

Chapter 5. Factorisation

Exercise 5(A)

Solution 1:

$$\begin{aligned}3a^2 - 9ab &= 3a \times a - 3a \times 3b \\ &= 3a(a - 3b)\end{aligned}$$

Solution 2:

[Taking $(x + y)$ common from both terms]

$$\begin{aligned}&= (x + y)[2(x + y)^2 - 6] \\ &= 2(x + y)[(x + y)^2 - 3] \\ &= 2(x + y)(x^2 + y^2 + 2xy - 3)\end{aligned}$$

Solution 3:

Taking $(2x - 3y)$ common from both terms

$$\begin{aligned}&= (2x - 3y)[x^3 - x^2(2x - 3y)] \\ &= x^2(2x - 3y)[x - (2x - 3y)] \\ &= x^2(2x - 3y)[x - 2x + 3y] \\ &= x^2(2x - 3y)[-x + 3y] \\ &= x^2(2x - 3y)(3y - x)\end{aligned}$$

Solution 4:

Taking $(2x - 5y)$ common from both terms

$$\begin{aligned}&= (2x - 5y)[2(3x + 4y) - 6(x - y)] \\ &= (2x - 5y)(6x + 8y - 6x + 6y) \\ &= (2x - 5y)(8y + 6y) \\ &= (2x - 5y)(14y) \\ &= (2x - 5y)14y\end{aligned}$$

Solution 5:

$$\begin{aligned}a^3 + a - 3a^2 - 3 &= a(a^2 + 1) - 3(a^2 + 1) \\ &= (a^2 + 1)(a - 3).\end{aligned}$$

Solution 6:

$$\begin{aligned}
16(a+b)^2 - 4a - 4b &= 16(a+b)^2 - 4(a+b) \\
&= 4(a+b)[4(a+b) - 1] \\
&= 4(a+b)(4a+4b-1)
\end{aligned}$$

Solution 7:

$$\begin{aligned}
a^4 - 2a^3 - 4a + 8 &= a^3(a-2) - 4(a-2) \\
&= (a^3 - 4)(a-2)
\end{aligned}$$

Solution 8:

$$\begin{aligned}
ab - 2b + a^2 - 2a &= b(a-2) + a(a-2) \\
&= (a+b)(a-2)
\end{aligned}$$

Solution 9:

$$\begin{aligned}
ab(x^2+1) + x(a^2+b^2) &= abx^2 + ab + a^2x + b^2x \\
&= ax(bx+a) + b(bx+a) \\
&= (ax+b)(bx+a)
\end{aligned}$$

Solution 10:

$$\begin{aligned}
a^2 + b - ab - a &= a^2 - a + b - ab \\
&= a(a-1) + b(1-a) \\
&= a(a-1) - b(a-1) \\
&= (a-1)(a-b)
\end{aligned}$$

Solution 11:

$$\begin{aligned}
(ax+by)^2 + (bx-ay)^2 &= a^2x^2 + b^2y^2 + 2axby + b^2x^2 + a^2y^2 - 2bxay \\
&= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2 \\
&= x^2(a^2+b^2) + y^2(a^2+b^2) \\
&= (x^2+y^2)(a^2+b^2)
\end{aligned}$$

Solution 12:

$$\begin{aligned}
a^2x^2 + (ax^2+1)x + a &= a^2x^2 + a + (ax^2+1)x \\
&= a(ax^2+1) + x(ax^2+1) \\
&= (a+x)(ax^2+1)
\end{aligned}$$

Solution 13:

$$\begin{aligned}(2a - b)^2 - 10a + 5b &= (2a - b)^2 - 5(2a - b) \\ &= (2a - b)(2a - b - 5)\end{aligned}$$

Solution 14:

$$\begin{aligned}a(a - 4) - a + 4 &= a(a - 4) - 1(a - 4) \\ &= (a - 4)(a - 1)\end{aligned}$$

Solution 15:

$$\begin{aligned}y^2 - (a + b)y + ab &= y^2 - ay - by + ab \\ &= y(y - a) - b(y - a) \\ &= (y - a)(y - b)\end{aligned}$$

Solution 16:

$$\begin{aligned}a^2 + \frac{1}{a^2} - 2 - 3a + \frac{3}{a} &= \left(a - \frac{1}{a}\right)^2 - 3\left(a - \frac{1}{a}\right) \\ &= \left(a - \frac{1}{a}\right)\left[\left(a - \frac{1}{a}\right) - 3\right] \\ &= \left(a - \frac{1}{a}\right)\left[a - \frac{1}{a} - 3\right]\end{aligned}$$

Solution 17:

$$\begin{aligned}&= (x^2 + y^2 + 2xy) + (x + y) \\ &[As (x + y)^2 = x^2 + 2xy + y^2] \\ &= (x + y)^2 + (x + y) \\ &= (x + y)(x + y + 1)\end{aligned}$$

Solution 18:

$$\begin{aligned}&= a^2 + 4b^2 - 4ab - 3a + 6b \\ &= a^2 + (2b)^2 - 2 \times a \times (2b) - 3(a - 2b) \\ &[As (a - b)^2 = a^2 - 2ab + b^2] \\ &= (a - 2b)^2 - 3(a - 2b) \\ &= (a - 2b)[(a - 2b) - 3] \\ &= (a - 2b)(a - 2b - 3)\end{aligned}$$

Solution 19:

$$= m(x - 3y)^2 - n(x - 3y) + 5(x - 3y)$$

[Taking $(x - 3y)$ common from all the three terms]

$$= (x - 3y)[m(x - 3y) - n + 5]$$

$$= (x - 3y)(mx - 3my - n + 5)$$

Solution 20:

$$= (6x - 5y)[x - 4(6x - 5y)]$$

[Taking $(6x - 5y)$ common from the three terms]

$$= (6x - 5y)(x - 24x + 20y)$$

$$= (6x - 5y)(-23x + 20y)$$

$$= (6x - 5y)(20y - 23x)$$

Exercise 5(B)**Solution 1:**

$$\begin{aligned} a^2 + 10a + 24 &= a^2 + 6a + 4a + 24 \\ &= a(a + 6) + 4(a + 6) \\ &= (a + 6)(a + 4) \end{aligned}$$

