Cubes and Cube Roots

EXERCISE : 4.1

i) 648

Expressing it in to prime factors

$648 = 2\times2\times2\times3\times3\times3\times3$

$= 2^3\times3^3\times3$

Since 3 is left after grouping in triplets

$\therefore$ 648 is not a perfect cube.

2	648
2	324
2	162
3	81
3	27
3	9
	3

ii) 8640

Expressing it in to prime factors

$8640 = 2\times2\times2\times2\times2\times2\times3\times3\times3\times5$

Since 5 is left after grouping in triplets

$\therefore$ 8640 is not a perfect cube .

2	8640
2	4320
2	2160
2	1080
2	540
2	270
5	135
3	27
3	9
	3

iii) $729 = 9\times9\times9 = 9^3$ is a perfect cube

iv) $8000 = 20\times20\times20 = 20^3$ is a perfect cube

2

i) 1728

Expressing it in to prime factors

$1728 = 2\times2\times2\times2\times2\times2\times3\times3\times3$

$= 2^3\times2^3\times3^3 = (2\times2\times3)^3$

$= 12^3$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27

3	9
	3

$\therefore 12^3 = 1728$ is a perfect cube

And 1728 is the cube of number 12.

ii) 5832

Expressing it into prime factors

$5832 = 2\times2\times2\times3\times3\times3\times3\times3\times3$

$= 2^3\times3^3\times3^3 = (2\times3\times3)^3$

$= 18^3$

$\therefore 18^3 = 5832$ is a perfect cube

And 5832 is the cube of number 18.

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

iii) 13824

Expressing it into prime factors

$13824 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times3$

$= 2^3\times2^3\times2^3\times3^3$

$= (2\times2\times2\times3)^3$

$13824 = 24^3$ is a perfect cube

2	13824
2	6912
2	3456
2	1728
2	864
3	432
3	144
3	48
2	16
2	8
2	4
2	2
	1

iv) 35937

3	35937
3	11979
3	3993
3	1331
3	447
	149

$3\times3\times3\times3\times3\times149$

$\therefore$ It is not a perfect cube

3.

i) 243

Expressing it in to prime factors.

3	243
3	81
3	27
3	9
3	3
	1

$243 = 3 \times 3 \times 3 \times 3 \times 3$

If we multiply above number with 3

then it becomes $= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^3 \times 3^3 = 9^3 = 729$, perfect Cube

$\therefore$ therefore the smallest number 3 is to be multiplied to make the number a perfect Cube

ii) 3072

Expressing it in to prime factors

2	3072
2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$3072 = \underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{2 \times 2 \times 2}_{2^3} \times 2 \times 2 \times 3$

If we multiply the above numbers with $2 \times 2 \times 3 \times 3$ i.e 36 it will become

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 2^3 \times 2^3 \times 2^3 \times 2^3 \times 3^3$

$= (2 \times 2 \times 2 \times 2 \times 3)^3$

$= 48^3 = 110592$ i.e 3072×36.

$\therefore$ therefore the smallest number 36 is to be multiplied with 3072 to make the number a perfect Cube.

iii) 11979

Expressing it in to prime factors

3	11979
3	3993
11	1331
11	121
11	11
	1

$11979 = 3 \times 3 \times 11 \times 11 \times 11$

In the above, prime factors 3 occurs twice 11 occurs thrice. Therefore the smallest number by which the given number must be multiplied so that the product is a perfect cube i.e 3

Then product $= 3 \times 3 \times 3 \times 11 \times 11 \times 11$

$= 3^3 \times 11^3 = 33^3 = 35937 = 11979 \times 3$

iv) 19652

Expressing it in to prime factors

2	19652
2	9826
17	4913
17	289
	17

$19652 = 2 \times 2 \times 17 \times 17 \times 17$

2 occurs twice, 17 occurs thrice

Therefore the smallest number by which given number must be multiplied so that product is a perfect cube is 2

Then product $= 2 \times 2 \times 2 \times 17 \times 17 \times 17$

$= 2^3 \times 17^3 = 34^3 = 39{,}304 = 19652 \times 2$

4.

i) 1536 expressing it in to prime factors

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
	6

$2 \mid 6$, 3

$1536 = \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times 3$

Since 3 is left after grouping in triplets

1536 is not a perfect Cube.

