Triangles and its Properties

Exercise 11.1

1.

i) R

ii) PR

iii) P

iv) PQ

2.

a)

i. Isosceles triangle

ii. Scalene triangle

iii. Equilateral triangle

iv. Isosceles triangle

v. Scalene triangle

vi. Isosceles triangle

b) i. Acute angled triangle

ii. Right angled triangle

iii. Acute angled triangle

iv. Obtuse angled triangle

v. Obtuse angled triangle

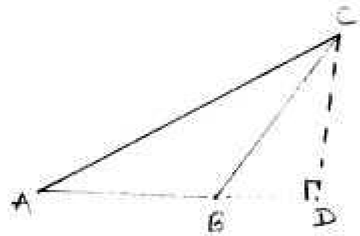
vi. Right angled triangle.

3.

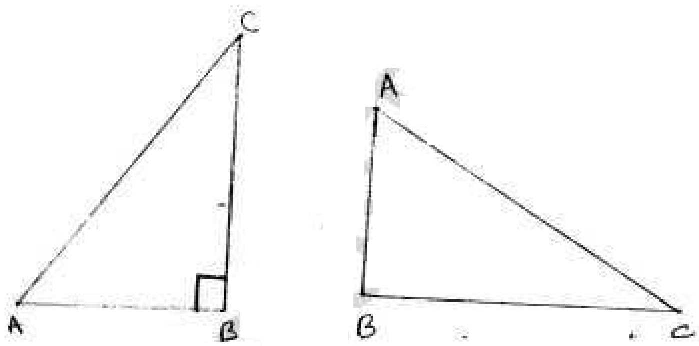
i. $\overline{PM}$ is Altitude

ii. $\overline{PD}$ is Median

4. No

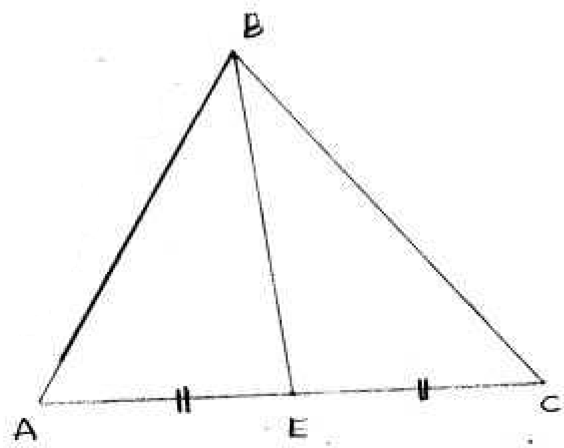


5. Yes

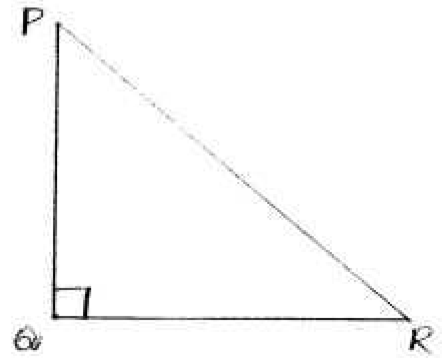


6.

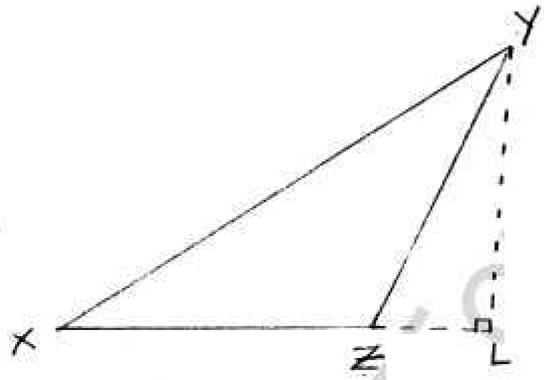
i)



ii)

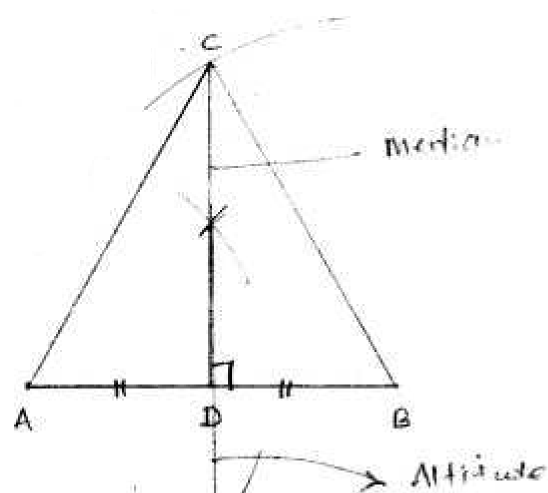


iii)



7.

