Matrices

EXERCISE - 9.1

Q1 classify the following matrices :

(i) $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -7 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$ (v) $\begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$ (vi) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Sol. (i) It is Square matrix of order 2

(ii) It is row matrix of order 1×3

(iii) It is column matrix of order 3×1

(iv) It is matrix of order 3×2

(v) It is matrix of order 2×3

(vi) It is zero matrix of order 2×3.

Q2. (i) If a matrix has 4 elements what are the possible orders it can have?

(ii) If a matrix has 8 elements, what are the possible orders it can have?

Sol. (i) It can have 1×4, 4×1 or 2×2 order

(ii) It can have 1×8, 8×1, 2×4 or 4×2 order.

Q3. Construct a 2×2 matrix whose elements a_{ij} are given by.

(i) $a_{ij} = 2i - j$ (ii) $a_{ij} = i \cdot j$

Sol. (i) It can be $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

(ii) It can be $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Q4. Find the values of x and y if $\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Sol. Comparing corresponding elements,

$$2x+y=5 \longrightarrow (i)$$
$$3x-2y=4 \longrightarrow (ii)$$

Multiply (i) by 2 and (ii) by 1

$$4x+2y=10$$
$$3x-2y=4$$

Adding we get, $7x=14 \Rightarrow x=2$

Substituting the value of x in (i)

$2\times 2+y=5 \Rightarrow y=1$

Hence $x=2,\ y=1$

Q5 Find the values of x, y and z if $\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$

Sol. Comparing the corresponding elements of equal determinants,

$x+2=-5 \Rightarrow x=-7$

$5z=-20 \Rightarrow z=-4$

$y^2+y=6 \Rightarrow y^2+y-6=0 \Rightarrow y^2+3y-2y-6=0$

$\Rightarrow y(y+3)-2(y+3)=0 \Rightarrow (y+3)(y-2)=0$

Either $y+3=0$, then $y=-3$ or $y-2=0$, then $y=2$

Hence $x=-7,\ y=-3, 2,\ z=-4$

Q6. Find the values of x, y, a and b if $\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$

Sol. Comparing corresponding elements,

$x-2=3 \Rightarrow x=5$

$y=1$

$$a+2b=5 \longrightarrow (i)$$
$$3a-b=1 \longrightarrow (ii)$$

Multiplying (i) by 1 and (ii) by 2

$a+2b=5$

$6a-2b=2$

Adding, we get $7a=7 \Rightarrow a=1$

Substitute the value of 'a' in (i)

$1+2b=5 \Rightarrow b=2$

Hence $x=5,\ y=1,\ a=1,\ b=2.$

Q7. Find the values of x, y, a and b if $\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$

Sol. Comparing the corresponding elements:

$3x+4y=2 \longrightarrow (i)$

$x-2y=4 \longrightarrow (ii)$

Multiply (i) by 1 and (ii) by 2

$3x+4y=2$

$2x-4y=8$

Adding, we get $5x=10 \Rightarrow x=2$

Substituting the value of x in (i)

$3\times 2+4y=2 \Rightarrow 4y=-4 \Rightarrow y=-1$

$a+b=5 \longrightarrow (iii)$

$2a-b=-5 \longrightarrow (iv)$

Adding we get $3a=0 \Rightarrow a=0$

Substitute the value of 'a' in (iii)

$0+b=5 \Rightarrow b=5$

$\therefore$ Hence $x=2,\ y=-1,\ a=0,\ b=5$

EXERCISE - 9.2

Q1 Given that $M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$. find $M + 2N$.

Sol.

$$M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}, \quad N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$M + 2N = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$$

Q2. If $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. find $2A - 3B$.

Sol.

$$2A - 3B = 2\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

Q3. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. find $3A + 4B$.

Sol.

$$3A + 4B = 3\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 20 \\ 18 & 13 \end{bmatrix}$$

Q4. Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$ (i) find the matrix $2A + B$.

(ii) Find the matrix C such that $C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol. (i)

$$2A + B = 2\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$$

(ii)

$$C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Q5. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$. find $A+2B-3C$.

Sol.

$$A+2B-3C = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

Q6. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$. find the matrix X if

(i) $3A + X = B$ (ii) $X - 3B = 2A$

Sol.

(i) $3A + X = B \Rightarrow X = B - 3A$

$$X = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -4 & -5 \end{bmatrix}$$

(ii) $X - 3B = 2A \Rightarrow X = 2A + 3B$

$$X = 2\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$$

Q7. Solve the matrix equation $\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$

Sol.

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} = 3X \Rightarrow \begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = 3X$$

$$\Rightarrow X = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Q8. If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$. find the matrix M.