To make it perfect Cube, we should divide the given number by 3, then the prime factorisation of the quotient will not contain 3.

In that case

$1536 \div 3 = 2\times2\times2\times2\times2\times2\times2\times2\times2 = 512$, which is a perfect Cube

So the smallest number by which 1536 must be divided so that quotient is a perfect Cube is 3

ii. 10985

5	10985
13	2197
13	169
13	13
	1

Expressing it in to prime factors, we have

$10985 = 5\times\underbrace{13\times13\times13}$

10985 is not a perfect Cube.

To make it perfect Cube, we should divide the given number by 5, then the prime factorisation of the quotient will not contain 5.

In that case

$10985 \div 5 = 13\times13\times13 = 2197$, which is a perfect Cube

So, the smallest number by which 10985 must be divided so that quotient is a perfect Cube is 5.

iii) 28672

Expressing it in to prime factors

$28672 = \underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times 7$

28672 is not a perfect Cube

To make it a perfect Cube, we should divide the given number by 7

Then the prime factorisation will not 7

In that Case

$28672 \div 7 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$

$= 4096$, is a perfect Cube

So the smallest number by which given number must be ~~multiplied~~ divided so that product will be-come perfect Cube is '7'

2	28672
2	14336
2	7168
2	3584
2	1792
2	896
2	448
2	224
2	112
2	56
2	28
2	14
	7

iv. 13718

Expressing it in to prime factor

$13718 = 19\times19\times19\times2$.

It is not a perfect Cube

To make it a perfect Cube, we should divide the given number by 2, then prime factorisation will not Contain '2'

In that Case $13718 \div 2 = 19\times19\times19 = 6859$ is a perfect Cube

So the smallest number 2 must be divided from given number to make it perfect Cube

2	13718
19	6859
19	361
19	19
	1

5.

The Volume occupied by one Cuboid is $3 \times 3 \times 5 = 45$

45 is not a perfect cube

In order to make it a Cube, the number which is to multiplied is $45 \times 3 \times 5 \times 5$ i.e $3 \times 5 \times 5 = 75$ is to be multiplied in order to make a Cube

So total number of Cuboids are needed to form a Cube are 75.

6. Given Surface area of a Cubical box is $486\,Cm^2$

we have, Volume of Cubical box is $(Side)^3$ and

Surface area of Cubical box is $6 \times (Side)^2$

i.e Let Side of a box is "a" Cm.

$$6a^2 = 486 \Rightarrow a^2 = \frac{486}{6}\ 81$$

$$a^2 = 81 = 9 \times 9$$

$$\boxed{a = 9\,Cm}$$

Volume of a Cubical box is $a^3 = 9^3 = 729\,Cm^3$.

7. i) $125 = 5 \times 5 \times 5 = 5^3$, Cube of odd natural number

$$\begin{array}{r|l} 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

ii) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^3 \times 2^3 \times 2^3 = 8^3$, Cube of even natural number

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline & 32 \end{array} \qquad \begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

iii) $1000 = 10 \times 10 \times 10 = 10^3$, Cube of even natural number.

iv) $2197 = 13 \times 13 \times 13 = 13^3$, Cube of odd natural number

v) $4096 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^3 \times 4^3$

$= 16^3$, Cube of even natural number

$$\begin{array}{r|l} 4 & 4096 \\ \hline 4 & 1024 \\ \hline 4 & 256 \\ \hline 4 & 64 \\ \hline 4 & 16 \\ \hline 4 & 4 \\ \hline & 1 \end{array}$$

vi) $6859 = 19 \times 19 \times 19 = 19^3$, Cube of odd natural number

8.

i) 231, unit's digit of Cube of number is 1

ii) 358, one's digit of Cube of number is 2

iii) 419 one's digit of Cube of number is 9

iv) 725. one's digit of Cube of number is 5

v. 854 one's digit of Cube of number is 4

vi. 987 one's digit of Cube is 3

vii. 752 one's digit of Cube is 8

viii) 893 one's digit of Cube is 7.

