QUADRATIC EQUATIONS

EXERCISE - 5.1

Q1. Determine whether $x = \frac{1}{2}$ and $x = \frac{3}{2}$ are the solutions of the equation $2x^2 - 5x + 3 = 0$ or not.

Sol. Given equation is $2x^2 - 5x + 3 = 0$

If $x = \frac{1}{2}$ is its solution, then it will satisfy the equation. Now substituting the value of $x = \frac{1}{2}$ in the given equation

$$= 2\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 3$$
$$= 2\left(\frac{1}{4}\right) - \frac{5}{2} + 3$$
$$= \frac{1}{2} - \frac{5}{2} + 3$$
$$= -2 + 3$$
$$= 1 \neq 0$$

$\therefore$ It is not its solution.

Again substituting the $x = \frac{3}{2}$ in the equation

$$= 2\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 3$$
$$= 2\left(\frac{9}{4}\right) - \frac{15}{2} + 3$$
$$= \frac{9}{2} - \frac{15}{2} + 3$$
$$= -3 + 3$$
$$= 0$$

$\therefore$ Hence $x = \frac{3}{2}$ is its solution.

Solve the following equations (2 to 25) by factorization:

Q2. (i) $4x^2 = 3x$

Sol. $4x^2 = 3x \Rightarrow 4x^2 - 3x = 0 \Rightarrow x(4x-3) = 0$

either $x = 0$ or $4x - 3 = 0 \Rightarrow x = \frac{3}{4}$

$\therefore x = 0, \frac{3}{4}$

(ii) $\frac{x^2-5x}{2} = 0 \Rightarrow x^2 - 5x = 0 \Rightarrow x(x-5) = 0$

either $x = 0$ or $x - 5 = 0 \Rightarrow x = 5$

Hence $x = 0, 5$

Q3. (i) $(x-3)(2x+5) = 0$

Sol. $(x-3)(2x+5) = 0$

either $x - 3 = 0$ or $2x + 5 = 0$

$\Rightarrow x = 3$ $\quad x = -\frac{5}{2}$

Hence $x = 3, \frac{-5}{2}$

(ii) $x(2x+1) = 6$

$2x^2 + x = 6$

$2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0 \Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (2x-3)(x+2) = 0$

either $x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow x = -2$ $\quad \Rightarrow 2x = 3$

$\Rightarrow x = 3/2$

Hence $x = -2, 3/2$

Q4. (i) $x^2 - 3x - 10 = 0$

Sol. $x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2)=0$

either $x-5=0$ or $x+2=0$

$x=5$ $\quad$ $x=-2$

Hence $x = 5, -2$

(ii) $x(2x+5)=3$

$\Rightarrow 2x^2+5x=3 \Rightarrow 2x^2+5x-3=0$

$\Rightarrow 2x^2+6x-x-3=0 \Rightarrow 2x(x+3)-1(x+3)=0$

$\Rightarrow (x+3)(2x-1)=0$

either $x+3=0$ or $2x-1=0$

$x=-3$ $\quad$ $x=\frac{1}{2}$

Hence $x=-3, \frac{1}{2}$

Q5. (i) $3x^2-5x-12=0$

sol. $\Rightarrow 3x^2-9x+4x-12=0$

$\Rightarrow 3x(x-3)+4(x-3)=0$

$\Rightarrow (x-3)(3x+4)=0$

either $x-3=0$ or $3x+4=0$.

$x=3$ $\quad$ $x=-4/3$

Hence $x=3, -4/3$

(ii) $21x^2-8x-4=0$

$\Rightarrow 21x^2-14x+6x-4=0$

$\Rightarrow 7x(3x-2)+2(3x-2)=0$

$\Rightarrow (3x-2)(7x+2)=0$

either $3x-2=0$ or $7x+2=0$

$x=2/3$ $\quad$ $x=-2/7$.

Hence $x=\frac{2}{3}, \frac{-2}{7}$

Q6. (i) $3x^2 = x + 4$

sol. $3x^2 - x - 4 = 0 \Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x-4) + 1(3x-4) = 0 \Rightarrow (3x-4)(x+1) = 0$

either $3x - 4 = 0$ or $x + 1 = 0$

$x = 4/3$ $\quad$ $x = -1$

Hence $x = -1, \frac{4}{3}$.

(ii) $x(6x-1) = 35$

$\Rightarrow 6x^2 - x - 35 = 0 \Rightarrow 6x^2 - 15x + 14x - 35 = 0$

$\Rightarrow 3x(2x-5) + 7(2x-5) = 0$

$\Rightarrow (2x-5)(3x+7) = 0$

either $2x - 5 = 0$ or $3x + 7 = 0$

$x = 5/2$ $\quad$ $x = -7/3$

Hence $x = \frac{5}{2}, \frac{-7}{3}$.

Q7. (i) $6p^2 + 11p - 10 = 0$

sol. $\Rightarrow 6p^2 + 15p - 4p - 10 = 0$

$\Rightarrow 3p(2p+5) - 2(2p+5) = 0$

$\Rightarrow (2p+5)(3p-2) = 0$

either $2p + 5 = 0$ or $3p - 2 = 0$

$p = -5/2$ $\quad$ $p = 2/3$

Hence $x = \frac{-5}{2}, \frac{2}{3}$

(ii) $\frac{2}{3}x^2 - \frac{1}{3}x = 1$

$\Rightarrow 2x^2 - x - 3 = 0 \Rightarrow 2x^2 - 3x + 2x - 3 = 0$

$\Rightarrow x(2x-3) + 1(2x-3) = 0 \Rightarrow (2x-3)(x+1) = 0$

either $2x - 3 = 0$ or $x + 1 = 0$

$x = 3/2$ $\quad$ $x = -1$

Hence $x = \frac{3}{2}, -1$

Q8. (i) $(x-4)^2+5^2=13^2$

sol. $x^2-8x+16+25=169$

$\Rightarrow x^2-8x-128=0$

$\Rightarrow x^2-16x+8x-128=0$

$\Rightarrow x(x-16)+8(x-16)=0$

$\Rightarrow (x-16)(x+8)=0$

either $x-16=0$ or $x+8=0$

$x=16$ $\quad x=-8$

Hence $x=16, -8$

(ii) $3(x-2)^2=147$

$3(x^2-4x+4)=147$

$\Rightarrow 3x^2-12x+12=147$

$\Rightarrow 3x^2-12x-135=0$

$\Rightarrow x^2-4x-45=0$ {∵ dividing by 3}

$\Rightarrow x^2-9x+5x-45=0$

$\Rightarrow x(x-9)+5(x-9)=0$

$\Rightarrow (x-9)(x+5)=0$

either $x-9=0$ or $x+5=0$

$x=9$ $\quad x=-5$

Hence $x=9, -5$.

Q9. (i) $\frac{1}{7}(3x-5)^2=28$

sol. $(3x-5)^2=28\times7=196$

$\Rightarrow 9x^2-30x+25=196$

$\Rightarrow 9x^2-30x-171=0$

$\Rightarrow 3x^2-10x-57=0$ {dividing by 3}

$\Rightarrow 3x^2-19x+9x-57=0$

$\Rightarrow x(3x-19)+3(3x-19)=0$

$\Rightarrow (3x-19)(x+3)=0$

either $3x-19=0$ or $x+3=0$

$x = 19/3$ $\quad$ $x=-3$

Hence $x = \frac{19}{3}, -3$

(ii) $3(y^2-6) = y(y+7)-3$

sol. $3y^2 - 18 = y^2+7y-3$

$\Rightarrow 2y^2-7y-15=0 \Rightarrow 2y^2-10y+3y-15=0$

$\Rightarrow 2y(y-5)+3(y-5)=0 \Rightarrow (y-5)(2y+3)=0$

either $y-5=0$ or $2y+3=0$

$y=5$ $\quad$ $y=-3/2$

Hence $y = 5, -3/2$

Q10. $x^2-4x-12=0$ when $x \in N$

sol. $x^2-6x+2x-12=0$

$\Rightarrow x(x-6)+2(x-6)=0$

$\Rightarrow (x-6)(x+2)=0$

either $x-6=0$ or $x+2=0$

$x=6$ $\quad$ $x=-2$

As $x \in N$, $x=6$.

Q11. $2x^2-8x-24=0$ when $x \in I$.

sol. $x^2-4x-12=0$ {dividing by '2'}

$\Rightarrow x^2-6x+2x-12=0$

$\Rightarrow (x-6)+2(x-6)=0 \Rightarrow (x-6)(x+2)=0$

either $x-6=0$ or $x+2=0 \Rightarrow x=-2$

$x=6$

$\therefore$ Hence $x=6, -2$

Q12. $5x^2 - 8x - 4 = 0$ when $x \in Q$

Sol. $\Rightarrow 5x^2 - 10x + 2x - 4 = 0$

$\Rightarrow 5x(x-2) + 2(x-2) = 0$

$\Rightarrow (x-2)(5x+2) = 0$

either $x - 2 = 0$ or $5x + 2 = 0$

$x = 2$ $\quad x = -2/5$

Hence $x = 2, \frac{-2}{5}$

Q13. $2x^2 - 9x + 10 = 0$ when (i) $x \in N$ (ii) $x \in Q$

Sol. $2x^2 - 9x + 10 = 0$

$\Rightarrow 2x^2 - 4x - 5x + 10 = 0 \Rightarrow 2x(x-2) - 5(x-2) = 0$

$\Rightarrow (x-2)(2x-5) = 0$

either $x - 2 = 0$ or $2x - 5 = 0$

$x = 2$ $\quad x = 5/2$

(i) when $x \in N$, then $x = 2$

(ii) when $x \in Q$, then $x = 2, \frac{5}{2}$.

Q14. (i) $a^2x^2 + 2ax + 1 = 0$, $a \neq 0$

(ii) $x^2 - (p+q)x + pq = 0$

Sol. (i) $a^2x^2 + 2ax + 1 = 0$

$a^2x^2 + ax + ax + 1 = 0$

$ax(ax+1) + 1(ax+1) = 0$

$(ax+1)(ax+1) = 0$

we get $ax + 1 = 0$

$x = -\frac{1}{a}, -\frac{1}{a}$

(ii) $x^2 - (p+q)x + pq = 0$

$x^2 - px - qx + pq = 0$

$x(x-p) - q(x-p) = 0$

$(x-p)(x-q) = 0$.

either $x - p = 0$. or $x - q = 0$

$x = p$ $\quad x = q$

Hence $x = p, q$

Q15. $a^2x^2 + (a^2+b^2)x + b^2 = 0$, $a \neq 0$

Sol. $a^2x^2 + a^2x + b^2x + b^2 = 0$

$\Rightarrow a^2x(x+1) + b^2(x+1) = 0$

$\Rightarrow (x+1)(a^2x + b^2) = 0$

either $x+1 = 0$ or $a^2x + b^2 = 0$

$x = -1$ $\quad$ $x = -b^2/a^2$

Hence $x = -1, -b^2/a^2$.

