

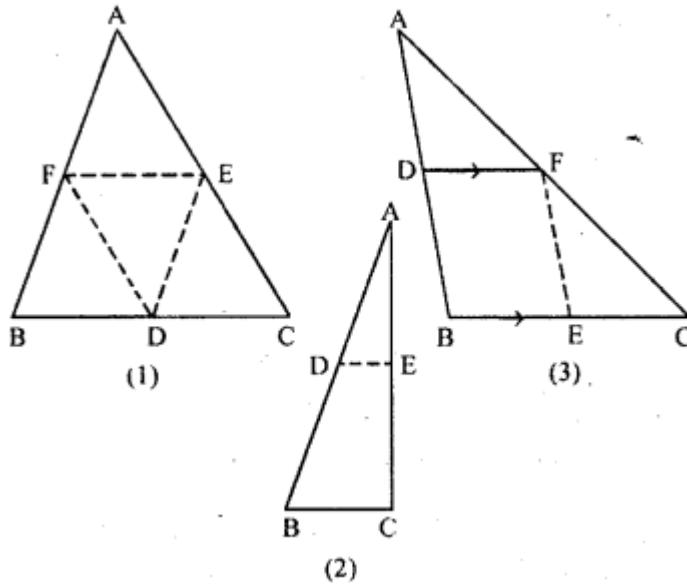
Mid Point Theorem

Question 1.

(a) In the figure (1) given below, D, E and F are mid-points of the sides BC, CA and AB respectively of $\triangle ABC$. If $AB = 6$ cm, $BC = 4.8$ cm and $CA = 5.6$ cm, find the perimeter of (i) the trapezium FBCE (ii) the triangle DEF.

(b) In the figure (2) given below, D and E are mid-points of the sides AB and AC respectively. If $BC = 5.6$ cm and $\angle B = 72^\circ$, compute (i) DE (ii) $\angle ADE$.

(c) In the figure (3) given below, D and E are mid-points of AB, BC respectively and $DF \parallel BC$. Prove that DBEF is a parallelogram. Calculate AC if $AF = 2.6$ cm.



Solution:

(a) (i) **Given :** $AB = 6 \text{ cm}$, $BC = 4.8 \text{ cm}$, and $CA = 5.6 \text{ cm}$

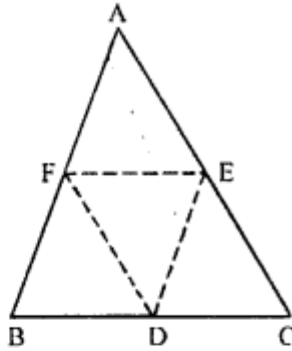
Required : The perimeter of trapezium FBCE.

\therefore F is the mid-point
of AB (given)

$$\begin{aligned}\therefore BF &= \frac{1}{2} AB = \frac{1}{2} \times 6 \text{ cm} \\ &= 3 \text{ cm} \quad \dots(1)\end{aligned}$$

\therefore E is the mid-point of AC
(given)

$$\begin{aligned}\therefore CE &= \frac{1}{2} AC \\ &= \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm} \quad \dots(2)\end{aligned}$$



Now F and E are the mid-point of the AB and CA

$$\therefore FE \parallel BC \text{ and } FE = \frac{1}{2} \times BC$$

$$\Rightarrow FE = \frac{1}{2} \times 4.8 \text{ cm} = 2.4 \text{ cm} \quad \dots(3)$$

\therefore Perimeter of trapezium FBCE

[substituting the value from (1), (2) and (3)]

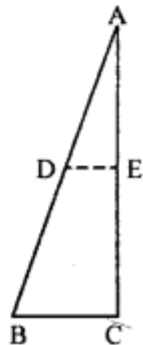
$$= BF + BC + CE + EF$$

$$= 3 \text{ cm} + 4.8 \text{ cm} + 2.8 \text{ cm} + 2.4 \text{ cm} = 13 \text{ cm}$$

Hence, perimeter of trapezium FBCE = 13 cm **Ans.**

(ii) \therefore D, E and F are the mid-points of the sides BC, CA and AB of ΔABC respectively.

$$\therefore EF \parallel BC \text{ and } EF = \frac{1}{2} BC$$



$$\Rightarrow EF = \frac{1}{2} \times 4.8 = 2.4 \text{ cm}$$

Similarly,

$$DE = \frac{1}{2} AB = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$$

$$\text{and } FD = \frac{1}{2} AC = \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm}$$

\therefore Perimeter of $\triangle DEF$

$$= DE + EF + FD$$

$$= 3 \text{ cm} + 2.4 \text{ cm} + 2.8 \text{ cm} = 8.2 \text{ cm Ans.}$$

(b) **Given :** D and E are mid-point of the sides AB and AC respectively. $BC = 5.6 \text{ cm}$ and $\angle B = 72^\circ$

Required : (i) DE (ii) $\angle ADE$

Sol. In $\triangle ABC$

\because D and E is mid-point of the sides AB and AC respectively.

\therefore By mid-point theorem

$$DE \parallel BC \text{ and } DE = \frac{1}{2} BC$$

$$(i) DE = \frac{1}{2} BC = \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm}$$

$$(ii) \angle ADE = \angle B$$

[(corresponding angles)]

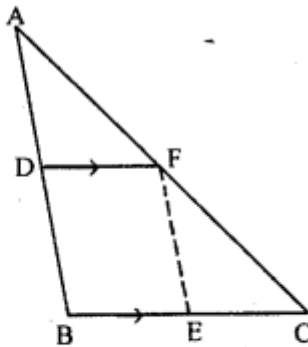
$$\therefore \angle ADE = 72^\circ$$

[$\because BC \parallel DE$]

[$\angle B = 72^\circ$ (given)]

(c) **Given :** D and E are mid-points of AB, BC respectively and $DF \parallel BC$, $AF = 2.6$ c.m.

