Pythagoras Theorem

Question 1.
Lengths of sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse:
(i) 3 cm, 8 cm, 6 cm
(ii) 13 cm, .12 cm, 5 cm
(iii) 1.4 cm, 4.8 cm, 5 cm
Solution:

We use Pythagoras Theorem's converse:

(*i*) Sides of a triangle are 3 cm, 8 cm, 6 cm

$3^2 + 6^2 = 9 + 36 = 45$

and $8^2 = 64$

$\because 45 \neq 64$

$\therefore$ It is not a right triangle.

(*ii*) Sides are 13 cm, 12 cm and 5 cm

$12^2 + 5^2 = 144 + 25 = 169$

and $13^2 = 169$

$\because 12^2 + 5^2 = 13^2$

$\therefore$ It is a right angled triangle.

(*iii*) 1.4 cm, 4.8 cm, 5 cm

and $(1.4)^2 + (4.8)^2 = 1.96 + 23.04 = 25$

and $(5)^2 = 25$

$\because (1.4)^2 + (4.8)^2 = 5^2$

$\therefore$ It is a right angled triangle

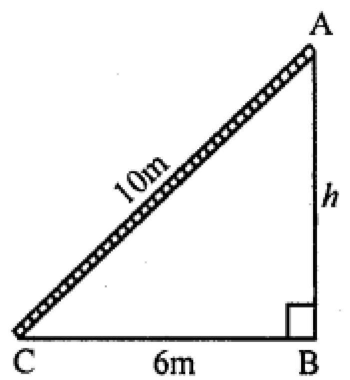
Question 2.
Foot of a 10 m long ladder leaning against a vertical well is 6 m away from the base of the wail. Find the height of the point on the wall where the top of the ladder reaches.
Solution:

Let AB be wall and AC be the ladder

Ladder AC = 10 m

BC = 6 m

Let height of wall AB = h



By Pythagoras Theorem,

$AC^2 = BC^2 + AB^2 \Rightarrow 10^2 = 6^2 + h^2$

$\Rightarrow 100 = 36 + h^2 \Rightarrow h^2 = 100 - 36 = 64 = (8)^2$

$\therefore h = 8$

$\therefore$ Height of wall = 8 cm

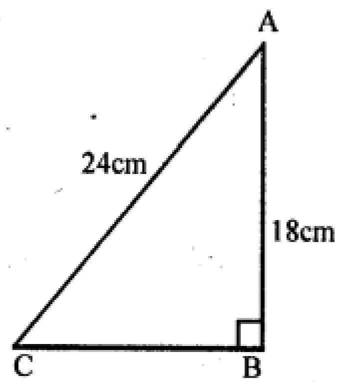
Question 3.

A guy attached a wire 24 m long to a vertical pole of height 18 m and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taught?

Solution:

Let AB be the pole and AC be the wire attached

AB = 18 m and AC = 24 m



In right ΔABC,

$$AC^2 = BC^2 + AB^2 \quad \text{(Pythagoras Theorem)}$$

$$24 = BC^2 + 18^2 \Rightarrow BC^2 = 24^2 - 18^2$$

$$\Rightarrow BC = \sqrt{576 - 324} = \sqrt{252}$$

$$= \sqrt{4 \times 9 \times 7} = 2 \times 3\sqrt{7} = 6\sqrt{7} \text{ m}$$

Question 4.

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

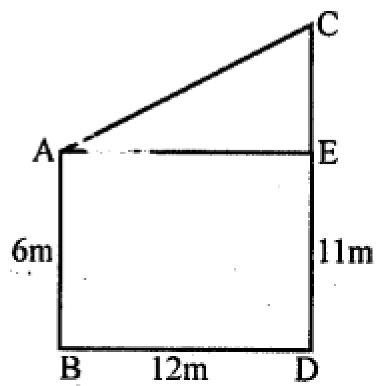
Solution:

Two poles AB and CD are 12 m apart

AB = 6 m, CD = 11 m

From A, draw AE || BD

Then AE = BD = 12 m



CE = CD – ED = CD – AB

= 11 – 6 = 5 m

Now in right ΔACE

$AC^2 = AE^2 + CE^2$ (Pythagoras Theorem)

$= 12^2 + 5^2 = 144 + 25 = 169 = (13)^2$

∴ AC = 13 m

∴ Distance between their tops = 13 m

Question 5.

In a right-angled triangle, if hypotenuse is 20 cm and the ratio of the other two sides is 4:3, find the sides.

Solution:

In the right angled triangle hypotenuse = 20 cm
ratio of other two sides = 4 : 3
Let First side = $4x$
then Second side = $3x$
By Pythagoras theorem,
$(\text{Hypotenuse})^2 = (\text{First side})^2 + (\text{Second side})^2$

$\therefore (20)^2 = (4x)^2 + (3x)^2$

$\Rightarrow (20)^2 = 16x^2 + 9x^2 \Rightarrow 400 = 25x^2$

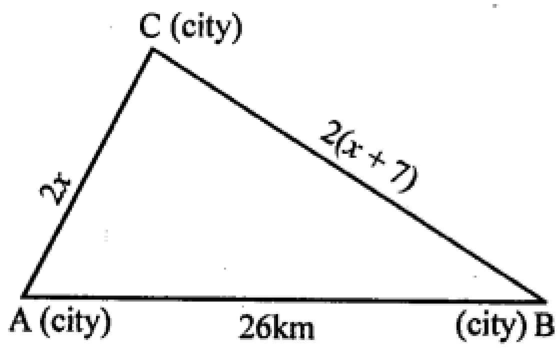
$\Rightarrow x^2 = \frac{400}{25} \Rightarrow x^2 = 16 \Rightarrow x = \sqrt{16} = 4$

$\therefore$ First side = $4x = 4 \times 4$ cm = 16 cm
Second side = $3x = 3 \times 4$ cm = 12 cm
Hence, other two sides of right angled triangle = 16 cm and 12 cm.

Question 6.
If the sides of a triangle are in the ratio 3:4:5, prove that it is right-angled triangle.
Solution:
Let three sides of given triangle ABC is AB, BC and CA = 3 : 4 : 5
Let AB = $3x$, BC = $4x$ and CA = $5x$
Here $(AB)^2 + (BC)^2 = (3x)^2 + (4x)^2$
$= 9x^2 + 16x^2 = 25x^2$
Also, $(CA)^2 = (5x)^2 = 25x^2$
i.e. $(AB)^2 + (BC)^2 = (CA)^2$
Hence, ABC is right angled triangle.

