Chapter 9. Triangles [Congruency in Triangles]

Exercise 9(A)

Solution 1:

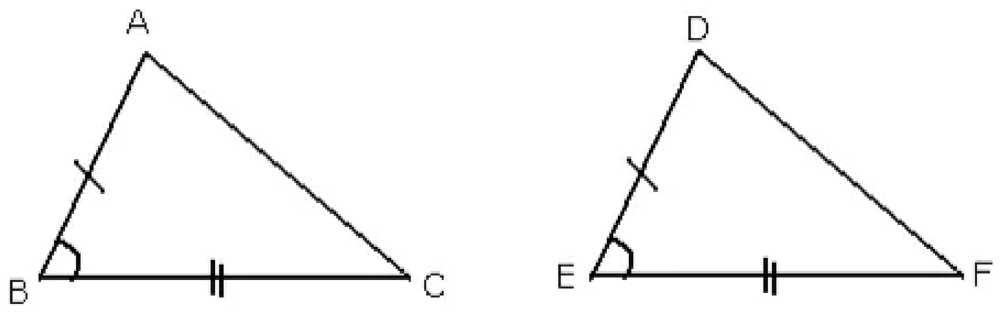
(a)

In ΔABC and ΔDEF

AB=DE [Given]

$\angle B = \angle E$ [Given]

BC=EF [Given]



By Side-Angle-Side criterion of congruency, the triangles ΔABC and ΔDEF are congruent to each other.

$\therefore \Delta ABC \cong \Delta DEF$

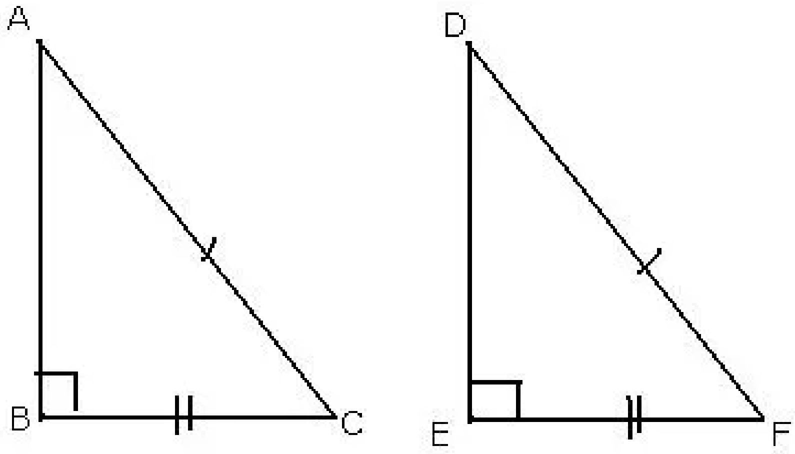
(b)

In ΔABC and ΔDEF

$\angle B = \angle E = 90^\circ$

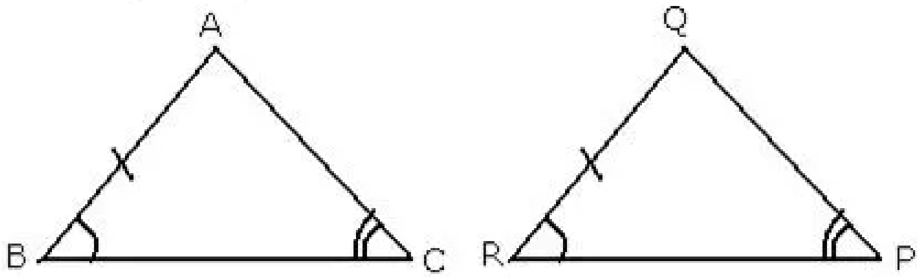
Hyp. AC=Hyp.DF

BC=EF



By Right Angle-Hypotenuse-Side criterion of congruency, the triangles
ΔABC and ΔDEF are congruent to each other.
$\therefore \Delta ABC \cong \Delta DEF$

(c)
In ΔABC and ΔQRP
$\angle B = \angle R$ [Given]
$\angle C = \angle P$ [Given]
$AB = QR$ [Given]



By Angle-Angle-Side criterion of congruency, the triangles
ΔABC and ΔQRP are congruent to each other.
$\therefore \Delta ABC \cong \Delta QRP$

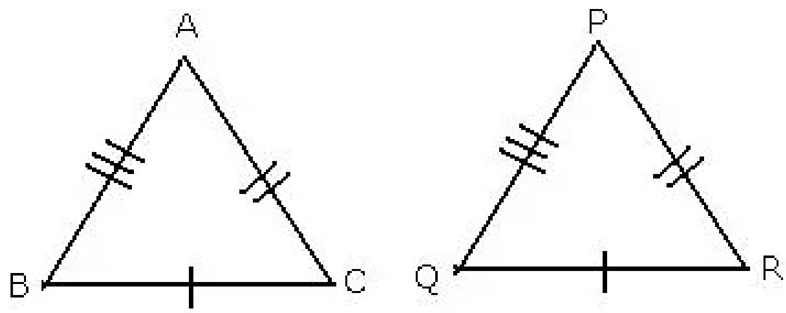
(d)

In ΔABC and ΔPQR

$AB=PQ$ [Given]

$AC=PR$ [Given]

$BC=QR$ [Given]



By Side-Side-Side criterion of congruency, the triangles ΔABC and ΔPQR are congruent to each other.

$\therefore \Delta ABC \cong \Delta PQR$

(e)

In ΔPQR

$\angle R=40°, \angle Q=50°$

$\angle P+\angle Q+\angle R=180°$ [Sum of all the angles in a triangle = 180°]

$\Rightarrow \angle P+50°+40°=180°$

$\Rightarrow \angle P+90°=180°$

$\Rightarrow \angle P=180° - 90°$

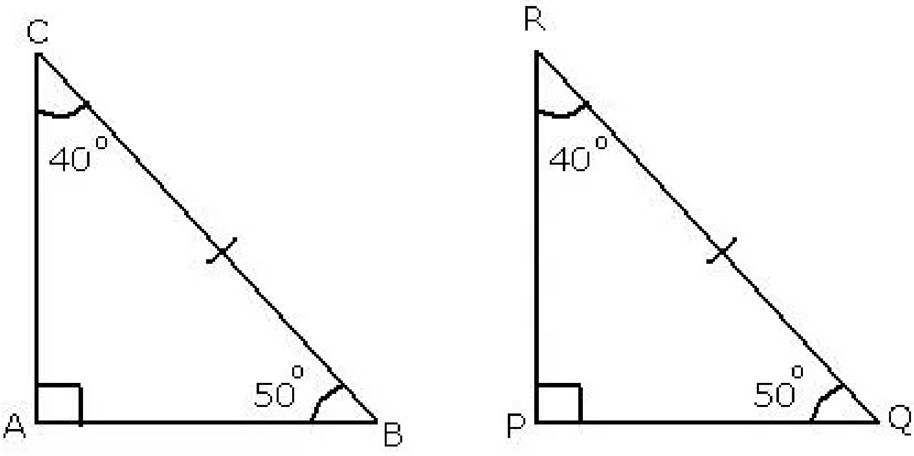
$\Rightarrow \angle P=90°$

In ΔABC and ΔPQR

$\angle A=\angle P$

$\angle C=\angle R$

$BC=QR$



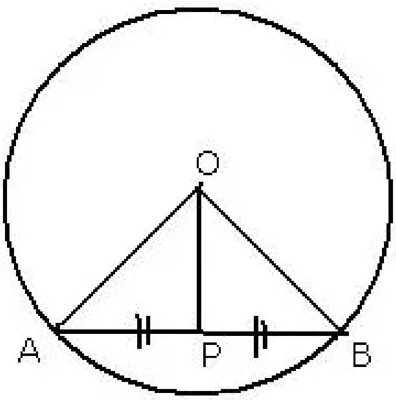
By Angle-Angle-Side criterion of congruency, the triangles ΔABC and ΔPQR are congruent to each other.

