Chapter 19. Mean and Median (For Ungrouped Data Only)

Exercise 19(A)

Solution 1:

The numbers given are 43,51,50,57,54

The mean of the given numbers will be

$$=\frac{43+51+50+57+54}{5}$$

$$=\frac{255}{5}$$

$$= 51$$

Solution 2:

The first six natural numbers are 1, 2, 3, 4, 5, 6

The mean of first six natural numbers

$$=\frac{1+2+3+4+5+6}{3}$$

$$=\frac{21}{3}$$

$$= 3.5$$

Solution 3:

The first ten odd natural numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

The mean of first ten odd numbers

$$= \frac{1+3+5+7+9+11+13+15+17+19}{-}$$

$$=\frac{100}{10}$$

$$= 10$$

Solution 4:

The all factors of 10 are 1, 2, 5, 10

The mean of all factors of 10 are

$$=\frac{1+2+5+10}{4}$$

$$=\frac{18}{4}$$

$$= 4.5$$

Solution 5:

The given values are x + 3, x + 5, x + 7, x + 9, x + 11

The mean of the values are

$$=\frac{x+3+x+5+x+7+x+9+x+11}{5}$$

$$=\frac{5x+35}{5}$$

$$=\frac{5(x+7)}{5}$$

$$= x + 7$$

Solution 6:

(i) The given numbers are 9.8, 5.4, 3.7, 1.7, 1.8, 2.6, 2.8, 8.6, 10.5, 11.1

$$\frac{x}{x} = \frac{x1 + x2 + x3 + x4 + x5 + \dots + xn}{n}$$

$$= \frac{9.8 + 5.4 + 3.7 + 1.7 + 1.8 + 2.6 + 2.8 + 8.6 + 10.5 + 11.1}{10}$$

$$= 5.8$$

(ii) The value of
$$\sum_{i=1}^{10} (x_i - \overline{x})$$

We know that

$$\sum_{i=1}^{n} \left(xi - \bar{x} \right) = \left(x1 - \bar{x} \right) + \left(x2 - \bar{x} \right) + \left(xn - \bar{x} \right) = 0$$

Here

$$\bar{x} = 5.8$$

Therefore

$$\sum_{i=1}^{10} (x_i - \overline{x})$$
= $(9.8-5.8) + (5.4-5.8) + (3.7-5.8) + (1.7-5.8) + (1.8-5.8)$
+ $(2.6-5.8) + (2.8-5.8) + (8.6-5.8) + (10.5-5.8) + (11.1-5.8)$
= $4-.4-2.1-4.1-4-3.2-3+2.8+4.7+5.3$
= 0

Solution 7:

Given that the mean of 15 observations is 32

- (i)resulting mean increased by 3
- =32 + 3
- =35
- (ii)resulting mean decreased by 7
- =32 7
- = 25
- (iii)resulting mean multiplied by 2
- =32*2
- =64
- (iv)resulting mean divide by 0.5

$$=\frac{32}{5}$$

- = 64
- (v)resulting mean increased by 60%

$$=32 + \frac{60}{100} \times 32$$

$$=32 + 19.2$$

$$=51.2$$

(vi)resulting mean decreased by 20%

$$=32 - \frac{20}{100} \times 32$$

$$=32-6.4$$

Solution 8:

Given the mean of 5 numbers is 18

Total sum of 5 numbers

- =18*5
- =90

On excluding an observation, the mean of remaining 4 observation is 16

- =16*4
- =64

Therefore sum of remaining 4 observations

- total of 5 observations-total of 4 observations
- = 90 64
- = 26

Solution 9:

(i) Given that the mean of observations x, x + 2, x + 4, x + 6 and x + 8 is 11

$$_{\text{Mean=}} \frac{observations}{n}$$

$$11 = \frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5}$$

$$11 = \frac{5x + 20}{5}$$

$$x = \frac{35}{7}$$

$$x = 7$$

(ii)The mean of first three observations are

$$= \frac{x + x + 2 + x + 4}{3}$$

$$=\frac{3x+6}{3}$$

$$=\frac{3*7+6}{3}$$
 [since x=7]

$$=\frac{21+6}{3}$$

Solution 10:

Given the mean of 100 observations is 40.

$$\frac{\sum x}{x} = x$$

$$\Rightarrow \frac{\sum x}{x} = 40$$

$$\Rightarrow x = 40*100$$

$$\Rightarrow x = 4000$$

Incorrect value of x=4000

Correct value of x=Incorrect value of x-Incorrect observation + correct observation

Correct mean

$$= \frac{\text{correct value of } \sum x}{x}$$

$$=\frac{3970}{100}$$

$$= 39.7$$

Solution 11:

