Similarity

Exercise -13

1. Given that ΔABC and ΔPQR are Similar. find
(i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1:3.
(ii) The ratio of their corresponding sides if area of ΔABC : area of ΔPQR = 25 : 36.

Sol. (i) $\because \Delta ABC \sim \Delta PQR$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \frac{BC^2}{QR^2}$$

(The ratio of the area of the similar triangles is equal to the ratio of the squares of any two corresponding sides.)

But $BC = QR = 1:3$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \frac{(1)^2}{(3)^2} = \frac{1}{9}$$

(ii) $\because \Delta ABC \sim \Delta PQR$, $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \dfrac{BC^2}{QR^2}$

But, area of ΔABC : area of ΔPQR = 25 : 36

$$\frac{BC^2}{QR^2} = \frac{25}{36} \Rightarrow \left(\frac{BC}{QR}\right)^2 = \left(\frac{5}{6}\right)^2 \Rightarrow \frac{BC}{QR} = \frac{5}{6}$$

$\therefore BC : QR = 5 : 6$

2. $\Delta ABC \sim \Delta DEF$. If area of $\Delta ABC = 9 cm^2$, area of $\Delta DEF = 16 cm^2$ and $BC = 2.1 cm$, find the length of EF.

Sol.

let $EF = x$, $\Delta ABC \sim \Delta DEF$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2} \Rightarrow \frac{9}{16} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{(2.1)^2}{x^2} = \frac{9}{16} \Rightarrow \frac{2.1}{x} = \frac{3}{4} \Rightarrow 3x = 4 \times 2.1$$

$$\Rightarrow x = \frac{4 \times 2.1}{3} = 2.8 cm.$$

Hence $EF = 2.8 cm$.

3. $\Delta ABC \sim \Delta DEF$. If $BC = 3cm$, $EF = 4cm$ and area of $\Delta ABC = 54 cm^2$. Determine the area of ΔDEF.

Sol.

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{\text{area of } \Delta DEF} = \frac{3^2}{4^2}$$

$$\Rightarrow \frac{54}{\text{area of } \Delta DEF} = \frac{9}{16} \Rightarrow \text{area of } \Delta DEF = \frac{54 \times 16}{9} = 96 cm.$$

4. The area of two similar triangles are $36 cm^2$ and $25 cm^2$. If altitude of the first triangle is $2.4 cm$, find the corresponding altitude of the other triangle.

Sol. let $ABC \sim DEF$, AL and DM are their altitudes

then area of $\Delta ABC = 36 cm^2$.

area of $\Delta DEF = 25 cm^2$ and $AL = 2.4 cm$.

let $DM = x$, Now $\Delta ABC \sim \Delta DEF$

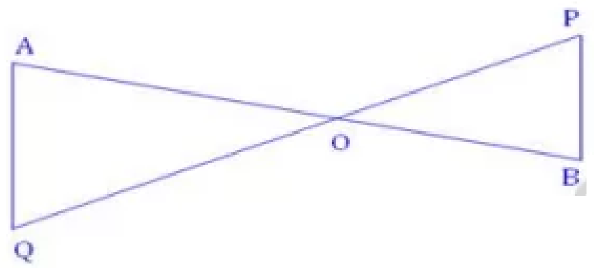
$$\therefore \frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{AL^2}{DM^2} \Rightarrow \frac{36}{25} = \frac{(2.4)^2}{x^2}$$

$$\Rightarrow \frac{2.4}{x} = \frac{6}{5} \Rightarrow x = \frac{2.4 \times 5}{6} = 2cm.$$

Hence altitude of the other triangle = 2cm.

5. In the adjoining fig. PB and QA are perpendiculars to the line segment AB. If PO = 6 cm and QO = 9cm and the area of $\Delta POB = 120cm^2$. find the area of ΔQOA.

Sol.



We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$$\frac{\text{Area of } \Delta AOQ}{\text{Area of } \Delta POB} = \frac{OQ^2}{OP^2} \quad (\because \angle A = \angle B,\ \angle AOQ = \angle BOP\ )$$

$$\Rightarrow \frac{\text{Area of } \Delta AOQ}{120} = \frac{9^2}{6^2}$$

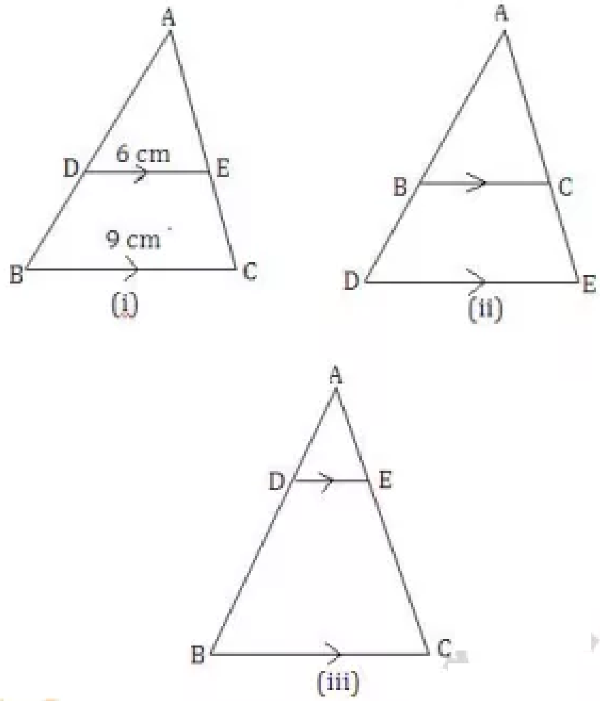
$$\Rightarrow \text{Area of } \Delta AOQ = \frac{81}{36} \times 120 = 270cm^2$$

6. (a) In the fig.(i) given below, DE || BC. If DE = 6cm, BC = 9cm, and area of $\Delta ADE = 28\ cm^2$, find the area of ΔABC.