To prove : (i) BEF is a parallelogram
(ii) To calculate the value of AC



Proof : (i) In $\triangle ABC$

\therefore D is the mid-point of AB and $DF \parallel BC$

\therefore F is the mid-point of AC(1)

Now, F and E are mid-point of AC and BC respectively.

$\therefore EF \parallel AB$ (2)

Now, $DF \parallel BC$

$\Rightarrow DF \parallel BE$ (3)

$\therefore EF \parallel AB$ [From (2)]

$\Rightarrow EF \parallel DB$ (4)

From (3) and (4), DBEF is a parallelogram

(ii) \therefore F is mid-point of AC

$\therefore AC = 2 \times AF = 2 \times 2.6 \text{ cm} = 5.2 \text{ cm}.$

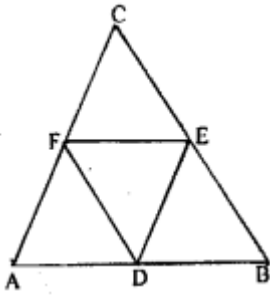
Question 2.

Prove that the four triangles formed by joining in pairs the mid-points of the sides of a triangle are congruent to each other.

Solution:

Given: In $\triangle ABC$, D, E and F,

are mid-points of AB, BC and CA respectively. Join DE, EF and FD.



To prove :

$$\triangle ADF \cong \triangle DBE \cong \triangle ECF \cong \triangle DEF.$$

Proof : In $\triangle ABC$, D and E are mid-point of AB and BC respectively

$$\therefore DE \parallel AC \text{ or } FC$$

Similarly, $DF \parallel EC$

\therefore DECF is a parallelogram.

\therefore Diagonal FE divides the parallelogram DECF in two congruent triangle DEF and CEF.

$$\therefore \triangle DEF \cong \triangle CEF \quad \dots(1)$$

Similarly we can prove that,

$$\triangle DBE \cong \triangle DEF \quad \dots(2)$$

$$\text{and } \triangle DEF \cong \triangle ADF \quad \dots(3)$$

From (1), (2) and (3),

$$\triangle ADF \cong \triangle DBE \cong \triangle ECF \cong \triangle DEF$$

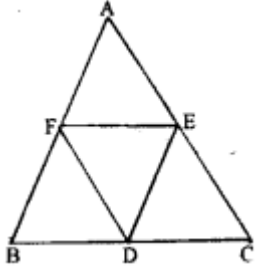
(Q.E.D.)

Question 3.

If D, E and F are mid-points of sides AB, BC and CA respectively of an isosceles triangle ABC, prove that $\triangle DEF$ is also isosceles.

Solution:

Given : ABC is an isosceles triangle in which $AB = AC$



D, E and F are mid point of the sides BC, CA and AB respectively D, E, F are joined

To prove : $\triangle DEF$ is an isosceles triangle.

Proof : D and E are the mid points of BC and AC

$$\therefore DE \parallel AB \text{ and } DE = \frac{1}{2} AB \quad \dots(1)$$

Again, D and F are the mid-points of BC and AB respectively.

$$\therefore DF \parallel AC \text{ and } DF = \frac{1}{2} AC \quad \dots(2)$$

$$\because AB = BC \quad \text{(given)}$$

$$\therefore DE = DF$$

$\therefore \triangle DEF$ is an isosceles triangle

(Q.E.D.)

Question 4.

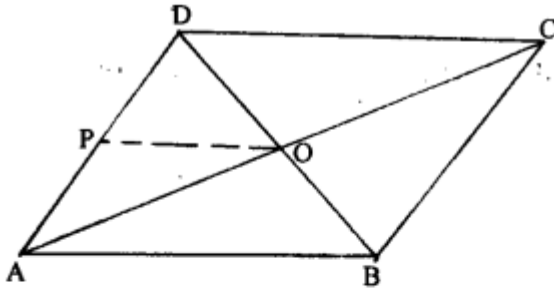
The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the mid-point of AD, prove that

(i) $PQ \parallel AB$

(ii) $PO = \frac{1}{2} CD$.

Solution:

(i) Given : ABCD is a parallelogram in which diagonals AC and BD intersect each other. At point O, P is the mid-point of AD. Join OP.



To Prove : (i) $PQ \parallel AB$ (ii) $PQ = \frac{1}{2} CD$.

Proof : We know that in parallelogram diagonals bisect each other.

$$\therefore BO = OD$$

i.e. O is the mid-point of BD

Now, in $\triangle ABD$,

P and O is the mid-point of AD and BD respectively

$$\therefore PO \parallel AB \text{ and } PO = \frac{1}{2} AB \quad \dots(1)$$

i.e. $PO \parallel AB$ [Proved (i) part]

(ii) Now $\because ABCD$ is a parallelogram

$$\therefore AB = CD \quad \dots(2)$$

From (1) and (2),

$$PO = \frac{1}{2} CD \quad \text{(Q.E.D.)}$$

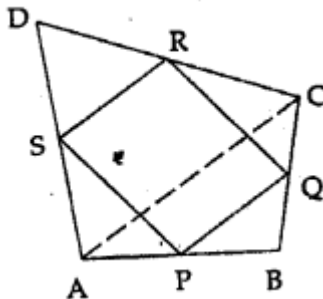
Question 5.

In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA respectively. AC is its diagonal. Show that

(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.



Solution:

Given : In quadrilateral ABCD

P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively AC is the diagonal.

To prove : (i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram

Proof : (i) In $\triangle ADC$

S and R are the mid-points of AD and DC

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \dots(i)$$

(Mid-points theorem)

(ii) Similarly in $\triangle ABC$,

P and Q are mid-points of AB and BC

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \dots(ii)$$

From (i) and (ii),

$$PQ = SR \text{ and } PQ \parallel SR$$

(iii) $\therefore PQ = SR$ and $PQ \parallel SR$

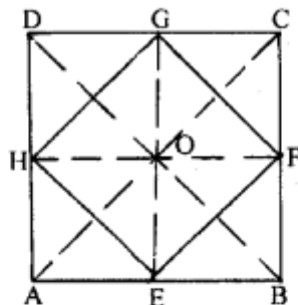
\therefore PQRS is a parallelogram

Question 6.

