

Ratios and Proportions

EXERCISE-7.1

Solution-01:

$$\begin{aligned}\text{Weight of Alloy} &= 27\frac{1}{2} \text{ kg} + 2\frac{3}{4} \text{ kg} \\ &= \frac{55}{2} \text{ kg} + \frac{11}{4} \text{ kg} = \frac{121}{4} \text{ kg}.\end{aligned}$$

$$\begin{aligned}\text{Weight of tin} &= 2\frac{3}{4} \text{ kg} \\ &= \frac{11}{4} \text{ kg}.\end{aligned}$$

$$\begin{aligned}\therefore \text{Ratio of tin to the alloy} &= \frac{\frac{11}{4} \text{ kg}}{\frac{121}{4} \text{ kg}} \\ &= \frac{11}{121} \\ &= 1:11.\end{aligned}$$

Solution-02:

$$\begin{aligned}\text{(i) Required ratio} &= \frac{2}{3} \times \frac{4}{9} \\ &= \frac{8}{27} = 8:27\end{aligned}$$

$$\begin{aligned}\text{(ii) Required ratio} &= \frac{4}{9} \times \frac{4}{7} \times \frac{9}{11} \\ &= \frac{36}{77} = 36:77\end{aligned}$$

$$\begin{aligned}\text{(iii) Required ratio} &= \frac{(a-b)}{(a+b)} \times \frac{(a+b)^2}{(a^2+b^2)} \times \frac{(a^2-b^2)}{(a^2+b^2)} \\ &= 1:1\end{aligned}$$

Solution-03:

- (i) The duplicate ratio of $2:3$ is $2^2:3^2$ i.e. $4:9$
(ii) The duplicate ratio of $\sqrt{5}:7$ is $(\sqrt{5})^2:7^2$ i.e. $5:49$
(iii) The duplicate ratio of $5a:6b$ is $(5a)^2:(6b)^2$
i.e. $(25a^2):(36b^2)$

Solution-04:

- (i) The triplicate ratio of $3:4$ is $3^3:4^3$
i.e. $27:64$.
(ii) The triplicate ratio of $\frac{1}{2}:\frac{1}{3}$ is $(\frac{1}{2})^3:(\frac{1}{3})^3$
i.e. $\frac{1}{8}:\frac{1}{27}$
(iii) The triplicate ratio of $1^3:2^3$ is $(1)^3:(2)^3$
i.e. $1:8$.

Solution-05:-

- (i) The subduplicate ratio of $9:16$ is $\sqrt{9}:\sqrt{16}$
i.e. $3:4$
(ii) The subduplicate ratio of $\frac{1}{4}:\frac{1}{9}$ is $\sqrt{\frac{1}{4}}:\sqrt{\frac{1}{9}}$
i.e. $\frac{1}{2}:\frac{1}{3}$
(iii) The subduplicate ratio of $9a^2:49b^2$ is $\sqrt{9a^2}:\sqrt{49b^2}$
i.e. $\sqrt{(3a)^2}:\sqrt{(7b)^2}$
 \therefore subduplicate ratio is $3a:7b$.

Solution-06:

(i) The subtriplicate ratio of $1:216$ i.e. $\sqrt[3]{1} : \sqrt[3]{216}$

$$\sqrt[3]{(1)^3} : \sqrt[3]{(6)^3}$$

$$\text{i.e. } 1:6$$

(ii) The subtriplicate ratio of $\frac{1}{8} : \frac{1}{125}$

$$\text{i.e. } \sqrt[3]{\left(\frac{1}{2}\right)^3} : \sqrt[3]{\left(\frac{1}{5}\right)^3}$$

$$\text{i.e. } \frac{1}{2} : \frac{1}{5}$$

(iii) The subtriplicate ratio of $27a^3 : 64b^3$

$$\text{i.e. } \sqrt[3]{(3a)^3} : \sqrt[3]{(4b)^3}$$

$$\text{i.e. } 3a : 4b.$$

Solution-07:

(i) The reciprocal ratio of $4:7$ is $7:4$

(ii) The reciprocal of ratio of $3^2:4^2$ is $4^2:3^2$

(iii) The reciprocal ratio of $\frac{1}{9}:2$ is $2:\frac{1}{9}$.