Question 7.
For going to a city B from city A, there is route via city C such that AC ⊥ CB, AC = 2x km and CB=2(x+ 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of highway.
Solution:

In right ΔABC, $\angle C = 90°$
$(2x)^2 + [2(x + 7)]^2 = 26^2$



$\Rightarrow 4x^2 + 4(x^2 + 14x + 49) = 676$
$\Rightarrow 4x^2 + 4x^2 + 56x + 196 - 676 = 0$
$\Rightarrow 8x^2 + 56x - 480 = 0$
$\Rightarrow x^2 + 7x - 60 = 0$ (Dividing by 8)
$\Rightarrow x^2 + 12x - 5x - 60 = 0$
$\Rightarrow x(x + 12) - 5(x + 12) = 0$
$\Rightarrow (x + 12)(x - 5) = 0$
Either $x + 12 = 0$, then $x = -12$ which is not possible being negative
or $x - 5 = 0$, then $x = 5$
Now distance between AC = $2x$
$= 2 \times 5 = 10$ km
and between BC = $2(x + 7) = 2(5 + 7)$
$= 2 \times 12 = 24$
$\therefore$ Distance from A to C and B to C = 10 + 24 = 34 km
$\therefore$ Distance saved = 34 – 26 = 8 km

Question 8.
The hypotenuse of right triangle is 6m more than twice the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Solution:

Let the shortest side of right angled triangle
$= x$ m
Hypotenuse $= (2x + 6)$ m.
Third side $= [(2x + 6) - 2]$ m
By Pythagoras theorem,
$(2x + 6)^2 = x^2 + [(2x + 6) - 2]^2$

$\Rightarrow \quad 4x^2 + 36 + 24x = x^2 + (2x + 4)^2$

$\Rightarrow \quad 4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$

$\Rightarrow \quad 36 + 24x = x^2 + 16 + 16x$

$\Rightarrow \quad 0 = x^2 + 16 + 16x - 36 - 24x$

$\Rightarrow \quad 0 = x^2 - 8x - 20 \quad \Rightarrow \quad x^2 - 8x - 20 = 0$

$\Rightarrow \quad x - 10x + 2x - 20 = 0$

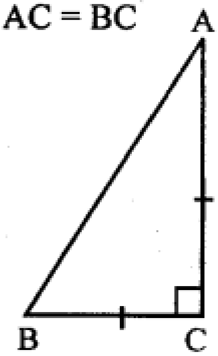
$\Rightarrow \quad x(x - 10) + 2(x - 10) = 0$

$\Rightarrow \quad (x + 2)(x - 10) = 0$

Either $x + 2 = 0$ or $x - 10 = 0$
$x = -2$ (Which is not possible)
or $x = 10$
Hence, shortest $= x = 10$ m
Hypotenuse $= (2x + 6)$ m $= (2 \times 10 + 6) = 26$ m
Third side $= (2x + 6) - $ m $= 26$m $- 2$m $= 24$ m

Question 9.
ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.
Solution:

ΔABC is an isosceles right triangle, right angle at C, AC = BC



To prove : $AB^2 = 2AC^2$

Proof : In right ΔABC

$\angle C = 90°$

$AB^2 = AC^2 + BC^2$ (Pythagoras Theorem)

$= AC^2 + AC^2$ ($\because BC = AC$)

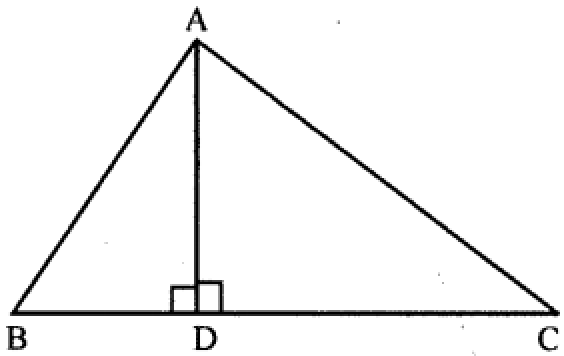
$= 2AC^2$

Question 10.

In a triangle ABC, AD is perpendicular to BC. Prove that $AB^2 + CD^2 = AC^2 + BD^2$.

Solution:

In ΔABC, $AD \perp BC$



To prove : $AB^2 + CD^2 = AC^2 + BD^2$

Proof : In ΔABC, $AD \perp BC$

$\therefore$ ΔABD and ΔACD are right triangles

In right ΔADB,

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem)

$\Rightarrow AD^2 = AB^2 - BD^2$ ...(*i*)

Similarly in right ΔADC

$AC^2 = AD^2 + CD^2$

$\Rightarrow AD^2 = AC^2 - CD^2$ ...(*ii*)

From (*i*) and (*ii*),

$AB^2 - BD^2 = AC^2 - CD^2$

$\Rightarrow AB^2 + CD^2 = AC^2 + BD^2$

Question 11.

In $\triangle PQR$, $PD \perp QR$, such that D lies on QR. If PQ = a, PR = b, QD = c and DR = d, prove that (a + b) (a – b) = (c + d) (c – d).

Solution:

In ΔPQR, $PD \perp QR$

$PQ = a$, $PR = b$, $QD = c$, $DR = d$

To prove : $(a + b)(a - b) = (c + d)(c - d)$

Proof : In ΔPQR, $PD \perp QR$

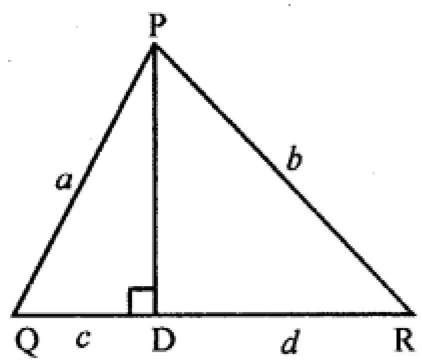
Now in right ΔPQD

$PQ^2 = PD^2 + QD^2$ (Pythagoras Theorem)

$\Rightarrow PD^2 = PQ^2 - QD^2 = a^2 - c^2$ ...(*i*)

Similarly in right ΔPDR

$PR^2 = PD^2 + DR^2$



$\Rightarrow PD^2 = PR^2 - DR^2$

$b^2 - d^2$ ...(*ii*)

From (*i*) and (*ii*),

$a^2 - c^2 = b^2 - d^2$

$\Rightarrow a^2 - b^2 = c^2 - d^2$

$\Rightarrow (a + b)(a - b) = (c + d)(c - d)$

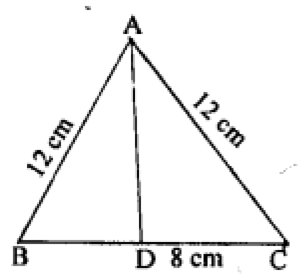
Question 12.

