= \frac{5\times5\times5}{2\times2\times2} = \frac{125}{8} = 15\frac{5}{8}.$$

Question 5.

Find the cubes of :

(i) -3

(ii) -7
(iii) -12
(iv) -18
(v) -25
(vi) -30
(vii) -50

Solution:

$(i)\ -3 = (-3)^3 = -3 \times -3 \times -3$
$= -(3 \times 3 \times 3) = -27$

$(ii)\ -7 = (-7)^3 = -7 \times -7 \times -7$
$= -(7 \times 7 \times 7) = -343$

$(iii)\ -12 = (-12)^3 = -12 \times -12 \times -12$
$= -(12 \times 12 \times 12) = -1728$

$(iv)\ -18 = (-18)^3 = -18 \times -18 \times -18$
$= -(18 \times 18 \times 18) = -5832$

$(v)\ -25 = (-25)^3 = -25 \times -25 \times -25$
$= -(25 \times 25 \times 25) = -15625$

$(vi)\ -30 = (-30)^3 = -30 \times -30 \times -30$
$= -(30 \times 30 \times 30) = -27000$

$(vii)\ -50 = (-50)^3 = -50 \times -50 \times -50$
$= -(50 \times 50 \times 50) = -125000$

Question 6.

Which of the following are cubes of:
(i) an even number
(ii) an odd number
216, 729, 3375, 8000, 125, 343, 4096 and 9261.

Solution:

$\because 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

2	216
2	108
2	54
3	27
3	9
3	3
	1

$= (2)^3 \times (3)^3 = (6)^3$

$\because 729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

3	729
3	243
3	81
3	27
3	9
3	3
	1

$= (3)^3 \times (3)^3 = (9)^3$

$\because 3375 = 5 \times 5 \times 5 \times 3 \times 3 \times 3$

5	3375
5	675
5	135
3	27
3	9
3	3
	1

$= (5)^3 \times (3)^3 = (15)^3$

$\because 8000 = 20 \times 20 \times 20 = (20)^3$

20	8000
20	400
20	20
	1

$125 = 5 \times 5 \times 5 = (5)^3$

5	125
5	25
5	5
	1

$\because 343 = 7 \times 7 \times 7 = (7)^3$

7	343
7	49
7	7
	1

$\because 4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= (2)^3 \times (2)^3 \times (2)^3 \times (2)^3 = (16)^3$

(i) Cubes of an even number are 216, 8000, 4096.
(ii) Cubes of an odd number are 729, 3375, 125, 343, 9261.

Question 7.
Find the least number by which 1323 must be multiplied so that the product is a perfect

cube.

Solution:

The prime factor of 1323 are =3 x 3 x 3 x 7 x 7

= (3 x 3 x 3) x 7 x 7

Clearly, 1323 must be multiplied by 7.

Question 8.

Find the smallest number by which 8768 must be divided so that the quotient is a perfect cube.

Solution:

The prime factor of 8768 are

2	8768
2	4384
2	2192
2	1096
2	548
2	274
137	137
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 137$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 137$

Clearly, 8768 must be divided by 137.

Question 9.

Find the smallest number by which 27783 be multiplied to get a perfect square number.

Solution:

3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= (3 \times 3 \times 3) \times (7 \times 7 \times 7) \times 3$

Clearly, 27783 must be multiplied by 3×3

$= 9$

Question 10.

With what least number must 8640 be divided so that the quotient is a perfect cube?

Solution:

The prime factors of 8640 are

2	8640
2	4320
2	2160
2	540
2	270
3	135
3	45
3	15
5	5
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5$

Clearly, 8640 must be divided by 5.

Question 11.

Which is the smallest number that must be multiplied to 77175 to make it a perfect cube?

Solution:

The prime factors of 77175 are

3	77175
3	25725
5	8575
5	1715
7	343
7	49
7	7
	1

$= 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 7$

$= (7 \times 7 \times 7) \times 3 \times 3 \times 5 \times 5$

Clearly, 77175 must be multiplied by 3×5

$= 15$

EXERCISE 4(B)

Question 1.