Show that the quadrilateral formed by joining the mid-points of the adjacent sides of a square, is also a square,

Solution:

Given : A square ABCD in which E, F, G and H are mid-points of AB, BC, CD and DA respectively join EF, FG, GH and HE.



To Prove : EFGH is a square

Construction : Join AC and BD.

Proof : In $\triangle ACD$, G and H are mid-points of CD and AC respectively.

$$\therefore GH \parallel AC \text{ and } GH = \frac{1}{2} AC \quad \dots(1)$$

Now, in $\triangle ABC$, E and F are mid-points of AB and BC respectively.

$$\therefore EF \parallel AC \text{ and } EF = \frac{1}{2} AC \quad \dots(2)$$

From (1) and (2),

$$EF \parallel GH \text{ and } EF = GH = \frac{1}{2} AC \quad \dots(3)$$

Similarly, we can prove that

$$EF \parallel GH \text{ and } EH = GF = \frac{1}{2} BD$$

But $AC = BD$ (\because Diagonals of square are equal)

Dividing both sides by 2,

$$\frac{1}{2} AC = \frac{1}{2} BD \quad \dots(4)$$

From (3) and (4),

$$EF = GH = EH = GF \quad \dots(5)$$

\therefore EFGH is a parallelogram

Now, in $\triangle GOH$ and $\triangle GOF$

$OH = OF$

(Diagonals of parallelogram bisect each other)

$OG = OG$ (Common)
 $GH = GF$ [From (5)]

$\therefore \triangle GOH \cong \triangle GOF$
 [By S.S.S. axiom of congruency]

$\therefore \angle GOH = \angle GOF$ (c.p.c.t.)

Now $\angle GOH + \angle GOF = 180^\circ$ (Linear pair)

or $\angle GOH + \angle GOH = 180^\circ$

or $2\angle GOH$

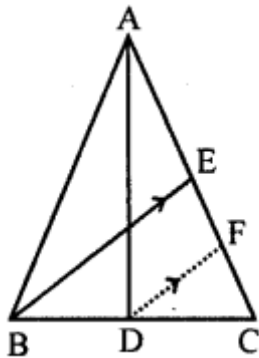
$\therefore \angle GOH = \frac{180^\circ}{2} = 90^\circ$

\therefore Diagonals of parallelogram ABCD bisect and perpendicular to each other.

\therefore EFGH is a square (Q.E.D.)

Question 7.

In the adjoining figure, AD and BE are medians of $\triangle ABC$. If $DF \perp BE$, prove that $CF = \frac{1}{4} AC$.



Solution:

Given : In the given figure,
 AD and BE are the medians of $\triangle ABC$
 DF \parallel BE is drawn

To prove : $CF = \frac{1}{4} AC$

Proof:

In $\triangle BCE$

\therefore D is the mid-point of BC and $DF \parallel BE$

\therefore F is the mid-point of EC

$$\Rightarrow CF = \frac{1}{2} EC \quad \dots(i)$$

\because E is the mid-point of AC

$$\therefore EC = \frac{1}{2} AC \quad \dots(ii)$$

From (i) and (ii),

$$CF = \frac{1}{2} EC = \frac{1}{2} \left(\frac{1}{2} AC \right)$$

$$= \frac{1}{4} AC$$

Question 8.

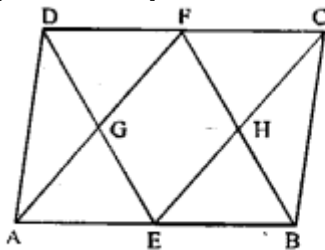
(a) In the figure (1) given below, ABCD is a parallelogram. E and F are mid-points of the sides AB and CO respectively. The straight lines AF and BF meet the straight lines ED and EC in points G and H respectively. Prove that

(i) $\triangle HEB = \triangle HCF$

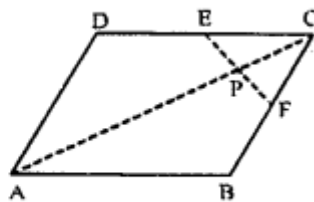
(ii) GEHF is a parallelogram.

(b) In the diagram (2) given below, ABCD is a parallelogram. E is mid-point of CD and P is a point on AC such that $PC = \frac{1}{4} AC$. EP produced meets BC at F. Prove that

(i) F is mid-point of BC (ii) $2EF = BD$



(1)



(2)

Solution:

Given : ABCD is a parallelogram. E and F are mid-point of the side AB and CD respectively.

To prove :

(i) $\triangle HEB \cong \triangle HCF$

(ii) GEHF is a parallelogram.

Proof : (i) ABCD is a parallelogram.

$FC \parallel BE$

$\therefore \angle CEB = \angle FCE$ (Alternate angles)

$\Rightarrow \angle HEB = \angle FCH$ (1)

Also $\angle EBF = \angle CFB$ (Alternate angles)

$\Rightarrow \angle EBH = \angle CFM$ (2)

E and F are mid-points of AB and CD respectively.

$\therefore BE = \frac{1}{2} AB$ (3)

and $CF = \frac{1}{2} CD$ (4)

But ABCD is a parallelogram

$\therefore AB = CD$

$$\frac{1}{2} AB = \frac{1}{2} CD \quad (\text{Dividing both sides by } \frac{1}{2})$$

$$BE = CF \quad [\text{From (3) and (4)}] \quad \dots(5)$$

Now, in $\triangle HEB$ and $\triangle HCF$

$$\angle HEB = \angle FCH \quad [\text{From (1)}]$$

$$\angle EBH = \angle CFH \quad [\text{From (2)}]$$

$$BE = CF \quad [\text{From (5)}]$$

$$\therefore \triangle HEB \cong \triangle HCF$$

(By A.S.A axiom of congruency)

[(i) part is proved]

(ii) Since E and F are mid-points of AB and CD

$$\therefore AE = CF \quad [\because AB = CD]$$

Now $AE \parallel CF$ (given)

$$\therefore AE = CF \text{ and } AE \parallel CF$$

$$\therefore AECF \text{ is a } \parallel \text{ gm.}$$

Now, G and H points on the AF and CE respectively.

$$\therefore GF \parallel EH \quad \dots(6)$$

Similarly we can prove that GFHE is a \parallel gm.