Solution 2:

$$\begin{aligned} a^2 - 3a - 40 &= a^2 - 8a + 5a - 40 \\ &= a(a - 8) + 5(a - 8) \\ &= (a - 8)(a + 5) \end{aligned}$$

Solution 3:

$$\begin{aligned} 1 - 2a - 3a^2 &= 1 - 3a + a - 3a^2 \\ &= 1(1 - 3a) + a(1 - 3a) \\ &= (1 + a)(1 - 3a) \end{aligned}$$

Solution 4:

$$\begin{aligned} x^2 - 3ax - 88a^2 &= x^2 - 11ax + 8ax - 88a^2 \\ &= x(x - 11a) + 8a(x - 11a) \\ &= (x + 8a)(x - 11a) \end{aligned}$$

Solution 5:

$$\begin{aligned}6a^2 - a - 15 &= 6a^2 - 10a + 9a - 15 \\ &= 2a(3a - 5) + 3(3a - 5) \\ &= (2a + 3)(3a - 5)\end{aligned}$$

Solution 6:

$$\begin{aligned}24a^3 + 37a^2 - 5a &= a(24a^2 + 37a - 5) \\ &= a(24a^2 + 40a - 3a - 5) \\ &= a \times [8a(3a + 5) - 1(3a + 5)] \\ &= a[(8a - 1)(3a + 5)] \\ &= a(8a - 1)(3a + 5)\end{aligned}$$

Solution 7:

$$\begin{aligned}a(3a - 2) - 1 &= 3a^2 - 2a - 1 \\ &= 3a^2 - 3a + a - 1 \\ &= 3a(a - 1) + 1(a - 1) \\ &= (3a + 1)(a - 1)\end{aligned}$$

Solution 8:

$$\begin{aligned}a^2b^2 + 8ab - 9 &= a^2b^2 + 9ab - ab - 9 \\ &= ab(ab + 9) - 1(ab + 9) \\ &= (ab + 9)(ab - 1)\end{aligned}$$

Solution 9:

$$\begin{aligned}3 - a(4 + 7a) &= 3 - 4a - 7a^2 \\ &= 3 - 7a + 3a - 7a^2 \\ &= 1(3 - 7a) + a(3 - 7a) \\ &= (3 - 7a)(a + 1)\end{aligned}$$

Solution 10:

$$(2a + b)^2 - 6a - 3b - 4 = (2a + b)^2 - 3(2a + b) - 4$$

Assume that $2a + b = x$

Therefore,

$$\begin{aligned} (2a + b)^2 - 6a - 3b - 4 &= x^2 - 3x - 4 \\ &= x^2 - 4x + x - 4 \\ &= 1(x - 4) + x(x - 4) \\ &= (x + 1)(x - 4) \\ &= (2a + b + 1)(2a + b - 4) \\ &\quad \text{[resubstitute the value of } x\text{]} \end{aligned}$$

Solution 11:

Assume that $a + b = x$;

$$\begin{aligned} 1 - 2(a + b) - 3(a + b)^2 &= 1 - 2x - 3x^2 \\ &= 1 - 3x + x - 3x^2 \\ &= 1(1 - 3x) + x(1 - 3x) \\ &= (1 - 3x)(1 + x) \\ &= (1 - 3(a + b))(1 + (a + b)) \\ &= (1 - 3a - 3b)(1 + a + b) \end{aligned}$$

Solution 12:

$$\begin{aligned} 3a^2 - 1 - 2a &= 3a^2 - 2a - 1 \\ &= 3a^2 - 3a + a - 1 \\ &= 3a(a - 1) + 1(a - 1) \\ &= (3a + 1)(a - 1) \end{aligned}$$

Solution 13:

$$\begin{aligned} x^2 + 3x + 2 + ax + 2a &= x^2 + 2x + x + 2 + ax + 2a \\ &= x(x + 2) + 1(x + 2) + a(x + 2) \\ &= (x + 2)(x + a + 1) \end{aligned}$$

Solution 14:

Assume that $3x - 2y = a$

Therefore,

$$\begin{aligned}(3x - 2y)^2 + 3(3x - 2y) - 10 &= a^2 + 3a - 10 \\ &= a^2 + 5a - 2a - 10 \\ &= a(a + 5) - 2(a + 5) \\ &= (a + 5)(a - 2) \\ &= (3x - 2y + 5)(3x - 2y - 2)\end{aligned}$$

Solution 15:

$$5 - (3a^2 - 2a)(6 - 3a^2 + 2a) = 5 - (3a^2 - 2a)[6 - (3a^2 - 2a)]$$

Assume that $3a^2 - 2a = x$

Therefore,

$$\begin{aligned}5 - (3a^2 - 2a)(6 - 3a^2 + 2a) &= 5 - x(6 - x) \\ &= 5 - 6x + x^2 \\ &= 5 - 5x - x + x^2 \\ &= 5(1 - x) - x(1 - x) \\ &= (5 - x)(1 - x) \\ &= (x - 5)(x - 1) \\ &= (3a^2 - 2a - 5)(3a^2 - 2a - 1) \\ &= (3a^2 - 5a + 3a - 5)(3a^2 - 3a + a - 1) \\ &= (a(3a - 5) + 1(3a - 5))(3a(a - 1) + 1(a - 1)) \\ &= (3a - 5)(a + 1)(3a + 1)(a - 1)\end{aligned}$$

Solution 16:

(i) Given expression: $x^2 - 3x - 54$

Comparing with $ax^2 + bx + c$, we get $a = 1$, $b = -3$ and $c = -54$

$\therefore b^2 - 4ac = (-3)^2 - 4(1)(-54) = 9 + 216 = 225$, which is a perfect square.