Both Altitudes and medians Co-incide.



Exercise 11.2

1.

i) Exterior angle = Sum of two interior opposite angles

$x = 45 + 65$

$x = 110^\circ$

ii) Exterior angle = Sum of two interior opposite angles

$x = 55 + 40$

$x = 95^\circ$

iii) Exterior angle = Sum of two interior opposite angles.

$x = 50 + 50$

$x = 100^\circ$

2.

i) Exterior angle = Sum of two interior opposite angles

$115 = x + 50$

$x = 115 - 50$

$x = 65^\circ$

ii) Exterior angle = Sum of two interior opposite angles

$$80 = x + 30$$

$$x = 80 - 30$$

$$x = 50^\circ$$

iii) Exterior angle = Sum of two interior opposite angles

$$70 = x + 36^\circ$$

$$x = 70 - 36$$

$$x = 34^\circ$$

5.

i) Exterior angle = Sum of two interior opposite angles

$$105 = x + 2x$$

$$3x = 105$$

$$x = \frac{105}{3}$$

$$x = 35^\circ$$

ii) Exterior angle = Sum of two interior opposite angles

$$125 = 2x + 3x$$

$$125 = 5x$$

$$x = \frac{125}{5}$$

$$x = 25^\circ$$

iii) Sum of angles in triangle $= 180°$

$$\therefore \quad x + 60 + 50 = 180°$$

$$x + 110 = 180$$

$$x = 180 - 110$$

$$x = 70°$$

4.

i) Sum of angles in triangle $= 180°$

$$50 + x + x = 180$$

$$2x + 50 = 180$$

$$2x = 180 - 50$$

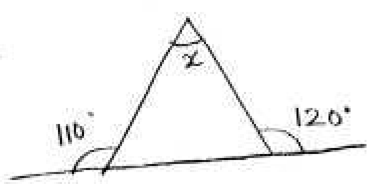
$$2x = 130$$

$$x = \frac{130}{2}$$

$$x = 65°$$

ii) Exterior angles $= 110°, 120°$

Corresponding ~~ans~~ Interior angles



$= 180 - 110, \ 180 - 120$

$= 70, 60$ ($\because$ forms linear pair)

$\therefore$ Angles of triangle $x, 70°, 60°$

$$\therefore \quad x + 70 + 60 = 180$$

$$x + 130 = 180$$

$$x = 180 - 130.$$

$$\boxed{x = 50°}$$

(iii) Sum of angles in triangle $= 180^\circ$

$$x+2x+90 = 180$$

$$3x+90 = 180$$

$$3x = 180-90$$

$$3x = 90$$

$$x = \frac{90}{3}$$

$$x = 30^\circ$$

5.

(i) Sum of angles in triangle $= 180^\circ$

$$x+y+50 = 180^\circ$$

$$x+y = 180-50$$

$$x+y = 130 \rightarrow (1)$$

Exterior angles = sum of interior angles

$$120 = 50+x$$

$$x = 120-50$$

$$\boxed{x = 70^\circ} \rightarrow (2)$$

Subtitude x value in eq (1)

$$x+y = 130$$

$$70+y = 130$$

$$y = 130-70$$

$$\boxed{y = 60^\circ}$$

(ii)

$x = 60°$ ($\because$ Vertically opposite angles)

Sum of angles in triangles = 180°

$40 + x + y = 180°$

$40 + 60 + y = 180°$

$y + 100 = 180$

$y = 180 - 100$

$\boxed{y = 80°}$

(iii)

$y = 90°$ ($\because$ Vertically opposite angles)

Sum of angles in triangles = 180°

$y + x + x = 180°$

$90 + 2x = 180°$

$2x = 180 - 90$

$2x = 90$

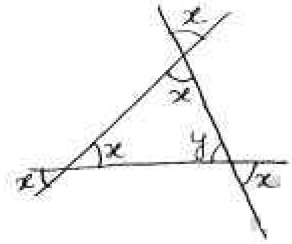
$x = \frac{90}{2}$

$\boxed{x = 45°}$

6.