$\therefore \Delta ABC \cong \Delta PQR$

Solution 2:

Given: In the figure, O is centre of the circle, and AB is chord. P is a point on AB such that AP =PB.

We need to prove that, $OP \perp AB$



Construction: Join OA and OB

Proof:

In ΔOAP and ΔOBP

$OA = OB$ [radii of the same circle]

$OP = OP$ [common]

$AP = PB$ [given]

$\therefore$ By Side-Side-Side criterion of congruency,

$\Delta OAP \cong \Delta OBP$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle OPA = \angle OPB$ [by c.p.c.t]

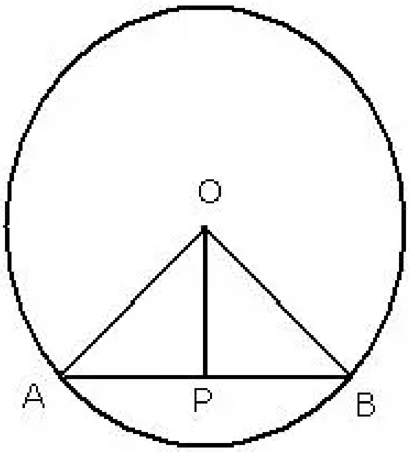
But $\angle OPA + \angle OPB = 180°$ [linear pair]

$\therefore \angle OPA = \angle OPB = 90°$

Hence $OP \perp AB$.

Solution 3:

Given: In the figure, O is centre of the circle,
and AB is chord. P is a point on AB such that AP =PB.
We need to prove that, $AP = BP$



Construction: Join OA and OB
Proof:
In right triangles ΔOAP and ΔOBP
Hypotenuse OA=OB [radii of the same circle]
Side $OP = OP$ [common]
∴ By Right angle-Hypotenuse-Side criterion of congruency,
ΔOAP ≅ ΔOBP
The corresponding parts of the congruent triangles are congruent.
∴ $AP=BP$ [by c.p.c.t]
Hence proved.

Solution 4:

Given: A ΔABC in which D is the mid-point of BC.

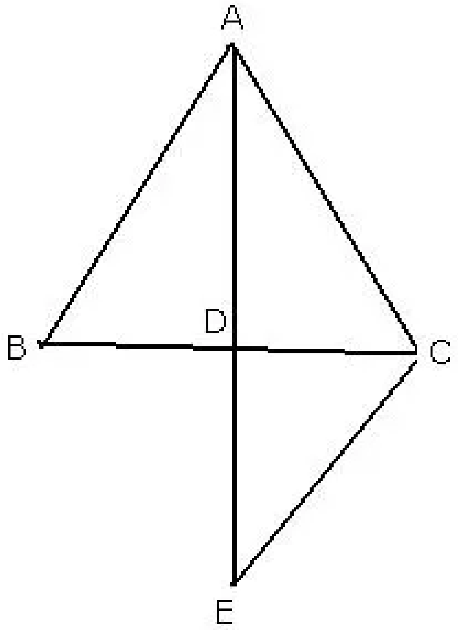
AD is produced to E so that DE=AD

We need to prove that

(i) $\Delta ABD \cong \Delta ECD$

(ii) $AB = EC$

(iii) $AB \parallel EC$



(i)In ΔABD and ΔECD

BD=DC	[D is the midpoint of BC]
$\angle ADB = \angle CDE$	[vertically opposite angles]
$AD = DE$	[Given]

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$\Delta ABD \cong \Delta ECD$

(ii)The corresponding parts of the congruent triangles are congruent.

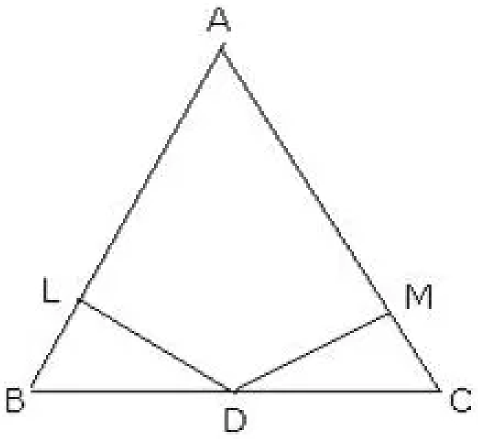
$\therefore$ AB=EC [c.p.c.t]

(iii)Also, $\angle DAB = \angle DEC$ [c.p.c.t]

$AB \parallel EC$ [$\angle DAB$ and $\angle DEC$ are alternate angles]

Solution 5:

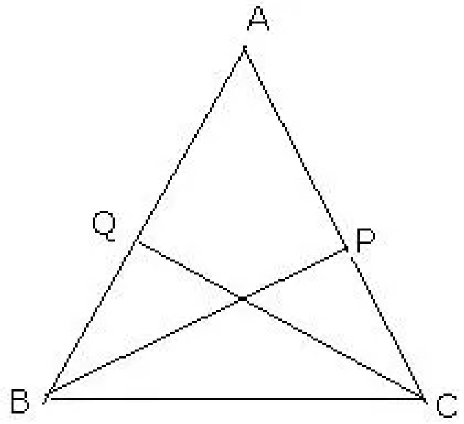
(i)Given: A ΔABC in which $\angle B = \angle C$.
DL is the perpendicular from D to AB
DM is the perpendicular from D to AC



We need to prove that
$DL = DM$
Proof:
In ΔDLB and ΔDMC

$\angle DLB = \angle DMC = 90°$	$[DL \perp AB$ and $DM \perp AC]$
$\angle B = \angle C$	[Given]
$BD = DC$	[D is the midpoint of BC]

$\therefore$ By Angle-Angle-Side criterion of congruence,
$\Delta DLB \cong \Delta DMC$
The corresponding parts of the congruent triangles are congruent.
$\therefore DL = DM$ [c.p.c.t]
(ii)Given: A ΔABC in which $\angle B = \angle C$.
BP is the perpendicular from D to AC
CQ is the perpendicular from C to AB



We need to prove that

$BP = CQ$

Proof:

In ΔBPC and ΔCQB

$\angle B = \angle C$ [Given]

$\angle BPC = \angle CQB = 90^\circ$ [$BP \perp AC$ and $CQ \perp AB$]

$BC = BC$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta BPC \cong \Delta CQB$

The corresponding parts of the congruent triangles are congruent.

