Chapter 1. Rational and Irrational Numbers

Selina ICSE Solutions for Class 10 Maths Chapter 1 Rational and Irrational Numbers

Exercise 1(A)

Solution 1:

(i)

Any rational number between x and y is given as $\frac{x+y}{2}$.

Thus the rational number between

$$\frac{3}{8} \text{ and } \frac{7}{12} = \frac{\frac{3}{8}+\frac{7}{12}}{2} = \frac{\frac{9+14}{24}}{2} = \frac{23}{24\times 2} = \frac{23}{48}$$

Similarly the rational number between

$$\frac{3}{8} \text{ and } \frac{23}{48} = \frac{\frac{3}{8}+\frac{23}{48}}{2} = \frac{\frac{18}{48}+\frac{23}{48}}{2} = \frac{\frac{18+23}{48}}{2} = \frac{41}{96}$$

Thus the rational numbers between $\frac{3}{8}$ and $\frac{7}{12}$ are: $\frac{23}{48}, \frac{41}{96}$

Thus, we have, $\frac{3}{8} < \frac{41}{96} < \frac{23}{48} < \frac{7}{12}$

(ii)

Any rational number between x and y is given as $\frac{x+y}{2}$.

Thus the rational number between

$$\frac{1}{3} \text{ and } \frac{1}{4} = \frac{\frac{1}{3}+\frac{1}{4}}{2}$$

$$= \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12 \times 2}$$

$$= \frac{7}{24}$$

Similarly, the rational number between

$$\frac{7}{24} \text{ and } \frac{1}{4} = \frac{\frac{7}{24}+\frac{1}{4}}{2}$$

$$= \frac{\frac{7}{24}+\frac{6}{24}}{2}$$

$$= \frac{13}{24 \times 2}$$

$$= \frac{13}{48}$$

Thus, the rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{13}{48}$

Thus, we have $\frac{1}{3} < \frac{7}{24} < \frac{13}{48} < \frac{1}{4}$

Solution 2:

(i)

L.C.M of 5 and 7 = 35

$\frac{2}{5}$ and $\frac{3}{7} = \frac{2 \times 7}{5 \times 7}$ and $\frac{3 \times 5}{5 \times 7} = \frac{14}{35}$ and $\frac{15}{35}$

However, to find more rational numbers let us multiply the numerator and denominator by multiples of 35.

Thus, we have $\frac{2}{5} = \frac{2 \times 7 \times 5}{5 \times 7 \times 5} = \frac{70}{175}$

and $\frac{3}{7} = \frac{3 \times 5 \times 5}{7 \times 5 \times 5} = \frac{75}{175}$

Since $\frac{70}{175} < \frac{75}{175}$

Thus, we have $\frac{70}{175} < \frac{71}{175} < \frac{72}{175} < \frac{73}{175} < \frac{74}{175} < \frac{75}{175}$

Thus, we have $\frac{2}{5} < \frac{71}{175} < \frac{72}{175} < \frac{73}{175} < \frac{3}{7}$.

(ii)

L.C.M of 11 and 16 = 176

$\frac{4}{11}$ and $\frac{9}{16} = \frac{4 \times 16}{11 \times 16}$ and $\frac{9 \times 11}{16 \times 11} = \frac{64}{176}$ and $\frac{99}{176}$

Since $\frac{64}{176} < \frac{99}{176}$

Thus, we have $\frac{64}{176} < \frac{65}{176} < \frac{66}{176} < \frac{67}{176} < \frac{99}{176}$.

Thus, the three rational numbers between $\frac{4}{11}$ and $\frac{9}{16}$ are given below:

$\frac{4}{11} < \frac{65}{176} < \frac{66}{176} < \frac{67}{176} < \frac{9}{16}$

Solution 3:

(i)

Both 5 and -2 are integers as well as rational numbers.

Since the set of integers is the subset of rational numbers, we have -2 < -1 < 0 < 1 < 2 < 3 < 4 < 5.

Thus, any three rational numbers between 5 and -2 are given below:

-2, -1 and 0

(ii)

$-\frac{3}{4}$ and $\frac{1}{2}$

L.C.M of 4 and 2 = 4

$-\frac{3}{4}$ and $\frac{1}{2} = \frac{-3}{4}$ and $\frac{2}{4}$

Since $\frac{-3}{4} < \frac{2}{4}$

Thus, we have, $\frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < 0 < \frac{1}{4} < \frac{2}{4}$

Thus, the three rational numbers between $-\frac{3}{4}$ and $\frac{1}{2}$ are given below:

$\frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < \frac{1}{4} < \frac{2}{4}$

Solution 4:

Given rational numbers are 5 and 8.

Here, $5 < 8$.

$\Rightarrow x = 5$ and $y = 8$

To insert 4 rational numbers between 5 and 8, $n = 4$

$$\Rightarrow d = \frac{y-x}{n+1} = \frac{8-5}{4+1} = \frac{3}{5}$$

Hence,

$$x + d = 5 + \frac{3}{5} = \frac{25+3}{5} = \frac{28}{5} = 5\frac{3}{5}$$

$$x + 2d = 5 + 2 \times \frac{3}{5} = 5 + \frac{6}{5} = \frac{25+6}{5} = \frac{31}{5} = 6\frac{1}{5}$$

$$x + 3d = 5 + 3 \times \frac{3}{5} = 5 + \frac{9}{5} = \frac{25+9}{5} = \frac{34}{5} = 6\frac{4}{5}$$

$$x + 4d = 5 + 4 \times \frac{3}{5} = 5 + \frac{12}{5} = \frac{25+12}{5} = \frac{37}{5} = 7\frac{2}{5}$$

$\therefore$ Required rational numbers are $5\frac{3}{5}$, $6\frac{1}{5}$, $6\frac{4}{5}$ and $7\frac{2}{5}$.

Solution 5:

Given rational numbers are $\frac{1}{3}$ and $\frac{5}{9}$.

Here, $\frac{1}{3} < \frac{5}{9}$.