(i) Angles of triangle = x, x, y

(∵ Vertically opposite angles are equal)



$y = x$ (∵ Vertically opposite angle)

Sum of angles in triangle = 180

$$x + x + y = 180$$

$$x + x + x = 180$$

$$3x = 180$$

$$x = \frac{180}{3}$$

$$\boxed{x = 60^\circ}$$

(ii) From given figure

$125 + x = 180^\circ$ (∵ forms linear pair)

$$x = 180 - 125$$

$$\boxed{x = 55^\circ}$$

Sum of angles in triangle = 180

$$x + x + y = 180^\circ$$

$$2x + y = 180$$

$$2 \times 55 + y = 180$$

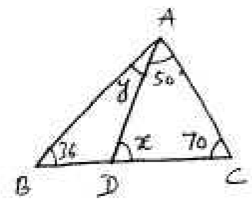
$$110 + y = 180$$

$$y = 180 - 110$$

$$\boxed{y = 70^\circ}$$

(iii)

In $\triangle^{le}$ ADC



$50 + x + 70 = 180°$

$x + 120 = 180$

$x = 180 - 120$

$\boxed{x = 60°}$

In $\triangle^{le}$ ABC

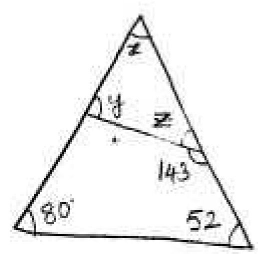
$(50+y) + 36 + 70 = 180$

$y + 156 = 180$

$y = 180 - 156$

$\boxed{y = 24°}$

7. Sum of angles in triangle $= 180°$



$x + 80 + 52 = 180$

$x + 132 = 180$

$x = 180 - 132$

$\boxed{x = 48°}$

$z + 143 = 180$ ($\because$ forms a linear pair)

$x = 180 - 143$

$\boxed{x = 37°}$

Sum of angles in triangle = 180

$x+y+z=180$

$48^\circ+37^\circ+z=180$

$85+z=180$

$z=180-85$

$\boxed{z=95^\circ}$

8.

Let the unknown angles are x, x.

One angle = 80°

Sum of angles in triangle = 180°

$x+x+80=180$

$2x+80=180$

$2x=180-80$

$2x=100$

$x=\frac{100}{2}$

$\boxed{x=50^\circ}$

9. Given

Angle in triangle = 60°

Other two angles ratio = 2 : 3

Let two angles = $2x, 3x$

Sum of angles = 180

$60+2x+3x=180$

$$60+5x=180$$

$$5x=180-60$$

$$5x=120$$

$$x=\frac{120}{5}$$

$$\boxed{x=24^\circ}$$

$\therefore$ Angles are $2x = 2\times24 = 48^\circ$

$3x = 3\times24 = 72^\circ$

10.

Given

Angles ratio $= 1:2:3$

Let Angles be $x, 2x, 3x$

Sum of angles in triangle $= 180^\circ$

$$x+2x+3x=180^\circ$$

$$6x=180$$

$$x=\frac{180}{6}$$

$$\boxed{x=30^\circ}$$

Angle $\Rightarrow$ $x = 30^\circ$

$2x = 2\times30^\circ = 60^\circ$

$3x = 3\times30^\circ = 90^\circ$

$\therefore$ It is Scalene triangle according to sides

It is Right angled triangle according to angles

11.

(i) $65^\circ, 74^\circ, 39^\circ$?

Sum of angles = $65^\circ + 74^\circ + 39 = 178^\circ \neq 180^\circ$

No, These are not angles of triangle

(ii) $\frac{1}{3}$ right angle = $\frac{1}{3} \times 90 = 30^\circ$

Right angle = 90°

Another angle = 60°

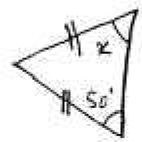
Sum of angles = $30 + 90 + 60 = 180^\circ$

$\therefore$ These are angles of triangle

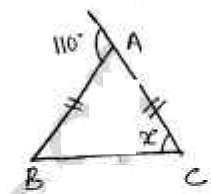
Exercise 11.3

(i) In, triangle angles opposite to equal sides are equal.