Now point G and H on the line DE and BF respectively.

$$\therefore GE \parallel HF \quad \dots(7)$$

From (6) and (7)

GEHF is a parallelogram. (Q.E.D.)

(b) **Given** : ABCD is a parallelogram in which E is the mid-point of DC and P is a point on AC such

that $PC = \frac{1}{4} AC$. EP produced meets BC at F.

To Prove : (i) F is the mid-point of BC.

$$(ii) 2 EF = BD$$

Construction : Join BD to intersect AC at O.

Proof : Diagonals of \parallel gm bisect each other.

$$\therefore AO = CO$$

But $CP = \frac{1}{4} AC$ (given)

$$\therefore CP = \frac{1}{4} (2 CO) \Rightarrow CP = \frac{1}{2} CO$$

i.e. P is mid-point of CO.

\therefore In $\triangle COD$, E and P are mid-points of DC & CO.

$\therefore EP \parallel DO$

i.e. $EF \parallel DO$

Further, in $\triangle CBD$, E is mid-point of DC and $EF \parallel BD$

\therefore F is the mid-point of BC and $EF = \frac{1}{2} BD$

i.e. $2EF = BD$. (Q.E.D.)

Question 9.

ABC is an isosceles triangle with $AB = AC$. D, E and F are mid-points of the sides BC, AB and AC respectively. Prove that the line segment AD is perpendicular to EF and is bisected by it.

Solution:

Given : ABC is an isosceles triangle with $AB = AC$. D, E and F are mid-points of the sides BC, AB and AC respectively.

To prove : $AD \perp EF$ and AD bisect the EF.

Proof : In $\triangle ABD$ and $\triangle ACD$

$\angle ABD = \angle ACD$ (ABC is an isosceles triangle)

$BD = DC$ (given D is mid-point of BC)

$AB = AC$ (given)

$\therefore \triangle ABD \cong \triangle ACD$

(By S.A.S. axiom of congruency)

$\therefore \angle ADB = \angle ADC$ (c.p.c.t.)

Also, $\angle ADB + \angle ADC = 180^\circ$ (Linear pair)

$\Rightarrow \angle ADB + \angle ADB = 180^\circ$ (By above)

$\Rightarrow 2 \angle ADB = \frac{180^\circ}{2} \Rightarrow \angle ADB = 90^\circ$

$\therefore AD \perp BC$... (1)

Now D and E are mid-points of BC and AB (given)

$\therefore DE \parallel AF$... (2)

Again D and F are mid-point of BC and AC

$\therefore DF \parallel AB$... (3)

From (2) and (3)

AEDF is a || gm.

\therefore Diagonals of a parallelogram bisect each other

\therefore AD and EF bisect each other

From (1) and (3)

$AD \perp EF$ (EF || BC) (Q.E.D.)

Question 10.

(a) In the quadrilateral (1) given below, $AB \parallel DC$, E and F are mid-points of AD and BD respectively. Prove that:

(i) G is mid-point of BC (ii) $EG = \frac{1}{2} (AB + DC)$.

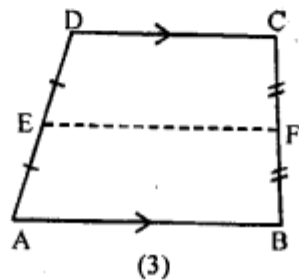
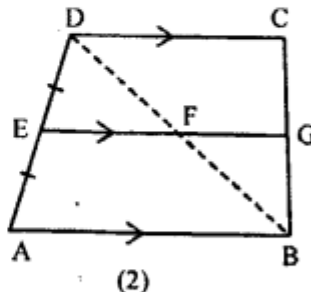
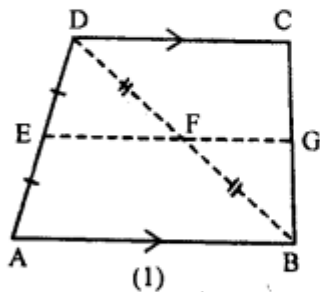
(b) In the quadrilateral (2) given below, $AB \parallel DC \parallel EG$. If E is mid-point of AD prove that :

(i) G is mid-point of BC (ii) $2EG = AB + CD$.

(c) In the quadrilateral (3) given below, $AB \parallel DC$. E and F are mid-point of non-parallel sides AD and BC respectively. Calculate :

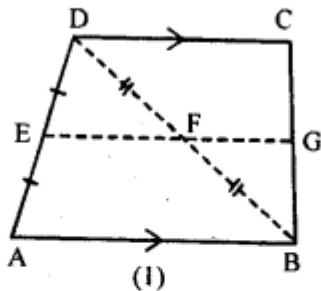
(i) EF if $AB = 6$ cm and $DC = 4$ cm

(ii) AB if $DC = 8$ cm and $EF = 9$ cm.



Solution:

(a) Given : $AB \parallel DC$, E and F are mid-points of AD and BD respectively



To Prove :

(i) G is mid-point of BC

(ii) $EG = \frac{1}{2} (AB + DC)$.

Proof :

In $\triangle ABD$, $DF = BF$ (\because F is mid-point of BD)

Also E is the mid-point of AD (given)

$$\therefore EF \parallel AB \text{ and } EF = \frac{1}{2} AB \quad \dots(1)$$

$$\Rightarrow EG \parallel CD \quad [AB \parallel CD \text{ (given)}]$$

Now F is mid-point of BD and $FG \parallel DC$

\therefore G is mid-point of BC.

$$\Rightarrow FG = \frac{1}{2} DC \quad \dots(2)$$

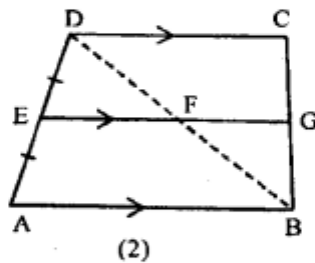
Adding (1) and (2), we get

$$EF + FG = \frac{1}{2} AB + \frac{1}{2} DC \Rightarrow EG = \frac{1}{2} (AB + DC)$$

Hence, the result.

