

# Exponents and Powers

## Exercise 2.1

### Solution 1

$$(i) \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2$$

$$= \frac{5 \times 5}{3 \times 3}$$

$$= \frac{25}{9}$$

$$\left(\because \left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^m\right)$$

$$(ii) (-3)^{-3} = \frac{1}{(-3)^3}$$

$$= \frac{1}{(-3) \times (-3) \times (-3)}$$

$$= \frac{1}{-27} = -\frac{1}{27}$$

$$\left(\because (a)^{-m} = \frac{1}{(a)^m}\right)$$

$$(iii) \left(\frac{2}{7}\right)^{-4} = \left(\frac{7}{2}\right)^4$$

$$= \frac{7 \times 7 \times 7 \times 7}{2 \times 2 \times 2 \times 2}$$

$$= \frac{2401}{16}$$

Solution 2

$$(i) [(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1} = \left[ \frac{1}{2} + \frac{1}{4} + \frac{1}{3} \right]^{-1} \quad (\because a^{-m} = \frac{1}{a^m})$$

$$= \left[ \frac{6 + 3 + 4}{12} \right]^{-1}$$

$$= \left( \frac{13}{12} \right)^{-1}$$

$$= \frac{12}{13}$$

$$(ii) [(4)^{-1} - (5)^{-1}]^2 \times \left( \frac{5}{8} \right)^{-1} = \left( \frac{1}{4} - \frac{1}{5} \right)^2 \times \left( \frac{8}{5} \right)^1$$

$$= \left( \frac{5-4}{20} \right)^2 \times \frac{8}{5}$$

$$= \frac{1}{20 \times 20} \times \frac{8}{5}$$

$$= \frac{1}{250}$$

$$(iii) [4^0 + 4^2 - 2^3] \times 3^{-2} = [1 + 16 - 8] \times \frac{1}{3^2} \quad [\because a^0 = 1]$$

$$= (9) \times \frac{1}{9}$$

$$= 1.$$

$$\begin{aligned}
 \text{(iv)} \quad \left[ (5)^2 - \left(\frac{1}{4}\right)^{-2} \right] \times \left(\frac{3}{4}\right)^{-2} &= \left[ (25) - (4)^2 \right] \times \left(\frac{4}{3}\right)^2 \\
 &= [25 - 16] \times \frac{16}{9} \\
 &= 9 \times \frac{16}{9} \\
 &= 16.
 \end{aligned}$$

Solution 3.

(i) As the multiplicative inverse of  $a^m = a^m$ ,

$\therefore$  the multiplicative inverse of  $\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{81}{16}\right)^{\frac{3}{4}}$

$$\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2}\right)^{\frac{3}{4}}$$

$$= \left(\frac{3^4}{2^4}\right)^{\frac{3}{4}}$$

$$= \left(\frac{3}{2}\right)^{4 \times \frac{3}{4}}$$

$$= \left(\frac{3}{2}\right)^3 = \frac{3 \times 3 \times 3}{2 \times 2 \times 2}$$

$$= \frac{27}{8}$$

$$(ii) \left[ \left( \frac{-3}{2} \right)^{-4} \right]^{\frac{1}{2}} = \left[ \left( \frac{-3}{2} \right)^{-4 \times \frac{1}{2}} \right]$$

$$= \left( \frac{-3}{2} \right)^{-2}$$

Multiplicative inverse of  $\left( \frac{-3}{2} \right)^{-2} = \left( \frac{-3}{2} \right)^2$

$$\left( \frac{-3}{2} \right)^2 = \frac{-3 \times -3}{2 \times 2} = \frac{9}{4}$$

$$(iii) \left( \frac{5}{7} \right)^{-2} \times \left( \frac{5}{7} \right)^4 \div \left( \frac{5}{7} \right)^3 = \left( \frac{5}{7} \right)^{-2+4-3} \quad \left( \begin{array}{l} \because a^m \times a^n = a^{m+n}, \\ a^m \div a^n = a^{m-n}. \end{array} \right)$$

$$= \left( \frac{5}{7} \right)^{-1}$$

Multiplicative inverse of  $\left( \frac{5}{7} \right)^{-1} = \left( \frac{5}{7} \right)$ .