$\therefore BP = CQ$ [c.p.c.t]

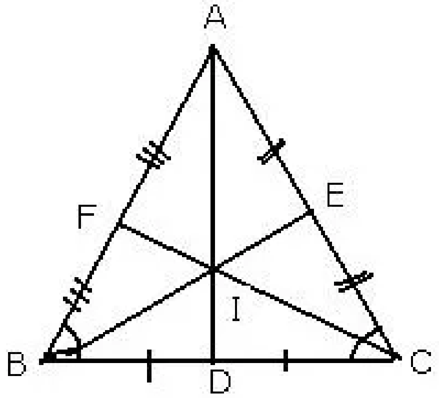
Solution 6:

Given: A ΔABC in which AD is the perpendicular bisector of BC

BE is the perpendicular bisector of CA

CF is the perpendicular bisector of AB

AD, BE and CF meet at I



We need to prove that

$IA = IB = IC$

Proof:

In ΔBID and ΔCID

$BD = DC$ [Given]

$\angle BDI = \angle CDI = 90°$ [AD is the perpendicular bisector of BC]

$BC = BC$ [Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta BID \cong \Delta CID$

The corresponding parts of the congruent triangles are congruent.

$\therefore IB = IC$ [c.p.c.t]

Similarly, in ΔCIE and ΔAIE

$CE = AE$ [Given]

$\angle CEI = \angle AEI = 90°$ [AD is the perpendicular bisector of BC]

$IE = IE$ [Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta CIE \cong \Delta AIE$

The corresponding parts of the congruent triangles are congruent.

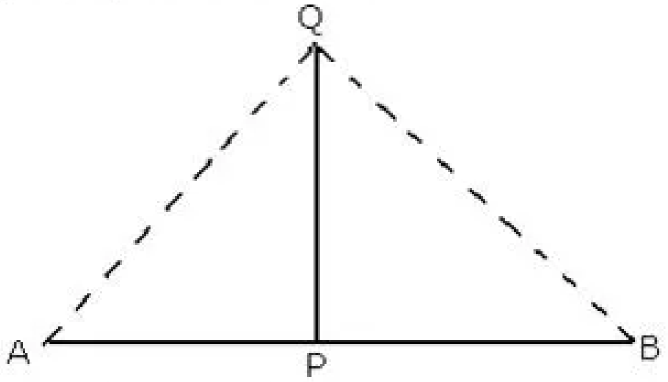
$\therefore IC = IA$ [c.p.c.t]

Thus, $IA = IB = IC$

Solution 7:

Given: A ΔABC in which AB is bisected at P

PQ is perpendicular to AB



We need to prove that

$QA = QB$

Proof:

In ΔAPQ and ΔBPQ

$AP = PB$ [P is the mid-point of AB]

$\angle APQ = \angle BPQ = 90^\circ$ [PQ is perpendicular to AB]

$PQ = PQ$ [Common]

$\therefore$ By Side-Angle-Side criterion of congruence,

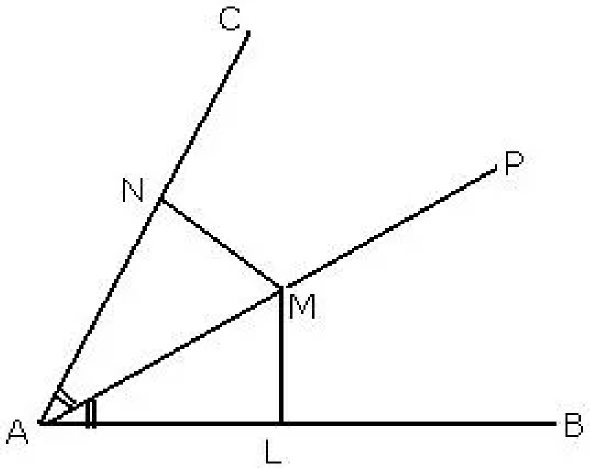
$\Delta APQ \cong \Delta BPQ$

The corresponding parts of the congruent triangles are congruent.

$\therefore QA = QB$ [c.p.c.t]

Solution 8:

From M, draw ML such that ML is perpendicular to AB and MN is perpendicular to AC



In ΔALM and ΔANM

∠LAM=∠MAN	[∵ AP is the bisector of ∠BAC]
∠ALM=∠ANM=90°	[∵ $ML \perp AB, MN \perp AC$]
$AM = AM$	[Common]

∴ By Angle-Angle-Side criterion of congruence,

ΔALM ≅ ΔANM

The corresponding parts of the congruent triangles are congruent.

∴ ML=MN [c.p.c.t]

Hence proved.

Solution 9:

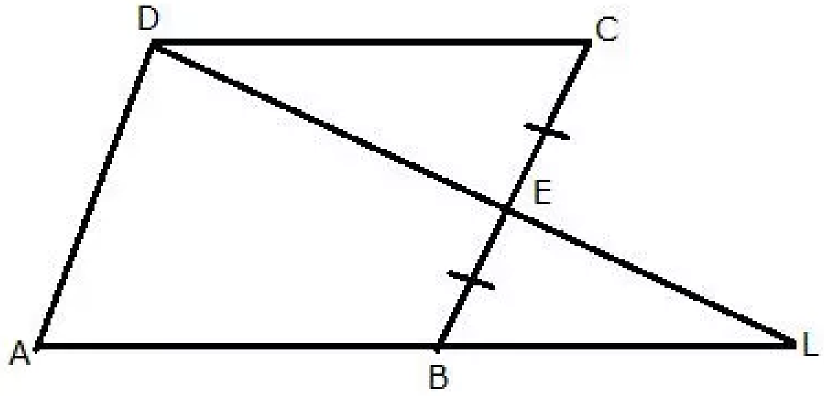
Given: ABCD is a parallelogram in which E is the mid-point of BC.

We need to prove that

(i) $\Delta DCE \cong \Delta LBE$

(ii) $AB = BL$

(iii) $AL = 2DC$



(i) In ΔDCE and ΔLBE

$\angle DCE = \angle EBL$ [DC || AB, alternate angles]

CE=EB [E is the midpoint of BC]

$\angle DEC = \angle LEB$ [vertically opposite angles]

$\therefore$ By Angle-Side-Angle criterion of congruence, we have,

$\Delta DCE \cong \Delta LBE$

The corresponding parts of the congruent triangles are congruent.

$\therefore$ DC=LB [c.p.c.t] ...(1)

(ii) DC=AB [opposite sides of a parallelogram]...(2)

From (1) and (2), AB=BL ...(3)

(iii) AL=AB+BL ...(4)

From (3) and (4), AL=AB+AB

$\Rightarrow$ AL=2AB

$\Rightarrow$ AL=2DC [from (2)]

Solution 10:

Given: In the figure AB=DB, AC=DC, $\angle ABD=58°$,

$\angle DBC=(2x-4)°$, $\angle ACB=(y+15)°$ and $\angle DCB=63°$

We need to find the values of x and y.