Given that the mean of 200 items was 50.

$$Mean = \frac{\sum x}{n}$$

$$\Rightarrow 50 = \frac{\sum x}{200}$$

$$\Rightarrow x = 10000$$

Incorrect value of $\sum x = 10000$

Correct value of

$$\sum x = 10000 - (92 + 8) + (192 + 88)$$

$$= 10000 - 100 + 280$$

$$= 10180$$

Correct mean

$$= \frac{\text{correct value of } \sum x}{x}$$

$$=\frac{10180}{200}$$

$$= 50.9$$

Solution 12:

Mean of 45 numbers = 18

$$\Rightarrow$$
 Sum of 45 numbers = $18 \times 45 = 810$

Mean of remaining (75 - 45)30 numbers = 13

$$\Rightarrow$$
 Sum of remaining 30 numbers = $13 \times 30 = 390$

$$\Rightarrow$$
 Sum of all the 75 numbers = 810 + 390 = 1200

$$\Rightarrow$$
 Mean of all the 75 numbers = $\frac{1200}{75}$ = 16

Solution 13:

Mean weight of 120 students = 52.75 kg

 \Rightarrow Sum of the weight of 120 students = 120 \times 52.75 = 6330 kg

Mean weight of 50 students = 51 kg

- \Rightarrow Sum of the weight of 50 students = $50 \times 51 = 2550 \text{ kg}$
- \Rightarrow Sum of the weight of remaining (120 50) 70 students
- = Sum of the weight of 120 students Sum of the weight of 50 students
- =(6330-2550) kg
- =3780 kg
- \Rightarrow Mean weight of remaining 70 students = $\frac{3780}{70}$ = 54 kg

Solution 14:

Let the number of boys and girls be x and y respectively.

Now,

Given, Mean marks of x boys in the examination = 70

 \Rightarrow Sum of marks of x boys in the examination = 70x

Given, Mean marks of y girls in the examination = 73

 \Rightarrow Sum of marks of y girls in the examination = 73y

Given, Mean marks of all students (x + y) in the examination = 71

 \Rightarrow Sum of marks of all students (x + y) students in the examination = 71(x + y)

Now, Sum of marks of all students (x + y) students in the examination

=Sum of marks of \times boys in the examination

+ Sum of marks of y girls in the examination

$$\Rightarrow$$
 71(x + y) = 70x + 73y

$$\Rightarrow$$
 71× + 71y = 70× + 73y

$$\Rightarrow x = 2y$$

$$\Rightarrow \frac{x}{y} = \frac{2}{1}$$

$$\Rightarrow$$
 x : y = 2 : 1

Thus, the ratio of number of boys to the number of girls is 2:1.

Exercise 19(B)

Solution 1:

(i) Firstly arrange the numbers in ascending order

Now since

n=9(odd)

Therefore Median

$$= \left(\frac{n+1}{2}\right)^{th}$$
$$= \left(\frac{9+1}{2}\right)^{th}$$
$$= 5^{th}$$

Thus the median is 26

(ii)

Firstly arrange the numbers in ascending order

241, 243, 257, 258, 261, 271, 292, 299, 327, 347, 350

Now since n=11(Odd)

Median = value of
$$\left(\frac{n+1}{2}\right)^{th}$$
 term
= 6^{th} term
= 271

Thus median is 271.

(iii) Firstly arrange the numbers in ascending order

Now since n=10(even)

Median=
$$\frac{1}{2}$$
 value of $\left(\frac{n}{2}\right)^{th}$ term+value of $\left(\frac{n}{2}+1\right)^{th}$ term $\left[\frac{1}{2}\right]^{th}$ term $\left[\frac{1}{2}\right]^{th$

Thus the median is 29 5

(iv) Firstly arrange the numbers in ascending order

173,185,189,194,194,200,204,208,220,223

Median=
$$\frac{1}{2}$$
 value of $\left(\frac{n}{2}\right)^{th}$ term+value of $\left(\frac{n}{2}+1\right)^{th}$ term
$$=\frac{1}{2}\left[\text{value of }\left(\frac{10}{2}\right)^{th} \text{ term+value of }\left(\frac{10}{2}+1\right)^{th} \text{ term}\right]$$

$$=\frac{1}{2}\left[200+194\right]$$

$$=\frac{1}{2}\left[394\right]$$

$$=197$$

Thus the median is 197

Solution 2:

Given numbers are 34, 37, 53, 55, x, x+2, 77, 83, 89, 100

Here n = 10(even)

