

Squares and Square Roots

Exercise 3.1

1.

i. 729

$$729 = \underbrace{3 \times 3 \times 3} \times \underbrace{3 \times 3 \times 3} \times \underbrace{3 \times 3 \times 3}$$
$$= 3^3 \times 3^3 \times 3^3$$

$$729 = 27^2$$

\therefore 729 is perfect square

because 729 can be expressed as product of pairs of equal prime factors.

Prime factorisation

3	729
3	243
3	81
3	27
3	9
3	3
	1

ii. 5488

$$5488 = \underbrace{2 \times 2 \times 2 \times 2} \times \underbrace{7 \times 7 \times 7}$$

Since '7' left unpaired

5488 is not a perfect square.

2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

iii. 1024

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$1024 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \quad 2.$$

Since 1024 is expressed as the product of pairs of equal prime numbers it is a perfect square.

iv. 243

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$243 = \underbrace{3 \times 3} \times \underbrace{3 \times 3} \times 3 \times 3$$

As '3' left unpaired, so 243 is not a square (perfect)

2.

i. 1296

$$\begin{array}{r|l} 2 & 1296 \\ \hline 2 & 648 \\ \hline 2 & 324 \\ \hline 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$1296 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{3 \times 3} \times \underbrace{3 \times 3}$$

Since 1296 is expressed as the product of pairs of equal prime numbers so it is a perfect square.

$$1296 = 2^2 \times 2^2 \times 3^2 \times 3^2$$

$$1296 = (2 \times 2 \times 3 \times 3)^2 = 36^2$$

∴ 1296 is square of 36

ii. 1784

3

$$\begin{array}{r|l} 2 & 1784 \\ \hline 2 & 892 \\ \hline 2 & 446 \\ \hline 223 & 223 \\ \hline & 1 \end{array}$$

$$1784 = \underbrace{2 \times 2 \times 2}_{2^3} \times 223$$

As 1784 can not be expressed as products of pairs of equal prime factors, so 1784 is not a perfect square.

iii) 3025

$$\begin{array}{r|l} 5 & 3025 \\ \hline 5 & 605 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$3025 = \underbrace{5 \times 5}_{5^2} \times \underbrace{11 \times 11}_{11^2}$$

Since 3025 can be expressed as the product of pairs of equal prime factors.

$$3025 = (5 \times 11)^2 = 55^2$$

Hence, 55 is a number whose square is 3025.

iv) 3969

$$\begin{array}{r|l} 3 & 3969 \\ \hline 3 & 1323 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$3969 = \underbrace{3 \times 3} \times \underbrace{3 \times 3} \times \underbrace{7 \times 7}$$

34

3969 can be expressed as product of pairs of equal prime numbers.

$$3969 = 3^4 \times 7^2$$

$$3969 = (3 \times 3 \times 7)^2$$

$$3969 = 63^2$$

Hence, 63 is the number whose square is 3969

3.

$$\begin{array}{r} 1008 \\ 2 \overline{) 1008} \\ \underline{200} \\ 2 \overline{) 504} \\ \underline{400} \\ 2 \overline{) 104} \\ \underline{80} \\ 2 \overline{) 24} \\ \underline{24} \\ 7 \overline{) 63} \\ \underline{63} \\ 3 \overline{) 9} \\ \underline{9} \\ 3 \overline{) 3} \\ \underline{3} \\ 1 \end{array}$$

$$1008 = \underbrace{2 \times 2 \times 2 \times 2} \times 7 \times \underbrace{3 \times 3}$$

Since '7' is left unpaired, so to make 1008 a Perfect Square it should be multiplied by '7'

4. 5808

5

$$\begin{array}{r|l} 2 & 5808 \\ \hline 2 & 2904 \\ \hline 2 & 1452 \\ \hline 2 & 726 \\ \hline 3 & 363 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11$$

Since '3' left unpaired. To make 5808 a perfect square it should be divided by '3'.

So divide 5808 by '3'

$$\frac{5808}{3} = \frac{2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11}{3}$$

$$1936 = (2 \times 2 \times 11)^2 = (44)^2$$

So 44 is a number whose square is 1936

Exercise - 3.2

5

1. i. 2
ii. 13
iii. 27
iv. 88
v. 243

2. i) 1
ii) 4
iii) 1
iv) 9
v) 6
vi) 5
vii) 9
viii) 4
ix) 0
x) 6

3. i. 567

567 has '7' in its unit's place. ~~But~~ a perfect square

Should have 1, 4, 5, 6, 9, 0 in its unit's place.

So 567 is not a perfect square.

- ii. 2453

2453 has '3' in its unit's place. ~~But~~ a perfect square

Should have 0, 1, 4, 5, 6, 9 in its unit's place.

So 2453 is not a perfect square.

iii) 5298

5298 has '8' in its unit's place. But a perfect square should have 0, 1, 4, 5, 6, 9, in its unit's place.

So 5298 is not a perfect square.

iv) 46292

46292 has '2' in its unit's place. But a perfect square should have 0, 1, 4, 5, 6, 9 in its unit's place.

So 46292 is not a perfect square.

v) 74000

74000 has '0' in its unit's place but it has odd no. of zeros and '740' is not a perfect square.

So 74000 is not a perfect square.

4.

i) 573

Square of 573 is an odd number because, If a number has '3' in the unit's place, then its square ends in '9'.

ii) 4096

Square of 4096 is an even number because, If a number has '6' in the unit's place, then its square ends in '6'.

iii) 8267

Square of 8267 is an odd number because, If a number has '7' in the unit's place, then its square ends in '9'.

iv) 37916

Square of 37916 is an even number because, If a number has '6' in the unit's place, then its square ends in '6'.

