Equation Of A Straight Line

EXERCISE - 12.1

Q1. Find the slope of a line whose inclination is (i) 45° (ii) 30°

Sol. (i) $\tan 45^\circ = 1$ (ii) $\tan 30^\circ = \frac{1}{\sqrt{3}}$

Q2. Find the inclination of a line whose gradient is
(i) 1 (ii) $\sqrt{3}$ (iii) $\frac{1}{\sqrt{3}}$

Sol. (i) $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

(ii) $\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

(iii) $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

Q3. Find the equation of a straight line parallel to x-axis which is at a distance (i) 2 units above it (ii) 3 units below it.

Sol. (i) A line which is parallel to x-axis is $y = a \Rightarrow y = 2 \Rightarrow y - 2 = 0$

(ii) A line which is parallel to x-axis is $y = a \Rightarrow y = -3 \Rightarrow y + 3 = 0$

Q4. Find the equation of a st line parallel to y-axis and passing through the point $(-3, 5)$

Sol. The equation of the line parallel to y-axis passing through $(-3, 5)$ is $x = -3 \Rightarrow x + 3 = 0$

Q5. Find the equation of a st. line parallel to y-axis which is at a distance (i) 3 units to the right (ii) 2 units to the left.

(i) The equation of line parallel to y-axis is at a distance of 3 units to the right is $x = 3 \Rightarrow x - 3 = 0$

(ii) The equation of line parallel to y-axis at a distance of 2 units to the left is $x = -2 \Rightarrow x + 2 = 0$.

Q6. Find the equation of a line whose

(i) slope $= 3$, y- intercept $= -5$

(ii) slope $= \frac{-2}{7}$, y- intercept $= 3$

(iii) Gradient $= \sqrt{3}$, Y- intercept $= \frac{4}{3}$

(iv) Inclination $= 30^\circ$, y-intercept $= 2$.

Sol. Equation of a line whose Slope and y-intercept is given by $y = mx + c$ where m is the slope and c is the y-intercept.

(i) $y = mx + c \Rightarrow y = 3x + (-5) \Rightarrow y = 3x - 5$

(ii) $y = \frac{-2}{7}x + 3 \Rightarrow 7y = -2x + 21 \Rightarrow 2x + 7y - 21 = 0$

(iii) $y = \sqrt{3}x + \left(\frac{-4}{3}\right) \Rightarrow y = \sqrt{3}x - \frac{4}{3} \Rightarrow 3y = 3\sqrt{3}x - 4 \Rightarrow 3\sqrt{3}x - 3y - 4 = 0$

(iv) Inclination $= 30^\circ$

slope $= \tan 30^\circ = \frac{1}{\sqrt{3}}$

$y = mx + c \Rightarrow y = \frac{1}{\sqrt{3}}x + 2 \Rightarrow \sqrt{3}y = x + 2\sqrt{3} \Rightarrow x - \sqrt{3}y + 2\sqrt{3} = 0$

Q7. Find the slope and y-intercept of the following lines:

(i) $x - 2y - 1 = 0$ (ii) $4x - 5y - 9 = 0$ (iii) $3x + 5y + 7 = 0$

(iv) $\frac{x}{3} + \frac{y}{4} = 1$ (v) $y - 3 = 0$ (vi) $x - 3 = 0$

Sol. We know that in the equation, $y = mx + c$, m is the slope and c is the y-intercept.

(i) $x - 2y - 1 = 0 \Rightarrow y = \frac{1}{2}x - \frac{1}{2}$

Here slope $= \frac{1}{2}$, y-intercept $= \frac{-1}{2}$

(ii) $4x - 5y - 9 = 0 \Rightarrow y = \frac{4}{5}x - \frac{9}{5}$

Here slope $= 4/5$, y-intercept $= -9/5$

(iii) $3x + 5y + 7 = 0 \Rightarrow y = \frac{-3}{5}x - \frac{7}{5}$

Here slope $= \frac{-3}{5}$, y-intercept $= \frac{-7}{5}$

(iv) $\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12 \Rightarrow y = \frac{-4}{3}x + \frac{12}{3} = \frac{-4}{3}x + 4$

Here slope $= \frac{-4}{3}$, y-intercept $= 4$

(v) $y - 3 = 0 \Rightarrow y = 3$

Here slope $= 0$, y-intercept $= 3$

(vi) $x - 3 = 0$

Here in this equation, slope can't be defined and doesn't meet y-axis.

Q8. The equation of the line PQ is $3y - 3x + 7 = 0$

(i) writedown the slope of the line PQ.

(ii) calculate the angle that the line PQ makes with the +ve direction of x-axis.

Sol. Equation of Line PQ is $3y - 3x + 7 = 0$

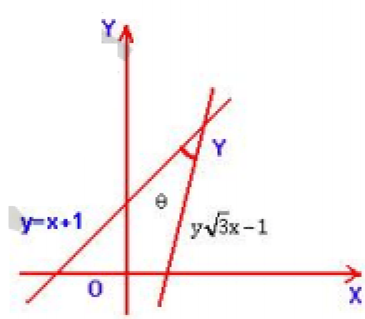
$$\Rightarrow y = \frac{3x}{3} - \frac{7}{3} \Rightarrow y = x - \frac{7}{3}$$

(i) Here slope $= 1$

(ii) Angle which makes PQ with x-axis is θ

But $\tan\theta = 1 \Rightarrow \theta = 45^\circ$.

Q9. The given fig. represents the lines $y = x + 1$ and $y = \sqrt{3}x - 1$. Writedown the angles which the lines make with the +ve direction of the x-axis. Hence determine θ.



Sol. $y = x + 1$ comparing it with $y = mx + c$.

slope $m = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^\circ$

$y = \sqrt{3}x - 1$

slope $m = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

Now in Δ formed by the given two lines and x-axis

Exterior angle = Sum of interior opposite angles

$60^\circ = \theta + 45^\circ \Rightarrow \theta = 15^\circ$

Q10. Find the value of P, given that the line $\frac{y}{2} = x - P$ passes through the point $(-4, 4)$.

Sol. Equation of Line is $\frac{y}{2} = x - P$

It passes through the point $(-4, 4)$

$\therefore$ It will satisfy the equation,

$\frac{4}{2} = -4 - P \Rightarrow 2 = -4 - P \Rightarrow P = -6$

Q11. Given that $(a, 2a)$ lies on the line $\frac{y}{2} = 3x - 6$, find the value of a.

Sol. point $(a, 2a)$ lies on the line $\frac{y}{2} = 3x - 6$

This point will satisfy the equation

$\frac{2a}{2} = 3a - 6 \Rightarrow a = 3a - 6 \Rightarrow a = 3$

Q12. The graph of the equation $y = mx + c$ passes through the points $(1, 4)$ and $(-2, -5)$. Determine the values of m & c.

Sol. $y = mx + c$

It passes through $(1, 4)$ i.e. $4 = m + c \longrightarrow (i)$

It also passes through $(-2, -5)$ i.e. $-5 = -2m + c$

$\Rightarrow 2m - c = 5 \longrightarrow (ii)$

Adding (i) & (ii), $3m = 9 \Rightarrow m = 3$

Substitute the value of m in (i)

$4 = 3 + c$

$\Rightarrow c = 1$

Hence $m = 3$, $c = 1$

Q13. Find the equation of the line passing through the point $(2,-5)$ and make an intercept of -3 on the y-axis.

Sol. slope of the line $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3+5}{0-2} = -1$

Equation of the line will be

$y - y_1 = m(x - x_1) \Rightarrow y - (-5) = -1(x-2)$

$\Rightarrow y + 5 = -x + 2 \Rightarrow x + y + 3 = 0$

Q14. Find the equation of a st line passing through $(-1, 2)$ and whose slope is $\frac{2}{5}$.

Sol. Equation of the line will be $y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{2}{5}(x+1) \Rightarrow 5y - 10 = 2x + 2$

$\Rightarrow 2x - 5y + 12 = 0$

Q15. Find the equation of a st. line whose inclination is 60° and which passes through the point $(0, -3)$.

