

Mensuration

EXERCISE -17.1

1. Find the length of diameter of a circle whose circumference is 44cm.

Sol. let radius of circle = r then Circumference = $2\pi r$
 $\therefore 2\pi r = 44 \Rightarrow r = \frac{44}{2} \times \frac{7}{22} = 7 \text{ cm.}$

$$\text{Diameter} = 2r = 2 \times 7 = 14 \text{ cm.}$$

2. Find the radius and area of a circle if its circumference is 18π cm.

Sol. let r be the radius of the circle.
Circumference = $2\pi r$

$$\therefore 18\pi = 2\pi r \Rightarrow r = 9 \text{ cm}$$

$$\text{Area} = \pi r^2 = \pi \times 9 \times 9 = 81\pi \text{ cm}^2.$$

3. Find the perimeter of a ^{Semi}Circle plate of radius 3.85 cm.

Sol. Radius of SemiCircular plate = 3.85 cm.

$$\begin{aligned} \text{perimeter} &= \pi r + 2r = 3.85 \left(\frac{22}{7} + 2 \right) = 3.85 \times \frac{36}{7} \\ &= 19.8 \text{ cm.} \end{aligned}$$

4. Find the radius and circumference of a circle whose area is 144π cm².

Sol. let radius = r
Area = $\pi r^2 = 144\pi$

$$\Rightarrow r^2 = 144 \Rightarrow r = 12 \text{ cm}$$

$$\text{Circumference} = 2\pi r = 2 \times \pi \times 12 = 24\pi \text{ cm.}$$

5. A sheet is 11cm long and 2cm wide. circular pieces - 0.5cm in diameter is cut from it to prepare discs. Calculate the no. of discs that can be prepared.

Sol.

$$\text{Area of the sheet} = 11 \times 2 = 22 \text{ cm}^2$$

$$\text{Area of square piece} = 0.5 \times 0.5 = 0.25 \text{ cm}^2 \quad (\text{Side} = 0.5 \text{ cm})$$

$$\text{No. of discs} = \frac{\text{Area of sheet}}{\text{Area of square piece}} = \frac{22}{0.25} = \frac{22 \times 100}{25} = 88$$

\therefore The no. of discs that can be prepared are 88.

6. If the area of semi circular region is 77 cm^2 , find its perimeter.

Sol.

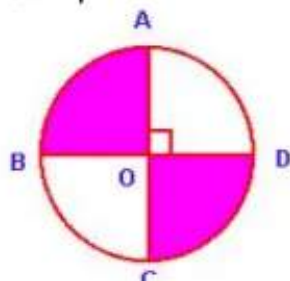
$$\text{Area of Semicircular region} = 77 \text{ cm}^2$$

$$\frac{\pi r^2}{2} = 77 \Rightarrow r = \sqrt{\frac{2 \times 77 \times 7}{22}} = 7 \text{ cm}$$

The perimeter of semicircular region = perimeter of semicircle + diameter of semicircle.

$$= \frac{2\pi r}{2} + 2r = r(\pi + 2) = 7 \times \frac{36}{7} = 36 \text{ cm.}$$

7. AC and BD are two \perp^{lar} diameters of a circle ABCD. given that the area of the shaded portion is 308 cm^2 , calculate: (i) The length of AC and (ii) The circumference of the circle.



Sol. Given that Area of shaded region = 308 cm^2

$$\Rightarrow \text{area of } \triangle AOB + \text{area of } \triangle COD = 308 \text{ cm}^2$$

$$\Rightarrow \frac{1}{4} \pi r^2 + \frac{1}{4} \pi r^2 = 308 \quad (\because \text{shaded region} = \frac{1}{4}^{\text{th}} \text{ of circle})$$

$$\Rightarrow \frac{1}{2} \pi r^2 = 308 \Rightarrow r = \sqrt{\frac{308 \times 2 \times 7}{2.2}} = 14 \text{ cm}$$

(i) The length of AC = $2r = 2 \times 14 = 28 \text{ cm}$.

(ii) Circumference of circle = $2\pi r = 2 \times \frac{22}{7} \times 14 = 88 \text{ cm}$.

8. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. given that the bucket ascends in 1 min. 28 sec. with a uniform speed of 1.1 m/sec, calculate the no. of complete revolutions the wheel makes in raising the bucket. Take π to be $\frac{22}{7}$.

Sol. Diameter of wheel = 77 cm

$$\text{radius} = \frac{77}{2} = 38.5 \text{ cm}$$

$$\text{Circumference} = 2\pi r = 2 \times \frac{22}{7} \times 38.5 = 242 \text{ cm}$$

length of rope = $1 \frac{28}{60}$ mins. at the speed of 1.1 m/sec.

$$= 88 \times 1.1 = 96.8 \text{ m} = 9680 \text{ cm}$$

$$\therefore \text{No. of revolutions} = \frac{9680}{242} = 40$$

9. The wheel of a cart is making 5 revolutions per second. if the diameter of the wheel is 84 cm, find its speed in km/hr. give your answer correct to the nearest km.

Sol. Diameter of wheel = 84 cm \Rightarrow radius = $\frac{84}{2} = 42 \text{ cm}$.

$$\text{Circumference of the wheel} = 2\pi r = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm.}$$

$$\text{Distance covered in 5 revolutions} = 264 \times 5 = 1320 \text{ cm.}$$

$$\text{time} = 1 \text{ sec.}$$

$$\begin{aligned} \text{Speed of the wheel} &= \frac{1320}{1} \times \frac{60 \times 60}{100 \times 1000} \text{ km/hr} \\ &= 47.52 \text{ km/hr} = 48 \text{ km/hr.} \end{aligned}$$

10. The circumference of a circle is 123.2 cm, taking $\pi = \frac{22}{7}$, calculate: (i) The radius of the circle in cm (ii) The area of the circle in cm^2 , correct to the nearest cm^2 (iii) The effect on the area of the circle if the radius is doubled.

Sol.

$$\text{Circumference of a circle} = 123.2$$

$$2\pi r = 123.2 \Rightarrow r = \frac{123.2 \times 7}{2 \times 22} = 19.6 \text{ cm}$$

(i) radius $r = 19.6 \text{ cm}$

(ii) Area of the circle $= \pi r^2 = \frac{22}{7} \times 19.6 \times 19.6 = 1207.36 \text{ cm}^2$
 $= 1207 \text{ cm}^2$

(iii) If radius is doubled i.e., $2r = 2 \times 19.6 = 39.2 \text{ cm}$.

$$\text{area of the circle} = \pi r^2 = \frac{22}{7} \times 39.2 \times 39.2 = 4829.44 \text{ cm}^2.$$

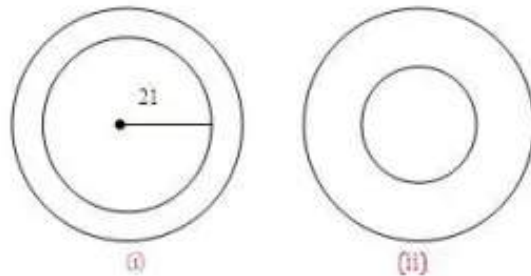
It is 4 times.

11.

(a) In the fig. (i) given below, the area enclosed between the concentric circle is 770 cm^2 given that the radius of the outer circle is 21 cm, calculate the radius of the inner circle.

(b) In the fig. (ii) given below, the area enclosed between the circumference of two concentric circles is 346.5 cm^2 . The circumference of the

inner circle is 88cm. Calculate the radius of the outer circle.



Sol.

(i) Radius of the outer circle (R) = 21 cm

let radius of inner circle = r cm

$$\text{Area of the ring} = \pi(R^2 - r^2) \Rightarrow 770 = \frac{22}{7}(441 - r^2)$$

$$\Rightarrow 441 - r^2 = \frac{770 \times 7}{22} \Rightarrow 441 - r^2 = 245$$

$$\Rightarrow r^2 = 196 \Rightarrow r = 14 \text{ cm.}$$

radius of inner circle = $r = 14$ cm.

(ii) Area of the ring = 346.5 cm^2

Circumference of inner circle = 88 cm.

$$2\pi r = 88 \Rightarrow r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm.}$$

let radius of outer circle = R

$$\text{Area of ring} = \pi(R^2 - r^2) \Rightarrow 346.5 = \frac{22}{7}(R^2 - 14^2)$$

$$\Rightarrow R^2 - 196 = \frac{346.5 \times 7}{22} \Rightarrow R^2 = 110.25 + 196 = 306.25$$

$$\Rightarrow R = \sqrt{306.25} = 17.5 \text{ cm.}$$

Radius of outer circle $R = 17.5$ cm.

12. A road 3.5m wide surrounds a circular plot whose circumference is 44m. find the cost paying the road at 50 rs. per m^2 .

Sol. Circumference of circular plot = 44m

$$2\pi r = 44 \Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7m.$$

width of the road = 3.5m.

Radius of outer circle = $7 + 3.5 = 10.5m$

$$\begin{aligned} \text{Area of the road} &= \pi(R^2 - r^2) = \frac{22}{7}(10.5^2 - 7^2) \\ &= \frac{22}{7} \times 17.5 \times 3.5 = 192.5m^2 \end{aligned}$$

Rate of paving the road = Rs. 50 per m^2 .

$$\text{total cost} = 192.5 \times 50 = \text{Rs. } 9625$$

13. The sum of diameters of two circles is 14cm and the difference of their circumference is 8cm. find the circumference of two circles.

Sol. $d_1 + d_2 = 14 \Rightarrow r_1 + r_2 = 7cm \quad \text{--- (i)}$

$$C_1 - C_2 = 8cm \quad (\because \text{Difference in Circumference})$$

$$\Rightarrow 2\pi r_1 - 2\pi r_2 = 8 \Rightarrow 2\pi(r_1 - r_2) = 8 \Rightarrow r_1 - r_2 = \frac{4}{\pi}$$

from (i) & (ii), $r_1 + r_2 = 7 \quad \text{--- (ii)}$

$$r_1 - r_2 = \frac{4}{\pi}$$

$$\underline{\hspace{10em}}$$

$$2r_1 = 7 + \frac{4}{\pi} \Rightarrow r_1 = \frac{7}{2} + \frac{2}{\pi}$$

$$r_2 = \frac{7}{2} - \frac{2}{\pi}$$

$$C_1 = 2\pi r_1 = 2\pi \left[\frac{7}{2} + \frac{2}{\pi} \right] = 7\pi + 4 = 7 \times \frac{22}{7} + 4 = 26cm$$

$$C_2 = 2\pi r_2 = 2\pi \left[\frac{7}{2} - \frac{2}{\pi} \right] = 7\pi - 4 = 7 \times \frac{22}{7} - 4 = 18cm.$$

14. Find the circumference of the circle whose area is equal to the area of three circles with radius 2cm, 3cm and 6cm.

sol. Radius of 1st circle = 2cm, Area = $\pi \times 2 \times 2 = 4\pi \text{ cm}^2$
Radius of 2nd circle = 3cm, Area = $\pi \times 3 \times 3 = 9\pi \text{ cm}^2$
Radius of 3rd circle = 6cm, Area = $\pi \times 6 \times 6 = 36\pi \text{ cm}^2$.

total area of 3 circles = $4\pi + 9\pi + 36\pi = 49\pi \text{ cm}^2$.

$$\pi r^2 = 49\pi \Rightarrow r = 7 \text{ cm.}$$

$$\text{Circumference} = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

15. A copper wire when bent in the form of a square encloses an area of 121 cm^2 . if the same wire is bent into the form of a circle, find the area of the circle.

sol. Area of the square = 121 \Rightarrow Side = 11 cm.

$$\text{Perimeter} = 4a = 4 \times 11 = 44 \text{ cm.}$$

Now Circumference of circle = 44 cm.

$$2\pi r = 44 \Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm.}$$

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2.$$

16. A copper wire when bent into an equilateral triangle has area $121\sqrt{3} \text{ cm}^2$. if the same wire is bent into the form of a circle, find the area enclosed by the wire.

sol. Area of the equilateral triangle = $121\sqrt{3} \text{ cm}^2$.

let the side of the triangle = a

$$\text{area} = \frac{\sqrt{3}}{4} a^2 \Rightarrow \frac{\sqrt{3}}{4} a^2 = 121\sqrt{3} \Rightarrow a^2 = 121 \times 4$$

$$\Rightarrow a = 22 \text{ cm.}$$

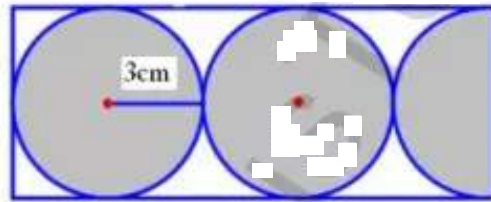
$$\text{length of wire} = 3a = 3 \times 22 = 66 \text{ cm}$$

$$\text{radius of circle} = \frac{66}{2\pi} = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm.}$$

$$\therefore \text{Area of circle} = \pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 346.5 \text{ cm}^2$$

17.

- (i) Find the circumference of the circle whose area is 16 times the area of the circle with diameter 7cm
 (ii) In the given fig. find the area of the unshaded portion within the rectangle. (take $\pi = 3.14$)



sol.

a) Diameter of circle = 7 cm \Rightarrow radius = $\frac{7}{2}$ cm.

$$\text{Area} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2.$$

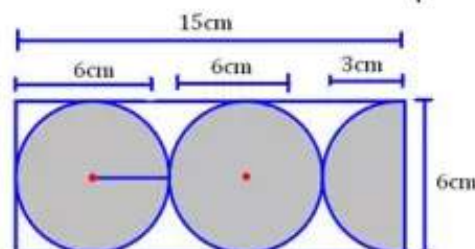
Now area of the bigger circle = $\frac{77}{2} \times 16 = 616 \text{ cm}^2.$

let radius = r cm, $\pi r^2 = 616 \Rightarrow r^2 = \frac{616 \times 7}{2} = 196$

$$\Rightarrow r = 14 \text{ cm}$$

$$\text{Circumference} = 2\pi r = 2 \times \frac{22}{7} \times 14 = 88 \text{ cm.}$$

b)



Area of shaded portion = $2 \times$ area of circle + $\frac{1}{2} \times$ area of circle.

$$= 2 \times \pi r^2 + \frac{1}{2} \times \pi r^2 = \frac{5}{2} \times \pi r^2$$

$$= \frac{5}{2} \times \frac{22}{7} \times 3 \times 3 = 70.71 \text{ cm}^2$$

Area of rectangle = $15 \times 6 = 90 \text{ cm}^2$.

Area of unshaded portion = Area of rectangle - Area of shaded portion.

$$= 90 - 70.71 = 19.29 \text{ cm}^2$$

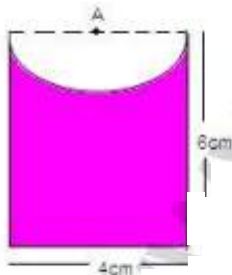
18. (a) In the fig(i) given below, A is the centre of the arc of the circle. find the perimeter and the area of the shaded region.

(b) In the fig(ii) given below, ABC is an equilateral triangle of side 14cm. side BC is the diameter of a circle. find the area of the shaded region. take

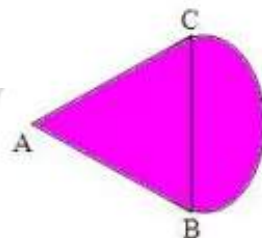
$$\sqrt{3} = 1.732$$

(c) find the perimeter and area of the shaded region in fig. (iii) given below in cm^2 correct to one place of decimal.

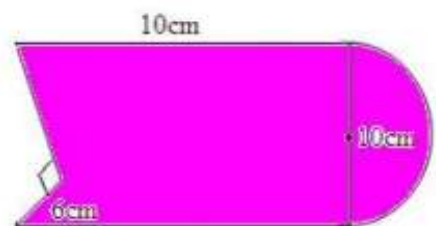
Sol.



(i)



(ii)



(iii)

a) from fig (i), radius of semicircle = $\frac{4}{2} = 2 \text{ cm}$.

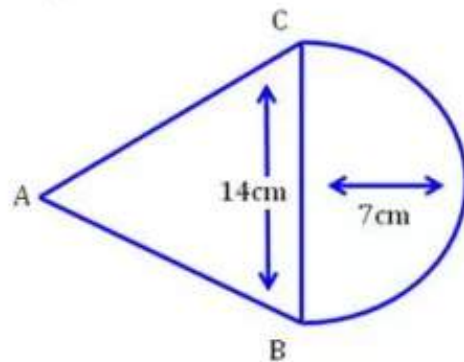
Area of shaded region = area of rectangle - Area of

Semi circle.

$$= 6 \times 4 - \frac{1}{2} \times \frac{22}{7} \times 2 \times 2 = 24 - \frac{44}{7}$$

$$= 24 - 6.28 = 17.72 \text{ cm}^2$$

b)



Area of shaded region = Area of equilateral triangle + Area of Semicircle.

$$= \frac{\sqrt{3}}{2} \times \text{side}^2 + \frac{\pi r^2}{2}$$

$$= \frac{\sqrt{3}}{2} \times 14 \times 14 + \frac{22}{7} \times \frac{7 \times 7}{2}$$

$$= 84.868 + 77 = 161.868 \text{ cm}^2.$$

(C) Radius of Semicircle = $\frac{10}{2} = 5 \text{ cm}$.