ABC is an isosceles triangle with AB = AC = 12 cm and BC = 8 cm. Find the altitude on BC and Hence, calculate its area.

Solution:

To find, Altitude on BC i.e. value of AD

In isosceles triangle perpendicular from vertex bisects the base



$\therefore$ BD = DC

$\therefore$ BD = $\frac{1}{2} \times 8$ cm = 4 cm.

In right angled triangle ABD

By Pythagoras theorem

$AD^2 + BD^2 = AB^2 \Rightarrow AD^2 + (4)^2 = (12)^2$

$AD^2 + 16 = 144 \Rightarrow AD^2 = 128$

$AD = \sqrt{128} = \sqrt{64 \times 2} = 8\sqrt{2}$

$\therefore$ Altitude on BC = $8\sqrt{2}$. Ans.

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ Altitude

$= \frac{1}{2} \times 8 \times 8\sqrt{2}$ cm^2 $= 4 \times 8\sqrt{2}$ cm^2

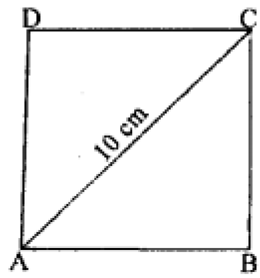
$= 32\sqrt{2}$ cm^2.

Question 13.

Find the area and the perimeter of a square whose diagonal is 10 cm long.

Solution:

Let ABCD be a square whose diagonal AC = 10 cm



Let length of sides of squared = x cm

In ΔABC

By Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (10)^2 = x^2 + x^2 \quad \Rightarrow \quad 2x^2 = 100$

$\Rightarrow \quad x^2 = \frac{100}{2}$

$\Rightarrow \quad x^2 = 50 \quad \Rightarrow \quad x = \sqrt{50}$

$\Rightarrow \quad x = \sqrt{25 \times 2} \quad \Rightarrow \quad x = 5\sqrt{2}$ cm

Area of square = side × side

$= 5\sqrt{2} \times 5\sqrt{2}$ cm² $= 25 \times 2$ cm²

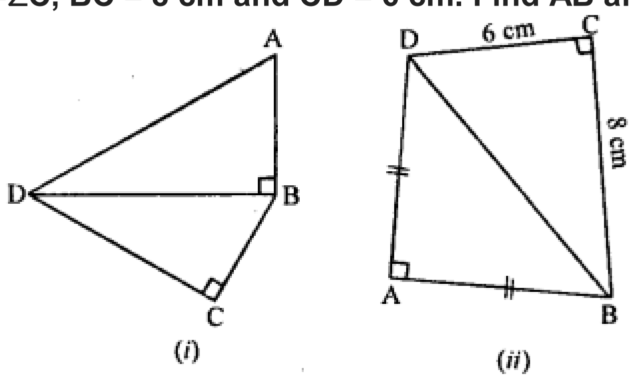
Perimeter of square = 4 × side

$= 4 \times 5\sqrt{2}$ cm $= 20\sqrt{2}$ cm Ans.

Question 14.

(a) In fig. (i) given below, ABCD is a quadrilateral in which AD = 13 cm, DC = 12 cm, BC = 3 cm, ∠ ABD = ∠BCD = 90°. Calculate the length of AB.

(b) In fig. (ii) given below, ABCD is a quadrilateral in which AB = AD, ∠A = 90° =∠C, BC = 8 cm and CD = 6 cm. Find AB and calculate the area of Δ ABD.



Solution:

(a) Given. ABCD is a quadrilateral in which $AD = 13$ cm, $DC = 12$ cm, $BC = 3$ cm and $\angle ABD = \angle BCD = 90^\circ$

To calculate : the length of AB

Sol. In right angled triangle BCD

By Pythagoras theorem,

$BD^2 = BC^2 + DC^2$

$\Rightarrow \quad BD^2 = (3)^2 + (12)^2$

$\Rightarrow \quad BD^2 = 9 + 144$

$\Rightarrow \quad BD^2 = 153$ (i)

Now, in right angled ΔABD,

By Pythagoras theorem,

$AD^2 = AB^2 + BD^2 = AB^2 = AD^2 - BD^2$

$= (13)^2 - 153 \quad (\because BD^2 = 153)$

$= 169 - 153 = 16 \Rightarrow AB = \sqrt{16} = 4$

Hence, length of AB = 4 cm.

(b) In right angled triangle BCD,

By Pythagoras theorem,

$BD^2 = BC^2 + CD^2 = (8)^2 + (6)^2 = 64 + 36 = 100$

$\Rightarrow \quad BD = \sqrt{100} = 10$

$\therefore \quad BD = 10$ cm.

In right angled triangle ABD,

$BD^2 = AB^2 + AD^2$

$\Rightarrow \quad BD^2 = AB^2 + AB^2 \quad (\because AB = AD \text{ (given)})$

$\Rightarrow \quad (10)^2 = 2AB^2$

$\Rightarrow \quad 2AB^2 = 100$

$\Rightarrow \quad AB^2 = \frac{100}{2} = 50$

$\Rightarrow \quad AB = \sqrt{50}$

$= \sqrt{25 \times 2} = 5\sqrt{2}$

$\therefore \quad AB = 5\sqrt{2}$ cm

Area of $\Delta ABD = \frac{1}{2} \times AB \times AD$

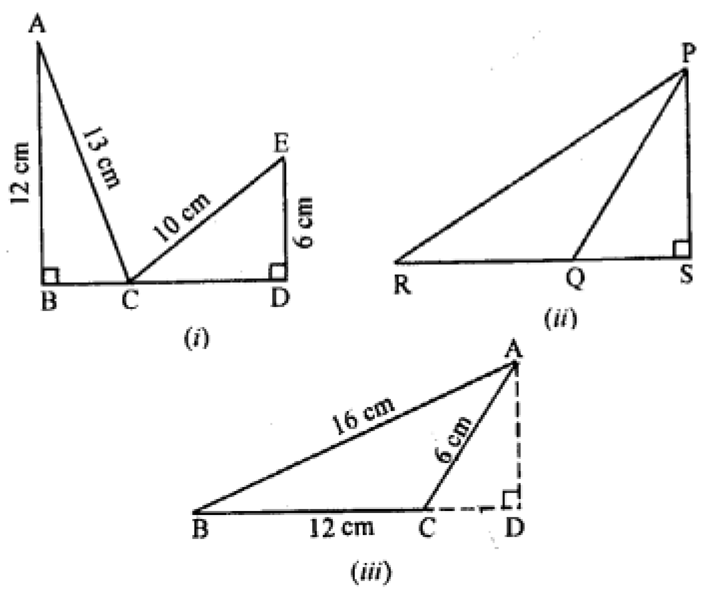
$= \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}\ cm^2$ $\quad (\because\ AB = AD)$

$= \frac{25 \times 2}{2}\ cm^2 = 25\ cm^2$

Question 15.