Sol. equation of the line will be $y - y_1 = m(x - x_1)$

Here $m = \tan 60^\circ = \sqrt{3}$ and point is $(0, -3)$

$y + 3 = \sqrt{3}(x - 0) \Rightarrow y + 3 = \sqrt{3}x \Rightarrow \sqrt{3}x - y - 3 = 0$

Q16. Find the gradient of a line passing through the following pairs of points: (i) $(0,-2), (3,4)$ (ii) $(3,-7), (-1,8)$.

Sol. $m = \frac{y_2 - y_1}{x_2 - x_1}$

(i) $m = \frac{4+2}{3-0} = 2$ gradient $= 2$

(ii) $m = \frac{8+7}{-1-3} = \frac{-15}{4}$

$\therefore$ Gradient $= \frac{-15}{4}$

Q17. The Co-ordinates of two points E and F are $(0,4)$ and $(3,7)$ respectively. find: (i) The gradient of EF (ii) The equation of EF (iii) The co-ordinates of the point where the line EF intersects the x-axis.

Sol.

(i) Gradient $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{3-0} = 1$

(ii) Equation of the line EF, $y - y_1 = m(x - x_1)$

$\Rightarrow y - 7 = 1(x-3) \Rightarrow y - 7 = x - 3$

$\Rightarrow x - y + 4 = 0$

(iii) co-ordinates of point of intersection EF and the x-axis will be $y = 0$. Substitute the value of y in

$x - y + 4 = 0 \Rightarrow x - 0 + 4 = 0 \Rightarrow x = -4$

∴ Hence co-ordinates are $(4, 0)$.

Q18 Find the intercepts made by the line $2x - 3y + 12 = 0$ on the co-ordinate axes.

Sol. putting $y = 0$, we will get the intercept made on x-axis in

$2x - 3y + 12 = 0$

$\Rightarrow 2x + 12 = 0 \Rightarrow x = -6$

and putting $x = 0$, we will get the intercept made on y-axis

$2x - 3y + 12 = 0 \Rightarrow -3y + 12 = 0$

$\Rightarrow y = +4$

Q19. Find the equation of the line passing through the point $P(5,1)$ and $Q(1,-1)$ Hence show that the points P, Q and $R(11,4)$ are collinear

Sol.

Slope $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{1-5} = \frac{1}{2}$

Equation of the line $y - y_1 = m(x - x_1)$

$y + 1 = \frac{1}{2}(x-1) \Rightarrow 2y + 2 = x - 1 \Rightarrow x - 2y - 3 = 0$ —(i)

If point R(11, 4) be on it, then it will satisfy it. Now

Substituting the value of x and y in (i)

$$11 - 2(4) - 3 = 0$$

∴ R satisfies it

∴ Hence P, Q and R collinear.

Q20. The graph of a linear equation in x and y passes through (4, 0) and (0, 3). Find the value of K, if the graph passes through (K, 1.5).

Sol. Slope $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{0-4} = \frac{-3}{4}$

Equation of the line will be $y - y_1 = m(x - x_1)$

$$y - 3 = \frac{-3}{4}(x - 0) \Rightarrow 4y - 12 = -3x + 0$$

$$\Rightarrow 3x + 4y - 12 = 0$$

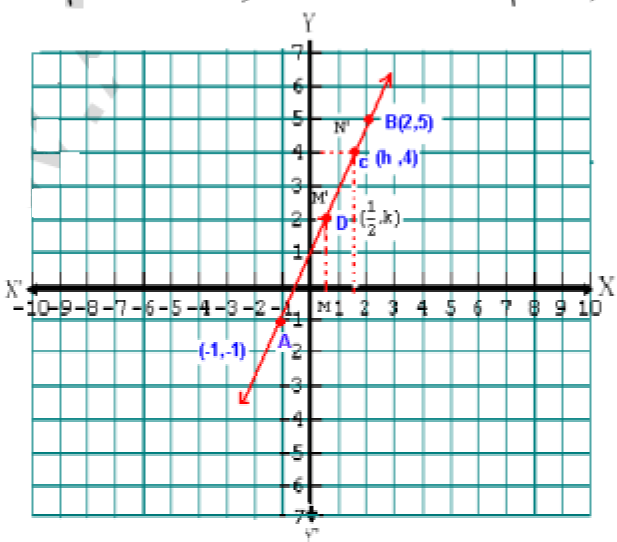
point (K, 1.5) or $(K, \frac{3}{2})$ lies on it

∴ It will satisfy the equation

Substituting the value of x and y in $3x + 4y - 12 = 0$

$$3K + 4\left(\frac{3}{2}\right) - 12 = 0 \Rightarrow 3K - 6 = 0 \Rightarrow K = 2$$

Q21. Use graph paper for this question. The graph of a linear equation in x and y passes through A(−1, −1) and B(2, 5). From your graph, find the values of h and K, if the line passes through (h, 4) and $(\frac{1}{2}, K)$.



Sol. Here we plotted two points A(-1,1) and B(2,5) and drawn a st. line. Now draw a $\perp^{lar}$ from y-axis at a distance of 4 units from origin to the st. line AN, which cut at C(h,k). So this point is at a distance of $\frac{3}{2}$ units from y-axis, Hence the value of h is $\frac{3}{2}$ units from y-axis. Hence the value of h is $\frac{3}{2}$ $\perp^{lar}$ from this point to x-axis. Cut x-axis at N.

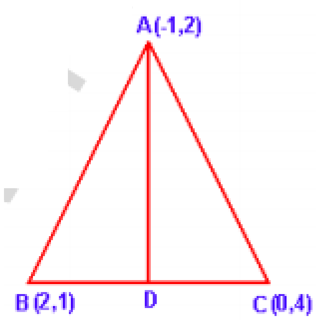
Secondly, $\frac{1}{2}$ units distance from origin from y-axis toward x-axis. draw a $\perp^{lar}$ on the line AB. which cut at $D\left(\frac{1}{2}, K\right)$. the distance of this point from x-axis is 2 units. hence the value of K is 2 units.

$\therefore \quad h = \frac{3}{2}$ and $K = 2$

Q22 If A(-1,2), B(2,1) and C(0,4) are the vertices of a ΔABC, find the equation of the median through A.

Sol. In ΔABC, vertices are A(-1,2), B(2,1) and C(0,4)

D is the mid point of BC.



co-ordinates of D will be $\left(\frac{2+0}{2}, \frac{1+4}{2}\right) = \left(1, \frac{5}{2}\right)$

Now slope of median AD $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{5}{2} - 2}{1-(-1)} = \frac{1}{4}$

equation of AD will be $y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 2 = \frac{1}{4}(x+1) \Rightarrow 4y - 8 = x + 1$

$\Rightarrow \quad x - 4y + 9 = 0$

Q23. Find the equation of a line passing through the point $(-2,3)$ and having x-intercept 4 units.

Sol. x-intercept $= 4$

slope $(m) = \frac{0-3}{4+2} = \frac{-1}{2}$

Equation of the line will be $y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \frac{-1}{2}(x-4) \Rightarrow 2y = -x+4$

$\Rightarrow x + 2y - 4 = 0$

Q24. Writedown the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where divides the lines segment joining $A(-2,6)$ and $B(3,-4)$ in the ratio $2:3$.

Sol. Co-ordinates of P will be

$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{2(3)+3(-2)}{2+3} = \frac{0}{5} = 0$$

$$y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2(-4)+3(6)}{2+3} = \frac{10}{5} = 2$$

$\therefore$ coordinates are $(0,2)$.

Now slope(m) of the line passing through $(0,2)$ $= \frac{3}{2}$

equation of the line will be

$y - 2 = \frac{3}{2}(x-0) \Rightarrow 2y - 4 = 3x$

$\Rightarrow 3x - 2y + 4 = 0$

Q25. Find the equation of the line passing through the point $(1,4)$ and intersecting the line $x - 2y - 11 = 0$ on the y-axis

Sol. line $x - 2y - 11 = 0$ passes through y-axis is $x = 0$

Substitute the value of x in $x - 2y - 11 = 0$

$-2y - 11 = 0 \Rightarrow y = \frac{-11}{2}$

$\therefore$ Co-ordinates of point will be $(0, -\frac{11}{2})$.