Sol. Given that

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix}$$

$$2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}.$$

$$M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$$

Q9. If $A = \begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix}$, find matrix X such that $3A + 5B - 2X = 0$

Sol. $3A + 5B - 2X = 0 \Rightarrow 2X = 3A + 5B$

$$\Rightarrow 2X = 3\begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix} = \begin{bmatrix} 27 & 3 \\ 15 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 25 \\ 35 & -55 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 32 & 28 \\ 50 & -46 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 16 & 14 \\ 25 & -23 \end{bmatrix}$$

Q10. find x and y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Sol.

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \longrightarrow (i)$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \longrightarrow (ii)$$

Adding (i) and (ii), we get $2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}.$$

Subtracting (ii) from (i)

$$2Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Q11

(i) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find the values of x and y.

(ii) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$. find the values of x, y and z

Sol.

(i) $\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$.

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

Comparing the corresponding elements,

$8+y=0 \Rightarrow y=-8$

$2x+1=5 \Rightarrow x=2$

Hence $x=2$, $y=-8$

(ii) Given that $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$\Rightarrow z=7$, $8+y=0 \Rightarrow y=-8$, $2x+1=5 \Rightarrow x=2$

Hence $x=2$, $y=-8$, $z=7$

Q12. If $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

Sol. $$\Rightarrow \begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 2-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+1 & b+2 \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements

$a+1=5 \Rightarrow a=4$

$b+2=0 \Rightarrow b=-2$

$-c=3 \Rightarrow c=-3$.

Q13. If $A=\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$, $B=\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, $C=\begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$ and $5A+2B=C$, find the values of a, b and c.

Sol. $A = \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, $C = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

NOW $5A + 2B = C$

$\Rightarrow 5\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} + 2\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 10 & 5a \\ -15 & 25 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 14 & 2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 5a+6 \\ -1 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

Comparing the corresponding elements of equal determinants,

$c = 6$

$5a + 6 = 9 \Rightarrow 5a = 3 \Rightarrow a = \frac{3}{5}$

$25 + 2b = -11 \Rightarrow 2b = -36 \Rightarrow b = -18$

$\therefore$ Hence $a = \frac{3}{5}$, $b = -18$, $c = 6$.

EXERCISE - 4.3

Q1. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$, find AB and BA. Is AB = BA ?

Sol.

$$A \times B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2-15 & -2+10 \\ 1-9 & -1+6 \end{bmatrix} = \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

and
$$B \times A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2-1 & 5-3 \\ -6+2 & -15+6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Hence $AB \neq BA$.

Q2. If $P = \begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$, find 2PQ ?

Sol.

$$2PQ = 2\begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix} = 2\begin{bmatrix} 8-6 & -12+6 \\ 4+8 & -6-8 \end{bmatrix} = 2\begin{bmatrix} 2 & -6 \\ 12 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -12 \\ 24 & -28 \end{bmatrix}$$

Q3. If $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}$, compute $(-A)^2$.

Sol.

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}, \quad -A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$(-A)^2 = (-A)(A) = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4+1 & -2-3 \\ -2-3 & 1+9 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$$

Q4. Given $A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$, evaluate $A^2 - 4A$.

Sol.

$$A^2 - 4A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Q8. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 1 \\ 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & -3 \\ 0 & 1 \end{pmatrix}$, find each of the following and state if they are equal.

(i) $CA + B$ (ii) $A + CB$.

Sol.

(i) $CA = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix}$

$CA + B = \begin{pmatrix} -11 & -16 \\ 3 & 4 \end{pmatrix} + \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -15 \\ 4 & 5 \end{bmatrix}$

(ii) $A + CB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -15 & -5 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -14 & -3 \\ 4 & 5 \end{pmatrix}$

we can say that $CA + B \neq A + CB$.

Q9. If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$, find $2B - A^2$.

Sol.

$2B = 2\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} = \begin{pmatrix} 6 & 4 \\ -4 & 2 \end{pmatrix}$

$A^2 = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

$2B - A^2 = \begin{pmatrix} 6 & 4 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ -4 & 5 \end{pmatrix}$

Q10. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ and $C = \begin{pmatrix} 5 & 1 \\ 7 & 4 \end{pmatrix}$. compute

(i) $A(B+C)$ (ii) $(B+C)A$.

Sol.

$(B+C) = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} + \begin{pmatrix} 5 & 1 \\ 7 & 4 \end{pmatrix} = \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix}$

$A(B+C) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix} = \begin{bmatrix} 29 & 14 \\ 65 & 30 \end{bmatrix}$

$$(B+C) = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} + \begin{pmatrix} 5 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 11 & 6 \end{pmatrix}$$

$$(B+C)A = \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{pmatrix} 13 & 22 \\ 29 & 46 \end{pmatrix}$$

Q11. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, find the matrix $C(B-A)$.