(b) **Given :** Quadrilateral ABCD in which $AB \parallel DC \parallel EG$. E is mid-point of AD.

To prove : (i) G is mid-point of BC.



$$(ii) 2 EG = AB + CD$$

Proof : $\because AB \parallel DC$ (given)

and $EG \parallel AB$ (given)

$$\Rightarrow EG \parallel DC$$

In $\triangle DAB$,

E is mid-point of AD and $EG \parallel AB$ (given)

$$\therefore F \text{ is the mid-point of } BD \text{ and } EF = \frac{1}{2} AB \quad \dots(1)$$

In $\triangle BCD$,

F is mid-point of BD and $FG \parallel DC$

$$\Rightarrow FG = \frac{1}{2} CD \quad \dots(2)$$

Adding (1) and (2)

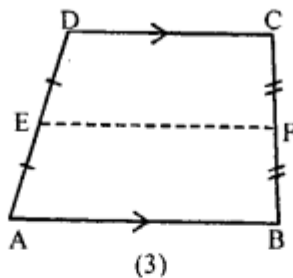
$$EF + FG = \frac{1}{2} AB + \frac{1}{2} CD$$

$$EG = \frac{1}{2} (AB + CD) \quad (\text{Q.E.D.})$$

(c) **Given :** A quadrilateral in which $AB \parallel DC$, E and F are mid-points of non parallel sides AD and BC respectively.

Required : (i) EF if $AB = 6$ cm and $DC = 4$ cm

(ii) AB if $DC = 8$ cm and $EF = 9$ cm



Now, The length of line segment joining the mid-points of two non-parallel sides is half the sum of the lengths of the parallel sides.

\therefore E and F are mid-points of AD and BC respectively.

$$\therefore EF = \frac{1}{2} (AB + DC) \quad \dots(1)$$

(i) $AB = 6$ cm and $DC = 4$ cm

Putting these in (1), we get

$$EF = \frac{1}{2} (6 + 4) = \frac{1}{2} \times 10 = 5 \text{ cm} \quad \text{Ans.}$$

(ii) $DC = 8$ cm and $EF = 9$ cm

Putting these in (1), we get

$$EF = \frac{1}{2} (AB + DC) \Rightarrow 9 = \frac{1}{2} (AB + 8)$$

$$\Rightarrow 18 = AB + 8 \Rightarrow 18 - 8 = AB$$

$\therefore AB = 10$ cm Ans.

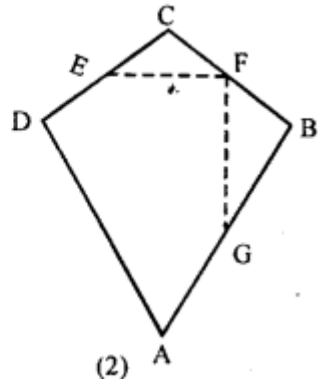
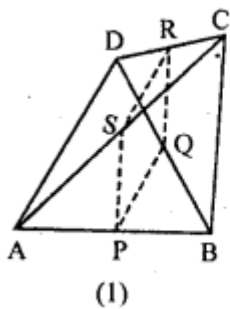
Question 11.

(a) In the quadrilateral (1) given below, $AD = BC$, P , Q , R and S are mid-points of AB , BD , CD and AC respectively. Prove that $PQRS$ is a rhombus.

(b) In the figure (2) given below, $ABCD$ is a kite in which $BC = CD$, $AB = AD$, E , F , G are mid-points of CD , BC and AB respectively. Prove that:

(i) $\angle EFG = 90^\circ$

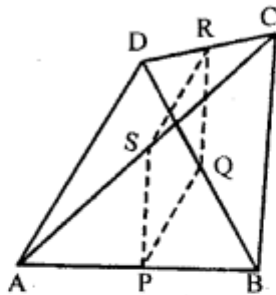
(ii) The line drawn through G and parallel to FE bisects DA .



Solution:

$AD = BC$. P, Q, R and S are mid-points of AB, BD, CD and AC respectively.

To Prove : PQRS is a rhombus.



Proof : In $\triangle ABD$, P and Q are mid-points of AB and BD respectively

(given)

$\therefore PQ \parallel AD$ and PQ

$$= \frac{1}{2} AB \quad \dots(1)$$

Again in $\triangle BCD$, R and Q are mid-points of DC and BD respectively (given)

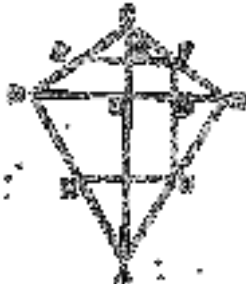
$$\therefore RQ \parallel BC \text{ and } RQ = \frac{1}{2} BC \quad \dots(2)$$

Also P and S are mid-points of AB and AC respectively (given)

$$PS \parallel BC \text{ and } PS = \frac{1}{2} BC \quad \dots(3)$$

$\therefore AD = BC$. (given)

Ex. 10.10 (i) $\angle EFG = 90^\circ$
 (ii) The line drawn through G and parallel to FE bisects DA.



To Prove : (i) $\angle EFG = 90^\circ$

(ii) The line drawn through G and parallel to FE bisects DA.

Construction : Join AC and BD and Draw GH through G parallel to FE.