9.

i) $(-13)^3 = -13 \times -13 \times -13 = (-13)^3 = -2197$

ii) $\left(3\frac{1}{5}\right)^3 = \left(\frac{16}{5}\right)^3 = \frac{16 \times 16 \times 16}{5 \times 5 \times 5} = \frac{4096}{125}$

iii) $\left(-5\frac{1}{7}\right)^3 = \left(-\frac{36}{7}\right)^3 = \frac{-36 \times -36 \times -36}{7 \times 7 \times 7} = -\frac{46656}{343}$

EXERCISE : 4.2

1.

i) 12167

Expressing it in to prime factors.

prime factorisation

$$\begin{array}{r|l} 23 & 12167 \\ \hline 23 & 529 \\ \hline & 23 \end{array}$$

$12167 = 23 \times 23 \times 23$

Hence, Cube root of 12167 is 23

ii) 35937

Expressing it in to prime factors

prime factorisation

$$\begin{array}{r|l} 33 & 35937 \\ \hline 33 & 1089 \\ \hline & 33 \end{array}$$

$35937 = 33 \times 33 \times 33$

Hence, Cube root of 35937 is 33

iii) 42875

Expressing it in to prime factors

$$\begin{array}{r|l} 35 & 42875 \\ \hline 35 & 1225 \\ \hline & 35 \end{array}$$

$42875 = 35 \times 35 \times 35$

Hence, Cube root of 42875 is 35.

iv) 21952

Expressing it in to prime factor.

$$\begin{array}{r|l} 2 & 21952 \\ \hline 2 & 10976 \\ \hline 2 & 5488 \\ \hline 2 & 2744 \\ \hline 2 & 1372 \\ \hline 2 & 686 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$$\begin{array}{r|l} 2 & 21952 \\ \hline 2 & 10976 \\ \hline 2 & 5238 \\ \hline 3 & 2619 \\ \hline 3 & 873 \\ \hline 3 & 291 \\ \hline & 97 \end{array}$$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$

$= (2 \times 2 \times 7)^3 = (28)^3$

Hence, Cube root of 21952 is 28

v) 373248

Expressing it in to prime factors

$373248 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times 3\times3\times3\times3\times3\times3$

$= (2\times2\times2\times3\times3)^3$

$= 72^3$

Hence, cube root of 373248 is 72

2	373248
2	186624
2	93312
2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

vi. 32768

Expressing it in to prime factors

$32768 = 2\times2\times2 \times 2\times2\times2 \times 2\times2\times2 \times 2\times2\times2 \times 2\times2\times2$

$= (2\times2\times2\times2\times2)^3$

$= 32^3$

Hence Cube root of 32768 is 32.

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

vii. 262144

Expressing it in to prime factors.

2	262144
2	131072
2	65536
2	32768
2	16384
2	8192
2	4096
	2048

2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
	8

2	8
2	4
2	2
	1

$262144 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times2\times2\times2$

$= (2\times2\times2\times2\times2\times2)^3$

$= 64^3$

Hence, Cube root of 262144 is 64

viii) 157464

Expressing it into prime factors

$$\begin{array}{r|l} 2 & 157464 \\ \hline 2 & 78732 \\ \hline 2 & 39366 \\ \hline 3 & 19683 \\ \hline 3 & 6561 \\ \hline 3 & 2187 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$157464 = 2\times2\times2\times3\times3\times3\times3\times3\times3\times3\times3\times3$

$= 2^3\times3^3\times3^3\times3^3$

$= (2\times3\times3\times3)^3$

$= 54^3$

Hence, Cube root of 157464 is 54

2.

i. 19683

$\underset{\text{Second group}}{\underline{19}}$ $\underset{\text{First group}}{\underline{683}}$

First group decides the unit digit of required Cube root So, The number 683 ends with 3. We know that 3 Comes at unit's place of a number only when it's Cube root ends in 7

Now Take second group 19, then it will decide the ten's digit of required Cube root.

Now that $2^3 = 8$ and $3^3 = 27$, Also $8 < 19 < 27$.

