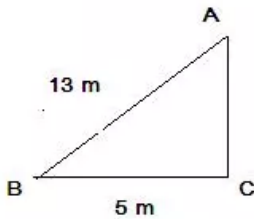


## Chapter 13. Pythagoras Theorem [Proof and Simple Applications with Converse]

### Exercise 13(A)

#### Solution 1:

The pictorial representation of the given problem is given below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

(i) Here, AB is the hypotenuse. Therefore applying the Pythagoras theorem we get,

$$AB^2 = BC^2 + CA^2$$

$$13^2 = 5^2 + CA^2$$

$$CA^2 = 13^2 - 5^2$$

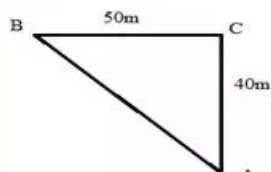
$$CA^2 = 144$$

$$CA = 12 \text{ m}$$

Therefore, the distance of the other end of the ladder from the ground is 12m

#### Solution 2:

Here, we need to measure the distance AB as shown in the figure below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore, in this case

$$AB^2 = BC^2 + CA^2$$

$$AB^2 = 50^2 + 40^2$$

$$AB^2 = 2500 + 1600$$

$$AB^2 = 4100$$

$$AB = 64.03$$

Therefore the required distance is 64.03 m.

#### Solution 3:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the  $\triangle PQS$  and applying Pythagoras theorem we get,

$$PQ^2 = PS^2 + QS^2$$

$$10^2 = PS^2 + 6^2$$

$$PS^2 = 100 - 36$$

$$PS = 8$$

Now, we consider the  $\triangle PRS$  and applying Pythagoras theorem we get,

$$PR^2 = RS^2 + PS^2$$

$$PR^2 = 15^2 + 8^2$$

$$PR = 17$$

The length of PR 17 cm

**Solution 4:**

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the  $\triangle BDC$  and applying Pythagoras theorem we get,

$$DB^2 = DC^2 + BC^2$$

$$DB^2 = 12^2 + 3^2$$

$$DB^2 = 144 + 9$$

$$DB^2 = 153$$

Now, we consider the  $\triangle ABD$  and applying Pythagoras theorem we get,

$$DA^2 = DB^2 + BA^2$$

$$13^2 = 153 + BA^2$$

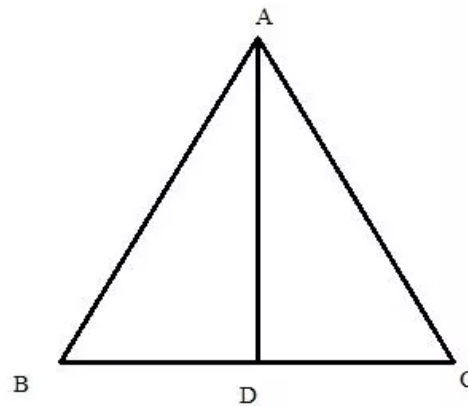
$$BA^2 = 169 - 153$$

$$BA = 4$$

The length of AB is 4 cm.

**Solution 5:**

Since ABC is an equilateral triangle therefore, all the sides of the triangle are of same measure and the perpendicular AD will divide BC in two equal parts.



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, we consider the  $\triangle ABD$  and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = 100 - 5^2$$

$$AD^2 = 100 - 25$$

$$AD^2 = 75$$

$$AD = 8.7$$

$$\left[ \begin{array}{l} \text{Given, } BC = 10 \text{ cm} = AB, \\ BD = \frac{1}{2} BC \end{array} \right] \text{ Therefore, the length of AD is 8.7 cm}$$