Circumference of Semicircle = $\pi r = \frac{22}{7} \times 5 = 15.7 \text{ cm}^2$

In the right $\triangle ABC$, $AC^2 = AB^2 - BC^2 = 10^2 - 6^2 = 64$

$\Rightarrow AC = 8 \text{ cm}$.

perimeter of the shaded fig. = $15.7 + 10 + 10 + 6 + 8 = 49.7 \text{ cm}$

Area of shaded fig. = Area of Square + area of Semicircle - area of $\triangle ABC$.

$$= \frac{1}{2} \pi r^2 + a^2 - \frac{1}{2} \times AC \times BC$$

$$= \frac{1}{2} \times \frac{22}{7} \times 5^2 + (10 \times 10) - \frac{1}{2} \times 8 \times 6$$

$$= \frac{11 \times 25}{7} + 100 - 24$$

$$= 39.3 + 100 - 24 = 115.3 \text{ cm}^2$$

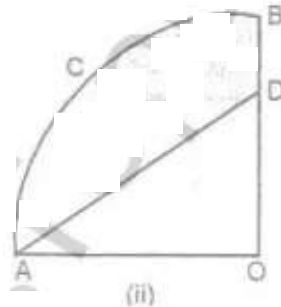
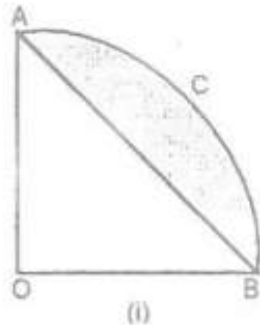
19.

a) In the fig.(i) given below, AOB is quadrant of a circle of radius 10m. Calculate the area of the shaded portion. Take $\pi = 3.14$ and give your answer correct to two significant fig.

b) The fig.(ii) given below, represents a quadrant of a circle of radius 3.5cm, centre O. Given $OD = 2$ cm, Calculate the area of the shaded portion.

(i) Calculate the area of quadrant.

(ii) Given $OD = 2$ cm, calculate the area of quadrant.



sol.

a) In the fig., shaded portion = quadrant - $\triangle AOB$.

Radius of the quadrant = 10m.

$$\text{Now area of quadrant} \Rightarrow \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{314}{4} = 78.5 \text{ m}^2.$$

$$\text{Area of } \triangle AOB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}.$$

$$\text{Area of shaded portion} = 78.5 - 50 = 28.5 \text{ m}^2$$

b) In fig(ii), radius of quadrant = 3.5cm.

$$\text{Area of quadrant} = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625 \text{ cm}^2.$$

$$\text{Area of } \triangle AOB = \frac{1}{2} \times AO \times OD = \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2$$

$$\text{Area of shaded portion} = 9.625 - 3.5 = 6.125 \text{ cm}^2.$$

20. A student takes a rectangular piece of paper 30cm long and 21cm wide. find the area of the bigger circle that can be cut out from the paper. Also find the area of the paper left after cutting out the circle.

Sol. length of rectangular piece = 30cm and width = 21cm.

$$\text{Area of rectangle} = l \times b = 30 \times 21 = 630 \text{ cm}^2.$$

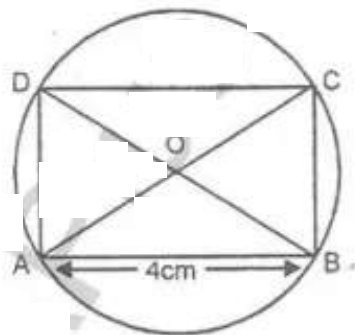
$$\text{Radius of the biggest circle} = \frac{21}{2} \text{ cm.}$$

$$\text{Area of biggest circle} = \frac{\pi}{4} \times \frac{21}{2} \times \frac{21}{2} = 346.5 \text{ cm}^2.$$

$$\text{Area of remaining part} = 630 - 346.5 = 283.5 \text{ cm}^2.$$

21. A rectangle with one side 4cm is inscribed in a circle of radius 2.5 m. find the area of the rectangle.

Sol.



In rectangle ABCD, $AB = 4\text{cm}$.

$$AC = \text{diameter of circle} = 2.5 \times 2 = 5\text{cm.}$$

$$BC = \sqrt{AC^2 - AB^2} = \sqrt{5^2 - 4^2} = 3\text{cm.}$$

$$\text{Area of rectangle} = AB \times BC = 4 \times 3 = 12 \text{ cm}^2.$$

22. (a) In the fig(i) given below, calculate the area of the shaded region correct to two decimal places.
 (b) In the fig(ii) given below, O is the centre of a circle. arc AOB is a straight line. find the

23. A circular field has perimeter 660 m. A plot in the shape of a square having its vertices on the circumference is marked in the field. Calculate the area of the square field.

sol. perimeter of circular field = 660 m.

$$2\pi r = 660 \Rightarrow r = \frac{660 \times 7}{2 \times 22} = 105 \text{ m.}$$

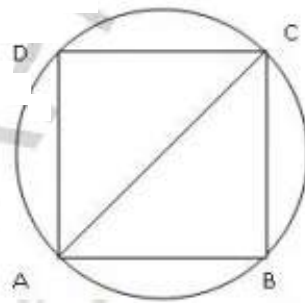
ABCD is a square which is inscribed in the circle whose diagonal is AC, which is the diameter of the circular field.

\therefore let 'a' be the side of the square, $AC = \sqrt{2} a$

$$\Rightarrow a = \frac{AC}{\sqrt{2}} = \frac{105 \times 2}{\sqrt{2}} = 105\sqrt{2} \text{ m.}$$

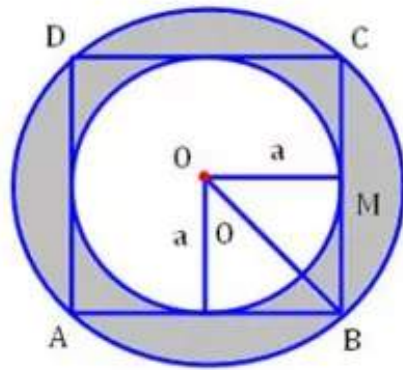
$$\therefore \text{Area of the square} = a^2 = (105\sqrt{2})^2 = 22050 \text{ m}^2.$$

24. In the adjacent fig. ABCD is a square find the ratio between (i) the circumferences (ii) the areas of the incircle and the circumcircle of the square.



sol. let a side of square be 2a circle then radius of incircle = a unit.

$$\begin{aligned} \text{radius of circumcircle (OB)} &= \sqrt{OM^2 + MB^2} \\ &= \sqrt{a^2 + a^2} \\ &= \sqrt{2} a. \end{aligned}$$

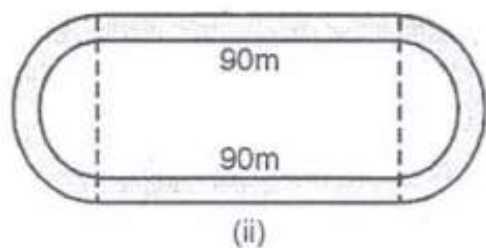
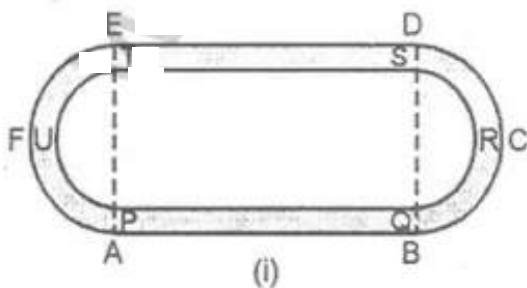


- (i) Ratio of Circumferences = $\frac{2\pi R}{2\pi r} = \frac{\sqrt{2}a}{a} = \sqrt{2} : 1$
- (ii) Area of incircle = $\pi r^2 = \pi a^2$
 Area of circumcircle = $\pi R^2 = 2\pi a^2$
 Ratio of areas = $\frac{2\pi a^2}{\pi a^2} = 2 : 1$

25. (a) The fig.(i) given below shows a running track surrounding a grassed enclosure PQRSU. The enclosure consists of a rectangle PQST with a semicircular region at each end PQ = 200m ; PT = 70m.

- (i) Calculate the area of the grassed enclosure in m^2 .
 (ii) Given that the tracks of constant width 7m, calculate the outer perimeter ABCDEF of the track (Take $\pi = \frac{22}{7}$).

(b) In the fig.(ii) given below, the inside perimeter of a practice running track with semicircular ends and straight ends and straight parallel sides is 312m. The length of the straight portion of the track is 90m. If the track has a uniform width of 2m throughout, find its area.



Sol (a) length of PQ = 200m, width PT = 70m.

(i) Area of PQST = $l \times b = 200 \times 70 = 14000 \text{ m}^2$.

Radius of each semicircular part on either side of the rectangle = $\frac{70}{2} = 35 \text{ m}$.

Area of both semicircular parts = $2 \times \frac{1}{2} \pi r^2 = \frac{22}{7} \times 35 \times 35 = 3850 \text{ m}^2$.

Total area of grassed enclosure = $14000 + 3850 = 17850 \text{ m}^2$.

(ii) width of the track around the enclosure = 7m.

outer length = 200m, and width = $70 + 7 \times 2 = 84 \text{ m}$.

outer radius = $\frac{84}{2} = 42 \text{ m}$.

Circumference of both semicircular part = $2 \times \pi r = 2 \times \frac{22}{7} \times 42 = 264 \text{ m}$.

outer perimeter = $264 + 200 \times 2 = 664 \text{ m}$.

(b) Inside perimeter = 312m.

Total length of parallel sides = $90 + 90 = 180 \text{ m}$.

Circumference of two semicircles = $312 - 180 = 132 \text{ m}$.

Radius of each semicircle = $\frac{132}{2\pi} = \frac{66}{3.14} = 21.02 \text{ m}$.

Diameter = $\frac{66}{\pi} \times 2 = \frac{132}{3.14} = 42.04 \text{ m}$.

width of track = 2m, outer diameter = $42.04 + 4 = 46.04$

Radius = $\frac{46.04}{2} = 23.02 \text{ m}$.

Now area of two semi circles = $2 \times \frac{1}{2} \times \pi r^2 = 3.14 \times (23.02)^2 = 1663.95 \text{ m}^2$.

Area of rectangle = $90 \times 46.04 = 4143.6 \text{ m}^2$

Total area = $1663.95 + 4143.60 = 5807.55 \text{ m}^2$

$$\text{area of two inner circles} = 2 \times \frac{1}{2} \times \pi r^2$$

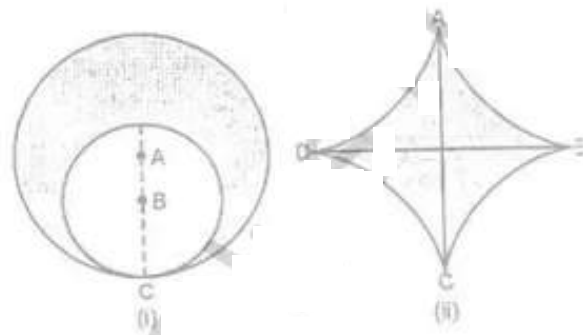
$$= 3.14 \times 21.02 \times 21.02 = 1387.38 \text{ m}^2$$

$$\text{area of inner rectangle} = 90 \times 42.04 = 3783.6 \text{ m}^2$$

$$\text{Total inner area} = 3783.6 + 1387.38 = 5170.98 \text{ m}^2$$

$$\text{Area of path} = 5807.55 - 5170.98 = 636.57 \text{ m}^2$$

26. (a) In the fig. (i) given below, two circles with centres A and B touch each other at the point C. If $AC = 8 \text{ cm}$ and $AB = 3 \text{ cm}$, find the area of the shaded region.
- (b) The quadrants shown in the fig. (ii) given below are each of radius 7 cm . Calculate the area of the shaded portion.



Sol. (a) $AC = 8 \text{ cm}$, $BC = AC - AB = 8 - 3 = 5 \text{ cm}$.

Area of bigger circle of radius $AC = \pi R^2 = \frac{22}{7} \times 8 \times 8 = 64 \times \frac{22}{7}$

and area of smaller circle = $\pi r^2 = \frac{22}{7} \times 5 \times 5 = \frac{25 \times 22}{7} \text{ cm}^2$

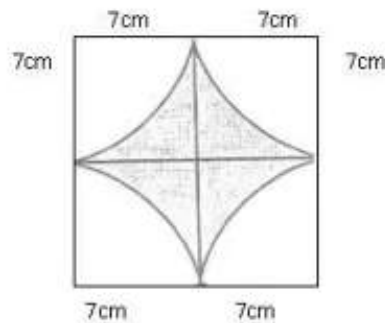
Area of shaded portion = $\frac{64 \times 22}{7} - \frac{25 \times 22}{7}$

$$= \frac{22}{7} (64 - 25)$$

$$= \frac{22}{7} \times 39$$

$$= 122.57 \text{ cm}^2$$

(b)

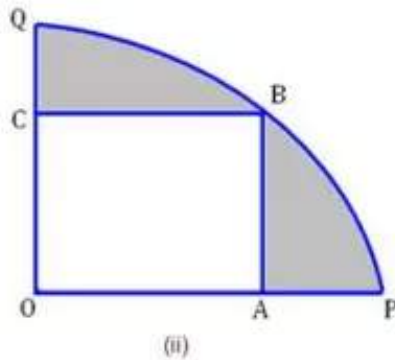
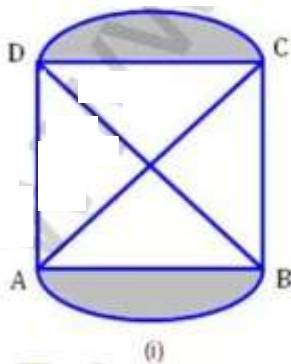


Radius of each quadrant = 7 cm.

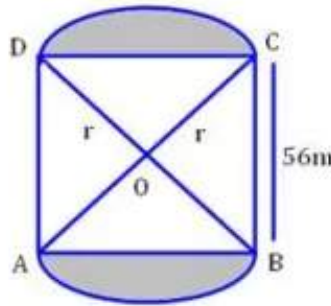
$$\begin{aligned}\text{Area of shaded region} &= \text{Area of Square} - 4 \times \text{area of quadrant} \\ &= (14)^2 - 4 \times \frac{1}{4} \times 7 \times 7 \\ &= 196 - 154 \\ &= 42 \text{ cm}^2.\end{aligned}$$

27. (a) In the fig(i), given below, two circular flower beds have been shown on the two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.

(b) In the fig(ii), given below a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region.



(a) Area of square lawn = $56 \times 56 = 3136 \text{ m}^2$.



let r be the radius of circular flower beds then

In $\triangle OBC$, $(56)^2 = r^2 + r^2 \Rightarrow 2r^2 = 56 \times 56$

$\Rightarrow r = \sqrt{\frac{56 \times 56}{2}} = 28\sqrt{2}$.

Area of flower beds = $\frac{1}{4} (\pi r^2 + \pi r^2) - (\text{Area of } \triangle OCD + \text{area of } \triangle OAB)$

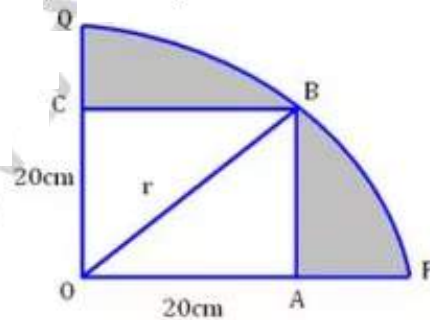
Shaded portion = $\frac{1}{2} (\pi r^2) - (\text{area of } \triangle OCD + \text{area of } \triangle OAB)$

$= \frac{1}{2} \times \frac{22}{7} \times 28 \times 28 \times 2 - \left[\frac{1}{4} (56 \times 56) + \frac{1}{4} (56 \times 56) \right]$

$= 2464 - 1568 = 896 \text{ m}^2$.

Sum of both areas = $3136 + 896 = 4032 \text{ m}^2$.

(b) In $\triangle OAB$ apply pythagoras $OB = r = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2} \text{ cm}$.