In ΔABC and ΔDBC

$AB = DB$ [given]

AC=DC [given]

$BC = BC$ [common]

$\therefore$ By Side-Side-Side criterion of congruence, we have,

$\Delta ABC \cong \Delta DBC$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle ACB=\angle DCB$ [c.p.c.t]

$\Rightarrow y° + 15°=63°$

$\Rightarrow y°=63° - 15°$

$\Rightarrow y°=48°$

and $\angle ABC=\angle DBC$[c.p.c.t]

But, $\angle DBC= (2x-4)°$

We have $\angle ABC+\angle DBC=\angle ABD$

$\Rightarrow (2x-4)° + (2x-4)° = 58°$

$\Rightarrow \quad 4x - 8° = 58°$

$\Rightarrow \quad 4x = 58° + 8°$

$\Rightarrow \quad 4x = 66°$

$\Rightarrow \quad x = \dfrac{66°}{4}$

$\Rightarrow \quad x = 16.5°$

Thus the values of x and y are:

$x = 16.5°$ and $y=48°$

Solution 11:

In the given figure AB||FD,

$\Rightarrow \angle ABC = \angle FDC$

Also AC||GE,

$\Rightarrow \angle ACB = \angle GEB$

Consider the two triangles ΔGBE and ΔFDC

$\angle B = \angle D$

$\angle C = \angle E$

Also given that

BD = CE

$\Rightarrow BD + DE = CE + DE$

$\Rightarrow BE = DC$

$\therefore$ By Angle – Side – Angle criterian of congruence

$\Delta GBE \cong \Delta FDC$

$$\therefore \frac{GB}{FD} = \frac{BE}{DC} = \frac{GE}{FC}$$

But BE = DC

$$\Rightarrow \frac{BE}{DC} = \frac{BE}{BE} = 1$$

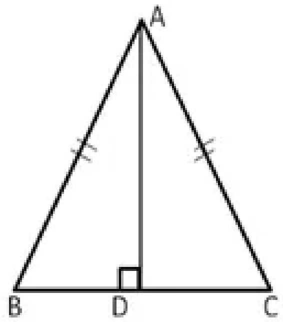
$$\therefore \frac{GB}{FD} = \frac{BE}{DC} = 1$$

$\Rightarrow GB = FD$

$$\therefore \frac{GE}{FC} = \frac{BE}{DC} = 1$$

$\Rightarrow GE = FC$

Solution 12:



In ΔADB and ΔADC,

AB = AC (Since ΔABC is an isosceles triangle)

AD = AD (common side)

$\angle ADB = \angle ADC$ (Since AD is the altitude so each is 90°)

$\Rightarrow \Delta ADB \cong \Delta ADC$ (RHS congruence criterion)

BD = DC (cpct)

$\Rightarrow$ AD is the median.

Solution 13:

In ΔDLB and ΔDMC,

$BL = CM$ (given)

$\angle DLB = \angle DMC$ (Both are 90°)

$\angle BDL = \angle CDM$ (vertically opposite angles)

$\therefore \Delta DLB \cong \Delta DMC$ (AAS congruence criterion)

$BD = CD$ (cpct)

Hence, AD is the median of ΔABC.

Solution 14:

(i) In ΔADB and ΔADC,

$\angle ADB = \angle ADC$ (Since AD is perpendicular to BC)

$AB = AC$ (given)

$AD = AD$ (common side)

$\therefore \Delta ADB \cong \Delta ADC$ (RHS congruence criterion)

$\Rightarrow BD = CD$ (cpct)

(ii) In ΔEFB and ΔEDB,

$\angle EFB = \angle EDB$ (both are 90°)

$EB = EB$ (common side)

$\angle FBE = \angle DBE$ (given)

$\therefore \Delta EFB \cong \Delta EDB$ (AAS congruence criterion)

$\Rightarrow EF = ED$ (cpct)

that is, $ED = EF$.

Solution 15:

In ΔABC and ΔEFD,

$AB \parallel EF \Rightarrow \angle ABC = \angle EFD$ (alternate angles)

$AC = ED$ (given)

$\angle ACB = \angle EDF$ (given)

$\therefore \Delta ABC \cong \Delta EFD$ (AAS congruence criterion)

$\Rightarrow AB = FE$ (cpct)

and $BC = DF$ (cpct)

$\Rightarrow BD + DC = CF + DC$ (B - D - C - F)

$\Rightarrow BD = CF$

Exercise 9(B)

Solution 1:

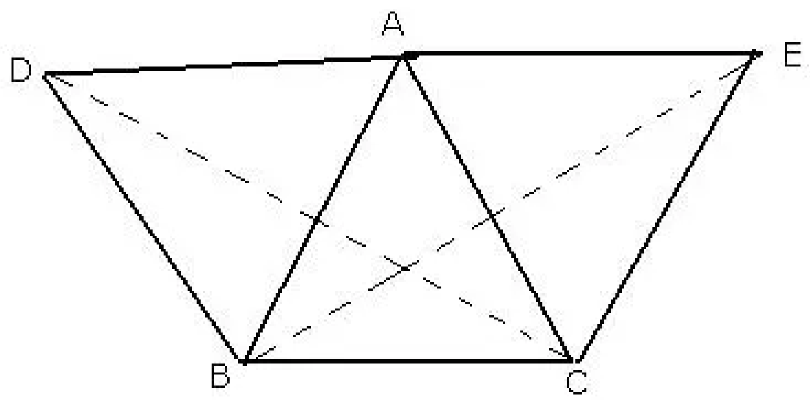
Given: ΔABD is an equilateral triangle

ΔACE is an equilateral triangle

We need to prove that

$(i) \angle CAD = \angle BAE$

$(ii) CD = BE$



Proof:

(i)

ΔABD is equilateral

$\therefore$ Each angle $= 60°$

$\Rightarrow \angle BAD = 60°$ ...(1)

Similarly,

ΔACE is equilateral

$\therefore$ Each angle $= 60°$

$\Rightarrow \angle CAE = 60°$ ...(2)

$\Rightarrow \angle BAD = \angle CAE$ [from (1) and (2)] ...(3)

Adding $\angle BAC$ to both sides, we have

$\angle BAD + \angle BAC = \angle CAE + \angle BAC$

$\Rightarrow \angle CAD = \angle BAE$ ...(4)

(ii)

In ΔCAD and ΔBAE

$AC = AE$ [ΔACE is equilateral]

$\angle CAD = \angle BAE$ [from (4)]

$AD = AB$ [ΔABD is equilateral]

$\therefore$ By Side-Angle-Side criterion of congruency,

$\Delta CAD \cong \Delta BAE$

The corresponding parts of the congruent triangles are congruent.

$\therefore CD = BE$ [by c.p.c.t]

Hence proved.

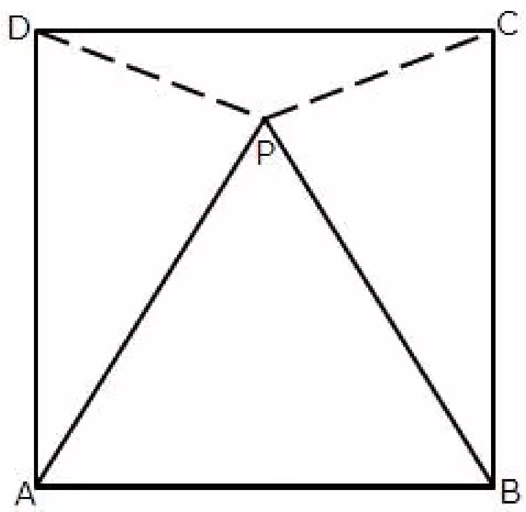
Solution 2:

Given: ABCD is a square and ΔAPB is an equilateral triangle.