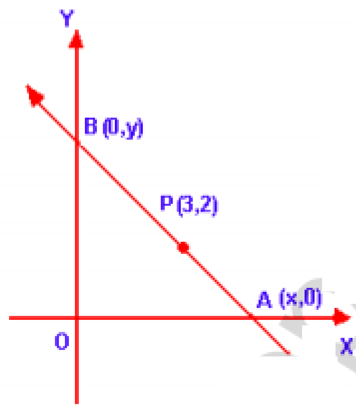
Now slope $(m) = \frac{-\frac{11}{2} - 4}{0-1} = \frac{-19/2}{-1} = \frac{19}{2}$

equation of the line will be

$$y + \frac{11}{2} = \frac{19}{2}(x - 0) \Rightarrow 2y + 11 = 19x$$

$$\Rightarrow \quad 19x - 2y - 11 = 0$$

Q26. Find the equation of the st. line containing the point (3, 2) and making +ve equal intercepts on axes.



Sol. let the line containing the point P(3, 2) passes through x-axis At A(x, 0) and y-axis at B(0, y)

$\because OA = OB$ given

$x = y$

Slope (m) $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - y}{x - 0} = -1 \quad (x = y)$

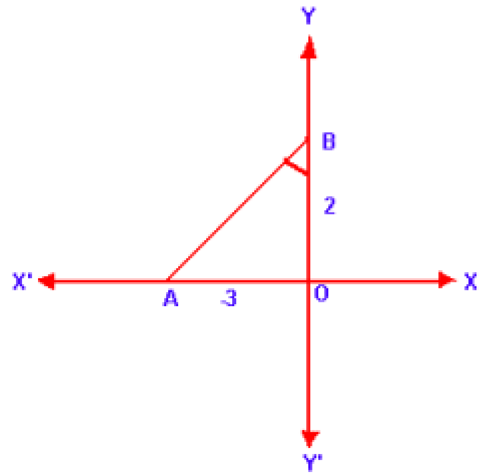
Equation of the line will be $y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 2 = -x + 3 \Rightarrow x + y - 5 = 0$

Q27. The intercepts made by a st line on the axes are -3 and 2 units. find : (i) The gradient of the line.

(ii) The equation of the line (iii) The area of the triangle enclosed between the line and the co-ordinate axes.

Sol. Two points A, B of the line which makes intercept on the axis are -3 and 2.



Co-ordinates of A are $(-3, 0)$ and $B(0, 2)$

(i) slope $(m) = \frac{2-0}{0+3} = \frac{2}{3}$

(ii) equation of the line will be

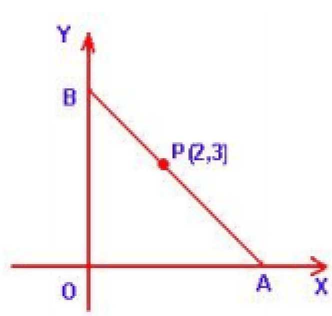
$$y - 2 = \frac{2}{3}(x - 0) \Rightarrow 3y - 6 = 2x$$

$$\Rightarrow 2x - 3y + 6 = 0$$

(iii) Area of $\Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 3 \times 2 = 3$ sq. units.

Q28. A line through the point $P(2, 3)$ meets the co-ordinate axes at point A and B. If $PA = 2PB$, find the co-ordinates of A and B. Also find the equation of the line AB.

Sol.



$$PA = 2PB \Rightarrow \frac{PA}{PB} = \frac{2}{1}$$

$$\Rightarrow PA : PB = 2 : 1$$

let the co-ordinates of A be $(x,0)$ and B be $(0,y)$

$$2 = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{2\times 0+1\times x}{2+1} = \frac{0+x}{3} \Rightarrow x = 6$$

and $$3 = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2\times y+1\times 0}{2+1} = \frac{2y}{3} \Rightarrow y = \frac{9}{2}$$

Hence co-ordinates of A are $(6,0)$ and $B\left(0, \frac{9}{2}\right)$

$$\text{slope} = \frac{\frac{9}{2}-0}{0-6} = \frac{-3}{4}$$

equation of the line passing through $P(2,3)$ will be

$$y-3 = \frac{-3}{4}(x-2) \Rightarrow 4y-12 = -3x+6$$

$$\Rightarrow 3x+4y-18 = 0$$

Q29. Calculate the co-ordinates of the point of intersection of the lines represented by $x+y=6$ and $3x-y=2$

Sol.
$$x+y = 6 \longrightarrow (i)$$
$$3x-y = 2 \longrightarrow (ii)$$

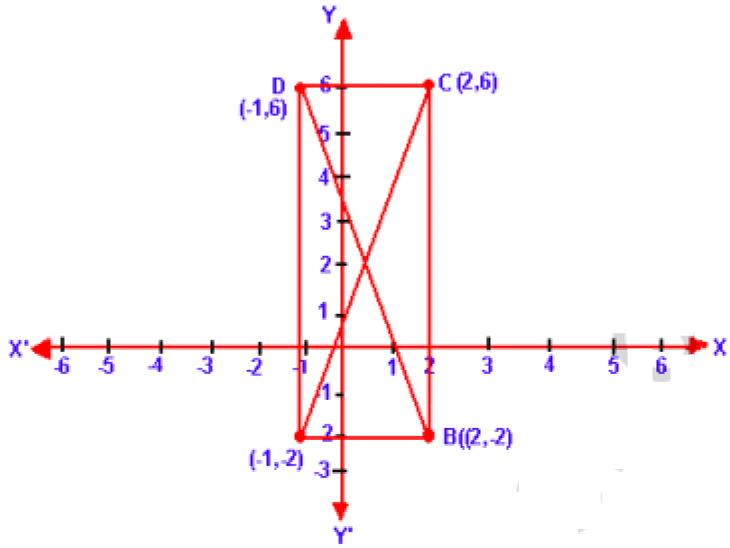
Adding (i) & (ii), we get $4x = 8 \Rightarrow x = 2$

Substitute the value of x in (i)

$$2+y = 6 \Rightarrow y = 4$$

$\therefore$ Hence co-ordinates of point will be $(2,4)$.

Q30. Find the equations of the diagonals of a rectangle whose sides are $x=-1$, $x=2$, $y=-2$ and $y=6$.



Sol. The equations of the sides of a rectangle whose equations are
$x = -1$, $x = 2$, $y = -2$, $y = 6$.
These lines form a rectangle when they intersect at A, B, C, D respectively.
$\therefore$ co-ordinates of A, B, C, D will be $(-1,-2)$, $(2,-2)$, $(2,6)$ and $(-1,6)$ respectively.
AC and BD are its diagonals.

(i) slope of the diagonal $AC = \frac{y_2-y_1}{x_2-x_1} = \frac{6+2}{2+1} = \frac{8}{3}$.

Equation of AC will be $y-y_1 = m(x-x_1)$

$\Rightarrow y+2 = \frac{8}{3}(x+1) \Rightarrow 3y+6 = 8x+8$

$\Rightarrow 8x-3y+2 = 0$

(ii) slope of $BD = \frac{6+2}{-1-2} = \frac{-8}{3}$

equation of BD will be $y+2 = \frac{-8}{3}(x-2)$

$\Rightarrow 3y+6 = -8x+16$

$\Rightarrow 8x+3y-10 = 0$

Q31. Find the equation of a St. line passing through the origin and through the point of intersection of the lines $5x+7y=3$ and $2x-3y=7$.

Sol.

$$5x+7y=3 \longrightarrow (i)$$
$$2x-3y=7 \longrightarrow (ii)$$

Multiply (i) by 3 and (ii) by 7,

$$15x+21y=9$$
$$14x-21y=49$$

on Adding we get, $29x=58 \Rightarrow x=2$.

Substitute the value of x in (i)

$$5(2)+7y=3 \Rightarrow 7y=-7 \Rightarrow y=-1$$

$\therefore$ point of intersection of lines is $(2,-1)$.

Now slope of the line joining the points $(2,-1)$ and the origin $(0,0)$.

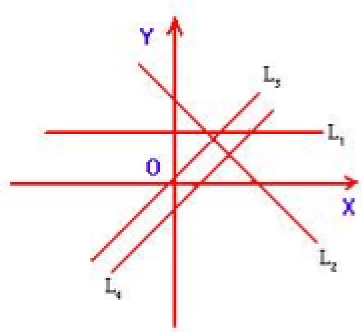
$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{0+1}{0-2}=\frac{-1}{2}.$$

Equation of line will be $y-y_1=m(x-x_1)$

$$\Rightarrow y-0=-\tfrac{1}{2}(x-0) \Rightarrow 2y=-x$$
$$\Rightarrow x+2y=0$$

Q32. Match the equations A, B, C, D with the lines L_1, L_2, L_3, L_4 whose graphs are roughly drawn in the adjoining diagram.