Solution-08:

Given ratios are $\frac{2}{3}$, $\frac{17}{21}$, $\frac{11}{14}$ and $\frac{5}{7}$

We convert them into equivalent like fractions

L.C.M of 3, 21, 14 and 7 is

$$\begin{array}{r} 7 \overline{) 3, 21, 14, 7} \\ 3 \overline{) 3, 3, 2, 1} \\ 1, 1, 2, 1 \end{array}$$

$$\text{L.C.M} = 21 \times 2 = 42.$$

$$\begin{aligned}\therefore A:D &= \frac{A}{B} \times \frac{B}{B} \\ &= \frac{2}{3} \times \frac{24}{35} \\ &= \frac{2 \times 8}{35}\end{aligned}$$

$$\therefore \text{Req. ratio} = \frac{16}{35}$$

(iii) Given that,

$$\frac{x}{y} = \frac{2}{3} \text{ \& } \frac{y}{z} = \frac{4}{7}$$

L.C.M of two values of y is 12
i.e 3 and 4 is 12

$$\text{Thus, } \frac{x}{y} = \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{y}{z} = \frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

$\therefore x:y:z$ is $8:12:21$ is the simplest form

Solution-10:

(i) Given that,

$$3A = 4B = 6C$$

$$\Rightarrow 3A = 4B$$

$$\Rightarrow \frac{A}{B} = \frac{4}{3}$$

$$4B = 6C$$

$$\Rightarrow \frac{B}{C} = \frac{6}{4} = \frac{3}{2} = \frac{3}{2}$$

$$\begin{aligned} \therefore A:D &= \frac{B}{D} \times \frac{A}{B} \times \frac{B}{D} \\ &= \frac{2}{\cancel{B}} \times \frac{\cancel{2}B}{3 \times 0} \frac{2A}{\cancel{3}5} \\ &= \frac{16}{35} \end{aligned}$$

Required ratio A:D is 16:35

9 (ii)

Given that $x:y = 2:3$, $y:z = 4:7$

L.C.M of two value of y is 12

i.e. 3 and 4 is 12.

$$\text{Thus, } x:y = 2:3 = 8:12$$

$$y:z = 4:7 = 12:21$$

$\therefore x:y:z = 8:12:21$, which is in simplest form

Solution - 10:

$$(i) \quad A:B = \frac{1}{4} : \frac{1}{5} = \frac{\frac{1}{4}}{\frac{1}{5}} = \frac{5}{4}$$

$$B:C = \frac{1}{5} : \frac{1}{6} = \frac{\frac{1}{5}}{\frac{1}{6}} = \frac{6}{5}$$

L.C.M of two values of B is $\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$

$$\text{i.e. } \frac{1}{5} \times \frac{1}{7} = \frac{1}{35} \quad \text{L.C.M} = 24$$

$$\text{Thus, } A:B = \frac{\frac{1}{4} \times \frac{1}{7}}{\frac{1}{5} \times \frac{1}{7}} = \frac{1}{28} : \frac{1}{35}$$

∴ Required ratio A : B : c is 4 : 3 : 2

(i) Given that,

$$A : B = \frac{1}{4} : \frac{1}{5} = \frac{5}{4}$$

$$B : c = \frac{1}{7} : \frac{1}{6} = \frac{6}{7}$$

L.C.M of two values of B is 4 and 6 is 24

i.e. $4 \times 6 = 24$

$$\therefore A : B = \frac{5 \times 6}{4 \times 6} = \frac{30}{24}$$

$$B : c = \frac{6 \times 4}{7 \times 4} = \frac{24}{28}$$

∴ Required ratio is A : B : c = 30 : 24 : 28
in simplest form 15 : 12 : 14.