$\therefore \quad x = 50^\circ$



(ii) In triangle



Exterior angle + interior angle = 180'

$110^\circ + \angle BAC = 180$

$\angle BAC = 180 - 110$

$\angle BAC = 70^\circ$

In triangle, angles opposite to equal sides are equal

$\therefore \angle ABC = \angle ACB = x$

Sum of angles in triangle = 180

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$

$x + x + 70 = 180^\circ$

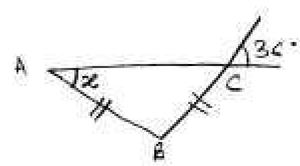
$2x = 180 - 70$

$2x = 110$

$x = \frac{110}{2}$

$\boxed{x = 55^\circ}$

(iii) In triangle, angles opposite to equal sides are equal.

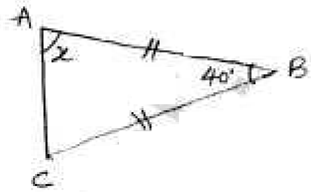


$\angle BAC = \angle ACB = x$.

$x = 36°$ [$\because$ vertically opposite angles.]

2.

(i) In a triangle, angles opposite to equal sides are equal.



$\angle BAC = \angle ACB = x$

Sum of angles in triangle $= 180$

$40 + x + x = 180$

$40 + 2x = 180$

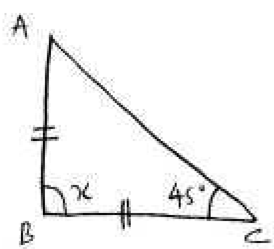
$2x = 180 - 40$

$2x = 140$

$x = \frac{140}{2}$

$\boxed{x = 70°}$

(ii) In a triangle angles opposite to equal sides are equal.



$\angle BCA = \angle CAB$

$\angle CAB = 45°$

Sum of angles in triangle $= 180°$

$\angle BCA + \angle CAB + \angle ABC = 180°$

$45 + 45 + \angle ABC = 180$

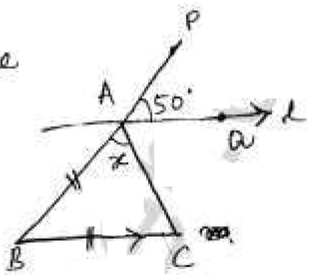
$90 + \angle ABC = 180$

$\angle ABC = 180 - 90$

$\boxed{\angle ABC = 90^\circ}$

(iii)

$l \parallel BC$, AB is transversal line



$\angle CBA = \angle PAQ$

$\angle CBA = 50^\circ$

In a triangle, angles opposite to equal sides are equal

$\cancel{\angle CBA = \angle C}$

$\angle BCA = \angle BAC = x$

Sum of angles in triangle $= 180^\circ$

$\angle BCA + \angle BAC + \angle CBA = 180^\circ$

$x + x + 50 = 180$

$2x + 50 = 180$

$2x = 180 - 50$

$2x = 130$

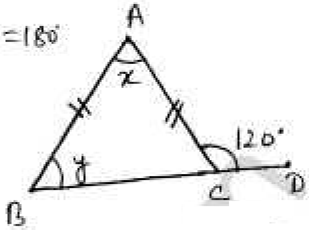
$x = \frac{130}{2}$

$x = 65^\circ$

3.

(i) In a triangle,

Sum of Exterior and interior angle $=180°$



$\angle ACB + \angle ACD = 180°$

$\angle ACB + 120 = 180$

$\angle ACB = 180 - 120$

$\angle ACB = 60°$

In a triangle, angles opposite to equal sides are equal

$\angle ABC = \angle ACB$

$\boxed{y = 60°}$

Sum of angles in triangle $= 180°$

$\angle ABC + \angle BCA + \cancel{\angle CAB}\ \angle CAB = 180°$

$60 + 60 + \cancel{\Delta}\ x = 180°$

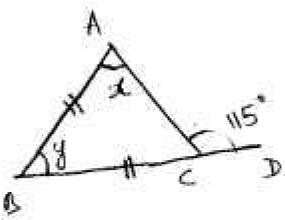
$x + 120 = 180$

$x = 180 - 120$

$\boxed{x = 60°}$

(ii) In a triangle

External angle + Internal Angle $= 180$



$115° + \angle ACB = 180°$

$\angle ACB = 180 - 115$

$\angle ACB = 65°$

In a triangle,

angles opposite to equal sides are equal.