$\Rightarrow x = \frac{1}{3}$ and $y = \frac{5}{9}$

To insert 5 rational numbers between $\frac{1}{3}$ and $\frac{5}{9}$, $n = 5$

$$\Rightarrow d = \frac{y-x}{n+1} = \frac{\frac{5}{9} - \frac{1}{3}}{5+1} = \frac{\frac{5-3}{9}}{6} = \frac{2}{9} \times \frac{1}{6} = \frac{1}{27}$$

Hence,

$$x + d = \frac{1}{3} + \frac{1}{27} = \frac{9+1}{27} = \frac{10}{27}$$

$$x + 2d = \frac{1}{3} + 2 \times \frac{1}{27} = \frac{9+2}{27} = \frac{11}{27}$$

$$x + 3d = \frac{1}{3} + 3 \times \frac{1}{27} = \frac{9+3}{27} = \frac{12}{27} = \frac{4}{9}$$

$$x + 4d = \frac{1}{3} + 4 \times \frac{1}{27} = \frac{9+4}{27} = \frac{13}{27}$$

$$x + 5d = \frac{1}{3} + 5 \times \frac{1}{27} = \frac{9+5}{27} = \frac{14}{27}$$

$\therefore$ Required rational numbers are $\frac{10}{27}$, $\frac{11}{27}$, $\frac{4}{9}$, $\frac{13}{27}$ and $\frac{14}{27}$.

Solution 6:

Given rational numbers are 4.6 and 8.4

$4.6 < 8.4$

$\Rightarrow \frac{46}{10} < \frac{84}{10}$

$\Rightarrow \frac{46}{10} < \frac{46+84}{10+10} < \frac{84}{10}$

$\Rightarrow \frac{46}{10} < \frac{130}{20} < \frac{84}{10}$

$\Rightarrow \frac{46}{10} < \frac{46+130}{10+20} < \frac{130}{20} < \frac{130+84}{20+10} < \frac{84}{10}$

$\Rightarrow \frac{46}{10} < \frac{176}{30} < \frac{130}{20} < \frac{214}{30} < \frac{84}{10}$

$\Rightarrow \frac{46}{10} < \frac{46+176}{10+30} < \frac{176}{30} < \frac{176+130}{30+20} < \frac{130}{20} < \frac{130+214}{20+30} < \frac{214}{30} < \frac{84}{10}$

$\Rightarrow \frac{46}{10} < \frac{222}{40} < \frac{176}{30} < \frac{306}{50} < \frac{130}{20} < \frac{344}{50} < \frac{214}{30} < \frac{84}{10}$

$\Rightarrow 4.6 < 5.6 < 5.9 < 6.1 < 6.5 < 6.9 < 7.1 < 8.4$

$\therefore$ Required rational numbers are 5.6, 5.9, 6.1, 6.5, 6.9 and 7.1

Solution 7:

Given rational numbers are 1 and 2.

Here, $1 < 2$.

$\Rightarrow x = 1$ and $y = 2$

To insert 7 rational numbers between 1 and 2, $n = 7$

$\Rightarrow d = \frac{y-x}{n+1} = \frac{2-1}{7+1} = \frac{1}{8}$

Hence,

$x + d = 1 + \frac{1}{8} = \frac{8+1}{8} = \frac{9}{8} = 1\frac{1}{8}$

$x + 2d = 1 + 2 \times \frac{1}{8} = \frac{8+2}{8} = \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$

$x + 3d = 1 + 3 \times \frac{1}{8} = \frac{8+3}{8} = \frac{11}{8} = 1\frac{3}{8}$

$x + 4d = 1 + 4 \times \frac{1}{8} = \frac{8+4}{8} = \frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$

$x + 5d = 1 + 5 \times \frac{1}{8} = \frac{8+5}{8} = \frac{13}{8} = 1\frac{5}{8}$

$x + 6d = 1 + 6 \times \frac{1}{8} = \frac{8+6}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$

$x + 7d = 1 + 7 \times \frac{1}{8} = \frac{8+7}{8} = \frac{15}{8} = 1\frac{7}{8}$

$\therefore$ Required rational numbers are $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$, $1\frac{5}{8}$, $1\frac{3}{4}$ and $1\frac{7}{8}$.

Solution 8:

Given rational numbers are 1.8 and 3.6

Here, $1.8 < 3.6$

$\Rightarrow x = 1.8$ and $y = 3.6$

To insert 8 rational numbers between 1.8 and 3.6, $n = 8$

$\Rightarrow d = \frac{y-x}{n+1} = \frac{3.6-1.8}{8+1} = \frac{1.8}{9} = 0.2$

Hence,

$x + d = 1.8 + 0.2 = 2.0$

$x + 2d = 1.8 + 2 \times 0.2 = 1.8 + 0.4 = 2.2$

$x + 3d = 1.8 + 3 \times 0.2 = 1.8 + 0.6 = 2.4$

$x + 4d = 1.8 + 4 \times 0.2 = 1.8 + 0.8 = 2.6$

$x + 5d = 1.8 + 5 \times 0.2 = 1.8 + 1.0 = 2.8$

$x + 6d = 1.8 + 6 \times 0.2 = 1.8 + 1.2 = 3.0$

$x + 7d = 1.8 + 7 \times 0.2 = 1.8 + 1.4 = 3.2$

$x + 8d = 1.8 + 8 \times 0.2 = 1.8 + 1.6 = 3.4$

$\therefore$ Required rational numbers are 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2 and 3.4.

Solution 9:

Consider the given numbers: $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ and $\frac{11}{18}$

The L.C.M of 9,12, and 18 is 36

Thus the given numbers are:

$$-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3} \text{ and } \frac{11}{18} = -\frac{5 \times 4}{9 \times 4}, \frac{7 \times 3}{12 \times 3}, -\frac{2 \times 12}{3 \times 12} \text{ and } \frac{11 \times 2}{18 \times 2}$$
$$= -\frac{20}{36}, \frac{21}{36}, -\frac{24}{36} \text{ and } \frac{22}{36}$$

Thus the numbers in ascending order are shown below:

$-\frac{24}{36}, -\frac{20}{36}, \frac{21}{36}$ and $\frac{22}{36}$

Thus the given numbers in ascending order are shown below:

$-\frac{2}{3}, -\frac{5}{9}, \frac{7}{12}$ and $\frac{11}{18}$

We need to find the difference between the largest and smallest of the above numbers.

$$\text{Thus, difference} = \frac{11}{18} - \left(-\frac{2}{3}\right)$$
$$= \frac{11}{18} + \frac{2}{3}$$
$$= \frac{11}{18} + \frac{2 \times 6}{3 \times 6}$$
$$= \frac{11}{18} + \frac{12}{18}$$
$$= \frac{11+12}{18}$$
$$= \frac{23}{18}$$

We need to express this fraction as a decimal, correct to one decimal place.

Thus, we have $\frac{23}{18} = 1.2\overline{7} \approx 1.3.$

Solution 10:

Consider the given numbers: $\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4}$ and $\frac{17}{32}$.

The LCM of 8, 16, 4 and 32 is 32.