We need to

(i)Prove that, $\Delta APD \cong \Delta BPC$

(ii) To find angles of ΔDPC



(a)

(i)Proof:

AP=PB=AB [ΔAPB is an equilateral triangle]

Also, we have,

$\angle PBA = \angle PAB = \angle APB = 60°$ ...(1)

Since ABCD is a square, we have

$\angle A = \angle B = \angle C = \angle D = 90°$ ...(2)

Since $\angle DAP = \angle A - \angle PAB$ ...(3)

$\Rightarrow \angle DAP = 90° - 60°$

$\Rightarrow \angle DAP = 30°$ [from (1) and (2)] ...(4)

Similarly $\angle CBP = \angle B - \angle PBA$

$\Rightarrow \angle CBP = 90° - 60°$

$\Rightarrow \angle CBP = 30°$ [from (1) and (2)] ...(5)

$\Rightarrow \angle DAP = \angle CBP$ [from (4) and (5)] ...(6)

In ΔAPD and ΔBPC

AD=BC [Sides of square ABCD]

$\angle DAP = \angle CBP$ [from (6)]

$AP = BP$ [Sides of equilateral ΔAPB]

∴ By Side-Angle-Side criterion of congruence, we have,

ΔAPD ≅ ΔBPC

(ii)

AP=PB=AB [ΔAPB is an equilateral triangle] ...(7)

$AB = BC = CD = DA$ [Sides of square ABCD] ...(8)

From (7) and (8), we have

AP=DA and PB=BC ...(9)

In ΔAPD,

AP=DA [from (9)]

$\therefore \angle ADP = \angle APD$ [Angles opposite to equal sides are equal] ...(10)

$\angle ADP + \angle APD + \angle DAP = 180°$ [Sum of angles of a triangle =180°]

$\Rightarrow \angle ADP + \angle ADP + 30° = 180°$ [from (3), $\angle DAP = 30°$ from (10), $\angle ADP = \angle APD$]

$\Rightarrow \angle ADP + \angle ADP = 180° - 30°$

$\Rightarrow 2\angle ADP = 150°$

$\Rightarrow \angle ADP = \dfrac{150°}{2}$

$\Rightarrow \angle ADP = 75°$

We have $\angle PDC = \angle D - \angle ADP$

$\Rightarrow \angle PDC = 90° - 75°$

$\Rightarrow \angle PDC = 15°$ ...(11)

In ΔBPC,

$PB = BC$ [from (9)]

$\therefore \angle PCB = \angle BPC$ $\begin{bmatrix}\text{Angles opposite to}\\ \text{equal sides are equal}\end{bmatrix}$ ...(12)

$\angle PCB + \angle BPC + \angle CBP = 180^\circ$ $\begin{bmatrix}\text{Sum of angles of}\\ \text{a triangle} = 180^\circ\end{bmatrix}$

$\Rightarrow \angle PCB + \angle PCB + 30^\circ = 180^\circ$ $\begin{bmatrix}\text{from (5), } \angle CBP = 30^\circ\\ \text{from (12), } \angle PCB = \angle BPC\end{bmatrix}$

$\Rightarrow 2\angle PCB = 180^\circ - 30^\circ$

$\Rightarrow \angle PCB = \dfrac{150^\circ}{2}$

$\Rightarrow \angle PCB = 75^\circ$

We have $\angle PCD = \angle C - \angle PCB$

$\Rightarrow \angle PCD = 90^\circ - 75^\circ$

$\Rightarrow \angle PCD = 15^\circ$ ...(13)

In ΔDPC,

$\angle PDC = 15^\circ$

$\angle PCD = 15^\circ$

$\angle PCD + \angle PDC + \angle DPC = 180^\circ$ $\begin{bmatrix}\text{Sum of angles of}\\ \text{a triangle} = 180^\circ\end{bmatrix}$

$\Rightarrow 15^\circ + 15^\circ + \angle DPC = 180^\circ$

$\Rightarrow \angle DPC = 180^\circ - 30^\circ$

$\Rightarrow \angle DPC = 150^\circ$

$\therefore$ Angles of ΔDPC, are: $15^\circ, 150^\circ, 15^\circ$

(b)

(i)Proof: In ΔAPB

$AP = PB = AB$ [ΔAPB is an equilateral triangle]

Also, we have,

$\angle PBA = \angle PAB = \angle APB = 60^\circ$ ...(1)

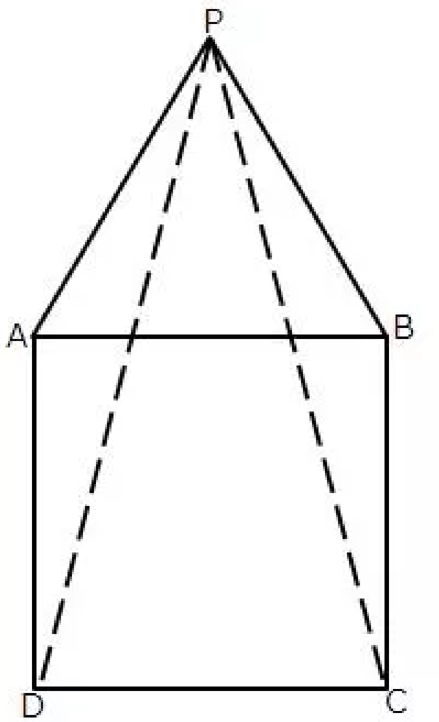
Since ABCD is a square, we have

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ ...(2)

Since $\angle DAP = \angle A + \angle PAB$ ...(3)

$\Rightarrow \angle DAP = 90^\circ + 60^\circ$

$\Rightarrow \angle DAP = 150^\circ$ [from (1) and (2)] ...(4)



Similarly $\angle CBP = \angle B + \angle PBA$

$\Rightarrow \angle CBP = 90^\circ + 60^\circ$

$\Rightarrow \angle CBP = 150^\circ$ [from (1) and (2)] ...(5)

$\Rightarrow \angle DAP = \angle CBP$ [from (4) and (5)] ...(6)

In ΔAPD and ΔBPC

$AD = BC$ [Sides of square ABCD]

$\angle DAP = \angle CBP$ [from (6)]

$AP = BP$ [Sides of equilateral ΔAPB]

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$\Delta APD \cong \Delta BPC$

(ii)

$AP = PB = AB$ [ΔAPB is an equilateral triangle] ...(7)

$AB = BC = CD = DA$ [Sides of square ABCD] ...(8)

From (7) and (8), we have

AP=DA and PB=BC ...(9)

In ΔAPD,

AP=DA [from (9)]

$\therefore \angle ADP = \angle APD$ $\begin{bmatrix}\text{Angles opposite to} \\ \text{equal sides are equal}\end{bmatrix}$ ...(10)