Proof : (i) Diagonals of a kite intersect at right angle

$$\therefore \angle MON = 90^\circ \quad \dots(1)$$

In $\triangle BCD$

E and F are mid-points of CD and BC respectively

$$\therefore EF \parallel DB \text{ and } EF = \frac{1}{2} DB \quad \dots(2)$$

$$\therefore EF \parallel DB \Rightarrow MF \parallel ON$$

$$\therefore \angle MON + \angle MFN = 180^\circ$$

$$\Rightarrow 90^\circ + \angle MFN = 180^\circ$$

$$\Rightarrow \angle MFN = 180^\circ - 90^\circ \Rightarrow \angle MFN = 90^\circ$$

$$\Rightarrow \angle EFG = 90^\circ \quad \text{(Proved)}$$

(ii) In $\triangle ABD$

G is mid-point of AB and $HG \parallel DB$

[From (2), $EF \parallel DB$ and $EF \parallel HG$ (given)]

$$\Rightarrow HG \parallel DB$$

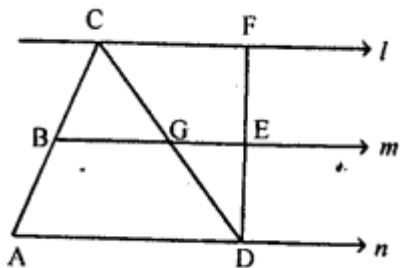
\therefore H is mid-point of DA

Hence, the line drawn through G and parallel to FE bisects DA. (Q.E.D.)

Question 12.

In the adjoining figure, the lines l , m and n are parallel to each other, and G is mid-point of CD . Calculate:

- (i) BG if $AD = 6$ cm
- (ii) CF if $GE = 2.3$ cm
- (iii) AB if $BC = 2.4$ cm
- (iv) ED if $FD = 4.4$ cm.



Solution:

Given : The straight line l , m and n are parallel to each other. G is the mid-point of CD .

To Calculate : (i) BG if $AD = 6$ cm



(1) $DE = \frac{1}{2} AC$
 (2) $DE = \frac{1}{4} AC$
 (3) $DE = \frac{1}{3} AC$
 (4) $DE = \frac{1}{2} AC$

Sol: In $\triangle ABC$,
 D is the mid-point of AB
 E is the mid-point of BC
 $\therefore DE \parallel AC$ (Mid-point theorem)

2. In $\triangle ABC$, D and E are mid-points of AB and AC respectively. If $BC = 6$ cm, then $DE =$

Sol: In $\triangle ABC$,
 D and E are mid-points of AB and AC respectively.

$\therefore DE \parallel BC$ (Mid-point theorem)

$\therefore DE = \frac{1}{2} BC$

$\Rightarrow DE = \frac{1}{2} \times 6 = 3$ cm

Sol: In $\triangle ABC$,
 D and E are mid-points of AB and AC respectively.

$\therefore DE \parallel BC$ (Mid-point theorem)

$\therefore DE = \frac{1}{2} BC$

$\Rightarrow DE = \frac{1}{2} \times 6 = 3$ cm

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 6):

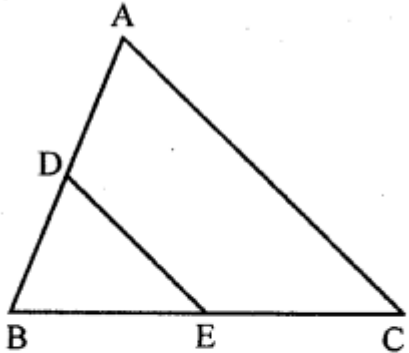
Question 1.

In a $\triangle ABC$, $AB = 3$ cm, $BC = 4$ cm and $CA = 5$ cm. If D and E are mid-points of AB and BC respectively, then the length of DE is

- (a) 1.5 cm
- (b) 2 cm
- (c) 2.5 cm
- (d) 3.5 cm

Solution:

In $\triangle ABC$, D and E are the mid-points of sides AB and BC



$$\therefore DE = \frac{1}{2} AC$$

But $AC = 5$ cm

$$\therefore DE = \frac{1}{2} \times 5 = \frac{5}{2} \text{ cm} = 2.5 \text{ cm} \quad (c)$$

Question 2.

In the given figure, ABCD is a rectangle in which $AB = 6$ cm and $AD = 8$ cm. If P and Q are mid-points of the sides BC and CD respectively, then the length of PQ is

- (a) 7 cm
- (b) 5 cm
- (c) 4 cm
- (d) 3 cm

Solution:

In the given figure,

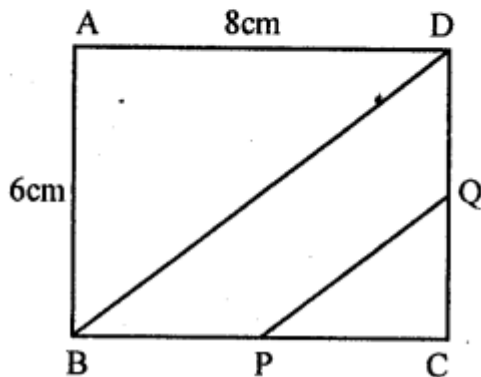
ABCD is a rectangle AB = 6 cm, AD = 8 cm

P and Q are mid-points of BC and CD

$$\therefore PQ = \frac{1}{2} BD$$

$$\text{But } BD = \sqrt{BC^2 + CD^2}$$

(Pythagoras Theorem)



$$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36} \text{ cm}$$

$$= \sqrt{100} = 10 \text{ cm}$$

$$\therefore PQ = \frac{1}{2} BD = \frac{1}{2} \times 10 = 5 \text{ cm} \quad (\text{b})$$

Question 3.