(a) In figure (i) given below, AB = 12 cm, AC = 13 cm, CE = 10 cm and DE = 6 cm.Calculate the length of BD.
(b) In figure (ii) given below, ∠PSR = 90°, PQ = 10 cm, QS = 6 cm and RQ = 9 cm. Calculate the length of PR.
(c) In figure (iii) given below, ∠ D = 90°, AB = 16 cm, BC = 12 cm and CA = 6 cm. Find CD.

(i)



Solution:

(*a*) Here AB = 12 cm, AC = 13 cm, CE = 10 cm and DE = 6 cm.
To calculate the length of BD.

Sol. In right angled ΔABC
By Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$\Rightarrow \quad (13)^2 = (12)^2 + BC^2$
$\Rightarrow \quad BC^2 = (13)^2 - (12)^2$
$\Rightarrow \quad BC^2 = 169 - 144$
$\Rightarrow \quad BC^2 = 25$
$\Rightarrow \quad BC = \sqrt{25} = 5$
$\therefore \quad BC = 5$ cm ...... (1)

In right angled ΔCED
By Pythagoras theorem,
$CE^2 = CD^2 + DE^2$
$\Rightarrow \quad (10)^2 = CD^2 + (6)^2 \quad \Rightarrow \quad CD^2 = 100 - 36$
$\Rightarrow \quad CD^2 = 64 \quad \Rightarrow \quad CD = \sqrt{64} \quad \Rightarrow \quad CD = 8$
$\therefore \quad CD = 8$ cm. ....(2)

Hence, length of BD = BC + CD
= 5 cm + 8 cm [Putting from (1) and (2)]
= 13 cm

(*b*) Here $\angle$ PSR = 90°

PQ = 10 cm, QS = 6 cm and RQ = 9 cm
To calculate the length of PR

Sol. In right angled ΔPQS.
By Pythagoras theorem,
$PQ^2 = PS^2 + QS^2$

$\Rightarrow (10)^2 = PS^2 + (6)^2 \quad \Rightarrow \quad (10)^2 - (6)^2 = PS^2$

$\Rightarrow 100 - 36 = PS^2 \Rightarrow PS^2 = 64 \Rightarrow PS = \sqrt{64} = 8$

$\therefore \quad PS = 8$ cm.

Now, in right angled ΔPSR
By Pythagoras theorem,
$PR^2 = PS^2 + RS^2$
$PR^2 = (8)^2 + (15)^2$ (RS = RQ + QS)
$PR^2 = 64 + 225 = (9 + 6)$ cm = 15 cm
$PR^2 = 289$

$PR = \sqrt{289} = 17$

$\therefore$ PR = 17 cm.

(*c*) Here $\angle D = 90°$
AB = 16 cm, BC = 12 cm and CA = 6 cm
To find CD

Sol. Let the value of CD = x cm,
By Pythagoras theorem,
$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad (16)^2 = AD^2 + (BC + CD)^2$

$\Rightarrow \quad (16)^2 = AD^2 + (12 + x)^2$

$\Rightarrow \quad AD^2 = (16)^2 - (12 + x)^2$ ..... (1)

Now, in right angle Δ ACD
By Pythagoras theorem,
$AC^2 = AD^2 + CD^2$

$\Rightarrow \quad (6)^2 = [(16)^2 - (12 + x)^2] + x^2$
($\because$ From (1) putting the value of AD)

$\Rightarrow \quad 36 = 256 - (144 + x^2 + 24x) + x^2$

$\Rightarrow \quad 36 = 256 - 144 - x^2 - 24x + x^2$

$\Rightarrow \quad 36 = 256 - 144 - 24x$

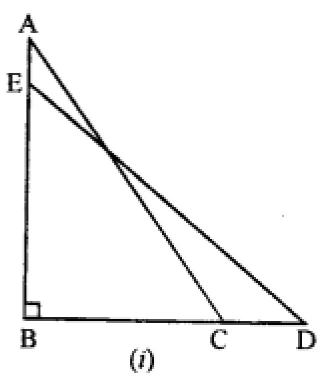
$\Rightarrow \quad 24x = 256 - 144 - 36 \quad \Rightarrow \quad 24x = 76$

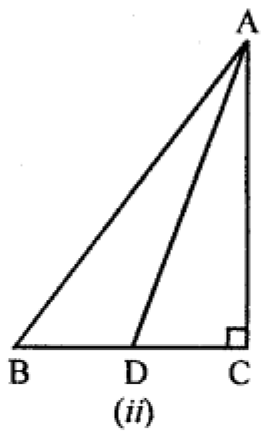
$\Rightarrow \quad x = \frac{76}{24} = \frac{19}{6} = 3\frac{1}{6}$

Hence, $CD = 3\frac{1}{6}$ cm.

Question 16.

(a) In figure (i) given below, BC = 5 cm,
$\angle B = 90°$, AB = 5AE, CD = 2AE and AC = ED. Calculate the lengths of EA, CD, AB and AC.
(b) In the figure (ii) given below, ABC is a right triangle right angled at C. If D is mid-point of BC, prove that AB2 = $4AD^2 - 3AC^2$.