Q16. (i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Sol. $\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$ $\quad \{\because \sqrt{3} \times 7\sqrt{3} = 21 = 7 \times 3\}$

$\Rightarrow \sqrt{3}x(x+\sqrt{3}) + 7(x+\sqrt{3}) = 0$

$\Rightarrow (x+\sqrt{3})(\sqrt{3}x + 7) = 0$

either $x + \sqrt{3} = 0$ or $\sqrt{3}x + 7 = 0$

$x = -\sqrt{3}$ $\quad$ $x = -7/\sqrt{3}$

Hence $x = -\sqrt{3}, -7/\sqrt{3}$

(ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Sol. $4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$ $\quad \{\because 4\sqrt{3}(-2\sqrt{3}) = -24 = 8(-3)\}$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$

either $\sqrt{3}x + 2 = 0$ or $4x - \sqrt{3} = 0$

$x = -2/\sqrt{3}$ $\quad$ $x = \sqrt{3}/4$

Hence $x = -2/\sqrt{3}, \sqrt{3}/4$

Q17. (i) $x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$

sol. $x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

$\Rightarrow x(x-1) - \sqrt{2}(x-1) = 0$

$\Rightarrow (x-1)(x-\sqrt{2}) = 0$

either $x-1=0$ or $x-\sqrt{2}=0$

$x=1$ $\qquad x=\sqrt{2}$

Hence $x = 1, \sqrt{2}$

(ii) $x + \frac{1}{x} = 2\frac{1}{20}$

sol. $\frac{x^2+1}{x} = \frac{41}{20} \Rightarrow 20x^2 + 20 = 41x$

$\Rightarrow 20x^2 - 41x + 20 = 0$

$\Rightarrow 20x^2 - 16x - 25x + 20 = 0$

$\Rightarrow 4x(5x-4) - 5(5x-4) = 0$

$\Rightarrow (5x-4)(4x-5) = 0$

either $5x-4=0$ or $4x-5=0$

$x = 4/5$ $\qquad x = 5/4$

$\therefore$ Hence $x = \frac{4}{5}, \frac{5}{4}$

Q18. (i) $3x - \frac{8}{x} = 2$

sol. $\frac{3x^2-8}{x} = 2 \Rightarrow 3x^2 - 8 = 2x$

$\Rightarrow 3x^2 - 2x - 8 = 0 \Rightarrow 3x^2 - 6x + 4x - 8 = 0$

$\Rightarrow 3x(x-2) + 4(x-2) = 0$

$\Rightarrow (x-2)(3x+4) = 0$

either $x-2=0$ or $3x+4=0$

$x=2$ $\qquad x = -4/3$

$\therefore$ Hence $x = 2, -4/3$

(ii) $\frac{x+2}{x+3} = \frac{2x-3}{3x-7}$

sol. $(x+2)(3x-7) = (2x-3)(x+3)$

$\Rightarrow 3x^2 - 7x + 6x - 14 = 2x^2 + 6x - 3x - 9$

$\Rightarrow 3x^2 - x - 14 - 2x^2 - 3x + 9 = 0$

$\Rightarrow x^2 - 4x - 5 = 0$

$\Rightarrow x^2 - 5x + x - 5 = 0$

$\Rightarrow x(x-5) + 1(x-5) = 0$

$\Rightarrow (x-5)(x+1) = 0$

either $x - 5 = 0$ or $x + 1 = 0$

$x = 5$ $\qquad x = -1$

$\therefore$ Hence $x = 5, -1$

Q19. (i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

$\Rightarrow 8(2-x) - 3(x+3) = 2(x+3)(2-x)$

$\Rightarrow 16 - 8x - 3x - 9 = 2(2x - x^2 + 6 - 3x)$

$\Rightarrow 7 - 11x = 4x - 2x^2 + 12 - 6x$

$\Rightarrow 2x^2 + 2x - 12 + 7 - 11x = 0$

$\Rightarrow 2x^2 - 9x - 5 = 0$

$\Rightarrow 2x^2 - 10x + x - 5 = 0$

$\Rightarrow 2x(x-5) + 1(x-5) = 0$

$\Rightarrow (x-5)(2x+1) = 0$

either $x - 5 = 0$ or $2x + 1 = 0$

$x = 5$ $\qquad x = -1/2$

$\therefore$ Hence $x = 5, -1/2$

(ii) $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

sol. $\Rightarrow \frac{x^2 + (x-1)^2}{x(x-1)} = \frac{5}{2}$

$$\Rightarrow \frac{x^2+x^2+1-2x}{x^2-x} = \frac{5}{2}$$

$$\Rightarrow \frac{2x^2+1-2x}{x^2-x} = \frac{5}{2}$$

$$\Rightarrow 4x^2+2-4x = 5x^2-5x$$

$$\Rightarrow 5x^2-5x-4x^2-2+4x = 0$$

$$\Rightarrow x^2-x-2 = 0$$

$$\Rightarrow x^2-2x+x-2 = 0$$

$$\Rightarrow x(x-2)+1(x-2) = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

either $x-2=0$ or $x+1=0$

$x = 2$ $\quad$ $x = -1$

$\therefore$ Hence $x = 2, -1$

Q20. (i) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$

Sol.

$$\Rightarrow \frac{x^2+x^2+2x+1}{x(x+1)} = \frac{34}{15}$$

$$\Rightarrow \frac{2x^2+2x+1}{x^2+x} = \frac{34}{15}$$

$$\Rightarrow 30x^2+30x+15 = 34x^2+34x$$

$$\Rightarrow -4x^2-4x+15 = 0$$

$$\Rightarrow 4x^2+4x-15 = 0$$

$$\Rightarrow 4x^2+10x-6x-15 = 0$$

$$\Rightarrow 2x(2x+5)-3(2x+5) = 0$$

$$\Rightarrow (2x+5)(2x-3) = 0$$

either $2x+5=0$ or $2x-3=0$

$x = -5/2$ $\quad$ $x = +3/2$

$\therefore$ Hence $x = -\frac{5}{2}, \frac{3}{2}$

(ii) $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3$

$\Rightarrow \frac{(x+1)(x+2) + (x-2)(x-1)}{(x-1)(x+2)} = 3$

$\Rightarrow \frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{3}{1}$

$\Rightarrow 2x^2+4 = 3x^2+3x-6$

$\Rightarrow -x^2-3x+10 = 0$

$\Rightarrow x^2+3x-10 = 0$

$\Rightarrow x^2+5x-2x-10 = 0$

$\Rightarrow x(x+5)-2(x+5) = 0$

$\Rightarrow (x+5)(x-2) = 0$

either $x+5=0$ or $x-2=0$

$x=-5$ $\quad$ $x=2$

$\therefore$ Hence $x = -5, 2$

Q21. (i) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$

Sol. $\frac{x+5-x+3}{(x-3)(x+5)} = \frac{1}{6}$

$\Rightarrow \frac{8}{x^2+2x-15} = \frac{1}{6}$

$\Rightarrow 48 = x^2+2x-15 \Rightarrow x^2+2x-15-48=0$

$\Rightarrow x^2+2x-63=0$

$\Rightarrow x^2+9x-7x-63=0$

$\Rightarrow x(x+9)-7(x+9)=0$

$\Rightarrow (x+9)(x-7)=0$

either $x+9=0$ or $x-7=0$

$x=-9$ $\quad$ $x=7$

$\therefore$ Hence $x=-9, 7$

(ii) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

Sol.

$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$

$\Rightarrow \frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8} = \frac{10}{3}$

$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8} = \frac{10}{3}$

$\Rightarrow 6x^2-30x+30 = 10x^2-60x+80$

$\Rightarrow 4x^2-30x+50=0$

$\Rightarrow 2x^2-15x+25=0$ {.. dividing by '2'}

$\Rightarrow 2x^2-10x-5x+25=0$

$\Rightarrow 2x(x-5)-5(x-5)=0$

$\Rightarrow (x-5)(2x-5)=0$

either $x-5=0$ or $2x-5=0$

$x=5$ $\quad$ $x=5/2$

$\therefore$ Hence $x=5, 5/2$

Q22. $\frac{a}{ax-1} + \frac{b}{bx-1} = a+b$, $a+b \neq 0$, $ab \neq 0$

Sol. $\left(\frac{a}{ax-1} - b\right) + \left(\frac{b}{bx-1} - a\right) = 0$

$\Rightarrow \frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$

$\Rightarrow (a+b-abx)\left[\frac{1}{ax-1} + \frac{1}{bx-1}\right] = 0$

$\Rightarrow (a+b-abx)\left[\frac{bx-1+ax-1}{(ax-1)(bx-1)}\right] = 0$

$\Rightarrow (a+b-abx)\left[\frac{ax+bx-2}{(ax-1)(bx-1)}\right] = 0$

$$\Rightarrow (a+b-abx)(ax+bx-2)=0$$

either $a+b-abx=0$ or $ax+bx-2=0$

$\Rightarrow a+b=abx$ $\qquad$ $\Rightarrow (a+b)x=2$

$\Rightarrow x=\dfrac{a+b}{ab}$ $\qquad$ $\Rightarrow x=\dfrac{2}{a+b}$

$\therefore$ Hence $x=\dfrac{a+b}{ab}, \dfrac{2}{a+b}$

Q23. $\dfrac{1}{p}+\dfrac{1}{q}+\dfrac{1}{x}=\dfrac{1}{p+q+x}$, $p+q\neq 0$, $p\neq 0$, $q\neq 0$

Sol.
$$\frac{qx+px+pq}{pqx}=\frac{1}{p+q+x}$$

$$\Rightarrow (p+q+x)(qx+px+pq)=pqx$$

$$\Rightarrow p^2x+pqx+p^2q+pqx+q^2x+pq^2+px^2+qx^2+pqx=pqx$$

$$\Rightarrow (p+q)x^2+(p+q)^2x+pq(p+q)=0$$

Dividing by $p+q$ as $p+q\neq 0$

$$\Rightarrow x^2+(p+q)x+pq=0$$

$$\Rightarrow x^2+px+qx+pq=0$$

$$\Rightarrow x(x+p)+q(x+p)=0$$

$$\Rightarrow (x+p)(x+q)=0$$

either $x+p=0$ or $x+q=0$

$x=-p$ $\qquad$ $x=-q$

$\therefore$ Hence $x=-p, -q$

Q24. $\dfrac{1}{x+6}+\dfrac{1}{x-10}=\dfrac{3}{x-4}$

Sol.
$$\frac{x-10+x+6}{(x+6)(x-10)}=\frac{3}{x-4}$$

$$\Rightarrow \frac{2x-4}{(x+6)(x-10)} = \frac{3}{x-4}$$

$$\Rightarrow (2x-4)(x-4) = 3(x+6)(x-10)$$

$$\Rightarrow 2x^2-8x-4x+16 = 3(x^2-4x-60)$$

$$\Rightarrow 2x^2-12x+16 = 3x^2-12x-180$$

$$\Rightarrow -x^2+196 = 0 \Rightarrow x^2-196 = 0$$

$$\Rightarrow x^2-(14)^2 = 0 \Rightarrow (x+14)(x-14) = 0$$

either $x+14 = 0$ or $x-14 = 0$

$x = -14$ $\quad$ $x = +14$

$\therefore$ Hence $x = +14, -14$

Q25. (i) $\sqrt{3x+4} = x$

Sol. squaring on bothsides,

$$\Rightarrow 3x+4 = x^2 \Rightarrow x^2-3x-4 = 0$$

$$\Rightarrow x^2-4x+x-4 = 0 \Rightarrow x(x-4)+1(x-4) = 0$$

$$\Rightarrow (x-4)(x+1) = 0$$

either $x-4 = 0$ or $x+1 = 0$

$x = 4$ $\quad$ $x = -1$

$\therefore$ Hence $x = 4, -1$

(ii) $\sqrt{x(x-7)} = 3\sqrt{2}$

Sol. squaring on bothsides

$$x(x-7) = 9\times 2 \Rightarrow x^2-7x = 18$$

$$\Rightarrow x^2-7x-18 = 0 \Rightarrow x^2-9x+2x-18 = 0$$

$$\Rightarrow x(x-9)+2(x-9) = 0$$

$$\Rightarrow (x-9)(x+2) = 0.$$

either $x-9 = 0$ or $x+2 = 0$

$x = 9$ $\quad$ $x = -2$ $\quad$ $\therefore$ Hence $x = 9, -2$

Q26. find the values of x if $p+1=0$ and $x^2+px-6=0$

Sol. $p+1=0 \Rightarrow p=-1$

Substituting the value of p in the given equation

$x^2-x-6=0 \Rightarrow x^2-3x+2x-6=0$

$\Rightarrow x(x-3)+2(x-3)=0 \Rightarrow (x-3)(x+2)=0$

either $x-3=0$ or $x+2=0$

$x=3$ $\quad$ $x=-2$

$\therefore$ Hence $x=3,-2$.