Solution- II:

$$\Rightarrow \frac{3x + 5y}{3x - 5y} = \frac{7}{3}$$

$$\Rightarrow 3(3x + 5y) = 7(3x - 5y)$$

$$\Rightarrow 9x + 15y = 21x - 35y$$

$$= 21x - 9x = 15y + 35y$$

$$\Rightarrow 12x = 50y$$

$$\Rightarrow \frac{x}{y} = \frac{50}{12} = \frac{25}{6}$$

$$(i) \quad a:b = 3:11$$

$$a = 3$$

$$b = 11$$

$$\therefore \frac{15a - 3b}{9a + 5b} = \frac{15(3) - 3(11)}{9(3) + 5(11)}$$

$$= \frac{45 - 33}{27 + 55}$$

$$= \frac{12}{82}$$

$$= \frac{6}{41}$$

Solution-12:

Given that

$$\frac{(4x^2 + xy)}{(3xy - y^2)} = \frac{12}{5}$$

$$\Rightarrow 5(4x^2 + xy) = 12(3xy - y^2)$$

$$\Rightarrow 20x^2 + 5xy = 36xy - 12y^2$$

$$\Rightarrow 20x^2 - 12y^2 - 31xy = 0$$

$$\Rightarrow 20x^2 + 12y^2 - 15xy - 16xy = 0$$

$$\Rightarrow 20x^2 - 16xy + 15xy + 12y^2 = 0$$

$$\Rightarrow 4x(5x - 4y) - 3y(5x - 4y) = 0$$

$$\Rightarrow (4x - 3y)(5x - 4y) = 0$$

$$\therefore 4x = 3y \text{ \& } 5x = 4y$$

EXERCISE - 7.2.

Solution-01:

(i) Given that

$$10:35 = x:42$$

$$\Rightarrow \frac{10}{35} = \frac{x}{42}$$

$$\Rightarrow 42 \times 10 = x \times 35 \quad [\because \text{By cross multiplication}]$$

$$\Rightarrow x \times 35 = 420$$

$$\Rightarrow x = \frac{420}{35}$$

$$\Rightarrow x = 12.$$

(ii) Given that

$$3:x = 24:2$$

$$\Rightarrow \frac{3}{x} = \frac{24}{2}$$

\Rightarrow By doing cross multiplication

$$24 \times x = 3 \times 2$$

$$\Rightarrow 24x = 6$$

$$\Rightarrow x = \frac{6}{24}$$

$$\Rightarrow x = \frac{1}{4}.$$

(iii) Given that

$$25:15 = x:3$$

$$\Rightarrow \frac{25}{15} = \frac{x}{3}$$

$$\Rightarrow 15x = 75 \quad \Rightarrow x = 5.$$

(iv) Given that

$$x:50 = 3:2$$

$$\Rightarrow x:50 = \frac{3}{2}$$

$$\Rightarrow x \times 2 = 50 \times 3$$

$$\Rightarrow 2x = 150$$

$$\Rightarrow x = \frac{150}{2}$$

$$\Rightarrow x = 75.$$

Solution-02:

(i) Given that,

3, 12, 15 and Let 4th proportion be 'x'

$$\frac{3}{12} = \frac{15}{x}$$

$$\Rightarrow 3x = 12 \times 15$$

$$\Rightarrow x = \frac{12 \times 15}{3}$$

$$\Rightarrow x = 4 \times 15$$

$$\Rightarrow x = 60.$$

(ii) Given that

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ Let 4th proportion be 'x' then

$$\Rightarrow \frac{1}{3} : \frac{1}{4} = \frac{1}{5} : x$$

$$\Rightarrow \frac{4}{3} = \frac{1}{5x}$$

$$\Rightarrow 20x = 3 \quad \Rightarrow x = \frac{3}{20}.$$

(iii) Given that,

1.5, 2.5, 4.5 Let 4th proportion be 'x' then.

$$1.5 : 2.5 = 4.5 : x$$

$$\Rightarrow \frac{1.5}{2.5} = \frac{4.5}{x}$$

$$\Rightarrow x \times 1.5 = 4.5 \times 2.5$$

$$\Rightarrow x = \frac{4.5 \times 2.5}{1.5}$$

$$\Rightarrow x = 3 \times 2.5$$

$$\Rightarrow x = 7.5.$$

(iv) Given that.

9.6 kg, 7.2 kg, 28.8 kg Let 4th proportion be 'x' then

$$\Rightarrow 9.6 \text{ kg} : 7.2 \text{ kg} = 28.8 \text{ kg} : x \text{ kg}$$

$$\Rightarrow \frac{9.6 \text{ kg}}{7.2 \text{ kg}} = \frac{28.8 \text{ kg}}{x \text{ kg}}$$

$$\Rightarrow 9.6 \times x = 7.2 \times 28.8$$

$$\Rightarrow 9.6x = \frac{7.2 \times 28.8}{9.6}$$

$$\Rightarrow x = 7.2 \times 3$$

$$\Rightarrow x = 21.6.$$

Solution - 03.