5.

7

i. 12 and 13

There are $2n$ non-square numbers between the squares of two consecutive numbers n and $n+1$

$$\therefore \text{natural numbers between } 12 \text{ and } (12+1) = 2 \times 12 = 24$$

Hence, there are 24 natural numbers between 12^2 and 13^2

ii. 90 and 91

There are $2n$ non-square numbers between the squares of two consecutive numbers n and $n+1$

$$\therefore \text{Natural numbers between } 90 \text{ and } 91 = 2 \times 90 = 180$$

Hence, there are 180 natural numbers between 90^2 and 91^2

6.

$$i. 1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = 49$$

$$ii. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + \dots + 29 = 15^2 = 225$$

$$\text{Sum of first 'n' odd numbers} = n^2$$

7. $\Rightarrow 64$

$$64 - 1 = 63 \quad ; \quad 63 - 3 = 60 \quad ; \quad 60 - 5 = 55$$

$$55 - 7 = 48 \quad ; \quad 48 - 9 = 39 \quad ; \quad 39 - 11 = 28$$

$$28 - 13 = 15 \quad ; \quad 15 - 15 = 0$$

$$\therefore 64 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2$$

ii) $\Rightarrow 121$

$$121 - 1 = 120 \quad ; \quad 120 - 3 = 117 \quad ; \quad 117 - 5 = 112 \quad ; \quad 112 - 7 = 105 \quad ;$$

$$105 - 9 = 96 \quad ; \quad 96 - 11 = 85 \quad ; \quad 85 - 13 = 72 \quad ; \quad 72 - 15 = 57 \quad ;$$

$$57 - 17 = 40 \quad ; \quad 40 - 19 = 21 \quad ; \quad 21 - 21 = 0$$

$$\therefore 121 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 11^2$$

8.

8

$$i. 19^2 = 361$$

A Perfect Square can be

"We can express the square of any odd number greater than 1 as the sum of two consecutive natural numbers."

$$\text{First number} = \frac{19^2 - 1}{2} = 180$$

$$\text{Second number} = \frac{19^2 + 1}{2} = 181$$

$$ii. 33^2 =$$

$$19^2 = 361 = 180 + 181$$

$$ii. 33^2 = 1089$$

$$\text{First number} = \frac{33^2 - 1}{2} = 544$$

$$\text{Second number} = \frac{33^2 + 1}{2} = 545$$

$$33^2 = 1089 = 544 + 545$$

$$iii. 47^2 = 2209$$

$$\text{First number} = \frac{47^2 - 1}{2} = 1104$$

$$\text{Second number} = \frac{47^2 + 1}{2} = 1105$$

$$47^2 = 2209 = 1104 + 1105$$

9.

$$i. 31^2 = (30+1)^2 = (30+1)(30+1)$$

$$= 30(30+1) + 1(30+1)$$

$$= 900 + 30 + 30 + 1$$

$$31^2 = 961$$

$$\begin{aligned}
 \text{ii)} \quad 42^2 &= (40+2)^2 = (40+2)(40+2) \\
 &= 40(40+2) + 2(40+2) \\
 &= 1600 + 80 + 80 + 4 \\
 42^2 &= 1764
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad 86^2 &= (80+6)^2 = (80+6)(80+6) \\
 &= 80(80+6) + 6(80+6) \\
 &= 6400 + 480 + 480 + 36 \\
 86^2 &= 7396
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} \quad 94^2 &= (90+4)^2 = (90+4)(90+4) \\
 &= 90(90+4) + 4(90+4) \\
 &= 8100 + 360 + 360 + 16 \\
 94^2 &= 8836
 \end{aligned}$$

10.

i) 45

Comparing with a^2 where $a=4$

$$\begin{aligned}
 (a^2)^2 &= a(a+1) \text{ hundreds} + 25 \\
 45^2 &= 4(4+1) \text{ hundreds} + 25 \\
 &= 20 \text{ hundreds} + 25 \\
 45^2 &= 2025
 \end{aligned}$$

ii) 305

Comparing a^2 where $a=30$

$$\begin{aligned}
 (a^2)^2 &= a(a+1) \text{ hundreds} + 25 \\
 (305)^2 &= 30(30+1) \text{ hundreds} + 25 \\
 &= 930 \text{ hundreds} + 25 \\
 (305)^2 &= 93025
 \end{aligned}$$

iii) 525

10

Comparing with a^2 where $a=52$

$$(a^2)^2 = a(a+1) \text{ hundreds} + 25$$

$$(525)^2 = 52(52+1) \text{ hundreds} + 25$$

$$= 2756 \text{ hundreds} + 25$$

$$(525)^2 = 275625$$

ii) $\rightarrow 8$

Given number = 8

Let us assume $m^2 - 1 = 8$

$$m^2 = 9$$

$$\boxed{m=3}$$

Remaining two numbers of Pythagorean triplet are

$$m^2 + 1, 2m$$

$$3^2 + 1, 2 \times 3$$

$$10, 6$$

\therefore The required triplet (6, 8, 10) with one number '8'

ii) 15

Given number = 15

Let us assume $m^2 - 1 = 15$

$$m^2 = 16$$

$$\boxed{m=4}$$

Remaining two numbers of Pythagorean triplet are

$$m^2 + 1, 2m$$

$$16 + 1, 2 \times 4$$

$$17, 8$$

\therefore The required triplet (8, 15, 17) with one number as 15.