Solution:

(a) Here BC = 5 cm, $\angle B = 90°$, AB = 5 AE,
CD = 2AE, AC = ED

To calculate the lengths of EA, CD, AB and AC

In right angled ΔABC

By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$ ...(*i*)

Also, in right angled ΔBED

ED^2, in right angled ΔBED

$ED^2 = BE^2 + BD^2$ ...(*ii*)

But AC = ED $\Rightarrow$ $AC^2 = ED^2$ ...(*iii*)

From (*i*), (*ii*) and (*iii*),

$AB^2 + BC^2 = BE^2 + BD^2$

$\Rightarrow$ $(5EA)^2 + (5)^2 = (4EA)^2 + (BE + CD)^2$

($\because$ BE = AB – EA = 5EA – EA = 4EA)

$\Rightarrow$ $25EA^2 + 25 = 16EA^2 + (5 + 2EA)^2$

($\because$ CD = 2EA)

$\Rightarrow$ $25EA^2 + 25 - 16EA^2 = 25 + 4EA^2 + 20EA$

$\Rightarrow$ $25x^2 + 25 - 16x^2 = 25 + 4x^2 + 30x$

(Let EA = x cm)

$\Rightarrow$ $9x^2 - 4x^2 = 20x \Rightarrow 5x^2 = 20x$

$\Rightarrow$ $x = 4$ cm ($\because x \neq 0$)

$\therefore$ EA = 4 cm

CD = 2AE = 2 × 4 cm = 8 cm

AB = 5AE = 5 × 4 cm = 20 cm

In ight angled ΔABC,

By Pythagoras Theorem,

$AC^2 - AB^2 + BC^2$

$\Rightarrow$ $AC^2 = (20)^2 + (5)^2 = 400 + 25 = 425$

$\Rightarrow$ $AC = \sqrt{425} = \sqrt{25 \times 17} = 5\sqrt{17}$

Hence, $AC = 5\sqrt{17}$ **Ans.**

(b) In right ΔABC, $\angle C = 90°$

D is mid-piont of BC

To prove : $AB^2 = 4AD^2 - 3AC^2$

Proof : In right ΔABC, $\angle C = 90°$

$AB^2 = AC^2 + BC^2$ ...(*i*)

(Pythagoras Theorem)

But in right ΔADC

$AD^2 = AC^2 + DC^2$

$\Rightarrow AC^2 = AD^2 - DC^2$ ...(ii)

From (i) and (ii),

$$AC^2 = AD^2 - \left(\frac{BC}{2}\right)^2$$

($\because$ D is mid-point of BC)

$$AC^2 = AD^2 - \frac{BC^2}{4}$$

$4AC^2 = 4AD^2 - BC^2$

$AC^2 + 3AC^2 = 4AD^2 - BC^2$

$AC^2 + BC^2 = 4AD^2 - 3AC^2$

But $BC^2 + AC^2 = AB^2$ [From (i)]

$\therefore AB^2 = 4AD^2 - 3AC^2$

Question 17.

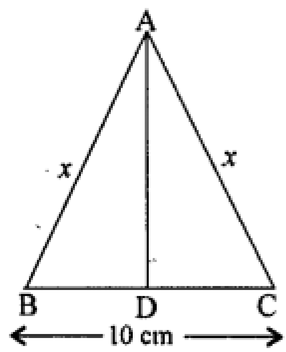
In Δ ABC, AB = AC = x, BC = 10 cm and the area of Δ ABC is 60 cm². Find x.

Solution:

Given. In ΔABC, $AB = AC = x$, $BC = 10$ cm. and area of $\Delta ABC = 60\ cm^2$

Required. Value of x.

Construction. Draw $AD \perp BC$



Sol. In isosceles triangle ABC

$BD = \frac{1}{2} \times BC$

$\Rightarrow \quad BD = \frac{1}{2} \times 10\ cm = 5\ cm$

In right angled ABD

By Pythagoras theorem,

$AB^2 = BD^2 + AD^2$

$x^2 = (5)^2 + AD^2$

$\Rightarrow \quad AD^2 = x^2 - 25 \quad \Rightarrow \quad AD = \sqrt{x^2 - 25}$

Area of $\Delta\ ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow \quad 60 = \frac{1}{2} \times 10 \times \sqrt{x^2 - 25}$

$\Rightarrow \quad \frac{60 \times 2}{10} = \sqrt{x^2 - 25} \quad \Rightarrow \quad 12 = \sqrt{x^2 - 25}$

Squaring both sides, we get

$(12)^2 = \left(\sqrt{x^2 - 25}\right)^2$

$\Rightarrow \quad 144 = x^2 - 25 \quad \Rightarrow \quad 144 + 25 = x^2$

$\Rightarrow \quad x^2 = 169 \quad \Rightarrow \quad x = \sqrt{169} = 13$

$\therefore$ Hence, $x = 13$ cm

Question 18.

In a rhombus, If diagonals are 30 cm and 40 cm, find its perimeter.

Solution:

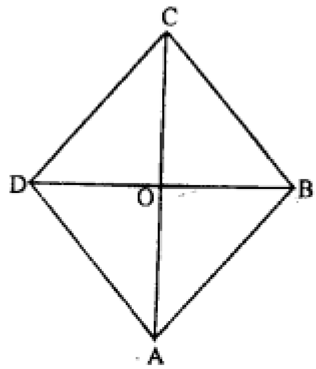
Given. AC = 30 cm and BD = 40 cm where AC and BD are diagonals of rhombus ABCD.

Required. Side of rhombus

Sol. We know that in rhombus diagonals are bisect each other also perpendicular to each other.

$\therefore \quad AO = \frac{1}{2} AC = \frac{1}{2} \times 30 \text{ cm} = 15 \text{ cm}$

and $BO = \frac{1}{2} BD = \frac{1}{2} \times 40 \text{ cm} = 20 \text{ cm}$



In right angled ΔAOB

By Pythagoras theorem,

$AB^2 = AO^2 + BO^2$

$= (15)^2 + (20)^2 \Rightarrow 225 + 400 = 625$

$AB = \sqrt{625} = 25$

Side of rhombus (a) = 25 cm

Perimeter of rhombus = $4a = 4 \times 25 = 100$ cm

Question 19.

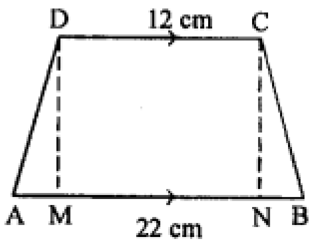
(a) In figure (i) given below, AB || DC, BC = AD = 13 cm. AB = 22 cm and DC = 12cm. Calculate the height of the trapezium ABCD.

(b) In figure (ii) given below, AB || DC, ∠ A = 90°, DC = 7 cm, AB = 17 cm and AC = 25 cm. Calculate BC.

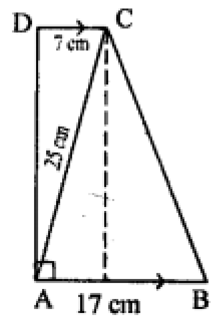
(c) In figure (iii) given below, ABCD is a square of side 7 cm. if AE = FC = CG = HA = 3 cm,

(i) prove that EFGH is a rectangle.