**Solution 6:**

We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the  $\triangle ABO$ , and applying Pythagoras theorem we get,

$$AB^2 = AO^2 + OB^2$$

$$AO^2 = AB^2 - OB^2$$

$$AO^2 = AB^2 - (BC + OC)^2$$

$$[\text{Let, } OC = x]$$

$$AO^2 = AB^2 - (BC + x)^2 \quad \dots\dots (i)$$

First, we consider the  $\triangle ACO$ , and applying Pythagoras theorem we get,

$$AC^2 = AO^2 + x^2$$

$$AO^2 = AC^2 - x^2 \quad \dots\dots (ii)$$

Now, from (i) and(ii),

$$AB^2 - (BC + x)^2 = AC^2 - x^2$$

$$8^2 - (6 + x)^2 = 3^2 - x^2 \quad [\text{Given, } AB = 8\text{cm, } BC = 8\text{cm}]$$

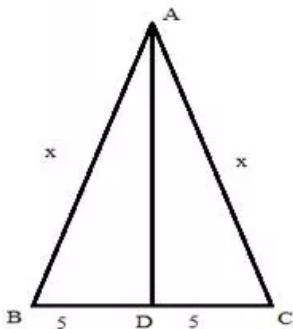
$$\text{and } AC = 3\text{cm}$$

$$x = 1\frac{7}{12}\text{cm}$$

Therefore, the length of OC will be  $1\frac{7}{12}$  cm

**Solution 7:**

Here, the diagram will be,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABC is an isosceles triangle, therefore perpendicular from vertex will cut the base in two equal segments.

First, we consider the  $\triangle ABD$ , and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = x^2 - 5^2$$

$$AD^2 = x^2 - 25$$

$$AD = \sqrt{x^2 - 25} \quad \dots\dots (i)$$

Now,

$$\text{Area} = 60$$

$$\frac{1}{2} \times 10 \times AD = 60$$

$$\frac{1}{2} \times 10 \times \sqrt{x^2 - 25} = 60$$

$$x = 13$$

Therefore, x is 13cm

**Solution 8:**

Let, the sides of the triangle be,  $x$ ,  $\sqrt{2}x$  and  $x$

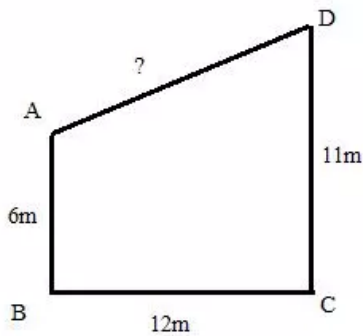
$$\text{Now, } x^2 + x^2 = 2x^2 = (\sqrt{2}x)^2 \quad \dots\dots(i)$$

Here, in (i) it is shown that, square of one side of the given triangle is equal to the addition of square of other two sides. This is nothing but Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore, the given triangle is a right angled triangle.

**Solution 9:**

The diagram of the given problem is given below,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here,  $11 - 6 = 5\text{m}$  (Since DC is perpendicular to BC)

base = 12m

Applying Pythagoras theorem we get,

$$\text{hypotenuse}^2 = 5^2 + 12^2$$

$$h^2 = 25 + 144$$

$$h^2 = 169$$

$$h = 13$$

Therefore, the distance between the tips will be 13m

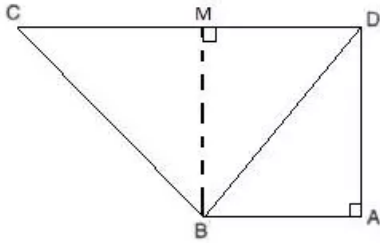
**Solution 10:**

Take M be the point on CD such that AB = DM.

So DM = 7cm and MC = 10 cm

Join points B and M to form the line segment BM.

So  $BM \parallel AD$  also  $BM = AD$ .



In right-angled  $\triangle BAD$

$$BD^2 = AD^2 + BA^2$$

$$(25)^2 = AD^2 + (7)^2$$

$$AD^2 = (25)^2 - (7)^2$$

$$AD^2 = 576$$

$$AD = 24$$

In right-angled  $\triangle CMB$

$$CB^2 = CM^2 + MB^2$$

$$CB^2 = (10)^2 + (24)^2 \quad [MB = AD]$$

$$CB^2 = 100 + 576$$

$$CB^2 = 676$$

$$CB = 26 \text{ cm}$$

### Solution 11:

Given that  $AX:XB = 1:2$ .