$\angle ADP + \angle APD + \angle DAP = 180^\circ$ $\begin{bmatrix}\text{Sum of angles of} \\ \text{a triangle } =180^\circ\end{bmatrix}$

$\Rightarrow \angle ADP + \angle ADP + 150^\circ = 180^\circ \begin{bmatrix}\text{from (3), } \angle DAP = 150^\circ \\ \text{from (10), } \angle ADP = \angle APD\end{bmatrix}$

$\Rightarrow \angle ADP + \angle ADP = 180^\circ - 150^\circ$

$\Rightarrow 2\angle ADP = 30^\circ$

$\Rightarrow \angle ADP = \dfrac{30^\circ}{2}$

$\Rightarrow \angle ADP = 15^\circ$

We have $\angle$PDC=$\angle$D $- \angle ADP$

$\Rightarrow \angle$PDC=$90^\circ - 15^\circ$

$\Rightarrow \angle$PDC=75° ...(11)

In ΔBPC,

PB=BC [from (9)]

$\therefore \angle PCB = \angle BPC$ $\begin{bmatrix}\text{Angles opposite to} \\ \text{equal sides are equal}\end{bmatrix}$ ...(12)

$\angle PCB + \angle BPC + \angle CBP = 180^\circ$ $\begin{bmatrix}\text{Sum of angles of} \\ \text{a triangle } =180^\circ\end{bmatrix}$

$\Rightarrow \angle PCB + \angle PCB + 150^\circ = 180^\circ \begin{bmatrix}\text{from (5), } \angle CBP = 150^\circ \\ \text{from (12), } \angle PCB = \angle BPC\end{bmatrix}$

$\Rightarrow 2\angle PCB = 180^\circ - 150^\circ$

$\Rightarrow \angle PCB = \dfrac{30^\circ}{2}$

$\Rightarrow \angle PCB = 15^\circ$

We have $\angle$PCD=$\angle$C $- \angle PCB$

$\Rightarrow \angle$PCD=$90^\circ - 15^\circ$

$\Rightarrow \angle PCD = 75°$ ...(13)

In ΔDPC,

$\angle PDC = 75°$

$\angle PCD = 75°$

$\angle PCD + \angle PDC + \angle DPC = 180°$ [Sum of angles of a triangle =180°]

$\Rightarrow 75° + 75° + \angle DPC = 180°$

$\Rightarrow \angle DPC = 180° - 150°$

$\Rightarrow \angle DPC = 30°$

$\therefore$ Angles of ΔDPC, are: 75°,30°,75°

Solution 3:

Given: A ΔABC is right angled at B.

ABPQ and ACRS are squares

We need to prove that

(i) $\Delta ACQ \cong \Delta ASB$

(ii) $CQ = BS$

Proof:

(i)

$\angle QAB = 90°$ [$ABPQ$ is a square] ...(1)

$\angle SAC = 90°$ [$ACRS$ is a square] ...(2)

From (1) and (2), we have

$\angle QAB = \angle SAC$ ...(3)

Adding $\angle BAC$ to both sides of (3), we have

$\angle QAB + \angle BAC = \angle SAC + \angle BAC$

$\Rightarrow \angle QAC = \angle SAB$ ...(4)

In ΔACQ and ΔASB,

$QA = QB$ [sides of a square ABPQ]

$\angle QAC = \angle SAB$ [from (4)]

$AC = AS$ [sides of a square ACRS]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta ACQ \cong \Delta ASB$

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore CQ = BS$ [c.p.c.t]

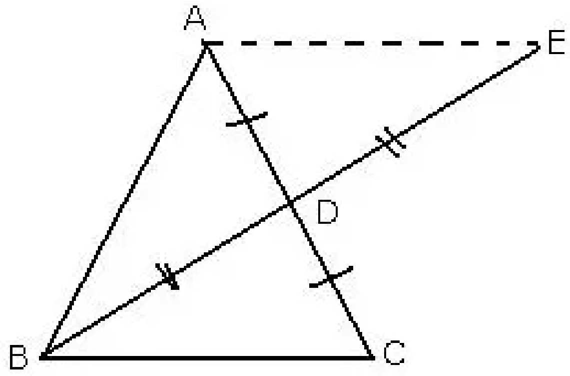
Solution 4:

Given: A ΔABC in which BD is the median to AC.

BD is produced to E such that BD=DE.

We need to prove that AE || BC.

Construction: Join AE



Proof:

AD=DC [*BD* is median to AC] ...(1)

In ΔBDC and ΔADE

$BD = DE$ [Given]

$\angle BDC = \angle ADE = 90°$ [vertically opposite angles]

$AD = DC$ [from (1)]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta BDC \cong \Delta ADE$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle EAD = \angle BCD$ [c.p.c.t]

But these are alternate angles and AC is the transversal

Thus, AE || BC

Solution 5:

Given: A ΔPQR in which QX is the bisector of $\angle Q$ and RX is the bisector of $\angle R$.

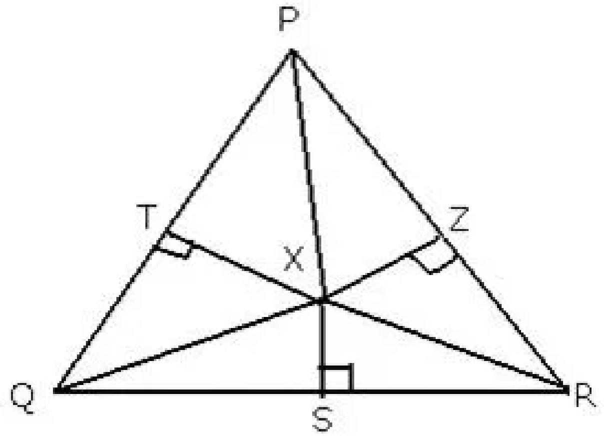
$XS \perp QR$ and $XT \perp PQ$.

We need to prove that

(i) $\Delta XTQ \cong \Delta XSQ$

(ii) PX bisects $\angle P$

Construction: Draw $XZ \perp PR$ and join PX.



Proof:

(i)

In ΔXTQ and ΔXSQ

$\angle QTX = \angle QSX = 90°$ [$XS \perp QR$ and $XT \perp PQ$]

$\angle TQX = \angle SQX$ [QX is bisector of $\angle Q$]

$QX = QX$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta XTQ \cong \Delta XSQ$ ...(1)

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore XT = XS$ [c.p.c.t]

In ΔXSR and ΔXZR

$\angle XSR = \angle XZR = 90°$ [$XS \perp QR$ and $\angle XSR = 90°$]

$\angle SRX = \angle ZRX$ [RX is bisector of $\angle R$]

$RX = RX$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\Delta XSR \cong \Delta XZR$

The corresponding parts of the congruent triangles are congruent.