iii) 63

Given number 63

Let us assume $m^2 - 1 = 63$

$$m^2 = 64$$

$$m = 8$$

Remaining two numbers of Pythagorean triplet are

$$m^2 + 1, \quad 2m$$

$$8^2 + 1, \quad 2 \times 8$$

$$65, \quad 16$$

\therefore The required triplet (16, 63, 65) with one number '63'

iv) 80

Given number 80

Let us assume $m^2 - 1 = 80 \Rightarrow m^2 = 81$

$$m = 9$$

Remaining two numbers of Pythagorean triplet are

$$m^2 + 1, \quad 2m$$

$$9^2 + 1, \quad 2 \times 9$$

$$82, \quad 18$$

\therefore The required triplet (18, 80, 82) with one number '80'

12.

$$21^2 = 441$$

$$201^2 = 40401$$

$$2001^2 = 4004001$$

$$20001^2 = 400040001$$

$$200001^2 = 40000400001$$

13.

$$9^2 = 81$$

$$99^2 = 9801$$

$$999^2 = 998001$$

$$9999^2 = 99980001$$

$$99999^2 = \underline{9999}8\underline{0000}1$$

$$999999^2 = \underline{99999}8\underline{00000}1$$

14.

$$7^n = 49$$

$$67^n = 4489$$

$$667^n = 444889$$

$$6667^n = 44448889$$

$$66667^n = \underline{4444}4\underline{8888}9$$

$$666667^n = \underline{44444}4\underline{88888}9$$

Exercise 3.3

13

i. 121

Given number = 121

$$121 - 1 = 120; 120 - 3 = 117; 117 - 5 = 112; 112 - 7 = 105$$

$$105 - 9 = 96; 96 - 11 = 85; 85 - 13 = 72; 72 - 15 = 57$$

$$57 - 17 = 40; 40 - 19 = 21; 21 - 21 = 0$$

∴ 121 is a perfect square

We have done '11' Subtractions

Hence, Square root of 121 is 11 $\Rightarrow \sqrt{121} = 11$

ii. 55

Given number = 55

$$55 - 1 = 54; 54 - 3 = 51; 51 - 5 = 46; 46 - 7 = 39$$

$$39 - 9 = 30; 30 - 11 = 19; 19 - 13 = 6; 6 - 15 = -9$$

$$-9 - 17 = -26$$

∴ 55 is not a perfect square.

iii) 36

Given number = 36

$$36 - 1 = 35; 35 - 3 = 32; 32 - 5 = 27; 27 - 7 = 20$$

$$20 - 9 = 11; 11 - 11 = 0$$

∴ 36 is a perfect square

We have done 6 Subtractions

Hence, Square root of 36 i.e. $\sqrt{36} = 6$.

iv) 90

Given number = 90

$$90 - 1 = 89; 89 - 3 = 86; 86 - 5 = 81; 81 - 7 = 74; 74 - 9 = 65$$

$$65 - 11 = 54; 54 - 13 = 41; 41 - 15 = 25; 25 - 17 = 10; 10 - 19 = -7$$

\therefore 90 is not a Perfect Square.

2.

i. 784

Given number = 784

$$\begin{array}{r|l} 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$$

$$\sqrt{784} = \sqrt{2^4 \times 7^2}$$

$$\sqrt{784} = 2 \times 2 \times 7 = 28$$

ii) 441

Given number = 441

$$\begin{array}{r|l} 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$441 = 3 \times 3 \times 7 \times 7$$

$$\sqrt{441} = \sqrt{3^2 \times 7^2}$$

$$\sqrt{441} = 3 \times 7 = 21$$

iii) 1849

Given number = 1849

$$\begin{array}{r|l} 43 & 1849 \\ 43 & 43 \\ \hline & 1 \end{array}$$

$$1849 = 43 \times 43$$

$$\sqrt{1849} = \sqrt{43 \times 43}$$

$$\sqrt{1849} = 43$$

iv) 4356

Given number = 4356

$$\begin{array}{r|l} 2 & 4356 \\ 2 & 2178 \\ 3 & 1089 \\ 3 & 363 \\ 11 & 121 \\ 11 & 11 \\ \hline & 1 \end{array}$$

$$4356 = 2 \times 2 \times 3 \times 3 \times 11 \times 11$$

$$\sqrt{4356} = \sqrt{2^2 \times 3^2 \times 11^2}$$

$$\sqrt{4356} = 2 \times 3 \times 11 = 66$$

v) 6241

Given number = 6241

$$\begin{array}{r|l} 79 & 6241 \\ 79 & 79 \\ \hline & 1 \end{array}$$

$$6241 = 79 \times 79$$

$$\sqrt{6241} = \sqrt{79^2} = 79$$

vi) 8836

Given number = 8836

$$\begin{array}{r}
 2 \overline{) 8836} \\
 \underline{2 \quad 4418} \\
 47 \overline{) 2209} \\
 \underline{47 \quad 47} \\
 1
 \end{array}$$

$$8836 = 2 \times 2 \times 47 \times 47$$

$$\sqrt{8836} = \sqrt{2^2 \times 47^2}$$

$$\sqrt{8836} = 2 \times 47 = 94$$

vii) 8281

Given number = 8281

$$\begin{array}{r}
 7 \overline{) 8281} \\
 \underline{7 \quad 1183} \\
 13 \overline{) 169} \\
 \underline{13 \quad 13} \\
 1
 \end{array}$$

$$8281 = 7 \times 7 \times 13 \times 13$$

$$\sqrt{8281} = \sqrt{7^2 \times 13^2}$$

$$\sqrt{8281} = 7 \times 13 = 91$$

viii) 9025

$$\begin{array}{r}
 5 \overline{) 9025} \\
 \underline{5 \quad 1805} \\
 19 \overline{) 361} \\
 \underline{19 \quad 19} \\
 1
 \end{array}$$

$$9025 = 5 \times 5 \times 19 \times 19$$

$$\sqrt{9025} = \sqrt{5^2 \times 19^2}$$

$$\sqrt{9025} = 95$$

3.