(i) Given that

5, 10 then let third proportion be 'x' then

$$\Rightarrow \frac{5}{10} = \frac{10}{x}$$

$$\Rightarrow 5x = 100$$

$$\Rightarrow x = \frac{100}{5}$$

$$\Rightarrow x = 20.$$

(ii) 0.24, 0.6

Let the third proportion be 'x' then

$$\Rightarrow \frac{0.24}{0.6} = \frac{0.6}{x}$$

$$\Rightarrow x \times 0.24 = 0.6 \times 0.6$$

$$\Rightarrow x = \frac{0.36}{0.24}$$

$$\Rightarrow x = \frac{3}{2}$$

(iii) ₹3, ₹12

Let the third proportion be 'x' then

$$\Rightarrow \frac{₹3}{₹12} = \frac{₹12}{x}$$

$$\Rightarrow ₹3x = 12 \times 12$$

$$\Rightarrow x = \frac{144}{3}$$

$$\Rightarrow x = 48.$$

(iv) Given that

$5\frac{1}{4}$ and 7

$$\Rightarrow 5\frac{1}{4} = 5.25 \text{ \& } 7$$

Let the third proportion be 'x' then

$$\Rightarrow \frac{5.25}{7} = \frac{7}{x}$$

$$\Rightarrow \frac{5.25}{1} = \frac{7}{x}$$

$$\Rightarrow 5.25x = 49$$

$$\Rightarrow x = \frac{49}{5.25}$$

$$\Rightarrow x = \frac{196}{525}$$

$$\rightarrow x = \frac{196}{21}$$

Solution-04.

(i) Given numbers are 5 & 80

$$\therefore \text{mean proportion of 5 \& } 80 = \sqrt{5 \times 80}$$

$$= \sqrt{400}$$

$$= \sqrt{4 \times 100}$$

$$= \sqrt{4} \times \sqrt{100}$$

$$= 2 \times 10$$

$$= 20.$$

\therefore mean proportion of 5 and 80 is 20.

(ii) Given numbers are, $\frac{1}{12}$ and $\frac{1}{75}$.

$$\begin{aligned}\text{mean proportion of } \frac{1}{12} \text{ and } \frac{1}{75} & \text{ is } \sqrt{\frac{1}{12} \times \frac{1}{75}} \\ & = \sqrt{\frac{1}{900}} \\ & = \sqrt{\left(\frac{1}{30}\right)^2} \\ & = \frac{1}{30}.\end{aligned}$$

(iii) Given that

$$\begin{aligned}\text{mean proportion of } 8.1 \text{ and } 2.5 & \text{ is } \sqrt{\frac{8.1}{8.1 \times 2.5}} \\ & = \sqrt{\frac{81 \times 25}{100}} \\ & = \sqrt{\frac{(9)^2 \times (5)^2}{(10)^2}} \\ & = \frac{45}{10} \\ & = 4.5.\end{aligned}$$

(iv) Given that

proportions are $(a-b)$ & $(a^3 - a^2b)$.

$$\begin{aligned}\text{mean proportion is } & \sqrt{(a-b)(a-b)a^2} \\ & = \sqrt{(a-b)^2 a^2} \\ & = a(a-b)\end{aligned}$$

mean proportion is $a(a-b)$.

Solution-05.

Given that $a, 12, 16$ and b are in continued proportion

$$\Rightarrow \frac{a}{12} = \frac{12}{16} = \frac{16}{b}$$

$$\Rightarrow \frac{a}{12} = \frac{12}{16}$$

$$\Rightarrow a = \frac{144}{16}$$

$$\Rightarrow a = 9$$

$$\Rightarrow b = \frac{16 \times 16}{12}$$

$$\Rightarrow b = \frac{256}{12}$$

$$\Rightarrow b = \frac{64}{3}$$

Solution-06.

Given numbers are $5, 11, 19$ and 37 .