Let  $n$  be the common multiple for which this proportion gets satisfied.

So,  $AX = 1(n)$  and  $XB = 2(n)$

$$AX + XB = 1(n) + 2(n)$$

$$\Rightarrow AB = n + 2n$$

$$\Rightarrow 12 = 3n$$

$$\Rightarrow n = 4$$

$AX = 1(n) = 4$  and  $XB = 2(n) = 8$

In  $\triangle ABC$ ,

$XY \parallel BC$

$$\frac{AB}{AX} = \frac{AC}{AY} = \frac{BC}{XY}$$

$$\Rightarrow \frac{AB}{AX} = \frac{AC}{AY}$$

$$\Rightarrow \frac{12}{4} = \frac{AC}{8}$$

$$\Rightarrow AC = 24 \text{ cm}$$

In right-angled  $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

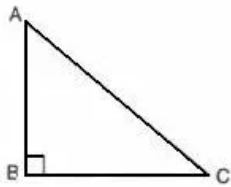
$$\Rightarrow (24)^2 = (12)^2 + BC^2$$

$$\Rightarrow BC^2 = (24)^2 - (12)^2$$

$$\Rightarrow BC^2 = 576 - 144$$

$$\Rightarrow BC^2 = 432$$

$$\Rightarrow BC = 12\sqrt{3} \text{ cm}$$

**Solution 12:**

(i) In right-angled  $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (x + 6)^2 = (x - 3)^2 + (x + 4)^2$$

$$\Rightarrow (x^2 + 12x + 36) = (x^2 - 6x + 9) + (x^2 + 8x + 16)$$

$$\Rightarrow x^2 - 10x - 11 = 0$$

$$\Rightarrow (x - 11)(x + 1) = 0$$

$$\Rightarrow x = 11 \text{ or } x = -1$$

But length of the side of a triangle can not be negative.

$$\Rightarrow x = 11 \text{ cm}$$

$$\therefore AB = (x - 3) = (11 - 3) = 8 \text{ cm}$$

$$BC = (x + 4) = (11 + 4) = 15 \text{ cm}$$

$$AC = (x + 6) = (11 + 6) = 17 \text{ cm}$$

(ii) In right-angled  $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (4x + 5)^2 = (x)^2 + (4x + 4)^2$$

$$\Rightarrow (16x^2 + 40x + 25) = (x^2) + (16x^2 + 32x + 16)$$

$$\Rightarrow x^2 - 8x - 9 = 0$$

$$\Rightarrow (x - 9)(x + 1) = 0$$

$$\Rightarrow x = 9 \text{ or } x = -1$$

But length of the side of a triangle can not be negative.

$$\Rightarrow x = 9 \text{ cm}$$

$$\therefore AB = x = 9 \text{ cm}$$

$$BC = (4x + 4) = (36 + 4) = 40 \text{ cm}$$

$$AC = (4x + 5) = (36 + 5) = 41 \text{ cm}$$

**Exercise 13(B)****Solution 1:**

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the  $\triangle ABD$  and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$

$$c^2 = h^2 + (a - x)^2$$

$$h^2 = c^2 - (a - x)^2 \quad \dots\dots (i)$$

First, we consider the  $\triangle ACD$  and applying Pythagoras theorem we get,

$$AC^2 = AD^2 + CD^2$$

$$b^2 = h^2 + x^2$$

$$h^2 = b^2 - x^2 \quad \dots\dots (ii)$$

From (i) and (ii) we get,

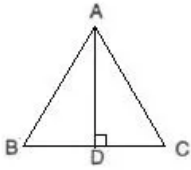
$$c^2 - (a - x)^2 = b^2 - x^2$$

$$c^2 - a^2 - x^2 + 2ax = b^2 - x^2$$

$$c^2 = a^2 + b^2 - 2ax$$

Hence Proved.

**Solution 2:**



In equilateral  $\Delta ABC$ ,  $AD \perp BC$ .