(b) In the fig(ii) given below, BC is parallel to DE. Area of $\Delta ABC = 25cm^2$, area of trapezium BCED = $24cm^2$, DE = 14cm. calculate the length of BC.

(c) In the fig.(iii) given below, DE || BC and AD:DB = 1:2 find the ratio of the areas of ΔADE and trapezium DBCE.

sol.



(a) In the fig.(i), DE || BC, $\angle D = \angle B$ and $\angle E = \angle C$ (corresponding angles)

Now in ΔADE and ΔABC, $\angle D = \angle B$, $\angle E = \angle C$ (proved)

$\angle A = \angle A$ (Common)

$\therefore \Delta ADE \sim \Delta ABC$ (AAA postulate)

$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{DE^2}{BC^2} \Rightarrow \frac{28}{\text{area of } \Delta ABC} = \frac{6^2}{9^2} = \frac{36}{81}$$

$$\Rightarrow \text{area of } \Delta ABC = \frac{28 \times 81}{36} = 63.$$

Area of $\Delta ABC = 63 \text{ cm}^2$.

(b) In the fig.(ii), BC || DE, area of $\Delta ABC = 25 \text{ cm}^2$.

area of trapezium BCED $= 24 \text{ cm}^2$

DE = 14 cm and BC = x (say)

$\therefore$ In ΔADE, $BC \parallel DE$, $\angle B = \angle D$, $\angle C = \angle E$

Now in ΔABC and ΔADE, $\angle B = \angle D$, $\angle C = \angle E$ (proved)

$\angle A = \angle A$ (common)

$\Delta ABC \sim \Delta ADE$ (AAA postulate)

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta ADE} = \frac{BC^2}{DE^2} \Rightarrow \frac{25}{25+24} = \frac{x^2}{(14)^2}$$

$$\Rightarrow \frac{25}{49} = \frac{x^2}{196} \Rightarrow x^2 = \frac{25 \times 196}{49} = 100 \Rightarrow x = 10$$

$\therefore$ Hence $BC = 10cm$.

(C) In the fig.(iii), $DE \parallel BC$

$\angle D = \angle B$ and $\angle E = \angle C$ (corresponding angles)

Now in ΔADE and ΔABC, $\angle D = \angle B$, $\angle E = \angle C$ (proved)

$\angle A = \angle A$ (common)

$\Delta ADE \sim \Delta ABC$ (AAA postulate)

But $\frac{AD}{DB} = \frac{1}{2} \Rightarrow \frac{DB}{AD} = \frac{2}{1} \Rightarrow \frac{DB}{AD} + 1 = \frac{2}{1} + 1$

$$\Rightarrow \frac{AD+DB}{AD} = \frac{2+1}{1} \Rightarrow \frac{AB}{AD} = \frac{3}{1} \Rightarrow \frac{AD}{AB} = \frac{1}{3}$$

$\therefore \Delta ADE \sim \Delta ABC$

$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{AD^2}{AB^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$\Rightarrow$ area of $\Delta ABC = 9 \times$ area of ΔADE.

$\therefore$ area of trapezium DBCE = area of ΔABC − area of ΔADE.

= $9 \times$ area of ΔADE − area of ΔADE

= $8 \times$ area of ΔADE.

$\therefore \dfrac{\text{area of } \Delta ADE}{\text{area of trapezium DBCE}} = \dfrac{1}{8}$

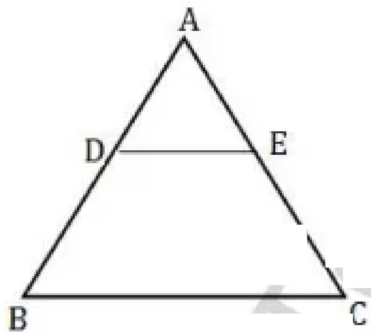
7. In the given fig, $DE \parallel BC$

(i) prove that ΔADE and ΔABC are similar.

(ii) Given that $AD = \frac{1}{2} BD$, calculate DE, if $BC = 4.5$

(iii) If area of $\Delta ABC = 18 cm^2$, find the area of trapezium DBCE.

Sol.



(i) In ΔADE and ΔABC, $\angle A = \angle A$ (common)

$\angle D = \angle B$, $\angle E = \angle C$ (corresponding angles)

$\therefore \Delta ADE \sim \Delta ABC$.

(ii) $\dfrac{AD}{DB} = \dfrac{1}{2} \Rightarrow \dfrac{DB}{AD} = \dfrac{2}{1} \Rightarrow \dfrac{DB}{AD} + 1 = \dfrac{2}{1} + 1$

$\Rightarrow \dfrac{AD + DB}{AD} = \dfrac{2+1}{1} \Rightarrow \dfrac{AB}{AD} = \dfrac{3}{1} \Rightarrow \dfrac{AD}{AB} = \dfrac{1}{3}$.