$\therefore x^2 - 3x - 54$ is factorisable.

$$\begin{aligned}\text{Now, } x^2 - 3x - 54 &= x^2 - 9x + 6x - 54 \\ &= x(x - 9) + 6(x - 9) \\ &= (x - 9)(x + 6)\end{aligned}$$

(ii) Given expression: $2x^2 - 7x - 15$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = -7$ and $c = -15$

$\therefore b^2 - 4ac = (-7)^2 - 4(2)(-15) = 49 + 120 = 169$, which is a perfect square.

$\therefore 2x^2 - 7x - 15$ is factorisable.

$$\begin{aligned}\text{Now, } 2x^2 - 7x - 15 &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x - 5) + 3(x - 5) \\ &= (2x + 3)(x - 5)\end{aligned}$$

(iii) Given expression: $2x^2 + 2x - 75$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = 2$ and $c = -75$

$\therefore b^2 - 4ac = (2)^2 - 4(2)(-75) = 4 + 600 = 604$, which is not a perfect square.

$\therefore 2x^2 + 2x - 75$ is not factorisable.

(iv) Given expression: $3x^2 + 4x - 10$

Comparing with $ax^2 + bx + c$, we get $a = 3$, $b = 4$ and $c = -10$

$\therefore b^2 - 4ac = (4)^2 - 4(3)(-10) = 16 + 120 = 136$, which is not a perfect square.

$\therefore 3x^2 + 4x - 10$ is not factorisable.

(v) Given expression: $x(2x - 1) - 1$

$$\text{Now, } x(2x - 1) - 1 = 2x^2 - x - 1$$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = -1$ and $c = -1$

$\therefore b^2 - 4ac = (-1)^2 - 4(2)(-1) = 1 + 8 = 9$, which is a perfect square.

$\therefore 2x^2 - x - 1$ is factorisable.

$$\begin{aligned}\text{Now, } 2x^2 - x - 1 &= 2x^2 - 2x + x - 1 \\ &= 2x(x - 1) + 1(x - 1) \\ &= (2x + 1)(x - 1)\end{aligned}$$

Exercise 5(C)**Solution 1:**

$$\begin{aligned}25a^2 - 9b^2 &= (5a)^2 - (3b)^2 \\ &= (5a - 3b)(5a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]\end{aligned}$$

Solution 2:

$$\begin{aligned}
a^2 - (2a + 3b)^2 &= (a)^2 - (2a + 3b)^2 \\
&= (a - 2a - 3b)(a + 2a + 3b) \quad [∵ a^2 - b^2 = (a + b)(a - b)] \\
&= (-a - 3b)(3a + 3b) \\
&= -3(a + 3b)(a + b)
\end{aligned}$$

Solution 3:

$$\begin{aligned}
a^2 - 81(b - c)^2 &= (a)^2 - [9(b - c)]^2 \\
&= (a - (9b - 9c))(a + (9b - 9c)) \quad [∵ a^2 - b^2 = (a + b)(a - b)] \\
&= (a - 9b + 9c)(a + 9b - 9c)
\end{aligned}$$

Solution 4:

$$\begin{aligned}
25(2a - b)^2 - 81b^2 &= [5(2a - b)]^2 - (9b)^2 \\
&= [5(2a - b) - 9b][5(2a - b) + 9b] \\
&\quad [∵ a^2 - b^2 = (a + b)(a - b)] \\
&= [10a - 5b - 9b][10a - 5b + 9b] \\
&= [10a - 14b][10a + 4b] \\
&= 2 \times (5a - 7b) \times 2 \times (5a + 2b) \\
&= 4(5a - 7b)(5a + 2b)
\end{aligned}$$

Solution 5:

$$\begin{aligned}
50a^3 - 2a &= 2a(25a^2 - 1) \\
&= 2a[(5a)^2 - 1^2] \\
&= 2a(5a + 1)(5a - 1) \quad [∵ a^2 - b^2 = (a + b)(a - b)]
\end{aligned}$$

Solution 6:

$$\begin{aligned}
4a^2b - 9b^3 &= b(4a^2 - 9b^2) \\
&= b[(2a)^2 - (3b)^2] \\
&= b(2a - 3b)(2a + 3b) \quad [∵ a^2 - b^2 = (a + b)(a - b)]
\end{aligned}$$

Solution 7:

$$\begin{aligned}
3a^5 - 108a^3 &= 3a^3(a^2 - 36) \\
&= 3a^3[(a)^2 - (6)^2] \\
&= 3a^3(a - 6)(a + 6) \quad ∵ a^2 - b^2 = (a + b)(a - b)
\end{aligned}$$

Solution 8:

$$\begin{aligned}
9(a-2)^2 - 16(a+2)^2 &= [3(a-2)]^2 - [4(a+2)]^2 \\
&= [3(a-2) - 4(a+2)][3(a-2) + 4(a+2)] \\
&\quad [\because a^2 - b^2 = (a+b)(a-b)] \\
&= [3a - 6 - 4a - 8][3a - 6 + 4a + 8] \\
&= (-a - 14)(7a + 2) \\
&= -(a + 14)(7a + 2)
\end{aligned}$$

Solution 9:

$$\begin{aligned}
a^4 - 1 &= (a^2)^2 - (1)^2 \\
&= (a^2 + 1)(a^2 - 1) [\because a^2 - b^2 = (a+b)(a-b)] \\
&= (a^2 + 1)((a)^2 - (1)^2) \\
&= (a^2 + 1)(a + 1)(a - 1)
\end{aligned}$$

Solution 10:

$$\begin{aligned}
a^3 + 2a^2 - a - 2 &= a^2(a + 2) - 1(a + 2) \\
&= (a^2 - 1)(a + 2) \\
&= (a + 1)(a - 1)(a + 2) [\because a^2 - b^2 = (a+b)(a-b)]
\end{aligned}$$