Therefore,  $BD = DC = x/2$  cm.

In right - angled  $\Delta ADC$

$$AC^2 = AD^2 + DC^2$$

$$\Rightarrow (x)^2 = AD^2 + \left(\frac{x}{2}\right)^2$$

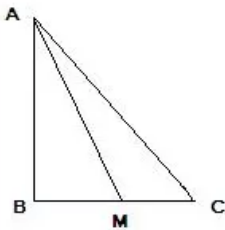
$$\Rightarrow AD^2 = (x^2) - \left(\frac{x}{2}\right)^2$$

$$\Rightarrow AD^2 = \left(\frac{x}{2}\right)^2$$

$$\Rightarrow AD = \left(\frac{x}{2}\right) \text{ cm}$$

**Solution 3:**

The pictorial form of the given problem is as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the  $\Delta ABM$  and applying Pythagoras theorem we get,

$$AM^2 = AB^2 + BM^2$$

$$AB^2 = AM^2 - BM^2 \quad \dots\dots(i)$$

Now, we consider the  $\Delta ABC$  and applying Pythagoras theorem we get,

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2 \quad \dots\dots(ii)$$

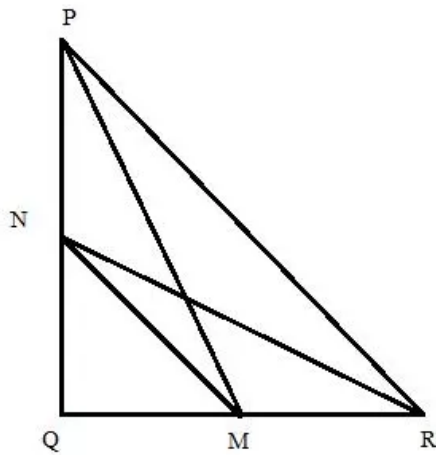
From (i) and (ii) we get,

$$AM^2 - BM^2 = AC^2 - BC^2$$

$$AM^2 + BC^2 = AC^2 + BM^2$$

Hence Proved

**Solution 4:**



We draw , PM,MN,NR

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, M and N are the mid-points of the sides QR and PQ respectively, therefore, PN=NQ,QM=RM

(i)

First, we consider the  $\Delta PQM$ , and applying Pythagoras theorem we get,

$$\begin{aligned}
 PM^2 &= PQ^2 + MQ^2 \\
 &= (PN + NQ)^2 + MQ^2 \\
 &= PN^2 + NQ^2 + 2PN \cdot NQ + MQ^2 \\
 &= MN^2 + PN^2 + 2PN \cdot NQ \quad \left[ \begin{array}{l} \text{From, } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2 \end{array} \right] \dots\dots (i)
 \end{aligned}$$

Now, we consider the  $\Delta RNQ$ , and applying Pythagoras theorem we get,

$$\begin{aligned}
 RN^2 &= NQ^2 + RQ^2 \\
 &= NQ^2 + (QM + RM)^2 \\
 &= NQ^2 + QM^2 + RM^2 + 2QM \cdot RM \\
 &= MN^2 + RM^2 + 2QM \cdot RM
 \end{aligned}$$

.....(ii)

Adding (i) and (ii) we get,

$$\begin{aligned}
 PM^2 + RN^2 &= MN^2 + PN^2 + 2PN \cdot NQ + MN^2 + RM^2 + 2QM \cdot RM \\
 PM^2 + RN^2 &= 2MN^2 + PN^2 + RM^2 + 2PN \cdot NQ + 2QM \cdot RM \\
 PM^2 + RN^2 &= 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2) \\
 PM^2 + RN^2 &= 2MN^2 + MN^2 + 2MN^2 \\
 PM^2 + RN^2 &= 5MN^2 \\
 \text{Hence Proved}
 \end{aligned}$$

(ii)