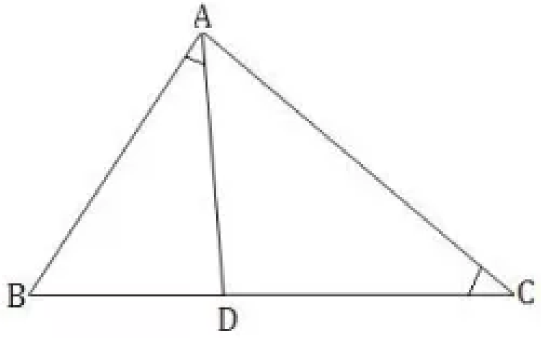
$\dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \dfrac{AD^2}{AB^2} \Rightarrow \dfrac{DE^2}{BC^2} = \dfrac{AD^2}{AB^2} \Rightarrow \dfrac{DE^2}{(4.5)^2} = \left(\dfrac{1}{3}\right)^2$

$\Rightarrow \dfrac{DE}{(4.5)} = \dfrac{1}{3} \Rightarrow DE = \dfrac{4.5}{3} = 1.5 \text{ cm}$.

(iii) area of $\Delta ABC = 9 \times \Delta ADE$

area of trapezium DBCE $= \Delta ABC - \Delta ADE = 16 cm^2$.

8. In the adjoining fig., D is a point on BC such that $\angle BAD = \angle C$ and $AB = 7cm$, $BD = 4cm$.



(i) prove that $\triangle ABD \sim \triangle ABC$

(ii) Find area of $\triangle ABC$: area of $\triangle ADC$.

Sol. In $\triangle ABD$ and $\triangle ABC$, $\angle BAD = \angle C$ (given)

$\angle B = \angle B$ (common)

$\therefore \triangle ABD \sim \triangle ABC$ (AA postulate)

$$\frac{\text{area of } \triangle ABD}{\text{area of } \triangle ABC} = \frac{BD^2}{AB^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

$\Rightarrow$ 16 area of $\triangle ABC$ = 49 area of $\triangle ABD$.

= 49 (area of $\triangle ABC$ − area of $\triangle ADC$)

= 49 area of $\triangle ABC$ − 49 area of $\triangle ADC$

$\Rightarrow$ 49 area of $\triangle ADC$ = 49 area of $\triangle ABC$ − 16 area of $\triangle ABC$

$\Rightarrow$ 49 area of $\triangle ADC$ = 33 area of $\triangle ABC$

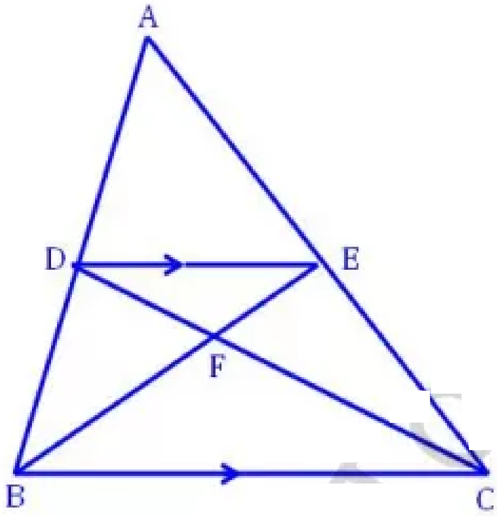
$\Rightarrow$ area of $\triangle ABC$: area of $\triangle ADC$ = 49 : 33.

9. In the given fig ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$

(i) Determine the ratios $\frac{AD}{AB}$, $\frac{DE}{BC}$.

(ii) prove that ΔDEF is smaller to ΔCBE, Hence find $\frac{EF}{FB}$?

(iii) what is the ratio of the areas of ΔDEF and ΔBFC?



sol. (i) $\frac{AD}{DB} = \frac{3}{2} \Rightarrow \frac{AD}{AD+DB} = \frac{3}{3+2} \Rightarrow \frac{AD}{AB} = \frac{3}{5}$

In ΔADE and ΔABC, We have

$\angle DAE = \angle BAC$ (common), $\angle ADE = \angle ABC$ (Corresponding angles)

$\therefore$ By AA criterion of Similarity, $\Delta ADE \sim \Delta ABC$

$$\frac{DE}{BC} = \frac{AD}{AB} = \frac{3}{5}$$

(ii) In ΔDEF and ΔCBF, we have

$\angle EDF = \angle FCB$, $\angle DEF = \angle FBC$

By AA Criterion of Similarity, $\Delta DEF \sim \Delta CBF$

$$\frac{EF}{BF} = \frac{DE}{CB} = \frac{3}{5}$$

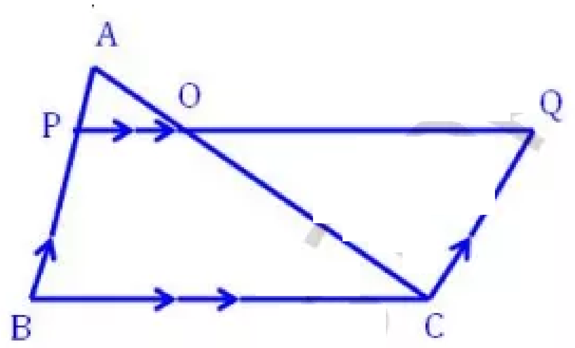
(iii) We know that the ratio of the areas of similar triangle is equal to the ratio of the squares of the corresponding sides. therefore

$$\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle BFC} = \frac{DE^2}{BC^2} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

10. In $\triangle ABC$, $AP : PB = 2 : 3$ PO is parallel to BC and extended to Q so that CQ is parallel to BA. find :

(i) area of $\triangle APO$: area of $\triangle ABC$

(ii) Area of $\triangle APO$: area of $\triangle CQO$



Sol. In $\triangle APO$ and $\triangle ABC$, $\angle A = \angle A$ (common)