Solution 11:

$$\begin{aligned}
(a+b)^3 - a - b &= (a+b)^3 - (a+b) \\
&= (a+b)[(a+b)^2 - 1] \\
&= (a+b)[(a+b)^2 - 1^2] \\
&= (a+b)((a+b) + 1)((a+b) - 1) \\
&\quad [\because a^2 - b^2 = (a+b)(a-b)] \\
&= (a+b)(a+b+1)(a+b-1)
\end{aligned}$$

Solution 12:

$$\begin{aligned}
a(a-1) - b(b-1) &= a^2 - a - b^2 + b \\
&= a^2 - b^2 - a + b \\
&= (a+b)(a-b) - (a-b) \\
&\quad [\because a^2 - b^2 = (a+b)(a-b)] \\
&= (a-b)[(a+b) - 1] \\
&= (a-b)[a+b-1]
\end{aligned}$$

Solution 13:

$$\begin{aligned}
4a^2 - (4b^2 + 4bc + c^2) &= (2a)^2 - (2b + c)^2 \\
&= [2a - (2b + c)][2a + (2b + c)] \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= [2a - 2b - c][2a + 2b + c]
\end{aligned}$$

Solution 14:

$$\begin{aligned}
4a^2 - 49b^2 + 2a - 7b &= [(2a)^2 - (7b)^2] + [2a - 7b] \\
&= [2a - 7b][2a + 7b] + [2a - 7b] \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= [2a - 7b][2a + 7b + 1]
\end{aligned}$$

Solution 15:

$$\begin{aligned}
9a^2 + 3a - 8b - 64b^2 &= 9a^2 - 64b^2 + 3a - 8b \\
&= (3a)^2 - (8b)^2 + 3a - 8b \\
&= (3a - 8b)(3a + 8b) + (3a - 8b) \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= (3a - 8b)(3a + 8b + 1)
\end{aligned}$$

Solution 16:

$$\begin{aligned}
4a^2 - 12a + 9 - 49b^2 &= (2a)^2 - 12a + (3)^2 - 49b^2 \\
&= (2a - 3)^2 - 49b^2 \\
&= (2a - 3)^2 - (7b)^2 \\
&= (2a - 3 - 7b)(2a - 3 + 7b) \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)]
\end{aligned}$$

Solution 17:

$$\begin{aligned}
4xy - x^2 - 4y^2 + z^2 &= z^2 - (x^2 + 4y^2 - 4xy) \\
&= z^2 - (x - 2y)^2 \\
&= [z - (x - 2y)][z + (x - 2y)] \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= [z - x + 2y][z + x - 2y]
\end{aligned}$$

Solution 18:

$$\begin{aligned}
& a^2 + b^2 - c^2 - d^2 + 2ab - 2cd \\
&= (a^2 + b^2 + 2ab) - (c^2 + d^2 + 2cd) \\
&= (a + b)^2 - (c + d)^2 \\
&= [(a + b) - (c + d)][(a + b) + (c + d)] \quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= (a + b - c - d)(a + b + c + d)
\end{aligned}$$

Solution 19:

$$\begin{aligned}
& 4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2 \\
&= 4x^2 + 9a^2 - 12ax - y^2 - z^2 - 2yz \\
&= (2x)^2 + (3a)^2 - 12ax - (y^2 + z^2 + 2yz) \\
&= (2x - 3a)^2 - (y + z)^2 \\
&= [(2x - 3a) - (y + z)][(2x - 3a) + (y + z)] \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= [2x - 3a - y - z][2x - 3a + y + z]
\end{aligned}$$

Solution 20:

$$\begin{aligned}
(a^2 - 1)(b^2 - 1) + 4ab &= a^2b^2 - a^2 - b^2 + 1 + 4ab \\
&= a^2b^2 + 1 + 2ab - a^2 - b^2 + 2ab \\
&= (a^2b^2 + 1 + 2ab) - (a^2 + b^2 - 2ab) \\
&= (ab + 1)^2 - (a - b)^2 \\
&= [(ab + 1) - (a - b)][(ab + 1) + (a - b)] \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= [ab + 1 - a + b][ab + 1 + a - b]
\end{aligned}$$

Solution 21:

$$\begin{aligned}
x^4 + x^2 + 1 &= x^4 + 2x^2 + 1 - x^2 \\
&= (x^2)^2 + 2x^2 + (1)^2 - x^2 \\
&= (x^2 + 1)^2 - (x)^2 \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= (x^2 + 1 - x)(x^2 + 1 + x)
\end{aligned}$$

Solution 22:

$$\begin{aligned}
(a^2 + b^2 - 4c^2)^2 - 4a^2b^2 &= (a^2 + b^2 - 4c^2)^2 - (2ab)^2 \\
&= (a^2 + b^2 - 4c^2 - 2ab)(a^2 + b^2 - 4c^2 + 2ab) \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= (a^2 + b^2 - 2ab - 4c^2)(a^2 + b^2 + 2ab - 4c^2) \\
&= ((a - b)^2 - (2c)^2)((a + b)^2 - (2c)^2) \\
&= (a - b + 2c)(a - b - 2c)(a + b + 2c)(a + b - 2c)
\end{aligned}$$

Solution 23:

$$\begin{aligned}
(x^2 + 4y^2 - 9z^2)^2 - 16x^2y^2 \\
&= (x^2 + 4y^2 - 9z^2)^2 - (4xy)^2 \\
&= (x^2 + 4y^2 - 9z^2 - 4xy)(x^2 + 4y^2 - 9z^2 + 4xy) \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= (x^2 + 4y^2 - 4xy - 9z^2)(x^2 + 4y^2 + 4xy - 9z^2) \\
&= [(x - 2y)^2 - (3z)^2][(x + 2y)^2 - (3z)^2] \\
&= [(x - 2y) - 3z][(x - 2y) + 3z][(x + 2y) - 3z][(x + 2y) + 3z] \\
&= [x - 2y - 3z][x - 2y + 3z][x + 2y - 3z][x + 2y + 3z]
\end{aligned}$$