Q27. find the values of x if $p+7=0$. $q-12=0$ and $x^2+px+q=0$

Sol.

$p+7=0$ $\quad$ $q-12=0$

$\Rightarrow p=-7$ $\quad$ $\Rightarrow q=12$

Substituting the values of p & q in the given equation

$x^2+(-7)x+12=0$

$\Rightarrow x^2-7x+12=0$

$\Rightarrow x^2-3x-4x+12=0$

$\Rightarrow x(x-3)-4(x-3)=0$

$\Rightarrow (x-3)(x-4)=0$

either $x-3=0$ or $x-4=0$

$x=3$ $\quad$ $x=4$

$\therefore$ Hence $x=3,4$

EXERCISE 5.2

Solve the following (1 to 7) equations by using formula:

Q1. (i) $2x^2 - 7x + 6 = 0$

Sol. Here $a = 2$, $b = -7$, $c = 6$

$\therefore D = b^2 - 4ac = (-7)^2 - 4(2)(6) = 49 - 48 = 1$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-(-7) \pm \sqrt{1}}{2 \times 2} = \dfrac{7 \pm 1}{4}$

$\therefore x_1 = \dfrac{7+1}{4} = \dfrac{8}{4} = 2$, $x_2 = \dfrac{7-1}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$\therefore x = 2, \dfrac{3}{2}$

(ii) $2x^2 - 6x + 3 = 0$

Sol. Here $a = 2$, $b = -6$, $C = 3$

then $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12$

Now $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-6) \pm \sqrt{12}}{2 \times 2} = \dfrac{6 \pm 2\sqrt{3}}{4}$

$\therefore x_1 = \dfrac{6 + 2\sqrt{3}}{4} = \dfrac{3 + \sqrt{3}}{2}$, $x_2 = \dfrac{6 - 2\sqrt{3}}{4} = \dfrac{3 - \sqrt{3}}{2}$

$\therefore x = \dfrac{3 + \sqrt{3}}{2}, \dfrac{3 - \sqrt{3}}{2}$

Q2. (i) $x^2 + 7x - 7 = 0$

Sol. Here $a = 1$, $b = 7$, $c = -7$

$\therefore D = b^2 - 4ac = (7)^2 - 4(1)(-7) = 49 + 28 = 77$

$\therefore x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-7 \pm \sqrt{77}}{2}$

$\therefore x_1 = \dfrac{-7 + \sqrt{77}}{2}$, $x_2 = \dfrac{-7 - \sqrt{77}}{2}$

(ii) $(2x+3)(3x-2)+2=0$

Sol. $\Rightarrow 6x^2-4x+9x-6+2=0$

$\Rightarrow 6x^2+5x-4=0$

Here $a=6,\ b=5,\ c=-4$

$\therefore D=b^2-4ac=(5)^2-4(6)(-4)=25+96=121$

$\therefore x=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-5\pm\sqrt{121}}{12}=\dfrac{-5\pm 11}{12}$

$\therefore x_1=\dfrac{-5+11}{12}=\dfrac{6}{12}=\dfrac{1}{2},\quad x_2=\dfrac{-5-11}{12}=\dfrac{-16}{12}=\dfrac{-4}{3}.$

$\therefore x=\dfrac{1}{2},\ \dfrac{-4}{3}$

Q3. (i) $256x^2-32x+1=0$

Sol. Here $a=256,\ b=-32,\ c=1$

$D=b^2-4ac=(-32)^2-4(256)(1)=1024-1024=0$

$\therefore x=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-(-32)\pm\sqrt{0}}{2\times 256}=\dfrac{32}{512}=\dfrac{1}{16}$

$\therefore$ Hence $x=\dfrac{1}{16},\ \dfrac{1}{16}$.

(ii) $25x^2+30x+7=0$

Sol. Here $a=25,\ b=30,\ c=7$

$D=b^2-4ac=(30)^2-4(25)(7)=900-700=200$

$\therefore x=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-30\pm\sqrt{200}}{2\times 25}=\dfrac{-30\pm 10\sqrt{2}}{50}.$

$=\dfrac{-3\pm\sqrt{2}}{5}$

$\therefore x_1=\dfrac{-3+\sqrt{2}}{5},\quad x_2=\dfrac{-3-\sqrt{2}}{5}.$

$\therefore x=\dfrac{-3+\sqrt{2}}{5},\ \dfrac{-3-\sqrt{2}}{5}$

Q4. (i) $2x^2 + \sqrt{5}x - 5 = 0$

Sol. Here $a = 2$, $b = \sqrt{5}$, $c = -5$

$D = b^2 - 4ac = (\sqrt{5})^2 - 4(2)(-5) = 5 + 40 = 45$

$\therefore x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\sqrt{5} \pm \sqrt{45}}{4} = \frac{-\sqrt{5} \pm 3\sqrt{5}}{4}$

$x_1 = \frac{-\sqrt{5} + 3\sqrt{5}}{4}$, $x_2 = \frac{-\sqrt{5} - 3\sqrt{5}}{4}$

$= \frac{\sqrt{5}}{2}$ $\qquad = -\sqrt{5}$

$\therefore x = \frac{\sqrt{5}}{2}, -\sqrt{5}$

(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Sol. Here $a = \sqrt{3}$, $b = 10$, $c = -8\sqrt{3}$

$D = b^2 - 4ac = (10)^2 - 4(\sqrt{3})(-8\sqrt{3})$

$= 100 + 96 = 196$

$\therefore x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-10 \pm \sqrt{196}}{2\sqrt{3}} = \frac{-10 \pm 14}{2\sqrt{3}}$

$\therefore x_1 = \frac{-10 + 14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\therefore x_2 = \frac{-10 - 14}{2\sqrt{3}} = \frac{-24}{2\sqrt{3}} = \frac{-12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = -4\sqrt{3}$

$\therefore$ Hence $x = \frac{2\sqrt{3}}{3}, -4\sqrt{3}$

Q5. (i) $\frac{x-2}{x+2} + \frac{x+2}{x-2} = 4$

Sol.

$\Rightarrow \frac{(x-2)^2 + (x+2)^2}{(x+2)(x-2)} = 4$

$\Rightarrow x^2 - 4x + 4 + x^2 + 4x + 4 = 4x^2 - 16$

$\Rightarrow 2x^2 + 8 - 4x^2 + 16 = 0$

$\Rightarrow -2x^2 + 24 = 0$

$\Rightarrow \quad x^2 - 12 = 0$

$\therefore$ Here $a = 1$, $b = 0$, $c = -12$

$D = b^2 - 4ac = 0^2 - 4(1)(-12) = 48$

$\therefore x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-0 \pm \sqrt{48}}{2 \times 1} = \dfrac{\pm 4\sqrt{3}}{2} = \pm 2\sqrt{3}$

$\therefore \quad x = \pm 2\sqrt{3}, -2\sqrt{3}$

(ii) $\dfrac{x+1}{x+3} = \dfrac{3x+2}{2x+3}$

Sol. $(x+1)(2x+3) = (3x+2)(x+3)$

$\Rightarrow \quad 2x^2 + 5x + 3 = 3x^2 + 11x + 6$

$\Rightarrow \quad x^2 + 6x + 3 = 0$

Here $a = 1$, $b = 6$, $c = 3$

$D = b^2 - 4ac = (6)^2 - 4(1)(3) = 36 - 12 = 24.$

$x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-6 \pm \sqrt{24}}{2 \times 1} = \dfrac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$

$\therefore \quad x = -3 + \sqrt{6}, -3 - \sqrt{6}$

Q6. (i) $a(x^2+1) = (a^2+1)x$, $a \neq 0$

Sol. $ax^2 + a = a^2x + x$

$\Rightarrow \quad ax^2 - (a^2+1)x + a = 0$

Here $a = a$, $b = -(a^2+1)$, $c = a$

$D = b^2 - 4ac = [-(a^2+1)]^2 - 4(a)(a)$

$= a^4 + 2a^2 + 1 - 4a^2 = a^4 - 2a^2 + 1 = (a^2-1)^2$

$\therefore x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{(a^2+1) \pm \sqrt{(a^2-1)^2}}{2a} = \dfrac{(a^2+1) \pm (a^2-1)}{2a}$

$\therefore x_1 = \frac{a^2+1+a^2-1}{2a} = \frac{2a^2}{2a} = a$

$\therefore x_2 = \frac{a^2+1-a^2+1}{2a} = \frac{2}{2a} = \frac{1}{a}$

$\therefore x = a, \frac{1}{a}$

(ii) $4x^2 - 4ax + (a^2-b^2) = 0$

Sol. Comparing the given equation with $ax^2+bx+c=0$, we get

$a = 4, \quad b = -4a, \quad c = (a^2-b^2)$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4a \pm \sqrt{(-4a)^2 - 4(4)(a^2-b^2)}}{2(4)}$$

$$= \frac{4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$$

$$= \frac{4a \pm 4b}{8} = \frac{a \pm b}{2}$$

$\therefore x = \frac{a+b}{2}, \frac{a-b}{2}$

Q7. $\frac{1}{x-2} + \frac{1}{x-3} + \frac{1}{x-4} = 0$

Sol.

$\Rightarrow \frac{1}{x-2} + \frac{1}{x-3} = -\frac{1}{x-4}$

$\Rightarrow \frac{x-3+x-2}{(x-2)(x-3)} = -\frac{1}{x-4}$

$\Rightarrow \frac{2x-5}{x^2-5x+6} = -\frac{1}{x-4}$

$\Rightarrow (2x-5)(x-4) = -1(x^2-5x+6)$

$\Rightarrow 2x^2-8x-5x+20 = -x^2+5x-6$

$\Rightarrow 3x^2-18x+26 = 0$

Here $a = 3$, $b = -18$, $c = 26$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18 \pm \sqrt{(-18)^2 - 4(3)(26)}}{2(3)}$$

$$= \frac{18 \pm \sqrt{324 - 312}}{6} = \frac{18 \pm \sqrt{12}}{6}$$

$$= \frac{18 \pm 2\sqrt{3}}{6} = \frac{9 \pm \sqrt{3}}{3}$$

$$x_1 = \frac{9 + \sqrt{3}}{3} = 3 + \frac{1}{\sqrt{3}}, \quad x_2 = \frac{9 - \sqrt{3}}{3} = 3 - \frac{1}{\sqrt{3}}$$

$$\therefore x = 3 + \frac{1}{\sqrt{3}}, \; 3 - \frac{1}{\sqrt{3}}$$

Q8. solve for x and given your answer correct to 2 decimal places : $x^2 - 10x + 6 = 0$

sol. Here $a = 1$, $b = -10$, $c = 6$

$$D = b^2 - 4ac = (-10)^2 - 4(1)(6) = 100 - 24 = 76$$

$$\therefore x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+10 \pm \sqrt{76}}{2} = \frac{10 \pm 2\sqrt{19}}{2} = 5 \pm \sqrt{19}$$

$$\therefore x_1 = 5 + \sqrt{19} = 5 + 4.36 = 9.36$$

$$x_2 = 5 - \sqrt{19} = 5 - 4.36 = 0.64$$

$$\therefore x = 9.36, \; 0.64$$

Q9. solve the following quadratic equations for x and given your answer correct to 2 decimal places :