$A \equiv y=2x$, $B \equiv y-2x+2=0$, $C \equiv 3x+2y=6$, $D \equiv y=2$.



Sol. (i) $A = y = 2x$

It passes through the origin $(0,0)$.

L_3 is the line of A.

(ii) $B = y - 2x + 2 = 0 \Rightarrow y = 2x - 2$

Slope of the given line will be $m = 2$ and slope of the line $y = 2x$ is also 2.

$\therefore$ Line $\parallel^{al}$ to L_3 is the required line which is L_4.

(iii) $D = y = 2$

this line is $\parallel^{al}$ to x-axis at a distance of $y = 2$.

$\therefore L_1$ is the line of this equation.

(iv) Now $C = 3x + 2y = 6$

$\therefore L_2$ is line of this equation.

Q33. point $A(3,-2)$ on reflection in the x-axis is mapped as A' and point B on reflection in the y-axis is mapped onto $B'(-4,3)$.

(i) writedown the co-ordinates of A' and B

(ii) Find the slope of the line $A'B$, hence find its inclination.

Sol. (i) A' is the image of $A(3,-2)$ on reflection in the x-axis.

co-ordinates of A' will be $A'(3,2)$.

Again $B'(-4,3)$ in the image of A', when reflected in the y-axis

$\therefore$ Co-ordinates B will be $(4,3)$.

(ii) slope of the line joining the points $A'(3,2)$ and $B(4,3)$

$$= \frac{2-3}{3-4} = \frac{-1}{-1} = 1$$

Now $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

$\therefore$ Hence angle of inclination $= 45^\circ$.

EXERCISE - 12.2

Q1 State which one of the following is true : the straight lines $y = 3x - 5$ and $2y = 4x + 7$ are (i) parallel (ii) perpendicular (iii) neither parallel nor perpendicular

Sol. Slope of the lines $y = 3x - 5$ is 3 and $2y = 4x + 7$

$\Rightarrow y = 2x + \frac{7}{2}$ is 2

Slope of both the lines are neither equal nor their product is -1.

$\therefore$ These lines are neither parallel nor perpendicular.

Q2. If $6x + 5y - 7 = 0$ and $2px + 5y + 1 = 0$ are parallel lines, find the value of p.

Sol. In equation $6x + 5y - 7 = 0 \Rightarrow 5y = -6x + 7$

$\Rightarrow y = \frac{-6}{5}x + \frac{7}{5}$

slope $(m) = \frac{-6}{5} \longrightarrow$ (i)

Again in equation $2px + 5y + 1 = 0$

$\Rightarrow 5y = -2px - 1 \Rightarrow y = \frac{-2p}{5}x - \frac{1}{5}$

Slope $(m) = \frac{-2p}{5} \longrightarrow$ (ii)

lines are parallel $\therefore m_1 = m_2$

from (i) & (ii), $\frac{-6}{5} = \frac{-2p}{5} \Rightarrow p = 3$

Q3. Lines $2x - by + 5 = 0$ and $ax + 3y = 2$ are parallel. find the relation connecting a and b.

Sol. In equation $2x - by + 5 = 0 \Rightarrow -by = -2x - 5$

$\Rightarrow y = \frac{2}{b} + \frac{5}{b}$

slope $(m) = \frac{2}{b}$

and in equation $ax + 3y = 2 \Rightarrow 3y = -ax + 2 \Rightarrow y = \frac{-a}{3}x + \frac{2}{3}$

slope $(m_2) = -\frac{a}{3}$

lines are parallel $\therefore m_1 = m_2$

$$\frac{2}{b} = -\frac{a}{3} \Rightarrow ab = -6.$$

Q4. Given that the line $\frac{y}{2} = x - p$ and the line $ax+5 = 3y$ are parallel, find the value of a.

Sol. In equation $\frac{y}{2} = x - P \Rightarrow y = 2x - 2P$

slope $(m_1) = 2$

In equation $ax+5 = 3y \Rightarrow y = \frac{a}{3}x + \frac{5}{3}$

slope $(m_2) = \frac{a}{3}$.

$\therefore$ lines are parallel $\therefore m_1 = m_2$

$$\frac{a}{3} = 2 \Rightarrow a = 6.$$

Q5. If the lines $y = 3x+7$ and $2y+px = 3$ are perpendicular to eachother, find the value of P.

Sol. Given $y = 3x+7 \longrightarrow (1)$

The slope of line (1) $= 3$

Given $2y + px = 3 \Rightarrow 2y = -px + 3 \Rightarrow y = \frac{-P}{2}x + \frac{3}{2} \longrightarrow (2)$

The slope of line (2) $= \frac{-P}{2}$

Since the given lines are $\perp^{lar}$ to each other, we get

$$3\left(-\frac{P}{2}\right) = -1 \Rightarrow P = \frac{2}{3}$$

Q6. Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are $\perp^{lar}$ to each other.

Sol. In equation, $kx - 5y + 4 = 0 \Rightarrow -5y = -kx - 4 \Rightarrow y = \frac{k}{5}x + \frac{4}{5}$

slope $(m_1) = \frac{k}{5}$

In equation $4x - 2y + 5 = 0 \Rightarrow 2y = 4x + 5 \Rightarrow y = 2x + \frac{5}{2}$

slope $(m_2) = 2$

$\therefore$ lines are $\perp$ to each other $\therefore m_1 \times m_2 = -1$

$$\frac{k}{5} \times 2 = -1$$

$$\Rightarrow k = -\frac{5}{2}$$

Q7. If the lines $3x+by+5=0$ and $ax-5y+7=0$ are $\perp$ to eachother. find the relation connecting a and b.

Sol. In equation $3x+by+5=0 \Rightarrow by = -3x-5 \Rightarrow y = \frac{-3}{b}x - \frac{5}{b}$

slope $m_1 = \frac{-3}{b}$

In equation $ax-5y+7=0 \Rightarrow 5y = ax+7 \Rightarrow y = \frac{a}{5}x + \frac{7}{5}$

slope $m_2 = \frac{a}{5}$

line are $\perp$ to each other $\therefore m_1 m_2 = -1$

$\left(\frac{-3}{b}\right)\left(\frac{a}{5}\right) = -1 \Rightarrow 3a = 5b.$

Q8. Is the line through $(-2,3)$ and $(4,1)$ $\perp$ to the line $3x = y+1$? Does the line $3x = y+1$ bisect the join of $(-2,3)$ and $(4,1)$?

Sol. slope of the line passing through the points $(-2,3)$ and $(4,1)$.

$= \frac{1-3}{4+2} = -\frac{1}{3}$

Slope of the line $3x = y+1 \Rightarrow y = 3x-1$

slope $= 3$

$\therefore m_1 m_2 = -\frac{1}{3} \times 3 = -1$

$\therefore$ these lines are $\perp$ to each other.

Co-ordinates of midpoint of line joining the points $(-2,3)$ and $(4,1)$ will be $\left(\frac{-2+4}{2}, \frac{3+1}{2}\right) = (1, 2)$

If mid-point $(1,2)$ lies on the line $3x = y+1$ then it will satisfy it.

Now substituting the value of x and y is $3x = y+1$

$\Rightarrow 3(1) = 2+1 \Rightarrow 3 = 3.$ which is true.

Hence the line $3x = y+1$ bisect the line joining the points $(-2,3)$, $(4,1)$.

Q9. Find the value of m, if the lines represented by $2mx - 3y = 1$ and $y = 1-2x$ are perpendicular to each other.

Sol. In the equation of line $2mx - 3y = 1$

$\Rightarrow 3y = 2mx - 1 \Rightarrow y = \frac{2m}{3}x - \frac{1}{3}$

slope $(m_1) = \frac{2m}{3}$ and in equation $y = 1 - 2x \Rightarrow y = -2x + 1$

Slope $(m_1) = \frac{2m}{3}$ and in equation $y = 1 - 2x \Rightarrow y = -2x + 1$

Slope $(m_2) = -2$

These lines are $\perp$ to each other $\therefore m_1 m_2 = -1$

$\frac{2m}{3} \times (-2) = -1 \Rightarrow m = \frac{3}{4}$

Q10. If the lines $3x + y = 4$, $x - ay + 7 = 0$ and $bx + 2y + 5 = 0$ for the three consecutive sides of a rectangle, find the values of a and b.

