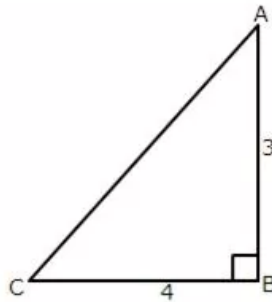# Chapter 22. Trigonometrical Ratios [Sine, Consine, Tangent of an Angle and their Reciprocals]

Exercise 22(A)

Solution 1:

Given angle $ABC = 90°$

$$\Rightarrow AC^2 = AB^2 + BC^2 \ (AC \text{ is hypotenuse})$$
$$\Rightarrow AC^2 = 3^2 + 4^2$$
$$\therefore \ AC^2 = 9 + 16 = 25 \text{ and } AC = 5$$

(i)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4}{5}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{3}{5}$$

(iii)

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3}{4}$$

(iv)

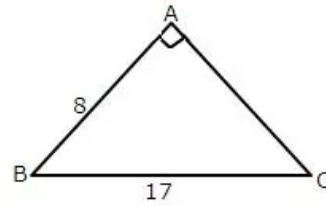$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{BC} = \frac{5}{4}$$

(v)

$$\csc C = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{AB} = \frac{5}{3}$$

(vi)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{3}{4}$$

Solution 2:

Given angle $BAC = 90°$

$\Rightarrow BC^2 = AB^2 + AC^2 \ (BC \text{ is hypotenuse})$

$\Rightarrow 17^2 = 8^2 + AC^2$

$\therefore \ AC^2 = 289 - 64 = 225$ and $AC = 15$

(i)

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{AC} = \frac{8}{15}$$

(iii)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\sin^2 B + \cos^2 B = \left(\frac{15}{17}\right)^2 + \left(\frac{8}{17}\right)^2$$

$$= \frac{225 + 64}{289}$$

$$= \frac{289}{289}$$

$$= 1$$

(iv)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

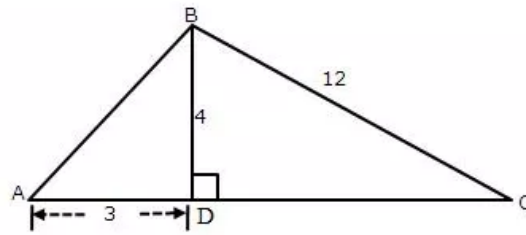$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\sin B \cdot \cos C + \cos B \cdot \sin C = \frac{15}{17} \cdot \frac{15}{17} + \frac{8}{17} \cdot \frac{8}{17}$$

$$= \frac{225 + 64}{289}$$

$$= \frac{289}{289}$$

$$= 1$$

Solution 3:

Consider the diagram as

Given angle $ADB = 90°$ and $BDC = 90°$

$\Rightarrow AB^2 = AD^2 + BD^2 \left(AB \text{ is hypotenuse in } \triangle ABD \right)$

$\Rightarrow AB^2 = 3^2 + 4^2$

$\therefore \quad AB^2 = 9 + 16 = 25$ and $AB = 5$

$\Rightarrow BC^2 = BD^2 + DC^2 \left(BC \text{ is hypotenuse in } \triangle BDC \right)$

$\Rightarrow DC^2 = 12^2 - 4^2$

$\therefore \quad DC^2 = 144 - 16 = 128$ and $DC = 8\sqrt{2}$

(i)

$\cos A = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{AD}{AB} = \dfrac{3}{5}$

(ii)

$\operatorname{cosec} A = \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{AB}{BD} = \dfrac{5}{4}$

(iii)

$\tan A = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{BD}{AD} = \dfrac{4}{3}$

$\sec A = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AB}{AD} = \dfrac{5}{3}$

$\tan^2 A - \sec^2 A = \left(\dfrac{4}{3} \right)^2 - \left(\dfrac{5}{3} \right)^2$

$\qquad\qquad = \dfrac{16}{9} - \dfrac{25}{9}$

$\qquad\qquad = \dfrac{-9}{9}$

$\qquad\qquad = -1$

(iv)

$\sin C = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BD}{BC} = \dfrac{4}{12} = \dfrac{1}{3}$

(v)

$\sec C = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{BC}{DC} = \dfrac{12}{8\sqrt{2}} = \dfrac{3}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$
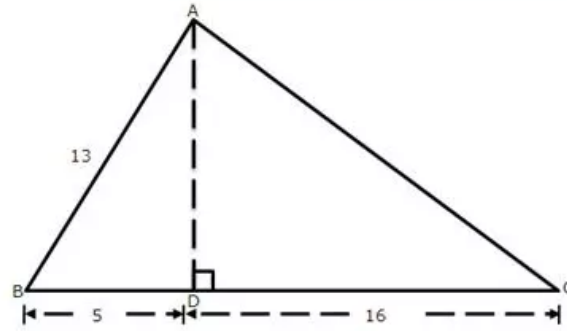
(vi)

$\cot C = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{DC}{BD} = \dfrac{8\sqrt{2}}{4} = 2\sqrt{2}$

$\sin C = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BD}{BC} = \dfrac{4}{12} = \dfrac{1}{3}$

$\cot^2 C - \dfrac{1}{\sin^2 C} = \left(2\sqrt{2} \right)^2 - \dfrac{1}{\left(\dfrac{1}{3} \right)^2}$

$\qquad\qquad = 8 - 9$

$\qquad\qquad = -1$

Solution 4:

Given angle $ADB = 90°$ and $ADC = 90°$

$\Rightarrow AB^2 = AD^2 + BD^2 \left(AB \text{ is hypotenuse in } \Delta ABD \right)$

$\Rightarrow 13^2 = AD^2 + 5^2$

$\therefore \quad AD^2 = 169 - 25 = 144 \text{ and } AD = 12$

$\Rightarrow AC^2 = AD^2 + DC^2 \left(AC \text{ is hypotenuse in } \Delta ADC \right)$

$\Rightarrow AC^2 = 12^2 + 16^2$

$\therefore \quad AC^2 = 144 + 256 = 400 \text{ and } AC = 20$

(i)
$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{12}{13}$$

(ii)
$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$$

(iii)
$$\sec B = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{BD} = \frac{13}{5}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{12}{5}$$

$$\sec^2 B - \tan^2 B = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2$$

$$= \frac{169 - 144}{25}$$
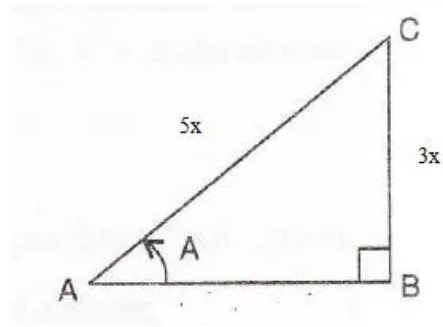
$$= \frac{25}{25}$$

$$= 1$$

(iv)
$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$$

$$\sin^2 C + \cos^2 C = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

$$= \frac{9 + 16}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

Solution 5:

Consider the diagram below:

$$\sin A = \frac{3}{5}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{BC}{AC} = \frac{3}{5}$$

Therefore if length of $BC = 3x$, length of $AC = 5x$

Since

$$AB^2 + BC^2 = AC^2 \quad \left[\text{Using Pythagoras Theorem}\right]$$

$$AB^2 + \left(3x\right)^2 = \left(5x\right)^2$$

$$AB^2 = 25x^2 - 9x^2 = 16x^2$$

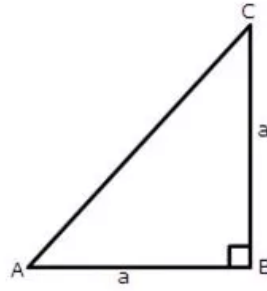$$\therefore AB = 4x \left(\text{base}\right)$$

Now

(i)

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3x}{4x} = \frac{3}{4}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

Solution 6:

Given angle $ABC = 90°$ in the figure

$\Rightarrow AC^2 = AB^2 + BC^2 \left(AC\, \text{is hypotenuse in}\, \Delta ABC\right)$

$\Rightarrow AC^2 = a^2 + a^2$

$\therefore \quad AC^2 = 2a^2 \text{ and } AC = \sqrt{2}a$

Now

(i) $\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

(ii) $\sec A = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AC}{AB} = \dfrac{\sqrt{2}a}{a} = \sqrt{2}$
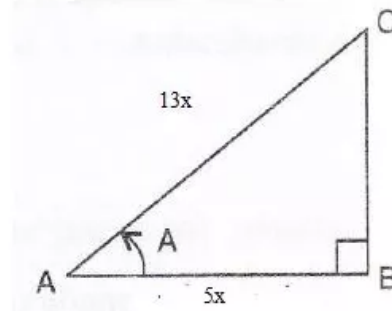
(iii) $\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