17

$$i. \quad 9 \frac{67}{121} = \frac{1156}{121}$$

$$9 \frac{67}{121} = \frac{2^2 \times 2 \times 17 \times 17}{11 \times 11}$$

$$\sqrt{9 \frac{67}{121}} = \sqrt{\frac{2 \times 2 \times 17 \times 17}{11 \times 11}}$$

$$= \frac{2 \times 17}{11}$$

$$\sqrt{9 \frac{67}{121}} = \frac{34}{11}$$

$$ii) \quad 17 \frac{13}{36} = \frac{625}{36}$$

$$17 \frac{13}{36} = \frac{5 \times 5 \times 5 \times 5}{6 \times 6}$$

$$\sqrt{17 \frac{13}{36}} = \sqrt{\frac{5 \times 5 \times 5 \times 5}{6 \times 6}}$$

$$= \frac{5 \times 5}{6}$$

$$\sqrt{17 \frac{13}{36}} = \frac{25}{6}$$

$$iii) \quad 1.96 = \frac{196}{100}$$

$$1.96 = \frac{2 \times 2 \times 7 \times 7}{10 \times 10}$$

$$\sqrt{1.96} = \sqrt{\frac{2 \times 2 \times 7 \times 7}{10 \times 10}}$$

$$\sqrt{1.96} = \frac{2 \times 7}{10} = 1.4$$

$$\begin{array}{r} 2 \overline{) 1156} \\ \underline{4} \\ 756 \\ \underline{70} \\ 56 \\ \underline{54} \\ 2 \\ \underline{2} \\ 0 \end{array}$$

$$1156 = 2 \times 2 \times 17 \times 17$$

$$\begin{array}{r} 5 \overline{) 625} \\ \underline{5} \\ 125 \\ \underline{100} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

$$625 = 5 \times 5 \times 5 \times 5$$

$$\begin{array}{r} 2 \overline{) 196} \\ \underline{4} \\ 98 \\ \underline{84} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

$$196 = 2 \times 2 \times 7 \times 7$$

$$10) 0.0064$$

$$0.0064 = \frac{64}{10000}$$

$$0.0064 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{10 \times 10 \times 10 \times 10}$$

$$\sqrt{0.0064} = \sqrt{\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{10 \times 10 \times 10 \times 10}}$$

$$= \frac{2 \times 2 \times 2}{10 \times 10} = 0.08$$

$$\sqrt{0.0064} = 0.08$$

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

4)

i. Given number = 588

Expressing in prime factors

$$588 = 2 \times 2 \times 7 \times 7 \times 3$$

Since '3' left Unpaired, so to make 187 588 it should multiplied by '3'

$$588 \times 3 = 2 \times 2 \times 7 \times 7 \times 3 \times 3$$

$$1764 = 2^2 \times 7^2 \times 3^2$$

$$\sqrt{1764} = \sqrt{2^2 \times 7^2 \times 3^2}$$

$$\sqrt{1764} = 2 \times 7 \times 3 = 42$$

$$\begin{array}{r|l} 2 & 588 \\ \hline 2 & 294 \\ \hline 7 & 147 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

ii) Given number = 720

Expressing in prime factors

$$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

Since 5 left unpaired, to make 720 a perfect square, it should be multiplied by 5

$$720 \times 5 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

$$3600 = 2^4 \times 3^2 \times 5^2 \times 2^2$$

$$\sqrt{3600} = \sqrt{2^4 \times 3^2 \times 5^2} \times \sqrt{2^2 \times 2^2 \times 3^2 \times 5^2}$$

$$\sqrt{3600} = 2 \times 2 \times 3 \times 5 = 60$$

$$\begin{array}{r} 2 \overline{)720} \\ \underline{360} \\ 2 \overline{)360} \\ \underline{180} \\ 2 \overline{)180} \\ \underline{90} \\ 3 \overline{)90} \\ \underline{45} \\ 3 \overline{)45} \\ \underline{15} \\ 5 \overline{)15} \\ \underline{5} \\ 1 \end{array}$$

iii) Given number 2178

Expressing in prime factors

$$2178 = 2 \times 3 \times 3 \times 11 \times 11$$

Since 2 left unpaired, to make 2178 a perfect square, it should be multiplied by 2

$$2178 \times 2 = 2 \times 2 \times 3 \times 3 \times 11 \times 11$$

$$4356 = 2^2 \times 3^2 \times 11^2$$

$$\sqrt{4356} = \sqrt{2^2 \times 3^2 \times 11^2}$$

$$= 2 \times 3 \times 11$$

$$\sqrt{4356} = 66$$

$$\begin{array}{r} 2 \overline{)2178} \\ \underline{1089} \\ 3 \overline{)1089} \\ \underline{363} \\ 3 \overline{)363} \\ \underline{121} \\ 11 \overline{)121} \\ \underline{11} \\ 1 \end{array}$$

i) Given number = 3042

Expressing in prime factors

$$3042 = 2 \times \underbrace{3 \times 3}_{3^2} \times \underbrace{13 \times 13}_{13^2}$$

Since '2' left unpaired, so to make 3042 a perfect square, it should be multiplied by '2'

$$3042 \times 2 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

$$6084 = 2^2 \times 3^2 \times 13^2$$

$$\sqrt{6084} = \sqrt{2^2 \times 3^2 \times 13^2}$$

$$\sqrt{6084} = 2 \times 3 \times 13 = 78$$

2	3042
3	1521
3	507
13	169
13	13
	1

ii) 6300

Given number = 6300

Expressing in prime factors

$$6300 = 2 \times 2 \times \underbrace{5 \times 5}_{5^2} \times 7 \times \underbrace{3 \times 3}_{3^2}$$