Sol. In the line $3x + y = 4 \longrightarrow (i)$

$y = -3x + 4$ slope $(m_1) = -3$

In the line $x - ay + 7 = 0 \longrightarrow (ii)$

$\Rightarrow ay = x + 7 \Rightarrow y = \frac{1}{a}x + \frac{7}{a}$ slope $(m_2) = \frac{1}{a}$

and in the line $bx + 2y + 5 = 0 \longrightarrow (iii)$

$\Rightarrow 2y = -bx - 5 \Rightarrow y = \frac{-b}{2}x - \frac{5}{2}$ Slope $(m_3) = \frac{-b}{2}$

$\therefore$ These lines are consecutive three sides of a rectangle.

(i) and (ii) are $\perp^{lar}$ to each other $\therefore m_1 m_2 = -1$

$(-3)\left(\frac{1}{a}\right) = -1 \Rightarrow a = 3$

and (i) and (iii) are $\perp$ to each other

$(-3)\left(\frac{-b}{2}\right) = -1 \Rightarrow b = \frac{-2}{3}$.

Q11. Find the equation of a line which has the y-intercept 4 and is parallel to the line $2x - 3y - 7 = 0$. find the co-ordinates of the point where it cuts the x-axis.

Sol. In the given line $2x - 3y - 7 = 0 \Rightarrow 3y = 2x - 7$.

$\Rightarrow y = \frac{2}{3}x - \frac{7}{3}$

$\therefore$ Hence slope $(m_1) = \frac{2}{3}$

Equation of the line $\parallel^{er}$ to the given line will be

$y - y_1 = m(x - x_1)$

It passes through $(0,4)$, then

$y - 4 = \frac{2}{3}(x-0) \Rightarrow 3y - 12 = 2x$.

$\Rightarrow 2x - 3y + 12 = 0 \longrightarrow (ii)$

Now let it intersect x-axis at (x, y) $\therefore y = 0$

substitute the value of y in (ii)

$2x - 3(0) + 12 \Rightarrow 2x = -12 \Rightarrow x = -6$.

Q12. Find the equation of a st line $\perp$ to the line $2x + 5y + 7 = 0$ and with y-intercept -3 units.

Sol. In the line $2x + 5y + 7 = 0 \Rightarrow 5y = -2x - 7 \Rightarrow y = \frac{-2}{5}x - \frac{7}{5}$

slope $(m_1) = \frac{-2}{5}$

let the slope of the line $\perp$ to the given line $= m_2$

$m_1 m_2 = -1 \Rightarrow \frac{-2}{5} \times m_2 = -1 \Rightarrow m_2 = \frac{5}{2}$

It make y-intercept -3 units

equation of the new line $y - (-3) = \frac{5}{2}(x - 0)$

$\Rightarrow 2y + 6 = 5x \Rightarrow 5x - 2y - 6 = 0$

Q13. Find the equation of a st. line $\perp$ to the line $3x - 4y + 12 = 0$ and having same y-intercept as $2x - y + 5 = 0$

Sol. In the given line $3x - 4y + 12 = 0$

$\Rightarrow 4y = 3x + 12 \Rightarrow y = \frac{3}{4}x + 3$

Here slope $(m_1) = \frac{3}{4}$

let the slope of the line $\perp$ to given line be $= m_2$

$m_1 m_2 = -1$

$\frac{3}{4} \times m_2 = -1 \Rightarrow m_2 = \frac{-4}{3}$

y-intercept in the equation $2x - y + 5 = 0$

$\Rightarrow 2(0) - y + 5 = 0 \Rightarrow y = 5$

The equation of line passing through $(0, 5)$ will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y-5 = \frac{-4}{3}(x-0)$

$\Rightarrow 3y-15 = -4x$

$\Rightarrow 4x+3y-15=0$

Q14. Find the equation of the line which is $\parallel^{el}$ to $3x-2y=-4$ and passes through the point $(0,3)$.

Sol. In the given line $3x-2y=-4 \Rightarrow 2y = 3x+4$

$\Rightarrow y = \frac{3}{2}x+2$

Here slope $= \frac{3}{2}$

Equation of the line will be $y-y_1 = m(x-x_1)$

$y-3 = \frac{3}{2}(x-0) \Rightarrow 2y-6 = 3x$

$\Rightarrow 3x-2y+6=0$

Q15. Find the equation of the line passing through $(0,4)$ and $\parallel^{el}$ to the line $3x+5y+15=0$

Sol. In the given line $3x+5y+15=0 \Rightarrow 5y = -3x-15$

$\Rightarrow y = \frac{-3}{5}x-3$

Here slope $(m_1) = \frac{-3}{5}$

Equation of the line will be $y-y_1 = m(x-x_1)$

$y-4 = \frac{-3}{5}(x-0) \Rightarrow 5y-20 = -3x$

$\Rightarrow 3x+5y-20=0$

Q16. The equation of a line is $y=3x-5$. writedown the slope of this line and the intercept made by it on the y-axis. Hence or otherwise, writedown the equation of a line which is parallel to the line and which passes through the point $(0,5)$.

Sol. In the given line $y=3x-5$, Here slope $(m_1) = 3$

Substituting $x=0$, then $y=-5$

$\therefore$ y-intercept $= -5$

Equation of the line will be

$$y-5 = 3(x-0) \Rightarrow 3x-y+5=0$$

Q17. writedown the equation of the line $\perp$ to $3x+8y=12$ and passing through the point $(-1,-2)$.

Sol. In the given line $3x+8y=12 \Rightarrow 8y = -3x+12$

$$\Rightarrow y = -\frac{3}{8}x + \frac{12}{8}$$

Here slope $(m_1) = -\frac{3}{8}$.

let the slope of the line $\perp$ to the given line be $= m_2$

$$m_1m_2 = -1 \Rightarrow -\frac{3}{8} \times m_2 = -1 \Rightarrow m_2 = \frac{8}{3}$$

equation of the line will be $y-(-2) = \frac{8}{3}(x-(-1))$

$$\Rightarrow 3y+6 = 8x+8 \Rightarrow 8x-3y+2=0$$

Q18. (i) The line $4x-3y+12=0$ meet the x-axis at A. writedown the co-ordinates of A.

(ii) Determine the equation of the line passing through A and $\perp$ to $4x-3y+12=0$

Sol. (i) In the line $4x-3y+12=0 \Rightarrow 3y = 4x+12 \Rightarrow y = \frac{4}{3}x+4$

Here slope $(m_1) = \frac{4}{3}$

let the slope of the line $\perp$ to given line be $= m_2$

$$m_1m_2 = -1 \Rightarrow \frac{4}{3} \times m_2 = -1 \Rightarrow m_2 = \frac{-3}{4}$$

let the point on x-axis be $A(x,0)$

Substituting the value of x and y in $4x-3y+12=0$

$$\Rightarrow 4x-3(0)+12 = 0 \Rightarrow x = -3$$

$\therefore$ Coordinates of A will be $(-3,0)$

(ii) Equation of the line $\perp$ to the given line passing through A will be $y-0 = \frac{-3}{4}(x+3)$

$$\Rightarrow 4y = -3x-9$$

$$\Rightarrow 3x+4y+9=0$$

Q19. Find the equation of the line that is $\parallel^{lel}$ to $2x+5y-7=0$ and passes through the mid-point of the line segment joining the points (2,7) and (-4,1)

Sol. The given line $2x+5y-7=0 \Rightarrow 5y=-2x+7 \Rightarrow y=-\frac{2}{5}x+\frac{7}{5}$

Here slope $(m_1) = \frac{-2}{5}$

Co-ordinates of the mid point joining the points (2,7) and (-4,1) will be $= \left(\frac{2-4}{2}, \frac{7+1}{2}\right) = (-1,4)$

Equation of the line will be $y-y_1 = m(x-x_1)$

$$y-4 = -\frac{2}{5}(x+1) \Rightarrow 5y-20 = -2x-2$$

$$\Rightarrow 2x+5y-18=0$$

Q20. Find the equation of the line that is $\perp$ to $3x+2y-8=0$ and passes through the mid-point of the line segment joining the points (5,-2) and (2,2).