$\angle 1 = \angle 2$, $\angle 3 = \angle 4$ (corresponding angles)

$\triangle APO \sim \triangle ABC$ ($\because$ By AAA Similarity Axiom)

Also, $\frac{AP}{PB} = \frac{2}{3} \Rightarrow \frac{PB}{AP} = \frac{3}{2} \Rightarrow \frac{PB}{AP} + 1 = \frac{3}{2} + 1$

$\Rightarrow \frac{PB + AP}{AP} = \frac{3+2}{2} \Rightarrow \frac{AB}{AP} = \frac{5}{2} \Rightarrow \frac{AP}{AB} = \frac{2}{5}$

Now, $\frac{\text{area of } \triangle APO}{\text{area of } \triangle ABC} = \frac{AP^2}{AB^2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

Again in $\triangle APO$ and $\triangle CQO$.

$\angle 1 = \angle 6$ (alternate internal angles)

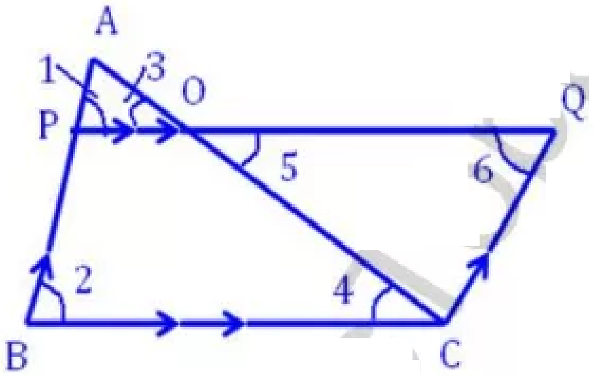
$\angle 3 = \angle 5$ (vertically opposite angles)

$\Delta APO \sim \Delta CQO$ (By AA similarity Axiom)

$$\frac{AP}{CQ} = \frac{AO}{OC}$$

Also, $\frac{AP}{PB} = \frac{AP}{CQ} = \frac{2}{3}$ [$\because PB = CQ$ opp. sides of a parallelogram]

$$\frac{\text{area of } \Delta APO}{\text{area of } \Delta CQO} = \frac{AO^2}{OC^2} = \left(\frac{AO}{OC}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

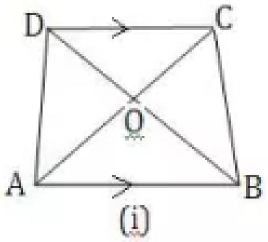


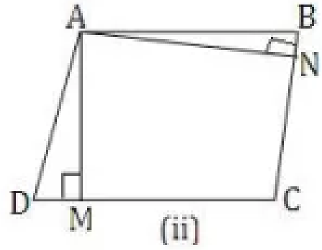
11. (a) In the fig.(i) given below, ABCE is a trapezium in which $AB \parallel DC$ and $AB = 2CD$. Determine the ratio of the areas of ΔAOB and ΔCOD.

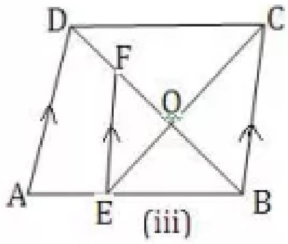
(b) In the fig.(ii) given below, ABCD is a parallelogram. $AM \perp DC$ and $AN \perp CB$. if $AM = 6cm$, $AN = 10cm$ and the area of parallelogram ABCD is $45cm^2$, find :
(i) AB (ii) BC (iii) area of ΔADM : area of ΔANB.

(c) In the fig (iii) given below, ABCD is a parallelogram E is a point on AB, CE intersects the diagonal BD at O and $EF \parallel BC$. If $AE : EB = 2 : 3$, find :
(i) EF : AD (ii) areas of ΔBEF : area of ΔABD.
(iii) area of ΔABD : area of trapezium AFED.

(iv) area of ΔFEO : area of ΔOBC.







Sol. (a) In trapezium ABCD, $AB \parallel DC$

$\angle OAB = \angle OCD$ (alternate angles)

$\angle OBA = \angle ODC$, $\Delta AOB \sim \Delta COD$.

$$\frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} \quad (\because AB = 2CD)$$

$$= \frac{4CD^2}{CD^2} = \frac{4}{1}$$

area of ΔAOB : area of ΔCOD = 4 : 1

(b) In parallelogram ABCD, $AM \perp DC$ and $AN \perp CB$

Now area of parallelogram ABCD = $DC \times AM$ or $BC \times AN$

$DC \times AM = BC \times AN$ = Area of parallelogram

$\Rightarrow DC \times 6 = BC \times 10 = 45$

(i) $DC = \frac{45}{6} = \frac{15}{2} = 7.5\text{ cm} = AB$.

$(\because AB = DC)$

(ii) $BC = \frac{45}{10} = 4.5 cm.$

(iii) Now in $\triangle ADM$ and $\triangle ABN$,

$\angle D = \angle B$ (opp. angles of a parallelogram)

$\angle M = \angle N$ (each $90°$)

$\therefore \triangle ADM \sim \triangle ABN$, $\frac{\text{area of } \triangle ADM}{\text{area of } \triangle ABN} = \frac{AD^2}{AB^2}$

$$\Rightarrow \frac{BC^2}{AB^2} = \frac{(4.5)^2}{(7.5)^2} = \frac{20.25}{56.25} = \frac{2025}{5625} = \frac{9}{25}$$

$\therefore$ Area of $\triangle ADM$: area of $\triangle ABN = 9 : 25$

(c) In parallelogram ABCD, E is a point on AB, CE intersects the diagonal BD at O. $EF \parallel BC$ and $AE : EB = 2 : 3$.