Solution 24:

$$\begin{aligned}
(a + b)^2 - a^2 + b^2 \\
&= a^2 + 2ab + b^2 - a^2 + b^2 \\
&= 2ab + 2b^2 \\
&= 2b(a + b)
\end{aligned}$$

Solution 25:

$$\begin{aligned}
a^2 - b^2 - (a + b)^2 \\
&= a^2 - b^2 - (a^2 + 2ab + b^2) \\
&= a^2 - b^2 - a^2 - 2ab - b^2 \\
&= -2ab - 2b^2 \\
&= -2b(a + b)
\end{aligned}$$

Solution 26:

$$\begin{aligned} & 9a^2 - (a^2 - 4)^2 \\ &= (3a)^2 - (a^2 - 4)^2 \\ &= [3a - (a^2 - 4)][3a + (a^2 - 4)] \\ &= [3a - a^2 - 4][3a + a^2 - 4] \\ &= [-a^2 + 3a - 4][a^2 + 3a - 4] \\ &= [-a^2 + 4a - a - 4][a^2 + 4a - a - 4] \\ &= [a(-a + 4) + 1(-a + 4)][a(a + 4) - 1(a + 4)] \\ &= [(a + 1)(4 - a)][(a + 4)(a - 1)] \\ &= (a + 1)(4 - a)(a + 4)(a - 1) \end{aligned}$$

Solution 27:

$$\begin{aligned} & x^2 + \frac{1}{x^2} - 11 \\ &= x^2 + \frac{1}{x^2} - 2 - 9 \\ &= x^2 + \frac{1}{x^2} - 2x \times x \times \frac{1}{x} - 9 \\ &= \left(x - \frac{1}{x}\right)^2 - (3)^2 \\ &= \left(x - \frac{1}{x} + 3\right)\left(x - \frac{1}{x} - 3\right) \end{aligned}$$

Solution 28:

$$\begin{aligned} & 4x^2 + \frac{1}{4x^2} + 1 \\ &= 4x^2 + \frac{1}{4x^2} + 2 - 1 \\ &= 4x^2 + \frac{1}{4x^2} + 2 \times 2x \times \frac{1}{2x} - 1 \\ &= \left(2x + \frac{1}{2x}\right)^2 - (1)^2 \\ &= \left(2x + \frac{1}{2x} + 1\right)\left(2x + \frac{1}{2x} - 1\right) \end{aligned}$$

Solution 29:

$$\begin{aligned}
& 4x^4 - x^2 - 12x - 36 \\
&= 4x^4 - (x^2 + 12x + 36) \\
&= (2x^2)^2 - (x^2 + 2x \times 6 + 6^2) \\
&= (2x^2)^2 - (x + 6)^2 \\
&= (2x^2 + x + 6)(2x^2 - x - 6) \\
&= (2x^2 + x + 6)(2x^2 - 4x + 3x - 6) \\
&= (2x^2 + x + 6)[2x(x - 2) + 3(x - 2)] \\
&= (2x^2 + x + 6)[(x - 2)(2x + 3)] \\
&= (2x^2 + x + 6)(x - 2)(2x + 3)
\end{aligned}$$

Solution 30:

$$\begin{aligned}
& a^2(b + c) - (b + c)^3 \\
&= (b + c)[a^2 - (b + c)^2] \\
&= (b + c)[(a + b + c)(a - b - c)] \\
&= (b + c)(a + b + c)(a - b - c)
\end{aligned}$$

Exercise 5(D)**Solution 1:**

$$\begin{aligned}
a^3 - 27 &= (a)^3 - (3)^3 \\
&= (a - 3)[(a)^2 + a \times 3 + (3)^2] \quad [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
&= (a - 3)[a^2 + 3a + 9]
\end{aligned}$$

Solution 2:

$$\begin{aligned}
1 - 8a^3 &= (1)^3 - (2a)^3 \\
&= (1 - 2a)[(1)^2 + 1 \times 2a + (2a)^2] \\
&\quad [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
&= (1 - 2a)[1 + 2a + 4a^2]
\end{aligned}$$

Solution 3:

$$\begin{aligned}
64 - a^3b^3 &= (4)^3 - ab^3 \\
&= (4 - ab)[(4)^2 + 4 \times ab + (ab)^2] \\
&\quad [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
&= (4 - ab)[16 + 4ab + a^2b^2]
\end{aligned}$$

Solution 4:

$$\begin{aligned}
a^6 + 27b^3 &= (a^2)^3 + (3b)^3 \\
&= (a^2 + 3b) \left[(a^2)^2 - a^2 \times 3b + (3b)^2 \right] \\
&\quad [\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \\
&= (a^2 + 3b) [a^4 - 3a^2b + 9b^2]
\end{aligned}$$

Solution 5:

$$\begin{aligned}
3x^7y - 81x^4y^4 &= 3xy (x^6 - 27x^3y^3) \\
&= 3xy \left((x^2)^3 - (3xy)^3 \right) \\
&= 3xy (x^2 - 3xy) \left[(x^2)^2 + x^2 \times 3xy + (3xy)^2 \right] \\
&\quad [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
&= 3xy (x^2 - 3xy) [x^4 + 3x^3y + 9x^2y^2] \\
&= 3xy \{ x(x - 3y)x^2 [x^2 + 3xy + 9y^2] \} \\
&= 3x^4y (x - 3y) [x^2 + 3xy + 9y^2]
\end{aligned}$$

Solution 6:

$$\begin{aligned}
a^3 - \frac{27}{a^3} &= (a)^3 - \left(\frac{3}{a} \right)^3 \\
&= \left(a - \frac{3}{a} \right) \left(a^2 + a \times \frac{3}{a} + \left(\frac{3}{a} \right)^2 \right) \\
&\quad [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
&= \left(a - \frac{3}{a} \right) \left(a^2 + 3 + \frac{9}{a^2} \right)
\end{aligned}$$

Solution 7:

$$\begin{aligned}
a^3 + 0.064 &= (a)^3 + (0.4)^3 \\
&= (a + 0.4) \left[(a)^2 - a \times 0.4 + (0.4)^2 \right] \\
&\quad [\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \\
&= (a + 0.4) [a^2 - 0.4a + 0.16]
\end{aligned}$$