Median =
$$\frac{1}{2} \left[value \ of \left(\frac{n}{2} \right)^{th} term + value \ of \left(\frac{n}{2} + 1 \right)^{th} term \right]$$

= $\frac{1}{2} \left[value \ of \left(\frac{10}{2} \right)^{th} term + value \ of \left(\frac{10}{2} + 1 \right)^{th} term \right]$
= $\frac{1}{2} \left[value \ of \ (5)^{th} term + value \ of \ (5 + 1)^{th} term \right]$
= $\frac{1}{2} \left[value \ of \ (5)^{th} term + value \ of \ (6)^{th} term \right]$
63 = $\frac{1}{2} \left[x + x + 2 \right]$
 $\Rightarrow \frac{\left[2 + 2x \right]}{2} = 63$
 $\Rightarrow x + 1 = 63$
 $\Rightarrow x = 62$

Solution 3:

For any given set of data, the median is the value of its middle term.

Here, total observations = n = 10 (even)

If n is even, we have

Median =
$$\frac{1}{2} \left[\text{value of } \left(\frac{n}{2} \right)^{\text{th}} \text{ term} + \text{value of } \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term} \right]$$

Thus, for n = 10, we have

Median =
$$\frac{1}{2} \left[\text{value of } \left(\frac{10}{2} \right)^{\text{th}} \text{ term + value of } \left(\frac{10}{2} + 1 \right)^{\text{th}} \text{ term} \right]$$

= $\frac{1}{2} \left[\text{value of } 5^{\text{th}} \text{ term + value of } 6^{\text{th}} \text{ term} \right]$

Hence, if 7th number is diminished by 8, there is no change in the median value.

Solution 4:

Here, total observations = n = 10 (even)

Thus, we have

Median =
$$\frac{1}{2} \left[\text{value of } \left(\frac{10}{2} \right)^{\text{th}} \text{ term + value of } \left(\frac{10}{2} + 1 \right)^{\text{th}} \text{ term} \right]$$

= $\frac{1}{2} \left[\text{value of 5}^{\text{th}} \text{ term + value of 6}^{\text{th}} \text{ term} \right]$

According to given information, data in ascending order is as follows:

	1 st Term	2 nd Term	3 rd Term	4 th Term	5 th Term	6 th Term	7 th Term	8 th Term	9 th Term	10 th Term
Marks	Less than 30		35	40	48	66	More than 75		75	

: Median =
$$\frac{1}{2}$$
 (40 + 48) = $\frac{88}{2}$ = 44

Hence, the median score of the whole group is 44.

Solution 5:

Total number of observations = 9(odd)

Now, if
$$n = odd$$

Median =
$$\left(\frac{n+1}{2}\right)^{th}$$
 term

$$\Rightarrow$$
 Median = $\left(\frac{9+1}{2}\right)^{th}$ term = 5^{th} term = $x + 5$

Now, Median = 18 (given)

$$x + 5 = 18$$

$$\Rightarrow \times = 13$$

Exercise 19(C)

Solution 1:

Mean of the given data =
$$\frac{8 + 12 + 16 + 22 + 10 + 4}{6}$$

= $\frac{72}{6}$ = 12

(i) Multiplied by 3

If \overline{x} is the mean of n number of observations $x_1, x_2, x_3, ..., x_n$, then mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $a\overline{x}$. Thus, when each of the given data is multiplied by 3, the mean is also multiplied by 3. Mean of the original data is 12. Hence, the new mean = 12 \times 3 = 36.

(ii) Divided by 2

If $\overline{\mathbf{x}}$ is the mean of n number of observations $\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \dots, \mathbf{x_n},$

then mean of
$$\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \dots, \frac{x_n}{a}$$
 is $\frac{\overline{x}}{a}$.

Thus, when each of the given data is divided by 2, the mean is also divided by 3.

Mean of the original data is 12.

Hence, the new mean = $\frac{12}{2}$ = 6.

(iii) multiplied by 3 and then divided by 2

If $\overline{\times}$ is the mean of n number of observations $\times_1, \times_2, \times_3, \ldots, \times_n$,

then mean of
$$\frac{a}{b} \times_1$$
, $\frac{a}{b} \times_2$, $\frac{a}{b} \times_3$, ..., $\frac{a}{b} \times_n$ is $\frac{a}{b} \overline{\times}$.

Thus, when each of the given data is multiplied by $\frac{3}{2}$,

the mean is also multiplied by $\frac{3}{2}$.

Mean of the original data is 12.