Thus, the given numbers are given below:

$$\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4} \text{ and } \frac{17}{32} = \frac{5 \times 4}{8 \times 4}, -\frac{3 \times 2}{16 \times 2}, -\frac{1 \times 8}{4 \times 8} \text{ and } \frac{17 \times 1}{32 \times 1}$$

$$= \frac{20}{32}, -\frac{6}{32}, -\frac{8}{32} \text{ and } \frac{17}{32}$$

Thus, the numbers in descending order are shown below:

$\frac{20}{32}, \frac{17}{32}, -\frac{6}{32}$ and $-\frac{8}{32}$.

Thus, the given numbers in descending order are listed below:

$\frac{5}{8}, \frac{17}{32}, -\frac{3}{16}$ and $-\frac{1}{4}$.

We need to find the sum of the largest and the smallest of the above numbers.

$$\text{Thus, sum } = \frac{5}{8} + \left(-\frac{1}{4}\right)$$

$$= \frac{5}{8} - \frac{1}{4}$$

$$= \frac{5}{8} - \frac{1 \times 2}{4 \times 2}$$

$$= \frac{5}{8} - \frac{2}{8}$$

$$= \frac{3}{8}$$

We need to express this fraction as a decimal, correct to two decimal places.

Thus, we have $\frac{3}{8} = 0.375 \approx 0.38$.

Exercise 1(B)

Solution 1:

In a recurring decimal, if all the digits in the decimal part are not repeating, it is called a mixed recuring decimal and if all the digits in the decimal part are repeating, it is called a pure recuring decimal.

Thus, we have

(i) $0.\overline{083}$: Pure recuring decimal

(ii) $0.0\overline{83}$: Mixed recuring decimal

(iii) $0.\overline{227}$: Pure recuring decimal

(iv) $3.5\dot{4}$: Mixed recuring decimal

(v) $2.\overline{81}$: Pure recuring decimal

Solution 2:

(i) $\frac{4}{15} = 0.26666.... = 0.2\overline{6}$

(ii) $\frac{2}{7} = 0.285714285714....... = 0.\overline{285714}$

(iii) $\frac{4}{9} = 0.44444..... = 0.\overline{4}$

(iv) $\frac{5}{24} = 0.2083333..... = 0.208\overline{3}$

(v) $\frac{8}{13} = 0.615384615384.... = 0.\overline{615384}$

Solution 3(i):

Given decimal number is $0.5\dot{3}$

$x = 0.5\dot{3}$ ...(1)

The number of digits after the decimal point which do not have bar on them is 1.

Thus multiplying both sides of equation (1) by $10^1 = 10$

$\Rightarrow 10x = 5.\dot{3}$ ...(2)

$\therefore$ The right hand side of the number is only the repeating decimal part.

And the number of repeating decimal parts is 1.

Thus, multiplying both sides of equation (2) by $10^1 = 10$

$100x = 53.\dot{3}$ ...(3)

Subtracting equation (2) from equation (3), we have,

$90x = 48$

$\Rightarrow x = \frac{48}{90}$

$\Rightarrow x = \frac{8}{15}$

$\therefore 0.5\dot{3} = \frac{8}{15}$

Solution 3(ii):

Given decimal number is $0.2\overline{27}$

$x = 0.2\overline{27}$ ...(1)

The number of digits after the decimal point which do not have the bar on them is 1.

Thus, multiplying both sides of equation (1) by $10^1 = 10$

$\Rightarrow 10x = 2.\overline{27}$ ...(2)

$\therefore$ The right hand side of the number is only the repeating decimal part.

The number of repeating decimal parts is 2.

Thus, multiplying both sides of equation (2) by $10^2 = 100$.

$1000x = 227.\overline{27}$ ...(3)

Subtracting equation (2) from equation (3), we have

$990x = 225$

$\Rightarrow x = \frac{225}{990}$

$\Rightarrow x = \frac{5}{22}$

$\therefore 0.2\overline{27} = \frac{5}{22}$

Solution 3(iii):

Given decimal number is $0.2\overline{104}$

$x = 0.2\overline{104}$ ...(1)

The number of digits after the decimal point which do not have the bar on them is 1.

Thus, multiplying both sides of equation (1) by $10^1 = 10$

$\Rightarrow 10x = 2.\overline{104}$ ...(2)

$\therefore$ The right hand side of the number is only the repeating decimal part.

The number of repeating decimal parts is 3.

Thus, multiplying both sides of equation (2) by $10^3 = 1000$

$10000x = 2104.\overline{104}$ ...(3)

Subtracting equation (2) from equation (3), we have

$9990x = 2102$

$\Rightarrow x = \frac{2102}{9990}$

$\Rightarrow x = \frac{1051}{4995}$

$\therefore 0.2\overline{104} = \frac{1051}{4995}$

Solution 3(iv):

Given decimal number is $3.5\dot{2}$

Now, $3.5\dot{2} = 3 + 0.5\dot{2}$

For $0.5\dot{2}$, numerator $= 52 - 5 = 47$

And, denominator $= 90$

$$\therefore\ 3.5\dot{2} = 3 + 0.5\dot{2} = 3 + \frac{47}{90} = 3\frac{47}{90}$$

Solution 3(v):

Given decimal number is $2.24\overline{689}$

Now, $2.24\overline{689} = 2 + 0.24\overline{689}$

For $0.24\overline{689}$, numerator $= 24689 - 24 = 24665$

And, denominator $= 99900$

$$\therefore\ 2.24\overline{689} = 2 + 0.24\overline{689} = 2 + \frac{24665}{99900} = 2 + \frac{4933}{19980} = 2\frac{4933}{19980}$$

Solution 3(vi):

Given decimal number is $0.\overline{572}$

For $0.\overline{572}$, numerator $= 572 - 0 = 572$

And, denominator $= 999$

$$\therefore\ 0.\overline{572} = \frac{572}{999}$$

Solution 3(vii):

Given decimal number is $0.15\dot{8}$

For $0.15\dot{8}$, numerator $= 158 - 15 = 143$

And, denominator $= 900$

$$\therefore\ 0.15\dot{8} = \frac{143}{900}$$

Solution 3(viii):

Given decimal number is $0.0\overline{384}$

For $0.0\overline{384}$, numerator $= 0384 - 03 = 381$

And, denominator $= 9990$

$$\therefore\ 0.0\overline{384} = \frac{381}{9990} = \frac{127}{3330}$$

Solution 4:

$\frac{1}{7} = 0.142857142857...... = 0.\overline{142857}$

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

Solution 5(i):

Given number is $\frac{7}{16}$

Since $16 = 2 \times 2 \times 2 \times 2 = 2^4 = 2^4 \times 5^0$

i.e. 16 can be expressed as $2^m \times 5^n$

$\therefore \frac{7}{16}$ is convertible into the terminating decimal.