Solution 8:

$$\begin{aligned}
a^4 - 343a &= a(a^3 - 7^3) \\
&= a(a-7)[(a)^2 + a \times 7 + (7)^2] \\
&\quad [\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)] \\
&= a(a-7)[a^2 + 7a + 49]
\end{aligned}$$

Solution 9:

$$\begin{aligned}
&= (x-y)^3 - (2x)^3 \\
&= (x-y-2x)[(x-y)^2 + 2x(x-y) + (2x)^2] \\
&\quad [\text{Using identity } (a^3 - b^3) = (a-b)(a^2 + ab + b^2)] \\
&= (-x-y)[x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2] \\
&= -(x+y)[7x^2 - 4xy + y^2]
\end{aligned}$$

Solution 10:

$$\begin{aligned}
\frac{8a^3}{27} - \frac{b^3}{8} &= \left(\frac{2a}{3}\right)^3 - \left(\frac{b}{2}\right)^3 \\
&= \left(\frac{2a}{3} - \frac{b}{2}\right) \left[\left(\frac{2a}{3}\right)^2 + \frac{2a}{3} \times \frac{b}{2} + \left(\frac{b}{2}\right)^2 \right] \\
&\quad [\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)] \\
&= \left(\frac{2a}{3} - \frac{b}{2}\right) \left[\frac{4a^2}{9} + \frac{ab}{3} + \frac{b^2}{4} \right]
\end{aligned}$$

Solution 11:

We know that,

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2) \dots (1)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \dots (2)$$

$$\begin{aligned}
a^6 - b^6 &= (a^3)^2 - (b^3)^2 \\
&= (a^3 + b^3)(a^3 - b^3) \\
&= (a+b)(a^2 - ab + b^2)(a-b)(a^2 + ab + b^2) \quad [\text{from (1) and (2)}] \\
&= (a+b)(a-b)(a^2 - ab + b^2)(a^2 + ab + b^2)
\end{aligned}$$

Solution 12:

We know that,

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \dots(1)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \dots(2)$$

$$\begin{aligned} a^6 - 7a^3 - 8 &= a^6 - 8a^3 + a^3 - 8 \\ &= a^3(a^3 - 8) + 1(a^3 - 8) \\ &= (a^3 + 1)(a^3 - 8) \\ &= (a^3 + 1^3)(a^3 - 2^3) \\ &= (a + 1)(a^2 - a + 1)(a - 2)(a^2 + 2a + 4) \text{ [from (1) and (2)]} \\ &= (a + 1)(a - 2)(a^2 - a + 1)(a^2 + 2a + 4) \end{aligned}$$

Solution 13:

We know that,

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \dots(1)$$

$$\begin{aligned} a^3 - 27b^3 + 2a^2b - 6ab^2 &= (a)^3 - (3b)^3 + 2ab(a - 3b) \\ &= (a - 3b)[a^2 + a \times 3b + (3b)^2] + 2ab(a - 3b) \text{ [from (1)]} \\ &= (a - 3b)[a^2 + 3ab + 9b^2] + 2ab(a - 3b) \\ &= (a - 3b)[a^2 + 3ab + 9b^2 + 2ab] \\ &= (a - 3b)[a^2 + 5ab + 9b^2] \end{aligned}$$

Solution 14:

We know that,

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \dots(1)$$

$$\begin{aligned} 8a^3 - b^3 - 4ax + 2bx &= [(2a)^3 - (b)^3] - 2x(2a - b) \\ &= (2a - b)[(2a)^2 + 2a \times b + (b)^2] - 2x(2a - b) \\ &\quad \text{[from (1)]} \\ &= (2a - b)[4a^2 + 2ab + b^2] - 2x(2a - b) \\ &= (2a - b)[4a^2 + 2ab + b^2 - 2x] \end{aligned}$$

Solution 15:

We know that,

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \dots(1)$$

$$\begin{aligned} a - b - a^3 + b^3 &= a - b - (a^3 - b^3) \\ &= (a - b) - (a - b)[a^2 + ab + b^2] \quad [\text{from (1)}] \\ &= (a - b)[1 - a^2 - ab - b^2] \end{aligned}$$

Solution 16:

$$= 2(x^3 + 27y^3 - 2x - 6y)$$

$$= 2\{[(x)^3 + (3y)^3] - 2(x + 3y)\}$$

$$[\text{Using identity } (a^3 + b^3) = (a + b)(a^2 - ab + b^2)]$$

$$= 2\{(x + 3y)(x^2 - 3xy + 9y^2) - 2(x + 3y)\}$$

$$= 2(x + 3y)(x^2 - 3xy + 9y^2 - 2)$$

Solution 17:

$$(i) (13^3 - 5^3)$$

$$[\text{Using identity } (a^3 - b^3) = (a - b)(a^2 + ab + b^2)]$$

$$= (13 - 5)(13^2 + 13 \times 5 + 5^2)$$

$$= 8(169 + 65 + 25)$$

Therefore, the number is divisible by 8.

$$(ii) (35^3 + 27^3)$$

$$[\text{Using identity } (a^3 + b^3) = (a + b)(a^2 - ab + b^2)]$$

$$= (35 + 27)(35^2 + 35 \times 27 + 27^2)$$

$$= 62 \times (35^2 + 35 \times 27 + 27^2)$$

Therefore, the number is divisible by 62.