Area of shaded portion = Area of OPBQ part - Area of OACB

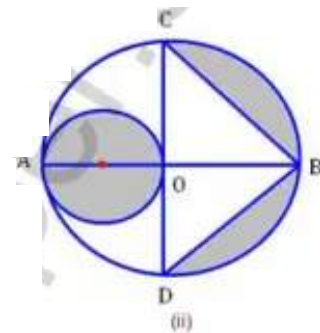
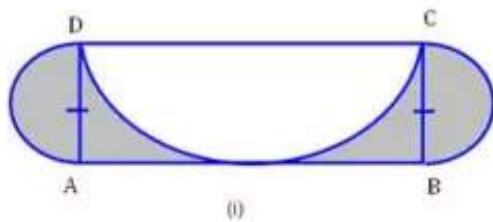
$= \frac{1}{4} \pi r^2 - 20 \times 20 = \frac{1}{4} \times \frac{22}{7} \times 20 \times 20 - 20 \times 20$

$= \frac{4400}{7} - 400 = 228.6 \text{ cm}^2$

28. (a) In the fig. given below, ABCD is a rectangle.

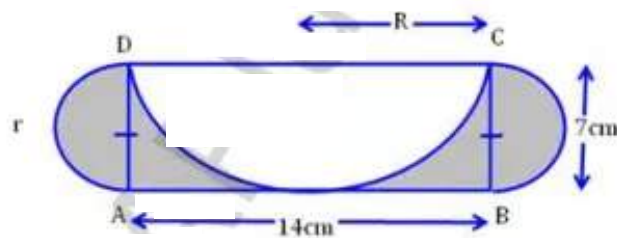
$AB = 14\text{cm}$ and $BC = 7\text{cm}$. Taking DC, BC and AD as diameters of 3 semicircles are drawn as shown in the fig. find the area of the shaded portion.

(b) In the fig. (ii) given below, AB and CD are diameters of the circle with centre O. If $OA = 7\text{cm}$ find the area of the shaded region.



sol. (a) here $r = \frac{7}{2} = 3.5\text{cm}$ [Radius of small circle]

$R = \frac{14}{2} = 7\text{cm}$ [Radius of big circle]



Area of shaded portion = 2 (Area of small semicircle) +
Area of ABCD - (Area of big semicircle)

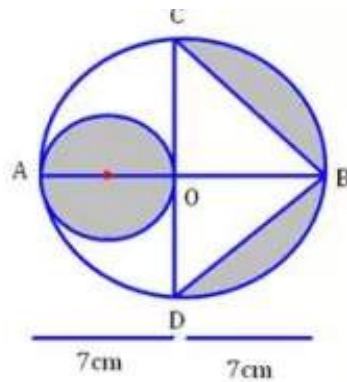
$$= \frac{2}{2} [\pi \times (3.5)^2] + 14 \times 7 - \frac{1}{2} (\pi \times 7 \times 7)$$

$$= \frac{2}{2} \times \frac{22}{7} \times 3.5 \times 3.5 + 14 \times 7 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{77}{2} + 98 - 77$$

$$= 49.5\text{cm}^2$$

(b)

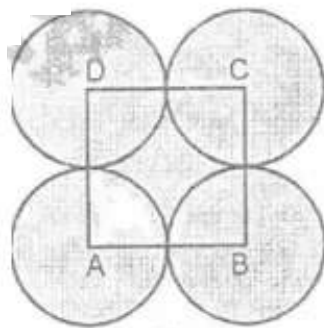


Area of shaded portion = Area of circle + Area of Semicircle $ODBC$
- area of $\triangle DBC$.

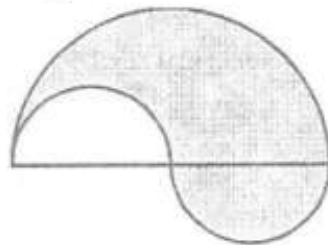
$$\begin{aligned} &= \pi \left(\frac{7}{2}\right)^2 + \frac{1}{2} \pi \times 7 \times 7 - \frac{1}{2} \times DB \times DC \\ &= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 - \frac{1}{2} \times 7 \times 14 \\ &= \frac{77}{2} + 77 - 49 = 66.5 \text{ cm}^2. \end{aligned}$$

29. (a) In the fig (i) given below, ABCD is a square of side 14 cm. A, B, C and D are centres of the equal circles which touch externally in pairs. Find the area of the shaded region.

(b) In the fig (ii) given below, the boundary of the shaded region in the given diagram consists of 3 semi-circular arcs, the smaller being equal. If the diameter of the larger one is 10 cm, calculate
(i) The length of the boundary (ii) The area of shaded region.



(i)



(ii)

Sol. (a) Side of the square = 7 cm, radius of each circle = 3.5 cm.

Area of shaded portion = Area of 4 circles + area of square -
Area of 4 quadrants at the vertices.

$$= 4 \times \pi r^2 + a^2 - 4 \times \frac{1}{4} \pi r^2 = 3\pi r^2 + a^2$$

$$= 3 \times \frac{22}{7} \times 3.5 \times 3.5 + 7 \times 7 = 164.5 \text{ cm}^2$$

(b) Radius of big semicircle = $\frac{10}{2} = 5$ cm, radius = $\frac{5}{2}$ cm.

(i) length of the boundary = Circumference of bigger semi
circle + 2 circumference of smaller semicircle.

$$= \pi R + \pi r + \pi r = 3.14 (R + 2r)$$

$$= 3.14 \left(5 + 2 \times \frac{5}{2} \right) = 31.4 \text{ cm.}$$

(ii) Area of shaded region = Area of bigger semicircle +
Area of one small circle - area of other small semicircle

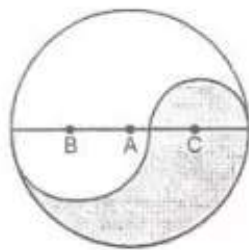
= Area of bigger semicircle.

$$= \frac{1}{2} \pi R^2 = 3.14 \times \frac{5 \times 5}{2} = 39.25 \text{ cm}^2$$

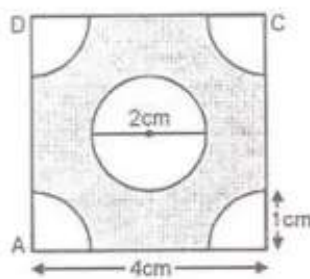
30. (a) In the figure given below, the points A, B and C are centres of arcs of circle of radii 5 cm, 3 cm and 2 cm respectively. Find the perimeter and the area of the shaded region. (leave the answer in π).

(b) In the fig (ii) given below, ABCD is a square of side 4 cm. At each corner of the square a quarter circle of radius 1 cm and at the centre of circle of diameter 2 cm are drawn. Find the perimeter and the area of

The shaded region.



(i)



(ii)

Sol. (a) Radius of bigger circle = 5 cm, Radius of smaller circle = 3 cm
radius of smaller circle = 2 cm.

(i) perimeter of the shaded region = Circumference of bigger semicircle + Circumference of smaller semicircle + Circumference of smaller semicircle.

$$= \pi R + \pi r_1 + \pi r_2 = \pi(5 + 3 + 2) = 10\pi \text{ cm.}$$

(ii) Area of the shaded region = Area of bigger semicircle + area of smaller semicircle - area of small semicircle

$$= \frac{1}{2} \pi R^2 + \frac{1}{2} \pi r_2^2 + \frac{1}{2} \pi r_1^2 = \frac{1}{2} \pi (R^2 + r_2^2 + r_1^2)$$

$$= \frac{1}{2} \pi (5^2 + 2^2 - 3^2) = \frac{1}{2} \pi (25 + 4 - 9) = 10\pi \text{ cm}^2$$

(b) Side of square ABCD = 4 cm, Radius of each quadrant circle = 1 cm and radius of circle in the square = $\frac{2}{2} = 1$ cm

(i) perimeter of shaded portion = Circumference of 4 quadrants + Circumference of circle + $4 \times \frac{1}{2}$ Side of square.

$$= 4 \times \frac{1}{4} (2\pi r) + (2\pi r) + 4 \times 2$$

$$= 2\pi r + 2\pi r + 8 = 4 \times 3.14 \times 1 + 8$$

$$= 20.56 \text{ cm.}$$

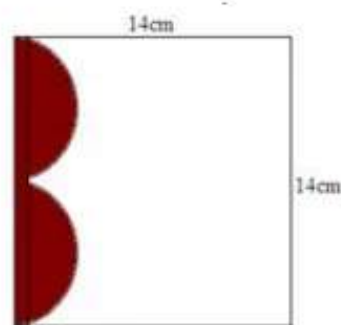
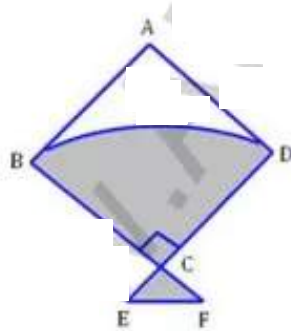
(ii) Area of shaded region = Area of Square - area of 4 quad - area of circle.

$$= \text{Side}^2 - 4 \times \frac{1}{4} \pi r^2 - \pi r^2$$

$$= 4^2 - \pi r^2 - \pi r^2 = 16 - 2 \times 3.14 \times 1^2 = 9.72 \text{ cm}^2$$

31. (a) The fig. (i) given below shows a kite in which BCD is in the shape of a quadrant of a circle of radius 42cm. ABCD is a square and $\triangle CEF$ is an isosceles right angled triangle whose equal sides are 6cm long. find the area of the shaded region.

(b) In the fig. (ii) given below from a sheet of cardboard in the shape of a square of side 14cm a piece in the shape of the letter B is cut off. The curved side of the letter consists of two equal semicircle and the breadth of the rectangular piece is 1cm. find the area of the remaining part of the cardboard.

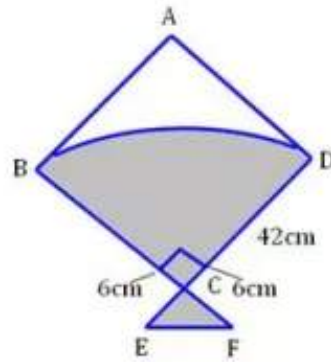


sol. (a) Area of shaded region = Area of circular part BCD + area of $\triangle CEF$.

$$= \frac{1}{4} \pi r^2 + \frac{1}{2} \times CF \times CE$$

$$= \frac{1}{4} \times \frac{22}{7} \times 42 \times 42 + \frac{1}{2} \times 6 \times 6$$

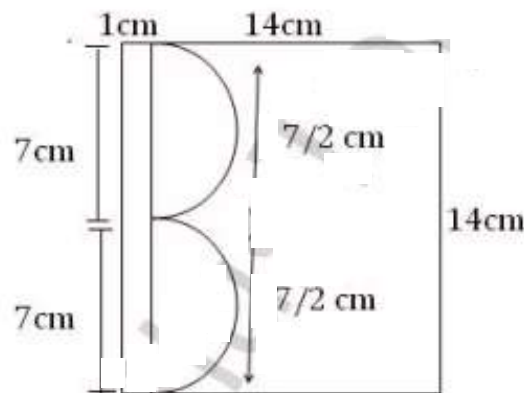
$$= 1404 \text{ cm}^2$$



(b) Area of remaining part = Area of Square - Area of rectangular piece - Area of two Semicircles

$$= 14 \times 14 - 14 \times 1 - 2 \left(\frac{1}{2} \pi \times \frac{7}{2} \times \frac{7}{2} \right)$$

$$= 196 - 14 - \frac{77}{2} = 143.5 \text{ cm}^2.$$

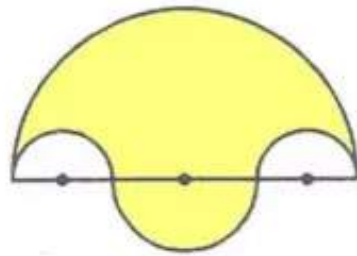


32. (a) In the fig.(i) given below, the boundary of the shaded region in the given diagram consists of four semi circular arcs the smaller two being equal. if the diameter of the largest is 14cm and the smallest is 3.5cm, calculate (i) The length of the boundary.

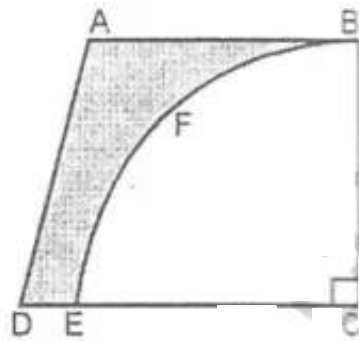
(ii) the area of the shaded region.

(b) In the fig.(ii) given below, a piece of cardboard in the shape of a trapezium ABCD, and $AB \parallel DC$ and $\angle C = 90^\circ$, quarter circle BFEC is removed.

Given $AB = BC = 3.5 \text{ cm}$ and $DE = 2 \text{ cm}$. Calculate the area of the remaining piece of the cardboard.



(i)



(ii)

(a) Sol. (i) length of boundary = circumference of bigger semicircle + circumference of small semicircle + 2 x circumference of the smaller semicircle.

$$= \pi R + \pi r_1 + 2 \pi r_2 = \pi (R + r_1 + 2r_2)$$

$$= \frac{22}{7} (7 + 3.5 + 2 \times \frac{3.5}{2}) = 33 + 11 = 44 \text{ cm.}$$

(ii) Area of shaded region = Area of bigger semicircle + area of small semicircle - 2 x area of smaller semicircle.

$$= \frac{1}{2} \pi (7)^2 + \frac{1}{2} \pi (3.5)^2 - 2 \times \frac{1}{2} \pi (1.75)^2$$

$$= 77 + 19.25 - 9.625 = 86.625 \text{ cm}^2.$$

(b) ABCD is a trapezium in which $AB \parallel DC$ and $\angle C = 90^\circ$.

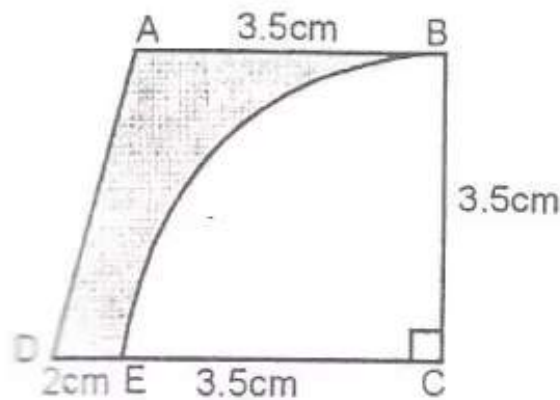
$AB = BC = 3.5 \text{ cm}$, $DE = 2 \text{ cm}$.

Radius of quadrant = 3.5 cm .

$$\text{Area of trapezium} = \frac{1}{2} (AB + DC) \times BC$$

$$= \frac{1}{2} (3.5 + 3.5 + 2) \times 3.5$$

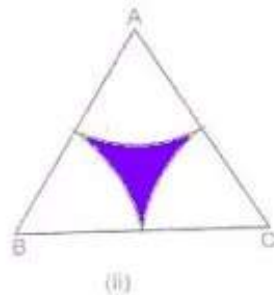
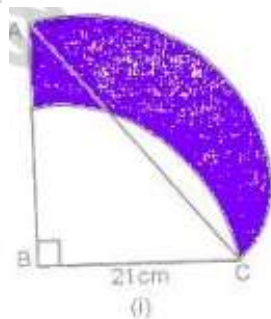
$$= 15.75 \text{ cm}^2$$



$$\text{Area of quadrant} = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625 \text{ cm}^2$$

$$\therefore \text{Area of shaded portion} = 15.75 - 9.625 = 6.125 \text{ cm}^2$$

33. (a) In the fig(i) given below, ABC is a right angle triangle, $\angle B = 90^\circ$, AB = 28 cm and BC = 21 cm. with AC as diameter a semicircle is drawn and with BC as radius a quarter circle is drawn. find the area of the shaded region correct to two decimal places.
- (b). In the fig(ii) given below, ABC is an equilateral Δ of side 8 cm. A, B and C are the centres of circular arcs of radius 4 cm. find the area of the shaded portion correct upto 2 decimal places.



sol. (a) In right $\triangle ABC$, $\angle B = 90^\circ$

$$AC^2 = AB^2 + BC^2 = (28)^2 + (21)^2 = 784 + 441 = 1225.$$

$$\therefore AC = \sqrt{1225} = 35 \text{ cm.}$$

$$\text{Radius of semicircle (R)} = \frac{35}{2}$$

$$\text{Radius of quadrant (r)} = 21 \text{ cm}$$

Area of shaded region = Area of $\triangle ABC$ + Area of Semicircle -
area of quadrant.

$$= \frac{1}{2} \times 28 \times 21 + \frac{1}{2} \pi R^2 - \frac{1}{2} \pi r^2$$

$$= 294 + \frac{1}{2} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{2} \times \frac{22}{7} \times 21 \times 21$$

$$= 294 + 481.25 - 346.5$$

$$= 428.75 \text{ cm}^2.$$

(b) $\triangle ABC$ is an equilateral \triangle of side 8m. At A, B and C as centre three sectors are drawn with radius 4cm

$$\begin{aligned} \text{Now area of } \triangle ABC &= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 8 \times 8 = 16\sqrt{3} \text{ cm}^2 \\ &= 27.712 \text{ cm}^2 \end{aligned}$$

Area of 3 equal sector of 60° whose radius 4cm

$$= 3 \times \pi r^2 \times \frac{60^\circ}{360^\circ}$$

$$= 3 \times \frac{1}{6} \times 3.142 \times 4 \times 4$$

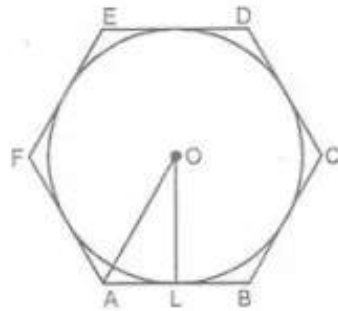
$$= 25.136 \text{ cm}^2$$

$$\text{Area of shaded region} = 27.712 - 25.136$$

$$= 2.58 \text{ cm}^2.$$

34. A Circle inscribed in a regular hexagon of side $2\sqrt{3}$ cm. find (i) The circumference of the inscribed circle, (ii) The area of the inscribed circle.