(ii) find the area and perimeter of EFGH.

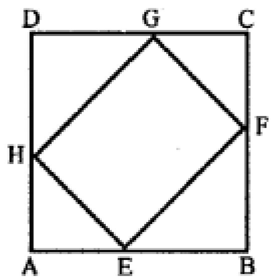


(i)



(ii)

Solution:



(iii)

(a) Given. AB || DC, BC = AD = 13 cm, AB = 22 cm and DC = 12 cm

Required. Height of trapezium ABCD.

Sol. Here CD = MN = 12 cm.

Also, AM = BN

$\therefore$ AB = AM + MN + BN

$\Rightarrow$ 22 = AM + 12 + AM

$\Rightarrow$ 22 – 12 = 2 AM

$\Rightarrow$ 10 = 2 AM

$\Rightarrow$ $AM = \frac{10}{2} = 5$

$\therefore$ AM = 5 cm.

In right angled ΔAMD

$AD^2 = AM^2 + DM^2$

$\Rightarrow (13)^2 = (5)^2 + DM^2 \Rightarrow DM^2 = (13)^2 - (5)^2$

$\Rightarrow DM^2 = 169 - 25 \Rightarrow DM^2 = 144$

$\Rightarrow DM = \sqrt{144} = 12$ cm.

Hence, height of trapezium = 12 cm.

(b) Given. AB || DC, $\angle A = 90°$, DC = 7cm, AB = 17 cm and AC = 25 cm.

Required. BC

In right angled triangle

$AC^2 = AD^2 + CD^2$ (By Pythagoras theorem)

$\Rightarrow$ $(25)^2 = AD^2 + (7)^2$

$\Rightarrow$ $AD^2 = 625 - 49$

$\Rightarrow$ $AD^2 = 576$

$\Rightarrow$ $AD = \sqrt{576} = 24$

$\therefore$ AD = 24 cm.

Also, AD = MC = 24 cm ($\because$ AB || DC)

Also AM = DC = 7 cm

i.e. AM = 7 cm

$\therefore$ BM = AB – AM = 10 cm

In right angled ΔBMC

$BC^2 = MC^2 + BM^2$

$= (24)^2 + (10)^2$

$= 576 + 100 = 676 = (26)^2$

$\Rightarrow \quad BC = 26$

$\therefore \quad BC = 26$ cm **Ans.**

(c) Given. ABCD is a square of side = 7 cm.
AE = FC = CG = HA = 3 cm.

To prove. ***(i)*** EFGH is a rectangle.

(ii) To find the area and perimeter of EFGH.

Proof. BE = BF = DG = DH = 7 – 3 = 4 cm

In right angled ΔAEH

$HE^2 = HA^2 + AE^2$

$= (3)^2 + (3)^2$

$= 9 + 9 = 18$

$\Rightarrow \quad HE = \sqrt{18} = 3\sqrt{2}$ cm.

$\therefore \; HE = GF = 3\sqrt{2}$ cm.

Again In right angled ΔEBF

$EF^2 = EB^2 + BF^2$

$= (4)^2 + (4)^2$

$= 16 + 16 = 32$

$EF = \sqrt{32} = \sqrt{16\times 2} = 4\sqrt{2}$ cm.

$\therefore \; EF = HG = 4\sqrt{2}$ cm.

Join EG

In ΔEFG

$EF^2 + GF^2 = (3\sqrt{2})^2 + (4\sqrt{2})^2 = 18 + 32 = 50$

Also, $EH^2 + HG^2 = (3\sqrt{2})^2 + (4\sqrt{2})^2 = 18 + 32 = 50$

$\therefore \; EF^2 + GF^2 = EH^2 + HG^2$

i.e $EG^2 = HF^2$

i.e EG = HF

i.e Diagonals of quadrilateral are equal.

$\therefore$ EFGH is a rectangle.

Area of rectangle EFGH = HE × EF

$= 3\sqrt{2} \times 4\sqrt{2}\ cm^2 = 24\ cm^2$ Ans.

Perimeter of rectangle EFGH = 2 (EF + HE)

$= 2\,(4\sqrt{2} + 3\sqrt{2})$

$= 2 \times 7\sqrt{2}$ cm

$= 14\sqrt{2}$ cm

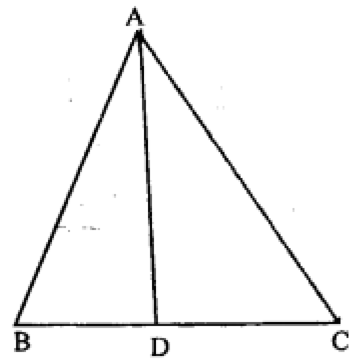
Question 20.

AD is perpendicular to the side BC of an equilateral Δ ABC. Prove that $4AD^2 = 3AB^2$.

Solution:

Given. ABC is an equilateral triangle and $AD \perp BC$

To prove. $4AD^2 = 3AB^2$



Proof. Since ABC is an equilateral triangle

$\therefore \quad AB = BC = CA$

In right angled triangle ABD

$AB^2 = BD^2 + AD^2$ (By Pythagoras theorem)

$$\Rightarrow \quad AB^2 = \left(\frac{BC}{2}\right)^2 + AD^2$$

$\left[\because BD = \frac{BC}{2}\right]$

$$\Rightarrow \quad AB^2 = \frac{(AB)^2}{4} + AD^2 \quad [\because AB = BC]$$

$$\Rightarrow \quad AB^2 - \frac{AB^2}{4} = AD^2$$

$$\Rightarrow \quad \frac{4AB^2 - AB^2}{4} = AD^2$$

$$\Rightarrow \quad \frac{3AB^2}{4} = AD^2$$

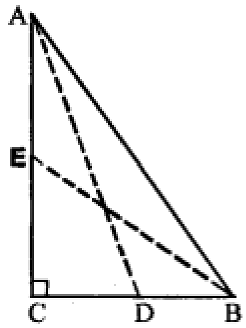
$\Rightarrow 3AB^2 = 4AD^2$

$\Rightarrow 4AD^2 = 3AB^2$

Hence, the result is proved.

Question 21.

In figure (i) given below, D and E are mid-points of the sides BC and CA respectively of a ΔABC, right angled at C.



Solution:

Prove that :

(*i*) $4AD^2 = 4AC^2 + BC^2$

(*ii*) $4BE^2 = 4BC^2 + AC^2$

(*iii*) $4(AD^2 + BE^2) = 5AB^2$.

Ans. (*a*) Given. In ΔABC, right angled at C. D and E are mid-points of the sides BC and CA respectively.