$$\angle ACB = \angle BAC = 65^\circ$$

$$\boxed{x = 65^\circ}$$

Sum of angle in a triangle = 180°

$$\angle ABC + \angle ACB + \angle BAC = 180$$

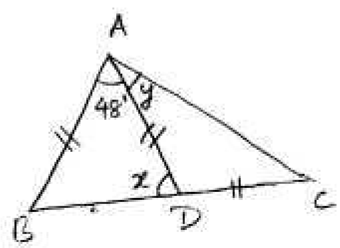
$$y + 65^\circ + 65^\circ = 180$$

$$y + 130 = 180$$

$$y = 180 - 130$$

$$\boxed{y = 50^\circ}$$

3. In $\triangle ABD$



$\angle ABD = \angle ADB = x$

(∵ In a triangle, angles opposite to the equal sides are equal)

Sum of angle in triangle = 180°

$$x + x + 48 = 180$$

$$2x + 48 = 180$$

$$2x = 180 - 48$$

$$2x = 132$$

$$x = \frac{132}{2}$$

$$\boxed{x = 66^\circ}$$

~~In a tria~~

Externd angle + Internal angle $= 180°$

$\angle ADC + 66 = 180°$

$\angle ADC = 180 - 66$

$\angle ADC = 114°$

In $\triangle ADC$.

$\angle ACD = \angle DAC = y$

($\because$ In a triangle, angles opposite to equal sides are equal)

Sum of angles in triangle $= 180$

$y + y + 114° = 180°$

$2y + 114 = 180$

$2y = 180 - 114$

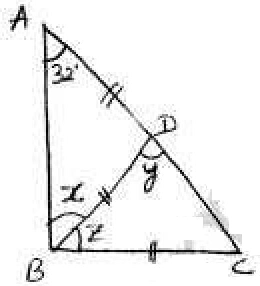
$2y = 66°$

$y = \frac{66}{2}$

$\boxed{y = 33°}$

4.

i. In Δ^{le} ADB



$\angle BAD = \angle ABD$ $(\because \overline{AD} = \overline{BD})$

$x = 32°$

In Δ^{le} BDC

$\angle BCD = \angle BDC = y$ $(\because \overline{BD} = \overline{BC})$

Sum of angles in triangle = 180°

$\angle BDC + \angle DBC + \angle DCB = 180°$

$y + z + y = 180°$

$z + 2y = 180 \rightarrow (1)$

In Δ^{le} ABC

$\angle A + \angle ABC + \angle BCA = 180°$

$32° + x + z + y = 180°$

$32 + 32 + z + y = 180°$

$z + y = 180 - 64$

$z + y = 116 \rightarrow (2)$

Solving (1) and (2)

$$\begin{array}{r} z + 2y = 180 \\ -z - y = -116 \\ \hline y = 64° \end{array}$$

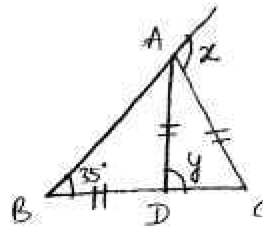
$z + y = 116$

$z + 64 = 116$

$z = 116 - 64 \Rightarrow \underline{\underline{z = 52°}}$

(ii)

In $\triangle ABD$



$\angle ABD = \angle BAD$ ($\because \overline{BD} = \overline{AD}$)

$\angle BAD = 35°$

Somof angles in triangle = 180°

$\angle ABD + \angle BAD + \angle ADB = 180$

$35 + 35 + \angle ADB = 180$

$\angle ADB = 180 - 70$

$\angle ADB = 110°$

External angle + Internal angle = 180

$110 + y = 180$

$y = 180 - 110$

$\boxed{y = 70°}$

In $\triangle ADC$

$\angle ADC = \angle ACD$

$y = \angle ACD$

External angle = Somof other two interior angle

$x = \angle ADC + \angle ACD$

$x = y + y$

$x = 2y$

$x = 2 \times 70$

$\boxed{x = 140°}$

5. Given

Angles ratio = 1:2:1

Let Angles = $x, 2x, x$

Sum of angles = $180°$

$x + 2x + x = 180$

$4x = 180$

$x = \frac{180}{4}$

$\boxed{x = 45°}$

$\therefore$ Angles = $45°, 2 \times 45°, 45$

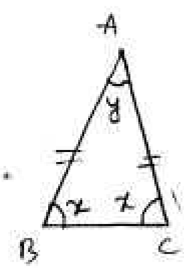
$= 45°, 90°, 45°$

$\therefore$ Right-angled triangle, Isosceles triangle.