Solution 4.

$$(i) (16)^{-2} = (2 \times 2 \times 2 \times 2)^{-2}$$

$$= (2^4)^{-2}$$

$$= 2^{4 \times -2}$$

$$= 2^{-8}$$

$$\left( \because (a^m)^n = a^{mn} \right)$$

$$\begin{aligned}
 \text{(ii)} \quad (125)^{-4} &= (5 \times 5 \times 5)^{-4} \\
 &= (5^3)^{-4} \\
 &= 5^{3 \times (-4)} \\
 &= 5^{-12}
 \end{aligned}$$

### Solution 5

$$\begin{aligned}
 \text{(i)} \quad 2789.453 &= 2 \times 1000 + 7 \times 100 + 8 \times 10 + 9 \times 1 + \frac{4}{10} + \frac{5}{100} + \frac{3}{1000} \\
 &= 2 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2} + 3 \times 10^{-3}
 \end{aligned}$$

$$\text{(ii)} \quad 3007.805$$

$$\begin{aligned}
 &= 3 \times 1000 + 0 \times 100 + 0 \times 10 + 7 \times 1 + \frac{8}{10} + \frac{0}{100} + \frac{5}{1000} \\
 &= 3 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 7 \times 10^0 + 8 \times 10^{-1} + 0 \times 10^{-2} + 5 \times 10^{-3}
 \end{aligned}$$

### Solution 6

$$\begin{aligned}
 \text{(i)} \quad \left[ \left( \left( \frac{5}{7} \right)^2 \right)^{-1} \right]^{-3} &= \left( \frac{5}{7} \right)^{2 \times -1 \times -3} \quad \left( \because (a^m)^n = a^{m \times n} \right) \\
 &= \left( \frac{5}{7} \right)^6
 \end{aligned}$$

$$(ii) \left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{ \left(\frac{7}{5}\right)^{-2} \right\}^{-4}$$

$$= \left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^3 \div \left(\frac{7}{5}\right)^{(-2) \times (-4)} \quad \left[ \because \left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^m \right]$$

$$= \left(\frac{2}{7}\right)^5 \div \left(\frac{7}{5}\right)^8$$

$$= \left(\frac{2}{7}\right)^5 \times \left(\frac{5}{7}\right)^8$$

$$= \frac{2^5 \times 5^8}{7^5 \times 7^8} = \frac{2^5 \times 5^8}{7^{(5+8)}} \quad (\because a^m \times a^n = a^{m+n})$$

$$= \frac{2^5 \times 5^8}{7^{13}}$$

$$(iii) \left(\frac{4}{5}\right)^2 \times (5)^4 \times \left(\frac{2}{5}\right)^{-2} \div \left(\frac{5}{2}\right)^{-3}$$

$$= \left(\frac{4}{5}\right)^2 \times (5)^4 \times \left(\frac{5}{2}\right)^2 \div \left(\frac{2}{5}\right)^3$$

$$= \frac{4^2 \times 5^4}{5^2} \times \frac{5^2}{2^2} \times \frac{5^3}{2^3}$$

$$= \frac{5^{4+2+3-2}}{2^{2+3}} \times (2^4)^2 = \frac{5^7}{2^{5-4}} = \frac{5^7}{2^1}$$

$$(iv) \frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}}$$

$$= 2^{-3+4} \times 5^3$$

$$= 5^3 \times 2^1$$

Solution 7

$$(i) ((-2)^3)^2 + (5)^{-3} \div (5)^{-5} - \left(-\frac{1}{2}\right)^0$$

$$= (-2)^{3 \times 2} + (5)^{-3} \times (5)^{+5} - 1 \quad (a^0 = 1; (a^m)^n = a^{mn})$$