Solution 5(ii):

Given number is $\frac{23}{125}$

Since $125 = 5 \times 5 \times 5 = 5^3 = 2^0 \times 5^3$

i.e. 125 can be expressed as $2^m \times 5^n$

$\therefore \frac{23}{125}$ is convertible into the terminating decimal.

Solution 5(iii):

Given number is $\frac{9}{14}$

Since $14 = 2 \times 7 = 2^1 \times 7^1$

i.e. 14 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{9}{14}$ is not convertible into the terminating decimal.

Solution 5(iv):

Given number is $\frac{32}{45}$

Since $45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

i.e. 45 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{32}{45}$ is not convertible into the terminating decimal.

Solution 5(v):

Given number is $\frac{43}{50}$

Since $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

i.e. 50 can be expressed as $2^m \times 5^n$

$\therefore \frac{43}{50}$ is convertible into the terminating decimal.

Solution 5(vi):

Given number is $\frac{17}{40}$

Since $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

i.e. 40 can be expressed as $2^m \times 5^n$

$\therefore \frac{17}{40}$ is convertible into the terminating decimal.

Solution 5(vii):

Given number is $\frac{61}{75}$

Since $75 = 3 \times 5 \times 5 = 3^1 \times 5^2$

i.e. 75 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{61}{75}$ is not convertible into the terminating decimal.

Solution 5(viii):

Given number is $\frac{123}{250}$

Since $250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$

i.e. 250 can be expressed as $2^m \times 5^n$

$\therefore \frac{123}{250}$ is convertible into the terminating decimal.

Exercise 1(C)

Solution 1:

(i) $\left(2+\sqrt{2}\right)^2 = 2^2 + 2(2)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2$

$= 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2}$

Irrational

(ii) $\left(3-\sqrt{3}\right)^2 = (3)^2 - 2(3)\left(\sqrt{3}\right) + \left(\sqrt{3}\right)^2$

$= 9 - 6\sqrt{3} + 3$

$= 12 - 6\sqrt{3} = 6\left(2-\sqrt{3}\right)$

Irrational

(iii) $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right) = (5)^2 - \left(\sqrt{5}\right)^2$

$= 25 - 5 = 20$

Rational

(iv) $\left(\sqrt{3}-\sqrt{2}\right)^2 = \left(\sqrt{3}\right)^2 - 2\left(\sqrt{3}\right)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2$

$= 3 - 2\sqrt{6} + 2 = 5 - 2\sqrt{6}$ Irrational

(v) $\left(\frac{3}{2\sqrt{2}}\right)^2 = \frac{(3)^2}{\left(2\sqrt{2}\right)^2} = \frac{9}{4\times 2} = \frac{9}{8}$ Rational

(vi) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2 = \frac{\left(\sqrt{7}\right)^2}{\left(6\sqrt{2}\right)^2} = \frac{7}{36\times 2} = \frac{7}{72}$ Rational

Solution 2:

(i)

$\left(\frac{3\sqrt{5}}{5}\right)^2 = \frac{3^2\left(\sqrt{5}\right)^2}{5^2}$

$= \frac{9\times 5}{25}$

$= \frac{9}{5}$

$= 1\frac{4}{5}$

(ii)

$\left(\sqrt{3}+\sqrt{2}\right)^2 = \left(\sqrt{3}\right)^2 + 2\left(\sqrt{3}\right)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2$

$= 3 + 2\sqrt{6} + 2 = 5 + 2\sqrt{6}$

(iii)

$\left(\sqrt{5}-2\right)^2 = \left(\sqrt{5}\right)^2 - 2\left(\sqrt{5}\right)(2) + (2)^2$

$= 5 - 4\sqrt{5} + 4$

$= 9 - 4\sqrt{5}$

(iv)

$\left(3+2\sqrt{5}\right)^2 = 3^2 + 2(3)\left(2\sqrt{5}\right) + \left(2\sqrt{5}\right)^2$

$= 9 + 12\sqrt{5} + 20$

$= 29 + 12\sqrt{5}$

Solution 3:

(i) False

(ii) $2\sqrt{4}+2=2\times 2+2=4+2=6$ which is true

(iii) $3\sqrt{7}-2\sqrt{7}=\sqrt{7}$ True.

(iv) False because

$$\frac{2}{7}=0.\overline{285714}$$

which is recurring and non-terminating and hence it is rational

(v) True because $\frac{5}{11}=0.\overline{45}$ which is recurring and non-terminating

(vi) True

(vii) False

(viii) True.

Solution 4:

Given Universal set is

$$\left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

(i)

We need to find the set of rational numbers.

Rational numbers are numbers of the form $\frac{p}{q}$, where $q\neq 0$.

$$U=\left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

Clearly, $-5\frac{3}{4}$, $-\frac{3}{5}$, $-\frac{3}{8}$, $\frac{4}{5}$ and $1\frac{2}{3}$ are of the form $\frac{p}{q}$.

Hence, they are rational numbers.

Since the set of integers is a subset of rational numbers,

-6, 0 and 1 are also rational numbers.

Thus, decimal numbers 3.01 and 8.47 are also rational numbers

because they are terminating decimals.

Hence, from the above set, the set of rational

numbers is Q, and $Q=\left\{-6,\ -5\frac{3}{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ 3.01,\ 8.47\right\}$

(ii)

We need to find the set of irrational numbers.

Irrational numbers are numbers which are not rational.

From the above subpart, the set of rational

numbers is Q,

and $Q=\left\{-6,\ -5\frac{3}{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ 3.01,\ 8.47\right\}$

Set of irrational numbers is the set of complement of

the rational numbers over real numbers.

Here the set of irrational numbers is $U-Q=\left\{\sqrt{8},\ \pi\right\}$

(iii)

We need to find the set of integers.
Set of integers consists of zero, the natural numbers and their additive inverses.
The set of integers is Z
$Z=\{\ldots -3, \ -2, \ -1, \ 0, \ 1, \ 2, \ 3, \ldots\}$
Here the set of integers is $U \cap Z = \{-6, \ \sqrt{4}, \ 0, \ 1\}$.