In $\triangle ABD$, $EF \parallel BC$ or AD

(i) $\frac{AB}{BE} = \frac{AD}{EF} \Rightarrow \frac{EF}{AD} = \frac{BE}{AB}$

But

$$\frac{AE}{EB} = \frac{2}{3} \Rightarrow \frac{AE}{EB} + 1 = \frac{2}{3} + 1 \Rightarrow \frac{AE + EB}{EB} = \frac{2+3}{3}$$

$$\Rightarrow \frac{AB}{EB} = \frac{5}{3} \Rightarrow \frac{EB}{AB} = \frac{3}{5} \Rightarrow \frac{EF}{AD} = \frac{BE}{AB} = \frac{3}{5}$$

$\therefore EF : AD = 3 : 5$

(ii) $\triangle BEF \sim \triangle ABD$,

$$\frac{\text{area of } \triangle BEF}{\text{area of } \triangle ABD} = \frac{(EF)^2}{(AD)^2} = \frac{3^2}{5^2} = \frac{9}{25}$$

$\therefore$ area of $\triangle BEF$: area of $\triangle ABD = 9 : 25$

(iii) $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta BEF} = \frac{25}{9}$ $\{\because$ from (ii) $\}$

25 area of ΔBEF = 9 area of ΔABD

$\Rightarrow$ 25(area of ΔABD − area of trap. AFED) = 9 area of ΔABD

$\Rightarrow$ 25 area of ΔABD − 9 area of ΔABD = 25 area of trap. AFED.

$\Rightarrow$ 16 area of ΔABD = 25 Area of trap. AFED.

$\Rightarrow$ $\frac{\text{area of } \Delta ABD}{\text{area of trap. AFED}} = \frac{25}{16}$

(iv) In ΔFEO and ΔOBC, $\angle EOF = \angle BOC$ (Vertically opp. angles)

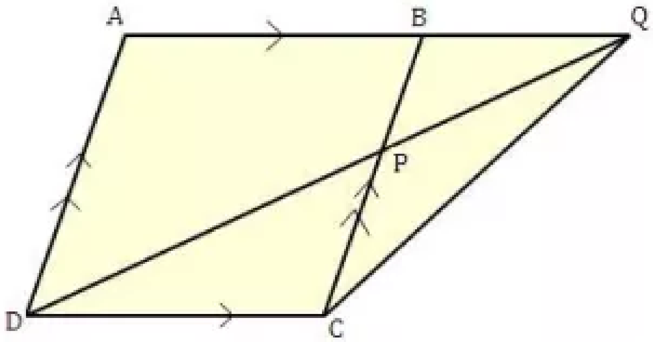
$\angle F = \angle OBC$ (alternate angles)

$\therefore \Delta FEO \sim \Delta OBC$

$\frac{\text{area of } \Delta FEO}{\text{area of } \Delta OBC} = \frac{EF^2}{BC^2} = \frac{EF^2}{AD^2} = \frac{9}{25}$.

$\therefore$ area of ΔFEO : area of ΔOBC = 9 : 25

12. In the adjoining fig ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2 and DP produced meets AB produced at Q. If the area of $\Delta CPQ = 20 cm^2$, find (i) area of ΔBPQ, (ii) area of ΔCDP, (iii) area of parallelogram ABCD.



Sol. $\dfrac{\text{Area of } \Delta BPQ}{\text{Area of } \Delta CPQ} = \dfrac{\frac{1}{2} \times QN \times PB}{\frac{1}{2} \times QN \times PC}$

let area of $\Delta BPQ = x$

$\Rightarrow \dfrac{x}{20} = \dfrac{PB}{PC}$ [$PB : PC = 1:2$ (given)]

$\Rightarrow \dfrac{x}{20} = \dfrac{1}{2} \Rightarrow x = 10 \text{ cm}^2$

(i) Area of $\Delta BPQ = 10 \text{ cm}^2$

In ΔBPQ and ΔCDP,

$\angle P = \angle P$ (vertically opp. angle)

$\angle PBC = \angle BCD$ (interior alternate angles)

$\Delta BPQ \sim \Delta CPD$, $\dfrac{\text{Area of } \Delta BPQ}{\text{Area of } \Delta CPD} = \dfrac{BP^2}{CP^2}$

$\Rightarrow \dfrac{10}{\text{area of } \Delta CPD} = \dfrac{1}{4} \Rightarrow$ area of $\Delta CPD = 40 \text{ cm}^2$.

Area of ΔCDQ = Area of ΔCPD + Area of ΔCPQ

$= 40 + 20 = 60 \text{ cm}^2$.

In ΔCDQ, It has a similar base of parallelogram ABCD; i.e. CD; also the same height.

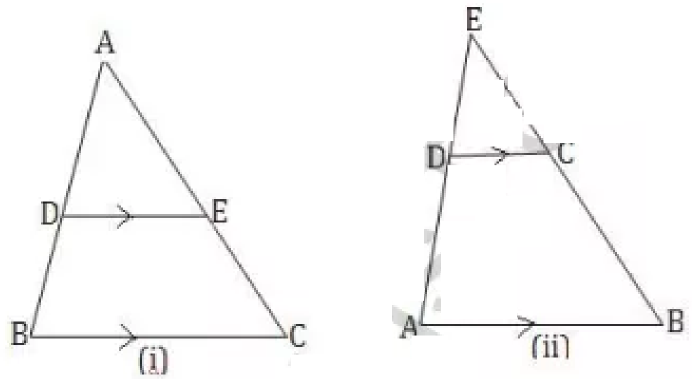
Area of $\Delta CDQ \times 2$ = area of parallelogram ABCD.