Since '7' left unpaired, so to make 6300 a perfect square, it should be multiplied by '7'

$$6300 \times 7 = 2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 3 \times 3$$

$$\sqrt{44100} = \sqrt{2^2 \times 5^2 \times 7^2 \times 3^2}$$

$$\sqrt{44100} = 2 \times 5 \times 7 \times 3$$

$$\sqrt{44100} = 210$$

2	6300
2	3150
5	1575
5	315
7	63
3	9
3	3
	1

5.

i. Given number 1872

Expressing in prime factors

1872 = 2x2x2x2x3x3x13

Since 13 left unpaired, so to make 1872 a perfect square, it should be divided by 13

1872 / 13 = 2x2x2x2x3x3x13 / 13

144 = 2x2x2x2x3x3

sqrt(144) = sqrt(2^4 x 3^2)

sqrt(144) = 2x2x3 = 12

Handwritten division table for 1872: 2|1872, 2|936, 2|468, 2|234, 3|117, 3|39, 13|13, 1.

ii) Given number 2592

Expressing in prime factor

2592 = 2x2x2x2x2x3x3x3x3

Since '2' left unpaired, so to make 2592 a perfect square, it should be divided '2'

2592 / 2 = 2x2x2x2x2x3x3x3x3 / 2

1296 = 2x2x2x2x3x3x3x3

sqrt(1296) = sqrt(2^6 x 3^4)

= sqrt(2x2x3x3) x 2x2x3x3

sqrt(1296) = 36

Handwritten division table for 2592: 2|2592, 2|1296, 2|648, 2|324, 2|162, 3|81, 3|27, 3|9, 3|3, 1.

iii) Given number 3380

Expressing in terms of prime factors

$$3380 = 2 \times 2 \times 5 \times 13 \times 13$$

Since '5' is left unpaired, so to make a perfect square, it should be divided by '5'

$$3380 \div 5 = \frac{2 \times 2 \times 5 \times 13 \times 13}{5}$$

$$676 = 2 \times 2 \times 13 \times 13$$

$$\sqrt{676} = \sqrt{2 \times 2 \times 13 \times 13}$$

$$\sqrt{676} = 2 \times 13 = 26$$

2	3380
2	1690
5	845
13	169
13	13
	1

iv) 16244

Expressing in terms of prime factors

16244

2	16244
2	8122
	4061

2	16244
2	8122
11	4061
3	371
3	117
3	39
13	13

1

8.

let breadth of rectangle = x m.
(b)

24

g:ven

length of rectangle is equal to 4 times its breadth

$$\therefore l = 4 \cdot x$$

Area of rectangle = 1936 sq. m

$$l \times b = 1936$$

$$4x \times x = 1936$$

$$x^2 = \frac{1936}{4}$$

$$x^2 = 484$$

$$\boxed{x = 22 \text{ m}}$$

Hence, breadth of rectangle = $x = 22$ m

length of rectangle = $4x = 4 \times 22 = 88$ m

9.

let the no. of columns = x

g:ven no. of rows equal to no. of column.

\therefore no. of column \times rows = x

Total students equal to 2000 but 64 students could not be accommodated in these rows & columns

$$\therefore x \times x = 2000 - 64$$

$$x^2 = 1936$$

$$x = \sqrt{1936}$$

$$x = 44$$

Hence, no. of rows = 44

$$\begin{array}{r|l} 2 & 1936 \\ \hline 2 & 868 \\ \hline 2 & 434 \\ \hline 2 & 242 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$1936 = 2 \times 2 \times 2 \times 2 \times 11 \times 11$$

$$\sqrt{1936} = 2 \times 2 \times 11 = 44$$

v) Given number 61347

Expressing in terms of prime numbers factors

$$61347 = 3 \times 11 \times 11 \times 13 \times 13$$

Since '3' left unpaired, so to make 61347 a perfect square, it should be divided by '3'

$$61347 \div 3 = \frac{3 \times 11 \times 11 \times 13 \times 13}{3}$$

$$20449 = 11 \times 11 \times 13 \times 13$$

$$\sqrt{20449} = \sqrt{11^2 \times 13^2}$$

$$\sqrt{20449} = 11 \times 13 = \underline{\underline{143}}$$

3	61347
11	20449
11	1859
13	169
13	13
	1

6.

Let no. of rows of plants in garden = x

Given each row contains as many plants as the no. of rows.

\therefore no. of plants in each row = x .

Total no. of plants = $x \times x = x^2$

Given Total no. of plants in garden = 4225

$$\therefore x^2 = 4225$$

$$x = \sqrt{4225}$$

$$x = 65$$

\therefore Hence, no. of rows in garden = 65.

no. of plants in a row = 65

10. Let no. of students = x

25

Given Contribution of each student = no. of students

∴ Contribution of each student = x

Total collected for picnic = ₹ 2304

$$\therefore x \times x = 2304 \quad \text{2304}$$

$$x^2 = 2304$$

$$x = \sqrt{2304}$$

$$x = 48$$

11.

Let one number to be ' x '

Given one number is 15 times the other

∴ Second number = $15x$.