Exercise 5(E)

Solution 1:

$$\begin{aligned}
x^2 + \frac{1}{4x^2} + 1 - 7x - \frac{7}{2x} &= (x)^2 + \frac{1}{(2x)^2} + 2 \times x \times \frac{1}{2x} - 7\left(x + \frac{1}{2x}\right) \\
&= \left(x + \frac{1}{2x}\right)^2 - 7\left(x + \frac{1}{2x}\right) \\
&= \left(x + \frac{1}{2x}\right)\left(x + \frac{1}{2x} - 7\right) \\
&= \left(x + \frac{1}{2x}\right)\left(x - 7 + \frac{1}{2x}\right)
\end{aligned}$$

Solution 2:

$$\begin{aligned}
9a^2 + \frac{1}{9a^2} - 2 - 12a + \frac{4}{3a} &= (3a)^2 + \frac{1}{(3a)^2} - 2 \times 3a \times \frac{1}{3a} - 4\left(3a - \frac{1}{3a}\right) \\
&= \left(3a - \frac{1}{3a}\right)^2 - 4\left(3a - \frac{1}{3a}\right) \\
&= \left(3a - \frac{1}{3a}\right)\left[\left(3a - \frac{1}{3a}\right) - 4\right] \\
&= \left(3a - \frac{1}{3a}\right)\left(3a - 4 - \frac{1}{3a}\right)
\end{aligned}$$

Solution 3:

$$\begin{aligned}
x^2 + \frac{a^2 + 1}{a}x + 1 &= x^2 + ax + \frac{1}{a}x + 1 \\
&= x(x + a) + \frac{1}{a}(x + a) \\
&= (x + a)\left(x + \frac{1}{a}\right)
\end{aligned}$$

Solution 4:

$$\begin{aligned}
x^4 + y^4 - 27x^2y^2 &= (x^2)^2 + (y^2)^2 - 2x^2y^2 - 25x^2y^2 \\
&= (x^2 - y^2)^2 - 25x^2y^2 \\
&= (x^2 - y^2)^2 - (5xy)^2 \quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= [(x^2 - y^2) + 5xy][(x^2 - y^2) - 5xy] \\
&= [x^2 + 5xy - y^2][x^2 - 5xy - y^2]
\end{aligned}$$

Solution 5:

$$\begin{aligned}
4x^4 + 9y^4 + 11x^2y^2 &= (2x^2)^2 + (3y^2)^2 + 12x^2y^2 - x^2y^2 \\
&= (2x^2 + 3y^2)^2 - x^2y^2 \\
&= (2x^2 + 3y^2)^2 - (xy)^2 \\
&= (2x^2 + 3y^2 - xy)(2x^2 + 3y^2 + xy) \\
&\quad [\because a^2 - b^2 = (a + b)(a - b)]
\end{aligned}$$

Solution 6:

$$\begin{aligned}
x^2 + \frac{1}{x^2} - 3 &= x^2 + \frac{1}{x^2} - 2 - 1 \\
&= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 1 \\
&= \left(x - \frac{1}{x}\right)^2 - 1 \\
&= \left(x - \frac{1}{x}\right)^2 - (1)^2 \\
&= \left(x - \frac{1}{x} - 1\right)\left(x - \frac{1}{x} + 1\right) \quad [\because a^2 - b^2 = (a + b)(a - b)]
\end{aligned}$$

Solution 7:

$$\begin{aligned}
a - b - 4a^2 + 4b^2 &= (a - b) - 4(a^2 - b^2) \\
&= (a - b) - 4(a - b)(a + b) \quad [\because a^2 - b^2 = (a + b)(a - b)] \\
&= (a - b)[1 - 4(a + b)] \\
&= (a - b)[1 - 4a - 4b]
\end{aligned}$$

Solution 8:

$$\begin{aligned}
(2a - 3)^2 - 2(2a - 3)(a - 1) + (a - 1)^2 \\
&= [(2a - 3) - (a - 1)]^2 \\
&= [2a - 3 - a + 1]^2 \\
&= (a - 2)^2
\end{aligned}$$

Solution 9:

Let us assume, $a^2 - 3a = x$

Then the given expression is,

$$\begin{aligned}
 (a^2 - 3a)(a^2 - 3a + 7) + 10 &= x(x + 7) + 10 \\
 &= x^2 + 7x + 10 \\
 &= x^2 + 5x + 2x + 10 \\
 &= x(x + 5) + 2(x + 5) \\
 &= (x + 5)(x + 2) \\
 &= (a^2 - 3a + 5)(a^2 - 3a + 2) \\
 &\quad \text{[resubstitute the value of } x\text{]} \\
 &= (a^2 - 3a + 5)(a^2 - 2a - a + 2) \\
 &= (a^2 - 3a + 5)(a(a - 2) - 1(a - 2)) \\
 &= (a^2 - 3a + 5)[(a - 1)(a - 2)]
 \end{aligned}$$

Solution 10:

Let us assume $a^2 - a = x$

Then the given expression is

$$\begin{aligned}
 (a^2 - a)(4a^2 - 4a - 5) - 6 &= x(4x - 5) - 6 \\
 &= 4x^2 - 5x - 6 \\
 &= 4x^2 - 8x + 3x - 6 \\
 &= 4x(x - 2) + 3(x - 2) \\
 &= (4x + 3)(x - 2) \\
 &= (4(a^2 - a) + 3)(a^2 - a - 2) \\
 &\quad \text{[resubstitute the value of } x\text{]} \\
 &= (4a^2 - 4a + 3)(a^2 - a - 2) \\
 &= (4a^2 - 4a + 3)(a^2 - 2a + a - 2) \\
 &= (4a^2 - 4a + 3)(a(a - 2) + 1(a - 2)) \\
 &= (4a^2 - 4a + 3)(a - 2)(a + 1)
 \end{aligned}$$

Solution 11:

$$\begin{aligned}
 x^4 + y^4 - 3x^2y^2 &= x^4 + y^4 - 2x^2y^2 - x^2y^2 \\
 &= (x^2)^2 + (y^2)^2 - 2x^2y^2 - x^2y^2 \\
 &= (x^2 - y^2)^2 - (xy)^2 \\
 &= (x^2 - y^2 - xy)(x^2 - y^2 + xy) \\
 &\quad [\because a^2 - b^2 = (a + b)(a - b)]
 \end{aligned}$$