$\therefore XS = XZ$ [c.p.c.t] ...(2)

From (1) and (2)

$XT = XZ$ ...(3)

In ΔXTP and ΔXZP

$\angle XTP = \angle XZP = 90°$ [Given]

Hyp. XP=Hyp.XP [Common]

$XT = XZ$ [from (3)]

$\therefore$ By Right angle-Hypotenuse-Side criterion of congruence,

$\Delta XTP \cong \Delta XZP$

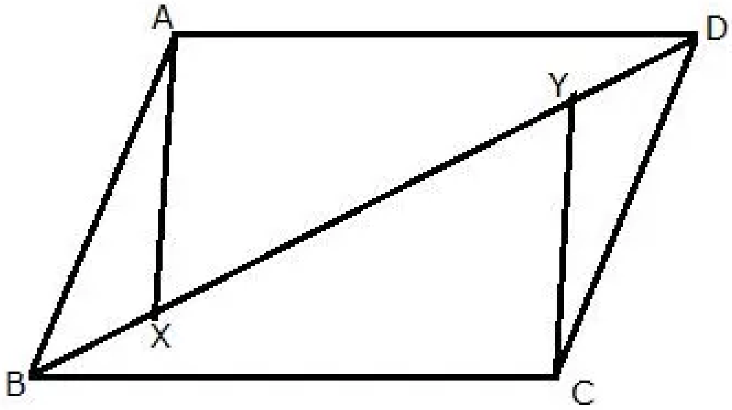
The corresponding parts of the congruent triangles are congruent.

$\therefore \angle XPT = \angle XPZ$ [c.p.c.t]

$\therefore PX$ bisects $\angle P$

Solution 6:

ABCD is a parallelogram in which ∠A and ∠C are obtuse.



Points X and Y are taken on the diagonal BD such that ∠XAD=∠YCB=90°.

We need to prove that XA=YC

Proof:

In ΔXAD and ΔYCB

$\angle XAD=\angle YCB=90°$ [Given]

AD=BC [Opposite sides of a parallelogram]

$\angle ADX=\angle CBY$ [Alternate angles]

∴ By Angle-Side-Angle criterion of congruence,

$\Delta XAD \cong \Delta YCB$

The corresponding parts of the congruent triangles are congruent.

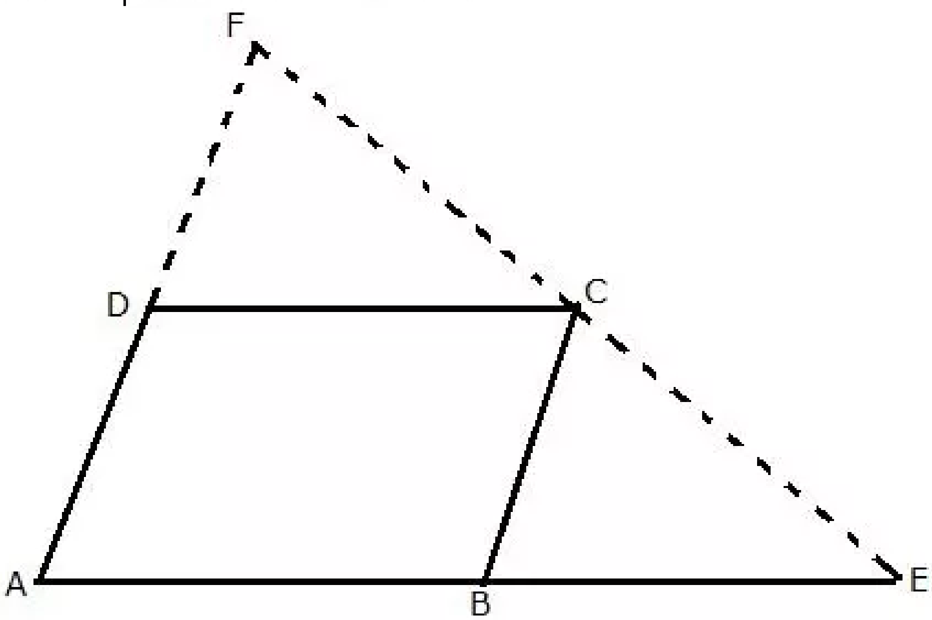
∴ $XA=YC$ [c.p.c.t]

Hence proved.

Solution 7:

ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively, such that AB = BE and AD = DF.

We need to prove that $\Delta BEC \cong \Delta DCF$



Proof:

AB=DC [Opposite sides of a parallelogram] ...(1)

$AB = BE$ [Given] ...(2)

From (1) and (2), we have

$BE = DC$ ...(3)

AD=BC [Opposite sides of a parallelogram] ...(4)

$AD = DF$ [Given] ...(5)

From (4) and (5), we have

BC=DF ...(6)

Since AD || BC, the corresponding angles are equal.

$\therefore \angle DAB = \angle CBE$ ...(7)

Since AB || DC, the corresponding angles are equal.

$\therefore \angle DAB = \angle FDC$ ...(8)

From (7) and (8), we have

$\angle CBE = \angle FDC$ ...(9)

In ΔBEC and ΔDCF

In ΔBEC and ΔDCF

$BE = DC$ [from (3)]

$\angle CBE = \angle FDC$ [from (9)]

$BC = DF$ [from (6)]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\Delta BEC \cong \Delta DCF$

Hence proved.

Solution 8:

Since, BC=QR, we have

$BD = QS$ and $DC = SR$ $\left[\begin{array}{l}\text{D is the midpoint of BC and} \\ \text{S is the midpoint of QR}\end{array}\right]$

In ΔABD and ΔPQS

$AB = PQ$ ...(1)

$AD = PS$ ...(2)

$BD = QS$ ...(3)

Thus, by Side-Side-Side criterion of congruence,

we have $\Delta ABD \cong \Delta PQS$

Similarly, in ΔADC and ΔPSR

$AD = PS$ ...(4)

$AC = PR$ ...(5)

$DC = SR$ ...(6)

Thus, by Side-Side-Side criterion of congruence,

we have $\Delta ADC \cong \Delta PSR$

We have

$BC = BD + DC$ [D is the midpoint of BC]

$= QS + SR$ [from (3) and (6)]

$= QR$ [S is the midpoint of QR] ...(7)

Now consider the triangles ΔABC and ΔPQR

$AB = PQ$ [from (1)]

$BC = QR$ [from (7)]

$AC = PR$ [from (7)]

$\therefore$ By Side-Side-Side criterion of congruence, we have $\Delta ABC \cong \Delta PQR$

Hence proved.

Solution 9:

In the figure, AP and BQ are equal and parallel to each other. $\therefore$ AP=BQ and AP $\parallel$ BQ.

We need to prove that

(i) $\Delta AOP \cong \Delta BOQ$

(ii) AB and PQ bisect each other

(i) $\because$ AP $\parallel$ BQ

$\therefore \angle APO = \angle BQO$ [Alternate angles] ...(1)

and $\angle PAO = \angle QBO$ [Alternate angles] ...(2)

Now in ΔAOP and ΔBOQ,

$\angle APO = \angle BQO$ [from (1)]

$AP = BQ$ [given]

$\angle PAO = \angle QBO$ [from (2)]

$\therefore$ By Angle-Side-Angle criterion of congruence, we have

$\Delta AOP \cong \Delta BOQ$

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore OP = OQ$ [c.p.c.t]

$OA = OB$ [c.p.c.t]

Hence AB and PQ bisect each other.