Hence, the new mean =
$$\frac{3}{2} \times 12 = \frac{36}{2} = 18$$

(iv) increased by 25%

New mean = Original mean + 25% of original mean

$$\Rightarrow$$
 New mean = 12 + $\frac{25}{100}$ × 12

$$\Rightarrow$$
 New mean = 12 + $\frac{1}{4}$ x 12

(v) decreased by 40%

New mean = Original mean - 40% of original mean

$$\Rightarrow$$
 New mean = 12 - 40% of 12

$$\Rightarrow$$
 New mean = 12 - $\frac{40}{100} \times 12$

$$\Rightarrow$$
 New mean = 12- $\frac{2}{5}$ x 12

$$\Rightarrow$$
 New mean = 12 - 0.4 \times 12

$$\Rightarrow$$
 New mean = 12 - 4.8

Solution 2:

Mean of given data =
$$\frac{18 + 24 + 15 + 2x + 1 + 12}{5}$$

$$\Rightarrow 21 = \frac{70 + 2x}{5}$$

$$\Rightarrow 5x21 = 70 + 2x$$

$$\Rightarrow 105 = 70 + 2x$$

$$\Rightarrow 2x = 105 - 70$$

$$\Rightarrow 2x = 35$$

$$\Rightarrow x = \frac{35}{2}$$

$$\Rightarrow x = 17.5$$

Solution 3:

Let $\overline{\times}$ be the mean of n number of observations $\times_1, \times_2, \times_3, \ldots, \times_n$

Mean of given data=
$$\frac{\times_1 + \times_2 + \times_3 + \dots + \times_n}{n}$$

Given that mean of 6 numbers is 42.

That is,

$$\frac{\times_1 + \times_2 + \times_3 + \dots + \times_6}{6} = 42$$

$$\Rightarrow \times_1 + \times_2 + \times_3 + \dots + \times_6 = 6 \times 42$$

$$\Rightarrow \times_1 + \times_2 + \times_3 + \times_4 + \times_5 = 252 - \times_6 \dots (1)$$

Also, given that the mean of 5 numbers is 45.

That is,

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 45$$

$$\Rightarrow \times_1 + \times_2 + \times_3 + \times_4 + \times_5 = 5 \times 45$$

$$\Rightarrow \times_1 + \times_2 + \times_3 + \times_4 + \times_5 = 225....(2)$$

From equations (1) and (2), we have,

$$x_1 + x_2 + x_3 + x_4 + x_5 = 252 - x_6 = x_1 + x_2 + x_3 + x_4 + x_5 = 225$$

$$252 - x_6 = 225$$

 $\Rightarrow x_6 = 252 - 225 = 27$

Solution 4:

Let \overline{x} be the mean of n number of observations $x_1, x_2, x_3, ..., x_n$

Mean of given data=
$$\frac{X_1 + X_2 + X_3 + \dots + X_n}{n}$$

Given that mean of 10 numbers is 24.

That is,

$$\frac{X_1 + X_2 + X_3 + \dots + X_{10}}{10} = 24$$

$$\Rightarrow x_1 + x_2 + x_3 + ... + x_{10} = 10 \times 24$$

$$\Rightarrow x_1 + x_2 + x_3 + ... + x_{10} = 240$$

$$\Rightarrow x_1 + x_2 + x_3 + ... + x_{10} + x_{11} = 240 + x_{11}(1)$$

Also, given that mean of 11 numbers is 25.

That is,

$$\frac{X_1 + X_2 + X_3 + \dots + X_{10} + X_{11}}{11} = 25$$

$$\Rightarrow \times_1 + \times_2 + \times_3 + \dots + \times_{10} + \times_{11} = 11 \times 25$$

$$\Rightarrow x_1 + x_2 + x_3 + ... + x_{10} + x_{11} = 275....(2)$$

From equations (1) and (2), we have:

$$X_1 + X_2 + X_3 + \dots + X_{10} + X_{11} = 240 + X_{11} = 275$$

$$240 + x_{11} = 275$$

$$\Rightarrow x_{11} = 275 - 240 = 35$$

Solution 5:

Consider the given data:

Here the number of observations is 9, which is odd.

Thus, the median of the given data is $\left(\frac{n+1}{2}\right)^{th}$ observation.

From the given data, $\left(\frac{9+1}{2}=5\right)^{\text{th}}$ observation is x+13

Also, given that the median is 78.

Thus, we have

$$x + 13 = 78$$

$$\Rightarrow x = 78 - 13$$

Solution 6:

Consider the given data:

Here the number of observations is 10, which is even.

Thus, the median of given data is
$$\frac{1}{2} \left[\left(\frac{n}{2} \right)^{th} term + \left(\frac{n}{2} + 1 \right)^{th} term \right]$$
.