Product of two numbers = 7260

$$15x \cdot x = 7260$$

$$x^2 = \frac{7260}{15}$$

$$x^2 = 484$$

$$x = \sqrt{484}$$

$$x = 22$$

$$\begin{array}{r} 2 \overline{) 484} \\ \underline{242} \\ 121 \\ \underline{110} \\ 110 \\ \underline{110} \\ 0 \end{array}$$

$$484 = 2 \times 2 \times 11 \times 11$$

$$\sqrt{484} = 2 \times 11 = 22$$

12. given numbers are in ratio of 2:3:5 25

let numbers be $2x, 3x, 5x$

given sum of squares of numbers = 950

$$(2x)^2 + (3x)^2 + (5x)^2 = 950$$

$$4x^2 + 9x^2 + 25x^2 = 950$$

$$x^2 = \frac{950}{38}$$

$$x^2 = 25$$

$$\boxed{x = 5}$$

Hence,

Numbers are $2x, 3x, 5x$

10, 15, 25

13.

Perimeters of two squares = 60m, 144m

$$P_1 = 60m; P_2 = 144m$$

perim let lengths of sides of squares are x_1, x_2

$$\therefore P_1 = 4x_1$$

$$P_2 = 4x_2$$

$$60 = 4x_1$$

$$144 = 4x_2$$

$$\boxed{x_1 = 15m}$$

$$\boxed{x_2 = 36m}$$

$$\text{Area } A_1 = x_1^2$$

$$\text{Area } A_2 = x_2^2$$

$$A_1 = 15^2$$

$$A_2 = 36^2$$

$$A_1 = 225 \text{ m}^2$$

$$A_2 = 1296 \text{ m}^2$$

let a square of side x with area $A_1 + A_2$

$$A = A_1 + A_2$$

$$x^2 = 225 + 1296$$

$$x^2 = 1521$$

$$\begin{aligned}x^2 &= 1521 \\x &= \sqrt{1521} \\x &= 39\text{m}\end{aligned}$$

Perimeter of square

$$\begin{aligned}P &= 4x \\&= 4 \times 39 \\P &= 156\text{m}\end{aligned}$$

\therefore Hence, Perimeter = 156m

$$\begin{array}{r|l}3 & 1521 \\ \hline 3 & 507 \\ \hline 13 & 169 \\ \hline 13 & 13 \\ \hline & 1\end{array}$$

26

$$1521 = 3 \times 3 \times 13 \times 13$$

$$\sqrt{1521} = 3 \times 13 = 39$$

Exercise 34

27

1.

i) Given number 2401

$$\begin{array}{r|l} & 49 \\ \hline 4 & \overline{2401} \\ & -16 \\ \hline 89 & 801 \\ & -801 \\ \hline & 0 \end{array}$$

$$\therefore \sqrt{2401} = 49$$

ii) 4489

$$\begin{array}{r|l} & 67 \\ \hline 6 & \overline{4489} \\ & 36 \\ \hline 127 & 889 \\ & (-) 889 \\ \hline & 0 \end{array}$$

$$\therefore \sqrt{4489} = 67$$

iii)

$$\begin{array}{r|l} & 327 \\ \hline 3 & \overline{106929} \\ & -9 \\ \hline 62 & 159 \\ & 124 \\ \hline 647 & 4529 \\ & (-) 4529 \\ \hline & 0 \end{array}$$

$$\therefore \sqrt{106929} = 327$$

iv) given number 167281

$$\begin{array}{r|l}
 & 409 \\
 \hline
 4 & \overline{167281} \\
 & -16 \\
 \hline
 80 & 072 \\
 & 00 \\
 \hline
 809 & 7281 \\
 & (-) 7281 \\
 \hline
 & 0
 \end{array}$$

$$\therefore \sqrt{167281} = 409$$

v) given number 53824

$$\begin{array}{r|l}
 & 232 \\
 \hline
 2 & \overline{53824} \\
 & -4 \\
 \hline
 23 & 138 \\
 & (-) 129 \\
 \hline
 462 & 924 \\
 & (-) 924 \\
 \hline
 & 0
 \end{array}$$

$$\therefore \sqrt{53824} = 232$$

vi) given number 213444

$$\begin{array}{r|l}
 & 462 \\
 \hline
 4 & \overline{213444} \\
 & (-) 16 \\
 \hline
 86 & 534 \\
 & (-) 516 \\
 \hline
 922 & 1844 \\
 & (-) 1844 \\
 \hline
 & 0
 \end{array}$$

$$\therefore \sqrt{213444} = 462$$

2.

i) Given number 81 = 2 (even)

The number of digits in its square root = $\frac{2}{2} = 1$

ii) Given number 169 = 3 (odd)

The number of digits in its square root = $\frac{3+1}{2} = 2$

iii) Given number 4761 = 4 (even)

The number of digits in its square root = $\frac{4}{2} = 2$

iv) Given number 27889 = 5 (odd)

 \therefore The number of digits in its square root = $\frac{5+1}{2} = 3$

v) Given number 525625 = 6 (even)

 \therefore The number of digits in its square root = $\frac{6}{2} = 3$

3.

i. Given number 51.84

$$\begin{array}{r}
 7.2 \\
 \hline
 7 \overline{) 51.84} \\
 \underline{49} \\
 142 \\
 \underline{142} \\
 0
 \end{array}$$

$$\therefore \sqrt{51.84} = 7.2$$

ii) 42.25

$$\begin{array}{r}
 65 \\
 \hline
 6 \overline{) 42.25} \\
 \underline{-36} \\
 125 \overline{) 625} \\
 \underline{(-) 625} \\
 0
 \end{array}$$

$$\sqrt{42.25} = 6.5$$

iii) Given number 18.4041

$$\begin{array}{r}
 4.29 \\
 \hline
 4 \overline{) 18.4041} \\
 \underline{-16} \\
 82 \overline{) 240} \\
 \underline{-164} \\
 849 \overline{) 7641} \\
 \underline{-7641} \\
 0
 \end{array}$$

$$\sqrt{18.4041} = 4.29$$

iv) Given number 5.774409

$$\begin{array}{r}
 2.403 \\
 \hline
 2 \overline{) 5.774409} \\
 \underline{-4} \\
 44 \overline{) 177} \\
 \underline{(-) 176} \\
 480 \overline{) 144} \\
 \underline{(-) 000} \\
 4803 \overline{) 14409} \\
 \underline{(-) 14409} \\
 0
 \end{array}$$

$$\therefore \sqrt{5.774409} = 2.403$$

4.