Solution 10:

Given:

In the figure, OA=OC, AB=BC

We need to prove that,

(i) $\angle AOB = 90°$

(ii) $\Delta AOD \cong \Delta COD$

(iii) $AD = CD$

(i) In ΔABO and ΔCBO,

$AB=BC$ [given]

$AO=CO$ [given]

$OB=OB$ [common]

$\therefore$ By Side-Side-Side criterion of congruence, we have

$\Delta ABO \cong \Delta CBO$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle ABO=\angle CBO$ [c.p.c.t]

$\Rightarrow \angle ABD=\angle CBD$

and $\angle AOB=\angle COB$ [c.p.c.t]

We have

$\angle AOB+\angle COB=180°$ [Linear pair]

$\Rightarrow \angle AOB=\angle COB=90°$ and $AC \perp BD$

(ii) In ΔAOD and ΔCOD,

$OD=OD$ [common]

$\angle AOD=\angle COD$ [each=90°]

$AO=CO$ [given]

$\therefore$ By Side-Angle-Side criterion of congruence, we have

$\Delta AOD \cong \Delta COD$

(iii)

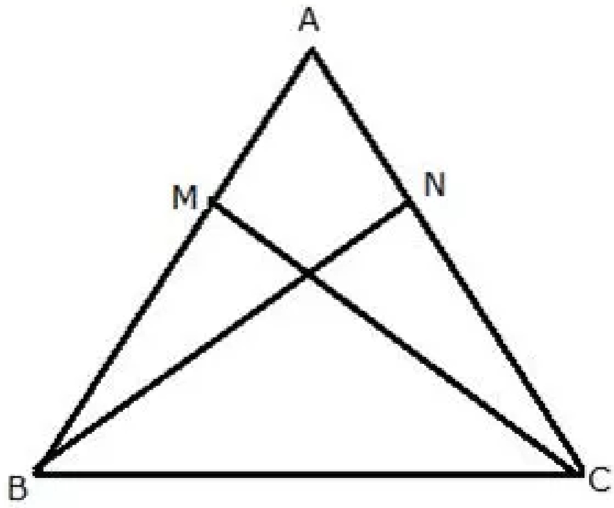
The corresponding parts of the congruent triangles are congruent.

$\therefore AD=CD$ [c.p.c.t]

Hence proved.

Solution 11:

In ΔABC, AB=AC. M and N are points on AB and AC such tht BM=CN.
BN and CM are joined.



(i) In ΔAMC and ΔANB

$AB = AC$ [Given] ...(1)

$BM = CN$ [Given] ...(2)

Subtracting (2) from (1), we have

$AB - BM = AC - CN$

$\Rightarrow AM = AN$ ...(3)

(ii) Consider the triangles ΔAMC and ΔANB

$AC = AB$ [given]

$\angle A = \angle A$ [common]

$AM = AN$ [from (3)]

$\therefore$ By Side-Angle-Side criterion of congruence, we have $\Delta AMC \cong \Delta ANB$

(iii)

The corresponding parts of the congruent triangles are congruent.

$\therefore$ $CM = BN$ [c.p.c.t] ...(4)

(iv)Consider the triangles ΔBMC and ΔCNB

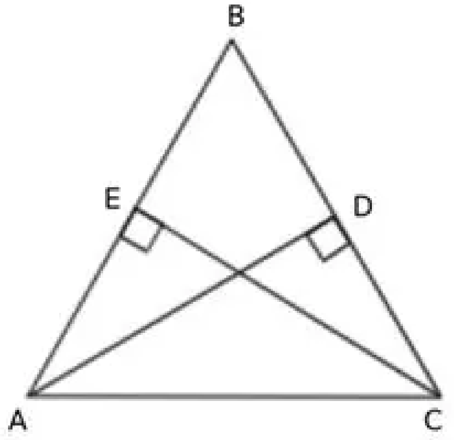
$BM = CN$ [given]

$BC = BC$ [common]

$CM = BN$ [from (4)]

$\therefore$ By Side-Side-Side criterion of congruence, we have $\Delta BMC \cong \Delta CNB$

Solution 12:



In ΔABD and ΔCBE,

AB = BC (given)

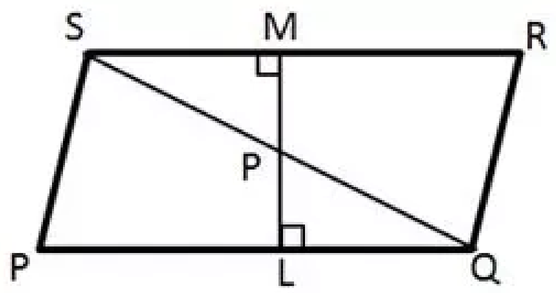
$\angle ADB = \angle CEB = 90^\circ$

$\angle B = \angle B$ (common angle)

$\therefore$ $\Delta ABD \cong \Delta CBE$ (by SAS congruence)

$\Rightarrow$ AD = CE (cpct)

Solution 13:



Given : PL = RM

To prove: SP = PQ and MP = PL

Proof :

Since SR and PQ are opposite sides of a parallelogram,

PQ = SR ....(1)

Also, PL = RM ....(2)

Subtracting (2) from (1),

PQ − PL = SR − RM

⇒ LQ = SM ....(3)

Now, in ΔSMP and ΔQLP,

∠MSP = ∠PQL (alternate interior angles)

∠SMP = ∠PLQ (alternate interior angles)

SM = LQ [From (3)]

∴ ΔSMP ≅ ΔQLP (by ASA congruence)

⇒ SP = PQ and MP = PL (cpct)

⇒ LM and QS bisect each other.

Solution 14:

ΔABC is an equilateral triangle.

So, each of its angles equals 60°.

QP is parallel to AC,

$\Rightarrow \angle PQB = \angle RAQ = 60^\circ$

In ΔQBP,

$\angle PBQ = \angle BQP = 60^\circ$

So, $\angle PBQ + \angle BQP + \angle BPQ = 180^\circ$ (angle sum property)

$\Rightarrow 60^\circ + 60^\circ + \angle BPQ = 180^\circ$

$\Rightarrow \angle BPQ = 60^\circ$

So, ΔBPQ is an equilateral triangle.

$\Rightarrow QP = BP$

$\Rightarrow QP = CR$ ...(i)

Now, $\angle QPM + \angle BPQ = 180^\circ$ (linear pair)

$\Rightarrow \angle QPM + 60^\circ = 180^\circ$

$\Rightarrow \angle QPM = 120^\circ$

Also, $\angle RCM + \angle ACB = 180^\circ$ (linear pair)

$\Rightarrow \angle RCM + 60^\circ = 180^\circ$

$\Rightarrow \angle RCM = 120^\circ$

In ΔRCM and ΔQMP,

$\angle RCM = \angle QPM$ (each is 120°)

$\angle RMC = \angle QMP$ (vertically opposite angles)

$QP = CR$ (from (i))

$\Rightarrow \Delta RCM \cong \Delta QMP$ (AAS congruence criterion)

So, $CM = PM$

$\Rightarrow$ QR bisects PC.