Chapter 8. Logarithms

Exercise 8(A)

Solution 1:

(i)

$5^3 = 125$

$\Rightarrow \log_5 125 = 3 \quad \left[a^b = c \Rightarrow \log_a c = b\right]$

(ii)

$3^{-2} = \frac{1}{9}$

$\Rightarrow \log_3 \frac{1}{9} = -2 \quad \left[a^b = c \Rightarrow \log_a c = b\right]$

(iii)

$10^{-3} = 0.001$

$\Rightarrow \log_{10} 0.001 = -3 \quad \left[a^b = c \Rightarrow \log_a c = b\right]$

(iv)

$(81)^{\frac{3}{4}} = 27$

$\Rightarrow \log_{81} 27 = \frac{3}{4}$ [By definition of logarithm, $a^b = c \Rightarrow \log_a c = b$]

Solution 2:

(i)

$\log_8 0.125 = -1$

$\Rightarrow 8^{-1} = 0.125 \quad \left[\log_a c = b \Rightarrow a^b = c\right]$

(ii)

$\log_{10} 0.01 = -2$

$\Rightarrow 10^{-2} = 0.01 \quad \left[\log_a c = b \Rightarrow a^b = c\right]$

(iii)

$\log_a A = x$

$\Rightarrow a^x = A \quad \left[\log_a c = b \Rightarrow a^b = c\right]$

(iv)

$\log_{10} 1 = 0$

$\Rightarrow 10^0 = 1 \quad \left[\log_a c = b \Rightarrow a^b = c\right]$

Solution 3:

$\log_{10} x = -2$

$\Rightarrow 10^{-2} = x \quad [\log_a c = b \Rightarrow a^b = c]$

$\Rightarrow x = 10^{-2}$

$\Rightarrow x = \frac{1}{10^2}$

$\Rightarrow x = \frac{1}{100}$

$\Rightarrow x = 0.01$

Solution 4:

(i)

Let $\log_{10}100 = x$

$\therefore 10^x = 100$

$\Rightarrow 10^x = 10 \times 10$

$\Rightarrow 10^x = 10^2$

$\Rightarrow x = 2 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\therefore \log_{10}100 = 2$

(ii)

Let $\log_{10}0.1 = x$

$\therefore 10^x = 0.1$

$\Rightarrow 10^x = \frac{1}{10}$

$\Rightarrow 10^x = 10^{-1}$

$\Rightarrow x = -1 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\therefore \log_{10}0.1 = -1$

(iii)

Let $\log_{10}0.001 = x$

$\therefore 10^x = 0.001$

$\Rightarrow 10^x = \frac{1}{1000}$

$\Rightarrow 10^x = \frac{1}{10^3}$

$\Rightarrow 10^x = 10^{-3}$

$\Rightarrow x = -3 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\therefore \log_{10}0.001 = -3$

(iv)

Let $\log_4 32 = x$

$\therefore 4^x = 32$

$\Rightarrow (2^2)^x = 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^{2x} = 2^5$

$\Rightarrow 2x = 5 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\Rightarrow x = \frac{5}{2}$

$\therefore \log_4 32 = \frac{5}{2}$

(v)

Let $\log_2 0.125 = x$

$\therefore 2^x = 0.125$

$\Rightarrow 2^x = \frac{125}{1000}$

$\Rightarrow 2^x = \frac{1}{8}$

$\Rightarrow 2^x = 8^{-1}$

$\Rightarrow 2^x = (2 \times 2 \times 2)^{-1}$

$\Rightarrow 2^x = (2^3)^{-1}$

$\Rightarrow 2^x = 2^{-3}$

$\Rightarrow x = -3$ [if $a^m = a^n$; then m=n]

$\therefore \log_2 0.125 = -3$

(vi)

Let $\log_4 \frac{1}{16} = x$

$\therefore 4^x = \frac{1}{16}$

$\Rightarrow 4^x = \frac{1}{4 \times 4}$

$\Rightarrow 4^x = (4 \times 4)^{-1}$

$\Rightarrow 4^x = (4^2)^{-1}$

$\Rightarrow 4^x = 4^{-2}$

$\Rightarrow x = -2$ [if $a^m = a^n$; then m=n]

$\therefore \log_4 \frac{1}{16} = -2$

(vii)

Let $\log_9 27 = x$

$\therefore 9^x = 27$

$\Rightarrow (3 \times 3)^x = 3 \times 3 \times 3$

$\Rightarrow \quad (3^2)^x = (3^3)$

$\Rightarrow \quad 3^{2x} = (3^3)$

$\Rightarrow \quad 2x = 3$ [if $a^m = a^n$; then m=n]

$\Rightarrow \quad x = \frac{3}{2}$

$\therefore \log_9 27 = \frac{3}{2}$

(viii)

Let $\log_{27} \frac{1}{81} = x$

$\therefore 27^x = \frac{1}{81}$

$\Rightarrow (3 \times 3 \times 3)^x = \frac{1}{3 \times 3 \times 3 \times 3}$

$\Rightarrow \quad (3^3)^x = \frac{1}{3^4}$

$\Rightarrow \quad (3^3)^x = (3^4)^{-1}$

$\Rightarrow \quad 3^{3x} = (3^{-4})$

$\Rightarrow \quad 3x = -4$ [if $a^m = a^n$; then m=n]

$\Rightarrow \quad x = \frac{-4}{3}$

$\therefore \log_{27} \frac{1}{81} = \frac{-4}{3}$

Solution 5:

(i)

Consider the equation

$\log_{10} x = a$

$\Rightarrow 10^a = x$

Thus the statement, $10^x = a$ is false

(ii)

Consider the equation

$x^y = z$

$\Rightarrow \log_x z = y$

Thus the statement, $\log_z x = y$ is false

(iii)

Consider the equation

$\log_2 8 = 3$

$\Rightarrow 2^3 = 8....(1)$

Now consider the equation

$\log_8 2 = \frac{1}{3}$

$\Rightarrow 8^{\frac{1}{3}} = 2$

$\Rightarrow (2^3)^{\frac{1}{3}} = 2....(2)$

Both the equations (1) and (2) are correct

Thus the given statements, $\log_2 8 = 3$ and $\log_8 2 = \frac{1}{3}$ are true

Solution 6:

(i)

Consider the equation

$\log_3 x = 0$

$\Rightarrow 3^0 = x$

$\Rightarrow 1 = x$ or $x=1$

(ii)

Consider the equation

$\log_x 2 = -1$

$\Rightarrow x^{-1} = 2$

$\Rightarrow \frac{1}{x} = 2$

$\Rightarrow x=\frac{1}{2}$

(iii)

Consider the equation

$\log_9 243 = x$

$\Rightarrow 9^x = 243$

$\Rightarrow (3^2)^x = 3^5$

$\Rightarrow 3^{2x} = 3^5$

$\Rightarrow 2x=5$

$\Rightarrow x=\frac{5}{2}$

$\Rightarrow x=2\frac{1}{2}$

(iv)

Consider the equation

$\log_5(x-7)=1$

$\Rightarrow 5^1 = x-7$

$\Rightarrow 5 = x-7$

$\Rightarrow x = 5+7$

$\Rightarrow x=12$

(v)

Consider the equation

$\log_4 32 = x-4$

$\Rightarrow 4^{x-4} = 32$

$\Rightarrow (2^2)^{x-4} = 2^5$

$\Rightarrow 2^{2(x-4)} = 2^5$

$\Rightarrow 2x-8=5$

$\Rightarrow 2x = 5+8$

$\Rightarrow 2x = 13$

$\Rightarrow x = \dfrac{13}{2}$

$\Rightarrow x = 6\dfrac{1}{2}$

(vi)

Consider the equation

$\log_7(2x^2-1)=2$

$\Rightarrow 7^2 = 2x^2-1$

$\Rightarrow 7 \times 7 = 2x^2-1$

$\Rightarrow 2x^2-1-49=0$

$\Rightarrow 2x^2-50=0$

$\Rightarrow 2x^2=50$

$\Rightarrow x^2 = \dfrac{50}{2}$

$\Rightarrow x^2 = 25$

$\Rightarrow x = \pm\sqrt{25}$

$\Rightarrow x = 5$ [neglecting the negative value]