Let the number to be added 'x' then

$5+x, 11+x, 19+x$ and $37+x$ are in proportion.

$$\Rightarrow \frac{5+x}{11+x} = \frac{19+x}{37+x}$$

$$\Rightarrow (5+x)(37+x) = (19+x)(11+x)$$

$$\Rightarrow \cancel{165} + 5x + 37x + \cancel{x^2} = 209 + 11x + \cancel{x^2} + 19x$$

$$\Rightarrow 42x - 30x = 209 - 165$$

$$\Rightarrow 12x = 24$$

$$\Rightarrow x = 2.$$

Solution-07:

Given that numbers are 23, 30, 57 and 78

Let the 'x' ^{number} be subtracted to be in proportion

$$\Rightarrow \frac{23-x}{30-x} = \frac{57-x}{78-x}$$

$$\Rightarrow (23-x)(78-x) = (57-x)(30-x)$$

$$\Rightarrow 1794 - 23x - 78x + x^2 = 1710 - 57x - 30x + x^2$$

$$\Rightarrow 1794 - 1710 = -87x + 101x$$

$$\Rightarrow 14x = 84$$

$$\Rightarrow x = \frac{84}{14} = 6$$

Solution-08:

Given that,

$$\frac{2x-1}{5x-6} = \frac{6x+2}{15x-9}$$

$$\Rightarrow (2x-1)(15x-9) = (6x+2)(5x-6)$$

$$\Rightarrow 30x^2 - 15x + 9 - 18x = 30x^2 - 36x + 10x - 12$$

$$\Rightarrow -33x + 9 = -26x - 12$$

$$\Rightarrow 33x - 26x = 12 + 9$$

$$\Rightarrow 7x = 21$$

$$\Rightarrow x = \frac{21}{7}$$

$$\Rightarrow x = 3.$$

$$\frac{2}{3} = \frac{2 \times 14}{3 \times 14} = \frac{28}{42}$$

$$\frac{17}{21} = \frac{17 \times 2}{21 \times 2} = \frac{34}{42}$$

$$\frac{11}{14} = \frac{11 \times 3}{14 \times 3} = \frac{33}{42}$$

$$\frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$$

∴ The given ratios are $\frac{28}{42}, \frac{34}{42}, \frac{33}{42}, \frac{30}{42}$

Hence, the given ratios in the ascending order of magnitude

$$\frac{28}{42}, \frac{30}{42}, \frac{33}{42}, \frac{34}{42}$$

i.e. 2 : 3, 5 : 7, 11 : 14, 17 : 21.

Solution-09:

Given that,

$$A : B = 2 : 3$$

$$B : C = 4 : 5$$

$$C : D = 6 : 7$$

$$\text{then } B : D = \frac{B \times C}{C \times D}$$

$$= \frac{4 \times 6}{5 \times 7}$$

$$= \frac{4}{5} \times \frac{6}{7} = \frac{24}{35}$$

Solution-09:

Mean proportion of $x+2$ and $x+9$ is

$$\sqrt{(x+2)(x+9)} = x+5$$

Rooting on b/s we get

$$\Rightarrow \left[\sqrt{(x+2)(x+9)} \right]^2 = [x+5]^2$$

$$\Rightarrow (x+2)(x+9) = x^2 + 25 + 2(5)(x)$$

$$\Rightarrow x^2 + 2x + 9x + 18 = x^2 + 10x + 25$$

$$\Rightarrow 11x - 10x = 25 - 18$$

$$\Rightarrow x = 7.$$

Solution-10.

Given that

$$\frac{16+y}{26+y} = \frac{26+y}{40+y} = \frac{40+y}{x+y}$$

$$\Rightarrow \frac{26+y}{40+y} = \frac{40+y}{x+y}$$

$$\Rightarrow 26x = 1600 \quad (26+y)(x+y) = (40+y)^2$$

$$\Rightarrow x = \frac{1600}{26}$$

$$26x + x^2 + xy + 26y = 1600 + y^2 + 80y$$

$$\Rightarrow 26x + xy - 54y - 1600 = 0$$

$$\Rightarrow \frac{16+y}{26+y} = \frac{40+y}{x+y}$$

$$\Rightarrow (16+y)(x+y) = (40+y)(26+y)$$

$$\Rightarrow 16x + xy + 16y + y^2 = 1040 + 26y + y^2 + 26y + y^2$$

$$\Rightarrow 16x = 10y + 1040.$$