Sol. In the given line $3x+2y-8=0 \Rightarrow 2y=-3x+8 \Rightarrow y=-\frac{3}{2}x+4$

Here slope $(m_1) = \frac{-3}{2}$

Co-ordinates of the mid point of line segment joining points (5,-2) and (2,2) will be $\left(\frac{5+2}{2}, \frac{-2+2}{2}\right) = \left(\frac{7}{2}, 0\right)$ and let slope of the line $\perp$ to given line be $= m_2$

$$m_1 m_2 = -1 \Rightarrow \frac{-3}{2} \times m_2 = -1 \Rightarrow m_2 = \frac{2}{3}.$$

Equation of the line $\perp$ to the given line and passing through $\left(\frac{7}{2}, 0\right)$ will be $y-y_1 = m(x-x_1)$

$$\Rightarrow y-0 = \frac{2}{3}\left(x-\frac{7}{2}\right)$$

$$\Rightarrow 3y = 2x-7$$

$$\Rightarrow 2x-3y-7=0$$

Q21. Find the equation of a straight line passing through the intersection of $2x+5y-4=0$ with x-axis and ||^{el} to the line $3x-7y+8=0$

Sol. let the point of intersection of the line $2x+5y-4=0$ and x-axis be $(x,0)$.

Substitute the value of y in the equation

$2x+5(0)-4=0 \Rightarrow x=2$.

∴ co-ordinates of the point of intersection will be $(2,0)$

Now in the line $3x-7y+8=0 \Rightarrow 7y=3x+8$

$\Rightarrow y=\frac{3}{7}x+\frac{8}{7}$

Slope $(m_1)=\frac{3}{7}$

slope of the line ||^{el} to the above line will be $=\frac{3}{7}$

Equation of the line will be $y-y_1=m(x-x_1)$

$\Rightarrow y-0=\frac{3}{7}(x-2) \Rightarrow 7y=3x-6$

$\Rightarrow 3x-7y-6=0$

Q22. Find the equation of the ⊥ from the point $(1,-2)$ on the line $4x-3y-5=0$. Also find the co-ordinates of the foot of ⊥.

Sol. In the equation $4x-3y-5=0 \Rightarrow 3y=4x-5 \Rightarrow y=\frac{4}{3}x-\frac{5}{3}$

slope$(m_1)=\frac{4}{3}$

let the slope of ⊥ $=m_2$.

$m_1m_2=-1 \Rightarrow \frac{4}{3}\times m_2=-1 \Rightarrow m_2=-\frac{3}{4}$

Equation of the ⊥ where slope is $\frac{-3}{4}$ and drawn through the point $(1,-2)$.

$y+2=\frac{-3}{4}(x-1)$

$\Rightarrow 4y+8=-3x+3$

$\Rightarrow 3x+4y+5=0$

For find the co-ordinates of the foot of the ⊥, we have to solve the equations

$4x-3y-5=0$ ——— (i)

$3x+4y+5=0$ ——— (ii)

Multiplying (i) by 4 and (ii) by 3, we get

$$16x - 12y - 20 = 0$$
$$9x + 12y + 15 = 0$$

on Adding we get, $25x = 5 \Rightarrow x = \frac{1}{5}$

Substituting the value of x in (i).

$4\left(\frac{1}{5}\right) - 3y - 5 = 0 \Rightarrow 3y = \frac{-21}{5} \Rightarrow y = \frac{-7}{5}$

$\therefore$ Co-ordinates are $\left(\frac{1}{5}, \frac{-7}{5}\right)$

23. prove that the line through (0,0) and (2,3) is $\parallel^{al}$ to the line through (2,-2) and (6,4).

Sol. slope of the line through (0,0) and (2,3)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{2-0} = \frac{3}{2}.$$

and Slope of the line through (2,-2) and (6,4)

$$m_2 = \frac{4+2}{6-2} = \frac{3}{2}.$$

$\therefore m_1 = m_2 = \frac{3}{2}.$

$\therefore$ These lines are parallel to eachother.

Q24. prove that the line through (-2,6) and (4,8) is $\perp$ to the line through (8,12) and (4,24).

Sol. slope of the line through (-2,6) and (4,8)

$$m_1 = \frac{8-6}{4+2} = \frac{1}{3}$$

and slope of the line through (8,12) and (4,24)

$$m_2 = \frac{24-12}{4-8} = -3$$

$\therefore m_1 \times m_2 = \frac{1}{3}(-3) = -1$

$\therefore$ These lines are $\perp$ to eachother.

Q25. Show that the Δ^{le} formed by the points $A(1,3)$, $B(3,-1)$ and $C(-5,-5)$ is a right angled triangle by using slopes.

Sol.

Slope of the line by joining the points $A(1,3)$, $B(3,-1)$

$$m_1 = \frac{-1-3}{3-1} = -2$$

Slope of the line by joining the points $B(3,-1)$ and $C(-5,-5)$

$$m_2 = \frac{-5+1}{-5-3} = \frac{1}{2}$$

$$\therefore m_1 \times m_2 = (-2)\left(\frac{1}{2}\right) = -1$$

$\therefore$ lines AB and BC are $\perp$ to each other.

Hence ΔABC is a right angle Δ^{le}.

Q26. Find the equation of the line through the point $(-1,3)$ and $\parallel^{el}$ to the line joining the points $(0,-2)$ and $(4,5)$.

Sol.

Slope of the line joining the points $(0,-2)$ and $(4,5)$

$$m_1 = \frac{5+2}{4-0} = \frac{7}{4}$$

Equation of the line $y - y_1 = m(x - x_1)$

$$y - 3 = \frac{7}{4}(x+1) \Rightarrow 4y - 12 = 7x + 7$$

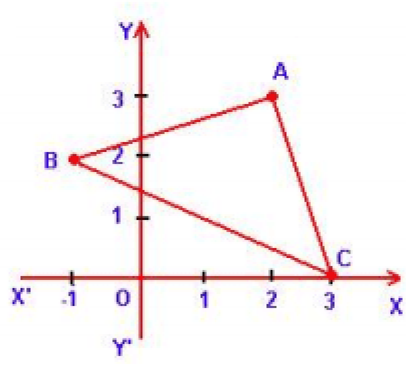
$$\Rightarrow 7x - 4y + 19 = 0$$

Q27. $A(1,4)$, $B(3,2)$ and $C(7,5)$ are the vertices of a ΔABC. Find

(i) The Co-ordinates of the Centroid G of ΔABC.

(ii) The equation of a line through G and parallel to AB.

Sol.



Sol. Vertices of a ΔABC are $A(1,4)$, $B(3,2)$ and $C(7,5)$

$\therefore$ Co-ordinates of Centroid G will be $\left(\frac{1+3+7}{3}, \frac{4+2+5}{3}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

slope of the line $AB(m_1) = \frac{2-4}{3-1} = -1$

slope of the line $AB = -1$ and passes through $G\left(\frac{11}{3}, \frac{11}{3}\right)$

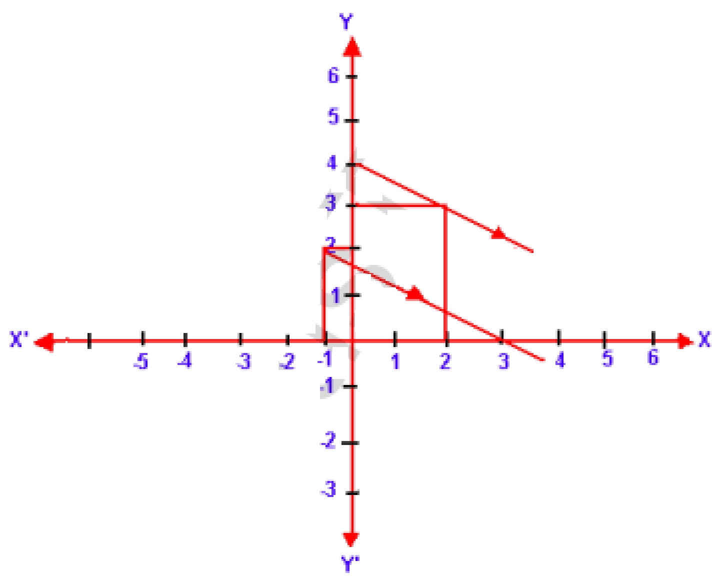
Equation of the line will be $y - y_1 = m(x - x_1)$

$\Rightarrow y - \frac{11}{3} = -1\left(x - \frac{11}{3}\right) \Rightarrow y - \frac{11}{3} = -x + \frac{11}{3}$

$\Rightarrow x + y - \frac{22}{3} = 0 \Rightarrow 3x + 3y - 22 = 0$

Q28. In the adjoining diagram, writedown

(i) The Co-ordinates of the points A, B and C.