(i) $x^2 - 3x - 9 = 0$

sol. Here $a = 1$, $b = -3$, $c = -9$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9+36}}{2}$$

$$= \frac{3 \pm \sqrt{45}}{2}$$

$$= \frac{3 \pm 3\sqrt{5}}{2}$$

$$\therefore x_1 = \frac{3+3\sqrt{3}}{2} = 4.85$$

$$\therefore x_2 = \frac{3-3\sqrt{3}}{2} = -1.85$$

(ii) $5x(x+2) = 3$

Sol. $5x^2 + 10x - 3 = 0$

Here $a = 5$, $b = 10$, $c = -3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-10 \pm \sqrt{(10)^2 - 4(5)(-3)}}{2(5)}$$

$$= \frac{-10 \pm \sqrt{100+60}}{10} = \frac{-10 \pm \sqrt{160}}{10} = \frac{-10 \pm 4\sqrt{10}}{10}$$

$$= \frac{-5 \pm 2\sqrt{10}}{5}$$

$$x_1 = \frac{-5+2\sqrt{10}}{5} = 0.26, \quad x_2 = \frac{-5-2\sqrt{10}}{5} = -2.26$$

$$\therefore x = 0.26, -2.26$$

Q10. Solve the following quadratic equations by using formula and give your answer correct to 2 decimal places :

(i) $x^2 - 5x - 10 = 0$

Sol. Here $a = 1$, $b = -5$, $c = -10$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)} = \frac{5 \pm \sqrt{25+40}}{2}$$

$$= \frac{5 \pm \sqrt{65}}{2}$$

$$\therefore x_1 = \frac{5+\sqrt{65}}{2} = 6.53$$

$$x_2 = \frac{5-\sqrt{65}}{2} = -1.53$$

(ii) $2x - \frac{1}{x} = 7$

Sol. $\Rightarrow 2x^2 - 1 = 7x$

$\Rightarrow 2x^2 - 7x - 1 = 0$

Here $a = 2$, $b = -7$, $c = -1$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2(2)} = \frac{7 \pm \sqrt{49+8}}{4}$$

$$= \frac{7 \pm \sqrt{57}}{4}$$

$$\therefore x_1 = \frac{7+\sqrt{57}}{4} = 3.64$$

$$\therefore x_2 = \frac{7-\sqrt{57}}{4} = -0.14$$

EXERCISE 5.3

Q1. find the discriminant of the following quadratic equations and hence find the nature of roots :

(i) $3x^2 - 5x - 2 = 0$

Sol. Here $a = 3$, $b = -5$, $c = -2$

Discriminant $D = b^2 - 4ac = (-5)^2 - 4(3)(-2) = 25 + 24$
$= 49$

As $D > 0$, so the equation has two distinct real roots.

(ii) $2x^2 - 3x + 5 = 0$

Here $a = 2$, $b = -3$, $c = 5$

$D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$

As $D < 0$, so the equation has no real roots.

(iii) $7x^2 + 8x + 2 = 0$

Here $a = 7$, $b = 8$, $c = 2$

$D = b^2 - 4ac = (8)^2 - 4(7)(2) = 64 - 56 = 8$

As $D > 0$, so the equation has two distinct real roots.

(iv) $3x^2 + 2x - 1 = 0$

Here $a = 3$, $b = 2$, $c = -1$

$D = b^2 - 4ac = (2)^2 - 4(3)(-1) = 4 + 12 = 16$

As $D > 0$, so the equation has two distinct real roots

(v) $16x^2 - 40x + 25 = 0$

Here $a = 16$, $b = -40$, $c = 25$

$D = (-40)^2 - 4(16)(25) = 1600 - 1600 = 0$

As $D = 0$, so the equation has two equal real roots.

(vi) $2x + 15x + 30 = 0$

Here $a = 2$, $b = 15$, $c = 30$

$D = (15)^2 - 4(2)(30) = 225 - 240 = -15$

As $D < 0$, so the equation has no real roots.

Q2. Discuss the nature of the roots of the following quadratic equations:

(i) $x^2 - 4x - 1 = 0$

Here $a = 1$, $b = -4$, $c = -1$

$D = (-4)^2 - 4(1)(-1) = 16 + 4 = 20$

As $D > 0$, so the equation has two distinct real roots.

(ii) $3x^2 - 2x + \frac{1}{3} = 0$

Here $a = 3$, $b = -2$, $c = \frac{1}{3}$

$D = (-2)^2 - 4(3)(\frac{1}{3}) = 4 - 4 = 0$

As $D = 0$, so the equation has two equal real roots.

(iii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Here $a = 3$, $b = -4\sqrt{3}$, $c = 4$

$D = (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

As $D = 0$, so the equation has two real equal roots

(iv) $x^2 - \frac{1}{2}x + 4 = 0$

Here $a = 1$, $b = \frac{-1}{2}$, $c = 4$

$D = (\frac{-1}{2})^2 - 4(1)(4) = \frac{1}{4} - 16 = \frac{-63}{4}$

As $D < 0$, so the equation has no real roots.

(v) $-2x^2 + x + 1 = 0$

Here $a = -2$, $b = 1$, $c = 1$

$D = (1)^2 - 4(-2)(1) = 1 + 8 = 9$

As $D > 0$, so the equation has two distinct real roots.

(vi) $2\sqrt{3}\,x^2 - 5x + \sqrt{3} = 0$

Here $a = 2\sqrt{3}$, $b = -5$, $c = \sqrt{3}$

$D = (-5)^2 - 4(2\sqrt{3})(\sqrt{3}) = 25 - 24 = 1$

As $D > 0$, so the equation has two distinct real roots.

Q3. Find the nature of the roots of the following quadratic equations; if real roots exist, find them

(i) $x^2 - \frac{1}{2}x - \frac{1}{2} = 0$

Here $a = 1$, $b = \frac{-1}{2}$, $c = \frac{-1}{2}$

$D = b^2 - 4ac = \left(\frac{-1}{2}\right)^2 - 4(1)\left(\frac{-1}{2}\right) = \frac{1}{4} + 2 = \frac{9}{4}$

As $D > 0$, so the equation has two real distinct roots.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\left(\frac{-1}{2}\right) \pm \sqrt{\frac{9}{4}}}{2(1)} = \frac{\frac{1}{2} \pm \frac{3}{2}}{2} = \frac{1}{4} \pm \frac{3}{4}$$

$\therefore x = 1, \frac{-1}{2}$

(ii) $x^2 - 2\sqrt{3}\,x - 1 = 0$

Here $a = 1$, $b = -2\sqrt{3}$, $c = -1$

$D = (-2\sqrt{3})^2 - 4(1)(-1) = 12 + 4 = 16$

As $D > 0$, so the equation has two distinct real roots.

$$x = \frac{2\sqrt{3} \pm \sqrt{16}}{2(1)} = \frac{2\sqrt{3} \pm 4}{2} = \sqrt{3} \pm 2$$

$\therefore x = 2 + \sqrt{3}, -2 + \sqrt{3}$

Q4. Find the values of k for which each of the following quadratic equation has equal roots:

(i) $kx^2 - 4x + 3 = 0$

Sol. Here $a = k$, $b = -4$, $c = 3$

$D = 0$ {∵ roots are equal}

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-4)^2 - 4(k)(3) \Rightarrow 16 - 12k = 0$

$\Rightarrow 16 = 12k \Rightarrow k = \frac{16}{12} = \frac{4}{3}$

(ii) $2x^2 + kx + 3 = 0$

Here $a = 2$, $b = k$, $c = 3$

Given that roots are equal. so $D = 0$

$b^2 - 4ac = 0$

$k^2 - 4(2)(3) = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(iii) $(k-4)x^2 + 2(k-4)x + 4 = 0$.

Here $a = k-4$, $b = 2(k-4)$, $c = 4$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [2(k-4)]^2 - 4(k-4)(4) = 0$

$\Rightarrow 4(k-4)^2 - 16(k-4) = 0$

$\Rightarrow (k-4)^2 - (k-4) = 0$

$\Rightarrow (k-4)[k-4-4] = 0$

$\Rightarrow (k-4)(k-8) = 0$

As $k \neq 4$. so the value of $k = 8$

(iv) $kx(x-2)+6=0$

$\Rightarrow \quad kx^2-2kx+6=0$

Here $a=k$, $b=-2k$, $c=6$

$\Rightarrow \quad b^2-4ac=0$

$\Rightarrow \quad (-2k)^2-4(k)(6)=0$

$\Rightarrow \quad 4k^2-24k=0$

$\Rightarrow \quad 4k(k-6)=0$

As $k \neq 0$, so $k=6$

Q5. Find the value of k for which each of the following quadratic equation has equal roots:

(i) $(3k+1)x^2+2(k+1)x+k=0$

Sol. Here $a=3k+1$, $b=2(k+1)$, $c=k$

Given that roots are equal, so $b^2-4ac=0$

$[2(k+1)]^2-4(3k+1)(k)=0$

$\Rightarrow \quad 4(k+1)^2 \quad 4k(3k+1)=0$

$\Rightarrow \quad 4k^2+4+8k-12k^2-4k=0$

$\Rightarrow \quad -8k^2+4k+4=0$

$\Rightarrow \quad 8k^2-4k-4=0$

$\Rightarrow \quad 2k^2-k-1=0$

$\Rightarrow \quad 2k^2-2k+k-1=0$

$\Rightarrow \quad 2k(k-1)+1(k-1)=0$

$\Rightarrow \quad (k-1)(2k+1)=0$

$k=1, -\frac{1}{2}$

(ii) $x^2 - 2(5+2k)x + 3(7+10k) = 0$

Sol. Here $a = 1$, $b = -2(5+2k)$, $c = 3(7+10k)$

Given that roots are equal, $b^2 - 4ac = 0$

$$[-2(5+2k)]^2 - 4(1)[3(7+10k)] = 0$$

$$\Rightarrow 4(25+4k^2+20k) - 4[21+30k] = 0$$

$$\Rightarrow 100 + 16k^2 + 80k - 84 - 120k = 0$$

$$\Rightarrow 16k^2 - 40k + 16 = 0$$

$$\Rightarrow 2k^2 - 5k + 2 = 0$$

$$\Rightarrow 2k^2 - 4k - k + 2 = 0$$

$$\Rightarrow 2k(k-2) - 1(k-2) = 0$$

$$\Rightarrow (k-2)(2k-1) = 0$$

$$\Rightarrow k = 2, \frac{1}{2}$$

Q6. Find the values of p for which each of the following quadratic equations has real roots:

(i) $px^2 + 4x + 1 = 0$

Here $a = p$, $b = 4$, $c = 1$

given that equation has real roots i.e. $D \geq 0$

$$\Rightarrow b^2 - 4ac \geq 0$$

$$\Rightarrow 16 - 4(p)(1) \geq 0$$

$$\Rightarrow 16 - 4p \geq 0$$

$$\Rightarrow p \leq 4$$

(ii) $4x^2 + 8x - p = 0$

Here $a = 4$, $b = 8$, $c = -p$

$b^2 - 4ac \geq 0 \Rightarrow 64 - 4(4)(-p) \geq 0$

$\Rightarrow 64 + 16p \geq 0 \Rightarrow 16p \geq -64$

$\Rightarrow p \geq -4$

Q7. Find the values of p for which the equation $3x^2 - px + 5 = 0$ has real roots.

Sol.

$$3x^2 - px + 5 = 0$$

Here $a = 3$, $b = -p$, $c = 5$

Given that roots of the given equation are real

i.e, $D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow (-p)^2 - 4(3)(5) \geq 0$

$\Rightarrow p^2 - 60 \geq 0$

$\Rightarrow p^2 \geq 60$

$\Rightarrow p \geq \pm 2\sqrt{15}$

Q8. Find the values of k for which each of the following quadratic equation has equal roots:

(i) $9x^2 + kx + 1 = 0$

Sol. Here $a = 9$, $b = k$, $c = 1$

$\because$ given that roots are equal ie $D = 0$

$\Rightarrow b^2 - 4ac = 0 \Rightarrow k^2 - 4(9)(1) = 0$

$\Rightarrow k^2 = 36 \Rightarrow k = \pm 6$

(ii) $x^2 - 2kx + 7k - 12 = 0$

Sol. Here $a = 1$, $b = -2k$, $c = 7k - 12$

Given that roots are equal, $D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4(1)(7k - 12) = 0$

$\Rightarrow 4k^2 - 28k + 48 = 0$

$\Rightarrow k^2 - 7k + 12 = 0$

$\Rightarrow k^2 - 3k - 4k + 12 = 0$

$\Rightarrow (k-3)(k-4) = 0 \Rightarrow k = 3, 4.$

EXERCISE-5.4

Q1. Find two consecutive natural numbers such that the sum of their squares is 61.