Find the cube-roots of :

(i) 64

(ii) 343

(iii) 729

(iv) 1728

(v) 9261

(vi) 4096

(vii) 8000

(viii) 3375

Solution:

$(i)\ 64 = \sqrt[3]{64} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 = 4$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$(ii)\ 343 = \sqrt[3]{343} = 7 \times 7 \times 7 = 7$

7	343
7	49
7	7
	1

(*iii*) $729 = \sqrt[3]{729} = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$= 3 \times 3 = 9$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(*iv*) $1728 = \sqrt[3]{1728} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\times (3 \times 3 \times 3)$

$= 2 \times 2 \times 3 = 12$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(*v*) $9261 = \sqrt[3]{9261} = (3 \times 3 \times 3) \times (7 \times 7 \times 7)$

$= 3 \times 7 = 21$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

(*vi*) $4096 = \sqrt[3]{4096} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 = 16$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(*vii*) $8000 = \sqrt[3]{8000} = (4 \times 4 \times 4) \times (5 \times 5 \times 5)$

$= 4 \times 5 = 20$

4	8000
4	2000
4	500
5	125
5	25
5	5
	1

$(viii) 3375 = \sqrt[3]{3375} = (5 \times 5 \times 5) \times (3 \times 3 \times 3)$

$= 5 \times 3 = 15$

5	3375
5	675
5	135
3	27
3	9
3	3
	1

Question 2.

Find the cube-roots of :

(i) $\frac{27}{64}$

(ii) $\frac{125}{216}$

(iii) $\frac{343}{512}$

(iv) 64 x 729

(v) 64 x 27

(vi) 729 x 8000

(vii) 3375 x 512

Solution:

$$(i)\ \frac{27}{64} = \sqrt[3]{\frac{27}{64}} = \frac{\sqrt{3\times3\times3}}{\sqrt{4\times4\times4}} = \frac{3}{4}$$

$$(ii)\ \frac{125}{216} = \sqrt[3]{\frac{125}{216}} = \frac{\sqrt{5\times5\times5}}{\sqrt{6\times6\times6}} = \frac{5}{6}$$

$$(iii)\ \frac{343}{512} = \sqrt[3]{\frac{343}{512}} = \frac{\sqrt{7\times7\times7}}{\sqrt{8\times8\times8}} = \frac{7}{8}$$

$$(iv)\ 64 \times 729 = \sqrt[3]{64\times729}$$

$$= \sqrt{4\times4\times4\times9\times9\times9} = 4 \times 9 = 36$$

$$(v)\ 64 \times 27 = \sqrt[3]{64\times27}$$

$$= \sqrt{4\times4\times4\times3\times3\times3} = 4 \times 3 = 12$$

$$(vi)\ 729 \times 8000 = \sqrt[3]{729\times8000}$$

$$= \sqrt{9\times9\times9\times20\times20\times20}$$

$$= 9 \times 20 = 180$$

$$(vii)\ 3375 \times 512 = \sqrt[3]{3375\times512}$$

$$= \sqrt{15\times15\times15\times8\times8\times8}$$

$$= 15 \times 8 = 120$$

Question 3.

Find the cube-roots of :

(i) -216

(ii) -512

(iii) -1331

(iv) $\frac{-27}{125}$

(v) $\frac{-64}{343}$

(vi) $\frac{-512}{343}$

(vii) -2197

(viii) -5832

(ix) -2744000

Solution:

(*i*) $-216 = \sqrt[3]{-216} = \sqrt{-6\times-6\times-6} = -6$

(*ii*) $-512 = \sqrt[3]{-512} = \sqrt{-8\times-8\times-8} = -8$

(*iii*) $-1331 = \sqrt[3]{-1331}$

$= \sqrt{-11\times-11\times-11} = -11$

(*iv*) $-\frac{27}{125} = -\frac{\sqrt{27}}{\sqrt{125}} = -\sqrt{\frac{3\times3\times3}{5\times5\times5}} = -\frac{3}{5}$

(*v*) $\frac{-64}{343} = \frac{\sqrt[3]{-64}}{\sqrt[3]{343}} = \frac{\sqrt[3]{-4\times-4\times-4}}{\sqrt[3]{7\times7\times7}} = \frac{-4}{7}$

(*vi*) $-\frac{512}{343} = -\sqrt[3]{\frac{512}{343}} = -\sqrt[3]{\frac{8\times8\times8}{7\times7\times7}} = -\frac{8}{7}$