(iv)

We need to find the set of non-negative integers.
Set of non-negative integers consists of zero and the natural numbers.
The set of non-negative integers is Z^+ and
$Z^+=\{0, 1, 2, 3, \ldots\}$
Here the set of integers is $U \cap Z^+ = \{0, 1\}$

Solution 5:

```
              1.73209...
       1 | 3.0000000000
         | -1
      27 |  200
         | -189
     343 |   1100
         |  -1029
    3462 |     7100
         |    -6924
  346409 |       17160000
         |       -311841
         |       144815900...
```

$\Rightarrow \sqrt{3} = 1.73209....$ which is an irrational number.

```
             2.23606....
      1 | 5.0000000000...
        | -4
     42 |  100
        | - 84
    443 |   1600
        |  -1329
   4466 |     27100
        |    -26796
 447206 |      3040000
        |     -2683236
        |       356764...
```

$\sqrt{5} = 2.23606...$ which is an irrational number.

Solution 6:

Let us suppose that $\sqrt{3}$ and $\sqrt{5}$ are rational numbers

$\therefore \sqrt{3} = \frac{a}{b}$ and $\sqrt{5} = \frac{x}{y}$ (Where a, b $\in$ 7 and b, y $\neq$ 0 x , y)

Squaring both sides

$3 = \frac{a^2}{b^2} \qquad ,5 = \frac{x^2}{y^2}$

$\left.3b^2 = a^2 \qquad ,5y^2 = x^2\right\} \quad \ldots(*)$

$\Rightarrow a^2$ and x^2 are odd as $3b^2$ and $5y^2$ are odd .

$\Rightarrow$ a and x are odd....(1)

Let a = 3c, x = 5z

$a^2 = 9c^2, x^2 = 25z^2$

$3b^2 = 9c^2, 5y^2 = 25z^2$(From equation $(*)$)

$\Rightarrow b^2 = 3c^2, y^2 = 5z^2$

$\Rightarrow b^2$ and y^2 are odd as $3c^2$ and $5z^2$ are odd .

$\Rightarrow$ b and y are odd...(2)

From equation (1) and (2) we get a, b, x, y are odd integers.

i.e., a, b, and x, y have common factors 3 and 5 this contradicts our assumption that $\frac{a}{b}$ and $\frac{x}{y}$ are rational i.e, a, b and x, y do not have any common factors other than.

$\Rightarrow \frac{a}{b}$ and $\frac{x}{y}$ is not rational

$\Rightarrow \sqrt{3}$ and $\sqrt{5}$ are irrational.

Solution 7:

$\sqrt{3}+5$ and $\sqrt{5}-3$ are irrational numbers whose sum is irrational.

$(\sqrt{3}+5)+(\sqrt{5}-3) = \sqrt{3}+\sqrt{5}+5-3 = \sqrt{3}+\sqrt{5}+2$ which is irrational.

Solution 8:

$\sqrt{3}+5$ and $4-\sqrt{3}$ are two irrational numbers whose sum is rational.

$(\sqrt{3}+5)+(4-\sqrt{3}) = \sqrt{3}+5+4-\sqrt{3} = 9$

Solution 9:

$\sqrt{3}+2$ and $\sqrt{2}-3$ are two irrational numbers whose difference is irrational.

$(\sqrt{3}+2)-(\sqrt{2}-3) = \sqrt{3}+2-\sqrt{2}+3 = \sqrt{3}-\sqrt{2}+5$ which is irrational.

Solution 10:

$\sqrt{5}-3$ and $\sqrt{5}+3$ are irrational numbers whose difference is rational.

$(\sqrt{5}-3)-(\sqrt{5}+3) = \sqrt{5}-3-\sqrt{5}-3 = -6$ which is rational.

Solution 11:

Consider two irrational numbers $(5+\sqrt{2})$ *and* $(\sqrt{5}-2)$

Thus, the product, $(5+\sqrt{2}) \times (\sqrt{5}-2) = 5\sqrt{5}-10+\sqrt{10}-2\sqrt{2}$ *is irrational.*

Solution 12:

$(\sqrt{3}+\sqrt{2})$ and $(\sqrt{3}-\sqrt{2})$ are irrational numbers whose product is rational.

$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2-(\sqrt{2})^2 = 3-2 = 1$

Solution 13:

(i) $3\sqrt{5} = \sqrt{3^2 \times 5} = \sqrt{45}, 4\sqrt{3} = \sqrt{4^2 \times 3} = \sqrt{48}$

and $45 < 48 \therefore \sqrt{45} < \sqrt{48} \Rightarrow 3\sqrt{5} < 4\sqrt{3}$

(ii) $2\sqrt[3]{5} = \sqrt[3]{2^3 \times 5} = \sqrt[3]{40}, 3\sqrt[3]{2} = \sqrt[3]{3^3 \times 2} = \sqrt[3]{54}$

and $40 < 54 \Rightarrow \sqrt[3]{40} < \sqrt[3]{54}$

$\Rightarrow 2\sqrt[3]{5} < 3\sqrt[3]{2}$

(iii) $6\sqrt{5} = \sqrt{6^2 \times 5} = \sqrt{180}$

$7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{147}$

$8\sqrt{2} = \sqrt{8^2 \times 2} = \sqrt{128}$

and $128 < 147 < 180$

$\therefore \sqrt{128} < \sqrt{147} < \sqrt{180}$

$\Rightarrow 8\sqrt{2} < 7\sqrt{3} < 6\sqrt{5}$

Solution 14:

(i) $2\sqrt[4]{6} = \sqrt[4]{2^4 \times 6} = \sqrt[4]{96}$

$3\sqrt[4]{2} = \sqrt[4]{3^4 \times 2} = \sqrt[4]{162}$

Since $162 > 96$

$\Rightarrow \sqrt[4]{162} > \sqrt[4]{96}$

$\Rightarrow 3\sqrt[4]{2} > 2\sqrt[4]{6}$

(ii) $7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{141}$

$3\sqrt{7} = \sqrt{3^2 \times 7} = \sqrt{63}$

$141 > 63 \Rightarrow \sqrt{141} > \sqrt{63}$

$\Rightarrow 7\sqrt{3} > 3\sqrt{7}$

Solution 15:

(i) $\sqrt[6]{15} = (15)^{\frac{1}{6}}$ and $\sqrt[4]{12} = (12)^{\frac{1}{4}}$

Make powers $\frac{1}{6}$ and $\frac{1}{4}$ same

L.C.M. of 6,4 is 12

$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$

and $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

$\Rightarrow \sqrt[6]{15} = (15)^{\frac{1}{6}} = (15)^{\frac{2}{12}} = \left(15^2\right)^{\frac{1}{12}} = (225)^{\frac{1}{12}}$

and $\sqrt[4]{12} = (12)^{\frac{1}{4}} = (12)^{\frac{3}{12}} = \left(12^3\right)^{\frac{1}{12}} = (1728)^{\frac{1}{12}}$

$\Rightarrow 1272 > 225$

$\Rightarrow (1728)^{\frac{1}{12}} > (225)^{\frac{1}{12}}$

$\Rightarrow \sqrt[4]{12} > \sqrt[6]{15}$

(ii) $\sqrt{24} = (24)^{\frac{1}{2}}$ and $\sqrt[3]{35} = (35)^{\frac{1}{3}}$

L.C.M. of 2 and 3 is 6.