$$= 2^6 + 5^2 - 1$$

$$= 64 + 25 - 1$$

$$= 88$$

$$= 8 \times 11$$

$$= 2^3 \times 11$$

$$(ii) 3^{-5} \times 3^2 \div 3^{-6} + (2^2 \times 3^2)^2 + \left(\frac{2}{3}\right)^{-1} + 2^{-1} + \left(\frac{1}{19}\right)^{-1}$$

$$= 3^{-5+2} \times 3^6 + 2^4 \times 3^2 + \left(\frac{3}{2}\right)^1 + \left(\frac{1}{2}\right)^1 + (19)^1$$

$$= 3^{-3+6} + 2^4 \times 3^2 + \frac{3+1}{2} + 19$$

$$= 3^3 + 2^4 \times 3^2 + \frac{4}{2} + 19$$

$$= 27 + 144 + 2 + 19$$

$$= 192 = 3 \times 64$$

$$= 2^6 \times 3.$$

Solution 8

$$(i) 5^3 \times \left(\frac{4}{5}\right)^3 = 5^3 \times \frac{4^3}{5^3}$$

$$= 4^3 \times 5^{3-3}$$

$$= 4^3$$

$$= \frac{1}{(4)^{-3}}$$



$$(ii) \left[ \left( \frac{3}{7} \right)^{-2} \right]^{-3} = \left( \frac{3}{7} \right)^{-2 \times -3} \quad (\because (a^m)^n = a^{m \times n})$$

$$= \left( \frac{3}{7} \right)^6$$

$$= \left( \frac{7}{3} \right)^{-6}$$

$$(iii) \left( \frac{5}{9} \right)^{-2} \times \left( \frac{5}{3} \right)^2 \div \left( \frac{1}{5} \right)^{-2} = \frac{\left( \frac{9}{5} \right)^2 \times \left( \frac{5}{3} \right)^2}{\left( \frac{1}{5} \right)^{-2}}$$

$$= \frac{9^2}{5^2} \times \frac{5^2}{3^2} \times \frac{1^2}{5^2}$$

$$= 3^{4-2} \times 5^{2-4}$$

$$= 3^{+2} \times 5^{-2} = 5^{-2} \times \frac{1}{(3)^{-2}}$$

$$= \left( \frac{5}{3} \right)^{-2}$$

$$(iv) 2^{-1} \left[ \left( \frac{5}{3} \right)^4 + \left( \frac{3}{5} \right)^{-2} \right] \div \frac{17}{9}$$

$$= \frac{1}{2} \left[ \left( \frac{5}{3} \right)^4 + \left( \frac{5}{3} \right)^2 \right] \times \frac{9}{17}$$

$$= \frac{1}{2} \left[ \left( \frac{5}{3} \right)^2 \left[ \left( \frac{5}{3} \right)^2 + 1 \right] \right] \times \frac{9}{17}$$

$$= \frac{1}{2} \left[ \frac{25}{9} \left( \frac{25}{9} + 1 \right) \right] \times \frac{9}{17}$$

$$= \frac{1}{2} \left[ \frac{25}{9} \left( \frac{25+9}{9} \right) \right] \times \frac{9}{17}$$

$$= \frac{1}{2} \left[ \frac{25}{9} \times \frac{34}{9} \right] \times \frac{9}{17}$$

$$= \frac{1}{2} \left[ \frac{25}{9} \times \frac{17 \times 2}{9} \times \frac{9}{17} \right]$$