Sol.



ABCDEF is a regular hexagon of side $2\sqrt{3}$ cm and a circle is inscribed in it with Centre O.

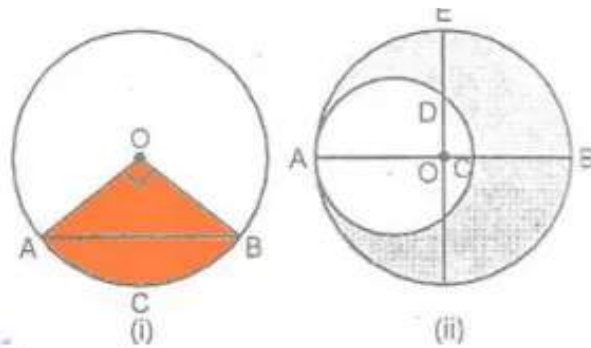
$$\begin{aligned} \text{Radius of inscribed circle} &= \frac{\sqrt{3}}{2} \times \text{Side of regular hexagon} \\ &= \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3 \text{ cm.} \end{aligned}$$

(i) Circumference = $2\pi r = 2\pi \times 3 = 6\pi$ cm.

(ii) Area = $\pi r^2 = \pi \times 3 \times 3 = 9\pi$ cm².

35. (a) In the fig.(i) given below, a chord AB of a circle of radius 10 cm, subtends a right angle at the Centre O. find the area of the Sector OACB and of the major segment.

(b) In the fig.(ii) given below, Crescent is formed by two circle with touch at the point A. O is the Centre of the bigger circle. If CB = 9 cm and ED = 5 cm, find the area of the shaded region.



Sol. (a) Radius of circle = 10 cm.

Angle at the Centre subtended by a chord $AB = 90^\circ$.

$$\text{Area of sector } OACB = \pi r^2 \times \frac{90^\circ}{360^\circ} = 3.14 \times 10 \times 10 \times \frac{1}{4} = 78.5 \text{ cm}^2$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

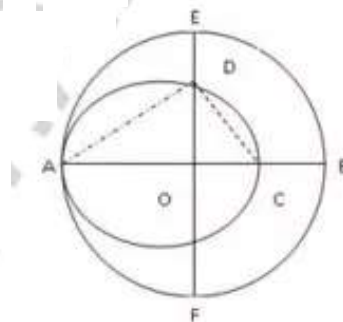
$$\begin{aligned} \text{Area of minor segment} &= \text{Area of sector } OACB - \text{Area of } \triangle OAB \\ &= 78.5 - 50 = 28.5 \text{ cm}^2 \end{aligned}$$

$$\text{Area of circle} = \pi r^2 = 3.14 \times 10 \times 10 = 314 \text{ cm}^2$$

$$\begin{aligned} \text{Area of major segment} &= \text{Area of circle} - \text{Area of minor segment} \\ &= 314 - 28.5 = 285.5 \text{ cm}^2 \end{aligned}$$

(b) Two circles touch each other internally, O is the centre of bigger circle. $CB = 9 \text{ cm}$, $ED = 5 \text{ cm}$.

Let R and r be the radii of the circles.



$$\therefore AB - AC = 9 \text{ cm} \Rightarrow 2R - 2r = 9 \text{ cm} \Rightarrow R - r = \frac{9}{2} \text{ cm}$$

Join AD and CD. Now in $\triangle AOD$ and $\triangle COD$,

$$\angle AOD = \angle COD \quad (\text{each } 90^\circ)$$

$$\angle DAO = \angle ODC$$

$\therefore \triangle AOD \sim \triangle COD$ (AA criterion of similarity)

$$\frac{OD}{OA} = \frac{OC}{OD} \Rightarrow OD^2 = OA \cdot OC \Rightarrow (R-5)^2 = R(R-9)$$

$$\Rightarrow R^2 - 10R + 25 = R^2 - 9R \Rightarrow R = 25$$

$$\text{But } R - r = \frac{9}{2} \Rightarrow 25 - r = \frac{9}{2} \Rightarrow r = 25 - \frac{9}{2} = \frac{41}{2} \text{ cm.}$$

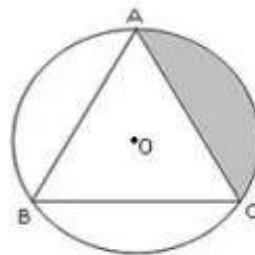
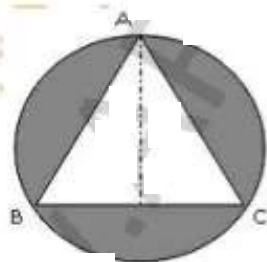
$$\text{Now area of shaded region} = \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$$

$$= 3.14 \left(25^2 - \left(\frac{41}{2} \right)^2 \right) = 3.14 \left[25 + \frac{41}{2} \right] \left[25 - \frac{41}{2} \right]$$

$$= 3.14 \times \frac{91}{2} \times \frac{9}{2} = 642.92 \text{ cm}^2.$$

36. (a) In the fig (i) given below, ABC is an equilateral triangle inscribed in a circle of radius 4cm. find the area of the shaded region.

(b) In the fig (ii) given below, ABC is an equilateral triangle inscribed a circle of radius 4cm with centre O. find the area of the shaded region.



Sol. (a) $\triangle ABC$ is an equilateral triangle which is inscribed in a circle radius = 4cm.

let R be the radius of circumcircle, and 'a' be the side of the equilateral triangle ABC and h is its height.

$$R = \frac{2}{3} h \text{ and } h = \frac{\sqrt{3}}{2} a$$

$$R = \frac{2}{3} \times \frac{\sqrt{3}}{2} a = \frac{1}{\sqrt{3}} a = 4 \Rightarrow a = 4\sqrt{3} \text{ cm.}$$

Now area of shaded region = area of circle - area of $\triangle ABC$

$$\begin{aligned} &= \pi R^2 - \frac{\sqrt{3}}{4} a^2 = \pi(4)^2 - \frac{\sqrt{3}}{4} (4\sqrt{3})^2 \\ &= 16\pi - \frac{\sqrt{3}}{4} \times 4\sqrt{3} \times 4\sqrt{3} = 16\pi - 3\sqrt{3} \times 4 \\ &= 4(4\pi - 3\sqrt{3}) \text{ cm}^2. \end{aligned}$$

(b) $\triangle ABC$ which is an equilateral triangle is inscribed in centre of radius = 4cm. O is the centre of circle. Join AO and OC. $\angle AOC = 2 \angle ABC = 2 \times 60^\circ = 120^\circ$.

Area of shaded region = $\frac{1}{3}$ (Area of circle - area of $\triangle ABC$)

$$\begin{aligned} &= \frac{1}{3} \left[\pi R^2 - \frac{\sqrt{3}}{4} a^2 \right] \\ &= \frac{1}{3} \left[\pi(4)^2 - \frac{\sqrt{3}}{4} (4\sqrt{3})^2 \right] \\ &= \frac{1}{3} \left[16\pi - \frac{\sqrt{3}}{4} \times 4\sqrt{3} \times 4\sqrt{3} \right] \\ &= \frac{1}{3} [16\pi - 12\sqrt{3}] \\ &= \frac{4}{3} [4\pi - 3\sqrt{3}] \text{ cm}^2. \end{aligned}$$

EXERCISE - 17.2

1. The diameter of the base of a right circular cylinder is 28cm and its height is 21cm. find its
(i) Curved Surface area (ii) Total Surface area
(iii) Volume.

Sol. Radius of the base of cylinder (r) = $\frac{28}{2} = 14$ cm.

height (h) = 21 cm.

(i) Curved Surface area = $2\pi rh = 2 \times \frac{22}{7} \times 14 \times 21 = 1848 \text{ cm}^2$

(ii) Total Surface area = $2\pi r(r+h) = 2 \times \frac{22}{7} \times 14(21+14)$
 $= 3080 \text{ cm}^2$.

(iii) volume = $\pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 21 = 12936 \text{ cm}^3$.

2. The height of a circular cylindrical pillar is 10.5m and the diameter of its base is 90cm. what will be cost of painting the curved surface of the pillar at Rs. 40/m²?

Sol. Radius of the base of pillar (r) = $\frac{90}{2} = 45$ cm.

Height (h) = 10.5 m.

Curved Surface area = $2\pi rh = 2 \times \frac{22}{7} \times \frac{45}{100} \times 10.5 = 29.7 \text{ m}^2$

Rate of painting = 40 rs. per m²

total cost of painting = $29.7 \times 40 = 1188$.

3. A log of wood is 3.5m long and 40cm in diameter, find its weight if the woods weight 225 kg. per m³.

Sol. Radius of the log (r) = $\frac{40}{2} = 20 \text{ cm} = \frac{1}{5} \text{ m}$

$$\text{length of log (h)} = 3.5 \text{ m}$$

$$\text{volume} = \pi r^2 h = \frac{22}{7} \times \frac{1}{5} \times \frac{1}{5} \times 3.5 = 0.44 \text{ m}^3.$$

$$\text{weight of the wood} = 225 \text{ kg per m}^3.$$

$$\text{Total weight of the log} = 0.44 \times 225 = 99 \text{ kg.}$$

4. A solid metallic circular cylinder of radius 14cm and height 12cm is melted and recast into small cubes of edge 2cm. How many such cubes can be made from the solid cylinder?

Sol. Radius of the solid cylinder (r) = 14cm.

$$\text{height (h)} = 12 \text{ cm.}$$

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 12 = 7392 \text{ cm}^3.$$

$$\text{volume of one cube of edge 2cm} = (2)^3 = 8 \text{ cm}^3.$$

$$\text{Total no. of cubes} = \frac{\text{Volume of solid cylinder}}{\text{Volume of one cube}} = \frac{7392}{8} = 924.$$

5. A hollow copper pipe of inner diameter 6cm and outer diameter 10cm is melted and changed into a solid circular cylinder of the same height as that of the pipe. Find the diameter of the solid cylinder.

Sol. outer radius of the pipe (R) = $\frac{10}{2} = 5 \text{ cm.}$

$$\text{Inner radius (r)} = \frac{6}{2} = 3 \text{ cm.}$$

$$\text{let height of the pipe} = h.$$

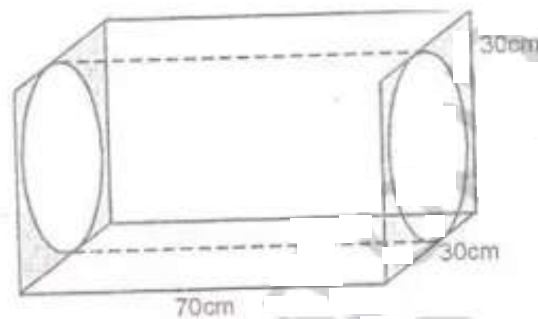
$$\text{Volume of pipe} = \pi (R^2 - r^2) h = \pi \cdot h (5^2 - 3^2) = 16\pi h \text{ cm}^3$$

let the diameter of the solid cylinder = $2r$

$$\text{Volume} = \pi r^2 h = 16\pi h \Rightarrow r^2 = 16 \Rightarrow r = 4$$

\therefore diameter of solid cylinder = $2 \times 4 = 8$ cm.

6. The adjoining fig. shows a cuboidal block of wood through which a circular cylindrical hole of the biggest size is drilled. find the volume of the wood left in the block.



sol. Diameter of the biggest hole = 30cm.

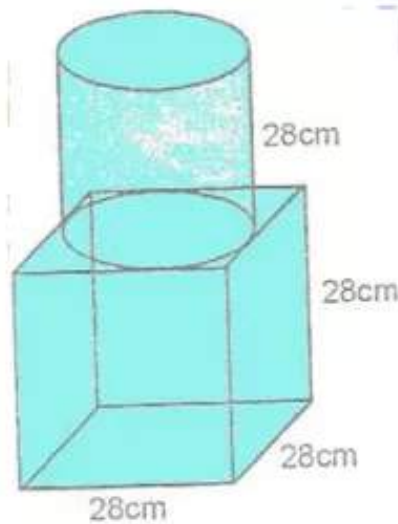
$$\text{radius } (r) = \frac{30}{2} = 15\text{cm}, \text{ height } (h) = 70\text{cm}.$$

$$\begin{aligned} \text{Volume of the hole made} &= \pi r^2 h = \frac{22}{7} \times 15 \times 15 \times 70 \\ &= 49500 \text{ cm}^3. \end{aligned}$$

$$\text{Total volume of the log} = 70 \times 30 \times 30 = 63000 \text{ cm}^3.$$

$$\text{Volume of the wood left} = 63000 - 49500 = 13500 \text{ cm}^3.$$

7. The adjoining fig. shows a solid trophy made of shining glass. If one cubic centimeter of glass cost Rs. 0.75. find the cost of the glass for making the trophy.



sol. Edge of cubical part = 28cm, radius of cylinder (r) = 14cm.
Height (h) = 28cm.

$$\text{Total volume of the trophy} = (\text{edge})^3 + \pi r^2 h$$

$$= (28)^3 + \frac{22}{7} \times 14 \times 14 \times 28 = 39200 \text{ cm}^3$$

Rate of cost of glass = Rs. 0.75 per cm^3 .

$$\text{Total cost} = 39200 \times 0.75 = \text{Rs. } 29400$$

8. An Agriculture field is in the form of a rectangle of length 20m and width 14m. A 10m deep well of diameter 7m is dug in the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in its level, correct to 2 decimal places.

sol. length of field = 20m and width = 14m.

$$\text{Area} = l \times b = 20 \times 14 = 280 \text{ m}^2$$

Radius of the well (r) = $\frac{7}{2}$ m and depth (h) = 10m.

$$\text{Volume of earth so dug out} = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10$$

$$= 385 \text{ m}^3$$

$$\text{Surface area of the well in the field} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 38.5 \text{ m}^2$$

$$\text{Area of remaining part of the field} = 80 - 38.5 = 41.5 \text{ m}^2$$

let height of earth spread = h

$$41.5 h = 385 \Rightarrow h = \frac{385}{41.5} = 9.28 \text{ m.}$$

9. A copper rod of diameter 1cm and length 8cm is drawn into a wire of length 18m of uniform thickness. find the thickness of the wire.

Sol. Radius of rod = $\frac{1}{2}$ cm, length of rod = 8cm.

$$\text{Volume of rod} = \pi r^2 h = \pi \times \frac{1}{2} \times \frac{1}{2} \times 8 = 2\pi \text{ cm}^3.$$

let the thickness of wire = $2r$, radius = r

$$\text{length} = 18 \text{ m} = 1800 \text{ cm.}$$

$$\text{Volume of the wire} = \pi r^2 h = \pi r^2 \times 1800 = 1800\pi r^2 \text{ cm}^3$$

$$\therefore 1800\pi r^2 = 2\pi \Rightarrow r^2 = \frac{2\pi}{1800\pi} = \frac{1}{900}$$

$$\Rightarrow r = \frac{1}{30} \Rightarrow 2r = 2 \times \frac{1}{30} = \frac{1}{15} \text{ cm.}$$

$$\text{Thickness of wire} = 2r = \frac{1}{15} \text{ cm.}$$

10. Two right circular cylinders have equal volume. If their heights are in the ratio 4:9, find the ratio of their radii.

Sol. Let r_1 and r_2 be the radii of the cylinders and h_1 and h_2 be their heights respectively then

$$h_1 : h_2 = 4 : 9 \Rightarrow \frac{h_1}{h_2} = \frac{4}{9}$$

Their volumes are equal

$$\pi r_1^2 h_1 = \pi r_2^2 h_2 \Rightarrow \frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$$

$$\Rightarrow \frac{4}{9} = \frac{r_2^2}{r_1^2} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$$

\therefore Ratio of their radii 3:2.