Solution 7:

(i)

Let $\log_{10} 0.01 = x$

$\Rightarrow 10^x = 0.01$

$\Rightarrow 10^x = \frac{1}{100}$

$\Rightarrow 10^x = \frac{1}{10 \times 10}$

$\Rightarrow 10^x = \frac{1}{10^2}$

$\Rightarrow 10^x = 10^{-2}$

$\Rightarrow x = -2$

Thus, $\log_{10} 0.01 = -2$

(ii)

Let $\log_2 \frac{1}{8} = x$

$\Rightarrow 2^x = \frac{1}{8}$

$\Rightarrow 2^x = \frac{1}{2 \times 2 \times 2}$

$\Rightarrow 2^x = \frac{1}{2^3}$

$\Rightarrow 2^x = 2^{-3}$

$\Rightarrow x = -3$

Thus, $\log_2 \frac{1}{8} = -3$

(iii)

Let $\log_5 1 = x$

$\Rightarrow 5^x = 1$

$\Rightarrow 5^x = 5^0$

$\Rightarrow x = 0$

Thus, $\log_5 1 = 0$

(iv)

Let $\log_5 125 = x$

$\Rightarrow 5^x = 125$

$\Rightarrow 5^x = 5 \times 5 \times 5$

$\Rightarrow 5^x = 5^3$

$\Rightarrow x = 3$

Thus, $\log_5 125 = 3$

(v)

Let $\log_{16} 8 = x$

$\Rightarrow 16^x = 8$

$\Rightarrow (2 \times 2 \times 2 \times 2)^x = 2 \times 2 \times 2$

$\Rightarrow (2^4)^x = 2^3$

$\Rightarrow 2^{4x} = 2^3$

$\Rightarrow 4x = 3$

$\Rightarrow x = \frac{3}{4}$

Thus, $\log_{16} 8 = \frac{3}{4}$

(vi)

Let $\log_{0.5} 16 = x$

$\Rightarrow 0.5^x = 16$

$\Rightarrow \left(\frac{5}{10}\right)^x = 2 \times 2 \times 2 \times 2$

$\Rightarrow \left(\frac{1}{2}\right)^x = 2^4$

$\Rightarrow \frac{1}{2^x} = 2^4$

$\Rightarrow 2^{-x} = 2^4$

$\Rightarrow -x = 4$

$\Rightarrow x = -4$

Thus, $\log_{0.5} 16 = -4$

Solution 8:

$$\log_a m = n$$
$$\Rightarrow a^n = m$$
$$\Rightarrow \frac{a^n}{a} = \frac{m}{a}$$
$$\Rightarrow a^{n-1} = \frac{m}{a}$$

Solution 9:

$$\log_2 x = m \text{ and } \log_5 y = n$$
$$\Rightarrow 2^m = x \text{ and } 5^n = y$$
$$\text{(i) Consider } 2^m = x$$
$$\Rightarrow \frac{2^m}{2^3} = \frac{x}{2^3}$$
$$\Rightarrow 2^{m-3} = \frac{x}{8}$$
$$\text{(ii) Consider } 5^n = y$$
$$\Rightarrow \left(5^n\right)^3 = y^3$$
$$\Rightarrow 5^{3n} = y^3$$
$$\Rightarrow 5^{3n} \times 5^2 = y^3 \times 5^2$$
$$\Rightarrow 5^{3n+2} = 25y^3$$

Solution 10:

Given that :

$$\log_2^x = a \text{ and } \log_3^y = a$$
$$\Rightarrow 2^a = x \text{ and } 3^a = y \qquad \left[\begin{array}{l} \because \log_a^m = n \\ \Rightarrow a^n = m \end{array}\right]$$

Now prime factorization of 72 is

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

Hence,

$$(72)^a = (2 \times 2 \times 2 \times 3 \times 3)^a$$
$$= \left(2^3 \times 3^2\right)^a$$
$$= 2^{3a} \times 3^{2a}$$
$$= \left(2^a\right)^3 \times \left(3^a\right)^2 \qquad \left[\begin{array}{l} \text{as } 2^a = x \\ \quad 3^a = y \end{array}\right]$$
$$= x^3 y^2$$

Solution 11:

$\log(x-1)+\log(x+1)=\log_2 1$

$\Rightarrow \log(x-1)+\log(x+1)=0$

$\Rightarrow \log[(x-1)(x+1)]=0$

$\Rightarrow (x-1)(x+1)=1....(\text{Since} \log 1=0)$

$\Rightarrow x^2-1=1$

$\Rightarrow x^2=2$

$\Rightarrow x=\pm\sqrt{2}$

$-\sqrt{2}$ cannot be possible, since log of a negative number is not defined.

So, $x=\sqrt{2}$.

Solution 12:

$\log(x^2-21)=2$

$\Rightarrow x^2-21=10^2$

$\Rightarrow x^2-21=100$

$\Rightarrow x^2=121$

$\Rightarrow x=\pm 11$

Exercise 8(B)

Solution 1:

(i)

$$\log 36 = \log(2 \times 2 \times 3 \times 3)$$
$$= \log(2^2 \times 3^2)$$
$$= \log(2^2) + \log(3^2) \quad [\log_a mn = \log_a m + \log_a n]$$
$$= 2\log 2 + 2\log 3 \quad [\log_a m^n = n\log_a m]$$

(ii)

$$\log 144 = \log(2 \times 2 \times 2 \times 2 \times 3 \times 3)$$
$$= \log(2^4 \times 3^2)$$
$$= \log(2^4) + \log(3^2) \quad [\log_a mn = \log_a m + \log_a n]$$
$$= 4\log 2 + 2\log 3 \quad [\log_a m^n = n\log_a m]$$

(iii)

$$\log 4.5 = \log \frac{45}{10}$$
$$= \log \frac{5 \times 3 \times 3}{5 \times 2}$$
$$= \log \frac{3^2}{2}$$
$$= \log 3^2 - \log 2 \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$
$$= 2\log 3 - \log 2 \quad [\log_a m^n = n\log_a m]$$

(iv)

$$\log \frac{26}{51} - \log \frac{91}{119} = \log \frac{\frac{26}{51}}{\frac{91}{119}} \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= \log \frac{26}{51} \times \frac{119}{91}$$
$$= \log \frac{2 \times 13}{3 \times 17} \times \frac{7 \times 17}{7 \times 13}$$
$$= \log \frac{2}{3}$$
$$= \log 2 - \log 3 \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$

(v)

$$\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$$

$$= \log\frac{75}{16} - \log\left(\frac{5}{9}\right)^2 + \log\frac{32}{243} \quad [n\log_a m = \log_a m^n]$$

$$= \log\frac{75}{16} - \log\frac{5}{9}\times\frac{5}{9} + \log\frac{32}{243}$$

$$= \log\frac{75}{16} - \log\frac{25}{81} + \log\frac{32}{243}$$

$$= \log\left(\frac{\frac{75}{16}}{\frac{25}{81}}\right) \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$

$$= \log\frac{75}{16}\times\frac{81}{25} + \log\frac{32}{243}$$

$$= \log\frac{3\times 25}{16}\times\frac{81}{25} + \log\frac{32}{243}$$

$$= \log\frac{3\times 81}{16} + \log\frac{32}{243}$$

$$= \log\frac{243}{16} + \log\frac{32}{243}$$

$$= \log\frac{243}{16}\times\frac{32}{243} \quad [\log_a m + \log_a n = \log_a mn]$$

$$= \log\frac{32}{16}$$

$$= \log 2$$

Solution 2:

(i)

Consider the given equation

$2\log x - \log y = 1$

$\Rightarrow \log x^2 - \log y = 1$

$\Rightarrow \log \frac{x^2}{y} = \log 10$

$\Rightarrow \frac{x^2}{y} = 10$

$\Rightarrow x^2 = 10y$

(ii)