From the given data, $\left(\frac{10}{2} = 5\right)^{\text{th}}$ observation is x - 1

and
$$\left(\frac{10}{2} + 1 = 6\right)^{\text{th}}$$
 observation is $x + 3$.

Also, given that the median is 58.

Thus, we have

$$\frac{1}{2}[x-1+x+3] = 116$$

$$\Rightarrow$$
 2x + 2 = 116

$$\Rightarrow$$
 2x = 116 - 2

$$\Rightarrow$$
 2x = 114

$$\Rightarrow x = \frac{114}{2}$$

$$\Rightarrow x = 57$$

Solution 7:

Let \bar{x} be the mean of n number of observations $x_1, x_2, x_3, ..., x_n$

$$Mean = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

Therefore,

Mean of given data=
$$\frac{30 + 32 + 24 + 34 + 26 + 28 + 30 + 35 + 33 + 25}{10}$$
$$= \frac{297}{10}$$
$$= 29.7$$

(i)

Let us tabulate the observations and their deviations from the mean

Observations	Devaiations
×i	$\times_i - \overline{\times}$
30	0.3
32	2.3
24	-5.7
34	4.3
26	-3.7
28	-1.7
30	0.3
35	5.3
33	3.3
25	-4.7
Total	0

From the table, it is clear that the sum of the deviations from

Consider the given data:

Let us rewrite the above data in ascending order.

There are 10 observations, which is even.

Therefore, median=
$$\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} term + \left(\frac{n}{2} + 1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} term + \left(\frac{10}{2} + 1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[\left(5\right)^{th} term + \left(5 + 1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[5^{th} term + 6^{th} term\right]$$

$$= \frac{1}{2}\left[30 + 30\right]$$

$$= \frac{1}{2}\left[60\right]$$

$$= 30$$

Solution 8:

Let $\bar{\times}$ be the mean of n number of observations $\times_1, \times_2, \times_3, ..., \times_n$

$$Mean = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

Therefore,

Mean of given data= $\frac{35 + 48 + 92 + 76 + 64 + 52 + 51 + 63 + 71}{9}$

$$=\frac{552}{9}$$

= 61.33

Let us rewrite the given data in ascending order:

Thus, we have

There are 9 observations, which is odd.

Therefore, median = $\left(\frac{n+1}{2}\right)^{th}$ observation

$$\Rightarrow$$
 Median = $\left(\frac{9+1}{2}\right)^{th}$ observation

$$\Rightarrow$$
 Median = $\left(\frac{10}{2}\right)^{\text{th}}$ observation

If 51 is replaced by 66, the new set of data in ascending order is:

Since median = 5thobservation,

We have, new median = 64

Solution 9:

Let $\bar{\times}$ be the mean of n number of observations $\times_1, \times_2, \times_3, \ldots, \times_n$

Mean
$$=\frac{X_1 + X_2 + X_3 + \dots + X_n}{n}$$

Therefore,

Mean of given data=
$$\frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5}$$

$$=\frac{5x + 20}{5}$$
$$= x + 4$$

Also, it's given that mean of the given data is 11.

$$\Rightarrow x + 4 = 11$$

$$\Rightarrow x = 7$$

Hence the mean of the first three observations = $\frac{x + x + 2 + x + 4}{3}$

$$=\frac{3x+6}{3}$$

Solution 10:

Let us find the factors of 72:

$$72 = 1 \times 72$$

$$=2 \times 36$$

$$= 3 \times 24$$

$$= 4 \times 18$$

$$= 6 \times 12$$

$$= 8 \times 9$$

$$= 9 \times 8$$

$$= 12 \times 6$$

$$= 18 \times 4$$

$$= 24 \times 3$$

$$= 36 \times 2$$

$$= 72 \times 1$$

Therefore, the data set is:

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Mean of the above data set= $\frac{1+2+3+4+6+8+9+12+18+24+36+72}{12}$

$$= \frac{195}{12}$$
$$= 16.25$$

Since the number of observations is 12, which is even, median is given by

Median =
$$\frac{1}{2} \left[\left(\frac{n}{2} \right)^{th} \operatorname{term} + \left(\frac{n}{2} + 1 \right)^{th} \operatorname{term} \right]$$

= $\frac{1}{2} \left[\left(\frac{12}{2} \right)^{th} \operatorname{term} + \left(\frac{12}{2} + 1 \right)^{th} \operatorname{term} \right]$
= $\frac{1}{2} \left[6^{th} \operatorname{term} + 7^{th} \operatorname{term} \right]$
= $\frac{1}{2} \left[8 + 9 \right]$
= $\frac{1}{2} \times 17$
= 8.5