i) 645.8

	25.412
2	645.800000
	4
45	245
	225
504	2080
	2016
5081	6400
	5081
50822	131900
	101644
	30300

$\therefore \sqrt{645.8} = 25.412 \approx 25.41$ (Correct to 2 decimals)

ii)

	10.365
1	107.450000
	00
20	007
	000
203	745
	-609
2066	13600
	12396
20725	120400
	-103625
	16775

$\sqrt{107.45} = 10.365 \approx 10.36$

iii) Given number 5.462

$$\begin{array}{r}
 2.337 \\
 \hline
 2 \quad 5.462000 \\
 \quad -4 \\
 \hline
 43 \quad 146 \\
 \quad \rightarrow 129 \\
 \hline
 463 \quad 1720 \\
 \quad \quad 1389 \\
 \hline
 4667 \quad 33100 \\
 \quad \quad - 32669 \\
 \hline
 \quad \quad \quad 431
 \end{array}$$

$$\therefore \sqrt{5.462} = 2.337 \approx 2.34 \text{ (corrected to '2' decimals)}$$

iv) Given number 2

$$\begin{array}{r}
 1.414 \\
 \hline
 1 \quad 2.000000 \\
 \quad 1 \\
 \hline
 24 \quad 100 \\
 \quad - 96 \\
 \hline
 281 \quad 400 \\
 \quad \quad - 281 \\
 \hline
 2824 \quad 19100 \\
 \quad \quad - 11296 \\
 \hline
 \quad \quad \quad 7804
 \end{array}$$

$$\sqrt{2} = 1.414 \approx 1.41$$

v) Given number 3

$$\begin{array}{r}
 1.732 \\
 \hline
 1 \quad \overline{3.00\ 00\ 00} \\
 \hline
 \quad 1 \\
 \hline
 27 \quad 200 \\
 \quad -189 \\
 \hline
 343 \quad 1100 \\
 \quad 1029 \\
 \hline
 3462 \quad 7100 \\
 \quad -6924 \\
 \hline
 \quad 176
 \end{array}$$

$$\sqrt{3} = 1.732 \approx 1.73 \text{ (Corrected to 2 decimals)}$$

5.

i. $\frac{841}{1521}$

$$= \frac{\sqrt{841}}{\sqrt{1521}} = \frac{29}{39}$$

$$\begin{array}{r}
 29 \\
 \hline
 2 \quad \overline{841} \\
 \quad 4 \\
 \hline
 49 \quad 441 \\
 \quad 441 \\
 \hline
 \quad 0
 \end{array}$$

$$\begin{array}{r}
 39 \\
 \hline
 3 \quad \overline{1521} \\
 \quad 9 \\
 \hline
 69 \quad 621 \\
 \quad -621 \\
 \hline
 \quad 0
 \end{array}$$

ii) $8 \frac{257}{529} = \frac{4489}{529}$

$$\sqrt{8 \frac{257}{529}} = \sqrt{\frac{4489}{529}}$$

$$\sqrt{8 \frac{257}{529}} = \frac{67}{23}$$

$$\begin{array}{r}
 67 \\
 \hline
 6 \quad \overline{4489} \\
 \quad 36 \\
 \hline
 127 \quad 889 \\
 \quad -889 \\
 \hline
 \quad 0
 \end{array}$$

$$\begin{array}{r}
 23 \\
 \hline
 2 \quad \overline{529} \\
 \quad 4 \\
 \hline
 43 \quad 129 \\
 \quad 129 \\
 \hline
 \quad 0
 \end{array}$$

$$\text{ii)} \quad 16 \frac{169}{441} = \frac{7225}{441}$$

85	
8	7225
	-64
165	825
	-825
	0

21	
2	44
	4
41	041
	41
	0

$$\sqrt{16 \frac{169}{441}} = \frac{\sqrt{7225}}{\sqrt{441}} = \frac{85}{21}$$

6.

i) Given number 2000

44	
4	2000
	16
84	400
	336
	64

Hence, The least number that must be subtracted from 2000 so as to make it a perfect square is 64.

$$\therefore \text{Required Perfect Square number} = 2000 - 64 = 1936 = 44^2$$

ii) Given number 984

31	
3	984
	9
61	84
	61
	23

Hence, The least number that must be subtracted from 984 so as to make it a perfect square is 23.

$$\therefore \text{Required Perfect Square number} = 984 - 23 = 961 = 31^2$$

iii) Given number 8934

$$\begin{array}{r|l}
 & 94 \\
 \hline
 9 & \overline{8934} \\
 & 81 \\
 \hline
 184 & 834 \\
 & -736 \\
 \hline
 & 98
 \end{array}$$

Hence, The least number that must be subtracted from 8934 so as to make it a perfect square is 98

$$\therefore \text{The required square number } 8934 - 98 = 8836 = 94^2$$

iv) Given number 11021

$$\begin{array}{r|l}
 & 104 \\
 \hline
 1 & \overline{11021} \\
 & 1 \\
 \hline
 20 & 010 \\
 & 00 \\
 \hline
 204 & 1021 \\
 & 816 \\
 \hline
 & 205
 \end{array}$$

Hence, The least number that must be subtracted from 11021 so as to make it a perfect square is 205

$$\therefore \text{The required square number } 11021 - 205 = 10816 = 104^2$$

7.