$\therefore$ Area of $\parallel ABCD = 60 \times 2 = 120\,cm^2$.

13. (a) In the fig(i) given below, $DE \parallel BC$ and the ratio of the areas of ΔADE and trapezium DBCE is 4 : 5. find the ratio of DE : BC.

(b) In the fig(ii) given below, $AB \parallel DC$ and AB = 2 DC. If AD = 3cm, BC = 4cm and AD, BC produced meet at E, find (i) ED (ii) BE

(iii) Area of ΔEDC : area of trapezium ABCD.



Sol.

(a) In ΔABC, $DE \parallel BC$

Now in ΔADC and ΔADE

$\angle A = \angle A$ (Common), $\angle B = \angle D$, $\angle E = \angle C$ (corresponding angles)

$\Delta ADE \sim \Delta ABC$, $\dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \dfrac{DE^2}{BC^2}$ —— (i)

But $\dfrac{\text{area of } \Delta ADE}{\text{area of trap. DBCE}} = \dfrac{4}{5} \Rightarrow \dfrac{\text{Area of trap. DBCE}}{\text{area of } \Delta ADE} = \dfrac{5}{4}$.

$\Rightarrow \dfrac{\text{Area of trap. DBCE}}{\text{Area of } \Delta ADE} + 1 = \dfrac{5}{4} + 1$.

$$\Rightarrow \frac{\text{Area of trap. DBCE} + \text{area of } \Delta ADE}{\text{area of } \Delta ADE} = \frac{5+4}{4} = \frac{9}{4}$$

$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \frac{9}{4} \Rightarrow \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{4}{9}$$

Now from(i), $\frac{DE^2}{BC^2} = \frac{4}{9} = \left(\frac{2}{3}\right)^2 \Rightarrow \frac{DE}{BC} = \frac{2}{3}$

(b) In the fig., $DC \parallel AB$, $AB = 2DC$.

$AD = 3cm$, $BC = 4cm$.

In ΔEAB, $DC \parallel AB$

$$\frac{EA}{DA} = \frac{EB}{CB} = \frac{AB}{DC} = \frac{2DC}{DC} = \frac{2}{1}$$

(i) $EA = 2DA = 2 \times 3 = 6$ cm.

$ED = EA - DA = 6 - 3 = 3cm$.

(ii) $\frac{EB}{CB} = \frac{2}{1} \Rightarrow EB = 2CB = 2 \times 4 = 8cm$.

$\therefore BE = 8cm$.

(iii) In ΔEAB, $DC \parallel AB$

$\Delta EDC \sim \Delta EAB$, $\frac{\text{area of } \Delta EDC}{\text{area of } \Delta ABE} = \frac{DC^2}{AB^2} = \frac{DC^2}{(2DC)^2}$

$$\frac{\text{area of } \Delta EDC}{\text{area of } \Delta ABE} = \frac{DC^2}{4DC^2} = \frac{1}{4}$$

$\therefore$ Area of $\Delta ABE = 4 \times$ Area of ΔEDC.

$\Rightarrow$ area of ΔEDC + area of trap. ABCD $= 4 \times$ Area of ΔEDC.

$\Rightarrow$ Area of trap. ABCD $= 3 \times$ Area of ΔEDC.

$\Rightarrow$ Area of ΔEDC : area of trop. ABCD = 1 : 3

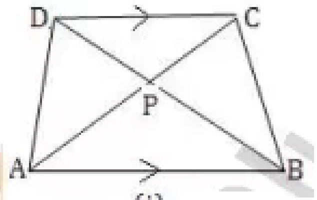
14. (a) In the fig(i) given below, ABCD is a trapezium in which DC // AB. If AB = 9cm, DC = 6cm and BD = 12cm, find (i) BP (ii) The ratio of areas of ΔAPB and ΔDPC.

(b) In the fig(ii) given below, M is mid point of AB, $\angle A = \angle B = 90° = \angle CMD$, prove that

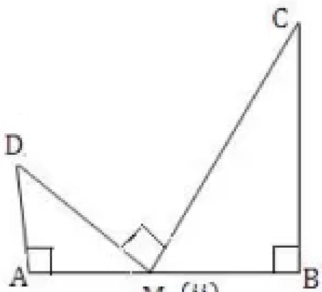
(i) ΔDAM is similar to ΔCMB.

(ii) $\dfrac{\text{area of } \Delta DAM}{\text{area of } \cancel{\text{trap}}\ \Delta CMB} = \dfrac{AD}{BC}$

(iii) $\dfrac{AD}{BC} = \dfrac{MD^2}{MC^2}$

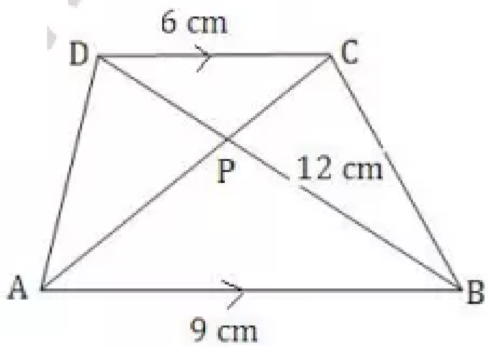


(i)



(ii)

Sol. In trapezium ABCD, DC || AB.