(ii) The equation of the line through A, parallel to BC.

Sol. (i) Co-ordinates of points A, B and C are $A(2,3)$, $B(-1,2)$ and $C(3,0)$.

(ii) slope of BC $(m) = \frac{0-2}{3+1} = -\frac{1}{2}$.

Equation of the line will be $y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{2}(x - 2)$

$\Rightarrow 2y - 6 = -x + 2$

$\Rightarrow x + 2y - 8 = 0$

Q29. Find the equation of the line through $(0,-3)$ and $\perp$ to the line joining the points $(-3,2)$ and $(9,1)$.

Sol. The slope of the line joining the point $(-3,2)$ and $(9,1)$

$$m_1 = \frac{1-2}{9+3} = \frac{-1}{12}.$$

let slope of the line $\perp$ to the line $= m_2$

$$\therefore m_1 m_2 = -1 \Rightarrow \frac{-1}{12} \times m_2 = -1 \Rightarrow m_2 = 12.$$

Equation of the line passing through $(0,-3)$ and slope $m_2 = 12$

$$y+3 = 12(x-0) \Rightarrow 12x - y - 3 = 0$$

Q30. The vertices of a Δ^{le} are $A(10,4)$, $B(4,-9)$ and $C(-2,-1)$. find the equation of the altitude through A.

Sol. Slope of the line BC $(m_1) = \frac{-1+9}{-2-4} = \frac{-4}{3}$.

let the slope of the altitude from $A(10,4)$ to BC $= m_2$

$$\therefore m_1 m_2 = -1 \Rightarrow -\frac{4}{3} \times m_2 = -1 \Rightarrow m_2 = \frac{3}{4}$$

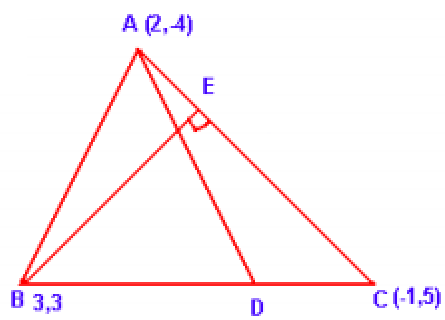
Equation of the line will be

$$y - 4 = \frac{3}{4}(x-10) \Rightarrow 4y - 16 = 3x - 30$$

$$\Rightarrow 3x - 4y - 14 = 0$$

Q31. $A(2,-4)$, $B(3,3)$ and $C(-1,5)$ are the vertices of ΔABC. find the equation of (i) the median of the Δ^{le} through A.
(ii) The altitude of the Δ^{le} through B.

Sol.



(i) D is the mid point of BC

$\therefore$ co-ordinates of D will be $\left(\frac{3-1}{2}, \frac{3+5}{2}\right) = (1, 4)$

slope of median AD $(m) = \frac{4+4}{1-2} = -8$

then equation of AD will be $y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 4 = -8(x-1) \Rightarrow y - 4 = -8x + 8$

$\Rightarrow \quad 8x + y - 12 = 0$

(ii) BE is the altitude from B to AC

slope of AC $(m_1) = \frac{5+4}{-1-2} = -3$

let slope of BE $= m_2$

$m_1 m_2 = -1 \Rightarrow -3 \times m_2 = -1 \Rightarrow m_2 = \frac{1}{3}$.

Equation of BE will be $y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{3}(x-3) \Rightarrow 3y - 9 = x - 3$

$\Rightarrow \quad x - 3y + 6 = 0$

Q32. Find the equation of the right bisector of the line segment joining the points (1, 2) and (5, −6)

Sol. slope of line joining the points (1, 2) and (5, −6).

$m_1 = \frac{-6-2}{5-1} = -2$.

let m_2 be the right bisector of the line

$m_1 m_2 = -1 \Rightarrow -2 \times m_2 = -1 \Rightarrow m_2 = \frac{1}{2}$

midpoint of the line segment joining (1, 2) and (5, −6) will be

$= \left(\frac{1+5}{2}, \frac{2-6}{2}\right) = (3, -2)$.

equation of the line, the right bisector will be $y - y_1 = m(x - x_1)$

$y + 2 = \frac{1}{2}(x - 3)$

$\Rightarrow \quad 2y + 4 = x - 3$

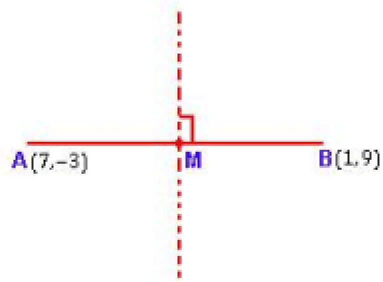
$\Rightarrow \quad x - 2y - 7 = 0$

Q33. points A and B have Co-ordinates $(7,-3)$ and $(1,9)$ respectively. find (i) the slope of AB (ii) the equation of the $\perp$ bisector of the line segment AB. (iii) the value of p if $(-2, p)$ lies on it.

Sol. Given points $A(7,-3)$ and $B(1,9)$

(i) The slope of line $AB = \frac{9-(-3)}{1-7} = -2$.

(ii) The slope of $\perp$ bisector of $AB = \frac{1}{2}$.



The Co-ordinate of midpoint $M = \left(\frac{7+1}{2}, \frac{-3+9}{2}\right) = M(4,3)$

The Slope of the line through in $(4,3)$ and having slope $\frac{1}{2}$ is

$$y-3 = \frac{1}{2}(x-4) \Rightarrow 2y-6 = x-4 \Rightarrow x-2y+2=0$$

(iii) If the point $(-2, p)$ lies on the $\perp$ bisector $x-2y+2=0$

$$\Rightarrow -2-2p+2=0 \Rightarrow p=0$$

Q34. The points $B(1,3)$ and $D(6,8)$ are two opposite vertices of a Square ABCD. find the equation of the diagonal AC.

Sol. slope of BD $(m_1) = \frac{8-3}{6-1} = 1$

Diagonal AC is $\perp$ bisector of diagonal BD.

slope of $AC = -1$ $\quad (m_1 m_2 = -1)$

Co-ordinates of midpoint of BD will be $\left(\frac{1+6}{2}, \frac{3+8}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right)$

Equation of AC, $y - \frac{11}{2} = -1\left(x - \frac{7}{2}\right)$

$$\Rightarrow 2y-11 = -2x+7$$

$$\Rightarrow 2x+2y-18=0 \Rightarrow x+y-9=0$$

Q35. ABCD is a rhombus. The co-ordinates of A and C are (3,6) and (−1,2) respectively. write down the equation of BD.

Sol. slope of AC $(m_1) = \frac{2-6}{-1-3} = 1$

But line BD is the right bisector of AC.

slope of BD $= -1$ $(m_1 m_2 = -1)$

Co-ordinates of midpoint of AC will be $\left(\frac{3-1}{2}, \frac{6+2}{2}\right) = (1,4)$

Equation of BD will be $y-4 = -1(x-1)$

$\Rightarrow y-4 = -x+1 \Rightarrow x+y-5 = 0$

Q36. Find the equation of the line passing through the intersection of the lines $4x+3y=1$ and $5x+4y=2$ and

(i) parallel to the line $x+2y-5=0$ (ii) $\perp$ to the x-axis.

Sol. the given equations are

$4x+3y=1 \quad \times 4 \Rightarrow 16x+12y=4$

$5x+4y=2 \quad \times 3 \Rightarrow \underline{15x+12y=6}$ (subtracting)

$x = -2$

put $x=-2$ in $4x+3y=1 \Rightarrow 4(-2)+3y=1$

$\Rightarrow 3y = 9 \Rightarrow y = 3.$

The point of intersection is $(-2,3)$.