$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}, \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$

$\Rightarrow (24)^{\frac{1}{2}} = (24)^{\frac{3}{6}} = \left(24^3\right)^{\frac{1}{6}} = (13824)^{\frac{1}{6}}$

$(35)^{\frac{1}{3}} = (35)^{\frac{2}{6}} = \left(35^2\right)^{\frac{1}{6}} = (1225)^{\frac{1}{6}}$

$\Rightarrow 13824 > 1225$

$\Rightarrow (13824)^{\frac{1}{6}} > \sqrt[3]{35}$

$\Rightarrow \sqrt{24} > \sqrt[3]{35}$

Solution 16:

We know that $5 = \sqrt{25}$ *and* $6 = \sqrt{36}$.

Thus consider the numbers,

$\sqrt{25} < \sqrt{26} < \sqrt{27} < \sqrt{28} < \sqrt{29} < \sqrt{30} < \sqrt{31} < \sqrt{32} < \sqrt{33} < \sqrt{34} < \sqrt{35} < \sqrt{36}$

Therefore, any two irrational numbers between 5 *and* 6 *is* $\sqrt{27}$ *and* $\sqrt{28}$

Solution 17:

We know that $2\sqrt{5} = \sqrt{4 \times 5} = \sqrt{20}$ *and* $3\sqrt{3} = \sqrt{27}$

Thus, we have, $\sqrt{20} < \sqrt{21} < \sqrt{22} < \sqrt{23} < \sqrt{24} < \sqrt{25} < \sqrt{26} < \sqrt{27}$

So any five irrational numbers between $2\sqrt{5}$ *and* $3\sqrt{3}$ *are:*

$\sqrt{21}, \sqrt{22}, \sqrt{23}, \sqrt{24}$ *and* $\sqrt{26}$

Solution 18:

We want rational numbers a/b and c/d such that: $\sqrt{2}$ < a/b < c/d < $\sqrt{3}$

Consider any two rational numbers between 2 and 3 such that they are perfect squares.

Let us take 2.25 and 2.56 as $\sqrt{2.25} = 1.5$ and $\sqrt{2.56} = 1.6$

Thus we have,

$$\sqrt{2} < \sqrt{2.25} < \sqrt{2.56} < \sqrt{3}$$
$$\Rightarrow \sqrt{2} < 1.5 < 1.6 < \sqrt{3}$$
$$\Rightarrow \sqrt{2} < \frac{15}{10} < \frac{16}{10} < \sqrt{3}$$
$$\Rightarrow \sqrt{2} < \frac{3}{2} < \frac{8}{5} < \sqrt{3}$$

Therefore any two rational numbers between $\sqrt{2}$ *and* $\sqrt{3}$ *are:* $\frac{3}{2}$ *and* $\frac{8}{5}$

Solution 19:

Consider some rational numbers between 3 and 5 such that they are perfect squares.

Let us take, 3.24, 3.61, 4, 4.41 and 4.84 as

$$\sqrt{3.24} = 1.8,\ \sqrt{3.61} = 1.9,\ \sqrt{4} = 2,\ \sqrt{4.41} = 2.1 \text{ and } \sqrt{4.84} = 2.2$$

Thus we have,

$$\sqrt{3} < \sqrt{3.24} < \sqrt{3.61} < \sqrt{4} < \sqrt{4.41} < \sqrt{4.84} < \sqrt{5}$$
$$\Rightarrow \sqrt{3} < 1.8 < 1.9 < 2 < 2.1 < 2.2 < \sqrt{5}$$
$$\Rightarrow \sqrt{3} < \frac{18}{10} < \frac{19}{10} < 2 < \frac{21}{10} < \frac{22}{10} < \sqrt{5}$$
$$\Rightarrow \sqrt{3} < \frac{9}{5} < \frac{19}{10} < 2 < \frac{21}{10} < \frac{11}{5} < \sqrt{5}$$

Therefore, any three rational numbers between $\sqrt{3}$ *and* $\sqrt{5}$ *are:*

$\frac{9}{5}, \frac{19}{10}$ *and* $\frac{21}{10}$

Exercise 1(D)

Solution 1:

(i) $\sqrt{180} = \sqrt{2\times2\times5\times3\times3} = 6\sqrt{5}$ Which is irrational

$\therefore \sqrt{180}$ is a surds

(ii) $\sqrt[4]{27} = \sqrt[4]{3\times3\times3}$ Which is irrational

$\therefore \sqrt[4]{24}$ is a surds

(iii) $\sqrt[5]{128} = \sqrt[5]{2\times2\times2\times2\times2\times2\times2} = 2\sqrt[5]{4}$

$\therefore \sqrt[5]{128}$ is a surds

(iv) $\sqrt[3]{64} = \sqrt[3]{2\times2\times2\times2\times2\times2} = 4$ which is rational

$\therefore \sqrt[3]{34}$ is not a surds

(v) $\sqrt[3]{25}.\sqrt[3]{40} = \sqrt[3]{5\times5\times2\times2\times2\times5} = 2\times5 = 10$

$\therefore \sqrt[3]{25}.\sqrt[3]{40}$ is not a surds

(vi) $\sqrt[3]{-125} = \sqrt[3]{-5\times-5\times-5} = -5$

$\therefore$ is is not a surds

(vii) $\sqrt{\pi}$ not a surds as π is irrational

(viii) $\sqrt{3+\sqrt{2}}$ is not a surds because $3+\sqrt{2}$ is irrational.