$AB = 9cm$, $DC = 6cm$, $BD = 12cm$.

(i) In ΔAPB and ΔCPD, $\angle APB = \angle CPD$ (Vertically opp. angles)

$\angle PAB = \angle PCD$ (alternate angles)

$\Delta APB \sim \Delta CPD$ (AA postulate)

$$\frac{BP}{PD} = \frac{AB}{CD} \Rightarrow \frac{BP}{12 - BP} = \frac{9}{6}$$

$$\Rightarrow 6BP = 108 - 9BP \Rightarrow 15BP = 108 \Rightarrow BP = 7.2cm.$$

Again $\Delta APB \sim \Delta CPD$.

$$\frac{\text{Area of } \Delta APB}{\text{area of } \Delta CPD} = \frac{AB^2}{CD^2} = \frac{9^2}{6^2} = \frac{81}{36} = \frac{9}{4}.$$

$\therefore$ Area of ΔAPB : Area of $\Delta CPD = 9 : 4$

(b)

(i) $\angle CMD = 90°$, $\angle A = \angle B = 90°$

$\angle AMD + \angle BMC = \angle AMD + \angle ADM = 90°$

$\angle BMC = \angle ADM$

Now in ΔDAM and ΔCMB, $\angle A = \angle B = 90°$

$\angle ADM = \angle BMC$ (proved)

$\therefore \Delta DAM \sim \Delta CMB$

(ii) $$\frac{\text{area of } \Delta DAM}{\text{area of } \Delta CMB} = \frac{\frac{1}{2} \times AM \times AD}{\frac{1}{2} \times BM \times BC} = \frac{\frac{1}{2} \times AM \times AD}{\frac{1}{2} \times AM \times BC}$$

$$= \frac{AD}{BC} \quad (\because AM = MB)$$

(iii) $\Delta DAM \sim \Delta CMB$ {prove in (i)}

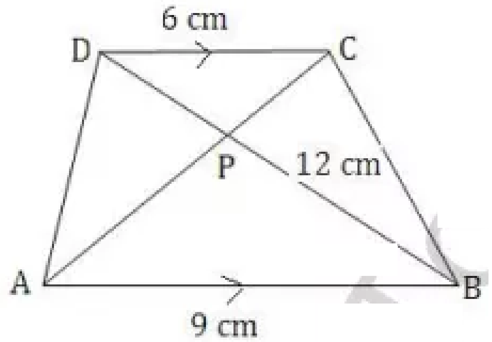
$$\frac{\text{area of } \Delta DAM}{\text{area of } \Delta CMB} = \frac{MD^2}{MC^2} \quad \text{But} \quad \frac{\text{area of } \Delta DAM}{\text{area of } \Delta CMB} = \frac{AD}{BC}$$

{prove in (ii)}

$$\therefore \frac{AD}{BC} = \frac{MD^2}{MC^2}$$

15. Two isosceles triangles have equal vertical angles and their areas are in the ratio 7:16. find the ratio of their corresponding height.

Sol.



In two isosceles ΔABC and ΔDEF, $\angle A = \angle D$ (given)

$\angle B + \angle C = \angle E + \angle F$ But $\angle B = \angle C$ and $\angle E = \angle F$

(opp. angles of equal sides)

$\angle B = \angle E$ and $\angle C = \angle F$

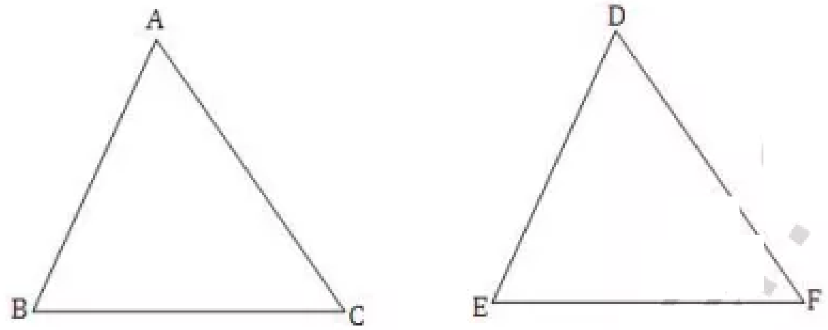
$\Delta ABC \sim \Delta DEF$, $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \dfrac{AL^2}{DM^2} \Rightarrow \dfrac{AL^2}{DM^2} = \dfrac{7}{16}$

$\Rightarrow \dfrac{AL}{DM} = \dfrac{\sqrt{7}}{4}$

Hence $AL : DM = \sqrt{7} : 4$

16. If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Given $\Delta ABC \sim \Delta DEF$



and area of ΔABC = area of ΔDEF.

To prove $\Delta ABC \cong \Delta DEF$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

But area of ΔABC = area of ΔDEF (given)

$BC^2 = EF^2 \Rightarrow BC = EF$

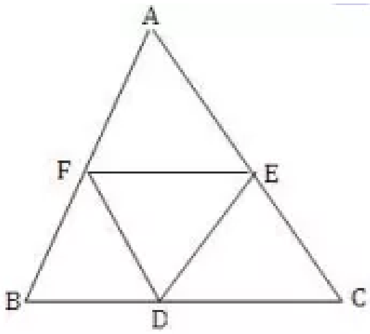
But $\angle B = \angle E$ and $\angle C = \angle F$ ($\because \Delta ABC \sim \Delta DEF$)

$\Delta ABC \cong \Delta DEF$ Hence proved.

17. D, E and F are midpoints of the sides of BC, CA and AB respectively of a ΔABC. find the ratio of the areas of ΔDEF and ΔABC.