We consider the  $\Delta PQM$ , and applying Pythagoras theorem we get,

$$\begin{aligned}
 PM^2 &= PQ^2 + MQ^2 \\
 4PM^2 &= 4PQ^2 + 4MQ^2 && \left[ \begin{array}{l} \text{Multiplying both} \\ \text{sides by 4} \end{array} \right] \\
 4PM^2 &= 4PQ^2 + 4 \cdot \left( \frac{1}{2} QR \right)^2 && \left[ MQ = \frac{1}{2} QR \right] \\
 4PM^2 &= 4PQ^2 + 4 \cdot \frac{1}{4} QR^2 \\
 4PM^2 &= 4PQ^2 + QR^2
 \end{aligned}$$

Hence Proved



(iii)

We consider the  $\Delta RQN$ , and applying Pythagoras theorem we get,

$$\begin{aligned}RN^2 &= NQ^2 + RQ^2 \\4RN^2 &= 4NQ^2 + 4QR^2 && \left[ \begin{array}{l} \text{Multiplying both} \\ \text{sides by 4} \end{array} \right] \\4RN^2 &= 4QR^2 + 4 \cdot \left( \frac{1}{2} PQ \right)^2 && \left[ MQ = \frac{1}{2} PQ \right] \\4RN^2 &= 4QR^2 + 4 \cdot \frac{1}{4} PQ^2 \\4RN^2 &= PQ^2 + 4QR^2 \\ \text{Hence Proved}\end{aligned}$$

(iv)

First, we consider the  $\Delta PQM$ , and applying Pythagoras theorem we get,

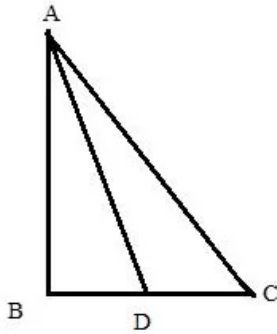
$$\begin{aligned}PM^2 &= PQ^2 + MQ^2 \\ &= (PN + NQ)^2 + MQ^2 \\ &= PN^2 + NQ^2 + 2PN \cdot NQ + MQ^2 \\ &= MN^2 + PN^2 + 2PN \cdot NQ && \left[ \begin{array}{l} \text{From } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2 \end{array} \right] \dots (i)\end{aligned}$$

Now, we consider the  $\Delta RNQ$ , and applying Pythagoras theorem we get,

$$\begin{aligned}RN^2 &= NQ^2 + RQ^2 \\ &= NQ^2 + (QM + RM)^2 \\ &= NQ^2 + QM^2 + RM^2 + 2QM \cdot RM \\ &= MN^2 + RM^2 + 2QM \cdot RM \\ &\dots (ii)\end{aligned}$$

Adding (i) and (ii) we get,

$$\begin{aligned}PM^2 + RN^2 &= MN^2 + PN^2 + 2PN \cdot NQ + MN^2 + RM^2 + 2QM \cdot RM \\ PM^2 + RN^2 &= 2MN^2 + PN^2 + RM^2 + 2PN \cdot NQ + 2QM \cdot RM \\ PM^2 + RN^2 &= 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2) \\ PM^2 + RN^2 &= 2MN^2 + MN^2 + 2MN^2 \\ PM^2 + RN^2 &= 5MN^2 \\ 4(PM^2 + RN^2) &= 4 \cdot 5 \cdot (NQ^2 + MQ^2) \\ 4(PM^2 + RN^2) &= 4 \cdot 5 \cdot \left[ \left( \frac{1}{2} PQ \right)^2 + \left( \frac{1}{2} QR \right)^2 \right] \\ &\left[ \because NQ = \frac{1}{2} PQ, MQ = \frac{1}{2} QR \right] \\ 4(PM^2 + RN^2) &= 5PR^2 \\ \text{Hence Proved}\end{aligned}$$

**Solution 5:**

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

In triangle ABC,  $\angle B = 90^\circ$  and D is the mid-point of BC. Join AD. Therefore,  $BD = DC$