Consider the given equation

$2\log x + 3\log y = \log a$

$\Rightarrow \log x^2 + \log y^3 = \log a$

$\Rightarrow \log x^2 y^3 = \log a$

$\Rightarrow x^2 y^3 = a$

(iii)

Consider the given equation

$a\log x - b\log y = 2\log 3$

$\Rightarrow \log x^a - \log y^b = \log 3^2$

$\Rightarrow \log \frac{x^a}{y^b} = \log 9$

$\Rightarrow \frac{x^a}{y^b} = 9$

$\Rightarrow x^a = 9y^b$

Solution 3:

(i) Consider the given expression

$\log 5+\log 8-2\log 2=\log 5+\log 8\times 8-\log 2^2$ $[n\log_a m=\log_a m^n]$

$=\log 5\times 8-\log 2^2$ $[\log_a m+\log_a n=\log_a mn]$

$=\log 40-\log 4$

$=\log\frac{40}{4}$ $[\log_a m-\log_a n=\log_a \frac{m}{n}]$

$=\log 10$

$=1$

(ii) Consider the given expression

$\log_{10}8+\log_{10}25+2\log_{10}3-\log_{10}18$

$=\log_{10}8+\log_{10}25+\log_{10}3^2-\log_{10}18$

$[n\log_a m=\log_a m^n]$

$=\log_{10}8+\log_{10}25+\log_{10}9-\log_{10}18$

$=\log_{10}8\times 25\times 9-\log_{10}18$

$[\log_a \ell+\log_a m+\log_a n=\log_a \ell mn]$

$=\log_{10}1800-\log_{10}18$

$=\log_{10}\frac{1800}{18}$ $[\log_a m-\log_a n=\log_a \frac{m}{n}]$

$=\log_{10}100$

$=2$ $[\because \log_{10}100=2]$

(iii) Consider the given expression

$\log 4+\frac{1}{3}\log 125-\frac{1}{5}\log 32$

$=\log 4+\log(125)^{\frac{1}{3}}-\log(32)^{\frac{1}{5}}$ $[n\log_a m=\log_a m^n]$

$=\log 4+\log(5^3)^{\frac{1}{3}}-\log(2^5)^{\frac{1}{5}}$

$=\log 4+\log 5-\log 2$

$=\log 4\times 5-\log 2$ $[\log_a m+\log_a n=\log_a mn]$

$=\log\frac{20}{2}$ $[\log_a m-\log_a n=\log_a \frac{m}{n}]$

$=\log 10$

$=1$

Solution 4:

We need to prove that

$$2\log\frac{15}{18}-\log\frac{25}{162}+\log\frac{4}{9}=\log 2$$

$$L.H.S = 2\log\frac{15}{18}-\log\frac{25}{162}+\log\frac{4}{9}$$

$$=\log\left(\frac{15}{18}\right)^2-\log\frac{25}{162}+\log\frac{4}{9} \qquad [n\log_a m=\log_a m^n]$$

$$=\log\left[\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\right]-\log\frac{25}{162}+\log\frac{4}{9}$$

$$=\log\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}-\log\frac{25}{162} \qquad [\log_a m+\log_a n=\log_a mn]$$

$$=\log\frac{\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}}{\frac{25}{162}} \qquad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$=\log\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}\times\frac{162}{25}$$

$$=\log\frac{72}{36}$$

$$=\log 2$$

$$=R.H.S$$

Solution 5:

Consider the given equation

$$x-\log 48+3\log 2=\frac{1}{3}\log 125-\log 3$$

$$\Rightarrow x=\frac{1}{3}\log 125-\log 3+\log 48-3\log 2$$

$$\Rightarrow x=\log(125)^{\frac{1}{3}}-\log 3+\log 48-\log 2^3 \qquad [n\log_a m=\log_a m^n]$$

$$\Rightarrow x=\log(5\times5\times5)^{\frac{1}{3}}-\log 3+\log 48-\log 8$$

$$\Rightarrow x=\log(5^3)^{\frac{1}{3}}-\log 3+\log 48-\log 8$$

$$\Rightarrow x=\log 5-\log 3+\log 48-\log 8$$

$$\Rightarrow x=\log 5+\log 48-\log 3-\log 8$$

$$\Rightarrow x=(\log 5+\log 48)-(\log 3+\log 8)$$

$$\Rightarrow x=(\log 5\times 48)-(\log 3\times 8) \qquad [\log_a m+\log_a n=\log_a mn]$$

$$\Rightarrow x=\log\frac{5\times 48}{3\times 8} \qquad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$\Rightarrow x=\log\frac{5\times6\times8}{3\times8}$$

$$\Rightarrow x=\log 10$$

$$\Rightarrow x=1$$

Solution 6:

$$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10 \qquad [\because \log_{10} 10 = 1]$$
$$= \log_{10} 2 \times 10 \qquad [\log_a m + \log_a n = \log_a mn]$$
$$= \log_{10} 20$$

Solution 7:

(i)

$$log_{10}(x - 10) = 1$$
$$\Rightarrow log_{10}(x - 10) = log_{10}10$$
$$\Rightarrow x - 10 = 10$$
$$\Rightarrow x = 10 + 10$$
$$\Rightarrow x = 20$$

(ii)

$$log\left(x^2 - 21\right) = 2$$
$$\Rightarrow log\left(x^2 - 21\right) = log100$$
$$\Rightarrow x^2 - 21 = 100$$
$$\Rightarrow x^2 - 21 - 100 = 0$$
$$\Rightarrow x^2 - 121 = 0$$
$$\Rightarrow x^2 = 121$$
$$\Rightarrow x = \pm\sqrt{121}$$
$$\Rightarrow x = \pm 11$$

(iii)

$$log(x - 2) + \log(x + 2) = \log 5$$
$$\Rightarrow \log(x - 2)(x + 2) = \log 5 \quad [\log_a m + \log_a n = \log_a mn]$$
$$\Rightarrow log\left(x^2 - 4\right) = \log 5$$
$$\Rightarrow x^2 - 4 = 5$$
$$\Rightarrow x^2 = 9$$
$$\Rightarrow x = \pm\sqrt{9}$$
$$\Rightarrow x = \pm\sqrt{3^2}$$
$$\Rightarrow x = \pm 3$$

(iv)

$log(x+5)+\log(x-5)=4\log2+2\log3$

$\Rightarrow \log(x+5)(x-5)=4\log2+2\log3$ $\quad [\log_a m+\log_a n=\log_a mn]$

$\Rightarrow log(x^2-25)=\log2^4+\log3^2$ $\quad [n\log_a m=\log_a m^n]$

$\Rightarrow log(x^2-25)=\log16+\log9$

$\Rightarrow log(x^2-25)=\log16\times9$ $\quad [\log_a m+\log_a n=\log_a mn]$

$\Rightarrow log(x^2-25)=\log144$

$\Rightarrow x^2-25=144$

$\Rightarrow x^2=144+25$

$\Rightarrow x^2=169$

$\Rightarrow x=\pm\sqrt{169}$

$\Rightarrow x=\pm\sqrt{13^2}$

$\Rightarrow x=\pm13$

Solution 8:

(i)

$$\frac{\log 81}{\log 27} = x$$

$$\Rightarrow x = \frac{\log 81}{\log 27}$$

$$\Rightarrow x = \frac{\log 3 \times 3 \times 3 \times 3}{\log 3 \times 3 \times 3}$$

$$\Rightarrow x = \frac{\log 3^4}{\log 3^3}$$

$$\Rightarrow x = \frac{4\log 3}{3\log 3} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{4}{3}$$

$$\Rightarrow x = 1\frac{1}{3}$$

(ii)

$$\frac{\log 128}{\log 32} = x$$

$$\Rightarrow x = \frac{\log 128}{\log 32}$$

$$\Rightarrow x = \frac{\log 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\log 2 \times 2 \times 2 \times 2 \times 2}$$

$$\Rightarrow x = \frac{\log 2^7}{\log 2^5}$$

$$\Rightarrow x = \frac{7\log 2}{5\log 2} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{7}{5}$$

$$\Rightarrow x = 1.4$$

(iii)

$$\frac{\log 64}{\log 8} = \log x$$

$$\Rightarrow \log x = \frac{\log 64}{\log 8}$$

$$\Rightarrow \log x = \frac{\log 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\log 2 \times 2 \times 2}$$

$$\Rightarrow \log x = \frac{\log 2^6}{\log 2^3}$$

$$\Rightarrow \log x = \frac{6 \log 2}{3 \log 2} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log x = \frac{6}{3}$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10 \times 10$$

$$\Rightarrow x = 100$$

(iv)

$$\frac{\log 225}{\log 15} = \log x$$

$$\Rightarrow \log x = \frac{\log 225}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15 \times 15}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15^2}{\log 15}$$

$$\Rightarrow \log x = \frac{2 \log 15}{\log 15} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10 \times 10$$

$$\Rightarrow x = 100$$

Solution 9:

Given that

$\log x = m + n;$

$\log y = m - n;$

Consider the expression $\log \frac{10x}{y^2}$:

$$\log \frac{10x}{y^2} = \log 10x - \log y^2$$

$$= \log 10x - 2\log y \qquad [n \log_a m = \log_a m^n]$$

$$= \log 10 + \log x - 2\log y \quad [\log_a m + \log_a n = \log_a mn]$$

$$= 1 + \log x - 2\log y$$

$$= 1 + m + n - 2(m - n)$$

$$= 1 + m + n - 2m + 2n$$

$$\Rightarrow \log \frac{10x}{y^2} = 1 - m + 3n$$

Solution 10:

(i)

We have,

$\log 1 = 0$ and $\log 1000 = 3$

$\therefore \log 1 \times \log 1000 = 0 \times 3 = 0$

Thus the statement, $\log 1 \times \log 1000 = 0$ is true

(ii)

We know that

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\therefore \frac{\log x}{\log y} \neq \log x - \log y$$

Thus the statement, $\dfrac{\log x}{\log y} = \log x - \log y$ is false

(iii)

Given that

$$\frac{\log 25}{\log 5} = \log x$$

$$\Rightarrow \frac{\log 5 \times 5}{\log 5} = \log x$$

$$\Rightarrow \frac{\log 5^2}{\log 5} = \log x$$

$$\Rightarrow \frac{2\log 5}{\log 5} = \log x \qquad [\log_a m^n = n\log_a m]$$

$$\Rightarrow 2 = \log_{10} x$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 100$$

Thus the statement, $x = 2$ is false

(iv)

We know that

$\log x + \log y = \log xy$

$\therefore \log x + \log y \neq \log x \times \log y$

Thus the statement $\log x + \log y = \log x \times \log y$ is false

Solution 11:

Given that $\log_{10} 2 = a$ and $\log_{10} 3 = b$

(i)

$$\log 12 = \log 2 \times 2 \times 3$$
$$= \log 2 \times 2 + \log 3 \quad [\log_a mn = \log_a m + \log_a n]$$
$$= \log 2^2 + \log 3$$
$$= 2\log 2 + \log 3 \quad [n\log_a m = \log_a m^n]$$
$$= 2a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$

(ii)

$$\log 2.25 = \log \frac{225}{100}$$
$$= \log \frac{25 \times 9}{25 \times 4}$$
$$= \log \frac{9}{4}$$
$$= \log \left(\frac{3}{2}\right)^2$$
$$= 2\log\left(\frac{3}{2}\right) \quad [n\log_a m = \log_a m^n]$$
$$= 2(\log 3 - \log 2) \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= 2(b - a) \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$
$$= 2b - 2a$$

(iii)

$$\log 2\frac{1}{4} = \log \frac{9}{4}$$
$$= \log \left(\frac{3}{2}\right)^2$$
$$= 2\log\left(\frac{3}{2}\right) \quad [n\log_a m = \log_a m^n]$$
$$= 2(\log 3 - \log 2) \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= 2(b - a) \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$
$$= 2b - 2a$$

(iv)

$$\log 5.4 = \log\frac{54}{10}$$
$$= \log\left(\frac{2\times3\times3\times3}{10}\right)$$
$$= \log(2\times3\times3\times3) - \log_{10}10 \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= \log_{10}2 + \log_{10}3^3 - \log_{10}10 \quad [\log_a mn = \log_a m + \log_a n]$$
$$= \log_{10}2 + 3\log_{10}3 - \log_{10}10 \quad [n\log_a m = \log_a m^n]$$
$$= \log_{10}2 + 3\log_{10}3 - 1 \quad [\because \log_{10}10 = 1]$$
$$= a + 3b - 1 \quad [\because \log_{10}2 = a \text{ and } \log_{10}3 = b]$$

(v)

$$\log 60 = \log_{10}10\times2\times3$$
$$= \log_{10}10 + \log_{10}2 + \log_{10}3 \quad [\log_a mn = \log_a m + \log_a n]$$
$$= 1 + \log_{10}2 + \log_{10}3 \quad [\because \log_{10}10 = 1]$$
$$= 1 + a + b \quad [\because \log_{10}2 = a \text{ and } \log_{10}3 = b]$$

(vi)

$$\log 3\frac{1}{8} = \log_{10}\left(\frac{25}{8}\times\frac{4}{4}\right)$$
$$= \log_{10}\left(\frac{100}{32}\right)$$
$$= \log_{10}100 - \log_{10}32 \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$
$$= \log_{10}100 - \log_{10}2^5$$
$$= 2 - \log_{10}2^5 \quad [\because \log_{10}100 = 2]$$
$$= 2 - 5\log_{10}2 \quad [\log_a m^n = n\log_a m]$$
$$= 2 - 5a \quad [\because \log_{10}2 = a]$$

Solution 12:

We know that $\log 2 = 0.3010$ and $\log 3 = 0.4771$

(i)

$$\log 12 = \log 2 \times 2 \times 3$$
$$= \log 2 \times 2 + \log 3 \qquad [\log_a mn = \log_a m + \log_a n]$$
$$= \log 2^2 + \log 3$$
$$= 2\log 2 + \log 3 \qquad [n\log_a m = \log_a m^n]$$
$$= 2(0.3010) + 0.4771 \qquad \begin{bmatrix} \because \log 2 = 0.3010 \text{ and} \\ \log 3 = 0.4771 \end{bmatrix}$$
$$= 1.0791$$

(ii)

$$\log 1.2 = \log \frac{12}{10}$$
$$= \log 12 - \log 10 \qquad \left[\log_a \frac{m}{n} = \log_a m - \log_a n\right]$$
$$= \log 2 \times 2 \times 3 - 1 \qquad [\because \log 10 = 1]$$
$$= \log 2 \times 2 + \log 3 - 1 \qquad [\log_a mn = \log_a m + \log_a n]$$
$$= \log 2^2 + \log 3 - 1$$
$$= 2\log 2 + \log 3 - 1 \qquad [n\log_a m = \log_a m^n]$$
$$= 2(0.3010) + 0.4771 - 1 \qquad \begin{bmatrix} \because \log 2 = 0.3010 \\ \text{and } \log 3 = 0.4771 \end{bmatrix}$$
$$= 1.0791 - 1$$
$$= 0.0791$$

(iii)

$$\log 3.6 = \log\frac{36}{10}$$

$= \log 36 - \log 10$ $\quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$

$= \log 2 \times 2 \times 3 \times 3 - 1$ $\quad [\because \log 10 = 1]$

$= \log 2 \times 2 + \log 3 \times 3 - 1$ $\quad [\log_a mn = \log_a m + \log_a n]$

$= \log 2^2 + \log 3^2 - 1$

$= 2\log 2 + 2\log 3 - 1$ $\quad [n \log_a m = \log_a m^n]$

$= 2(0.3010) + 2(0.4771) - 1$ $\quad \begin{bmatrix} \because \log 2 = 0.3010 \\ \text{and } \log 3 = 0.4771 \end{bmatrix}$