6. Both base angles are equal because Isosceles triangle

x - base angle

y - Vertical angle



Given

Base angle = 4 × Vertical angle

$x = 4y$

Sum of angles in triangles = $180°$

$y + x + x = 180$

$y + 4y + 4y = 180°$

$9y = 180$

$y = 180/9$

$\boxed{y = 20°}$

Angles = $y, 4y, 4y \Rightarrow 20°, 80°, 80°$

Exercise 11·4

1.

i) $2cm + 3cm = 5cm \not> 5cm.$

" Sum of two sides must be greater than the third side, to form triangle"

$\therefore$ Triangle is not possible

ii)

$2.5cm + 4.5cm = 7cm \not> < 8cm.$

" In a triangle, sum of two sides must be greater than the third side".

$\therefore$ Triangle is not possible

iii)

$5.8 + 4.5 = 10.3cm > 10.2cm$

' In a triangle, sum of two sides must be greater than the third side'

$\therefore$ Triangle is possible.

iv)

$3.4cm + 4.7cm = 8.1cm > 6.2cm$

" In a triangle, sum of two sides must be greater than the third side"

$\therefore$ Triangle is possible

2. Given sides = 7cm, 10cm.

Difference of sides = 10 cm − 7 cm = 3 cm.

Sum of sides = 10 cm + 7 cm = 17 cm

Hence, the length of the third side of triangle must be greater than 3cm and less than 17cm.

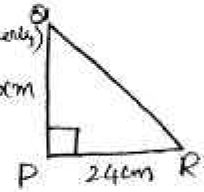
3. No

Sum of two angles

$30^\circ + 30^\circ = 60^\circ < 120^\circ$

Exercise 11.5

1.



$QR^2 = PQ^2 + PR^2$ (Pythagorus property)

$QR^2 = 10^2 + 24^2$

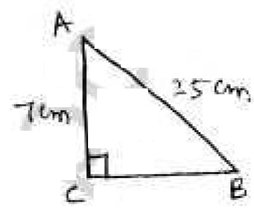
$QR^2 = 100 + 576$

$QR^2 = 676$

$QR = \sqrt{676}$

$QR = 26cm$

2.



$AB^2 = AC^2 + BC^2$

$25^2 = 7^2 + BC^2$

$625 = 49 + BC^2$

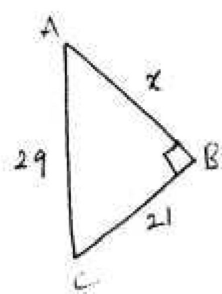
$BC^2 = 625 - 49$

$BC^2 = 576$

$BC = \sqrt{576}$

$BC = 24cm.$

3. (i)



$AC^2 = AB^2 + BC^2$ ($\because$ Pythagorus Property)

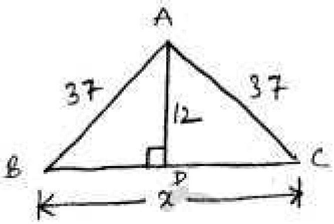
$29^2 = x^2 + 21^2$

$841 = x^2 + 441$

$x^2 = 841 - 441$

$x^2 = 400 \Rightarrow x = \sqrt{400} = 20 cm.$

(ii) In $\triangle ABD$



$AB^2 = AD^2 + BD^2$ ($\because$ Pythagoras theorem)

$37^2 = 12^2 + BD^2$

$BD^2 = 1369 - 144$

$BD^2 = 1225$

$BD = 35cm$

In $\triangle ADC$

$AC^2 = AD^2 + DC^2$

$37^2 = 12^2 + DC^2$

$DC^2 = 1369 - 144$

$DC^2 = 1225$

$DC = 35cm$

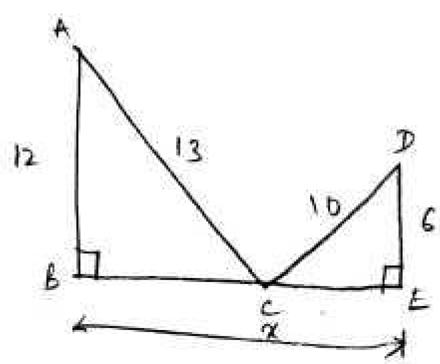
$x = BD + DC$

$= 35 + 35$

$x = 70cm$

iii)

In $\triangle ABC$



$AC^2 = AB^2 + BC^2$

$13^2 \neq 12^2 + BC^2$

$169 = 144 + BC^2$

$BC^2 = 169 - 144$

$BC^2 = 25$

$BC = 5cm.$

In Δ DCE

$DC^2 = DE^2 + CE^2$

$10^2 = 6^2 + CE^2$

$CE^2 = 100 - 36$

$CE^2 = 64$

$CE = \sqrt{64}$

$CE = 8cm.$

$x = \overline{BC} + \overline{CE}$

$= 5 + 8$

$x = 13cm.$

4.