11. A hollow right circular cylinder of length 32cm, external diameter 20cm, and thickness 4mm is to be made. What length of a solid right circular cylinder of diameter 8cm must be recast to make the hollow cylinder?

Sol. External diameter of hollow cylinder = 20cm and thickness = 4mm = 0.4cm.

$$\text{Internal diameter} = 20 - 0.8 = 19.2 \text{ cm}$$

$$\text{Length of cylinder (h)} = 32 \text{ cm}$$

Outer radius (r) = 10cm and inner radius (r) = 9.6cm

$$\text{Volume} = \pi (R^2 - r^2) h = \pi \times 32 [10^2 - (9.6)^2] = 32\pi \times 7.84 \text{ cm}^3$$

— (i)

let length of solid cylinder = h , radius = $\frac{8}{2} = 4\text{cm}$

$$\text{Volume} = \pi r^2 h = \pi \times 4 \times 4 \times h = 16\pi h \text{ cm}^3. \quad \text{--- (ii)}$$

From (i) and (ii),

$$16\pi h = 32\pi \times 7.84 \Rightarrow h = \frac{32\pi \times 7.84}{16\pi} = 15.68\text{cm}.$$

12. The difference between the outer and inner curved surfaces of a circular cylinder 14cm long is 88cm^2 . Find the outer and the inner radii of the cylinder, given that the volume of the metal is 176cm^3 .

Sol. Difference between the outer and inner curved surface area of a cylinder = 88cm^2

$$\text{length of cylinder (h)} = 14\text{cm}$$

$$\text{Volume of metal used} = 176\text{cm}^3.$$

let R and r be the radii of outer and inner of the base of cylinder

\therefore Difference between outer and inner curved surface area

$$2\pi R h - 2\pi r h = 2\pi h(R-r) = 88$$

$$\text{Volume} = \pi (R^2 - r^2) h = 176\text{cm}^3$$

$$\Rightarrow \frac{\pi h (R^2 - r^2)}{2\pi h (R-r)} = \frac{176}{88} \Rightarrow \frac{(R+r)(R-r)}{2(R-r)} = \frac{2}{1}$$

$$\Rightarrow \frac{R+r}{2} = \frac{2}{1} \Rightarrow R+r = 4 \quad \text{--- (i)}$$

$$\text{But } 2\pi h(R-r) = 88$$

$$\Rightarrow 2 \times \frac{22}{7} \times 14 (R-r) = 88$$

$$\Rightarrow R-r = \frac{88 \times 7}{2 \times 22 \times 14} = 1 \quad \text{--- (ii)}$$

Adding (i) & (ii), $2R = 5 \Rightarrow R = 2.5 \text{ cm}$

Subtracting (i) & (ii), $2r = 3 \Rightarrow r = 1.5 \text{ cm}$

Hence radii are 2.5 cm and 1.5 cm

13. water flows through a cylindrical pipe of internal diameter 7 cm at 36 km/hr. calculate the times in minutes it would take to fill a cylindrical tank, the radius of whose base is 35 cm and height 1 m.

Sol. Inner radius of cylindrical pipe = $\frac{7}{2}$ cm.

Speed of water = 36 km/hr

Radius of cylindrical tank = 35 cm, height (h) = 1 m
= 100 cm.

$$\text{Volume of tank} = \pi r^2 h = \frac{22}{7} \times 35 \times 35 \times 100 = 385000 \text{ cm}^3 \quad \text{--- (i)}$$

Now inner radius of pipe = $\frac{7}{2}$ cm.

let length of water in pipe = h_1

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1 = \frac{77}{2} h_1 \text{ cm}^3 \quad \text{--- (ii)}$$

From (i) & (ii), $\frac{77}{2} h_1 = 385000 \Rightarrow h_1 = \frac{385000 \times 2}{77}$

$$\Rightarrow h_1 = 10,000 \text{ cm} = 100 \text{ m}$$

Speed of water = 36 km/hr

$$\begin{aligned} \text{Time taken to fill the tank} &= \frac{100}{1000 \times 36} \text{ hr} = \frac{1}{360} \text{ hr} \\ &= \frac{1}{360 \times 60} = \frac{1}{6} \text{ minutes.} \end{aligned}$$

14. water flows through a circular pipe internal radius 7cm at 5m per second. if the pipe is always half full, find the number of litres of water delivered by the pipe in 10 minutes.

Sol.

$$\text{Internal radius of pipe} = 7 \text{ cm} = \frac{7}{100} \text{ m.}$$

$$\text{Flow of water} = 5 \text{ m in 1 sec.}$$

$$\text{Time taken} = 5 \text{ minutes.}$$

$$\text{length of pipe filled with water} = 5 \times 60 \times 5 = 1500 \text{ m.}$$

$$\text{Volume of water} = \pi r^2 h = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 1500 = 23.10 \text{ m}^3.$$

$$\text{Amount of water in litres} = 23.10 \times 1000 = 23100$$

15.

In the adjoining fig. a rectangular platform is given with a semi circular end on one side. It is 23.5 long from one end to the other. if the length of the half the circumference is 22m, find the cost of constructing is the platform $2\frac{1}{2}$ m high at the Rs. 180 per cubic meter. Take $\pi = \frac{22}{7}$.



sol.

Total of platform = 23.5 m.

Circumference of half circle = 22 m.

$$\text{radius} = \frac{22 \times 7}{2 \times 2} = 7 \text{ m and diameter} = 7 \times 2 = 14 \text{ m.}$$

width of plane = 14 m and height = $2\frac{1}{2}$ m = $\frac{5}{2}$ m.

length of rectangular platform = $23.5 - 7 = 16.5$ m

Now volume of platform = $lbh + \frac{1}{2} \pi r^2 \times h$

$$= 16.5 \times 14 \times 2.5 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \times 2.5$$

$$= 577.5 + 192.5$$

$$= 770 \text{ m}^3$$

Rate of constructing per m^3 = Rs. 180

\therefore Total cost = $770 \times 180 = \text{Rs. } 138600$.

EXERCISE -17.3

1. Calculate the curved surface area of a cone whose perpendicular height is 4.8 and the radius of whose base is 3.6m. give your answer in terms of π .

Sol. Radius of the cone (r) = 3.6m, height (h) = 4.8cm

$$\text{Slant height } (l) = \sqrt{r^2 + h^2} = \sqrt{(3.6)^2 + (4.8)^2} = \sqrt{36} = 6 \text{ cm.}$$

$$\text{Curved Surface area} = \pi r l = \pi \times 3.6 \times 6 = 21.6\pi \text{ cm}^2.$$

2. Find the volume of the cone given that its slant height is 17cm and radius is 8cm.

Sol. Radius of cone (r) = 8 cm, slant height (l) = 17 cm.

Let the height of the cone = h

$$\therefore l^2 = r^2 + h^2 \Rightarrow h^2 = l^2 - r^2 = 17^2 - 8^2$$

$$\Rightarrow h = \sqrt{289 - 64} = \sqrt{225} = 15 \text{ cm.}$$

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} \pi r^2 h = \frac{1}{3} (3.142) \times 8 \times 8 \times 15 \\ &= 1005.44 \text{ cm}^3. \end{aligned}$$

3. Find the volume and total surface area of a cone having slant height 17cm and the base diameter 30cm. Take $\pi = 3.14$

Sol. Radius of the base of a cone (r) = $\frac{30}{2} = 15 \text{ cm.}$

Slant height (l) = 17 cm.

Let h be the perpendicular height then

$$l^2 = r^2 + h^2$$

$$\Rightarrow h^2 = l^2 - r^2 = 17^2 - 15^2 = 64$$

$$\Rightarrow h = 8 \text{ cm.}$$

$$(i) \text{ volume of cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times 3.14 \times 15 \times 15 \times 8 = 1884 \text{ cm}^3.$$

$$(ii) \text{ Total Surface area} = \pi r^2 + \pi r l = \pi r (r + l) \\ = 3.14 \times 15 (15 + 17) = 3.14 \times 15 \times 32 \\ = 1507.20 \text{ cm}^2$$

4. Find the volume of a cone given that its height is 8cm and area of base is 156 cm^2 .

Sol.

$$\text{Area of the base of cone} = 156 \text{ cm}^2.$$

$$\text{and height (h)} = 8 \text{ cm.}$$

$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times 156 \times 8 = 416 \text{ cm}^3.$$

5. The perimeter of the base of a cone is 44cm and slant height is 25cm. find the volume and Curved Surface area of the cone.

Sol.

$$\text{perimeter of the base} = 44 \text{ cm.}$$

$$\text{radius (r)} = \frac{44}{2\pi} = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm.}$$

$$\text{slant height (l)} = 25 \text{ cm.}$$

let h be the perpendicular height on the cone then

$$l^2 = r^2 + h^2 \Rightarrow h^2 = l^2 - r^2 = (25)^2 - (7)^2$$

$$\Rightarrow h^2 = 576 \Rightarrow h = 24 \text{ cm.}$$

$$(i) \text{ Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 = 1232 \text{ cm}^3.$$

$$(ii) \text{ Curved Surface area} = \pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2.$$

6. The volume of a right circular cone is 660 cm^3 . If the diameter of its base is 12 cm , find (i) the height of the cone (ii) the slant height of the cone.

Sol.

$$\text{Volume of the cone} = 660 \text{ cm}^3$$

$$\text{Radius (r)} = \frac{12}{2} = 6 \text{ cm.}$$

Let h be the perpendicular height and l be the slant height, then

$$\frac{1}{3} \pi r^2 h = 660 \Rightarrow \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times h = 660$$

$$\Rightarrow h = \frac{660 \times 3 \times 7}{22 \times 6 \times 6} = 17.5 \text{ cm.}$$

$$\text{Slant height (l)} = \sqrt{r^2 + h^2} = \sqrt{(6)^2 + (7.5)^2} = \sqrt{342.25} \\ = 18.5 \text{ cm.}$$

7. The curved surface area of a cone is 550 cm^2 . Find the volume of the cone given that its base diameter is 14 cm .

Sol.

$$\text{Curved surface area} = 550 \text{ cm}^2, \text{ radius (r)} = \frac{14}{2} = 7 \text{ cm.}$$

$$\pi r l = 550 \Rightarrow \frac{22}{7} \times 7 \times l = 550$$

$$\Rightarrow l = \frac{550}{22} = 25 \text{ cm.}$$

Let h be the perpendicular height, then

$$l^2 = r^2 + h^2$$

$$\Rightarrow h^2 = d^2 - r^2 = (25)^2 - (7)^2 = 625 - 49$$

$$\Rightarrow h = \sqrt{576} = 24 \text{ cm.}$$

$$\text{volume of the cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 = 1232 \text{ cm}^3$$

8. The radius and the height of a cone are in the ratio 1:3 and its volume is 1078 cm^3 . find its diameter and the lateral surface area correct to two decimal places.

Sol.

let radius of cone = r and their height (h) = $3r$

$$\text{volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 \times 3r = \pi r^3.$$

$$\therefore \pi r^3 = 1078 \Rightarrow r^3 = \frac{1078 \times 7}{22} = 343 = (7)^3$$

$$\Rightarrow r = 7 \text{ cm.}$$

$$\text{height } h = 3r = 3 \times 7 = 21 \text{ cm.}$$

$$\text{but } l^2 = r^2 + h^2 = (7)^2 + (21)^2 = 490 \Rightarrow l = 22.136 \text{ cm.}$$

$$\text{lateral surface area} = \pi r l = \frac{22}{7} \times 7 \times 22.136 = 486.99 \text{ cm}^2$$

$$\text{and diameter of the cone} = 2r = 2 \times 7 = 14 \text{ cm.}$$

9. (a) the ratio of the base radii of two right circular cones of the same height is 3:4. find the ratio of their volumes.

- (b) The ratio of the heights of two right circular cones is 5:2 and that of their base radii is 2:5. find the ratio of their volumes.

- (c) The height and the radius of the base of a right circular cone is half the corresponding height and radius of another bigger cone. find:
- The ratio of their volumes.
 - the ratio of their lateral surface areas.

Sol.

(a) Given that $\frac{r_1}{r_2} = \frac{3}{4}$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} \quad (\because h_1 = h_2)$$

$$\frac{V_1}{V_2} = \frac{r_1^2}{r_2^2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

(b) $\frac{h_1}{h_2} = \frac{5}{2}$ & $\frac{r_1}{r_2} = \frac{2}{5}$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{2}{5}\right)^2 \times \frac{5}{2} = \frac{2}{5}$$

(c) Given $h_1 = \frac{1}{2} h_2$ — (i) & $r_1 = \frac{1}{2} r_2$ — (ii)

(i) $\frac{V_1}{V_2} = \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$

(ii) $\frac{S_1}{S_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{r_1}{r_2} \frac{\sqrt{r_1^2 + h_1^2}}{\sqrt{r_2^2 + h_2^2}} = \frac{r_1^2}{r_2^2} \frac{\sqrt{1 + \left(\frac{h_1}{r_1}\right)^2}}{\sqrt{1 + \left(\frac{h_2}{r_2}\right)^2}}$

$$= \left(\frac{1}{2}\right)^2 \cdot \frac{\sqrt{1 + \left(\frac{h_2}{r_2}\right)^2}}{\sqrt{1 + \left(\frac{h_2}{r_2}\right)^2}}$$

$$= 1:4 \quad \left[\because \text{By dividing 1 \& 2 we get } \frac{h_2}{r_2} = \frac{h_1}{r_1} \right]$$

10. Find what length of Canvas 2m in width is required to make a conical tent 20m in diameter and 42m is slant height allowing 10% for folds and sticking. Also find the cost of Canvas at the rate of Rs. 80 per meter.

Sol. Radius of conical tent (r) = $\frac{20}{2} = 10$ m.

Slant height (l) = 42 m.

Curved Surface area = $\pi r l = \frac{22}{7} \times 10 \times 42 = 1320$ m²

width of Canvas = 2 m.

length of Canvas required = $\frac{1320}{2} = 660$ m.

length of Canvas for structuring and cutting

= 10% of 660 = $\frac{10}{100} \times 660 = 66$ m.

Total length of Canvas = 660 + 66 = 726 m.

Rate of Canvas = Rs. 80 per m.

Total cost = 726 × 80 = Rs. 58080.

11. The volume of a cone is same as that of a cylinder whose height is 9cm and diameter 40cm. find the radius of the base of the cone if its slant height is 108cm.

Sol. Radius of cylinder (r) = $\frac{40}{2} = 20$ cm. and
height (h) = 9 cm.

Volume of cylinder = $\pi r^2 h = \pi \times 20 \times 20 \times 9 = 3600\pi$ cm³.

Let r be the radius of the cone.

$$\text{Volume of Cone} = \frac{1}{3} \pi r^2 h = 3600 \pi.$$

$$\Rightarrow \frac{1}{3} \times \pi \times r^2 \times 108 = 3600 \pi \Rightarrow 36r^2 = 3600$$

$$\Rightarrow r^2 = 100 \Rightarrow r = 10 \text{ cm.}$$

$$\text{radius} = 10 \text{ cm.}$$

12. A girl fills a cylindrical bucket 32cm in height and 18cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24cm, find (i) the radius (ii) the slant height of the heap.

Sol.

Radius of cylindrical bucket = 18cm and height (h) = 32cm

$$\text{Volume of cylinder} = \pi r^2 h = \pi \times 18 \times 18 \times 32 = 10368 \pi \text{ cm}^3.$$

$$\text{Volume of cone} = 10368 \pi \text{ cm}^3$$

height (h₁) = 24cm, let radius of base = r₁

$$\text{Volume} = \frac{1}{3} \pi r_1^2 h_1 = 10368 \pi.$$

$$\Rightarrow \frac{1}{3} \pi r_1^2 \times 24 = 10368 \pi \Rightarrow 8r_1^2 = 10368$$

$$\Rightarrow r_1^2 = 1296 \Rightarrow r_1 = 36.$$

(i) Radius r₁ = 36cm.

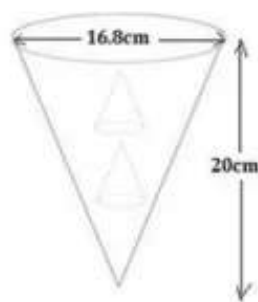
$$(ii) \text{ slant height (l)} = \sqrt{r^2 + h^2} = \sqrt{(36)^2 + (24)^2}$$

$$= \sqrt{1296 + 576} = \sqrt{1872}$$

$$= 43.3 \text{ cm.}$$

13. A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20cm and diameter is 16.8cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result one third of the water in the original cone overflows. What is the volume of each of the solid cones submerged?

sol.