First, we consider the  $\triangle ADB$ , and applying Pythagoras theorem we get,

$$AD^2 = AB^2 + BD^2$$

$$AB^2 = AD^2 - BD^2 \quad \dots\dots (i)$$

Similarly, we get from rt. angle triangles ABC we get,

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2 \quad \dots\dots (ii)$$

From (i) and (ii),

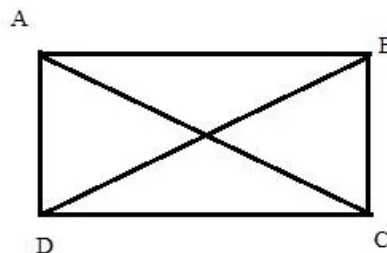
$$AC^2 - BC^2 = AD^2 - BD^2$$

$$AC^2 = AD^2 - BD^2 + BC^2$$

$$AC^2 = AD^2 - CD^2 + 4CD^2 \quad \left[ BD = CD = \frac{1}{2} BC \right]$$

$$AC^2 = AD^2 + 3CD^2$$

Hence proved.

**Solution 6:**

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABCD is a rectangle angles A, B, C and D are rt. angles.

First, we consider the  $\triangle ACD$ , and applying Pythagoras theorem we get,

$$AC^2 = DA^2 + CD^2 \quad \dots\dots (i)$$

Similarly, we get from rt. angle triangle BDC we get,

$$BD^2 = BC^2 + CD^2$$

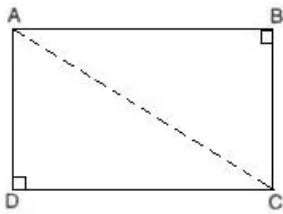
$$= BC^2 + AB^2 \quad \left[ \text{In a rectangle, opposite sides are equal, } \therefore CD = AB \right] \quad \dots\dots (ii)$$

Adding (i) and (ii),

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$$

Hence proved.

**Solution 7:**



In quadrilateral ABCD,  $\angle B = 90^\circ$  and  $\angle D = 90^\circ$ .  
So,  $\triangle ABC$  and  $\triangle ADC$  are right-angled triangles.

In  $\triangle ABC$  using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

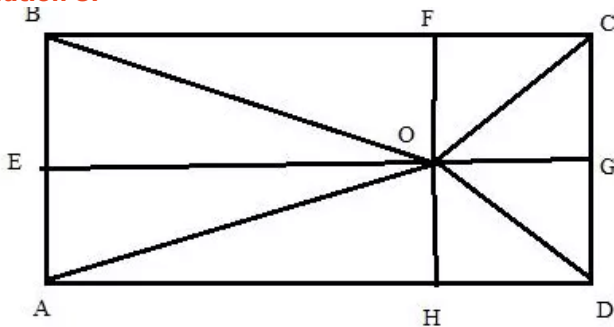
$$\Rightarrow AB^2 = AC^2 - BC^2 \dots\dots\dots(i)$$

In  $\triangle ADC$ , using Pythagoras theorem,

$$AC^2 = AD^2 + DC^2 \dots\dots\dots(ii)$$

$$\begin{aligned} LHS &= 2AC^2 - AB^2 \\ &= 2AC^2 - (AC^2 - BC^2) \quad [from(i)] \\ &= 2AC^2 - AC^2 + BC^2 \\ &= AC^2 + BC^2 \\ &= AD^2 + DC^2 + BC^2 \quad [from(ii)] \\ &= RHS \end{aligned}$$

**Solution 8:**



Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD. Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Using Pythagorean theorem we have from the above diagram:

$$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$$

$$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$$

$$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$$

$$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$$

Adding these equalities we get:

$$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$$

$$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$$

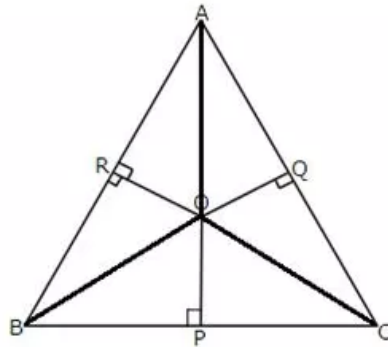
From which we prove that for any point within the rectangle there is the relation

$$OA^2 + OC^2 = OB^2 + OD^2$$

Hence Proved.