Sol. In ΔABC, D, E and F are midpoints of sides BC, CA and AB respectively. DE, EF and FD are joined.

$\because$ E and F are the midpoints of AC and AB.



$EF \parallel BC$ and $EF = \frac{1}{2} BC$.

Similarly $FD \parallel AC$

$\therefore$ CDEF is a parallelogram. $\angle C = \angle F$

Similarly we can prove that $\angle A = \angle D$ and $\angle B = \angle E$

$\therefore \Delta DEF \sim \Delta ABC$

$$\frac{\text{area of } \Delta DEF}{\text{area of } \Delta ABC} = \frac{EF^2}{BC^2} = \frac{(\frac{1}{2}BC)^2}{BC^2} = \frac{1}{4} \cdot \frac{BC^2}{BC^2} = \frac{1}{4}$$

$\therefore$ area of ΔDEF : area of ΔABC = 1 : 4.

18. The volume of a machine is 27000 cm^3. A model of the machine is made, the reduction factor being 2:15. find the volume of the model.

Sol. volume of machine = 27000cm^3.

and scale factor = 2 : 15 or $\frac{2}{15}$

volume of model = K (Volume of actual machine)

$$= \left(\frac{2}{15}\right)^3 \times 27000$$

$$= \frac{8}{15 \times 15 \times 15} \times 27000$$

$$= 64 \text{cm}^3$$

19. The scale of map is 1:200000. A plot of land of area $20 km^2$ is to be represented on the map. find
(i) The no. of km's on the ground which is represented by 1cm on the map.
(ii) The area in km^2 that can be represented by $1cm^2$.
(iii) The area on the map that represents the plot of land.

Sol. Scale factor $= 1:200000$ is $k = \frac{1}{200000}$

Area of a plot of land $= 20 km^2$

length on map $= 1cm$.

(i) length of actual plot $= \frac{1}{k}$ (length on map)

$= 200000 \times 1cm$

$= \frac{200000}{100 \times 1000} km$

$= 2km$.

(ii) Area of map $= 1 cm^2$

Area of actual plot $= \left(\frac{1}{k}\right)^2$ (area on map)

$= (200000)^2 (1cm)^2$

$= \frac{200000 \times 200000}{100000 \times 100000} km^2$

$= 4 km$

(iii) Area of plot $= 20 km^2$

Area of map $= k^2$ (Area of plot of land)

$= \frac{1}{(200000)^2} \times 20 \; km^2$

$$= \frac{20}{200000 \times 200000} \text{ km}^2$$

$$= \frac{20 \times 100000 \times 100000}{200000 \times 200000}$$

$$= 5 \text{cm}^2$$

20. on a map drawn to a Scale of 1:250000, a triangular plot of land has the following measurements:

AB = 3cm, BC = 4cm, angle ABC = 90°. Calculate

(i) The actual length of AB in km.

(ii) The area of the plot in km².

Sol. Scale factor $k = 1 : 250000 = \frac{1}{250000}$

length on map.

AB = 3cm, BC = 4cm

$\therefore$ length of AB of actual plot $= \frac{1}{k}$ (length of AB on the map)

$$= 250000 \ (3\text{cm})$$

$$= \frac{250000 \times 3}{100 \times 1000} \text{ km}$$

$$= \frac{15}{2} = 7.5 \text{ km}.$$

(ii) Area of plot on the map $= \frac{1}{2} \times AB \times BC$.

$$= \frac{1}{2} \times 3 \times 4 = 6\text{cm}^2.$$

Area of actual plot $= \frac{1}{k^2} \times 6 \text{cm}^2$

$$= (250000)^2 \times 6 \text{ cm}^2$$

$$= \frac{250000 \times 250000 \times 6}{100000 \times 100000} \text{ km}^2$$

$$= \frac{25}{4} \times 6 = \frac{75}{2} = 37.5 \text{ km}^2$$

21. On a map drawn to a scale of 1:250000, a rectangular plot of land, ABCD has the following measurements, AB = 12cm and BC = 16cm. Angles A, B, C and D are 90° each. Calculate:

(i) the distance of a diagonal of the plot in km.

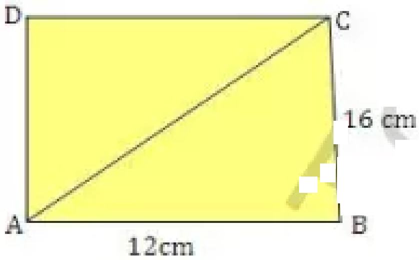
(ii) The area of the plot in km^2.

Sol.

Scale factor $k = \frac{1}{250000}$

Measurement of plot ABCD on the map are

AB = 12cm and BC = 16cm.



$$\text{Diagonal } AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{12^2 + 16^2}$$
$$= \sqrt{144 + 256}$$
$$= \sqrt{400}$$
$$= 20 \text{ cm.}$$

and area $= AB \times BC$
$$= 12 \times 16$$
$$= 192 \text{ cm}^2.$$

(i) Now actual length of AC $= \frac{1}{k}$ (length of AC on map)

$= 25000 \times 20 cm.$

$= \frac{25000 \times 20}{100 \times 1000} km.$

$= 5 km.$

(ii) Area of plot $= \left(\frac{1}{k}\right)^2$ (Area of plot on map)

$= (25000)^2 \times 192 \; cm^2$

$= \frac{25000 \times 25000 \times 192}{(100)^2 \times (1000)^2} km^2$

$= 12 km^2.$