Sol. let the first number $= x$

then second number $= x+1$

According to the condition,

$$x^2+(x+1)^2 = 61$$

$$\Rightarrow x^2+x^2+2x+1 = 61$$

$$\Rightarrow 2x^2+2x-60=0$$

$$x^2+x-30=0$$

$$\Rightarrow x^2+6x-5x-30=0$$

$$\Rightarrow x(x+6)-5(x+6)=0$$

$$\Rightarrow (x+6)(x-5)=0$$

either $x+6=0$ or $x-5=0$

$x=-6$ $\quad x=5$

$\because$ The numbers are positive. so $x=-6$ is not possible.

Hence the first number $=5$

second number $=5+1=6$.

Q2. (i) If the product of two positive consecutive even integers is 288, find the integers.

Sol. let the first positive even integer $= 2x$

then the second integer $= 2x + 2$

According to the condition,

$$(2x)(2x+2) = 288$$

$$\Rightarrow 4x^2 + 4x - 288 = 0$$

$$\Rightarrow x^2 + x - 72 = 0$$

$$\Rightarrow x^2 + 9x - 8x - 72 = 0$$

$$\Rightarrow x(x+9) - 8(x+9) = 0$$

$$(x+9)(x-8)=0$$

either $x+9=0$ or $x-8=0$

$x=-9$ $\quad$ $x=8$

$x=-9$, but it is not possible as it is negative

$\therefore$ first even integer $= 2\times 8 = 16$

and second even integer $= 16+2 = 18$.

(ii) If the product of two consecutive even integers is 224, find the integers.

Sol. let the first even integer $= 2x$

then second even integer $= 2x+2$

According to the condition,

$$(2x)(2x+2) = 224$$

$$\Rightarrow 4x^2+4x-224=0$$

$$\Rightarrow x^2+x-56=0$$

$$\Rightarrow x^2+8x-7x-56=0$$

$$\Rightarrow x(x+8)-7(x+8)=0$$

$$\Rightarrow (x+8)(x-7)=0$$

either $x+8=0$ or $x-7=0$

$x=-8$ $\quad$ $x=7$

when $x=-8$, first even integer $= 2(-8) = -16$

second even integer $= -16+2 = -14$

when $x=7$, first even integer $= 2(7) = 14$

second even integer $= 14+2 = 16$

Q3. The Sum of two numbers is 9 and the Sum of their squares is 41. Taking one number as x, form an equation in x and solve it to find the numbers.

sol. Sum of two numbers $= 9$

let the first number $= x$

then second number $= 9 - x$

Now according to the condition

$$x^2 + (9-x)^2 = 41$$

$$\Rightarrow x^2 + 81 - 18x + x^2 - 41 = 0$$

$$\Rightarrow 2x^2 - 18x + 40 = 0$$

$$\Rightarrow x^2 - 9x + 20 = 0$$

$$x^2 - 5x - 4x + 20 = 0$$

$$\Rightarrow x(x-5) - 4(x-5) = 0$$

$$\Rightarrow (x-5)(x-4) = 0$$

either $x - 5 = 0$, then $x = 5$ or $x - 4 = 0$ then $x = 4$

If $x = 5$, first number $= 5$, second number $= 4$

If $x = 4$, first number $= 4$, second number $= 5$

$\therefore$ Hence numbers are 4 & 5.

Q4. Five times a certain whole number is equal to three less than twice the square of the number. find the number.

sol. let number $= x$

Now according to the condition,

$$5x = 2x^2 - 3$$

$$\Rightarrow 2x^2 - 5x - 3 = 0$$

$$\Rightarrow 2x^2 - 6x + x - 3 = 0$$

$$\Rightarrow 2x(x-3) + 1(x-3) = 0$$

$$\Rightarrow (x-3)(2x+1) = 0$$

either $x-3=0$, then $x=3$

or $2x+1=0$, then $x=-1/2$

But it is not possible as the number is whole number.

$\therefore$ Number $=3$.

Q5. Divide 15 into two parts such that the sum of their reciprocals is 3/10.

Sol. let first part $= x$

then second part $= 15-x$

Now according to the condition,

$$\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$$

$$\Rightarrow \frac{15-x+\cancel{x}}{x(15-x)}=\frac{3}{10} \Rightarrow \frac{15}{15x-x^2}=\frac{3}{10}$$

$$\Rightarrow 150=45x-3x^2 \Rightarrow 3x^2-45x+150=0$$

$$\Rightarrow x^2-15x+50=0 \Rightarrow x^2-5x-10x+50=0$$

$$\Rightarrow x(x-5)-10(x-5)=0 \Rightarrow (x-5)(x-10)=0$$

Either $x-5=0$ or $x-10=0$.

$x=5$ $x=10$

If $x=5$, first part $=5$, second part $=10$

If $x=10$, first part $=10$, second part $=5$

$\therefore$ Hence parts are 5 & 10.

Q6. The difference of the squares of two numbers is 45. The square of the smaller number is 4 times the larger number. Determine the numbers.

Sol. let the larger number $= x$

the smaller number $= y$

Now according to the condition,

$x^2-y^2=45$ —— (i) $\quad y^2=4x$ —— (ii)

Substituting the value of y^2 from (ii) in (i),

$$x^2-4x=45 \Rightarrow x^2-4x-45=0$$

$$\Rightarrow x^2-9x+5x-45=0 \Rightarrow x(x-9)+5(x-9)=0$$

$$\Rightarrow (x-9)(x+5)=0$$

either $x-9=0$ or $x+5=0$

$x=9$ $\qquad$ $x=-5$

If $x=9$, the larger number $=9$

Smaller number $y=\sqrt{4x}=\sqrt{4\times 9}=6$

$\therefore$ Hence numbers are 6 & 9

If $x=-5$, the larger number $=-5$

smaller number $y=\sqrt{4(-5)}=\sqrt{-20}$

$\therefore$ which is not possible.

Q7. There are three consecutive positive integers such that the sum of the squares of the first and the product of other two is 154. What are the integers?

Sol. let the first integer $=x$

the second integer $=x+1$

and third integer $=x+2$

Now according to the condition,

$$x^2+(x+1)(x+2)=154.$$

$$\Rightarrow x^2+x^2+3x+2=154$$

$$\Rightarrow 2x^2+3x-152=0$$

$$\Rightarrow 2x^2+19x-16x-152=0$$

$$\Rightarrow x(2x+19)-8(2x+19)=0$$

$$\Rightarrow (2x+19)(x-8)=0$$

either $2x+19=0$, then $x=-19/2$

$\therefore$ But it is not possible as it is not an integer.

Or $x-8=0$, then $x=8$

$\therefore$ Numbers are 8, 9 and 10.

Q8. (i) Find three successive even natural numbers, the sum of whose squares is 308.

Sol. Let first even number $=2x$

second even number $=2x+2$

third even number $=2x+4$

Now according to the question,

$$(2x)^2+(2x+2)^2+(2x+4)^2=308$$

$$\Rightarrow 4x^2+4x^2+8x+4+4x^2+16+16x=308$$

$$\Rightarrow 12x^2+24x-288=0$$

$$\Rightarrow x^2+2x-24=0$$

$$\Rightarrow x^2+6x-4x-24=0$$

$$\Rightarrow x(x+6)-4(x+6)=0$$

$$\Rightarrow (x+6)(x-4)=0$$

either $x+6=0$, then $x=-6$

But it is not possible as it is not a natural number.

or $x-4=0$, then $x=4$.

$\therefore$ first even natural number $=2\times4=8$

second even natural number $=8+2=10$

third number $=10+2=12$.

(ii) Find three consecutive odd integers, the sum of whose squares is 83.

Sol. Let the three consecutive odd integers are

$x+1$, $x+3$, $x+5$.

According to the question,

$$(x+1)^2+(x+3)^2+(x+5)^2=83.$$

$$\Rightarrow x^2+2x+1+x^2+6x+9+x^2+10x+25=83$$

$$\Rightarrow 3x^2+18x+35=83$$

$$\Rightarrow 3x^2+18x-48=0$$

$$\Rightarrow x^2+6x-16=0 \Rightarrow x^2+8x-2x-16=0$$

$$\Rightarrow x(x+8)-2(x+8)=0 \Rightarrow (x+8)(x-2)=0$$

either $x+8=0$ or $x-2=0$

$x=-8$ $\quad$ $x=2$

when $x=-8$, The numbers are $-7, -5, -3$.

when $x=2$, The numbers are $3, 5, 7$.

Q9. In a certain positive fraction, the denominator is greaterthan the numerator by 3. If 1 is Subtracted from both the numerator and denominator, the fraction is decreased by 1/14. find the fraction?

Sol. let the numerator of a fraction $= x$

then denominator $= x+3$

then fraction $= \frac{x}{x+3}$

Now according to the condition,

$$\frac{x-1}{x+3-1} = \frac{x}{x+3} - \frac{1}{14}$$

$$\Rightarrow \frac{x-1}{x+2} = \frac{14x-x-3}{14x+42}$$

$$\Rightarrow \frac{x-1}{x+2} = \frac{13x-3}{14x+42}$$

$$\Rightarrow (x-1)(14x+42) = (13x-3)(x+2)$$

$$\Rightarrow 14x^2+42x-14x-42 = 13x^2+26x-3x-6$$

$$\Rightarrow x^2+5x-36=0$$

$$\Rightarrow x^2+9x-4x-36=0$$

$\Rightarrow \quad x(x+9) - 4(x+9) = 0$

$\Rightarrow \quad (x+9)(x-4) = 0$

either $x+9=0$. then $x=-9$. but it is not possible as the fraction is positive.

or $x-4=0$, then $x=4$

$\therefore$ Fraction $= \dfrac{x}{x+3} = \dfrac{4}{4+3} = \dfrac{4}{7}$

Q10. The sum of the numerator and denominator of a certain positive fraction is 8. If 2 is added to both the numerator and denominator, the fraction is increased by 4/35. find the fraction.

Sol. let the denominator of a positive fraction $= x$

then numerator $= 8-x$

$\therefore$ fraction $= \dfrac{8-x}{x}$

According to the condition,

$$\frac{8-x+2}{x+2} = \frac{8-x}{x} + \frac{4}{35}$$

$$\Rightarrow \frac{10-x}{x+2} - \frac{8-x}{x} = \frac{4}{35}$$

$$\Rightarrow \frac{10x - x^2 - 8x + x^2 - 16 + 2x}{x(x+2)} = \frac{4}{35}$$

$$\Rightarrow \frac{4x-16}{x^2+2x} = \frac{4}{35}$$

$\Rightarrow \quad 4x^2 + 8x = 140x - 560$

$\Rightarrow \quad 4x^2 - 132x + 560 = 0$

$\Rightarrow \quad x^2 - 33x + 140 = 0$

$\Rightarrow \quad x^2 - 28x - 5x + 140 = 0$

$\Rightarrow \quad x(x-28) - 5(x-28) = 0$

$\Rightarrow (x-28)(x-5)=0$

either $x-28=0$ then $x=28$, but it is not possible as sum of numerator and denominator is 8.

or $x-5=0$ then $x=5$

$\therefore$ Fraction $= \frac{8-x}{x} = \frac{8-5}{5} = \frac{3}{5}$

Q11. A two digit number contains the bigger at ten's place. The product of the digits is 27 and the difference between two digits is 6. find the number.