Let V be the volume of original big cone.

$$\Rightarrow V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{16.8}{2} \times 20 = 2956.8 \text{ cm}^3$$

Volume of small cones are equal (V')

$$\frac{1}{3} V = 2V' \Rightarrow \frac{1}{3} \times 2956.8 = 2V'$$

$$\Rightarrow V' = \frac{1}{2} \times \frac{2956.8}{3} = 492.8 \text{ cm}^3$$

14. A hollow metallic cylindrical tube has an internal radius of 3cm and height 21cm. The thickness of the metal of the tube is $\frac{1}{2}$ cm. The tube is melted and cast into a right circular cone of height 7cm. Find the radius of the cone correct to one decimal place.

Sol. Internal radius of hollow metallic cylindrical tube
 $r = 3 \text{ cm}$.

Thickness of tube $= \frac{1}{2} \text{ cm}$.

Height of tube (h) $= 21 \text{ cm}$.

outer radius (R) $= 3 + \frac{1}{2} = \frac{7}{2} \text{ cm}$.

$$\begin{aligned} \text{Volume of the metal used} &= \pi (R^2 - r^2) h = \pi \left(\left(\frac{7}{2} \right)^2 - 3^2 \right) \times 21 \\ &= 21 \pi \left(\frac{49}{4} - 9 \right) = 21 \pi \times \frac{13}{4} = \frac{273}{4} \pi \text{ cm}^3 \end{aligned}$$

Volume of the cone $= \frac{273}{4} \pi \text{ cm}^3$.

Height for cone $= 7 \text{ cm}$, let radius of base $= r$

$$\text{Volume} = \frac{1}{3} \pi r^2 h \Rightarrow \frac{1}{3} \pi r^2 \times 7 = \frac{273}{4} \pi$$

$$\Rightarrow \frac{7}{3} r^2 = \frac{273}{4} \Rightarrow r^2 = \frac{273}{4} \times \frac{3}{7} = \frac{117}{4}$$

$$\Rightarrow r = \sqrt{29.25} = 5.4 \text{ cm}$$

radius of cone $= 5.4 \text{ cm}$.

15. Two right circular cone x and y are made, x having three the radius of y and y having half the volume of x . find the ratio of heights of x and y .

Sol. let radius of y cone $= r$ then radius of x cone $= 3r$.

let h_1 and h_2 be the heights of cones x and y respectively, then

$$\text{Volume of cone } x = \frac{1}{3} \pi (3r)^2 h_1$$

$$\text{Volume of cone } y = \frac{1}{3} \pi r^2 h_2$$

But volume of y is half of volume of x .

$$\Rightarrow \frac{1}{3} \pi (3r)^2 h_1 = 2 \times \frac{1}{3} \pi r^2 h_2$$

$$\Rightarrow \frac{9}{3} \pi r^2 h_1 = \frac{2}{3} \pi r^2 h_2$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{2\pi r^2}{3 \times 3\pi r^2} = \frac{2}{9}$$

$$\therefore h_1 : h_2 = 2 : 9$$

16. A right circular cone of height 20cm and base diameter 30cm is cast into smaller cones of equal sizes with base radius 10cm and height 9cm. find how many cones are made?

Sol.

Base radius of bigger cone = $\frac{30}{2} = 15\text{cm}$.

and height of cone = 20cm.

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (15)^2 \times 20 = 1500\pi \text{ cm}^3.$$

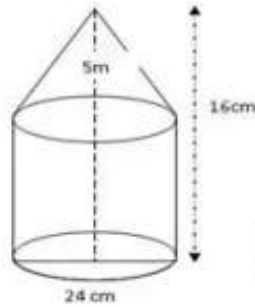
Radius of smaller cone = 10cm and height = 9cm.

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (10)^2 \times 9 = 300\pi \text{ cm}^3.$$

$$\therefore \text{No. of smaller cones} = \frac{\text{Volume of bigger cone}}{\text{Volume of smaller cone}} = \frac{1500\pi}{300\pi} = 5$$

17. A circus tent is in the shape of a cylinder surmounted by a cone. The diameter of cylindrical portion is 24m and its height is 11m. If the vertex of the cone is 16m above the ground, find the area of canvas used to make the tent.

Sol.



Radius of base of cylindrical portion of tent

$$r = \frac{24}{2} = 12 \text{ m}$$

Height of cylindrical portion (h) = 11 m.

Height of conical part (h_2) = $16 - 11 = 5$ m.

Radius of conical part = 12 m

$$\text{Slant height } (l) = \sqrt{r^2 + h_2^2} = \sqrt{(12)^2 + 5^2} = 13 \text{ m.}$$

Now surface area of tent = $\pi r l + 2\pi r h = \pi r (l + 2h)$

$$= \frac{22}{7} \times 12 (13 + 2 \times 11) = \frac{22}{7} \times 12 \times 35$$

$$= 1320 \text{ m}^2$$

18. An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching. Give your answer to the nearest m^2 .

Sol.

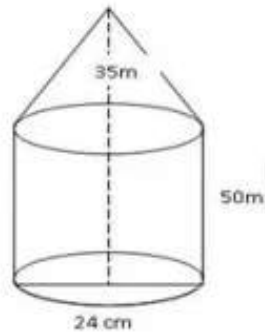
Total height of the tent = 85 m. and

Height of cylindrical part (1) 50 m.

Height of Conical part (h_2) = $85 - 50 = 35$ m.

Diameter of the base = 168 m.

$$\text{radius} = \frac{168}{2} = 84 \text{ m.}$$



$$\text{Slant height } (l) = \sqrt{r^2 + h_2^2} = \sqrt{(84)^2 + (35)^2} = \sqrt{8281} = 91 \text{ m}$$

Now Surface area of the tent = $\pi r l + 2\pi r h = \pi r (l + 2h)$

$$= \frac{22}{7} \times 84 \times (91 + 2 \times 50) = 50424 \text{ m}^2$$

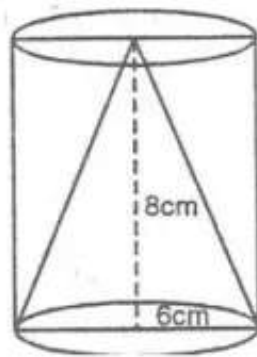
Extra canvas for fold and stitching @ 20%

$$= 50424 \times \frac{20}{100} = 10084.8 \text{ m}^2$$

$$\begin{aligned} \text{Total canvas required} &= 50424 + 10084.8 = 60508.8 \\ &= 60509 \text{ m}^2 \end{aligned}$$

19. From a solid cylinder whose height is 8 cm and radius is 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to 4 significant figures. Also find the total surface area of the remaining solid.

Sol.



Radius of cylinder (r) = 6cm and height (h) = 8cm.

Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 8 = 904.7808 \text{ cm}^3$.

Radius of cone curved out = 6cm and height = 8cm.

Volume = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8 = 301.5936 \text{ cm}^3$

Volume of remaining part = $904.7808 - 301.5936$
 $= 603.1872 \text{ cm}^3$

Total surface area of the cylinder = $2\pi r h + 2\pi r^2$

Curved surface area of cone = $\pi r l$

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ cm.}$$

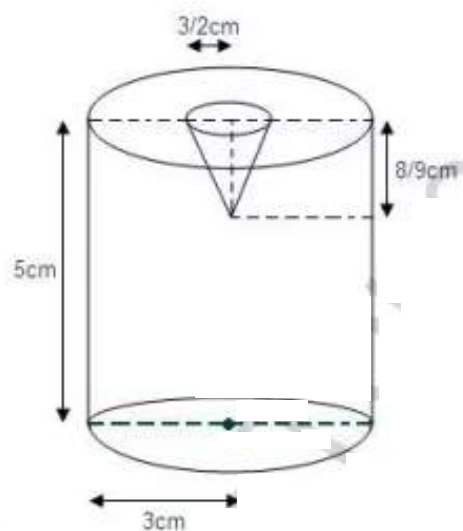
Total surface area of the remaining part

$$= 2\pi r h + 2\pi r^2 + \pi r l = \pi r (2h + r + l)$$

$$= 3.1416 \times 6 (2 \times 8 + 6 + 10) = 3.1416 \times 6 \times 32$$

$$= 603.1872 \text{ cm}^2$$

20. A metallic cylinder has radius 3cm and height 5cm. It is made of a metal A. To reduce its weight, a conical hole is drilled in the cylinder as shown and it is completely filled with a lighter metal B. The conical hole has a radius of $\frac{3}{2}$ cm and its depth is $\frac{8}{9}$ cm. Calculate the ratio of the volume of the metal A to the volume of the metal B in the solid.



sol. Radius of cylinder (r) = 3cm, Height (h) = 5cm.

Radius of conical hole (r) = $\frac{3}{2}$ cm, depth (h) = $\frac{8}{9}$ cm.

$$\text{Volume of cylinder} = \pi r^2 h = \pi \times 3 \times 3 \times 5 = 45\pi$$

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times \frac{3}{2} \times \frac{3}{2} \times \frac{8}{9} = \frac{2}{3} \pi \text{ cm}^3$$

$$\text{Volume of metal B} = \frac{2}{3} \pi \text{ cm}^3$$

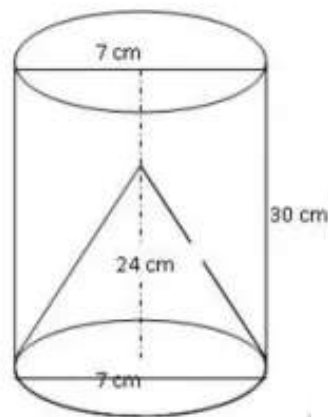
$$\text{Volume of remaining metal A} = 45\pi - \frac{2}{3} \pi = \frac{133}{3} \pi \text{ cm}^3$$

\therefore Ratio between volume of metal A and B

$$= \frac{133}{3} \pi : \frac{2}{3} \pi = 133 : 2$$

21. From a solid cylinder of height 30cm and radius 7cm conical cavity of height 24cm and of base radius 7cm is drilled out. find the volume & the total surface of the remaining solid.

Sol.



Radius of solid cylinder $r_1 = 7\text{ cm}$.

Height (h_1) = 30 cm

Radius of cone (r) = 7 cm and height (h_2) = 24 cm.

Volume of cylinder = $\pi r^2 h_1$, volume of cone = $\frac{1}{3} \pi r^2 h_2$

Volume of remaining part = $\pi r^2 h_1 - \frac{1}{3} \pi r^2 h_2 = \pi r^2 (h_1 - \frac{1}{3} h_2)$

$$= \frac{22}{7} \times 7 \times 7 \times (30 - \frac{1}{3} \times 24) = 154 (30 - 8)$$

$$= 3388 \text{ cm}^3.$$

$$l = \sqrt{r^2 + h_2^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25 \text{ cm}.$$

Surface area of remaining part = $2\pi r h_1 + \pi r^2 + \pi r l$

$$= \pi r (2h_1 + r + l)$$

$$= \frac{22}{7} \times 7 \times (2 \times 30 + 7 + 25)$$

$$= 22 \times 92 = 2024 \text{ cm}^2.$$

22. An open cylindrical vessel of internal diameter 7cm and height 8cm stands on a horizontal table. Inside this placed a solid metallic right circular cone, the diameter of whose base is $\frac{7}{2}$ cm and height 8cm. Find the volume of water required to fill the vessel. If the cone is replaced by another cone, whose height is $1\frac{3}{4}$ cm and the radius of whose base is 2cm, find the drop in the water level.

Sol. Diameter of open cylinder = 7cm, Radius $r_1 = \frac{7}{2}$ cm.
 Height of cylinder (h_1) = 8cm.
 Diameter of cone = $\frac{7}{2}$ cm, radius of cone $r_2 = \frac{7}{4}$ cm.
 and height of cone (h_2) = 8cm.

$$\begin{aligned} \text{Volume of water} &= \text{volume of cylinder} - \text{volume of cone} \\ &= \pi r_1^2 h - \frac{1}{3} \pi r_2^2 h = \pi h \left(r_1^2 - \frac{1}{3} r_2^2 \right) \\ &= \frac{22}{7} \times 8 \times \left[\left(\frac{7}{2} \right)^2 - \frac{1}{3} \left(\frac{7}{4} \right)^2 \right] \\ &= \frac{176}{7} \left[\frac{49}{4} - \frac{49}{48} \right] = \frac{176}{7} \times 49 \left[\frac{1}{4} - \frac{1}{48} \right] \\ &= 176 \times 7 \times \frac{11}{48} = 282 \frac{1}{3} \text{ cm}^3 \end{aligned}$$

(ii) Radius of smaller cone = 2cm and height = $1\frac{3}{4} = \frac{7}{4}$ cm.

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times \frac{7}{4} = \frac{22}{3} \text{ cm}^3$$

change in volume = volume of big cone - volume of small cone.

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times 8 - \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times \frac{7}{4} \right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \left[\frac{7}{4} \times 8 - 4 \right] = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times 10 \text{ cm}^3$$

let the drop in water level be x cm.

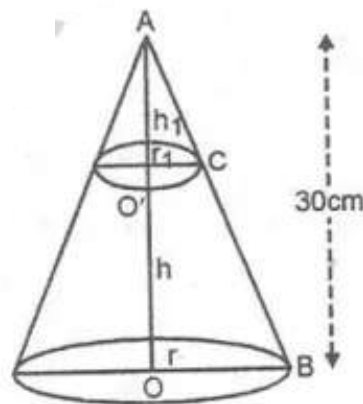
$$\frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times x = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times 10$$

$$\Rightarrow x = \frac{10}{21} \text{ cm.}$$

23. The height of a cone is 30cm. A small cone is cutoff at the top by a plane parallel to its base. if its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section cut?

Sol. Height of bigger cone (h) = 30cm

let r_1 and h_1 be the radius and height of the smaller cone and r, h be the radius and height of bigger cone.



Now $\triangle AO'C \sim \triangle AOB$, $\frac{r_1}{r} = \frac{h_1}{h}$ — (i)

$$\frac{\text{Volume of smaller cone}}{\text{Volume of bigger cone}} = \frac{1}{27} \quad (\text{given})$$

$$\frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r^2 h} = \frac{1}{27} \Rightarrow \frac{h_1^2 \cdot h_1}{h^2 \cdot h} = \frac{1}{27} \Rightarrow \frac{h_1^3}{h^3} = \frac{1}{27} = \left(\frac{1}{3}\right)^3$$

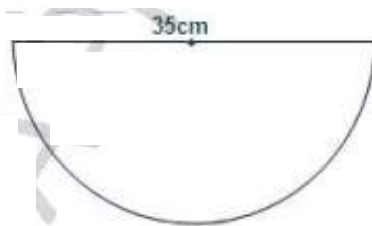
$$\Rightarrow \frac{h_1}{h} = \frac{1}{3} \Rightarrow h_1 = \frac{1}{3} h = \frac{1}{3} \times 30 = 10 \text{ cm.}$$

$$\therefore OO' = AO - AO' = 30 - 10 = 20 \text{ cm.}$$

Hence the height above the base = 20 cm.

24. A semi circular lamina of radius 35 cm is folded so that the two bounding radii are joined together to form a cone. find: (i) The radius of the cone
(ii) The total surface area of the cone.

Sol.



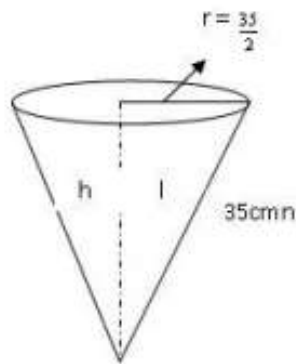
Radius of semi-circular lamina = 35 cm.

$$\text{Circumference} = \pi r = \frac{22}{7} \times 35 = 110 \text{ cm.}$$

and slant height $l = r = 35 \text{ cm.}$

$$(i) \text{ Radius of the cone so formed} = \frac{110 \times 7}{22 \times 2} = \frac{35}{2}$$

$$= 17.5 \text{ cm.}$$



$$(ii) \text{ Total surface area} = \pi r l + \pi r^2$$

$$= \pi r (l + r)$$

$$= \frac{22}{7} \times \frac{35}{2} \times \left(35 + \frac{35}{2}\right)$$

$$= 55 \times \frac{105}{2}$$

$$= 2887.5 \text{ cm.}$$

EXERCISE - 17.4

1. Find the volume and the curved surface area of the sphere if its diameter is 14cm. Take π to be $\frac{22}{7}$.

Sol. Radius of Sphere = $\frac{14}{2} = 7\text{cm}$.

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 1431.5\text{cm}^3$$

$$\text{Curved surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7 = 516\text{cm}^2$$

2. If the volume of a sphere is $179\frac{2}{3}\text{cm}^3$, find its radius and the surface area. Take $\pi = \frac{22}{7}$.