$\cos A = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

$$\cos^2 A + \sin^2 A = \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2$$
$$= \dfrac{1}{2} + \dfrac{1}{2}$$
$$= 1$$

Solution 7:

Consider the diagram below:

$$\cos A = \frac{5}{13}$$

$i.e.\ \dfrac{base}{hypotenuse} = \dfrac{5}{13} \Rightarrow \dfrac{AB}{AC} = \dfrac{5}{13}$

Therefore if length of $AB = 5x$, length of $AC = 13x$

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$(5x)^2 + BC^2 = (13x)^2$

$BC^2 = 169x^2 - 25x^2 = 144x^2$

$\therefore BC = 12x\ (perpendicular)$

Now

$\tan A = \dfrac{perpendicular}{base} = \dfrac{12x}{5x} = \dfrac{12}{5}$

$\sin A = \dfrac{perpendicular}{hypotenuse} = \dfrac{12x}{13x} = \dfrac{12}{13}$

$\cot A = \dfrac{base}{perpendicular} = \dfrac{5x}{12x} = \dfrac{5}{12}$

(i)

$\dfrac{\sin A - \cot A}{2\tan A}$

$= \dfrac{\dfrac{12}{13} - \dfrac{5}{12}}{2\left(\dfrac{12}{5}\right)}$

$= \dfrac{79}{156} \cdot \dfrac{5}{24}$

$= \dfrac{395}{3744}$
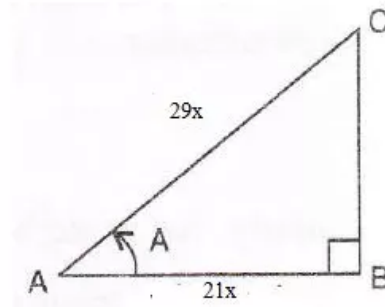
(ii)

$\cot A + \dfrac{1}{\cos A}$

$= \dfrac{5}{12} + \dfrac{1}{\dfrac{5}{13}}$

$= \dfrac{5}{12} + \dfrac{13}{5}$

$= \dfrac{181}{60}$

Solution 8:

Consider the diagram below:

$$\sec A = \frac{29}{21}$$

i.e. $\frac{\text{hypotenuse}}{\text{base}} = \frac{29}{21} \Rightarrow \frac{AC}{AB} = \frac{29}{21}$

Therefore if length of $AB = 21x$, length of $AC = 29x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(21x)^2 + BC^2 = (29x)^2$$

$$BC^2 = 841x^2 - 441x^2 = 400x^2$$

$$\therefore BC = 20x \, (\text{perpendicular})$$

Now

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{20x}{29x} = \frac{20}{29}$$

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{20x}{21x} = \frac{20}{21}$$

Therefore
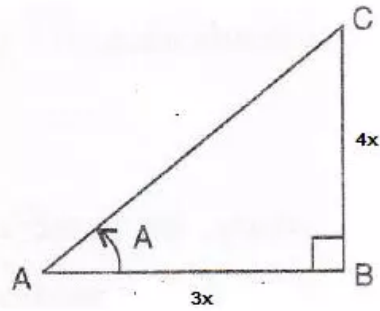
$$\sin A - \frac{1}{\tan A}$$

$$= \frac{20}{29} - \frac{1}{\frac{20}{21}}$$

$$= \frac{20}{29} - \frac{21}{20}$$

$$= -\frac{209}{580}$$

Solution 9:

Consider the diagram below:

$$\tan A = \frac{4}{3}$$

$$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3}$$

Therefore if length of $AB = 3x$, length of $BC = 4x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = AC^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore AC = 5x \, (\text{hypotenuse})$$

Now

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$$

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3x}{4x} = \frac{3}{4}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$$

Therefore

$$\frac{\csc A}{\cot A - \sec A}$$
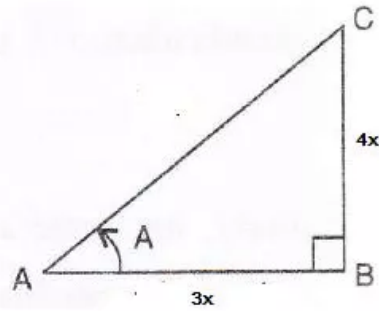
$$= \frac{\dfrac{5}{4}}{\dfrac{3}{4} - \dfrac{5}{3}}$$

$$= \frac{\dfrac{5}{4}}{\dfrac{-11}{12}}$$

$$= -\frac{60}{44}$$

$$= -\frac{15}{11}$$

Solution 10:

Consider the diagram below:

$4 \cot A = 3$

$\cot A = \dfrac{3}{4}$

$i.e. \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{3}{4} \Rightarrow \dfrac{AB}{BC} = \dfrac{3}{4}$

Therefore if length of AB = 3x, length of BC = 4x
Since

$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x \,(\text{hypotenuse})$

(i)

$\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{4x}{5x} = \dfrac{4}{5}$

(ii)

$\sec A = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AC}{AB} = \dfrac{5x}{3x} = \dfrac{5}{3}$

(iii)

$\operatorname{cosec} A = \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{AC}{BC} = \dfrac{5x}{4x} = \dfrac{5}{4}$

$\cot A = \dfrac{3}{4}$

$\operatorname{cosec}^2 A - \cot^2 A$

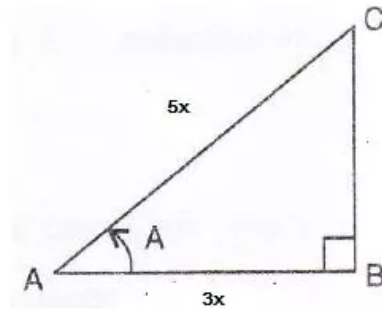$= \left(\dfrac{5}{4}\right)^2 - \left(\dfrac{3}{4}\right)^2$

$= \dfrac{25 - 9}{16}$

$= \dfrac{16}{16}$

$= 1$

Solution 11:

Consider the diagram below:

$\cos A = 0.6$

$\cos A = \dfrac{6}{10} = \dfrac{3}{5}$

$i.e.\ \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{3}{5} \Rightarrow \dfrac{AB}{AC} = \dfrac{3}{5}$

Therefore if length of AB = 3x, length of AC = 5x
Since

$AB^2 + BC^2 = AC^2 \quad \left[\text{Using Pythagoras Theorem}\right]$

$(3x)^2 + BC^2 = (5x)^2$

$BC^2 = 25x^2 - 9x^2 = 16x^2$

$\therefore BC = 4x\,(\text{perpendicular})$

Now all other trigonometric ratios are

$\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{4x}{5x} = \dfrac{4}{5}$

$\operatorname{cosec} A = \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{AC}{BC} = \dfrac{5x}{4x} = \dfrac{5}{4}$
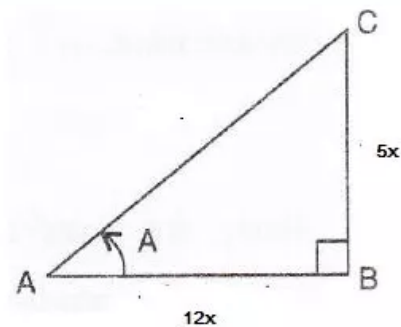
$\sec A = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AC}{AB} = \dfrac{5x}{3x} = \dfrac{5}{3}$

$\tan A = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{4x}{3x} = \dfrac{4}{3}$

$\cot A = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{3x}{4x} = \dfrac{3}{4}$

Solution 12:

Consider the diagram below:

$\tan A = \dfrac{5}{12}$

$i.e.\ \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{5}{12} \Rightarrow \dfrac{BC}{AB} = \dfrac{5}{12}$

Therefore if length of AB = 12x, length of BC = 5x
Since

$AB^2 + BC^2 = AC^2 \quad \left[\text{Using Pythagoras Theorem}\right]$

$(12x)^2 + (5x)^2 = AC^2$

$AC^2 = 144x^2 + 25x^2 = 169x^2$

$\therefore AC = 13x\,(\text{hypotenuse})$

(i)
$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12x}{13x} = \frac{12}{13}$$

(ii)
$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{5x}{13x} = \frac{5}{13}$$

(iii)
$$\frac{\cos A + \sin A}{\cos A - \sin A}$$

$$= \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$
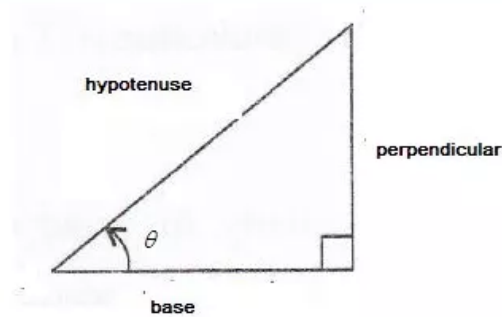
$$= \frac{\frac{17}{13}}{\frac{7}{13}}$$

$$= \frac{17}{7}$$

$$= 2\frac{3}{7}$$

Solution 13:

Consider the diagram below:

$$\sin \theta = \frac{p}{q}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{p}{q}$$

Therefore if length of perpendicular = px, length of hypotenuse = qx

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$$

$$\text{base}^2 + (px)^2 = (qx)^2$$

$$\text{base}^2 = q^2x^2 - p^2x^2 = (q^2 - p^2)x^2$$

$$\therefore \text{base} = \sqrt{q^2 - p^2}\, x$$

Now

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{q^2 - p^2}}{q}$$
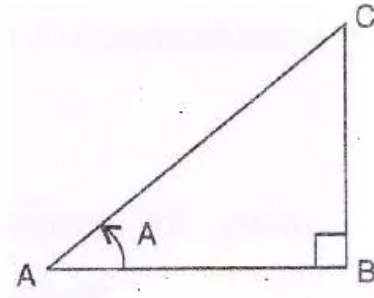
Therefore
$$\cos \theta + \sin \theta$$

$$= \frac{\sqrt{q^2 - p^2}}{q} + \frac{p}{q}$$

$$= \frac{p + \sqrt{q^2 - p^2}}{q}$$

Consider the diagram below:

$$\cos A = \frac{1}{2}$$

$$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{1}{2} \Rightarrow \frac{AB}{AC} = \frac{1}{2}$$
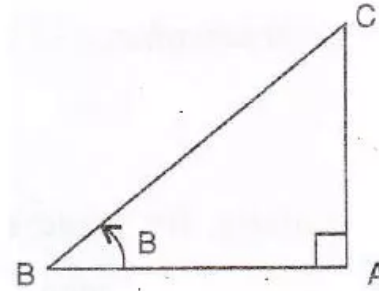
Therefore if length of AB = x, length of AC = 2x

Since

$$AB^2 + BC^2 = AC^2 \qquad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + BC^2 = (2x)^2$$

$$BC^2 = 4x^2 - x^2 = 3x^2$$

$$\therefore BC = \sqrt{3}x \, (\text{perpendicular})$$

Consider the diagram below:

$$\sin B = \frac{1}{\sqrt{2}}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AC}{BC} = \frac{1}{\sqrt{2}}$$

Therefore if length of AC = x, length of $BC = \sqrt{2}\,x$

Since

$$AB^2 + AC^2 = BC^2 \qquad [\text{Using Pythagoras Theorem}]$$

$$AB^2 + x^2 = \left(\sqrt{2}x\right)^2$$

$$AB^2 = 2x^2 - x^2 = x^2$$

$$\therefore AB = x \, (\text{base})$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$$

Therefore

$$\frac{\tan A - \tan B}{1 + \tan A \tan B}$$
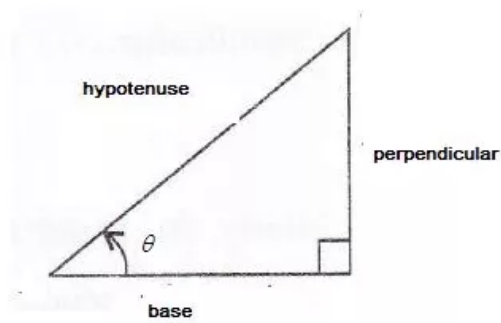
$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

Solution 15:

Consider the diagram below:

$5\cot\theta = 12$

$\cot\theta = \dfrac{12}{5}$

$i.e. \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{12}{5}$

Therefore if length of base = 12x, length of perpendicular = 5x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$

$(12x)^2 + (5x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = 144x^2 + 25x^2 = 169x^2$

$\therefore \text{hypotenuse} = 13x$

Now

$\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{13x}{5x} = \dfrac{13}{5}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{13x}{12x} = \dfrac{13}{12}$

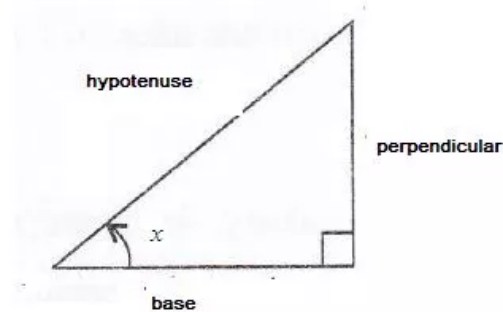Therefore

$\text{cosec}\,\theta + \sec\theta$

$= \dfrac{13}{5} + \dfrac{13}{12}$

$= \dfrac{221}{60}$

$= 3\dfrac{41}{60}$

Solution 16:

Consider the diagram below:

$$\tan x = 1\frac{1}{3}$$

$$\tan x = \frac{4}{3}$$

$$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3}$$

Therefore if length of base = 3x, length of perpendicular = 4x

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$$

$$(3x)^2 + (4x)^2 = \text{hypotenuse}^2$$

$$\text{hypotenuse}^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore \text{hypotenuse} = 5x$$

Now

$$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

$$\cos x = \frac{\text{base}}{\text{hypotenuse}} = \frac{3x}{5x} = \frac{3}{5}$$

Therefore

$$4\sin^2 x - 3\cos^2 x + 2$$

$$= 4\left(\frac{4}{5}\right)^2 - 3\left(\frac{3}{5}\right)^2 + 2$$
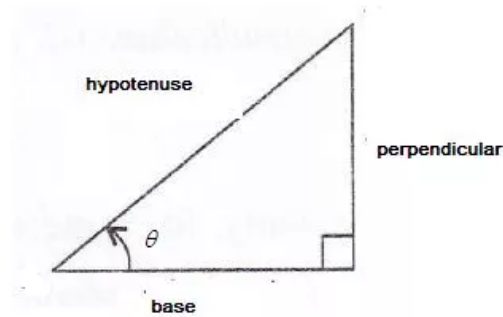
$$= \frac{64}{25} - \frac{27}{25} + 2$$

$$= \frac{87}{25}$$

$$= 3\frac{12}{25}$$

Solution 17:

Consider the diagram below:

$$\operatorname{cosec}\theta = \sqrt{5}$$

$$i.e. \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\sqrt{5}}{1}$$

Therefore if length of hypotenuse $= \sqrt{5}\,x$, length of perpendicular = x

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$$

$$\text{base}^2 + (x)^2 = \left(\sqrt{5}x\right)^2$$

$$\text{base}^2 = 5x^2 - x^2 = 4x^2$$

$$\therefore \text{base} = 2x$$

Now

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}}$$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}}$$

(i)

$$2 - \sin^2\theta - \cos^2\theta$$

$$= 2 - \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$$

$$= 2 - \frac{1}{5} - \frac{4}{5}$$

$$= \frac{5}{5}$$

$$= 1$$

(ii)

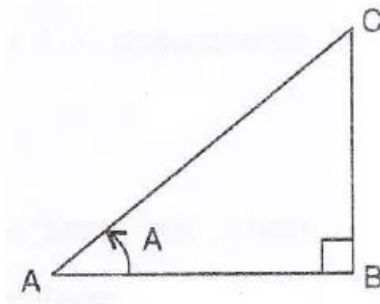$$2 + \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= 2 + \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2} - \frac{\left(\frac{2}{\sqrt{5}}\right)^2}{\left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= 2 + 5 - 4$$

$$= 3$$

Solution 18:

Consider the diagram below:

$$sec\,A = \sqrt{2}$$

$$i.e.\frac{hypotenuse}{base} = \frac{\sqrt{2}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{2}}{1}$$

Therefore if length of AB = x, length of $AC = \sqrt{2}\,x$

Since

$$AB^2 + BC^2 = AC^2 \quad [Using\,Pythagoras\,Theorem]$$

$$(x)^2 + BC^2 = \left(\sqrt{2}x\right)^2$$

$$BC^2 = 2x^2 - x^2 = x^2$$

$$\therefore\ BC = x\,(perpendicular)$$

Now

$$tan\,A = \frac{perpendicular}{base} = \frac{x}{x} = 1$$

$$sin\,A = \frac{perpendicular}{hypotenuse} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$$cos\,A = \frac{base}{hypotenuse} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