**Solution 9:**

Here, we first need to join OA, OB, and OC after which the figure becomes as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides. First, we consider the  $\triangle ARO$  and applying Pythagoras theorem we get,

$$\begin{aligned} AO^2 &= AR^2 + OR^2 \\ AR^2 &= AO^2 - OR^2 \quad \dots\dots (i) \end{aligned}$$

Similarly, from triangles, BPO,COQ,AOQ,CPO and BRO we get the following results,

$$\begin{aligned} BP^2 &= BO^2 - OP^2 \quad \dots\dots (ii) \\ CQ^2 &= OC^2 - OQ^2 \quad \dots\dots (iii) \\ AQ^2 &= AO^2 - OQ^2 \quad \dots\dots (iv) \\ CP^2 &= OC^2 - OP^2 \quad \dots\dots (v) \\ BR^2 &= OB^2 - OR^2 \quad \dots\dots (vi) \end{aligned}$$

Adding (i), (ii) and (iii),we get

$$\begin{aligned} AR^2 + BP^2 + CQ^2 &= AO^2 - OR^2 + BO^2 - OP^2 \\ &\quad + OC^2 - OQ^2 \quad \dots\dots (vii) \end{aligned}$$

Adding (iv), (v) and (vi),we get ,

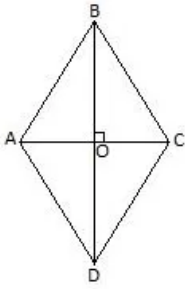
$$\begin{aligned} AQ^2 + CP^2 + BR^2 &= AO^2 - OR^2 + BO^2 - OP^2 \\ &\quad + OC^2 - OQ^2 \quad \dots\dots (viii) \end{aligned}$$

From (vii) and (viii), we get,

$$AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$$

Hence proved.

**Solution 10:**



Diagonals of the rhombus are perpendicular to each other.

In quadrilateral ABCD,  $\angle AOD = \angle COD = 90^\circ$ .  
So,  $\triangle AOD$  and  $\triangle COD$  are right-angled triangles.

In  $\triangle AOD$  using Pythagoras theorem,  
 $AD^2 = OA^2 + OD^2$   
 $\Rightarrow OA^2 = AD^2 - OD^2 \dots\dots\dots(i)$

In  $\triangle COD$  using Pythagoras theorem,  
 $CD^2 = OC^2 + OD^2$   
 $\Rightarrow OC^2 = CD^2 - OD^2 \dots\dots\dots(ii)$

$$\begin{aligned}
 LHS &= OA^2 + OC^2 \\
 &= AD^2 - OD^2 + CD^2 - OD^2 && [from(i)and(ii)] \\
 &= AD^2 + CD^2 - 2OD^2 \\
 &= AD^2 + AD^2 - 2\left(\frac{BD}{2}\right)^2 && \left[AD = CD \text{ and } OD = \frac{BD}{2}\right] \\
 &= 2AD^2 - \frac{BD^2}{2} \\
 &= RHS
 \end{aligned}$$

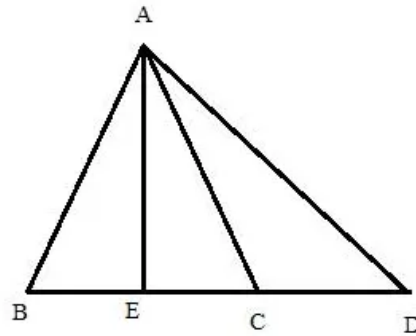
**Solution 11:**

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

We consider the  $\triangle ACD$  and applying Pythagoras theorem we get,

$$\begin{aligned}
 AC^2 &= AD^2 + DC^2 \\
 &= (AB^2 - DB^2) + (DB + BC)^2 \\
 &= BC^2 - DB^2 + DB^2 + BC^2 + 2DB \cdot BC \quad (\text{Given, } AB = BC) \\
 &= 2BC^2 + 2DB \cdot BC \\
 &= 2BC(BC + DB) \\
 &= 2BC \cdot DC
 \end{aligned}$$

Hence Proved.