Sol.

$$B-A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$C(B-A) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}$$

Q12. Let $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$. find $A^2 + AB + B^2$.

Sol.

$$A^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$A^2 + AB + B^2 = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ 5 & 4 \end{pmatrix}$$

Q13. Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 3 \\ 1 & -3 \end{bmatrix}$. find $A^2 - A + BC$.

Sol.

$$A^2 - A + BC = \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix} - \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix} + \begin{pmatrix} 2 & -6 \\ -3 & 6 \end{pmatrix}$$

$$= \begin{bmatrix} 4-4+2 & -2+2-6 \\ 6-6-3 & -3+3+6 \end{bmatrix}$$

$$= \begin{pmatrix} 2 & -6 \\ -3 & 6 \end{pmatrix}$$

Q14. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find A^2 and A^3. Also state which of these is equal to A.

Sol.

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

from above it is clear that $A^3 = A$.

Q15. If $x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, show that $6x - x^2 = 9I$, where I is the unit matrix.

Sol.

$$x^2 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$LHS = 6x - x^2 = 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I = RHS.$$

Hence proved.

Q16. Show that $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is a solution of the matrix equation $x^2 - 2x - 3I = 0$ where I is the unit matrix of order 2.

Sol.

$$x^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$\text{NOW } x^2 - 2x - 3I = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore x^2 - 2x - 3I = 0$ Hence proved.

Q17. Find the matrix X of order 2x2 which satisfies the equation:

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Sol.

$$\Rightarrow \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

Q18. If $A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$, find the value of x so that $A^2 = 0$

Sol.

$$A^2 = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix} = \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix}$$

$$\because A^2 = 0$$

$$\Rightarrow \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

on comparing. $1+x = 0 \Rightarrow x = -1$

Q19. If $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$. find the value of x.

Sol.

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-3 \\ 0+0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

Comparing the corresponding element.

$x = -1$.

Q20. Find x and y if $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

Sol.

$$\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3x+4 \\ 0-10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

Comparing the corresponding elements,

$-3x+4 = -5 \Rightarrow -3x = -9 \Rightarrow x = 3$

$-10 = y \Rightarrow y = -10$

Q21. Find the values of x and y if $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Sol.

$$\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+2y & -y \\ 4x & -x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Comparing the corresponding elements,

$2x+y = 3$ —— (i)

$3x+y = 2$ —— (ii)

on subtracting, we get

$-x = 1 \Rightarrow x = -1$

Substituting the value of x in (i)

$2(-1) + y = 3 \Rightarrow y = 5$

Q22. Find x and y if $\begin{bmatrix} 2 & x \\ y & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix}$

Sol.

$$\begin{bmatrix} 2+x & 4+x \\ y+3 & 2y+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix}$$

Comparing the corresponding elements

$2+x = 3 \Rightarrow x = 1$

$y+3 = 5 \Rightarrow y = 2$

Hence $x = 1$, $y = 2$

Q23. If $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$, find the values of x and y.

Sol.

$$\begin{bmatrix} x+0 & 0+2y \\ 3x+0 & 0+3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x & 2y \\ 3x & 3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$$

Comparing the corresponding elements

$2y = 0 \Rightarrow y = 0$

$3x = 9 \Rightarrow x = 3$

Q24. If $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ writedown the values of a, b, c & d.

Sol.

$$\Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Comparing the corresponding elements,

$a = 3$, $b = 4$, $c = 2$, $d = 5$

Q25. Find the value of x given that $A^2 = B$ where $A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

Sol. Given that $A^2 = B$

$$\Rightarrow \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{bmatrix} = \begin{pmatrix} 4 & x \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

on Comparing, $x = 36$.

Q26. If $A = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$, find the value of x, given that $A^2 = B$.

Sol. $A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 2x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix}$

$\therefore A^2 = B$

$\begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$.

Comparing the corresponding elements.

$3x = 36 \Rightarrow x = 12.$

Q27. Evaluate x, y if $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

Sol. $\Rightarrow \begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

Comparing the corresponding elements.

$6x-10 = 8 \Rightarrow 6x = 18 \Rightarrow x = 3$

$-2x+14 = 4y \Rightarrow -6+14 = 4y \Rightarrow 4y = 8 \Rightarrow y = 2.$

$\therefore$ Hence $x = 3$, $y = 2$

Q28. If $\begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$. find a, b and c.