Sol. let r be the radius of sphere.

$$\text{Volume} = \frac{4}{3}\pi r^3 = 179\frac{2}{3} \Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{539}{3}$$

$$\Rightarrow r^3 = \frac{539}{3} \times \frac{3 \times 7}{4 \times 22} = \frac{49 \times 7}{8} = \left(\frac{7}{2}\right)^3$$

$$\Rightarrow r = \frac{7}{2}\text{cm} = 3.5\text{cm}$$

$$\text{Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 154\text{cm}^2$$

3. 8 metallic spheres, each of radius 2mm, are melted and cast into a single sphere. Calculate the radius of new (single) sphere.

Sol. Radius of each equal sphere = 2mm

$$\begin{aligned} \text{Volume of 8 spheres} &= 8 \times \frac{4}{3}\pi r^3 = 8 \times \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 \\ &= \frac{256 \times 22}{21} \text{mm}^3 \end{aligned}$$

let r be the radius

$$\frac{4}{3}\pi r^3 = \frac{256 \times 22}{21} \Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{256 \times 22}{21}$$

$$\Rightarrow r^3 = \frac{256 \times 22 \times 21}{21 \times 88} = 64 \Rightarrow r = \sqrt[3]{64} = 4 \text{ cm.}$$

4. The radius of a sphere is 9 cm. It is melted and drawn into a wire of diameter 2 mm. Find the length of the wire in meters.

Sol.

Radius of Sphere = 9 cm

$$\text{Volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 9 \times 9 \times 9 = 972 \pi \text{ cm}^3$$

$$\text{Radius of wire} = \frac{2}{2} = 1 \text{ mm} = \frac{1}{10} \text{ cm.}$$

let length of wire = h.

$$\text{Volume of wire} = \pi r^2 h = \pi \times \frac{1}{10} \times \frac{1}{10} \times h \text{ cm}^3$$

$$\therefore \frac{\pi}{100} h = 972 \pi$$

$$\Rightarrow h = 97200 \text{ cm} = 972 \text{ m.}$$

5. A metallic disc in the shape of a right circular cylinder is of height 2.5 mm and base radius 12 cm. Metallic disc is melted and made into a sphere. Calculate the radius of the sphere.

Sol.

Radius of the base of circular disc = 12 cm.

$$\text{Height (h)} = 2.5 \text{ mm} = \frac{2.5}{10} \text{ cm} = 0.25 \text{ cm.}$$

$$\text{Volume} = \pi r^2 h = \pi \times 12 \times 12 \times 0.25 = 36 \pi \text{ cm}^3.$$

$$\text{Volume of Sphere} = 36 \pi \text{ cm}^3.$$

let radius of the sphere = r

$$\frac{4}{3} \pi r^3 = 36 \pi \Rightarrow r^3 = \frac{36 \times 3}{4} = 27 = (3)^3$$

$$\Rightarrow r = 3 \text{ cm.}$$

6. The radius of the base of a cone and the radius of a sphere are the same, each being 8cm. Given that the volume of these two solids is also the same, calculate the slant height of the cone, correct to one place of decimal.

Sol. Radius of cone = 8cm and radius of sphere = 8cm.

Volume of cone = volume of sphere

$$\text{Now, volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (8)^3 \text{ cm}^3$$

Let height of cone = h

$$\text{Then volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (8 \times 8) \times h = \frac{64}{3}\pi \times h$$

$$\therefore \frac{4}{3}\pi \times (8)^3 = \frac{64}{3}\pi \times h$$

$$\Rightarrow h = \frac{4\pi \times 8 \times 8 \times 8 \times 3}{3\pi \times 8 \times 8} = 32 \text{ cm.}$$

$$\begin{aligned} \text{Now slant height } l &= \sqrt{r^2 + h^2} = \sqrt{8^2 + 32^2} = \sqrt{64 + 1024} \\ &= \sqrt{1088} = 32.98 \text{ cm} = 33 \text{ cm.} \end{aligned}$$

7. Two spheres of the same metal weight 1kg and 7kg. The radius of the smaller sphere is 3cm. The two spheres are melted to form a single big sphere. Find the diameter of the big sphere.

Sol. weight of 1st sphere = 1kg and weight of 2nd sphere = 7kg.

Total weight of both spheres = $1 + 7 = 8$ kg.

Radius of smaller sphere = 3cm.

weight of smaller sphere = 8 times weight of single big sphere.

$$\begin{aligned} \text{Now volume of smaller sphere} &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (3)^3 \\ &= \frac{4}{3}\pi \times 27 = 36\pi \text{ cm}^3. \end{aligned}$$

$$\text{Volume of big single sphere} = 36\pi \times 8 = 288\pi \text{ cm}^3.$$

let r be the radius of the sphere, then

$$\frac{4}{3}\pi r^3 = 288\pi \Rightarrow r^3 = \frac{288\pi \times 3}{4\pi} = 72 \times 3$$

$$\Rightarrow r^3 = 216 = (6)^3 \Rightarrow r = 6 \text{ cm.}$$

$$\therefore \text{Diameter} = 2r = 2 \times 6 = 12 \text{ cm.}$$

8. A metallic sphere of radius 10.5 cm is melted and then recast into small cones each of radius 3.5 cm and height 3 cm. find the no. of cones thus obtained.

Sol.

$$r_1 = 10.5 \text{ cm (radius of sphere)}$$

$$r_2 = 3.5 \text{ cm (radius of cone)}$$

$$h_2 = 3 \text{ cm.}$$

$$\text{Volume of Sphere} = n \times \text{Volume of cone}$$

$$\Rightarrow \frac{4}{3}\pi r_1^3 = n \times \frac{1}{3}\pi r_2^2 h_2$$

$$\Rightarrow n = \frac{4r_1^3}{r_2^2 \cdot h_2} = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3} = 126.$$

9. what is the least number of solid metallic spheres each 6 cm in diameter, that should be melted to cast a solid metallic cylinder whose height is 53 cm and diameter 6 cm? Also, find the total surface area of the cylinder.

Sol. Diameter of cylinder = 6 cm and height (h) = 53 cm.
radius = $\frac{6}{2} = 3$ cm.

Volume of cylinder = $\pi r^2 h = \pi \times 3 \times 3 \times 53 = 477\pi$ cm³.

Diameter of small sphere = 6 cm, radius (r) = $\frac{6}{2} = 3$ cm.

Volume of one sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi$ cm³.

No. of sphere out of solid cylinder = $\frac{\text{Volume of cylinder}}{\text{Volume of one sphere}}$

$$= \frac{477\pi}{36\pi} = 13.25 = 14$$

Now, total surface of solid cylinder = $2\pi r h + 2\pi r^2$

$$= 2\pi r(h+r) = 2 \times \frac{22}{7} \times 3(53+3)$$

$$= 2 \times \frac{22}{7} \times 3 \times 56$$

$$= 1056 \text{ cm}^2$$

10. The surface area of a solid metallic sphere is 1256 cm². It is melted and recast into solid right circular cones of radius 2.5 cm, height 8 cm. Calculate:

(i) The radius of the solid spheres.

(ii) The number of cones recast (Take $\pi = 3.14$)

Sol. Surface area of metallic sphere = 1256 cm².

Let r be the radius

$$(i) \quad 4\pi r^2 = 1256 \Rightarrow r^2 = \frac{1256}{4 \times 3.14} = \frac{1256 \times 100}{4 \times 314}$$

$$\Rightarrow r^2 = 100$$

$$\Rightarrow r = 10 \text{ cm.}$$

$$\begin{aligned} \text{Volume of Sphere} &= \frac{4}{3} \pi r^3 = \frac{4}{3} (3.14) (10)^3 \\ &= \frac{4}{3} \times 1000 \times 3.14 = \frac{12560}{3} \text{ cm}^3. \end{aligned}$$

(ii) Radius of cone (r) = 2.5 cm and height (h) = 8 cm.

$$\begin{aligned} \text{Volume of height of one cone} &= \frac{1}{3} \pi r^2 h = \frac{1}{3} (3.14) (2.5)^2 \times 8 \\ &= \frac{3.14 \times 6.25 \times 8}{3} \text{ cm}^3 \end{aligned}$$

$$\text{No. of cones} = \frac{\text{Volume of Sphere}}{\text{Volume of One Cone}} = \frac{12560 \times 3}{3 \times 3.14 \times 6.25 \times 8} = 80.$$

11. (a) If the ratio of the radii of two spheres is 3:7 find (i) The ratio of their volumes (ii) The ratio of their surface areas
 (b) If the ratio of the volumes of the two spheres is 125:64. find the ratio of their surface areas.

Sol.

$$(a) \frac{r_1}{r_2} = \frac{3}{7} \quad (\text{given})$$

$$(i) \frac{V_1}{V_2} = \frac{\frac{4}{3} \pi r_1^3}{\frac{4}{3} \pi r_2^3} = \left(\frac{3}{7}\right)^3 = \frac{27}{343}$$

$$(ii) \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$$

$$(b) \frac{V_1}{V_2} = \frac{125}{64} \quad \text{--- (1)}$$

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} \quad \text{--- (2)}$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{125}{64} = \frac{5 \times 5 \times 5}{4 \times 4 \times 4}$$

from (1), $\frac{r_1}{r_2} = \frac{5}{4}$

put in (2), $\frac{S_1}{S_2} = \frac{r_1^2}{r_2^2} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

12. A sphere and a cube have the same surface. Show that the ratio of the volume of the sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$.

Sol.

let 'a' be the side of the cube

Surface area = $6a^2$, Volume = a^3

Surface area of sphere = $6a^2$

let r be radius of sphere, then

$$4\pi r^2 = 6a^2 \Rightarrow r^2 = \frac{6a^2}{4\pi} \Rightarrow r = \sqrt{\frac{6a^2}{4\pi}}$$

and volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\sqrt{\frac{6a^2}{4\pi}}\right)^3$

Ratio between the volume of sphere and cube will be

$$\Rightarrow \frac{4}{3}\pi \left(\frac{6a^2}{4\pi}\right)^{3/2} : a^3$$

$$\Rightarrow \frac{4}{3}\pi \frac{(6)^{3/2} \cdot a^3}{(2)^3 \cdot \pi^{3/2}} : a^3$$

$$\Rightarrow 4\pi 6^{3/2} \cdot a^3 : a^3 \times 24 \times \pi^{3/2}$$

$$\Rightarrow 4 \times 6^{3/2} : 24 \pi^{1/2}$$

$$\Rightarrow 4 \times 6 \times \sqrt{6} : 24 \sqrt{\pi}$$

$$\Rightarrow \sqrt{6} : \sqrt{\pi} \quad \therefore \text{Hence proved.}$$

13. An iron sphere of diameter 12cm is dropped into a cylinder can of diameter 24cm containing water. find the rise in the level of water when the sphere is completely immersed.

sol. Radius of Sphere = $\frac{12}{2} = 6\text{cm}$.

Volume of Sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (6)^3 = 288\pi \text{ cm}^3$.

Radius of cylinder = $\frac{24}{2} = 12\text{cm}$.

let rise of water level = h

Volume of water = $\pi r^2 h = \pi (12)^2 h = 144\pi h \text{ cm}^3$.

\therefore Volume of water = volume of sphere.

$144\pi h = 288\pi \Rightarrow h = 2\text{cm}$.

\therefore Height of water = 2cm.

14. (a) there is water to a height of 14cm in a cylindrical glass jar of radius 8cm. inside the water there is a sphere of diameter 12cm completely immersed. By what height will the water go down when the sphere is removed?

(b) A cylindrical can whose base is horizontal and of radius 3.5cm contains sufficient water so that when sphere is placed in the can, the water just covers the sphere. given that the just fits into the can, Calculate:

- (i) The total surface area of the can it contains with water when the sphere is in it.

(ii) The depth of the water in the can before the sphere was put into the can. Take π to be $\frac{22}{7}$ and give your answer as proper fractions.

Sol. (a) Radius of cylindrical jar = 8cm,
Height of water level = 14cm.

$$\text{Volume of water} = \pi r^2 h = \frac{22}{7} \times 8 \times 8 \times 14 = 2816 \text{ cm}^3.$$

$$\text{Radius of sphere} = \frac{12}{2} = 6 \text{ cm}.$$

$$\begin{aligned} \text{Volume of Sphere} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 = \frac{6336}{7} \\ &= 905 \frac{1}{7} \text{ cm}^3 \end{aligned}$$

when the sphere is removed, then

$$\text{Volume} = 2816 - 905 \frac{1}{7} = 1910 \frac{6}{7} = \frac{13376}{7} \text{ cm}^3.$$

let present height of water in the jar = h

$$\text{then volume} = \pi r^2 h = \frac{22}{7} \times 8 \times 8 \times h \text{ cm}^3$$

$$\Rightarrow \frac{22}{7} \times 64 \times h = \frac{13376}{7}$$

$$\Rightarrow h = \frac{13376 \times 7}{7 \times 22 \times 64} = 9.5 \text{ cm}.$$

level of water which was decreased = $14 - 9.5 = 4.5 \text{ cm}.$

(b) Radius of base of cylindrical can = 3.5cm.

Radius of Sphere which is inside the can = 3.5cm.

$$\begin{aligned} \text{Volume of Sphere} &= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{3} \text{ cm}^3. \end{aligned}$$

$$\begin{aligned} \text{Total Surface area of Can} &= 2\pi rh + \pi r^2 = \pi r(2h + r) \\ &= \frac{22}{7} \times \frac{7}{2} \left(2 \times 7 + \frac{7}{2} \right) \\ &= 11 \times \frac{35}{2} = \frac{385}{2} \text{ cm}^2 = 192.5 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \text{Volume of water with sphere in the can} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 = \frac{539}{2} \text{ cm}^3. \end{aligned}$$

$$\begin{aligned} \text{Volume of water without sphere in the can} &= \frac{539}{2} - \frac{539}{3} = 539 \left(\frac{1}{2} - \frac{1}{3} \right) = 539 \times \frac{1}{6} \\ &= 89 \frac{5}{6} \text{ cm}^3. \end{aligned}$$

let depth of water = h

$$\therefore \pi r^2 h = \frac{539}{6}$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{539}{6}$$

$$\Rightarrow \frac{77}{2} h = \frac{539}{6}$$

$$\Rightarrow h = \frac{539}{6} \times \frac{2}{77}$$

$$\Rightarrow h = \frac{7}{3} \text{ cm.}$$

15. Marbles of diameter 1.4 cm are dropped into a breaker containing some water and are fully submerged. The diameter of the breaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5.6 cm.

Sol. Diameter of each marble = 1.4 cm, radius = $\frac{1.4}{2} = 0.7$ cm.

$$\begin{aligned} \text{Volume of each marble} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (0.7)^3 = \frac{4}{3} \pi \times 0.343 \\ &= \frac{1.372}{3} \pi = \frac{1372}{3000} \pi \text{ cm}^3 \end{aligned}$$

$$\text{Radius of cylindrical breaker} = \frac{7}{2} \text{ cm.}$$

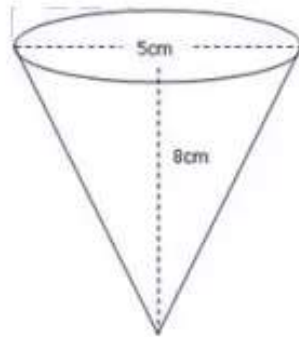
$$\text{Height of water} = 5.6 \text{ cm.}$$

$$\begin{aligned} \text{Volume of water} &= \pi r^2 h = \pi \times \left(\frac{7}{2}\right)^2 \times 5.6 = \pi \times \frac{49}{4} \times 5.6 \\ &= \pi \times \frac{49}{4} \times \frac{56}{10} = \frac{686}{10} \pi \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of marbles} &= \frac{\text{Volume of water}}{\text{Volume of one marble}} = \frac{686 \pi}{10} \times \frac{3000}{1372 \pi} \\ &= 150. \end{aligned}$$

16. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top which is open is 5 cm. It is filled with water upto the rim when lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the no. of lead shots dropped into the vessel.

Sol.



Radius of Conical vessel = 5cm and height = 8cm.

$$\begin{aligned}\text{Volume of water filled in} &= \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (5)^2 \times 8 \\ &= \frac{200}{3} \pi \text{ cm}^3\end{aligned}$$

Radius of each lead shot = 0.5cm = $\frac{1}{2}$ cm.

$$\begin{aligned}\text{Volume of each shot} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{1}{2}\right)^3 = \frac{4}{3} \pi \times \frac{1}{8} \\ &= \frac{1}{6} \pi \text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Volume of water flows out of the vessel} \\ &= \frac{1}{4} \times \frac{200}{3} \pi = \frac{50\pi}{3} \text{ cm}^3\end{aligned}$$

$$\therefore \text{No. of lead shots} = \frac{50\pi \times 6}{3 \times 1 \times \pi} = 100.$$

17. A solid sphere of radius 6cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4cm and height is 72cm, find the uniform thickness of the cylinder?