$$= \frac{5^2}{3^2} = \left( \frac{5}{3} \right)^2$$

$$= \left( \frac{3}{5} \right)^{-2}$$

$$(v) (-7)^3 \times \left(\frac{1}{-7}\right)^{-7} \div (-7)^{10}$$

$$= (-7)^3 \times (-7)^7 \div (-7)^{10}$$

$$= \frac{(-7)^3 \times (-7)^7}{(-7)^{10}}$$

$$= (-7)^{3+7-10}$$

$$= (-7)^2 = (7)^2$$

$$= \left(\frac{1}{-7}\right)^{-2} = \left(\frac{1}{7}\right)^{-2}$$

$$= \frac{1}{(7)^2}$$

Solution 9

$$(i) \frac{49 \times 2^{-3}}{7^{-3} \times 10 \times 7^{-5}} = \frac{7^2 \times 2^{-3}}{7^{-3} \times 10 \times 7^{-5}}$$

$$= \frac{7^{2+3} \times 2^{-3+5}}{10}$$

$$= \frac{7^5 \times 2^2}{10}$$

$$\left(\because \frac{a^m}{a^n} = a^{m-n}\right)$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{9^3 \times 27 \times t^4}{(3)^2 \times (3)^4 \times t^2} &= \frac{(3^2)^3 \times 3^3 \times t^4}{(3)^2 \times (3)^4 \times t^2} \\
 &= 3^6 \times 3^3 \times t^4 \times 3^{-2} \times 3^{-4} \times t^{-2} \\
 &= 3^{6+3-2-4} \times t^{4-2} \quad (\because a^m \times a^n = a^{m+n}) \\
 &= 3^3 \times t^2 \\
 &= 27 \times t^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3} &= \frac{3^{-4} \times 5^{-6} \times t^{-6}}{3^{-10} \times 5^{-6} \times t^{-12}} \quad (\because (a^m)^n = a^{m \times n}) \\
 &= 3^{-4+10} \times 5^{-6+6} \times t^{-6+12} \quad (\because \frac{a^m}{a^n} = a^{m-n}) \\
 &= 3^6 \times 5^0 \times t^6 \\
 &= 3^6 \times t^6 \quad (\because a^0 = 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{2^{-5} \times 15^{-5} \times 500}{5^{-6} \times 6^{-5}} &= \frac{2^{-5} \times (3 \times 5)^{-5} \times 5^3 \times 2^2}{5^{-6} \times (2 \times 3)^{-5}} \\
 &= \frac{2^{-5+2} \times 3^{-5} \times 5^{-5} \times 5^3}{5^{-6} \times 2^{-5} \times 3^{-5}} \\
 &= 2^{-5+2+5} \times 3^{-5+0+5} \times 5^{-5+6+3} = 2 \times 3 \times 5^4 \\
 &= 2500.
 \end{aligned}$$

Solution 10.

Let the number which should divide  $\left(\frac{3}{-2}\right)^{-3}$  to get

$\left(\frac{2}{3}\right)^2$  be  $x$ .

$$\Rightarrow \frac{\left(\frac{3}{-2}\right)^{-3}}{x} = \left(\frac{2}{3}\right)^2$$

$$\Rightarrow \frac{3^{-3}}{(-2)^{-3}} = \frac{x \times 2^2}{3^2}$$

cross multiply

$$\Rightarrow 3^{-3} \times 3^2 = x \times 2^2 \times (-2)^{-3}$$

$$\Rightarrow 3^{-3+2} = x \times (2)^{2-3}$$

$$(\because (-a)^2 = (a)^2)$$

$$\Rightarrow 3^{-1} = x \times (-2)^{-1}$$

$$\Rightarrow x = \frac{3^{-1}}{(-2)^{-1}}$$

$$\therefore x = -\frac{2}{3}$$

Solution 11:

$$9^m \div 3^{-2} = 9^4$$

$$\Rightarrow \frac{9^m}{3^{-2}} = 9^4$$

$$\Rightarrow 9^m \times 9 = 9^4 \quad (\because \frac{1}{a^m} = a^{-m})$$

$$\Rightarrow 9^{m+1} = 9^4$$

If  $a^m = a^n$  then  $m = n$

$$\Rightarrow m+1 = 4$$

$$\therefore m = 3.$$

Solution 12

$$\left(\frac{-5}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{12} = \left[\left(\frac{-5}{7}\right)^3\right]^{\lambda} \times \left(\frac{-5}{7}\right)^{-1}$$