Solution 2:

(i) $5\sqrt{2}\times\sqrt{2} = 5\times2 = 10$ which is rational

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(ii) $\sqrt{24} = \sqrt{2\times2\times2\times3} = 2\sqrt{6}$

$\therefore$ lowest rationalizing factor is $\sqrt{6}$

(iii) $\left(\sqrt{5}-3\right)\left(\sqrt{5}+3\right) = \left(\sqrt{5}\right)^2 - (3)^2 = 5-9 = -4$

$\therefore$ lowest rationalizing factor is $\left(\sqrt{5}+3\right)$

(iv) $7-\sqrt{7}$

$(7-\sqrt{7})(7+\sqrt{7}) = 49-7 = 42$

Therefore, lowest rationalizing factor is $\left(7+\sqrt{7}\right)$

(v) $\sqrt{18}-\sqrt{50}$

$$\sqrt{18}-\sqrt{50} = \sqrt{2\times3\times3} - \sqrt{5\times5\times2}$$
$$= 3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$$

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(vi) $\sqrt{5}-\sqrt{2}$

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 3$

Therefore lowest rationalizing factor is $\sqrt{5}+\sqrt{2}$

(vii) $\sqrt{13}+3$

$(\sqrt{13}+3)(\sqrt{13}-3) = (\sqrt{13})^2 - 3^2 = 13-9 = 4$

Its lowest rationalizing factor is $\sqrt{13}-3$

(viii) $15 - 3\sqrt{2}$

$$15 - 3\sqrt{2} = 3(5 - \sqrt{2})$$
$$= 3(5 - \sqrt{2})(5 + \sqrt{2})$$
$$= 3 \times \left[5^2 - (\sqrt{2})^2\right]$$
$$= 3 \times [25 - 2]$$
$$= 3 \times 23$$
$$= 69$$

Its lowest rationalizing factor is $5 + \sqrt{2}$

(ix) $3\sqrt{2} + 2\sqrt{3}$

$$3\sqrt{2} + 2\sqrt{3} = (3\sqrt{2} + 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})$$
$$= (3\sqrt{2})^2 - (2\sqrt{3})^2$$
$$= 9 \times 2 - 4 \times 3$$
$$= 18 - 12$$
$$= 6$$

its lowest rationalizing factor is $3\sqrt{2} - 2\sqrt{3}$

Solution 3:

(i)

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

(ii)

$$\frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2}{5}\sqrt{15}$$

(iii)

$$\frac{1}{\sqrt{3} - \sqrt{2}} \times \left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}\right) = \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{\sqrt{3} + \sqrt{2}}{3 - 2}$$

$$= \sqrt{3} + \sqrt{2}$$

(iv)

$$\frac{3}{\sqrt{5} + \sqrt{2}} \times \left(\frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}\right) = \frac{3(\sqrt{5} - \sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{3(\sqrt{5} - \sqrt{2})}{5 - 2}$$

$$= \sqrt{5} - \sqrt{2}$$

(v)

$$\frac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \left(\frac{2 - \sqrt{3}}{2 - \sqrt{3}}\right) = \frac{(2 - \sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = \frac{4 + 3 - 4\sqrt{3}}{4 - 3}$$

$$= \frac{7 - 4\sqrt{3}}{1} = 7 - 4\sqrt{3}$$

(vi)

$$\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - (1)^2} = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = \frac{4 + 2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2 + \sqrt{3})}{\cancel{2}} = 2 + \sqrt{3}$$

(vii)

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} = \frac{3+2-2\sqrt{6}}{3-2}$$

$$= 5-2\sqrt{6}$$

(viii)

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

$$= \frac{6+5-2\sqrt{30}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} = \frac{11-2\sqrt{30}}{6-5} = 11-2\sqrt{30}$$

(ix)

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{\left(2\sqrt{5}+3\sqrt{2}\right)^2}{\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$= \frac{4\times 5+9\times 2+12\sqrt{10}}{20-18}$$

$$= \frac{20+18+12\sqrt{10}}{2} = \frac{38+12\sqrt{10}}{2} = \frac{\cancel{2}\left(19+6\sqrt{10}\right)}{\cancel{2}}$$

$$= 19+6\sqrt{10}$$

Solution 4:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = a + b\sqrt{3}$

$\frac{\left(2+\sqrt{3}\right)^2}{(2)^2 - \left(\sqrt{3}\right)^2} = a + b\sqrt{3}$

$\frac{4+3+4\sqrt{3}}{4-3} = a + b\sqrt{3}$

$7 + 4\sqrt{3} = a + b\sqrt{3}$

$a = 7, \ b = 4$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2} = a\sqrt{7} + b$

$\frac{\left(\sqrt{7}-2\right)^2}{\left(\sqrt{7}\right)^2 - (2)^2} = a\sqrt{7} + b$

$\frac{7+4-4\sqrt{7}}{7-4} = a\sqrt{7} + b$

$\frac{11-4\sqrt{7}}{3} = a\sqrt{7} + b$

$a = \frac{-4}{3}, \ b = \frac{11}{3}$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2} = a\sqrt{3} - b\sqrt{2}$

$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{3-2} = a\sqrt{3} - b\sqrt{2}$

$\left(3\sqrt{3}+3\sqrt{2}\right) = a\sqrt{3} - b\sqrt{2}$

$\Rightarrow a = 3, b = -3$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} \times \frac{5+3\sqrt{2}}{5+3\sqrt{2}} = a + b\sqrt{2}$

$\frac{\left(5+3\sqrt{2}\right)^2}{(5)^2 - \left(3\sqrt{2}\right)^2} = a + b\sqrt{2}$

$\frac{25+18+30\sqrt{2}}{25-18} = a + b\sqrt{2}$

$\frac{43+30\sqrt{2}}{7} = a + b\sqrt{2}$

$a = \frac{43}{7}, \quad b = \frac{30}{7}$

Solution 5:

(i)

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$$

$$\frac{22\left(2\sqrt{3}-1\right)+17\left(2\sqrt{3}+1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-1}$$

$$=\frac{78\sqrt{3}-5}{12-1}=\frac{78\sqrt{3}-5}{11}$$

(ii)

$$\frac{\sqrt{2}}{\sqrt{6}-2}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{12}+2-\sqrt{18}+\sqrt{6}}{6-2}=\frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

Solution 6:

(i) $x^2=\left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)^2=\frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}}=\frac{9-4\sqrt{5}}{9+4\sqrt{5}}$

$$=\frac{9-4\sqrt{5}}{9+4\sqrt{5}}\times\left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right)=\frac{\left(9-4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

$$=\frac{81+80-72\sqrt{5}}{81-80}=161-72\sqrt{5}$$

(ii) $y^2=\left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)^2=\frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}}=\frac{9+4\sqrt{5}}{9-4\sqrt{5}}$

$$=\frac{9+4\sqrt{5}}{9-4\sqrt{5}}\times\frac{9+4\sqrt{5}}{9+4\sqrt{5}}=\frac{\left(9+4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}=\frac{81+80+72\sqrt{5}}{81-80}$$

$$=161+72\sqrt{5}$$

(iii) $xy=\frac{\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}=1$

(iv) $x^2+y^2+xy=161-72\sqrt{5}+161+72\sqrt{5}+1$

$=322+1=323$

Solution 7:

(i) $m = \frac{1}{3-2\sqrt{2}}$

$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$

$= \frac{3+2\sqrt{2}}{9-8}$

$= 3+2\sqrt{2}$

$\Rightarrow m^2 = (3+2\sqrt{2})^2$

$= (3)^2 + 2\times 3\times 2\sqrt{2} + (2\sqrt{2})^2$

$= 9 + 12\sqrt{2} + 8$

$= 17 + 12\sqrt{2}$

(ii) $n = \frac{1}{3+2\sqrt{2}}$

$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$

$= \frac{3+2\sqrt{2}}{9-8}$

$= 3-2\sqrt{2}$

$\Rightarrow n^2 = (3-2\sqrt{2})^2$

$= (3)^2 - 2\times 3\times 2\sqrt{2} + (2\sqrt{2})^2$

$= 9 - 12\sqrt{2} + 8$

$= 17 - 12\sqrt{2}$

(iii) $mn = (3+2\sqrt{2})(3-2\sqrt{2}) = (3)^2 - (2\sqrt{2})^2 = 9-8 = 1$

Solution 8:

(i) $\frac{1}{x} = \frac{1}{2\sqrt{3}+2\sqrt{2}} \times \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-2\sqrt{2}} = \frac{2\sqrt{3}-2\sqrt{2}}{12-8}$

$= \frac{\cancel{2}\left(\sqrt{3}-\sqrt{2}\right)}{\cancel{4}2} = \frac{\sqrt{3}-\sqrt{2}}{2}$

(ii) $x+\frac{1}{x} = 2\sqrt{3}+2\sqrt{2}+\frac{\sqrt{3}-\sqrt{2}}{2}$

$= 2\left(\sqrt{3}+\sqrt{2}\right)+\frac{\left(\sqrt{3}-\sqrt{2}\right)}{2}$

$= \frac{4\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)}{2}$

$= \frac{4\sqrt{3}+4\sqrt{2}+\sqrt{3}-\sqrt{2}}{2}$

$= \frac{5\sqrt{3}+3\sqrt{2}}{2}$

(iii) $\left(x+\frac{1}{x}\right)^2 = \left(\frac{5\sqrt{3}+3\sqrt{2}}{2}\right)^2 = \frac{75+18+30\sqrt{6}}{4}$

$= \frac{93+30\sqrt{6}}{4}$

Solution 9:

Given that $x = 1-\sqrt{2}$

We need to find the value of $\left(x-\frac{1}{x}\right)^3$.

Since $x = 1-\sqrt{2}$, we have

$\frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$\Rightarrow \frac{1}{x} = \frac{1+\sqrt{2}}{1^2-\left(\sqrt{2}\right)^2}$ $\left[\text{Since } (a-b)(a+b) = a^2-b^2\right]$

$\Rightarrow \frac{1}{x} = \frac{1+\sqrt{2}}{1-2}$

$\Rightarrow \frac{1}{x} = \frac{1+\sqrt{2}}{-1}$

$\Rightarrow \frac{1}{x} = -\left(1+\sqrt{2}\right) \ldots(1)$

Thus, $\left(x-\frac{1}{x}\right) = \left(1-\sqrt{2}\right)-\left(-\left(1+\sqrt{2}\right)\right)$

$\Rightarrow \left(x-\frac{1}{x}\right) = 1-\sqrt{2}+1+\sqrt{2}$

$\Rightarrow \left(x-\frac{1}{x}\right) = 2$

$\Rightarrow \left(x-\frac{1}{x}\right)^3 = 2^3$

$\Rightarrow \left(x-\frac{1}{x}\right)^3 = 8$

Solution 10:

Given $x = 5 - 2\sqrt{6}$

We need to find $x^2 + \frac{1}{x^2}$:

Since $x = 5 - 2\sqrt{6}$, we have

$$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{\left(5 - 2\sqrt{6}\right)\left(5 + 2\sqrt{6}\right)}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{5^2 - \left(2\sqrt{6}\right)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - 24}$$

$$\Rightarrow \frac{1}{x} = 5 + 2\sqrt{6} \ldots (1)$$

Thus, $\left(x - \frac{1}{x}\right) = \left(5 - 2\sqrt{6}\right) - \left(5 + 2\sqrt{6}\right)$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 5 - 2\sqrt{6} - 5 - 2\sqrt{6}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = -4\sqrt{6} \ldots (2)$$

Now consider $\left(x - \frac{1}{x}\right)^2$:

Thus

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x} \quad \left[\text{since } (a-b)^2 = a^2 - 2ab + b^2\right]$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = x^2 + \frac{1}{x^2} \ldots (3)$$

Thus, from equations (2) and (3), we have

$$x^2 + \frac{1}{x^2} = \left(-4\sqrt{6}\right)^2 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 96 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 98$$

Solution 11:

$$\text{L.H.S.} = \frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$

$$- \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2} + \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} - \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} + \frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

$$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$= 3+2$$

$$= 5$$

$$= \text{R.H.S.}$$

Solution 12:

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

$$= \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)+1} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{\left(\sqrt{3}-\sqrt{2}\right)-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}-\sqrt{2}\right)^2-(1)^2}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}\right)^2-2\sqrt{6}+\left(\sqrt{2}\right)^2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{3-2\sqrt{6}+2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{4-2\sqrt{6}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{2\left(2-\sqrt{6}\right)}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{2\left(2-\sqrt{6}\right)} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+\sqrt{18}-\sqrt{12}-\sqrt{6}}{2\left[(2)^2-\left(\sqrt{6}\right)^2\right]}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{2(4-6)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{2(-2)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{-4}$$

$$= \frac{1}{4}\left(2+\sqrt{6}-\sqrt{2}\right)$$

Solution 13(i):

$\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$

$$\frac{1}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$= \sqrt{3}+\sqrt{2}$$

$$= 1.7 + 1.4$$

$$= 3.1$$

Solution 13(ii):

$\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$

(ii) $\frac{1}{3+2\sqrt{2}}$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2 - \left(2\sqrt{2}\right)^2}$$

$$= \frac{3-2\sqrt{2}}{9-8}$$

$$= 3 - 2\sqrt{2}$$

$$= 3 - 2(1.4)$$

$$= 3 - 2.8$$

$$= 0.2$$