Sol. radius of sphere = 6cm.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (6)^3 = 288\pi \text{ cm}^3.$$

External radius of the base of cylinder = 4cm and
height = 72cm.

let x be the uniform thickness of the cylinder

$$\therefore \text{Internal radius} = (4-x) \text{ cm.}$$

$$\text{Volume of hollow cylinder} = \pi(R^2 - r^2)h$$

$$= \pi [4^2 - (4-x)^2] \times 72$$

$$= \pi [16 - 16 - x^2 + 8x] \times 72$$

$$= 72\pi [8x - x^2]$$

$$\therefore 72\pi [8x - x^2] = 288\pi$$

$$\Rightarrow 8x - x^2 = 4 \Rightarrow x^2 - 8x + 4 = 0$$

$$\text{Here } a = 1, b = -8, c = 4$$

$$D = b^2 - 4ac = (-8)^2 - 4(1)(4) = 64 - 16 = 48$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{8 \pm \sqrt{48}}{2(1)} = \frac{8 \pm 4\sqrt{3}}{2} = 4 \pm 2\sqrt{3}$$

$$x_1 = 4 + 2\sqrt{3} = 4 + 2(1.732) = 4 + 3.464 = 7.464.$$

but it is not possible as it is greater than 4 or

$$x_2 = 4 - 2\sqrt{3} = 4 - 2(1.732) = 4 - 3.464 = 0.536$$

\therefore Hence the thickness of cylinder = 0.536cm.

18. A hollow copper ball has an external diameter of 12cm, and a thickness of 0.1cm. find
- The outer surface area of the ball.
 - The weight of the ball if 1cm^3 of copper weighs 8.8 gm. (Take π to be 3.14)

Sol.

$$\text{External radius of copper ball} = \frac{12}{2} = 6\text{cm.}$$

$$\text{Thickness} = 0.1\text{cm}$$

$$\text{Internal radius} = 6 - 0.1 = 5.9\text{cm}$$

$$\begin{aligned} \text{(i) Outer surface area of ball} &= 4\pi r^2 = 4 \times (3.14) \times 6 \times 6 \\ &= 144 \times 3.14 = 452.16\text{cm}^2 \end{aligned}$$

$$\text{(ii) Volume of copper used} = \frac{4}{3}\pi [R^3 - r^3]$$

$$= \frac{4}{3} \times (3.14) [6^3 - (5.9)^3]$$

$$= \frac{4(3.14)}{3} [216 - 205.379]$$

$$= \frac{4 \times 3.14 \times 10.621}{3}$$

$$= 44.467$$

$$\text{Weight of } 1\text{cm}^3 \text{ copper} = 8.8$$

$$\text{Total weight} = 44.467 \times 8.8 = 391.31\text{ gm.}$$

19. Find the total surface area of a hemisphere of diameter 42cm. Take $\pi = \frac{22}{7}$.

Sol. Radius of hemisphere = $\frac{42}{2} = 21\text{cm}$.

$$\begin{aligned} \text{Total surface area} &= 3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 \\ &= 4158\text{cm}^2. \end{aligned}$$

$$\begin{aligned} \text{Volume of Sphere} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{3} \text{ cm}^3. \end{aligned}$$

$$\begin{aligned} \text{Total Surface area of Can} &= 2\pi r h + \pi r^2 = \pi r(2h + r) \\ &= \frac{22}{7} \times \frac{7}{2} \left(2 \times 7 + \frac{7}{2} \right) \\ &= 11 \times \frac{35}{2} = \frac{385}{2} \text{ cm}^2 = 192.5 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \text{Volume of water with sphere in the can} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 = \frac{539}{2} \text{ cm}^3. \end{aligned}$$

$$\begin{aligned} \text{Volume of water without sphere in the can} &= \frac{539}{2} - \frac{539}{3} = 539 \left(\frac{1}{2} - \frac{1}{3} \right) = 539 \times \frac{1}{6} \\ &= 89 \frac{5}{6} \text{ cm}^3. \end{aligned}$$

let depth of water = h

$$\therefore \pi r^2 h = \frac{539}{6}$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{539}{6}$$

$$\Rightarrow \frac{77}{2} h = \frac{539}{6}$$

$$\Rightarrow h = \frac{539}{6} \times \frac{2}{77}$$

$$\Rightarrow h = \frac{7}{3} \text{ cm.}$$

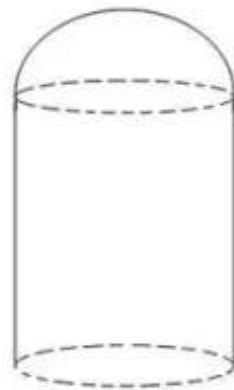
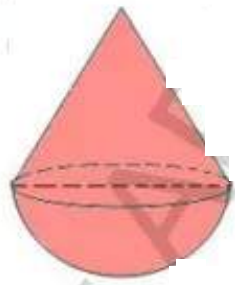
20. (a) The fig(i) given below shows a hemisphere of radius 5cm surmounted by a right circular cone of base radius 5cm. find the volume of the solid if the height of the cone is 7cm. give your answer correct to two decimal places.

(b) The fig(ii) given below shows a metal container in the form of a cylindrical surmounted by a hemisphere of the same radius. The internal height of the cylindrical is 7m the internal radius is 3.5m. Calculate :

(i) The total area of the internal surface, excluding the base.

(ii) The internal volume of the container in m^3 .

Sol.



(a) Radius of hemispherical part = 5cm and
radius of base of cone = 5cm and
height of the cone = 7cm.

$$\begin{aligned} \text{Volume of the solid} &= \frac{2}{3} \pi r^3 + \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi r^2 [2r + h] \end{aligned}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times (2 \times 5 + 7)$$

$$= \frac{550}{21} \times 17$$

$$= \frac{9350}{21}$$

$$= 445.24 \text{ cm}^3$$

(b) Internal height of cylinder = 7m.

$$\text{Internal radius} = \frac{7}{2} = 3.5 \text{ m}$$

(i) Total internal surface area = $2\pi r^2 + 2\pi r h$

$$= 2\pi r (r + h)$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \left(\frac{7}{2} + 7 \right) = 22 \left(\frac{21}{2} \right) = 231 \text{ m}^2$$

(ii) Internal volume = $\frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2 \left[\frac{2}{3}r + h \right]$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \left(\frac{2}{3} \times \frac{7}{2} + 7 \right)$$

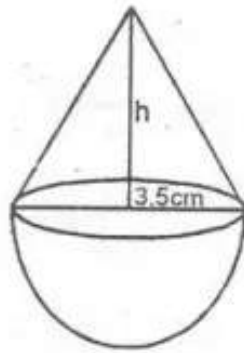
$$= \frac{77}{2} \times \frac{28}{3}$$

$$= \frac{1028}{3}$$

$$= 359 \frac{1}{3} \text{ m}^3$$

21. A toy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of the hemisphere. The radius of the base of the cone is 3.5m and its volume is $\frac{2}{3}$ of the hemisphere. Calculate the height of the cone and the surface area of the buoy correct to 2 decimal places.

sol.



Radius of base of hemisphere = $\frac{7}{2}$ m

$$\begin{aligned}\text{Volume of hemisphere} &= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \\ &= \frac{539}{6} \text{ m}^3\end{aligned}$$

$$\text{Volume of Cone} = \frac{539}{6} \times \frac{2}{3} = \frac{539}{9} \text{ m}^3$$

let height of the cone = h.

$$\frac{1}{3}\pi r^2 h = \frac{539}{9}$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{539}{9}$$

$$\Rightarrow \frac{77}{6} \times h = \frac{539}{9}$$

$$\Rightarrow h = \frac{539}{9} \times \frac{6}{77} = \frac{14}{3} = 4.67 \text{ m.}$$

Height of cone = 4.67 m.

$$l = \sqrt{r^2 + h^2} = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{14}{3}\right)^2} = \sqrt{\frac{49}{4} + \frac{196}{9}} = \sqrt{\frac{1225}{36}} = \frac{35}{6} \text{ m.}$$

Now, Surface area of the toy = $2\pi r^2 + \pi r l$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + \frac{22}{7} \times \frac{7}{2} \times \frac{35}{6}$$

$$= \frac{462 + 385}{6} = \frac{847}{6} = 141.17 \text{ m}^2$$

22. A circular hall (big room) has hemispherical roof. The greatest height is equal to the inner diameter. Find the area of the floor, given that the capacity of the hall is 48510 m^3 .

Sol. Let h be the greatest height and r be the radius of the base, then $2r = h + r \Rightarrow h = r$

$$\begin{aligned} \text{Volume of the hall} &= \frac{2}{3} \pi r^3 + \pi r^2 h = \frac{2}{3} \pi r^3 + \pi r^2 (r) \\ &= \frac{2}{3} \pi r^3 + \pi r^3 = \frac{5}{3} \pi r^3 \end{aligned}$$

$$\therefore \frac{5}{3} \pi r^3 = 48510$$

$$\Rightarrow r^3 = \frac{48510 \times 3 \times 7}{5 \times 22} = 9261 = (21)^3$$

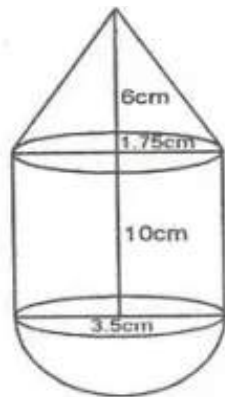
$$\Rightarrow r = 21 \text{ m.}$$

$$\text{Area of floor} = \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386 \text{ m}^2$$

23. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the height of the cylindrical and conical portions are 10 cm and 6 cm respectively. Find the volume of the solid.

Sol. Diameter = 3.5 cm , Height of cylindrical part (h_1) = 10 cm . and height of conical part (h_2) = 6 cm . and

$$\text{radius} = \frac{3.5}{2} = 1.75 \text{ cm.}$$



$$\text{Total Volume of the solid} = \frac{1}{3} \pi r^2 h_2 + \pi r^2 h_1 + \frac{2}{3} \pi r^3$$

$$= \pi r^2 \left(\frac{1}{3} h_2 + h_1 + \frac{2}{3} r \right)$$

$$= 3.14 \times (1.75)^2 \left[\frac{1}{3} \times 6 + 10 + \frac{2}{3} \times 1.75 \right]$$

$$= 3.14 \times (1.75)^2 \left[2 + 10 + \frac{3.5}{3} \right]$$

$$= 3.14 \times (1.75)^2 \times \frac{39.5}{3}$$

$$= \frac{379.84}{3}$$

$$= 126.61 \text{ cm}^3$$

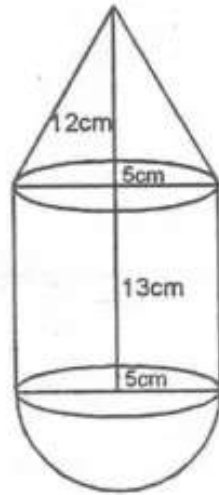
24. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of the conical part is 12 cm.

Sol.

Height of cylindrical part = 13cm.

Radius of Cone (r) = 5cm.

Height of Cone (h) = 12cm.



$$\text{Slant height } (l) = \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ cm.}$$

$$\text{Now Surface area of the top} = \pi r l + 2\pi r h + 2\pi r^2$$

$$= \pi r (l + 2h + 2r)$$

$$= \frac{22}{7} \times 5 [13 + 2 \times 13 + 2 \times 5]$$

$$= \frac{110}{7} (13 + 26 + 10)$$

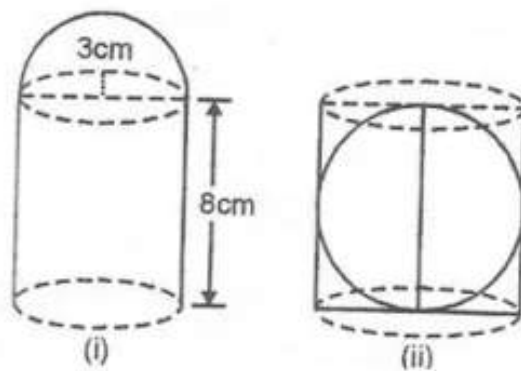
$$= 770 \text{ cm}^2.$$

25. (a) The fig(i) given below shows a model of a solid consisting of a cylinder surmounted by a hemisphere at one end. If the model is drawn to a scale of 1:200, find:

(i) The total surface area of the solid in $\pi \text{ m}^2$

(ii) The volume of the solid in π litres.

- (b) In the fig(ii) given below, a sphere is inscribed in a cylinder. Prove that the surface of the sphere is equal to the curved surface area of the cylinder.



Sol.

(a) (i) Radius of the base of hemi-spherical part $r = 3\text{cm}$
and radius of cylinder part $r = 3\text{cm}$.

height of cylinder (h) = 8cm.

$$\begin{aligned} \text{Total Surface area} &= 2\pi r^2 + 2\pi r h + \pi r^2 = 3\pi r^2 + 2\pi r h \\ &= \pi r (3r + 2h) \\ &= \pi \times 3 (3 \times 3 + 2 \times 8) \\ &= 3\pi \times 25 \\ &= 75\pi \text{ cm}^2 \end{aligned}$$

scale of model = 1:200

$$\begin{aligned} \text{Actual total Surface area} &= 75\pi \times (200)^2 \text{ cm}^2 \\ &= 75\pi \times \frac{(200)^2}{100 \times 100} \text{ m}^2 \\ &= 300\pi \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{(ii) Total volume of the solid} &= \frac{2}{3}\pi r^3 + \pi r^2 h \\ &= \frac{2}{3}\pi (3)^3 + \pi (3)^2 \times 8 \\ &= 18\pi + 72\pi = 90\pi \text{ cm}^3 \end{aligned}$$

Scale = 1:200

$$\begin{aligned}
 \therefore \text{Actual total volume} &= 90\pi \times (200)^3 \text{ cm}^3 \\
 &= \frac{90\pi \times (200)^3}{(100)^3} \text{ m}^3 = 90\pi \times (2)^3 \\
 &= 720\pi \text{ m}^3 \\
 &= 720\pi \times 1000 \text{ litre} \\
 &= 720000\pi \text{ litre} \\
 &= \approx 72 \times 10^4 \pi \text{ litre.}
 \end{aligned}$$

(b) let radius of cylinder = r and height $h = 2r$

radius of sphere = r

$$\begin{aligned}
 \text{Curved surface area of cylinder} &= 2\pi r h = 2\pi r \times 2r \\
 &= 4\pi r^2 \text{ sq. units}
 \end{aligned}$$

Hence surface area of sphere = Curved surface area of cylinder.

26. A hemisphere, cylinder and cone have equal base diameter and have same height. prove that their volumes are in the ratio 2:3:1

Sol. let the radius of each = r = height of each solid is equal.

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$\text{Volume of cylinder} = \pi r^2 h = \pi r^2 \times r = \pi r^3 \quad (\because h=r)$$

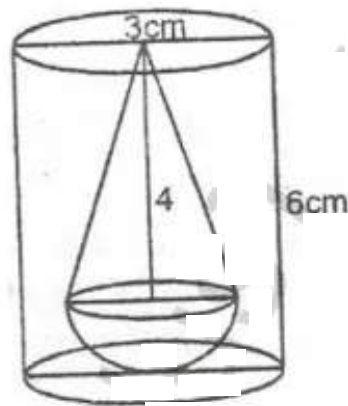
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \cdot r = \frac{1}{3}\pi r^3 \quad (\because h=r)$$

$$\text{Their ratio} \quad \frac{2}{3}\pi r^3 : \pi r^3 : \frac{1}{3}\pi r^3 = \frac{2}{3} : 1 : \frac{1}{3}$$

$\Rightarrow 2:3:1$ \therefore Hence proved.

27. A solid consisting of a right circular cone, standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. find the volume of the water left in the cylinder, having given that the radius of the cylinder is 3cm and its height is 6cm. the radius of the hemisphere is 2cm and the height of the cone is 4cm. give your answer to the nearest cm^3 .

Sol. Radius of cylinder = 3cm and height = 6cm.



$$\begin{aligned} \text{volume of water when it is full} &= \pi r^2 h = \pi (3)^2 \times 6 \text{ cm}^3 \\ &= 54\pi \text{ cm}^3 \end{aligned}$$

Volume of solid consisting cone and hemisphere

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 2r)$$

$$= \frac{1}{3} \pi (2)^2 (4 + 2 \times 2) = \frac{32}{3} \pi \text{ cm}^3$$

$$\text{Volume of water left in the cylinder} = 54\pi - \frac{32}{3}\pi$$

$$= \frac{130}{3} \pi = \frac{130}{3} \times \frac{22}{7} = \frac{2860}{21} \text{ cm}^3$$

$$= 136.19 = 136 \text{ cm}^3.$$