(i) Now slope of the line $x+2y-5=0 \Rightarrow y = -\frac{x}{2}+\frac{5}{2}$

slope $= -\frac{1}{2}$

Equation of the line will be $y-3 = \frac{-1}{2}(x+2)$

$\Rightarrow 2y-6 = -x-2 \Rightarrow x+2y-4=0$

(ii) The line $\perp$ to x-axis is $\parallel^{el}$ to y-axis. so the slope of the line will be infinite.

Hence the line having slope infinity and passing through the point $(-2,3)$ is $y-3 = \infty(x+2)$

$\Rightarrow x+2 = \frac{y-3}{\infty} = 0$

$\Rightarrow x+2 = 0$

Q37. (i) Write down the co-ordinates of the point P that divides the line joining A(−4, 1) and B(17, 10) in the ratio 1 : 2

(ii) Calculate the distance OP where O is the origin.

(iii) In what ratio does the y-axis divide the line AB?

Sol. (i) Let the co-ordinates of P will be (x, y)

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 17 + 2(-4)}{1+2} = \frac{9}{3} = 3.$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 10 + 2 \times 1}{1+2} = \frac{12}{3} = 4.$$

∴ co-ordinates of P will be $(3, 4)$.

(ii) Distance b/w O and P $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(0-3)^2 + (0-4)^2} = \sqrt{9+16} = 5 \text{ units}$$

(iii) Let y-axis divides AB in the ratio $m_1 : m_2$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \Rightarrow 0 = \frac{m_1 \times 17 + m_2(-4)}{m_1 + m_2}$$

$$\Rightarrow 17m_1 - 4m_2 = 0 \Rightarrow 17m_1 = 4m_2$$

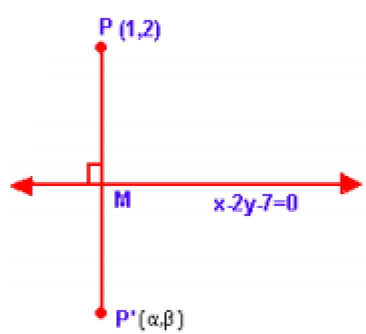
$$\Rightarrow m_1 : m_2 = 4 : 17$$

Q38. Find the image of the point $(1, 2)$ in the line $x - 2y - 7 = 0$

Sol. Draw a ⊥ from the point $P(1, 2)$ on the line $x - 2y - 7 = 0$

Let P' is the image of P and let its co-ordinates are (α, β)

slope of line $x - 2y - 7 = 0 \Rightarrow 2y = x - 7 \Rightarrow y = \frac{1}{2}x - \frac{7}{2}$ is $\frac{1}{2}$.



Slope of $PP' = -2$ $\quad (\because m_1 m_2 = -1)$

Equation of PP' $\quad y-2 = -2(x-1) \Rightarrow y-2 = -2x+2$

$\Rightarrow \quad 2x+y-4=0$

$P'(\alpha, \beta)$ lies on it $\quad 2\alpha+\beta = 4 \longrightarrow (i)$

P' is the image of P in the line $x-2y-7=0$

the line bisects PP' at M or M is the mid-point of PP'

$\therefore$ co-ordinates of M will be $\left(\frac{1+\alpha}{2}, \frac{2+\beta}{2}\right)$

M lies on the given line $x-2y-7=0$

Substituting the value of x, y

$\frac{1+\alpha}{2} - 2\left(\frac{2+\beta}{2}\right) - 7 = 0 \Rightarrow 1+\alpha-4-2\beta-14=0$

$\Rightarrow \alpha - 2\beta = 17 \longrightarrow (ii)$

$\Rightarrow \alpha = 2\beta+17$

Substituting the value of α in (i)

$2(2\beta+17)+\beta = 4 \Rightarrow 4\beta+34+\beta = 4$

$\Rightarrow 5\beta = -30 \Rightarrow \beta = -6$

Substituting the value of β in (i)

$2\alpha - 6 = 4 \Rightarrow 2\alpha = 10 \Rightarrow \alpha = 5$

Co-ordinates of P' will be $(5, -6)$.

Q39 If the line $x-4y-6=0$ is the $\perp$ bisector of the line segment PQ and the co-ordinates of P are $(1,3)$, find the co-ordinates of Q.

Sol. let the co-ordinates of Q be (α, β) and let the line $x-4y-6=0$ is the $\perp$ bisector of PQ and it intersects the line at M.

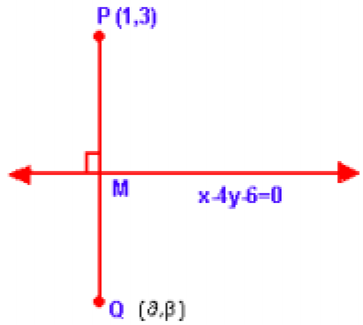
M is the mid point of PQ

Now slope of Line $x-4y-6=0$

$\Rightarrow 4y = x-6$

$\Rightarrow y = \frac{1}{4}x - \frac{3}{2}$

slope $= \frac{1}{4}$



Slope of $PQ = -4$ $\quad (\because m_1 m_2 = -1)$

equation of line PQ $\quad y-3 = -4(x-1)$

$\Rightarrow y-3 = -4x+4 \Rightarrow 4x+y-7=0$

$\therefore Q(\alpha, \beta)$ lies on it. $\quad 4\alpha+\beta = 7$ —— (i)

Now co-ordinates of M will be $\left(\frac{1+\alpha}{2}, \frac{3+\beta}{2}\right)$

$\therefore$ M lies on the line $x-4y-6=0$

$$\frac{1+\alpha}{2} - 4\left(\frac{3+\beta}{2}\right) - 6 = 0$$

$$\Rightarrow 1+\alpha-12-4\beta-12=0$$

$$\Rightarrow \alpha - 4\beta = 23 \quad \text{—— (ii)}$$

Multiply (i) by 4 and (ii) by 1

$$16\alpha + 4\beta = 28$$
$$\alpha - 4\beta = 23$$

Adding we get $\quad 17\alpha = 51 \Rightarrow \alpha = 3$

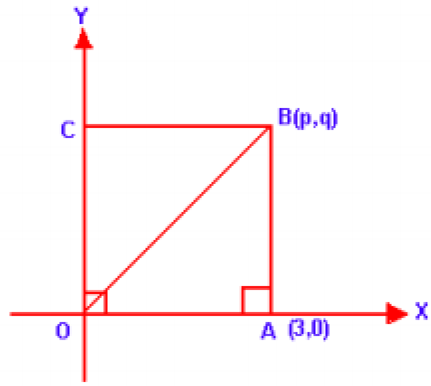
put the value of α in (i)

$$4(3) + \beta = 7 \Rightarrow \beta = -5$$

$\therefore$ co-ordinates of Q will be $(3, -5)$

Q40. OABC is a square, O is the origin and the points A and B are (3,0) and (p, q). Find the values of p and q. Write down the equation of AB and BC.

Sol. $OA = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3$



$AB = \sqrt{(3-p)^2+(0-q)^2} = \sqrt{(3-p)^2+q^2}$

$\therefore OA = AB$ (Side of a square)

$\therefore \sqrt{(3-p)^2+q^2} = 3 \Rightarrow (3-p)^2+q^2 = 9$

$\Rightarrow p^2+q^2-6p = 0 \longrightarrow (i)$

$OB = \sqrt{(p-0)^2+(q-0)^2} = \sqrt{p^2+q^2}$

But $OB^2 = OA^2 + AB^2$

$\Rightarrow \left(\sqrt{p^2+q^2}\right)^2 = 3^2 + \left(\sqrt{(3-p)^2+q^2}\right)^2$

$\Rightarrow p^2+q^2 = 9+9+p^2-6p+q^2$

$\Rightarrow 6p = 18 \Rightarrow p = 3$

Substituting the value of p in (i), $9+q^2-6(3) = 0 \Rightarrow q^2 = 9 \Rightarrow q = 3$

$\therefore p = 3, q = 3$

$\therefore$ AB parallel to y-axis

$\therefore$ Equation of AB will be $x = 3 \Rightarrow x-3 = 0$

and equation of BC will be $y = 3 \Rightarrow y-3 = 0$

$(\because BC \parallel x\text{-axis})$