To prove. (*i*) $4AD^2 = 4AC^2 + BC^2$

(*ii*) $4BE^2 = 4CB^2 + AC^2$

(*iii*) $4(AD^2 + BE^2) = 5AB^2$

Proof. In right angle ΔACD,

$AD^2 = AC^2 + CD^2$ (By Pythagoras theorem)

$4AD^2 = 4AC^2 + 4CD^2$

(Multiplying both sides by 4)

$4AD^2 = 4AC^2 + (2BD)^2$

$4AD^2 = 4AC^2 + BC^2$ ..... (1)

($\because$ $2BD = BC$ $\therefore$ D is mid-points of BC)

(*ii*) In right angled ΔBCE

$BE^2 = BC^2 + CE^2$ (By Pythagoras theorem)

$4BE^2 = 4BC^2 + 4CE^2$ (Multiplying both sides by 4)

$4BE^2 = 4BC^2 + (2CE)^2$

$4BE^2 = 4BC^2 + AC^2$ ..... (1)

($\because$ $2CE = AC$ $\therefore$ E is mid-points of AC)

Adding (1) and (2) , we get

$4AD^2 + 4BE^2 = 4AC^2 + BC^2 + 4BC^2 + AC^2$

$4(AD^2 + BE^2) = 5AC^2 + 5BC^2$

$= 5(AC^2 + BC^2)$

$= 5(AB^2)$

($\because$ In right angled ΔABC, $AC^2 + BC^2 = AB^2$)

lence, $4(AD^2 + BE^2) = 5AB^2$

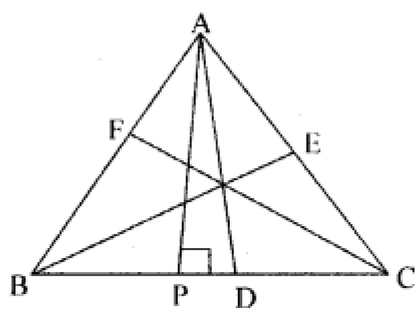
Question 22.

If AD, BE and CF are medians of EABC, prove that 3(AB² + BC² + CA²) = 4(AD² + BE² + CF²).

Solution:

Given : AD, BE and CF are medians of ΔABC.

To prove : $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$



Construction. Draw $AP \perp BC$.

Proof. In right angled ΔAPB.

$AB^2 = AP^2 + BP^2$

$= AP^2 + (BD - PD)^2$

$= AP^2 + BD^2 + PD^2 - 2BD.PD$

$= (AP^2 + PD^2) + BD^2 - 2BD.PD$

$= AD^2 + \left(\frac{1}{2}BC\right)^2 - 2 \times \left(\frac{1}{2}BC\right).PD$

$(\because AP^2 + PD^2 = AD^2$ and $BD = \frac{1}{2}BC)$

$= AD^2 + \frac{1}{4}BC^2 - BC.PD$ ..... (1)

Now, in ΔAPC

$AC^2 = AP^2 + PC^2$ (By Pythagoras theorem)

$= AP^2 + (PD + DC)^2$

$= AP^2 + PD^2 + DC^2 + 2PD.DC$

$= (AP^2 + PD^2) + \left(\frac{1}{2}BC\right)^2 + 2PD \times \left(\frac{1}{2}BC\right)$

$(\because DC = \frac{1}{2}BC)$

$$= AD^2 + \frac{1}{4} BC^2 + PD.BC \quad ..... (2)$$

Adding (1) and (2)

$$\therefore\ AB^2 + AC^2 = 2AD^2 + \frac{1}{2} BC^2 \quad ..... (3)$$

Similarly, Draw the perpendicular from B and C on AC and AB respectively, we get

$$BC^2 + CA^2 = 2CF^2 + \frac{1}{2} AB^2 \quad ..... (4)$$

$$AB^2 + BC^2 = 2BE^2 + \frac{1}{2} AC^2 \quad ..... (5)$$

Adding (3), (4) and (5), we get

$$2(AB^2 + BC^2 + CA^2)$$

$$= 2(AD^2 + BE^2 + CF^2) + \frac{1}{2}(BC^2 + AB^2 + AC^2)$$

$$\Rightarrow\ 2(AB^2 + BC^2 + CA^2) - \frac{1}{2}(AB^2 + BC^2 + CA^2) = 2(AD^2 + BE^2 + CF^2)$$

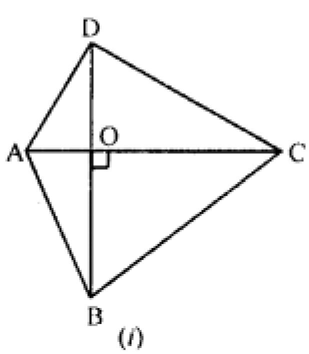
$$\Rightarrow\ \frac{3}{2}(AB^2 + BC^2 + CA^2) = 2(AD^2 + BE^2 + CF^2)$$

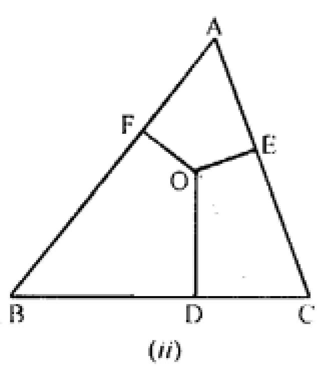
$$\therefore\ 3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence, the proved.

Question 23.

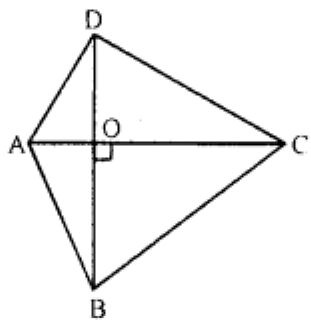
(a) In fig. (i) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that
$AB^2 + CD^2 = AD^2 + BC^2$.
(b) In figure (ii) given below, OD⊥BC, OE ⊥CA and OF ⊥ AB. Prove that :
(i) $OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OD^2 + OE^2 + OF^2$.
(ii) $OAF^2 + BD^2 + CE^2 = FB^2 + DC^2 + EA^2$.





Solution:

(*a*) **Given.** In quadrilateral ABCD the diagonals AC and BD intersect at O at right angles.