(i) $4^2 + 5^2 \Rightarrow 16 + 25 = 41 \neq 7^2$

$\therefore$ 4, 5, 7 are not sides of Right angled triangle

(ii) Biggest side = 2.5 cm = Hypotaneous

$1.5^2 + 2^2 \Rightarrow 2.25 + 4 = 6.25$

$2.5^2 = 6.25$

$\therefore 1.5^2 + 2^2 = 2.5^2$

$\therefore$ 1.5cm, 2cm, 2.5cm are sides of Right angled triangle.

Right angle is opposite to 2.5cm side

(iii) 7cm, 5.6cm, 4.2cm

Biggest side = 7cm = Hypotaneous

$\therefore 5.6^2 + 4.2^2 \Rightarrow 31.36 + 17.64 = 49.$

$7^2 = 49$

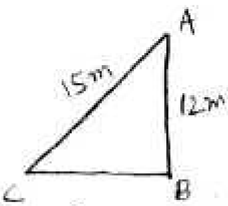
$\therefore 5.6^2 + 4.2^2 = 7^2$

$\therefore$ Given sides are sides of Right angled triangle

Right angle at opposite to side 7cm side.

5\. ladder length = 15m.

Height from the ground = 12 m



$AC^2 = AB^2 + BC^2$ ($\because$ Pythagorus theorem)

$15^2 = 12^2 + BC^2$

$BC^2 = 225 - 144$

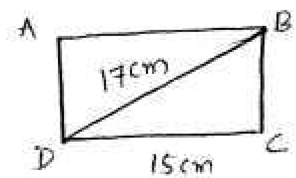
$BC^2 = 81$

$BC = \sqrt{81}$

$BC = 9$ m.

Distance b/w ladder foot to wall is 9m.

6.



$BD^2 = DC^2 + BC^2$

$17^2 = 15^2 + BC^2$

$BC^2 = 289 - 225$

$BC^2 = 64$

$BC = \sqrt{64}$

$BC = 8cm.$

Perimeter $= 2(l+b) \Rightarrow 2(DC+BC)$

$\Rightarrow 2(15+8)$

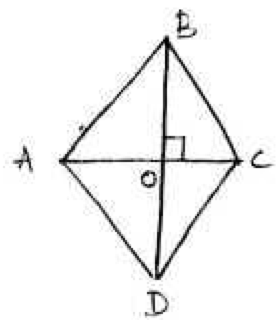
$= 2(23)$

Perimeter $= 46$ cm

Area $= l \times b = 15 \times 8$

$= 120\ cm^2$

7.



$AC = 10cm$

$BD = 24cm.$

$\overline{BO} = \overline{OD} = 12cm.$

$\overline{AO} = \overline{OC} = 5cm.$

$BC^2 = \overline{BO}^2 + \overline{OC}^2$

$BC^2 = 12^2 + 5^2$

$= 144 + 25$

$BC^2 = 169$

$BC = \sqrt{169}$

$BC = 13cm.$

Side of rohmbus = 13cm

Perimeter = 4 × side

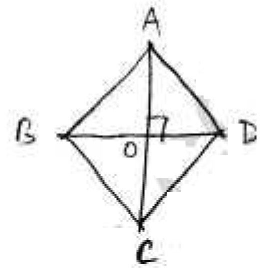
$= 4 \times 13 = 52cm$

8.

$AC = 8cm$

$\overline{AO} = \overline{OC} = 4cm.$

$\overline{AD} = 5cm.$



$\overline{AO}^2 + \overline{OD}^2 = \overline{AD}^2$

$4^2 + \overline{OD}^2 = 5^2$

$\overline{OD}^2 = 25 - 16$

$\overline{OD}^2 = 9$

$\overline{OD} = \sqrt{9}$

$\overline{OD} = 3cm.$

$BD = 2 \times \overline{OD}$

$= 2 \times 3$

$\overline{BD} = 6cm$