$= 1.5562 - 1$

$= 0.5562$

(iv)

$$\log 15 = \log\left(\frac{15}{10} \times 10\right)$$

$= \log\left(\frac{15}{10}\right) + \log 10$

$= \log\left(\frac{3}{2}\right) + 1$ $\quad [\because \log 10 = 1]$

$= \log 3 - \log 2 + 1$ $\quad [\because \log m - \log n = \log\left(\frac{m}{n}\right)]$

$= 0.4771 - 0.3010 + 1$

$= 1.1761$

(v)

$$\log 25 = \log\left(\frac{25}{4} \times 4\right)$$

$$= \log\left(\frac{100}{4}\right) \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 100 - \log(2 \times 2) \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$

$$= 2 - \log(2^2) \qquad [\log 100 = 2]$$

$$= 2 - 2\log 2 \qquad [\log_a m^n = n\log_a m]$$

$$= 2 - 2(0.3010) \qquad [\because \log 2 = 0.3010]$$

$$= 1.398$$

(vi)

$$\frac{2}{3}\log 8 = \frac{2}{3}\log 2 \times 2 \times 2$$

$$= \frac{2}{3}\log 2^3$$

$$= 3 \times \frac{2}{3}\log 2 \quad [\log_a m^n = n\log_a m]$$

$$= 2\log 2$$

$$= 2 \times 0.3010 \quad [\because \log 2 = 0.3010]$$

$$= 0.602$$

Solution 13:

(i)

Consider the given equation:

$$2\log_{10} x + 1 = \log_{10} 250$$

$$\Rightarrow \log_{10} x^2 + 1 = \log_{10} 250 \qquad [\log_a m^n = n\log_a m]$$

$$\Rightarrow \log_{10} x^2 + \log_{10} 10 = \log_{10} 250 \quad [\because \log_{10} 10 = 1]$$

$$\Rightarrow \log_{10}\left(x^2 \times 10\right) = \log_{10} 250 \qquad [\log_a m + \log_a n = \log_a mn]$$

$$\Rightarrow x^2 \times 10 = 250$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \sqrt{25}$$

$$\Rightarrow x = 5$$

(ii)

$x = 5$ (proved above in (i))

$$\log_{10} 2x = \log_{10} 2(5)$$

$$= \log_{10} 10$$

$$= 1 \qquad [\because \log_{10} 10 = 1]$$

Solution 14:

$$3\log x + \frac{1}{2}\log y = 2$$

$$\Rightarrow \log x^3 + \log\sqrt{y} = 2$$

$$\Rightarrow \log x^3\sqrt{y} = 2$$

$$\Rightarrow x^3\sqrt{y} = 10^2$$

$$\Rightarrow \sqrt{y} = \frac{10^2}{x^3}$$

Squaring both sides, we get

$$y = \frac{10000}{x^6}$$

$$\Rightarrow y = 10000x^{-6}$$

Solution 15:

$$x = (100)^a,\ \ y = (10000)^b \text{ and } z = (10)^c$$

$$\Rightarrow \log x = a\log 100,\ \ \log y = b\log 10000 \text{ and } \log z = c\log 10$$

$$\log\frac{10\sqrt{y}}{x^2z^3} = \log 10\sqrt{y} - \log\left(x^2z^3\right)$$

$$= \log\left(10y^{1/2}\right) - \log x^2 - \log z^3$$

$$= \log 10 + \log y^{1/2} - \log x^2 - \log z^3$$

$$= \log 10 + \frac{1}{2}\log y - 2\log x - 3\log z$$

$$= 1 + \frac{1}{2}\log(10000)^b - 2\log(100)^a - 3\log(10)^c \text{.......(Since } \log 10 = 1\text{)}$$

$$= 1 + \frac{b}{2}\log(10)^4 - a\log(10)^2 - 3c\log 10$$

$$= 1 + \frac{b}{2} \times 4\log 10 - 2 \times 2a\log 10 - 3c\log 10$$

$$= 1 + 2b - 4a - 3c$$

Solution 16:

$3(\log 5 - \log 3) - (\log 5 - 2\log 6) = 2 - \log x$

$\Rightarrow 3\log 5 - 3\log 3 - \log 5 + 2\log(2 \times 3) = 2 - \log x$

$\Rightarrow 3\log 5 - 3\log 3 - \log 5 + 2\log 2 + 2\log 3 = 2 - \log x$

$\Rightarrow 2\log 5 - \log 3 + 2\log 2 = 2 - \log x$

$\Rightarrow 2\log 5 - \log 3 + 2\log 2 + \log x = 2$

$\Rightarrow \log 5^2 - \log 3 + \log 2^2 + \log x = 2$

$\Rightarrow \log\left(\frac{25 \times 4 \times x}{3}\right) = 2$

$\Rightarrow \log\left(\frac{100x}{3}\right) = 2$

$\Rightarrow \frac{100x}{3} = 10^2$

$\Rightarrow \frac{x}{3} = 1$

$\Rightarrow x = 3$

Exercise 8(C)

Solution 1:

Given that $\log_{10}8 = 0.90$

$\Rightarrow \log_{10} 2 \times 2 \times 2 = 0.90$

$\Rightarrow \log_{10} 2^3 = 0.90$

$\Rightarrow 3\log_{10} 2 = 0.90$

$\Rightarrow \log_{10} 2 = \frac{0.90}{3}$

$\Rightarrow \log_{10} 2 = 0.30....(1)$

(i)

$\log 4 = \log_{10}(2 \times 2)$

$\Rightarrow \quad = \log_{10}(2^2)$

$\Rightarrow \quad = 2\log_{10} 2$

$\Rightarrow \quad = 2(0.30) \quad$ [from (1)]

$\Rightarrow \quad = 0.60$

(ii)

$\log \sqrt{32} = \log_{10}(32)^{\frac{1}{2}}$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(32)$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(2 \times 2 \times 2 \times 2 \times 2)$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(2^5)$

$\Rightarrow \quad = \frac{1}{2} \times 5\log_{10} 2$

$\Rightarrow \quad = \frac{1}{2} \times 5(0.30) \quad$ [from (1)]

$\Rightarrow \quad = 5 \times 0.15$

$\Rightarrow \quad = 0.75$

(iii)

$$\log 0.125 = \log_{10} \frac{125}{1000}$$
$$= \log_{10} \frac{1}{8}$$
$$= \log_{10} \frac{1}{2 \times 2 \times 2}$$
$$= \log_{10} \left(\frac{1}{2^3}\right)$$
$$= \log_{10} 2^{-3}$$
$$= -3 \times (0.30) \quad \text{[from (1)]}$$
$$= -0.9$$

Solution 2:

$$\log 27 = 1.431$$
$$\Rightarrow \log 3 \times 3 \times 3 = 1.431$$
$$\Rightarrow \log 3^3 = 1.431$$
$$\Rightarrow 3\log 3 = 1.431$$
$$\Rightarrow \log 3 = \frac{1.431}{3}$$
$$\Rightarrow \log 3 = 0.477 \ldots\ldots (1)$$

(i)

$$\log 9 = \log(3 \times 3)$$
$$= \log 3^2$$
$$= 2\log 3$$
$$= 2 \times 0.477 \quad \text{[from (1)]}$$
$$= 0.954$$

(ii)

$$\log 300 = \log(3 \times 100)$$
$$= \log 3 + \log 100$$
$$= \log 3 + 2 \quad [\because \log_{10} 100 = 2]$$
$$= 0.477 + 2 \quad \text{[from (1)]}$$
$$= 2.477$$

Solution 3:

$\log_{10} a = b$

$\Rightarrow 10^b = a$

$\Rightarrow \left(10^b\right)^3 = (a)^3$ [cubing both sides]

$\Rightarrow \frac{10^{3b}}{10^2} = \frac{a^3}{10^2}$ [dividing both sides by 10^2]