D and E are mid-points of the sides AB and AC of $\triangle ABC$ and O is any point on the side BC. O is joined to A. If P and Q are mid-points of OB and OC respectively, then DEQP is

- (a) a square
- (b) a rectangle
- (c) a rhombus
- (d) a parallelogram

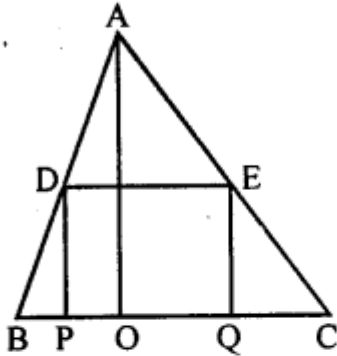
Solution:

D and E are mid-points of sides AB and AC respectively of $\triangle ABC$ O is any point on BC

and AO is joined P and Q are mid-points of OB and OC, EQ and DP are joined

D and E are mid-points of sides AB and AC respectively of ΔABC

O is any point on BC and AO is joined P and Q are mid-points of OB and OC, EQ and DP are joined



\therefore D and E are the mid-points of AB and AC

$$\therefore DE = \frac{1}{2} BC \text{ and } DE \parallel BC \quad \dots(i)$$

\therefore P and Q are mid-points of BO and OC

$$\therefore PQ = PO + OQ$$

$$= \frac{1}{2} BO + \frac{1}{2} OC = \frac{1}{2} (BO + OC)$$

$$= \frac{1}{2} BC \quad \dots(ii)$$

$$= DE$$

From EQPD is a \parallel gm (d)

Question 4.

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle if

(a) PQRS is a parallelogram

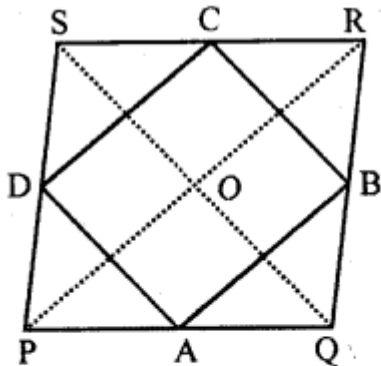
(b) PQRS is a rectangle

(c) the diagonals of PQRS are perpendicular to each other

(d) the diagonals of PQRS are equal.

Solution:

A, B, C and D are the mid-points of the sides PQ, QR, RS and SP respectively



The quadrilateral so formed by joining the mid-points A, B, C, D is ABCD
 ABCD will be rectangle, if the diagonals of PQRS bisect each other
 i.e., PR and QS bisect each other (c)

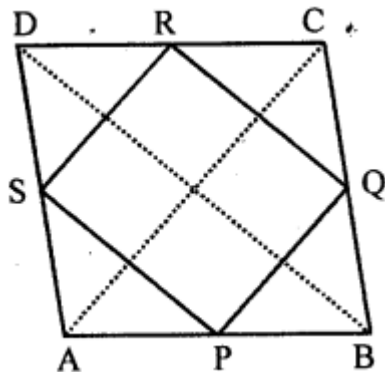
Question 5.

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a rhombus if

- (a) ABCD is a parallelogram
- (b) ABCD is a rhombus
- (c) the diagonals of ABCD are equal
- (d) the diagonals of ABCD are perpendicular to each other.

Solution:

P, Q, R and S are the mid-points of the quadrilateral ABCD and a quadrilateral is formed by joining the mid-points in order



PQRS will be a rhombus if the diagonals of ABCD are equal
 i.e., AC = BD (c)

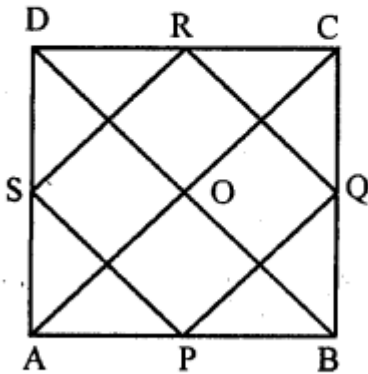
Question 6.

The figure formed by joining the mid points of the sides of a quadrilateral ABCD, taken in order, is a square only if

- (a) ABCD is a rhombus
- (b) diagonals of ABCD are equal
- (c) diagonals of ABCD are perpendicular to each other
- (d) diagonals of ABCD are equal and perpendicular to each other.

Solution:

P, Q, R and S are the mid-points of the quadrilateral ABCD and a quadrilateral is formed by joining them in order. The quadrilateral so formed will be a square if the diagonals of ABCD are equal and perpendicular to each other.



i.e., AC and BD are equal and bisect it perpendicular. (d)

Chapter Test

Question 1.

ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD respectively. Prove that $PQ \perp QR$.

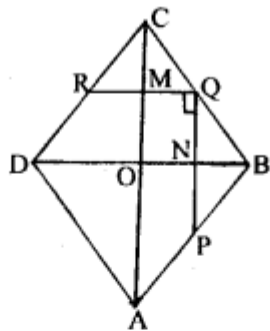
Solution:

Given : ABCD is a rhombus with P, Q and R as mid-points of AB, BC and CD respectively.

To Prove : $PQ \perp QR$

Construction : Join AC & BD.

Proof : Diagonals of rhombus intersect at right angle.



$$\therefore \angle MON = 90^\circ \quad \dots(1)$$

In $\triangle BCD$

Q and R are mid-points of BC and CD respectively.

$$\therefore RQ \parallel DB \text{ and } RQ = \frac{1}{2} DB \quad \dots(2)$$

$$\therefore RQ \parallel DB \Rightarrow MQ \parallel ON$$

$$\therefore \angle MQN + \angle MON = 180^\circ$$

$$\Rightarrow \angle MQN + 90^\circ = 180^\circ \Rightarrow \angle MQN = 180^\circ - 90^\circ$$

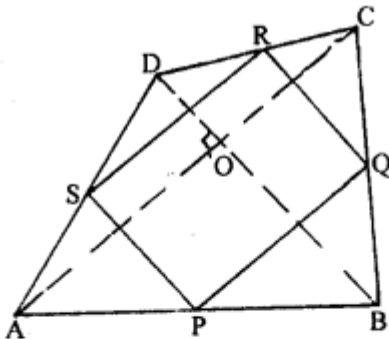
$$\Rightarrow \angle MQN = 90^\circ \Rightarrow NQ \perp MQ$$

or $PQ \perp QR$ (Q.E.D.)

Question 2.

The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral formed by joining the mid-points of its adjacent sides is a rectangle.