Sol. $\Rightarrow \begin{bmatrix} 4a-3 & 3a+2 \\ 4+0 & 3+0 \end{bmatrix} = \begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4a-3 & 3a+2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$

Comparing the corresponding elements

$3a+2 = 11 \Rightarrow a = 3$

$b = 4a-3 = 4(3)-3 = 9$

$c = 3$

Q29. If $A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{bmatrix}$, find the value of x if $AB = BA$.

Sol.

$$AB = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2+0 & x-2 \\ 0+0 & 0+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & x-2 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2+0 & 8-x \\ 0+0 & 0+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 8-x \\ 0 & \frac{1}{2} \end{bmatrix}$$

$\therefore AB = BA$

$$\begin{bmatrix} 2 & x-2 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 8-x \\ 0 & \frac{1}{2} \end{bmatrix}$$

Comparing the corresponding elements

$x - 2 = 8 - x \Rightarrow 2x = 10 \Rightarrow x = 5$

Q30. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, find x, y so that $A^2 = xA + yI$.

Sol.

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$\therefore A^2 = xA + yI$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = x\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2x & 3x \\ x & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2x+y & 3x \\ x & 2x+y \end{bmatrix}$$

comparing the corresponding elements

$3x = 12 \Rightarrow x = 4$

$2x + y = 7 \Rightarrow 2(4) + y = 7 \Rightarrow y = -1$

Q31. Given that $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and that $AB = A + B$, find the values of a, b and c.

Sol.

$A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$

$$AB = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{bmatrix} 3a+0 & 3b+0 \\ 0+0 & 0+4c \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix}$$

$$A+B = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

$\therefore AB = A+B$

$$\begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix} = \begin{pmatrix} 3+a & b \\ 0 & 4+c \end{pmatrix}$$

Comparing the corresponding elements

$3a = 3+a \Rightarrow 2a = 3 \Rightarrow a = \frac{3}{2}$

$3b = b \Rightarrow b = 0$

$4c = 4+c \Rightarrow c = \frac{4}{3}$.

Q32. If $P = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$, $Q = \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$, find x, y such that $PQ = 0$

Sol.

$$PQ = \begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix} \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix} = \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix}$$

$\therefore PQ = 0$

$$\begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the corresponding elements

$6+6y = 0 \Rightarrow y = -1$

$2x+12 = 0 \Rightarrow x = -6$

Q33. Let $M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$ where M is a matrix.

(i) State the order of the Matrix M.

(ii) Find the matrix M.

Sol.

(i) M is of the order 1×2

Let $M = \begin{bmatrix} x & y \end{bmatrix}$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} x+0 & x+2y \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

Comparing the corresponding elements

$x = 1$

$x + 2y = 2 \Rightarrow 1 + 2y = 2 \Rightarrow y = \frac{1}{2}$

$\therefore M = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}$

Q34. Given $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$, Write down (i) The order of the matrix X.
(ii) The matrix X.

Sol.
(i) order of matrix X is 2×1

let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

then $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8x - 2y \\ x + 4y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$

Comparing the corresponding elements

$8x - 2y = 12 \longrightarrow (i)$

$x + 4y = 10 \longrightarrow (ii)$

Multiply (i) by 2 and (ii) by 1.

$16x - 4y = 24$

$x + 4y = 10$

on Adding, $17x = 34 \Rightarrow x = 2$

Substituting the Value of x in (i)

$8 \times 2 - 2y = 12 \Rightarrow 16 - 2y = 12 \Rightarrow y = 2.$

$\therefore X = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Q35. Solve the matrix equation $\begin{bmatrix} 4 \\ 1 \end{bmatrix} X = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$

Sol. let matrix $X = \begin{bmatrix} x & y \end{bmatrix}$

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$$

Comparing the Corresponding elements

$4x = -4 \Rightarrow x = -1$

$4y = 8 \Rightarrow y = 2$

$\therefore X = \begin{bmatrix} -1 & 2 \end{bmatrix}$

Q36. If $A = \begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find matrix C such that $AC = B$.

Sol. let matrix $C = \begin{bmatrix} x \\ y \end{bmatrix}$

$$AC = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - y \\ -4x + 5y \end{bmatrix}$$

But $AC = B$

$$\begin{bmatrix} 2x - y \\ -4x + 5y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

Comparing the Corresponding elements

$2x - y = -3$ —— (i)

$-4x + 5y = 2$ —— (ii)

Multiply (i) by 5 and (ii) by 1

$10x - 5y = -15$

$-4x + 5y = 2$

on Adding, we get $6x = -13 \Rightarrow x = -13/6$

Substituting the value of x in (i), $2\left(-\frac{13}{6}\right) - y = -3$

$\Rightarrow y = -4/3.$

$\therefore$ Matrix $C = \begin{pmatrix} -13/6 \\ -4/3 \end{pmatrix}$