$$\Rightarrow \left(\frac{-5}{7}\right)^{-4+12} = \left(\frac{-5}{7}\right)^{3\lambda-1}$$

$$\Rightarrow -4+12 = 3\lambda-1 \quad (\because a^m = a^n \Rightarrow m=n)$$

$$\Rightarrow 3\lambda = 9$$

$$\therefore \lambda = 3.$$

Solution 13:

$$\left(\frac{-2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^8 = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-13} \times \left(\frac{-2}{3}\right)^{-8} = \left(\frac{-2}{3}\right)^{-2x+1} \quad \left(\because \left(\frac{p}{q}\right)^m = \left(\frac{q}{p}\right)^{-m}\right)$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-13-8} = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\therefore -21 = -2x+1 \quad (\because a^m = a^n \text{ then } m=n)$$

$$\Rightarrow -2x = -22$$

$$\therefore x = 11.$$

Solution 14:

$$(i) \quad 5^{2x-1} = \frac{1}{(125)^{x-3}} \Rightarrow 5^{2x-1} = \frac{1}{(5^3)^{x-3}}$$

$$\Rightarrow 5^{2x-1} \times 5^{3(x-3)} = 1$$

$$\Rightarrow 5^{(2x-1)+3(x-3)} = 5^0 \quad (\because a^0 = 1)$$

$$\therefore 2x-1+3x-9 = 0$$

$$\Rightarrow 5x = 10$$

$$\therefore x = 2.$$

$$(ii) \frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$$

$$\Rightarrow \frac{(3^2)^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3$$

$$\Rightarrow \frac{(2n+5+9)(-1-16)}{3 \times 3} = 3^3$$

$$\Rightarrow 3^{2n-3} = 3^3$$

$$\therefore 2n-3 = 3$$

$$\Rightarrow n = 3.$$

### Exercise 2.2

#### Solution 1

$$(i) 0.00000000000085 = \frac{85}{(10)^{13}}$$
$$= 8.5 \times 10^{-12}$$

$$(ii) 0.0000000000000942 = \frac{942}{(10)^{15}}$$
$$= 9.42 \times 10^{-13}$$

$$(iii) 6020000000000000 = 6.02 \times 10^{15}$$

$$(iv) 0.00000000837 = 8.37 \times 10^{-9}$$



### Solution-2

i)  $3.02 \times 10^{-6}$

0.00000302

ii)  $1.007 \times 10^{11}$

100700000000

iii)  $5.375 \times 10^{14}$

537500000000000

iv)  $7.579 \times 10^{-14} = 7.579 \times 10^{-14}$

0.00000000000007579

### Solution-3

i) The mass of a proton is  $1.673 \times 10^{-24}$  gram

ii) Thickness of a piece of paper is  $1.6 \times 10^{-3}$  cm

iii) Diameter of a wire on a computer chip is  $3 \times 10^{-6}$  m

iv) A helium atom has a diameter of  $32 \times 10^{-11}$  m

v) Mass of a molecule of hydrogen gas is about

$$3.34 \times 10^{-21} \text{ tons}$$

vi) Human body has  $10^{+12}$  of cells which vary

in shapes and sizes

vii) The distance from earth to the sun is  $149.6 \times 10^9$  m

viii) The speed of light is  $3 \times 10^8$  m/sec

ix) Mass of the Earth is  $5.97 \times 10^{24}$  kg

$$\begin{aligned}
 \text{x)} \quad 3 \text{ years} &= 3 \times 365 \text{ days} \\
 &= 3 \times 365 \times 24 \text{ Hours} \\
 &= 3 \times 365 \times 24 \times 60 \text{ Minutes} \\
 &= 3 \times 365 \times 24 \times 60 \times 60 \text{ Seconds} \\
 &= 94608000 \text{ Seconds} \\
 3 \text{ years} &= 9.4608 \times 10^7 \text{ Seconds}
 \end{aligned}$$