Solution:



Ans. Given : ABCD is a quadrilateral in which diagonals AC and BD are perpendicular to each other. P, Q, R and S are mid-points of AB, BC, CD and DA respectively.

To prove : PQRS is a rectangle.

Proof : P and Q are mid-points of AB and BC (given)

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(1)$$

Again S and R are mid-points of AD and DC (given)

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(2)$$

From (1) and (2)

$PQ \parallel SR$ and $PQ = SR$

\therefore PQRS is a parallelogram

Further AC and BD intersect at right angles

$\therefore SP \parallel BD$ and $BD \perp AC$.

$\therefore SP \perp AC$

i.e. $SP \perp SR$

i.e. $\angle RSP = 90^\circ$

$\therefore \angle RSP = \angle SRQ = \angle RQS = \angle SPQ = 90^\circ$

\therefore PQRS is a rectangle (Q.E.D.)

Question 3.

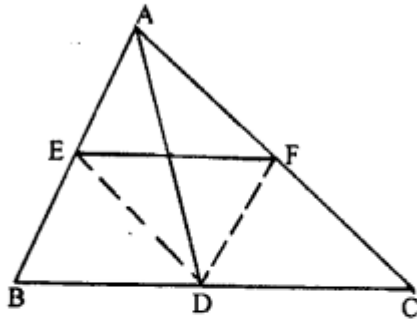
If D, E, F are mid-points of the sides BC, CA and AB respectively of a $\triangle ABC$, Prove that AD and FE bisect each other.

Solution:

Given : D, E, F are mid-points of the sides BC, CA and AB respectively of a $\triangle ABC$

To Prove : AD and FE bisect each other.

Const : Join ED and FD



Proof : D and E are mid-points of BC and AB respectively (given).

$$\therefore DE \parallel AC \Rightarrow DE \parallel AF \quad \dots(1)$$

Again D and F are mid-points of BC and AC respectively (given)

$$\therefore DF \parallel AB \Rightarrow DF \parallel AE \quad \dots(2)$$

From (1) and (2)

ADEF is a ||gm

\therefore Diagonals of a ||gm bisect each other

\therefore AD and EF bisect each other.

Hence, the result.

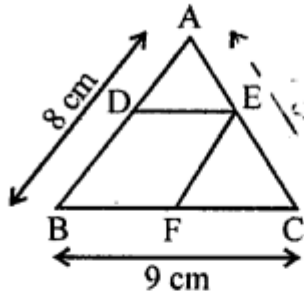
(Q.E.D.)

Question 4.

In $\triangle ABC$, D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If $AB = 8$ cm and $BC = 9$ cm, find the perimeter of the parallelogram BDEF.

Solution:

In $\triangle ABC$, D and E are the mid-points of sides AB and AC respectively. DE is joined and from E, $EF \parallel AB$ is drawn $AB = 8$ cm and $BC = 9$ cm.



To prove.

- (i) BDEF is a parallelogram.
- (ii) Find the perimeter of BDEF

Proof: In $\triangle ABC$,

\therefore D and E are the mid-points of AB and AC respectively

$$\therefore DE \parallel BC \text{ and } DE = \frac{1}{2} BC$$

$\therefore EF \parallel AB$

\therefore DEFB is a parallelogram.

$\therefore DE = BF$

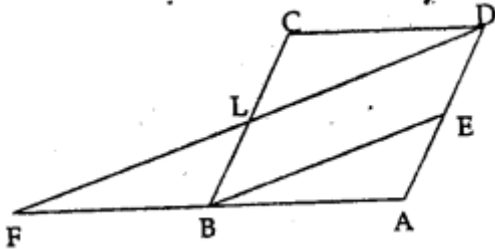
$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 9 = 4.5 \text{ cm}$$

$$EF = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4 \text{ cm.}$$

$$\begin{aligned} \therefore \text{Perimeter of BDEF} &= 2(DE + EF) \\ &= 2(4.5 + 4) \\ &= 8.5 \times 2 = 17 \text{ cm. Ans.} \end{aligned}$$

Question 5.

In the given figure, ABCD is a parallelogram and E is mid-point of AD. DL EB meets AB produced at F. Prove that B is mid-point of AF and EB = LF.



Solution:

Given In the figure

ABCD is a parallelogram

E is mid-point of AD

DL \parallel EB meets AB produced at F

To prove : EB = LF

B is mid-point of AF

Proof : \because BC \parallel AD and BE \parallel LD

\therefore BEDL is a parallelogram

\therefore BE = LD and BL = AE

\because E is mid-point of AD

\therefore L is mid-point of BC

In Δ FAD,

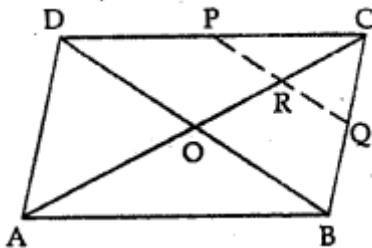
E is mid-point of AD and BE \parallel LD at FLD

\therefore B is mid point of AF

\therefore EB = $\frac{1}{2}$ FD = LF

Question 6.

In the given figure, ABCD is a parallelogram. If P and Q are mid-points of sides CD and BC respectively. Show that CR = $\frac{1}{2}$ AC.



Solution:

Given : In the figure, ABCD is a parallelogram P and Q are the mid-points of sides CD and BC respectively.

To prove : $CR = \frac{1}{4} AC$

Construction : Join AC and BD.

Proof : In ||gm ABCD, diagonals AC and BD bisect each other at O

$$AO = OC \text{ or } OC = \frac{1}{2} AC \quad \dots(i)$$

In $\triangle BCD$,

P and Q are mid points of CD and BC

$\therefore PQ \parallel BD$

\therefore In $\triangle BCO$,

Q is mid-point of BC and $PQ \parallel OB$

\therefore R is mid-point of CO

$$\therefore CR = \frac{1}{2} OC = \frac{1}{2} \left(\frac{1}{2} BC \right)$$

$$\therefore CR = \frac{1}{4} BC$$