Therefore

$$\frac{3\cos^2 A + 5\tan^2 A}{4\tan^2 A - \sin^2 A}$$

$$= \frac{3\left(\frac{1}{\sqrt{2}}\right)^2 + 5(1)^2}{4(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$$
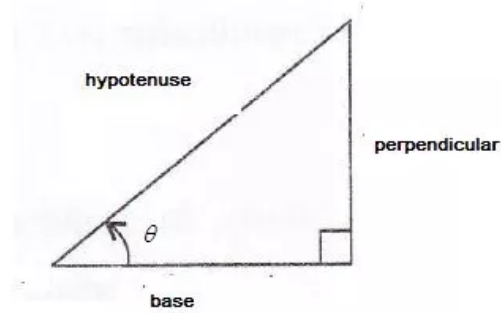
$$= \frac{\frac{13}{2}}{\frac{7}{2}}$$

$$= \frac{13}{7}$$

$$= 1\frac{6}{7}$$

Solution 19:

Consider the diagram below:

$\cot \theta = 1$

i.e. $\dfrac{\text{base}}{\text{perpendicular}} = \dfrac{1}{1}$

Therefore if length of base = x, length of perpendicular = x

Since

$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \left[\text{Using Pythagoras Theorem}\right]$

$(x)^2 + (x)^2 = \text{hypotenuse}^2$

$\text{hypotenuse}^2 = x^2 + x^2 = 2x^2$

$\therefore \text{hypotenuse} = \sqrt{2}x$

Now

$\sin \theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{x}{\sqrt{2}x} = \dfrac{1}{\sqrt{2}}$

$\tan \theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{x}{x} = 1$

Therefore

$5\tan^2 \theta + 2\sin^2 \theta - 3$

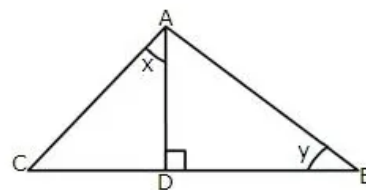$= 5(1)^2 + 2\left(\dfrac{1}{\sqrt{2}}\right)^2 - 3$

$= 5 + 1 - 3$

$= 3$

Solution 20:

Given angle $DAC = 90°$ and $\angle ADB = 90°$ in the figure

$\Rightarrow AC^2 = AD^2 + DC^2 \left(AC \text{ is hypotenuse in } \triangle ADC\right)$

$\Rightarrow AD^2 = 26^2 - 10^2$

$\therefore AD^2 = 576 \text{ and } AD = 24$

Again

$\Rightarrow AB^2 = AD^2 + BD^2 \left(AB \text{ is hypotenuse in } \triangle ABD\right)$

$\Rightarrow AB^2 = 24^2 + 32^2$

$\therefore AB^2 = 1600 \text{ and } AB = 40$

Now

(i)

$\cot x = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{AD}{CD} = \dfrac{24}{10} = 2.4$

(ii)

$\sin y = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AD}{AB} = \dfrac{24}{40} = \dfrac{3}{5}$

$\tan y = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{AD}{BD} = \dfrac{24}{32} = \dfrac{3}{4}$

Therefore

$$\frac{1}{\sin^2 y} - \frac{1}{\tan^2 y}$$

$$= \frac{1}{\left(\frac{3}{5}\right)^2} - \frac{1}{\left(\frac{3}{4}\right)^2}$$

$$= \frac{25}{9} - \frac{16}{9}$$

$$= \frac{9}{9}$$

$$= 1$$

(iii)

$$\tan y = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{24}{32} = \frac{3}{4}$$

$$\cos x = \frac{\text{base}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{24}{26} = \frac{12}{13}$$

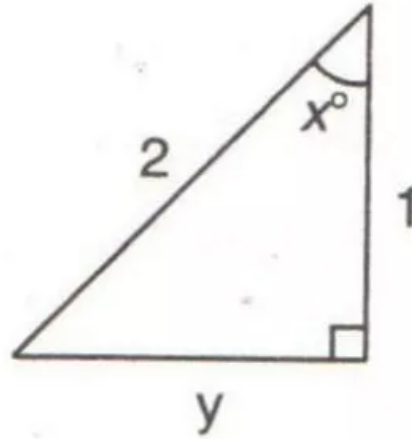$$\cos y = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{32}{40} = \frac{4}{5}$$

Therefore

$$\frac{6}{\cos x} - \frac{5}{\cos y} + 8\tan y$$

$$= \frac{6}{\frac{12}{13}} - \frac{5}{\frac{4}{5}} + 8\left(\frac{3}{4}\right)$$

$$= \frac{13}{2} - \frac{25}{4} + 6$$

$$= \frac{26 - 25 + 24}{4}$$

$$= \frac{25}{4}$$

$$= 6\frac{1}{4}$$

Solution 1:

Consider the given figure

(i)
Since the triangle is a right angled triangle, so using Pythagorean Theorem

$2^2 = y^2 + 1^2$

$y^2 = 4 - 1 = 3$

$y = \sqrt{3}$

(ii)

$\sin x° = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{\sqrt{3}}{2}$

(iii)

$\tan x° = \dfrac{\text{perpendicular}}{\text{base}} = \sqrt{3}$

$\sec x° = \dfrac{\text{hypotenuse}}{\text{base}} = 2$

Therefore

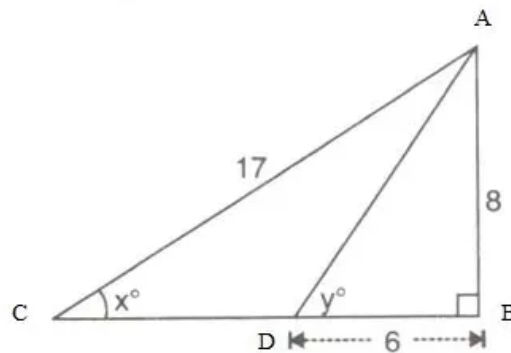$\left(\sec x° - \tan x°\right)\left(\sec x° + \tan x°\right)$

$= \left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)$

$= 4 - 3$

$= 1$

Solution 2:

Consider the given figure

Since the triangle is a right angled triangle, so using Pythagorean Theorem

$AD^2 = 8^2 + 6^2$

$AD^2 = 64 + 36 = 100$

$AD = 10$

Also

$BC^2 = AC^2 - AB^2$

$BC^2 = 17^2 - 8^2 = 225$

$BC = 15$

(i)

$\sin x° = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{8}{17}$

(ii)

$\cos y° = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{6}{10} = \dfrac{3}{5}$

(iii)

$\sin y° = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AB}{AD} = \dfrac{8}{10} = \dfrac{4}{5}$

$\cos y° = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{6}{10} = \dfrac{3}{5}$

$\tan x° = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{AB}{BC} = \dfrac{8}{15}$
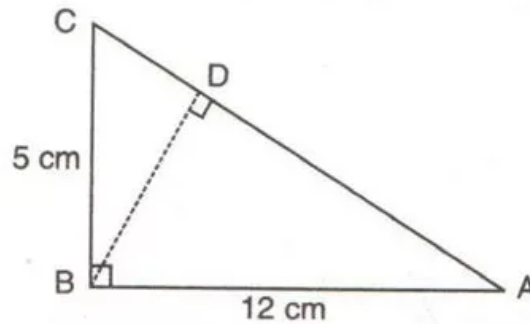
Therefore

$3\tan x° - 2\sin y° + 4\cos y°$

$= 3\left(\dfrac{8}{15}\right) - 2\left(\dfrac{4}{5}\right) + 4\left(\dfrac{3}{5}\right)$

$= \dfrac{8}{5} - \dfrac{8}{5} + \dfrac{12}{5}$

$= 2\dfrac{2}{5}$

Solution 3:

Consider the given figure

Since the triangle is a right angled triangle, so using Pythagorean Theorem

$AC^2 = 5^2 + 12^2$

$AC^2 = 25 + 144 = 169$

$AC = 13$

In $\triangle CBD$ and $\triangle CBA$, the $\angle C$ is common to both the triangles, $\angle CDB = \angle CBA = 90°$ so therefore $\angle CBD = \angle CAB$.
Therefore $\triangle CBD$ and $\triangle CBA$ are similar triangles according to AAA Rule
So

$\dfrac{AC}{BC} = \dfrac{AB}{BD}$

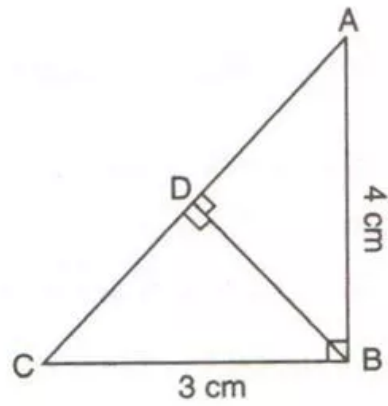$\dfrac{13}{5} = \dfrac{12}{BD}$

$BD = \dfrac{60}{13}$

(i)

$\cos \angle DBC = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{BD}{BC} = \dfrac{\frac{60}{13}}{5} = \dfrac{12}{13}$

(ii)

$\cot \angle DBA = \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{BD}{AB} = \dfrac{\frac{60}{13}}{12} = \dfrac{5}{13}$

Solution 4:

Consider the given figure

Since the triangle is a right angled triangle, so using Pythagorean Theorem

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9 = 25$

$AC = 5$

In $\triangle CBD$ and $\triangle CBA$, the $\angle C$ is common to both the triangles, $\angle CDB = \angle CBA = 90°$ so therefore $\angle CBD = \angle CAB$.