$$\begin{aligned}
 \text{xii)} \quad 7 \text{ hectares} &= 7 \times 10000 \text{ m}^2 \\
 &= 7 \times 10000 \times 10000 \text{ cm}^2 \\
 7 \text{ hectares} &= 7 \times 10^8 \text{ cm}^2
 \end{aligned}$$

xiii) A sugar factory has annual sales of 3,720,000,000 kilograms of sugar.

A sugar factory has annual sales of  $3.72 \times 10^9$  kg of sugar.

4. Solution 4  
Given

$$\text{Size of plant cell} = 0.00001275 \text{ m} = 1.275 \times 10^{-5} \text{ m}$$

$$\text{Thickness of piece of paper} = 0.0016 \text{ cm} = 1.6 \times 10^{-5} \text{ m}$$

Diameter of a wire on a Computer chip

$$= 0.000003 \text{ m} = 3 \times 10^{-6} \text{ m}$$

i)

Size of plant cell

Thickness of piece of paper

$$\frac{1.275 \times 10^{-5}}{1.6 \times 10^{-5}} = 0.796 \approx 0.8$$

∴ Size of plant cell 0.8 times the Thickness of piece of paper

ii)

Size of plant cell

Diameter of wire on computer chip

$$\frac{1.275 \times 10^{-5}}{3 \times 10^{-6}}$$
$$\underline{\underline{4.25}}$$

∴ Size of plant cell is 4.25 times bigger than the diameter of wire on computer chip.

iii)

Thickness of piece of paper

Diameter of wire on computer chip

$$\frac{1.6 \times 10^{-5}}{3 \times 10^{-6}}$$
$$\underline{\underline{5.33}}$$

∴ Thickness of piece of paper is 5.33 times bigger than the diameter of wire on computer chip.

8. Solution 5

no. of red blood cells per cubic millimeter

$$= 5.5 \times 10^6 / \text{mm}^3$$

$$= 5.5 \times 10^6 / \text{m.m}^3$$

total no. of red blood cells in 5 liters of blood

$$= 5.5 \times 10^6 / \text{mm}^3 \times 5 \text{ lit}$$

$$= 5.5 \times 10^6 / \text{mm}^3 \times 5 \times 10^5 \text{ mm}^3$$

$$= 5.5 \times 5 \times 10^6 \times 10^5$$

$$= 27.5 \times 10^{11} //$$

### Solution 6

$$\text{Mass of Mars} = 6.42 \times 10^{29} \text{ Kg}$$

$$\text{Mass of Sun} = 1.99 \times 10^{30} \text{ Kg}$$

$$\text{Total Mass} = \text{Mass of Mars} + \text{Mass of Sun}$$

$$= 6.42 \times 10^{29} + 1.99 \times 10^{30}$$

$$= 0.642 \times 10^{30} + 1.99 \times 10^{30}$$

$$\text{Total Mass} = 2.632 \times 10^{30} \text{ Kg} \quad (\because \text{make powers equal in both terms})$$

$$\therefore \text{Total mass} = 2.632 \times 10^{30} \text{ Kg}$$

### Solution 7

$$\text{distance between star and Earth} = 8.1 \times 10^{13} \text{ Km}$$

$$= 8.1 \times 10^{13} \times 10^3 \text{ m}$$

$$= 8.1 \times 10^{16} \text{ m}$$

$$\text{Speed of light} = 3 \times 10^8 \text{ m/sec}$$

$$\text{Time} = \frac{\text{distance}}{\text{Speed}}$$

$$= \frac{8.1 \times 10^{16}}{3 \times 10^8}$$

$$\text{Time} = 2.7 \times 10^8 \text{ sec}$$