Solution 12:

$$\begin{aligned}
& 5a^2 - b^2 - 4ab + 7a - 7b \\
&= 4a^2 + a^2 - b^2 - 4ab + 7a - 7b \\
&= a^2 - b^2 + 4a^2 - 4ab + 7a - 7b \\
&= (a^2 - b^2) + 4a(a - b) + 7(a - b) \\
&= (a - b)(a + b) + 4a(a - b) + 7(a - b) \quad [∵ a^2 - b^2 = (a + b)(a - b)] \\
&= (a - b)[(a + b) + 4a + 7] \\
&= (a - b)[a + b + 4a + 7] \\
&= (a - b)[5a + b + 7]
\end{aligned}$$

Solution 13:

$$12(3x - 2y)^2 - 3x + 2y - 1 = 12(3x - 2y)^2 - (3x - 2y) - 1$$

Let us assume that $3x - 2y = a$

Then the given expression is

$$\begin{aligned}
12(3x - 2y)^2 - 3x + 2y - 1 &= 12a^2 - 3a - 1 \\
&= 12a^2 - 4a + 3a - 1 \\
&= 4a(3a - 1) + 1(3a - 1) \\
&= (4a + 1)(3a - 1) \\
&= (4(3x - 2y) + 1)(3(3x - 2y) - 1) \\
&\quad \text{[resubstitute the value of a]} \\
&= (12x - 8y + 1)(9x - 6y - 1)
\end{aligned}$$

Solution 14:

$$4(2x - 3y)^2 - 8x + 12y - 3 = 4(2x - 3y)^2 - 4(2x - 3y) - 3$$

Let us assume that $2x - 3y = a$

Then the given expression is

$$\begin{aligned}
4(2x - 3y)^2 - 8x + 12y - 3 &= 4a^2 - 4a - 3 \\
&= 4a^2 - 6a + 2a - 3 \\
&= 2a(2a - 3) + 1(2a - 3) \\
&= (2a - 3)(2a + 1) \\
&= (2(2x - 3y) - 3)(2(2x - 3y) + 1) \\
&= (4x - 6y - 3)(4x - 6y + 1)
\end{aligned}$$

Solution 15:

$$3 - 5x + 5y - 12(x - y)^2 = 3 - 5(x - y) - 12(x - y)^2$$

Let us assume that $x - y = a$

Then the given expression is

$$\begin{aligned} 3 - 5x + 5y - 12(x - y)^2 &= 3 - 5a - 12a^2 \\ &= 3 - 9a + 4a - 12a^2 \\ &= 3(1 - 3a) + 4a(1 - 3a) \\ &= (3 + 4a)(1 - 3a) \\ &\quad \text{[resubstitute the value of } a\text{]} \\ &= (3 + 4(x - y))(1 - 3(x - y)) \\ &= (3 + 4x - 4y)(1 - 3x + 3y) \end{aligned}$$

Solution 16:

$$\begin{aligned} &9x^2 + 3x - 8y - 64y^2 \\ &= 9x^2 - 64y^2 + 3x - 8y \\ &= [(3x)^2 - (8y)^2] + (3x - 8y) \\ &= [(3x + 8y)(3x - 8y)] + (3x - 8y) \\ &= (3x - 8y)(3x + 8y + 1) \end{aligned}$$

Solution 17:

$$\begin{aligned} &2\sqrt{3}x^2 + x - 5\sqrt{3} \\ &= 2\sqrt{3}x^2 + 6x - 5x - 5\sqrt{3} \\ &= 2\sqrt{3}x(x + \sqrt{3}) - 5(x + \sqrt{3}) \\ &= (2\sqrt{3}x - 5)(x + \sqrt{3}) \end{aligned}$$

Solution 18:

$$\begin{aligned} &\frac{1}{4}(a+b)^2 - \frac{9}{16}(2a-b)^2 \\ &= \frac{1}{4} \left[(a+b)^2 - \frac{9}{4}(2a-b)^2 \right] \\ &= \frac{1}{4} \left[(a+b)^2 - \left(\frac{3}{2}(2a-b) \right)^2 \right] \\ &= \frac{1}{4} \left[\left(a+b + \frac{3}{2}(2a-b) \right) \left(a+b - \frac{3}{2}(2a-b) \right) \right] \\ &= \frac{1}{4} \left[\left(a+b + 3a - \frac{3b}{2} \right) \left(a+b - 3a + \frac{3b}{2} \right) \right] \\ &= \frac{1}{4} \left[\left(4a - \frac{b}{2} \right) \left(\frac{5b}{2} - 2a \right) \right] \\ &= \frac{1}{4} \left[\left(\frac{8a-b}{2} \right) \left(\frac{5b-4a}{2} \right) \right] \\ &= \frac{1}{4} \left[\frac{1}{4} (8a-b)(5b-4a) \right] \\ &= \frac{1}{16} (8a-b)(5b-4a) \end{aligned}$$

Solution 19:

$$\begin{aligned} & 2(ab + cd) - a^2 - b^2 + c^2 + d^2 \\ &= 2ab + 2cd - a^2 - b^2 + c^2 + d^2 \\ &= c^2 + d^2 + 2cd - a^2 - b^2 + 2ab \\ &= (c^2 + d^2 + 2cd) - (a^2 + b^2 - 2ab) \\ &= (c + d)^2 - (a - b)^2 \\ &= (c + d + a - b)(c + d - a + b) \end{aligned}$$

Solution 20:

$$\begin{aligned} \text{(i)} \quad & (987)^2 - (13)^2 \\ &= (987 + 13)(987 - 13) \\ &= 1000 \times 974 \\ &= 974000 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & (67.8)^2 - (32.2)^2 \\ &= (67.8 + 32.2)(67.8 - 32.2) \\ &= 100 \times 35.6 \\ &= 3560 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & \frac{(6.7)^2 - (3.3)^2}{6.7 - 3.3} \\ &= \frac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & \frac{(18.5)^2 - (6.5)^2}{18.5 + 6.5} \\ &= \frac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)} \\ &= 12 \end{aligned}$$