We take the one's place of smaller number 8 as ten's digit of required cube root (i.e 2)

Therefore $\sqrt[3]{19683} = 27$

ii) 59319

$\underset{\text{Second group}}{\underline{59}}$ $\underset{\text{First group}}{\underline{319}}$

First group decides the one's digit of required cube root

The number 319 ends with 9 we know that 9 comes at unit's place of a number only when its cube root ends in 9

Now second group decides the ten's digit of required cube root

59 lies in b/w $3^3 = 27$ and $4^3 = 64$. We take one's place of smaller number 27 as the ten's digit of required cube root

So $\sqrt[3]{59319} = 27$

iii) 85184

$\underset{\text{Second group}}{\underline{85}}$ $\underset{\text{First group}}{\underline{184}}$

184 ends with 4. we know that 4 comes at unit's place of number only when its cube root ends in 4

Second group decides tens digit

i.e 85 lies in between $4^3 = 64$ and $5^3 = 125$.

we know that one's place of smaller number 64 as ten's digit of required cube root

$$\text{So } \sqrt[3]{85184} = 44$$

iv. 148877

$\underset{\text{Second group}}{\underline{148}}$ $\underset{\text{First group}}{\underline{877}}$

Step: 1. First form groups of three digits starting from rightmost digit (i.e unit's digit) of number.

Step: 2 First group decides unit's digit of required root.

The number 877 ends with 7. We know that 7 comes at unit's place of a number only when it's cube root ends in 3.

So the unit digit of required cube root is 3.

Step: 3 If no group is left then number obtained is the cube root of given number.

But if second group exits (in this case 148) then it will decide the ten's digit of required cube root.

Now take second group i.e 148

We know that $5^3 = 125$ and $6^3 = 216$. Also $125 < 148 < 216$.

We take one's place, of the smaller number 125 as the ten's digit of required cube root (i.e 5)

Step: 4

If no group is left then the digit obtained in Step 2 and Step 3 decides the cube root of given number

i.e $\sqrt[3]{148877} = 53$.

3.

i. -250047

Expressing it in to prime factors

$$\begin{array}{r|l} 3 & 250047 \\ \hline 3 & 83349 \\ \hline 3 & 27783 \\ \hline 3 & 9261 \\ \hline 3 & 3087 \\ \hline 3 & 1029 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$$-250047 = -7 \times -7 \times -7 \times -3 \times -3 \times -3 \times -3 \times -3 \times -3$$

$$= (-7 \times -3 \times -3)^3$$

$$= (-63)^3.$$

Hence, cube root of -250047 is -63.

ii. $\dfrac{-64}{1331}$

Expressing 64 and 1331 in to prime factors.

$64 = 4 \times 4 \times 4 = 4^3$

$1331 = 11 \times 11 \times 11 = 11^3$

$$\frac{-64}{1331} = \frac{(-4)^3}{(11)^3} = \left(\frac{-4}{11}\right)^3 \Rightarrow \sqrt[3]{\frac{-64}{1331}} = \frac{-4}{11}.$$

iii) $4\frac{17}{27} = \frac{125}{27}$.

Expressing 125 and 27 it in to prime factors

$125 = 5 \times 5 \times 5 = 5^3$

$27 = 3 \times 3 \times 3 = 3^3$

Hence $\sqrt[3]{\frac{125}{27}} = \sqrt[3]{\left(\frac{5}{3}\right)^3} = \frac{5}{3}$.

iv) $5\frac{1182}{2197} = \frac{12167}{2197}$

$12167 = 23 \times 23 \times 23 = 23^3$

$2197 = 13 \times 13 \times 13 = 13^3$

$\frac{12167}{2197} = \frac{23^3}{13^3} = \left(\frac{23}{13}\right)^3$

Hence Cube root of $5\frac{1182}{2197}$ is $\frac{23}{13}$.

4.

i) $\sqrt[3]{512 \times 729}$

Expressing it in to prime factors.

$512 = 8 \times 8 \times 8 = 8^3$

$729 = 9 \times 9 \times 9 = 9^3$

$512 \times 729 = 8^3 \times 9^3 = (8 \times 9)^3 = 72^3$

Hence $\sqrt[3]{512 \times 729} = \sqrt[3]{72^3} = 72$

ii. $\sqrt[3]{(-1331)\times(3375)}$

Expressing it in to prime factors

11	1331
11	121
11	11
	1

5	3375
5	675
5	135
3	27
3	9
3	3
	1

$-1331 = (-11)^3$

$3375 = 5\times5\times5\times3\times3\times3 = (5\times3)^3$

$= 15^3$

$-1331\times3375 = (-11\times15)^3$

Hence $\sqrt[3]{(-1331\times3375)} = \sqrt[3]{(-11\times15)^3} = -11\times15$

$= -165$.