Sol. let unit's digit $= x$

then ten's digit $= x+6$

$\therefore$ Number $= x+10(x+6) = x+10x+60 = 11x+60$

According to the condition,

$x(x+6) = 27 \Rightarrow x^2+6x-27=0$

$\Rightarrow x^2+9x-3x-27=0 \Rightarrow x(x+9)-3(x+9)=0$

$\Rightarrow (x+9)(x-3)=0$

either $x+9=0$, then $x=-9$, but it is not possible as it is negative.

or $x-3=0$ then $x=3$

$\therefore$ Number $= 11x+60 = 11\times 3+60 = 93$.

Q12. A two digit number is such that the product of its digits is 24. when 18 is subtracted from this number, the digits interchanging their places. find the number.

Sol. let the unit's digit $= x$

then ten's digit $= \frac{24}{x}$

$\therefore$ Number $= x+10\left(\frac{24}{x}\right) = x+\frac{240}{x}$

on reversing the digits places.

the unit digit = $24/x$

and ten's digit = x

Then the new number = $\frac{24}{x} + 10x$

Now according to the condition,

$$x + \frac{240}{x} - 18 = \frac{24}{x} + 10x$$

$$\Rightarrow \frac{x^2+240}{x} - 18 = \frac{24+10x^2}{x}$$

$$\Rightarrow \frac{x^2+240-24-10x^2}{x} = 18$$

$$\Rightarrow -9x^2+216 = 18x$$

$$\Rightarrow -9x^2-18x+216=0$$

$$\Rightarrow x^2+2x-24=0$$

$$\Rightarrow x^2+6x-4x-24=0$$

$$\Rightarrow x(x+6)-4(x+6)=0$$

$$\Rightarrow (x+6)(x-4)=0$$

either $x+6=0$, then $x=-6$. but it is not possible as it is negative.

or $x-4=0$ then $x=4$

$\therefore$ Number $= x+\frac{240}{x} = 4+\frac{240}{4} = 64$.

Q13. A rectangle of area $105 cm^2$ has its length equal to x cm. write down its breadth interms of x. Give that the perimeter is 44cm, writedown an equation in x and solve it to determine the dimensions of the rectangle.

Sol. perimeter of the rectangle = 44cm.

$\therefore$ length + breadth $= \frac{44}{2} = 22$cm.

let length $= x$ then breadth $= 22 - x$

According to the condition,

$x(22-x) = 105 \Rightarrow 22x - x^2 = 105$

$\Rightarrow x^2 - 22x + 105 = 0 \Rightarrow x^2 - 15x - 7x + 105 = 0$

$\Rightarrow x(x-15) - 7(x-15) = 0 \Rightarrow (x-15)(x-7) = 0$

either $x - 15 = 0$ then $x = 15$

or $x - 7 = 0$ then $x = 7$

As length > breadth, $x = 7$ is not admissible.

$\therefore$ length $= 15$cm, breadth $= 22 - 15 = 7$cm.

Q14. A rectangular garden $10m \times 16m$ is to be surrounded by a concrete walk for uniform width. given that the area of the walk is $120\ m^2$, assuming the width of the walk to be x, form an equation in x and solve it to find the value of x.

Sol. length of garden $= 16m$ and width $= 10m$.

let width of walk $= x$ m.

$\therefore$ outer length $= 16 + 2x$, width $= 10 + 2x$.

Now according to the condition,

$(16+2x)(10+2x) - 16 \times 10 = 120$

$\Rightarrow 160 + 32x + 20x + 4x^2 - 160 = 120$

$\Rightarrow 4x^2 + 52x - 120 = 0 \Rightarrow x^2 + 13x - 30 = 0$

$\Rightarrow x^2 + 15x - 2x - 30 = 0 \Rightarrow x(x+15) - 2(x+15) = 0$

$\Rightarrow (x+15)(x-2) = 0$

either $x + 15 = 0$, then $x = -15$, but it is not possible.

or $x - 2 = 0$, then $x = 2$.

Q15. (i) Harish made a rectangular garden, with its length 5m more than its width. The next year, he increased the length by 3m and decreased by the width by 2m. If the area of the second garden was 119 sq.m., was the second garden larger or smaller?

Sol. In first case,

let length of the garden $= x$ m.

and width $= (x-5)$ m.

Area = length × width $= x(x-5)$ m^2

In second case, length $= x+3$, width $= x-7$ m.

according to the condition,

$(x+3)(x-7) = 119 \Rightarrow x^2 - 4x - 21 - 119 = 0$

$\Rightarrow x^2 - 4x - 140 = 0 \Rightarrow x^2 - 14x + 10x - 140 = 0$

$\Rightarrow x(x-14) + 10(x-14) = 0 \Rightarrow (x-14)(x+10) = 0$

either $x - 14 = 0$, then $x = 14$

or $x + 10 = 0$, then $x = -10$, but it is not possible as it is negative.

∴ length of first garden $= 14$m & width $= 9$m

Area $= l \times b = 14 \times 9 = 126$ m^2.

Difference of areas of two rectangles $= 126 - 119 = 7$ m^2

∴ Area of second garden is smaller than the area of the first garden by $7m^2$.

(ii) The length of a rectangle exceeds its breadth by 5m. If the breadth were doubled and the length reduced by 9m, the area of the rectangle would have increased by $140m^2$. Find its dimensions?

Sol. let the length of rectangle $= x$ m & width $= (x-5)$ m

Area $= x(x-5)$ m^2

In second case, length $= x-9$ & width $= 2(x-5)$ m

Area $= 2(x-9)(x-5)$ m^2

According to the condition,

$2(x-9)(x-5) = x(x-5)+140$

$\Rightarrow 2[x^2-14x+45] = x^2-5x+140$

$\Rightarrow 2x^2-28x+90-x^2+5x-140=0$

$\Rightarrow x^2-23x-50=0$

$\Rightarrow x^2-25x+2x-50=0$

$\Rightarrow x(x-25)+2(x-25)=0$

$\Rightarrow (x-25)(x+2)=0$

either $x-25=0$, then $x=25$

or $x+2=0$, then $x=-2$, but it is not possible as it is negative

$\therefore$ length of rectangle $= 25$ m.

width $= 25-5 = 20$ m.

Q16. The perimeter of a rectangular plot is 180m and its area is $1800\ m^2$. Take the length of the pole as x m. Use the perimeter 180m to write the value of the breadth interms of x. Use the values of length, breadth and the area to write an equation in x. solve the equation to calculate the length and breadth of the plot.

Sol. perimeter of a rectangular field $= 180$m

and area $= 1800\ m^2$

let length $= x$ m.

But length + breadth $= \frac{180}{2} = 90m$

$\therefore$ breadth $= (90-x)$ m

According to the condition,

$x(90-x) = 1800 \Rightarrow 90x - x^2 = 1800$

$\Rightarrow x^2 - 90x + 1800 = 0 \Rightarrow x^2 - 60x - 30x + 1800 = 0$

$\Rightarrow x(x-60) - 30(x-60) = 0 \Rightarrow (x-60)(x-30) = 0$

either $x - 60 = 0$, then $x = 60$

or $x - 30 = 0$, then $x = 30$

$\therefore$ length is greater than its breadth

$\therefore$ length $= 60m$, breadth $= 90 - 60 = 30m$

Q17. The lengths of the parallel sides of a trapezium are $(x+9)$ cm and $(2x-3)$ cm, and the distance between them is $(x+4)$ cm. if its area is $540 cm^2$, find x.

Sol. Area of a trapezium $= \frac{1}{2}$ (Sum of parallel sides) $\times$ height.

lengths of parallel sides are $(x+9)$ and $(2x-3)$ and

height $= x+4$

According to the condition,

$\frac{1}{2}(x+9+2x-3)(x+4) = 540$

$\Rightarrow (3x+6)(x+4) = 1080 \Rightarrow 3x^2 + 12x + 6x + 24 - 1080 = 0$

$\Rightarrow 3x^2 + 18x - 1056 = 0 \Rightarrow x^2 + 6x - 352 = 0$

$\Rightarrow x^2 + 22x - 16x - 352 = 0 \Rightarrow x(x+22) - 16(x+22) = 0$

$\Rightarrow (x+22)(x-16) = 0$

either $x + 22 = 0$, then $x = -22$.

But it is not possible as it is negative.

or $x - 16 = 0$, then $x = 16$.

Q18. If the perimeter of a rectangular plot is 68 m and length of its diagonal is 26 m, find its area.

Sol. perimeter = 68 m, diagonal = 26 m

$\therefore$ length + breadth = $\frac{68}{2}$ = 34 m

let length = x m and breadth = $(34 - x)$ m.

According to the condition,

$$l^2 + b^2 = h^2$$

$$x^2 + (34 - x)^2 = (26)^2$$

$\Rightarrow x^2 + 1156 + x^2 - 68x = 676$

$\Rightarrow 2x^2 - 68x + 1156 - 676 = 0 \Rightarrow 2x^2 - 68x + 480 = 0$

$\Rightarrow x^2 - 34x + 240 = 0 \Rightarrow x^2 - 24x - 10x + 240 = 0$

$\Rightarrow x(x - 24) - 10(x - 24) = 0 \Rightarrow (x - 24)(x - 10) = 0$

either $x - 24 = 0$, then $x = 24$

or $x - 10 = 0$, then $x = 10$

$\therefore$ length is greater than breadth.

length = 24 m and breadth = 34 − 24 = 10 m

Area = $l \times b$ = 24×10 = $240\,m^2$.

Q19. If the sum of two smaller sides of a right-angled triangle is 17 cm and the perimeter is 30 cm, then find the area of the triangle.

Sol. let one of the two smaller sides be x cm, then the other side is $(17 - x)$ cm

As we know

perimeter of triangle = length of two smaller sides + length of hypotenuse.

$\Rightarrow 30 = x + 17 - x +$ length of hypotenuse

$\Rightarrow$ length of hypotenuse = 30 − 17 = 13 cm

According to pythgoras theorem.

$$(13)^2 = (17-x)^2 + x^2$$

$\Rightarrow \quad 169 = 289 - 34x + x^2 + x^2$

$\Rightarrow \quad 2x^2 - 34x + 120 = 0 \Rightarrow x^2 - 17x + 60 = 0$

$\Rightarrow x^2 - 12x - 5x + 60 = 0 \Rightarrow x(x-12) - 5(x-12) = 0$

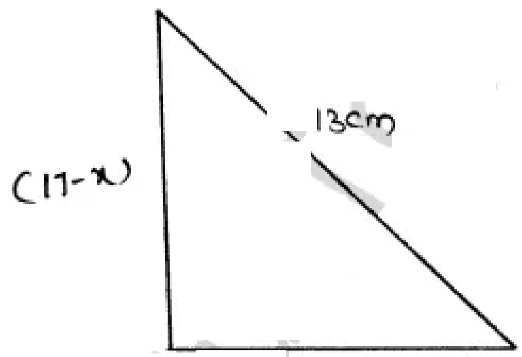
$\Rightarrow (x-12)(x-5) = 0$

$\Rightarrow x = 12, 5$

then smaller sides are 5cm, 12cm or 12cm, 5cm.

Area of the triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$



Q20. The hypotenuse of a grassy land in the shape of a right triangle is 1m morethan twice the shortest side. If the third side is 7m morethan the shortest side, find the sides of the grassy plot.

Sol. let the shortest side $= x$

Hypotenuse $= 2x+1$ and third side $= x+7$

According to the condition,

$(2x+1)^2 = x^2 + (x+7)^2$

$\Rightarrow 4x^2 + 4x + 1 = x^2 + x^2 + 14x + 49$

$\Rightarrow \quad 2x^2 - 10x - 48 = 0$

$\Rightarrow \quad x^2 - 5x - 24 = 0$

$\Rightarrow \quad x^2 - 8x + 3x - 24 = 0$

$\Rightarrow \quad x(x-8) + 3(x-8) = 0$

$\Rightarrow \quad (x-8)(x+3) = 0$

either $x - 8 = 0$, then $x = 8$

or $x + 3 = 0$, then $x = -3$, but it is not possible as it is negative.

$\therefore$ shortest side $= 8m$.