$\Rightarrow 10^{3b-2} = \frac{a^3}{100}$

Solution 4:

$\log_5 x = y$ [given]

$\Rightarrow 5^y = x$

$\Rightarrow \left(5^y\right)^2 = x^2$

$\Rightarrow 5^{2y} = x^2$

$\Rightarrow 5^{2y} \times 5^3 = x^2 \times 5^3$

$\Rightarrow 5^{2y+3} = 125x^2$

Solution 5:

Given that $\log_3 m = x$ and $\log_3 n = y$

$\Rightarrow 3^x = m$ and $3^y = n$

(i)

Consider the given expression:

$3^{2x-3} = 3^{2x} \cdot 3^{-3}$

$= 3^{2x} \cdot \dfrac{1}{3^3}$

$= \dfrac{3^{2x}}{3^3}$

$= \dfrac{(3^x)^2}{3^3}$

$= \dfrac{m^2}{27}$

Therefore, $3^{2x-3} = \dfrac{m^2}{27}$

(ii)

Consider the given expression:

$3^{1-2y+3x} = 3^1 \cdot 3^{-2y} \cdot 3^{3x}$

$= 3 \cdot \dfrac{1}{3^{2y}} \cdot 3^{3x}$

$= \dfrac{3}{(3^y)^2} \cdot (3^x)^3$

$= \dfrac{3}{(n)^2} \cdot (m)^3$

$= \dfrac{3m^3}{n^2}$

Therefore, $3^{1-2y+3x} = \dfrac{3m^3}{n^2}$

(iii)

Consider the given expression:

$2\log_3 A = 5x - 3y$

$\Rightarrow 2\log_3 A = 5\log_3 m - 3\log_3 n$

$\Rightarrow \log_3 A^2 = \log_3 m^5 - \log_3 n^3$

$\Rightarrow \log_3 A^2 = \log_3\left(\frac{m^5}{n^3}\right)$

$\Rightarrow A^2 = \left(\frac{m^5}{n^3}\right)$

$\Rightarrow A = \sqrt{\left(\frac{m^5}{n^3}\right)}$

Solution 6:

(i)

$\log(a)^3 - \log a = 3\log a - \log a$

$= 2\log a$

(ii)

$\log(a)^3 \div \log a = 3\log a \div \log a$

$= \frac{3\log a}{\log a}$

$= 3$

Solution 7:

$\log(a+b) = \log a + \log b$

$\Rightarrow \log(a+b) = \log ab$

$\Rightarrow a + b = ab$

$\Rightarrow a - ab = -b$

$\Rightarrow -ab + a = -b$

$\Rightarrow -a(b-1) = -b$

$\Rightarrow a(b-1) = b$

$\Rightarrow a = \frac{b}{b-1}$

Solution 8:

(i)

$$L.H.S = (\log a)^2 - (\log b)^2$$

$$\Rightarrow L.H.S = (\log a + \log b)(\log a - \log b)$$

$$\Rightarrow L.H.S = \log(ab)\log\left(\frac{a}{b}\right)$$

$$\Rightarrow L.H.S = \log\left(\frac{a}{b}\right) \times \log(ab)$$

$$\Rightarrow L.H.S = R.H.S$$

Hence proved.

(ii)

Given that

$$a\log b + b\log a - 1 = 0$$

$$\Rightarrow a\log b + b\log a = 1$$

$$\Rightarrow \log b^a + \log a^b = 1$$

$$\Rightarrow \log b^a + \log a^b = \log 10$$

$$\Rightarrow \log\left(b^a \cdot a^b\right) = \log 10$$

$$\Rightarrow b^a \cdot a^b = 10$$

Solution 9:

(i)

Given that

$$\log(a+1) = \log(4a-3) - \log 3$$

$$\Rightarrow \log(a+1) = \log\left(\frac{4a-3}{3}\right)$$

$$\Rightarrow a + 1 = \frac{4a-3}{3}$$

$$\Rightarrow 3a + 3 = 4a - 3$$

$$\Rightarrow 4a - 3a = 3 + 3$$

$$\Rightarrow a = 6$$

(ii)

$$2\log y - \log x - 3 = 0$$

$$\Rightarrow 2\log y - \log x = 3$$

$$\Rightarrow \log y^2 - \log x = 3$$

$$\Rightarrow \log y^2 - \log x = \log 1000$$

$$\Rightarrow \log\frac{y^2}{x} = \log 1000$$

$$\Rightarrow \frac{y^2}{x} = 1000$$

$$\Rightarrow x = \frac{y^2}{1000}$$

(iii)

$\log_{10} 125 = 3(1 - \log_{10} 2)$

$L.H.S. = \log_{10} 125$

$= \log_{10} 5 \times 5 \times 5$

$= \log_{10} 5^3$

$= 3\log_{10} 5 .....(1)$

$R.H.S = 3(1 - \log_{10} 2)$

$= 3(\log_{10} 10 - \log_{10} 2)$

$= 3\log_{10}\left(\frac{10}{2}\right)$

$= 3\log_{10} 5 .....(2)$

From (1) and (2), we have

L.H.S.=R.H.S.

Hence proved.

Solution 10:

Given $\log x = 2m - n$, $\log y = n - 2m$ and $\log z = 3m - 2n$

$$\log \frac{x^2y^3}{z^4} = \log x^2y^3 - \log z^4$$

$$= \log x^2 + \log y^3 - \log z^4$$

$$= 2\log x + 3\log y - 4\log z$$

$$= 2(2m - n) + 3(n - 2m) - 4(3m - 2n)$$

$$= 4m - 2n + 3n - 6m - 12m - 8n$$

$$= -14m - 7n$$

Solution 11:

$$\log_x 25 - \log_x 5 = 2 - \log_x \frac{1}{125}$$

$$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x \left(\frac{1}{5}\right)^3$$

$$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x 5^{-3}$$

$$\Rightarrow 2\log_x 5 - \log_x 5 = 2 + 3\log_x 5$$

$$\Rightarrow 2\log_x 5 - \log_x 5 - 3\log_x 5 = 2$$

$$\Rightarrow -2\log_x 5 = 2$$

$$\Rightarrow \log_x 5 = -1$$

$$\Rightarrow x^{-1} = 5$$

$$\Rightarrow \frac{1}{x} = 5$$

$$\Rightarrow x = \frac{1}{5}$$

Exercise 8(D)

Solution 1:

$$\frac{3}{2}\log a + \frac{2}{3}\log b - 1 = 0$$

$$\Rightarrow \log a^{\frac{3}{2}} + \log b^{\frac{2}{3}} = 1$$

$$\Rightarrow \log\left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right) = 1$$

$$\Rightarrow \log\left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right) = \log 10$$

$$\Rightarrow a^{\frac{3}{2}} \times b^{\frac{2}{3}} = 10$$

$$\Rightarrow \left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right)^6 = 10^6$$

$$\Rightarrow a^9 \cdot b^4 = 10^6$$

Solution 2:

Given that

$x = 1 + \log 2 - \log 5, y = 2\log 3$ and $z = \log a - \log 5$

Consider

$x = 1 + \log 2 - \log 5$

$= \log 10 + \log 2 - \log 5$

$= \log(10 \times 2) - \log 5$

$= \log 20 - \log 5$

$= \log \frac{20}{5}$

$= \log 4 \ldots (1)$

We have

$y = 2\log 3$

$= \log 3^2$

$= \log 9 \ldots (2)$

Also we have

$z = \log a - \log 5$

$= \log \frac{a}{5} \ldots (3)$

Given that $x + y = 2z$

$\therefore$ Substitute the values of x,y and z

from (1),(2) and (3), we have

$\Rightarrow \log 4 + \log 9 = 2\log \frac{a}{5}$

$\Rightarrow \log 4 + \log 9 = \log\left(\frac{a}{5}\right)^2$

$\Rightarrow \log 4 + \log 9 = \log \frac{a^2}{25}$

$\Rightarrow \log(4 \times 9) = \log \frac{a^2}{25}$

$\Rightarrow \log 36 = \log \frac{a^2}{25}$

$\Rightarrow \frac{a^2}{25} = 36$

$\Rightarrow a^2 = 36 \times 25$

$\Rightarrow a^2 = 900$

$\Rightarrow a = 30$

Solution 3:

Given that

$x = \log 0.6,\ y = \log 1.25, z = \log 3 - 2\log 2$

Consider

$z = \log 3 - 2\log 2$

$= \log 3 - \log 2^2$

$= \log 3 - \log 4$

$= \log \frac{3}{4}$

$= \log 0.75 \ldots (1)$

(i)

$x + y - z = \log 0.6 + \log 1.25 - \log 0.75$

$= \log \frac{0.6 \times 1.25}{0.75}$

$= \log \frac{0.75}{0.75}$

$= \log 1$

$= 0 \ldots (2)$

(ii)

$5^{x+y-z} = 5^0 \ldots [\because x + y - z = 0 \text{ from } (2)]$

$= 1$

Solution 4:

Given that

$a^2 = \log x, b^3 = \log y$ and $3a^2 - 2b^3 = 6\log z$

Consider the equation,

$3a^2 - 2b^3 = 6\log z$

$\Rightarrow 3\log x - 2\log y = 6\log z$

$\Rightarrow \log x^3 - \log y^2 = \log z^6$

$\Rightarrow \log\left(\frac{x^3}{y^2}\right) = \log z^6$

$\Rightarrow \frac{x^3}{y^2} = z^6$

$\Rightarrow \frac{x^3}{z^6} = y^2$

$\Rightarrow y^2 = \frac{x^3}{z^6}$

$\Rightarrow y = \left(\frac{x^3}{z^6}\right)^{\frac{1}{2}}$

$\Rightarrow y = \left(\frac{x^{\frac{3}{2}}}{z^{\frac{6}{2}}}\right)$

$\Rightarrow y = \frac{x^{\frac{3}{2}}}{z^3}$

Solution 5:

$\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow \log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log ab)$

$\Rightarrow \log\left(\frac{a-b}{2}\right) = \log(ab)^{\frac{1}{2}}$

$\Rightarrow \left(\frac{a-b}{2}\right) = (ab)^{\frac{1}{2}}$

Squaring both sides we have,

$\left(\frac{a-b}{2}\right)^2 = ab$

$\Rightarrow \frac{(a-b)^2}{4} = ab$

$\Rightarrow (a-b)^2 = 4ab$

$\Rightarrow a^2 + b^2 - 2ab = 4ab$

$\Rightarrow a^2 + b^2 = 4ab + 2ab$

$\Rightarrow a^2 + b^2 = 6ab$

Solution 6:

Given that

$a^2 + b^2 = 23ab$

$\Rightarrow a^2 + b^2 + 2ab = 23ab + 2ab$

$\Rightarrow a^2 + b^2 + 2ab = 25ab$

$\Rightarrow (a + b)^2 = 25ab$

$\Rightarrow \frac{(a + b)^2}{25} = ab$

$\Rightarrow \left(\frac{a + b}{5}\right)^2 = ab$

$\Rightarrow \log\left(\frac{a + b}{5}\right)^2 = \log ab$

$\Rightarrow 2\log\left(\frac{a + b}{5}\right) = \log ab$

$\Rightarrow \log\left(\frac{a + b}{5}\right) = \frac{1}{2}(\log a + \log b)$

Solution 7:

Given that

$m = \log 20$ and $n = \log 25$

We also have

$2\log(x - 4) = 2m - n$

$\Rightarrow 2\log(x - 4) = 2\log 20 - \log 25$

$\Rightarrow \log(x - 4)^2 = \log 20^2 - \log 25$

$\Rightarrow \log(x - 4)^2 = \log 400 - \log 25$

$\Rightarrow \log(x - 4)^2 = \log \frac{400}{25}$

$\Rightarrow (x - 4)^2 = \frac{400}{25}$

$\Rightarrow (x - 4)^2 = 16$

$\Rightarrow x - 4 = 4$

$\Rightarrow x = 4 + 4$

$\Rightarrow x = 8$

Solution 8:

$\log xy = \log\left(\frac{x}{y}\right) + 2\log 2 = 2$

$\log xy = 2$

$\Rightarrow \log xy = 2\log 10$

$\Rightarrow \log xy = \log 10^2$

$\Rightarrow \log xy = \log 100$

$\therefore xy = 100....(1)$

Now consider the equation

$\log\left(\frac{x}{y}\right) + 2\log 2 = 2$

$\Rightarrow \log\left(\frac{x}{y}\right) + \log 2^2 = 2\log 10$

$\Rightarrow \log\left(\frac{x}{y}\right) + \log 4 = \log 10^2$

$\Rightarrow \log\left(\frac{x}{y}\right) + \log 4 = \log 100$

$\Rightarrow \left(\frac{x}{y}\right) \times 4 = 100$

$\Rightarrow 4x = 100y$

$\Rightarrow x = 25y$

$\Rightarrow xy = 25y \times y$

$\Rightarrow xy = 25y^2$

$\Rightarrow 100 = 25y^2.....[\text{from}(1)]$

$\Rightarrow y^2 = \frac{100}{25}$

$\Rightarrow y^2 = 4$

$\Rightarrow y = 2 \quad [\because y>0]$

From (1),

$xy=100$

$\Rightarrow x \times 2=100$

$\Rightarrow x=\frac{100}{2}$

$\Rightarrow x=50$

Thus the values of x and y are x=50 and y=2

Solution 9:

(i)

$$\log_x 625 = -4$$

$$\Rightarrow 625 = x^{-4} \text{ [Removing Logarithm]}$$

$$\Rightarrow 5^4 = \left(\frac{1}{x}\right)^4$$

$$\Rightarrow 5 = \frac{1}{x} \text{ [Powers are same, bases are equal]}$$

$$\Rightarrow x = \frac{1}{5}$$

(ii)

$$\log_x (5x - 6) = 2$$

$$\Rightarrow 5x - 6 = x^2 \quad \text{[Removing Logarithm]}$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow x(x-3) - 2(x-3) = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\therefore x = 2, 3$$

Solution 10:

Given that

$p = \log 20$ and $q = \log 25$

we also have

$$2\log(x+1) = 2p - q$$

$$\Rightarrow 2\log(x+1) = 2\log 20 - \log 25$$

$$\Rightarrow \log(x+1)^2 = \log 20^2 - \log 25$$

$$\Rightarrow \log(x+1)^2 = \log 400 - \log 25$$

$$\Rightarrow \log(x+1)^2 = \log\frac{400}{25}$$

$$\Rightarrow \log(x+1)^2 = \log 16$$

$$\Rightarrow \log(x+1)^2 = \log 4^2$$

$$\Rightarrow x + 1 = 4$$

$$\Rightarrow x = 4 - 1$$

$$\Rightarrow x = 3$$

Solution 11:

$$\log_2(x+y) = \frac{\log 25}{\log 0.2}$$

$$\Rightarrow \log_2(x+y) = \log_{0.2} 25$$

$$\Rightarrow \log_2(x+y) = \log_{\frac{2}{10}} 25$$

$$\Rightarrow \log_2(x+y) = \log_{5^{-1}} 5^2$$

$$\Rightarrow \log_2(x+y) = -2\log_5 5$$

$$\Rightarrow \log_2(x+y) = -2$$

$$\Rightarrow x+y = 2^{-2} \text{[Removing logarithm]}$$

$$\Rightarrow x+y = \frac{1}{4} \ldots\ldots (i)$$

$$\log_3(x-y) = \frac{\log 25}{\log 0.2}$$

$$\Rightarrow \log_3(x-y) = \log_{0.2} 25$$

$$\Rightarrow \log_3(x-y) = \log_{\frac{2}{10}} 25$$

$$\Rightarrow \log_3(x-y) = \log_{5^{-1}} 5^2$$

$$\Rightarrow \log_3(x-y) = -2\log_5 5$$

$$\Rightarrow \log_3(x-y) = -2$$

$$\Rightarrow x-y = 3^{-2} \text{[Removing logarithm]}$$

$$\Rightarrow x-y = \frac{1}{9} \ldots\ldots (ii)$$

Solving (i) & (ii), we get

$$x = \frac{13}{72}, y = \frac{5}{72}$$

Solution 12:

$$\frac{\log x}{\log y} = \frac{3}{2}$$

$$\Rightarrow 2\log x = 3\log y$$

$$\Rightarrow \log y = \frac{2\log x}{3} \ldots\ldots\ldots(i)$$

$$\log(xy) = 5$$

$$\Rightarrow \log x + \log y = 5$$

$$\Rightarrow \log x + \frac{2\log x}{3} = 5 \quad [\text{Substituting } (i)]$$

$$\Rightarrow \frac{3\log x + 2\log x}{3} = 5$$

$$\Rightarrow \frac{5\log x}{3} = 5$$

$$\Rightarrow \log x = 3$$

$$\Rightarrow x = 10^3$$

$$\therefore x = 1000$$

Substituting $x = 1000$

$$\log y = \frac{2 \times 3}{3}$$

$$\Rightarrow \log y = 2$$

$$\Rightarrow y = 10^2$$

$$\therefore y = 100$$

Solution 13:

(i) $\log_{10}x = 2a$

$\Rightarrow x = 10^{2a}$ [Removing logarithm from both sides]

$\Rightarrow x^{1/2} = 10^a$

$\Rightarrow 10^a = x^{1/2}$

(ii) $\log_{10}y = \frac{b}{2}$

$\Rightarrow y = 10^{b/2}$

$\Rightarrow y^4 = 10^{2b}$

$\Rightarrow 10y^4 = 10^{2b} \times 10$

$\Rightarrow 10^{2b+1} = 10y^4$

(iii)

We know $10^a = x^{1/2}$

$10^{b/2} = y$

$\Rightarrow 10^b = y^2$

$\log_{10}^p = 3a - 2b$

$\Rightarrow p = 10^{3a-2b}$

$\Rightarrow p = \left(10^3\right)^a \div \left(10^2\right)^b$

$\Rightarrow p = (10^a)^3 \div (10^b)^2$

Substituting 10^a & 10^b, we get

$\Rightarrow p = (x^{1/2})^3 \div (y^2)^2$

$\Rightarrow p = x^{3/2} \div y^4$

$\Rightarrow p = \frac{x^{3/2}}{y^4}$

Solution 14:

$\log_5(x + 1) - 1 = 1 + \log_5(x - 1)$

$\Rightarrow \log_5(x + 1) - \log_5(x - 1) = 2$

$\Rightarrow \log_5 \frac{(x+1)}{(x-1)} = 2$

$\Rightarrow \frac{(x + 1)}{(x - 1)} = 5^2$

$\Rightarrow \frac{(x + 1)}{(x - 1)} = 25$

$\Rightarrow x + 1 = 25(x - 1)$

$\Rightarrow x + 1 = 25x - 25$

$\Rightarrow 25x - x = 25 + 1$

$\Rightarrow 24x = 26$

$\Rightarrow x = \frac{26}{24} = \frac{13}{12}$

Solution 15:

$\log_x 49 - \log_x 7 + \log_x \frac{1}{343} = -2$

$\Rightarrow \log_x \frac{49}{7 \times 343} = -2$

$\Rightarrow \log_x \frac{1}{49} = -2$

$\Rightarrow -\log_x 49 = -2$

$\Rightarrow \log_x 49 = 2$

$\Rightarrow 49 = x^2$ [Removing logarithm]

$\therefore x = 7$

Solution 16:

Given $a^2 = \log x, b^3 = \log y$

Now $\frac{a^2}{2} - \frac{b^3}{3} = \log c$

$\Rightarrow \frac{\log x}{2} - \frac{\log y}{3} = \log c$

$\Rightarrow \frac{3\log x - 2\log y}{6} = \log c$

$\Rightarrow 3\log x - 2\log y = 6\log c$

$\Rightarrow \log x^3 - \log y^2 = 6\log c$

$\Rightarrow \log\left(\frac{x^3}{y^2}\right) = \log c^6$

$\Rightarrow \frac{x^3}{y^2} = c^6$

$\Rightarrow c = \sqrt[6]{\frac{x^3}{y^2}}$

Solution 17:

$$x - y - z$$
$$= \log_{10}12 - \log_4 2 \times \log_{10}9 - \log_{10}0.4$$
$$= \log_{10}(4 \times 3) - \log_4 2 \times \log_{10}9 - \log_{10}0.4$$
$$= \log_{10}4 + \log_{10}3 - \log_4 2 \times 2\log_{10}3 - \log_{10}\left(\frac{4}{10}\right)$$
$$= \log_{10}4 + \log_{10}3 - \frac{\log_{10}2}{2\log_{10}2} \times 2\log_{10}3 - \log_{10}4 + \log_{10}10$$
$$= \log_{10}4 + \log_{10}3 - \frac{2\log_{10}3}{2} - \log_{10}4 + 1$$
$$= 1$$

(ii) $13^{x-y-z} = 13^1 = 13$

Solution 18:

$$\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$$
$$\Rightarrow \log_x 15\sqrt{5} + \log_x 3\sqrt{5} = 2$$
$$\Rightarrow \log_x\left(15\sqrt{5} \times 3\sqrt{5}\right) = 2$$
$$\Rightarrow \log_x 225 = 2$$
$$\Rightarrow \log_x 15^2 = 2$$
$$\Rightarrow 2\log_x 15 = 2$$
$$\Rightarrow \log_x 15 = 1$$
$$\Rightarrow x = 15$$

Solution 19:

$$(i)\log_b a \times \log_c b \times \log_a c$$

$$= \frac{\log_{10} a}{\log_{10} b} \times \frac{\log_{10} b}{\log_{10} c} \times \frac{\log_{10} c}{\log_{10} a}$$

$$= 1$$

$$(ii)\log_3 8 \div \log_9 16$$

$$= \frac{\log_3 8}{\log_9 16}$$

$$= \frac{\log_{10} 8}{\log_{10} 3} \times \frac{\log_{10} 9}{\log_{10} 16}$$

$$= \frac{3\log_{10} 2}{\log_{10} 3} \times \frac{2\log_{10} 3}{4\log_{10} 2}$$

$$= \frac{3}{2}$$

$$(iii)\ \frac{\log_5 8}{\log_{25} 16 \times \log_{100} 10}$$

$$= \frac{\dfrac{\log_{10} 8}{\log_{10} 5}}{\dfrac{\log_{10} 16}{\log_{10} 25} \times \dfrac{\log_{10} 10}{\log_{10} 100}}$$

$$= \frac{\dfrac{\log_{10} 2^3}{\log_{10} 5}}{\dfrac{\log_{10} 2^4}{\log_{10} 5^2} \times \dfrac{\log_{10} 10}{\log_{10} 10^2}}$$

$$= \frac{\log_{10} 2^3}{\log_{10} 5} \times \frac{\log_{10} 5^2}{\log_{10} 2^4} \times \frac{\log_{10} 10^2}{\log_{10} 10}$$

$$= \frac{3\log_{10} 2}{\log_{10} 5} \times \frac{2\log_{10} 5}{4\log_{10} 2} \times \frac{2\log_{10} 10}{\log_{10} 10}$$

$$= 3$$

Solution 20:

$$\log_a m \div \log_{ab} m = \frac{\log_a m}{\log_{ab} m}$$

$$= \frac{\log_m ab}{\log_m a} \quad \left[\because \log_b a = \frac{1}{\log_a b}\right]$$

$$= \log_a ab \quad \left[\because \frac{\log_x a}{\log_x b} = \log_b a\right]$$

$$= \log_a a + \log_a b$$

$$= 1 + \log_a b$$