i) Given number 1750

$$\begin{array}{r|l}
 41 & \\
 \hline
 4 & \overline{1750} \\
 & 16 \\
 \hline
 81 & \overline{150} \\
 & - 81 \\
 \hline
 & 69
 \end{array}$$

$$1750 > (41)^2 \Rightarrow \text{Remainder} = 69$$

$$(42)^2 = 1764$$

$$\therefore \text{Required number} = 1764 - 1750 = 14$$

Hence, the least number that must be added to 1750

So as to make it a perfect square is 14

ii) Given number 6412

$$\begin{array}{r|l}
 80 & \\
 \hline
 8 & \overline{6412} \\
 & 64 \\
 \hline
 160 & \overline{012} \\
 & \cdot 00 \\
 \hline
 & 12
 \end{array}$$

$$6412 > (80)^2$$

$$81^2 = 6561$$

$$\therefore \text{Required number} = 6561 - 6412 = 149$$

Hence, the least number that must be added to 6412

So as to make it a perfect square is 149

iii) Given number 6598

37

$$\begin{array}{r|l} & 81 \\ \hline 8 & \overline{6598} \\ & -64 \\ \hline 161 & 198 \\ & -161 \\ \hline & 37 \end{array}$$

$$6598 > (81)^2$$

$$(82)^2 = 6724$$

$$\therefore \text{Required number} = (82)^2 - 6598 = 126$$

Hence, The minimum number that must be added to 6598 so as to make it a perfect square is 126

iv) Given number 8000

$$\begin{array}{r|l} & 89 \\ \hline 8 & \overline{8000} \\ & -64 \\ \hline 169 & 1600 \\ & 1521 \\ \hline & 79 \end{array}$$

$$8000 > 89^2$$

$$90^2 = 8100$$

$$\therefore \text{Required number} = 90^2 - 8000 = 100$$

Hence, The minimum number that must be added to 8000 so as to make it a perfect square is 100

8. Smallest four digit number = 1000

$$\begin{array}{r|l}
 & 31 \\
 \hline
 3 & \overline{1000} \\
 & 9 \\
 \hline
 61 & 100 \\
 & 61 \\
 \hline
 & 39
 \end{array}$$

$$1000 > 31^2$$

32^2 will be next perfect square

$$32^2 = 1024$$

\therefore Hence, 1024 is Smallest four digit number which is a Perfect Square

9. Greatest ^{Six} digit number = 999999

$$\begin{array}{r|l}
 & 999 \\
 \hline
 9 & \overline{999999} \\
 & 81 \\
 \hline
 189 & 1899 \\
 & 1701 \\
 \hline
 1989 & 19899 \\
 & 17901 \\
 \hline
 & 1998
 \end{array}$$

To make 999999 a Perfect Square, we have to

Subtract 1998 from 999999

$$\therefore \text{The required number} = 998001$$

Hence, 998001 is greatest Six digit number which is a perfect square

10. $\triangle ABC$, $\angle B = 90^\circ$

i. $AB = 14 \text{ cm}$

$BC = 48 \text{ cm}$

According to Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$= 14^2 + 48^2$$

$$AC^2 = 2500$$

$$AC = \sqrt{2500}$$

$$AC = 50 \text{ cm.}$$

ii)

$AC = 37 \text{ cm}$, $BC = 35 \text{ cm}$, $AB = ?$

According to Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$37^2 = AB^2 + 35^2$$

$$1369 = AB^2 + 1225$$

$$AB^2 = 144$$

$$AB = 12 \text{ cm.}$$

	12
1	<u>144</u>
	1
22	<u>044</u>
	44
	<u>0</u>

11.

Total plants = 1400

let no. of rows = x

no. of rows = no. of columns

no. of columns = x

$$x^2 = 1400$$

so $1400 > (37)^2$

$$38^2 = 1444$$

so To make 1400 a perfect square, we have add minimum of 44

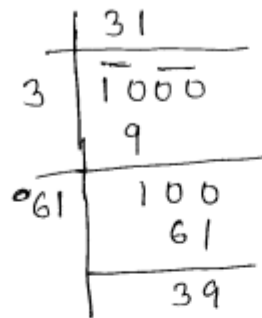
\therefore 44 plants needed more.

	37
3	<u>1400</u>
	9
67	<u>500</u>
	469
	<u>31</u>

12)

Total no. of students = 1000

40

let no. of row = no. of columns = x .Total students rows \times columns = 1000

$$x \times x = 1000$$

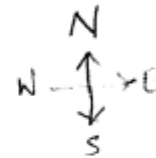
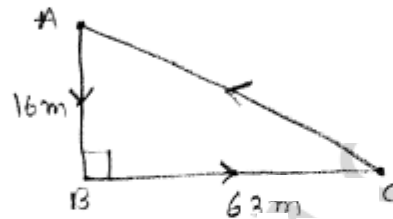
$$x^2 = 1000$$

$$x = \sqrt{1000}$$

So Remainder = 39

Hence, 39 children will be left out.

13)



Distance that he Amit walks while returning

$$= AC$$

A In $\triangle ABC$

According to Pythagoras Theorem;

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 16^2 + 63^2$$

$$AC^2 = 4225$$

$$AC = 65 \text{ m.}$$

Hence, Amit walks 65m while returning to his house

14. Length of ladder = 6m

Height of wall = 4.8m

In $\triangle ABC$

According to Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$6^2 = 4.8^2 + BC^2$$

$$BC^2 = 12.96$$

$$BC = \sqrt{12.96}$$

$$BC = 3.6\text{m}$$

∴ Hence, Distance between wall and Foot of ladder

is 3.6m