(*vii*) $-2197 = \sqrt[3]{-2197}$

13	2197
13	169
13	13
	1

$= \sqrt[3]{-13\times-13\times-13} = -13$

(*viii*) $-5832 = \sqrt[3]{-5832}$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$= \sqrt{-2 \times -2 \times -2 \times -3 \times -3 \times -3 \times -3 \times -3 \times -3}$$

$$= -2 \times -3 \times -3 = -18$$

$$(ix)\ -2744000 = \sqrt[3]{-2744000}$$

2	2744000
2	1372000
2	686000
7	343000
7	49000
7	7000
10	1000
10	100
10	10
	1

$$= \sqrt{-2 \times -2 \times -2 \times -7 \times -7 \times -7 \times -10 \times -10 \times -10}$$

$$= -2 \times -7 \times -10 = -140$$

Question 4.
Find the cube-roots of :
(i) 2.744
(ii) 9.261
(iii) 0.000027
(iv) -0.512
(v) -15.625
(vi) -125 x 1000
Solution:

(*i*) $2.744 = \sqrt[3]{\frac{2744}{1000}}$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$$= \sqrt[3]{\frac{2\times2\times2\times7\times7\times7}{10\times10\times10}}$$

$$= \frac{2\times7}{10} = \frac{14}{10} = 1.4$$

(*ii*) $9.261 = \sqrt[3]{\frac{9261}{1000}} = \sqrt{\frac{3\times3\times3\times7\times7\times7}{10\times10\times10}}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$= \frac{3\times7}{10} = \frac{21}{10} = 2.1$$

(*iii*) $0.000027 = \sqrt[3]{\frac{27}{1000000}}$

$$= \sqrt[3]{\frac{3\times3\times3}{100\times100\times100}} = \frac{3}{100} = 0.03$$

$$(iv)\ -0.512 = \sqrt[3]{\frac{-512}{1000}} = \sqrt{\frac{-8\times-8\times-8}{10\times10\times10}}$$

$$= \frac{-8}{10} = -0.8$$

$$(v)\ -15.625 = \sqrt[3]{\frac{-15625}{1000}}$$

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$$\sqrt{\frac{-(5\times5\times5)\times(5\times5\times5)}{10\times10\times10}}$$

$$= \frac{-5\times5}{10} = \frac{-25}{10} = -2.5$$

$$(vi)\ -125 \times 1000 = \sqrt{-125\times100}$$

$$= \sqrt{-(5\times5\times5)\times(10\times10\times10)}$$

$$= -5 \times 10 = -50$$

Question 5.
Find the smallest number by which 26244 may be divided so that the quotient is a perfect cube.
Solution:
The prime factors of 26244 are

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \times 3 \times 3 \times 2 \times 2$

Clearly, 26244 must be divided by
3 x 3 x 2 x 2 = 36

Question 6.
What is the least number by which 30375 should be multiplied to get a perfect cube?
Solution:
The prime factors of 30375 are

3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$= 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= (3 \times 3 \times 3) \times (5 \times 5 \times 5) \times 3 \times 3$

Clearly, 30375 must be multiplied with 3

Question 7.
Find the cube-roots of :
(i) 700 x 2 x 49 x 5
(ii) -216 x 1728
(iii) -64 x -125
(iv) $\frac{-27}{343}$
(v) $\frac{729}{-1331}$
(vi) 250.047

(vii) -175616

Solution:

(*i*) $700 \times 2 \times 49 \times 5$

2	700
2	350
5	175
5	35
7	7
	1

$= 2 \times 2 \times 5 \times 5 \times 7 \times 2 \times 7 \times 7 \times 5$

$= (2 \times 2 \times 2) \times (5 \times 5 \times 5) \times (7 \times 7 \times 7)$

$= 2 \times 5 \times 10 = 70$

(*ii*) -216×1728

2	216
2	108
2	54
3	27
3	9
3	3
	1

2	1728
2	864
2	432
	216

$= -(2 \times 2 \times 2 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3)$

$= -2 \times 3 \times 2 \times 2 \times 3 = -72$

(*iii*) -64×-125

$= -(4 \times 4 \times 4) \times -(5 \times 5 \times 5)$

$= -4 \times -5 = 20$

(*iv*) $-\frac{27}{343} = \frac{3 \times 3 \times 3}{7 \times 7 \times 7} = -\frac{3}{7}$

(*v*) $\frac{729}{-1331} = \frac{(9 \times 9 \times 9)}{-(11 \times 11 \times 11)} = -\frac{9}{11}$

(*vi*) $250.047 = \frac{250047}{1000}$

3	250047
3	83349
3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$= \frac{(3\times3\times3)\times(3\times3\times3)\times(7\times7\times7)}{(10\times10\times10)}$$

$$= \frac{3\times3\times7}{10} = \frac{63}{10} = 6.3$$

(*vii*) –175616

2	175616
2	27808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= -[(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$\times (7 \times 7 \times 7)]$
$= -[2 \times 2 \times 2 \times 7] = -56$