5.

i) 0.003375

$\sqrt[3]{0.003375} = \sqrt[3]{\dfrac{3375}{1000000}}$

$= \sqrt[3]{\dfrac{15\times15\times15}{100\times100\times100}}$

$= \dfrac{15}{100} = 0.15$

ii) 19.683

$\sqrt[3]{19.683} = \sqrt[3]{\dfrac{19683}{1000}}$

$= \sqrt[3]{\dfrac{27\times27\times27}{10\times10\times10}}$

$= \dfrac{27}{10} = 2.7$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

$= 3\times3\times3\times3\times3\times3\times3\times3\times3$

$= (3\times3\times3)^3 = 27^3$.

6.
$$\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$$

$27 = 3\times3\times3 \Rightarrow \sqrt[3]{27} = \sqrt[3]{3\times3\times3} = 3.$

$$\sqrt[3]{0.008} = \sqrt[3]{\frac{8}{1000}} = \sqrt[3]{\frac{2\times2\times2}{10\times10\times10}} = \frac{2}{10} = 0.2$$

$$\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \sqrt[3]{\frac{4\times4\times4}{10\times10\times10}} = \frac{4}{10} = 0.4$$

$$\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064} = 3 + 0.2 + 0.4 = 3.6.$$

7. 6561

Expressing it in to prime factors

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

$6561 = \underline{3\times3\times3} \times \underline{3\times3\times3} \times 3\times3$

$= 27 \times 27 \times 9$

9 is left in the above expansion, if we multiply the above numbers with 3 i.e. $9\times3 = 27$

ie it becomes

$$6561 \times 3 = 27\times27\times9\times3 = 27\times27\times27 = 19683 = 27^3$$

So the smallest number 3 must be multiplied to become the number a perfect cube

cube root of $19683 = 27$

8. 8748

Expressing it into prime factors

3	8748
3	2916
3	972
3	324
3	108
3	36
3	12
2	4
	2

$8748 = 3\times3\times3\times3\times3\times3\times3\times2\times2$

$= 27\times27\times3\times2\times2 = \cancel{9\times9\times4\times4\times3}$

If we ~~multiply~~ divide above numbers by $\frac{4}{9}$

then it becomes

$$= \frac{27\times27\times3\times4}{(4/9)} = 27\times27\times27 = 27^3$$

$$= 19683$$

$$\frac{8748\times9}{4} = 19683 = 27^3.$$

9. Given volume of Cubical box $= 21952\,m^3$

we know that Volume of Cube $= (\text{Side})^3$

Let Side length of Cube $= a$

4	21952
4	5488
4	1372
7	343
7	49
	7

$a^3 = 21952$

$= 4\times4\times4\times7\times7\times7$

$a^3 = (4\times7)^3$

$a = 28m$

$\therefore$ length of side of box $= 28\ m$.

10. Let the three number be $3x, 4x, 5x$, then

$$(3x)\times(4x)(5x) = 480.$$

$$60x^3 = 480$$

$$x^3 = \frac{480}{60} = 8$$

$$x^3 = 8 = 2^3$$

$$\therefore \boxed{x = 2} \Rightarrow 3x = 6,\ 4x = 8,\ 5x = 10$$

$\therefore$ The three numbers are 6, 8, 10.

11. Let the two numbers are $4x, 5x$, then

$$(5x)^3 - (4x)^3 = 61$$

$$125x^3 - 64x^3 = 61$$

$$61x^3 = 61$$

$$x^3 = \frac{61}{61} = 1$$

$$x^3 = 1 \Rightarrow \boxed{x = 1}$$

$\therefore$ The numbers are 4, 5

12. Let the cube root of smaller numbers be x

Given $8^3 - x^3 = 387$

$$512 - x^3 = 387$$

$$x^3 = 512 - 387 = 125$$

$$x^3 = 125 = 5\times5\times5 = 5^3$$

$$x^3 = 5^3$$

$$x = 5$$

$\therefore$ Therefore the smaller number is 5

Cube of this number is 125.