Therefore $\triangle CBD$ and $\triangle CBA$ are similar triangles according to AAA Rule

So

$\dfrac{AC}{BC} = \dfrac{AB}{BD}$

$\dfrac{5}{3} = \dfrac{4}{BD}$

$BD = \dfrac{12}{5}$

Now using Pythagorean Theorem

$DC^2 = 3^2 - \left(\dfrac{12}{5}\right)^2$

$DC^2 = 9 - \dfrac{144}{25} = \dfrac{81}{25}$

$DC = \dfrac{9}{5}$

Therefore

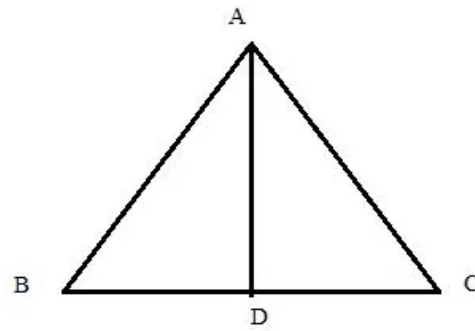$AD = AC - DC$

$= 5 - \dfrac{9}{5}$

$= \dfrac{16}{5}$

(i)

$\tan \angle DBC = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{DC}{BD} = \dfrac{\frac{9}{5}}{\frac{12}{5}} = \dfrac{3}{4}$

(ii)

$\sin \angle DBA = \dfrac{AD}{AB} = \dfrac{\frac{16}{5}}{4} = \dfrac{4}{5}$
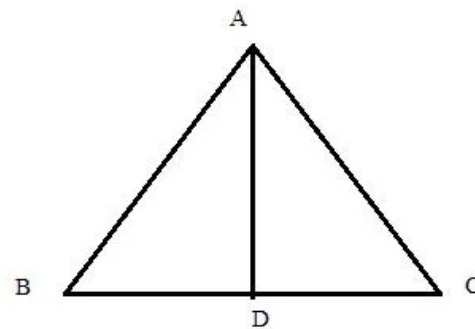
Solution 5:

Consider the figure below

In the isosceles $\triangle ABC$, $AB = AC = 15\,\text{cm}$ and $BC = 18\,\text{cm}$ the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 9\,\text{cm}$

$$\cos\angle ABC = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{9}{15} = \frac{3}{5}$$

Solution 6:

Consider the figure below

In the isosceles $\triangle ABC$, $AB = AC = 5\,\text{cm}$ and $BC = 8\,\text{cm}$ the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 4\,\text{cm}$

Since $\angle ADB = 90°$

$\Rightarrow AB^2 = AD^2 + BD^2 \left(AB \text{ is hypotenuse in } \triangle ABD\right)$

$\Rightarrow AD^2 = 5^2 - 4^2$

$\therefore\ AD^2 = 9$ and $AD = 3$

(i)

$$\sin B = \frac{AD}{AB} = \frac{3}{5}$$

(ii)

$$\tan C = \frac{AD}{DC} = \frac{3}{4}$$

(iii)

$$\sin B = \frac{AD}{AB} = \frac{3}{5}$$

$$\cos B = \frac{BD}{AB} = \frac{4}{5}$$

Therefore

$$\sin^2 B + \cos^2 B$$

$$= \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

$$= \frac{25}{25}$$

$$= 1$$

(iv)

$$\tan C = \frac{AD}{DC} = \frac{3}{4}$$

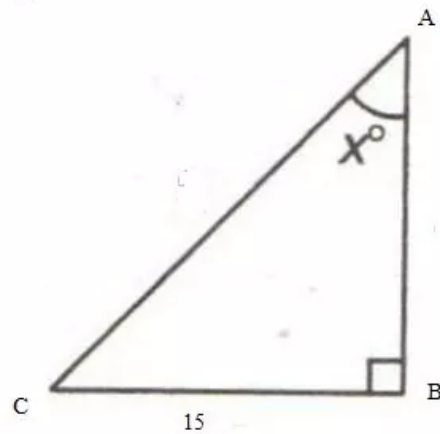$$\cot B = \frac{BD}{AD} = \frac{4}{3}$$

Therefore

$$\tan C - \cot B$$

$$= \frac{3}{4} - \frac{4}{3}$$

$$= -\frac{7}{12}$$

Solution 7:

Consider the figure

$$\tan x^{\circ} = \frac{3}{4}$$

$i.e. \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{BC}{AB} = \dfrac{3}{4}$

Therefore if length of base = 4x, length of perpendicular = 3x

Since

$BC^2 + AB^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x$

Now

$BC = 15$

$3x = 15$

$x = 5$

Therefore

$\begin{aligned} AB &= 4x \\ &= 4 \times 5 \\ &= 20\,cm \end{aligned}$

And

$\begin{aligned} AC &= 5x \\ &= 5 \times 5 \\ &= 25\,cm \end{aligned}$

Solution 8:

Consider the figure
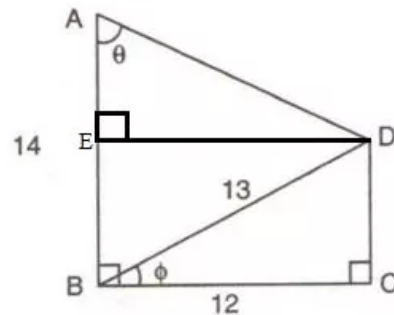
A perpendicular is drawn from D to the side AB at point E which makes BCDE is a rectangle.
Now in right angled triangle BCD using Pythagorean Theorem

$\Rightarrow BD^2 = BC^2 + CD^2 \left(AB\text{ is hypotenuse in } \Delta ABD\right)$

$\Rightarrow CD^2 = 13^2 - 12^2 = 25$

$\therefore CD = 5$

Since BCDE is rectangle so ED 12 cm, EB = 5 and AE = 14 - 5 = 9

(i)

$\sin \phi = \dfrac{CD}{BD} = \dfrac{5}{13}$

$\tan \theta = \dfrac{ED}{AE} = \dfrac{12}{9} = \dfrac{4}{3}$

(ii)

$\sec \theta = \dfrac{AD}{AE}$

$\sec \theta = \dfrac{AD}{9}$

$AD = 9\sec \theta$

Or

$\operatorname{cosec} \theta = \dfrac{AD}{ED}$

$\operatorname{cosec} \theta = \dfrac{AD}{12}$

$AD = 12\operatorname{cosec} \theta$

Solution 9:

Given

$$\sin B = \frac{4}{5}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{4}{5}$$

Therefore if length of perpendicular = 4x, length of hypotenuse = 5x

Since

$$BC^2 + AC^2 = AB^2 \qquad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 - (4x)^2 = BC^2$$

$$BC^2 = 9x^2$$

$$\therefore BC = 3x$$

Now

$$BC = 15$$

$$3x = 15$$

$$x = 5$$

(i)

$$AC = 4x$$

$$= 4 \times 5$$

$$= 20 \, \text{cm}$$

And

$$AB = 5x$$

$$= 5 \times 5$$

$$= 25 \, \text{cm}$$

(ii)

Given

$$\tan \angle ADC = \frac{1}{1}$$

$$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{AC}{CD} = \frac{1}{1}$$

Therefore if length of perpendicular = x, length of hypotenuse = x

Since

$$AC^2 + CD^2 = AD^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + (x)^2 = AD^2$$

$$AD^2 = 2x^2$$

$$\therefore AD = \sqrt{2}x$$

Now

$$AC = 20$$

$$x = 20$$

So

$$AD = \sqrt{2}x$$

$$= \sqrt{2} \times 20$$

$$= 20\sqrt{2} \text{ cm}$$

And

$$CD = 20 \text{ cm}$$

Now

$$\tan B = \frac{AC}{BC} = \frac{20}{15} = \frac{4}{3}$$

$$\cos B = \frac{BC}{AB} = \frac{15}{25} = \frac{3}{5}$$

So

$$\tan^2 B - \frac{1}{\cos^2 B}$$

$$= \left(\frac{4}{3}\right)^2 - \frac{1}{\left(\frac{3}{5}\right)^2}$$

$$= \frac{16}{9} - \frac{25}{9}$$

$$= -\frac{9}{9}$$

$$= -1$$

Solution 10:

$$\sin A + \text{cosec} A = 2$$

Squaring both sides

$$(\sin A + \text{cosec} A)^2 = 2^2$$

$$\sin^2 A + \text{cosec}^2 A + 2\sin A \cdot \text{cosec} A = 4$$

$$\sin^2 A + \text{cosec}^2 A + 2\sin A \cdot \frac{1}{\sin A} = 4$$

$$\sin^2 A + \text{cosec}^2 A = 2$$
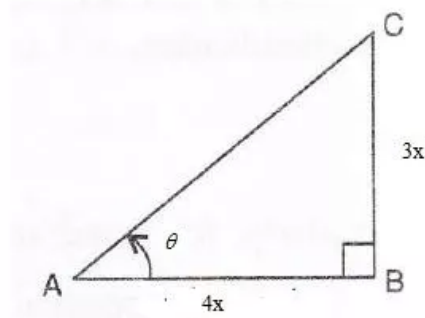
Solution 11:

$$\tan A + \cot A = 5$$

Squaring both sides

$$(\tan A + \cot A)^2 = 5^2$$

$$\tan^2 A + \cot^2 A + 2\tan A \cdot \cot A = 25$$

$$\tan^2 A + \cot^2 A + 2\tan A \cdot \frac{1}{\tan A} = 25$$

$$\tan^2 A + \cot^2 A = 23$$

Consider the diagram below:

$4\sin\theta = 3\cos\theta$

$\tan\theta = \dfrac{3}{4}$

$i.e.\ \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{3}{4} \Rightarrow \dfrac{BC}{AB} = \dfrac{3}{4}$

Therefore if length of BC = 3x, length of AB = 4x

Since

$AB^2 + BC^2 = AC^2$ \qquad [Using Pythagoras Theorem]

$(4x)^2 + (3x)^2 = AC^2$

$AC^2 = 25x^2$

$\therefore AC = 5x\ (\text{hypotenuse})$

(i)

$\sin\theta = \dfrac{BC}{AC} = \dfrac{3}{5}$

(ii)

$\cos\theta = \dfrac{AB}{AC} = \dfrac{4}{5}$

Solution 12:

(iii)

$\cot\theta = \dfrac{AB}{BC} = \dfrac{4}{3}$

$\operatorname{cosec}\theta = \dfrac{AC}{BC} = \dfrac{5}{3}$

Therefore

$\cot^2\theta - \operatorname{cosec}^2\theta$

$= \left(\dfrac{4}{3}\right)^2 - \left(\dfrac{5}{3}\right)^2$

$= \dfrac{16-25}{9}$

$= -\dfrac{9}{9}$

$= -1$

(iv)

$4\cos^2\theta - 3\sin^2\theta + 2$

$= 4\left(\dfrac{4}{5}\right)^2 - 3\left(\dfrac{3}{5}\right)^2 + 2$
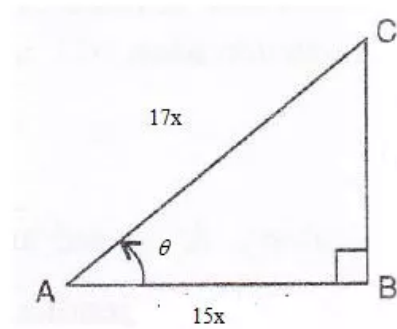
$= \dfrac{64}{25} - \dfrac{27}{25} + 2$

$= \dfrac{64-27+50}{25}$

$= \dfrac{87}{25}$

$= 3\dfrac{12}{25}$

Solution 13:

Consider the diagram below:

$17\cos\theta = 15$

$\cos\theta = \dfrac{15}{17}$

$i.e. \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{15}{17} \Rightarrow \dfrac{AB}{AC} = \dfrac{15}{17}$

Therefore if length of AB = 15x, length of AC = 17x

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$(17x)^2 - (15x)^2 = BC^2$

$BC^2 = 64x^2$

$\therefore BC = 8x \,(\text{perpendicular})$

Now

$\sec\theta = \dfrac{AC}{AB} = \dfrac{17}{15}$

$\tan\theta = \dfrac{BC}{AB} = \dfrac{8}{15}$

Therefore

$\tan\theta + 2\sec\theta$

$= \dfrac{8}{15} + 2 \cdot \dfrac{17}{15}$

$= \dfrac{42}{15}$

$= \dfrac{14}{5}$

$= 2\dfrac{4}{5}$

Solution 14:

$5\cos A - 12\sin A = 0$

$5\cos A = 12\sin A$

$\dfrac{\sin A}{\cos A} = \dfrac{5}{12}$

$\tan A = \dfrac{5}{12}$

Now

$\dfrac{\sin A + \cos A}{2\cos A - \sin A} = \dfrac{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\cos A}}{\dfrac{2\cos A}{\cos A} - \dfrac{\sin A}{\cos A}}$

$= \dfrac{\tan A + 1}{2 - \tan A}$

$= \dfrac{\dfrac{5}{12} + 1}{2 - \dfrac{5}{12}}$

$= \dfrac{\dfrac{17}{12}}{\dfrac{19}{12}}$

$= \dfrac{17}{19}$

Solution 15:

Since D is mid-point of AC so $AC = 2DC$

(i)

$$\frac{\tan \angle CAB}{\tan \angle CDB}$$

$$= \frac{\dfrac{BC}{AC}}{\dfrac{BC}{DC}}$$

$$= \frac{BC}{2\,DC} \cdot \frac{DC}{BC}$$

$$= \frac{1}{2}$$

(ii)

$$\frac{\tan \angle ABC}{\tan \angle DBC}$$

$$= \frac{\dfrac{AC}{BC}}{\dfrac{DC}{BC}}$$

$$= \frac{2\,DC}{BC} \cdot \frac{BC}{DC}$$

$$= 2$$

Solution 16:

Consider the diagram below:
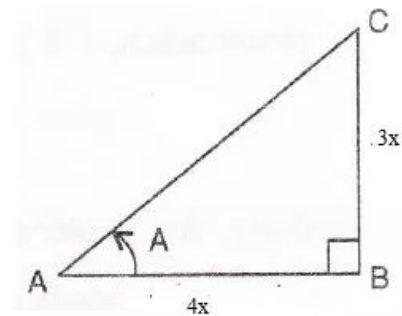
$$3\cos A = 4 \sin A$$

$$\cot A = \frac{4}{3}$$

$$i.e. \frac{\text{base}}{\text{perpendicular}} = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Therefore if length of AB = 4x, length of BC = 3x

Since

$$AB^2 + BC^2 = AC^2 \qquad \left[\text{Using Pythagoras Theorem}\right]$$

$$\left(4x\right)^2 + \left(3x\right)^2 = AC^2$$

$$AC^2 = 25x^2$$

$$\therefore AC = 5x \left(\text{hypotenuse}\right)$$

(i)

$$\cos A = \frac{AB}{AC} = \frac{4}{5}$$

(ii)

$$\text{cosec} A = \frac{AC}{BC} = \frac{5}{3}$$

Therefore

$3 - \cot^2 A + \csc^2 A$

$= 3 - \left(\dfrac{4}{3}\right)^2 + \left(\dfrac{5}{3}\right)^2$

$= \dfrac{27 - 16 + 25}{9}$

$= \dfrac{36}{9}$

$= 4$

Solution 17:

Consider the figure
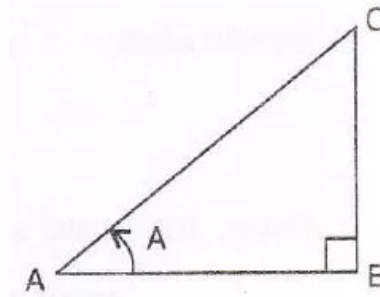
$\tan A = \dfrac{75}{100} = \dfrac{3}{4}$

$i.e. \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{BC}{AB} = \dfrac{3}{4}$

Therefore if length of base = 4x, length of perpendicular = 3x

Since

$BC^2 + AB^2 = AC^2 \qquad [\text{Using Pythagoras Theorem}]$

$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

$\therefore AC = 5x$

Now

$AC = 30$

$5x = 30$

$x = 6$

Therefore

$AB = 4x$
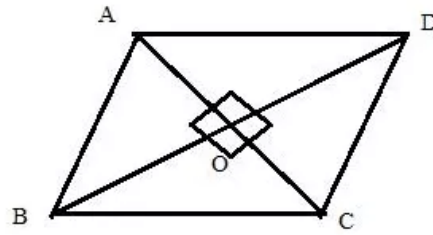
$\quad = 4 \times 6$

$\quad = 24 \, cm$

And

$BC = 3x$

$\quad = 3 \times 6$

$\quad = 18 \, cm$

Solution 18:

Consider the figure

The diagonals of a rhombus bisects each other perpendicularly

$$\cos \angle CAB = \frac{6}{10} = \frac{3}{5}$$

$$i.e. \frac{\text{base}}{\text{hypotenuse}} = \frac{OA}{AB} = \frac{3}{5}$$

Therefore if length of base = 3x, length of hypotenuse = 5x

Since

$$OB^2 + OA^2 = AB^2 \qquad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 - (3x)^2 = OB^2$$

$$OB^2 = 16x^2$$

$$\therefore OB = 4x$$

Now

$$OB = 8$$

$$4x = 8$$

$$x = 2$$

Therefore

$$AB = 5x$$

$$= 5 \times 2$$

$$= 10\,\text{cm}$$

And

$$OA = 3x$$

$$= 3 \times 2$$

$$= 6\,\text{cm}$$

Since the sides of a rhombus are equal so the length of the side of the rhombus $= 10\,\text{cm}$
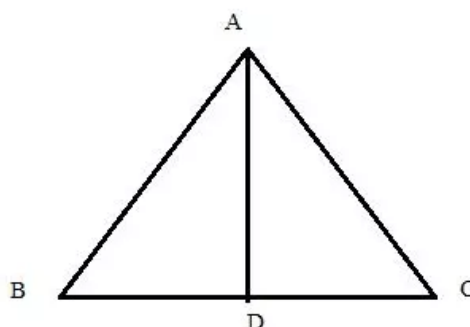
The diagonals are

$$BD = 8 \times 2$$

$$= 16\,\text{cm}$$

$$AC = 6 \times 2$$

$$= 12\,\text{cm}$$

Solution 19:

Consider the figure below

In the isosceles $\triangle ABC$, the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 9\,\text{cm}$

Since $\angle ADB = 90°$

$\Rightarrow AB^2 = AD^2 + BD^2 \left(AB \text{ is hypotenuse in } \triangle ABD\right)$

$\Rightarrow AD^2 = 15^2 - 9^2$

$\therefore \quad AD^2 = 144 \text{ and } AD = 12$

(i)

$\cos B = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{BD}{AB} = \dfrac{9}{15} = \dfrac{3}{5}$

(ii)

$\sin C = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AD}{AB} = \dfrac{12}{15} = \dfrac{4}{5}$

(iii)

$\tan B = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{AD}{BD} = \dfrac{12}{9} = \dfrac{4}{3}$

$\sec B = \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AB}{BD} = \dfrac{15}{9} = \dfrac{5}{3}$

Therefore

$\tan^2 B - \sec^2 B + 2$

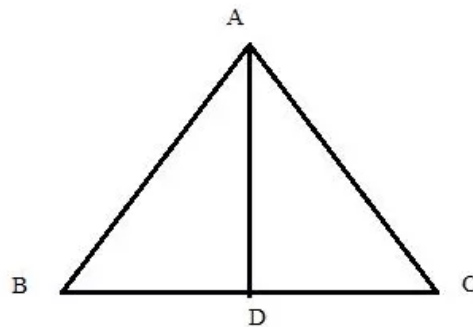$= \left(\dfrac{4}{3}\right)^2 - \left(\dfrac{5}{3}\right)^2 + 2$

$= \dfrac{16 - 25 + 18}{9}$

$= \dfrac{9}{9}$

$= 1$

Solution 20:

Consider the figure below

$\sin B = \dfrac{8}{10} = \dfrac{4}{5}$

$i.e. \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AD}{AB} = \dfrac{4}{5}$

Therefore if length of perpendicular = 4x, length of hypotenuse = 5x

Since

$AD^2 + BD^2 = AB^2 \quad \left[\text{Using Pythagoras Theorem}\right]$

$\left(5x\right)^2 - \left(4x\right)^2 = BD^2$

$BD^2 = 9x^2$

$\therefore BD = 3x$

Now

$BD = 9$

$3x = 9$

$x = 3$

Therefore

$AB = 5x$

$\quad = 5 \times 3$

$\quad = 15 \, cm$

And

$AD = 4x$

$\quad = 4 \times 3$

$\quad = 12 \, cm$

Again

$\tan C = \dfrac{1}{1}$

$i.e. \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{AD}{DC} = \dfrac{1}{1}$

Therefore if length of perpendicular = x, length of base = x

Since

$AD^2 + DC^2 = AC^2 \quad \left[\text{Using Pythagoras Theorem}\right]$

$(x)^2 + (x)^2 = AC^2$

$AC^2 = 2x^2$

$\therefore AC = \sqrt{2}x$

Now

$AD = 12$

$\quad x = 12$

Therefore

$DC = x$

$\quad = 12 \, cm$

And

$AC = \sqrt{2}x$

$\quad = \sqrt{2} \times 12$
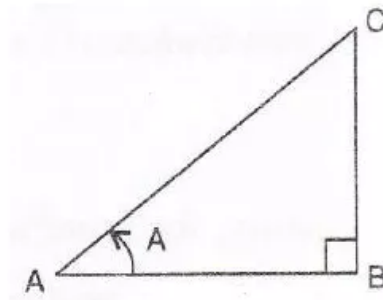
$\quad = 12\sqrt{2} \, cm$

Solution 21:

$q \tan A = p$

$\tan A = \dfrac{p}{q}$

Now

$$\dfrac{p \sin A - q \cos A}{p \sin A + q \cos A} = \dfrac{\dfrac{p \sin A}{\cos A} - \dfrac{q \cos A}{\cos A}}{\dfrac{p \sin A}{\cos A} + \dfrac{q \cos A}{\cos A}}$$

$$= \dfrac{p \tan A - q}{p \tan A + q}$$

$$= \dfrac{p\left(\dfrac{p}{q}\right) - q}{p\left(\dfrac{p}{q}\right) + q}$$

$$= \dfrac{\dfrac{p^2 - q^2}{q}}{\dfrac{p^2 + q^2}{q}}$$

$$= \dfrac{p^2 - q^2}{p^2 + q^2}$$

Solution 22:

Consider the figure

$\sin A = \cos A$

$\tan A = \dfrac{1}{1}$

$i.e. \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{BC}{AB} = \dfrac{1}{1}$

Therefore if length of perpendicular = x, length of base = x

Since

$AB^2 + BC^2 = AC^2 \qquad [\text{Using Pythagoras Theorem}]$

$(x)^2 + (x)^2 = AC^2$

$AC^2 = 2x^2$

$\therefore AC = \sqrt{2}x$

Now

$\sec A = \dfrac{AC}{AB} = \sqrt{2}$
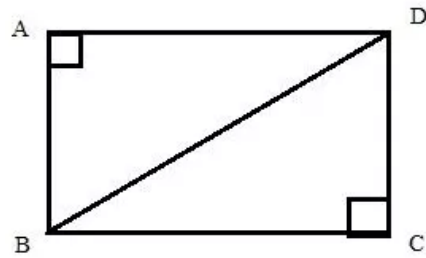
Therefore

$2\tan^2 A - 2\sec^2 A + 5$

$= 2(1)^2 - 2\left(\sqrt{2}\right)^2 + 5$

$= 2 - 4 + 5$

$= 3$

Solution 23:

Consider the diagram

$$\cot \angle ABD = \frac{15}{10} = \frac{3}{2}$$

$$i.e. \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BD} = \frac{3}{2}$$

Therefore if length of base = 3x, length of perpendicular = 2x

Since

$$AB^2 + AD^2 = BD^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (2x)^2 = BD^2$$

$$BD^2 = 13x^2$$

$$\therefore BD = \sqrt{13}x$$

Now

$$BD = 26$$

$$\sqrt{13}x = 26$$

$$x = \frac{26}{\sqrt{13}}$$

Therefore

$$AD = 2x$$

$$= 2 \times \frac{26}{\sqrt{13}}$$

$$= \frac{52}{\sqrt{13}} \, \text{cm}$$

$$AB = 3x$$

$$= 3 \times \frac{26}{\sqrt{13}}$$

$$= \frac{78}{\sqrt{13}} \, \text{cm}$$

Now

Area of rectangle $ABCD = AB \times AD$

$$= \frac{78}{\sqrt{13}} \times \frac{52}{\sqrt{13}}$$

$$= 312 \, \text{cm}^2$$
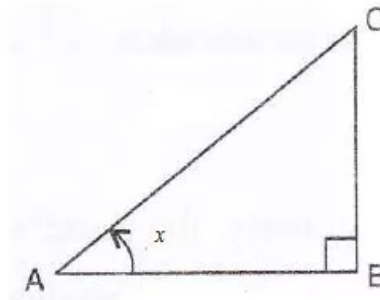
Perimeter of rectangle $ABCD = 2(AB + AD)$

$$= 2\left(\frac{78}{\sqrt{13}} + \frac{52}{\sqrt{13}}\right)$$

$$= \frac{260}{\sqrt{13}}$$

$$= 20\sqrt{13} \, \text{cm}$$

Solution 24:

Consider the figure

$2 \sin x = \sqrt{3}$

$\sin x = \dfrac{\sqrt{3}}{2}$

$i.e. \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{\sqrt{3}}{2}$

Therefore if length of $\text{perpendicular} = \sqrt{3}x$, length of $\text{hypotenuse} = 2x$

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$(2x)^2 - \left(\sqrt{3}x\right)^2 = AB^2$

$AB^2 = x^2$

$\therefore AB = x$

Now

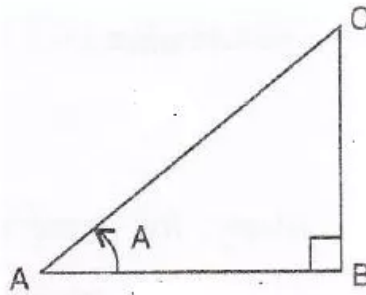$\cos x = \dfrac{AB}{AC} = \dfrac{1}{2}$

(i)

$4 \sin^3 x - 3 \sin x = 4 \left(\dfrac{\sqrt{3}}{2}\right)^3 - 3 \left(\dfrac{\sqrt{3}}{2}\right)$

$\qquad\qquad\qquad = \dfrac{3\sqrt{3}}{2} - \dfrac{3\sqrt{3}}{2}$

$\qquad\qquad\qquad = 0$

(ii)

$3 \cos x - 4 \cos^3 x = 3 \cdot \dfrac{1}{2} - 4 \cdot \left(\dfrac{1}{2}\right)^3$

$\qquad\qquad\qquad = \dfrac{3}{2} - \dfrac{1}{2}$

$\qquad\qquad\qquad = 1$

Consider the diagram below:

$$\sin A = \frac{\sqrt{3}}{2}$$

i.e. $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

Therefore if length of $BC = \sqrt{3}x$, length of $AC = 2x$

Since

$AB^2 + BC^2 = AC^2$ [Using Pythagoras Theorem]

$\left(\sqrt{3}x\right)^2 + AB^2 = \left(2x\right)^2$

$AB^2 = x^2$

$\therefore AB = x\left(\text{base}\right)$

Consider the diagram below:

$$\cos B = \frac{\sqrt{3}}{2}$$

i.e. $\frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{AB}{BC} = \frac{\sqrt{3}}{2}$

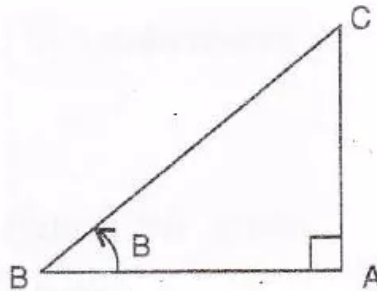Therefore if length of $AB = \sqrt{3}x$, length of $BC = 2x$

Since

$AB^2 + AC^2 = BC^2$ [Using Pythagoras Theorem]

$AC^2 + \left(\sqrt{3}x\right)^2 = \left(2x\right)^2$

$AC^2 = x^2$

$\therefore AC = x\left(\text{perpendicular}\right)$

Now

$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$

$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$
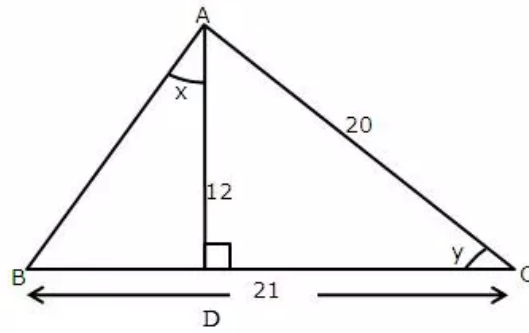
Therefore

$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{2}$$

$$= \frac{1}{\sqrt{3}}$$

Solution 26:

Consider the given diagram as

Using Pythagorean Theorem

$AD^2 + DC^2 = AC^2$

$DC^2 = 20^2 - 12^2 = 256$

$DC = 16$

Now

$BC = BD + DC$

$21 = BD + 16$

$BD = 5$

Again using Pythagorean Theorem

$AD^2 + BD^2 = AB^2$

$12^2 + 5^2 = AB^2$

$AB^2 = 169$

$AB = 13$

Now

$\sin x = \dfrac{BD}{AB} = \dfrac{5}{13}$

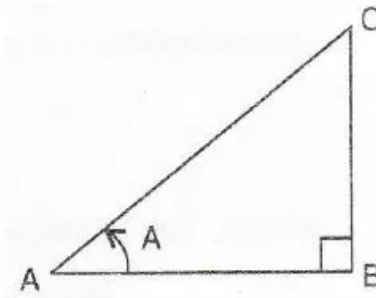$\sin y = \dfrac{AD}{AC} = \dfrac{12}{20} = \dfrac{3}{5}$

$\cot y = \dfrac{DC}{AD} = \dfrac{16}{12} = \dfrac{4}{3}$

Therefore

$\dfrac{10}{\sin x} + \dfrac{6}{\sin y} - 6\cot y = \dfrac{10}{\frac{5}{13}} + \dfrac{6}{\frac{3}{5}} - 6\left(\dfrac{4}{3}\right)$

$= \dfrac{130}{5} + \dfrac{30}{3} - \dfrac{24}{3}$

$= 26 + 10 - 8$

$= 28$

Solution 27:

Consider the figure

$\sec A = \sqrt{2}$

$i.e. \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{AC}{AB} = \sqrt{2}$

Therefore if length of base $= x$, length of hypotenuse $= \sqrt{2}x$

Since

$AB^2 + BC^2 = AC^2 \qquad [\text{Using Pythagoras Theorem}]$

$\left(\sqrt{2}x\right)^2 - (x)^2 = BC^2$

$BC^2 = x^2$

$\therefore BC = x$

Now

$\cos A = \dfrac{1}{\sec A} = \dfrac{1}{\sqrt{2}}$

$\sin A = \dfrac{BC}{AC} = \dfrac{1}{\sqrt{2}}$

$\tan A = \dfrac{BC}{AB} = 1$

$\cot A = \dfrac{1}{\tan A} = 1$

Therefore

$\dfrac{3\cot^2 A + 2\sin^2 A}{\tan^2 A - \cos^2 A} = \dfrac{3(1)^2 + 2\left(\dfrac{1}{\sqrt{2}}\right)^2}{1^2 - \left(\dfrac{1}{\sqrt{2}}\right)^2}$

$= \dfrac{3+1}{1-\dfrac{1}{2}}$

$= \dfrac{4}{\dfrac{1}{2}}$

$= 8$

Solution 28:

$$\cos \theta = \frac{3}{5}$$

Now

$$\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta} = \frac{\dfrac{1}{\sin \theta} - \dfrac{\cos \theta}{\sin \theta}}{\dfrac{1}{\sin \theta} + \dfrac{\cos \theta}{\sin \theta}}$$

$$= \frac{\dfrac{1 - \cos \theta}{\sin \theta}}{\dfrac{1 + \cos \theta}{\sin \theta}}$$

$$= \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$= \frac{1 - \dfrac{3}{5}}{1 + \dfrac{3}{5}}$$

$$= \frac{\dfrac{2}{5}}{\dfrac{8}{5}}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

Solution 29:

$$\csc A + \sin A = 5\frac{1}{5}$$

Squaring both sides

$$\left(\csc A + \sin A\right)^2 = \left(5\frac{1}{5}\right)^2$$

$$\csc^2 A + \sin^2 A + 2\,\cancel{\csc A} \cdot \frac{1}{\cancel{\csc A}} = \frac{26}{25}$$

$$\csc^2 A + \sin^2 A = \frac{626}{25}$$

$$\csc^2 A + \sin^2 A = 25\frac{1}{25}$$

Solution 30:

$$5\cos \theta = 6\sin \theta$$

$$\tan \theta = \frac{5}{6}$$

Now
(i)

$$\tan \theta = \frac{5}{6}$$

(ii)

$$\frac{12\sin \theta - 3\cos \theta}{12\sin \theta + 3\cos \theta} = \frac{\dfrac{12\sin \theta}{\cos \theta} - \dfrac{3\cos \theta}{\cos \theta}}{\dfrac{12\sin \theta}{\cos \theta} + \dfrac{3\cos \theta}{\cos \theta}}$$

$$= \frac{12\tan \theta - 3}{12\tan \theta + 3}$$

$$= \frac{12\left(\dfrac{5}{6}\right) - 3}{12\left(\dfrac{5}{6}\right) + 3}$$

$$= \frac{\dfrac{42}{6}}{\dfrac{78}{6}}$$

$$= \frac{42}{78}$$

$$= \frac{7}{13}$$