Third side $= x + 7 = 8 + 7 = 15m$ and

hypotenuse $= 2x + 1 = 2(8) + 1 = 17m$.

Q21. Mohini wishes to fit three rods together in the shape of a right triangle. if the hypotenuse is 2cm longer than the base and 4cm longer than the shortest side, find the lengths of the sides.

Sol. let the length of hypotenuse $= x$ cm

then base $= (x-2)$ cm and shortest side $= x - 4$

According to the condition,

$x^2 = (x-2)^2 + (x-4)^2 \Rightarrow x^2 = x^2 - 4x + 4 + x^2 - 8x + 16$

$\Rightarrow \quad x^2 = 2x^2 - 12x + 20 \Rightarrow x^2 - 12x + 20 = 0$

$\Rightarrow \quad x^2 - 10x - 2x + 20 = 0 \Rightarrow x(x-10) - 2(x-10) = 0$

$\Rightarrow \quad (x-10)(x-2) = 0$

either $x - 10 = 0$, then $x = 10$ or $x - 2 = 0$, then $x = 2$

but it is not possible as the hypotenuse is the longest side.

$\therefore$ Hypotenuse $= 10$cm.

Base $= 10 - 2 = 8$ cm.

and shortest side $= 10 - 4 = 6$ cm.

Q22. In a P.T display, 480 students are arranged in rows and columns. If there are 4 more students in each row than the number of rows, find the number of students in each row.

Sol. Total number of students = 480

let the number of students in each row = x

then the number of rows = $\frac{480}{x}$

According to the condition, $x = \frac{480}{x} + 4$

$\Rightarrow x^2 - 4x - 480 = 0 \Rightarrow x^2 - 24x + 20x - 480 = 0$

$\Rightarrow (x-24)(x+20) = 0$

$\Rightarrow x = 24, -20$

$\therefore x = -20$, is not possible as it is negative.

$\therefore$ Number of students in each row = 24.

Q23 In an auditorium, the number of rows was equal to the number of seats in each row. If the number of rows is doubled and the number of seats in each row is reduced by 5, then the total number of seats is increased by 375. How many rows were there?

Sol. let the number of rows = x

then the number of seats in each row = x and

total number of seats = $x \times x = x^2$

According to the condition,

$2x(x-5) = x^2 + 375$

$\Rightarrow 2x^2 - 10x - x^2 - 375 = 0$

$\Rightarrow x^2 - 10x - 375 = 0$

$\Rightarrow x^2 - 25x + 15x - 375 = 0$

$\Rightarrow (x-25)(x+15) = 0$

$\Rightarrow x = 25, -15.$

$x = -15$, but it is not possible as it is negative.

$\therefore$ Number of rows = 25.

Q24. At an annual function of a School, each students gives gift to every student. If the number of gifts is 1980, find the number of students.

Sol. let the number of Students $= x$

then the number of gifts given $= x-1$

$\therefore$ Total number of gifts $= x(x-1)$.

According to the condition,

$x(x-1) = 1980 \Rightarrow x^2 - x - 1980 = 0$

$\Rightarrow x^2 - 45x + 44x - 1980 = 0 \Rightarrow (x-45)(x+44) = 0$

$\Rightarrow x = 45, -44$.

$\therefore x = -44$, but it is not possible as it is negative.

Hence no. of students $= 45$.

Q25. An Express train makes a run of 240km at a Certain Speed. Another train, whose Speed is 12km/hr less, takes an hour longer to make the same trip. find the Speed of the Express train.

Sol. let the speed of the Express train $= x$ km/hr.

$\therefore$ Time taken by this train $= \frac{240}{x}$ hrs.

Speed of the other train $= (x-12)$ km/hr

$\therefore$ Time taken by this train $= \frac{240}{x-12}$ hrs.

According to the condition,

$\frac{240}{x-12} - \frac{240}{x} \Rightarrow 1$

$\Rightarrow 240x + 2880 - 240x = x(x-12)$

$\Rightarrow x^2 - 12x - 2880 = 0$

$\Rightarrow x^2 - 60x + 48x - 2880 \Rightarrow (x-60)(x+48) = 0$

$\Rightarrow x = 60, -48$

$\therefore$ Speed of the Express train $= 60$ km/hr.

Q26. A train covers a distance of 600km at x km/hr. Had the speed been $(x+20)$ km/hr, the time taken to cover the distance would have been reduced by 5 hours. Write down an equation in x and solve it to evaluate x.

Sol. Speed of train $= x$ km/hr.

Distance $= 600$ km

$\therefore$ Time taken $= \frac{600}{x}$ hrs.

In second case, Speed $= (x+20)$ km/hr

$\therefore$ Time taken $= \frac{600}{x+20}$ hrs.

According to the condition,

$$\frac{600}{x} - \frac{600}{x+20} = 5 \Rightarrow 600\left[\frac{1}{x} - \frac{1}{x+20}\right] = 5$$

$$\Rightarrow 600\left[\frac{x+20-x}{x(x+20)}\right] = 5 \Rightarrow x^2 + 20x = \frac{600 \times 20}{5}$$

$$\Rightarrow x^2 + 20x - 2400 = 0$$

$$\Rightarrow x^2 + 60x - 40x - 2400 = 0 \Rightarrow (x+60)(x-40) = 0$$

$$\Rightarrow x = 40, -60.$$

$x = -60$, but it is not possible as it is negative.

$\therefore x = 40.$

Q27. An Aeroplane travelled a distance of 400km at an average speed of x km/hr. On the return journey, the speed was increased by 40km/hr. Write down an expression for the time taken for: (i) The onward journey (ii) the return journey. If the return journey took 30 mins less than the onward journey, write down an equation in x and find its value.

Sol. Distance $= 400$ km.

Speed of aeroplane $= x$ km/hr

(i) time taken $= \frac{400}{x}$ hrs.

On increasing the speed by 40 km/hr, on the return journey, the speed $= (x+40)$ km/hr.

(ii) time taken $= \frac{400}{x+40}$ hrs

Now according to the condition,

$$\frac{400}{x} - \frac{400}{x+40} = 30 \text{ mins} = \frac{1}{2} \text{ hr}$$

$$\Rightarrow 400\left[\frac{x+40-x}{x(x+40)}\right] = \frac{1}{2} \Rightarrow x^2+40x = 400\times40\times2$$

$$\Rightarrow x^2+40x-32000=0 \Rightarrow x^2+200x-160x-32000=0$$

$$\Rightarrow (x+200)(x-160)=0$$

$$\Rightarrow x = 160, -200$$

$\therefore x = -200$, but it is not possible as it is negative.

$\therefore x = 160$.

Q28 The distance by road between two towns A & B is 216 km, and by rail it is 208 km. A car travels at a speed of x km/hr, and the train travels at a speed which is 16 km/hr faster than the car. Calculate:

(i) The time taken by the car, to reach town B from A, in terms of x.

(ii) The time taken by train, to reach town B from A, in terms of x.

(iii) If the train takes 2 hrs less than the car, to reach town B, obtain an equation in x, and solve it.

(iv) Hence find the speed of the train.

Sol. The distance by road between A & B = 216 km.

and the distance by rail = 208 km.

Speed of car = x km/hr.

(i) Time taken by car $= \frac{216}{x}$ hrs.

(ii) Time taken by train $= \frac{208}{x+16}$ hrs.

(iii) According to condition, $\frac{216}{x} - \frac{208}{x+16} = 2$

$\Rightarrow 216x + (216 \times 16) - 208x = 2(x+16)(x)$

$\Rightarrow 8x + 3456 = 2x^2 + 32x$

$\Rightarrow 2x^2 + 24x - 3456 = 0 \Rightarrow x^2 + 12x - 1728 = 0$

$\Rightarrow x^2 + 48x - 36x - 1728 = 0 \Rightarrow (x+48)(x-36) = 0$

either $x + 48 = 0 \Rightarrow x = -48$, but it is not possible as it is negative. or $x - 36 = 0 \Rightarrow x = 36$.

(iv) $\therefore$ Speed of train $= x + 16 = 36 + 16 = 52$ km/hr

Q29. An aeroplane flying with a wind of 30 km/hr takes 40 mins less to fly 3600 km, than what it would have taken to fly against the same wind. Find the plane's speed of flying in still air.

Sol. Let the speed of the plane in still ~~water~~ air $= x$ km/hr

Speed of wind $= 30$ km/hr, Distance $= 3600$ km.

$\therefore$ time taken with the wind $= \frac{3600}{x+30}$ and time taken against the wind $= \frac{3600}{x-30}$

According to the condition,

$\frac{3600}{x-30} - \frac{3600}{x+30} = 40 \text{ mins} = \frac{2}{3}$ hrs.

$\Rightarrow 3600\left[\frac{x+30-x+30}{(x-30)(x+30)}\right] = \frac{2}{3} \Rightarrow \frac{3600 \times 60}{x^2 - 900} = \frac{2}{3}$

$\Rightarrow 2x^2 - 1800 = 3600 \times 60 \times 3 \Rightarrow 2x^2 - 649800 = 0$

$\Rightarrow x^2 - 324900 = 0 \Rightarrow x^2 - (570)^2 = 0$

$\Rightarrow (x+570)(x-570) = 0$

$\therefore x = 570, -570$

$\therefore$ Hence speed of plane in still ~~water~~ = 570 km/hr

Q30. A school bus transported an Excursion party to picnic spot 150 km away. while returning, it was raining and the bus had to reduce its speed by 5 km/hr, and it took one hour longer to make the return trip. find the time taken to return.

Sol. let the speed of bus = x km/hr, distance = 150 km.

$\therefore$ time taken = $\frac{150}{x}$ hrs.

on returning speed of the bus = $(x-5)$ km/hr

$\therefore$ time taken = $\frac{150}{x-5}$

According to condition, $\frac{150}{x-5} - \frac{150}{x} = 1$

$\Rightarrow 150\left[\frac{1}{x-5} - \frac{1}{x}\right] = 1 \Rightarrow 150\left[\frac{x-x+5}{x(x-5)}\right] = 1$

$\Rightarrow 150 \times 5 = x^2 - 5x \Rightarrow x^2 - 5x - 750 = 0$

$\Rightarrow x^2 - 30x + 25x - 750 = 0 \Rightarrow (x-30)(x+25) = 0$

either $x+25=0$, then $x=-25$, which is not possible as it is negative or $x-30=0 \Rightarrow x = 30$.

$\therefore$ speed of bus = 30 km/hr.

time taken while returning = $\frac{150}{x-5} = \frac{150}{30-5} = 6$ hours.

Q31. Two pipes running together can fill a tank in $11\frac{1}{9}$ mins. if one pipe takes 5 mins more than the other to fill the tank. find the time in which each pipe would fill the tank.

Sol. let the time taken by one pipe = x mins.

then time taken by second pipe = $(x+5)$ mins.

time taken by both pipes = $11\frac{1}{9}$ mins.

Now according to condition, $\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$

$\Rightarrow \quad \frac{x+5+x}{x^2+5x} = \frac{9}{100} \Rightarrow 200x + 500 = 9x^2 + 45x$

$\Rightarrow \quad 9x^2 - 155x - 500 = 0 \Rightarrow 9x^2 - 180x + 25x - 500 = 0$

$\Rightarrow \quad 9x(x-20) + 25(x-20) = 0 \Rightarrow (x-20)(9x+25) = 0$

either $x-20$. $\Rightarrow x = 20$ or $9x+25=0$. $\Rightarrow x = -\frac{25}{9}$

but it is not possible as it is negative.

$\therefore x = 20.$

Hence the first pipe can fill the tank in 20mins and second pipe can do the same in $20+5 = 25$mins.

Q32. $2x$ articles cost $(5x+54)$ and $(x+2)$ similar articles cost $(10x-4)$; find x.

Sol. cost of $2x$ articles $= 5x + 54$.