**Solution 12:**

In an isosceles triangle ABC;  $AB = AC$  and D is point on BC produced. Construct AE perpendicular BC.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

We consider the rt. angled  $\triangle AED$  and applying Pythagoras theorem we get,

$$AD^2 = AE^2 + ED^2$$

$$AD^2 = AE^2 + (EC + CD)^2 \quad \dots (i)$$

$$[\because ED = EC + CD]$$

Similarly, in  $\triangle AEC$ ,

$$AC^2 = AE^2 + EC^2$$

$$AE^2 = AC^2 - EC^2 \quad \dots (ii)$$

putting  $AE^2 = AC^2 - EC^2$  in (i), we get,

$$AD^2 = AC^2 - EC^2 + (EC + CD)^2$$

$$= AC^2 + CD(CD + 2EC)$$

$$AD^2 = AC^2 + BD \cdot CD \quad [\because 2EC + CD = BD]$$

Hence Proved

**Solution 13:**

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

We consider the rt. angled  $\triangle ACD$  and applying Pythagoras theorem we get,

$$CD^2 = AC^2 + AD^2$$

$$CD^2 = AC^2 + (AB + BD)^2 \quad [\because AD = AB + BD]$$

$$CD^2 = AC^2 + AB^2 + BD^2 + 2AB \cdot BD \quad \dots (i)$$

Similarly, in  $\triangle ABC$ ,

$$BC^2 = AC^2 + AB^2$$

$$BC^2 = 2AB^2 \quad [AB = AC]$$

$$AB^2 = \frac{1}{2}BC^2 \quad \dots (ii)$$

Putting,  $AB^2$  from (ii) in (i) we get,

$$CD^2 = AC^2 + \frac{1}{2}BC^2 + BD^2 + 2AB \cdot BD$$

$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot (AD - AB)$$

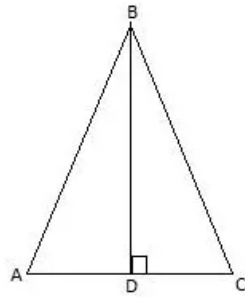
$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot AD - 2AB^2$$

$$CD^2 - BD^2 = 2AB \cdot AD$$

$$DC^2 - BD^2 = 2AB \cdot AD$$

Hence Proved.

**Solution 14:**



In right angled  $\triangle ADB$

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AD^2 = AB^2 - BD^2 \dots\dots\dots(i)$$

$$AC = AD + DC$$

$$\Rightarrow AC^2 = (AD + DC)^2$$

$$\Rightarrow AC^2 = AD^2 + DC^2 + 2AD \times DC$$

$$\Rightarrow AC^2 = AB^2 - BD^2 + DC^2 + 2AD \times DC \quad [from(i)]$$

$$\Rightarrow AC^2 = AC^2 - BD^2 + DC^2 + 2AD \times DC \quad [AB = AC]$$

$$\Rightarrow BD^2 - DC^2 = 2AD \times DC$$

**Solution 15:**

Here,

$$BD : DC = 1 : 3$$

$$\Rightarrow BD = \frac{1}{4} BC \text{ and } CD = \frac{3}{4} BC$$

$$AC^2 = AD^2 + CD^2 \text{ and } AB^2 = AD^2 + BD^2$$

Therefore,

$$AC^2 - AB^2 = CD^2 - BD^2$$

$$= \left(\frac{3}{4} BC\right)^2 - \left(\frac{1}{4} BC\right)^2$$

$$= \frac{9}{16} BC^2 - \frac{1}{16} BC^2$$

$$= \frac{1}{2} BC^2$$

$$\therefore 2AC^2 - 2AB^2 = BC^2$$

$$2AC^2 = 2AB^2 + BC^2$$

Hence proved