$\therefore$ cost of 1 article $= \frac{5x+54}{2x}$ —— (i)

Again cost $(x+2)$ articles $= 10x - 4$

$\therefore$ cost of 1 article $= \frac{10x-4}{x+2}$. —— (ii)

from (i) & (ii), $\frac{5x+54}{2x} = \frac{10x-4}{x+2}$

$\Rightarrow (5x+54)(x+2) = (10x-4)(2x)$

$\Rightarrow 5x^2 + 10x + 54x + 108 = 20x^2 - 8x$

$\Rightarrow 15x^2 - 72x - 108 = 0 \Rightarrow 5x^2 - 24x - 36 = 0$

$\Rightarrow 5x^2 - 30x + 6x - 36 = 0 \Rightarrow 5x(x-6) + 6(x-6) = 0$

$\Rightarrow (x-6)(5x+6) = 0$

either $x-6=0$. $\Rightarrow x = 6$ or $5x+6=0 \Rightarrow x = -6/5$

but it is not possible as it is negative.

$\therefore x = 6.$

Q33. A trader buys x articles for a total cost of 600 rs.
(i) writedown the cost of one article interms of x.
if the cost per article were 5 rs more, the number of articles that can be bought for 600 rs. would be four less.
(ii) writedown the equation in x for the above situation and solve it to find x.

Sol. Total cost $= 600$, No. of articles $= x$

(i) Cost of one article $= \frac{600}{x}$

In second case, price of one article.

$$\frac{600}{x} + 5 = \frac{600}{x-4} \Rightarrow \frac{600+5x}{x} = \frac{600}{x-4}$$

$\Rightarrow (600+5x)(x-4) = 600x \Rightarrow 600x - 2400 + 5x^2 - 20x = 600x$

$\Rightarrow 5x^2 - 20x - 2400 = 0 \Rightarrow x^2 - 4x - 480 = 0$

$\Rightarrow x^2 - 24x + 20x - 480 = 0 \Rightarrow (x-24)(x+20) = 0$

either $x - 24 = 0 \Rightarrow x = 24$ or $x + 20 = 0 \Rightarrow x = -20$

but it is not possible as it is negative.

$\therefore x = 24$.

Q34. A shopkeeper buys a certain no. of books for 750 rs. If the cost per book was 5 rs less, the no. of books that could be bought for 720 rs. would be 2 more. taking the original cost of each book be x rs., write an equation in x and solve it.

Sol. let the original cost of each book be x rs.

then no. of books bought $= \frac{720}{x}$

If the cost of one book becomes Rs. $(x-5)$ then the no. of books bought for Rs. 720 $= \frac{720}{x-5}$

As per question, $\frac{720}{x-5} - \frac{720}{x} = 2$

$\Rightarrow 360x - 360x + 1800 = x^2 - 5x$

$\Rightarrow x^2 - 5x - 1800 = 0 \Rightarrow x^2 - 45x + 40x - 1800 = 0$

$\Rightarrow (x-45)(x+40) = 0$

$\Rightarrow x = 45$ or $x = -40$ but x can't be negative.

$\therefore$ Hence the original cost of the book is Rs. 45.

Q35. The hotel bill for a no. of people for overnight stay is Rs. 4800. if there were 4 more, the bill each person had to pay would have reduced by Rs. 200. find the no. of people staying overnight.

Sol. let the no. of people - x

Amount of bill = 4800

Bill for each person = $\frac{4800}{x}$

In second case, the no. of people = $x+4$

then bill for each person = $\frac{4800}{x+4}$

According to condition, $\frac{4800}{x} - \frac{4800}{x+4} = 200$.

$\Rightarrow 4800\left[\frac{1}{x} - \frac{1}{x+4}\right] = 200 \Rightarrow 4800\left[\frac{x+4-x}{x^2+4x}\right] = 200$

$\Rightarrow x^2 + 4x = \frac{4800 \times 4}{200} \Rightarrow x^2 + 4x - 96 = 0$

$\Rightarrow x^2 + 12x - 8x - 96 = 0 \Rightarrow x(x+12) - 8(x+12) = 0$

$\Rightarrow (x+12)(x-8) = 0$

either $x + 12 = 0 \Rightarrow x = -12$ but x can't negative

so $x - 8 = 0 \Rightarrow x = 8$

$\therefore$ No. of people = 8

Q36. A person was given Rs. 3000 for a tour. if he Extends his tour programme by 5 days, he must Cutdown his daily Expenses by Rs. 20. find the no. of days of his tour programme.

Sol. let the no. of days of tour programme = x

Amount = Rs. 3000

$\therefore$ Expenses for each day = $\frac{3000}{x}$

In second case, no. of days = $x+5$ then

Expenses for each day = $\frac{3000}{x+5}$

Now. according to condition,

$$\frac{3000}{x} - \frac{3000}{x+5} = 20$$

$$\Rightarrow 3000\left[\frac{1}{x} - \frac{1}{x+5}\right] = 20 \Rightarrow 3000\left[\frac{x+5-x}{x^2+5x}\right] = 20$$

$$\Rightarrow x^2+5x = \frac{3000\times 5}{20} \Rightarrow x^2+5x-750=0$$

$$\Rightarrow x^2-25x+30x-750=0$$

$$\Rightarrow (x-25)(x+30)=0$$

either $x-25=0 \Rightarrow x=25$ or $x+30=0 \Rightarrow x=-30$

but x can't be negative.

$\therefore$ Number of days = 25.

Q37. Ritu bought a Saree for Rs. $60x$ and sold it Rs. $(500+4x)$ at a Loss of $x\%$ find the Costprice.

Sol. Cost price of Saree = $60x$

Selling price = $500+4x$, Loss = $x\%$

Now according to Condition.

$$S.P = C.P \times \frac{100 - Loss\%}{100}$$

$$\Rightarrow 500 + 4x = \frac{60x(100-x)}{100}$$

$$\Rightarrow 50{,}000 + 400x = 6000x - 60x^2$$

$$\Rightarrow 60x^2 - 5600x + 50000 = 0$$

$$\Rightarrow 3x^2 - 280x + 2500 = 0$$

$$\Rightarrow 3x^2 - 30x - 250x + 2500 = 0$$

$$\Rightarrow 3x(x-10) - 250(x-10) = 0$$

$$\Rightarrow (x-10)(3x-250) = 0$$

either $x - 10 = 0 \Rightarrow x = 10$ or $3x - 250 = 0 \Rightarrow x = \frac{250}{3}$

but it is not possible.

$\therefore$ Loss = 10%.

Cost price = $60x = 60 \times 10 =$ Rs. 600.

Q38. paul is x years old and his father's age is twice the square of paul's age. ten years hence, father's age will be four-times paul's age. find their present ages.

Sol. Age of paul = x, father's age = $2x^2$

10 years hence, Age of paul = $x+10$, father's age = $2x^2+10$

According condition,

$$2x^2+10 = 4(x+10) \Rightarrow 2x^2 - 4x - 30 = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0 \Rightarrow x^2 - 5x + 3x - 15 = 0$$

$$\Rightarrow (x-5)(x+3) = 0$$

either $x - 5 = 0 \Rightarrow x = 5$ or $x + 3 = 0 \Rightarrow x = -3$, but x can't -ve

$\therefore$ Age of paul = 5 years.

$\therefore$ father's age = $2x^2 = 2(5)^2$

= 50 years.

Q39. The age of man is twice the square of the age of his son. 8 years hence, the age of the man will be 4 years more than three times the age of his son. find their present age.

Sol. let the present ~~of~~ age of son $= x$

then present age of the man $= 2x^2$

8 years hence, the age of son $= x+8$ and the age of man $= 2x^2+8$

According to condition,

$2x^2+8 = 3(x+8)+4 \Rightarrow 2x^2+8 = 3x+28$

$\Rightarrow 2x^2-3x-20=0 \Rightarrow 2x^2-8x+5x-20=0$

$\Rightarrow (x-4)(2x+5)=0$

either $x-4=0 \Rightarrow x=4$ or $2x+5=0 \Rightarrow x=-5/2$

But it is not possible.

$\therefore$ present age of the son, $= 4$ years

present age of the man $= 2x^2 = 2(4)^2 = 32$ years.

Q40. 2 years ago, a man's age was three times the square of his daughter's age. 3 years hence, his age will be 4 times his daughter's age. find their present ages.

Sol. let the present age of man $= x$

present age of daughter $= y$

According to question,

$(x-2) = 3(y-2)^2$

$\Rightarrow x-2 = 3[y^2-4y+4]$

$\Rightarrow 3y^2-12y-x+14=0$ —— (i)

after these years the age will be $x+3 = 4(y+3)$

$\Rightarrow \quad x+3 = 4y+12$

$\Rightarrow \quad x = 4y+9 \quad \text{——— (ii)}$

Substitute the value of x from (ii) in (i)

$3y^2 - 12y - 4y - 9 + 14 = 0$

$\Rightarrow \quad 3y^2 - 16y + 5 = 0 \Rightarrow 3y^2 - 15y - y + 5 = 0$

$\Rightarrow \quad (y-5)(3y-1) = 0$

$\Rightarrow \quad y = 5, \frac{1}{3} \quad [\, y \neq \frac{1}{3} \because \text{age can't be fractional}]$

put value of y in (ii)

$x = 4(5)+9 = 29$

$\therefore$ Hence the age of person is 29 years and the age of his daughter is 5 years

Q41. The length of the hypotenuse of a right angled triangle Exceeds the length of one side by 2cm and Exceeds twice the length of other side by 1cm. find the length of Each side. Also find the perimeter and the area of the triangle.

Sol. let the length of one side $= x$ cm. and other side $= y$ cm. then hypotenuse $= x+2$ & $2y+1$

$\therefore \quad x+2 = 2y+1 \Rightarrow x - 2y + 1 = 0 \Rightarrow x = 2y-1 \quad \text{——— (i)}$

By using pythagoras theorem,

$x^2 + y^2 = (2y+1)^2$

$\Rightarrow \quad x^2 + y^2 = 4y^2 + 4y + 1$

$\Rightarrow \quad 4y^2 - 4y + 1 + y^2 = 4y^2 + 4y + 1 \quad \{\because \text{from (i)}\}$

$\Rightarrow \quad y^2 - 8y = 0 \Rightarrow y(y-8) = 0$

$\Rightarrow \quad y = 8 \quad (\because y = 0 \text{ not possible})$

Substituting the value of y in (i)

$x = 2y-1 = 2(8)-1 = 15$

$\therefore$ length of one side = 15cm, other side = 8cm.

and hypotenuse $= x+2 = 15+2 = 17$ cm.

$\therefore$ perimeter $= 15+8+17 = 40$ cm.

$\therefore$ Area $= \frac{1}{2} \times$ side $\times$ side $= \frac{1}{2} \times 15 \times 8 = 60 cm^2$

Q42. If twice the area of a smaller square is subtracted from the area of a larger square, the result is $14 cm^2$. However, if twice the area of the larger square is added to 3 times the area of the smaller square, the result is $203 cm^2$. Determine the sides of the two squares.

Sol. let the side of smaller square = x cm.

the side of bigger square = y cm.

According to condition,

$y^2 - 2x^2 = 14$ —— (i)

and $2y^2 + 3x^2 = 203$ —— (ii).

Multiply (i) by 2 and (ii) by 1

$2y^2 - 4x^2 = 28$

$2y^2 + 3x^2 = 203$

on subtracting we get, $-7x^2 = -175$

$\Rightarrow x^2 - 25 = 0 \Rightarrow (x+5)(x-5) = 0$

either $x+5=0 \Rightarrow x = -5$. but x can't be $-$ve

or $x-5=0 \Rightarrow x = 5$.

Substitute the value of x in (i)

$y^2 - 2(5)^2 = 14$

$$\Rightarrow \quad y^2 = 64 \Rightarrow y = 8$$

$\therefore$ Hence the Side of Smaller Square $= 5\,cm$

the Side of bigger Square $= 8\,cm$.