To prove. $AB^2 + CD^2 = AD^2 + BC^2$

Proof. In right angled ΔAOB

$AB^2 = AO^2 + OB^2$ ..... (1)

(By Pythagoras theorem)

In right angled Δ COD

$CD^2 = OD^2 + OC^2$ ..... (2)

Adding (1) and (2),

$AB^2 + CD^2 = (AO^2 + OB^2) + (OD^2 + OC^2)$

$AB^2 + CD^2 = (OA^2 + OD^2) + (OB^2 + OC^2)$ ... (3)

Now, in right angled triangle AOD and BOC

By Pythagoras theorem,

$OA^2 + OD^2 = AD^2$ ..... (4)

$OB^2 + OC^2 = BC^2$ ..... (5)

From (3), (4) and (5), we get

$AB^2 + CD^2 = AD^2 + BC^2$

Hence, the result.

(b) Given $OD \perp BC$, $OE \perp CA$ and $OF \perp AB$.

To prove.

(*i*) $OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OD^2 + OE^2 + OF^2$.

(*ii*) $AF^2 + BD^2 + CE^2 = FB^2 = DC^2 + EA^2$.

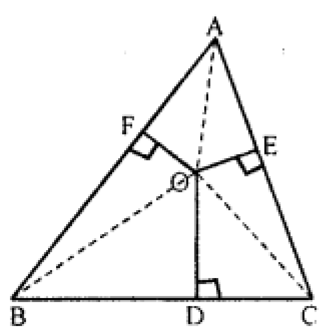
Proof.

In right angled ΔAOF

$OA^2 = AF^2 + OF^2$ ..... (1)

In right angled ΔBOD

$OB^2 = BD^2 + OD^2$ ..... (2)



In right angled Δ COE

$OC^2 = CE^2 + OE^2$ ..... (3)

Adding (1), (2) and (3), we get

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OD^2 + OE^2 + OF^2$ (Proved (*i*) part)

(*ii*) Also $OA^2 + OB^2 + OC^2$

$= AF^2 + BD^2 + CE^2 + OD^2 + OC^2 + OF^2$

$\Rightarrow \quad AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$ ..... (4)

Again in ΔBOF, ΔCOD, ΔAOE,

$BF^2 = OB^2 - OF^2$

$DC^2 = OC^2 - OD^2$

and $EA^2 = OA^2 - OE^2$

Adding above, we get

$BF^2 + DC^2 + EA^2 = OB^2 - OF^2 + OC^2 - OD^2 + OA^2 - OF^2$

$BF^2 + DC^2 + EA^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$ ..... (5)

From (4) and (5)

$AF^2 + BD^2 + CE^2 = BF^2 + DC^2 + EA^2$

Hence, the result.

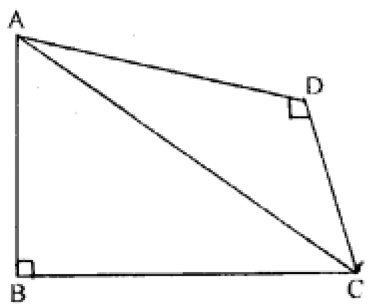
Question 24.

In a quadrilateral, ABCD∠B = 90° = ∠D. Prove that 2 AC^2 – BC2 = $AB^2 + AD^2 + DC^2$.

Solution:

Given. In quadrilateral ABCD, $\angle B = 90°$ and $\angle D = 90°$

To prove. $2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$



Construction. Join AC.

Proof. In right angled ΔABC

$AC^2 = AB^2 + BC^2$ ..... (1)

(By Pythagoras theorem)

In right angled ΔACD

$AC^2 = AD^2 + DC^2$ ..... (2)

(By Pythagoras theorem)

Adding (1) from (2), we get

$AC^2 + AC^2 = AB^2 + BC^2 + AD^2 + DC^2$

$2AC^2 = AB^2 + BC^2 + AD^2 + DC^2$

$2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$

Hence, the result.

Question 25.

In a Δ ABC, $\angle$ A = 90°, CA = AB and D is a point on AB produced. Prove that : $DC^2 - BD^2 = 2AB. AD$.

Solution:

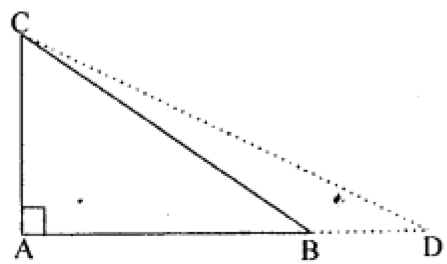
Given. ΔABC in which $\angle A = 90°$, $CA = AB$ and D is point on AB produced.

To prove. $DC^2 - BD^2 = 2AB.AD$

Proof. In right angled ΔACD,

$DC^2 = AC^2 + AD^2$

$DC^2 = AC^2 + (AB + BD)^2$



$DC^2 = AC^2 + AB^2 + BD^2 + 2AB.BD$

$DC^2 - BD^2 = AC^2 + AB^2 + 2AB.BD$

But $AC = AB$ (given)

$DC^2 - BD^2 = AB^2 + AB^2 + 2AB.BD$

$DC^2 - BD^2 = 2AB^2 + 2AB.BD$

$DC^2 - BD^2 = 2AB(AB + BD)$

$DC^2 - BD^2 = 2AB.AD$

Hence, the result.

Question 26.

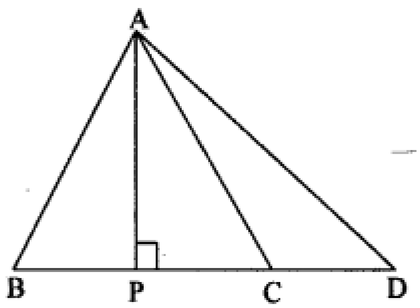
In an isosceles triangle ABC, AB = AC and D is a point on BC produced. Prove that $AD^2 = AC^2 + BD.CD$.

Solution:

Given. Isosceles ΔABC such that AB = AC.

D is mid-points on BC produced.

To prove. $AD^2 = AC^2 + BD.CD$



Const. Draw $AP \perp BC$

Proof. In right angled ΔAPD,

$AD^2 = AP^2 + PD^2$

$AD^2 = AP^2 + (PC + CD)^2$

$AD^2 = AP^2 + PC^2 + CD^2 + 2PC.CD$

In right angled ΔAPC

$AC^2 = AP^2 + PC^2$

$\therefore AD^2 = AC^2 + CD^2 + 2PC.CD$

But ΔABC is isosceles triangle and $AP \perp BC$

$\therefore PC = \frac{1}{2} BC$

$\therefore AD^2 = AC^2 + CD^2 + 2 \times \frac{1}{2} BC.CD$

$AD^2 = AC^2 + CD^2 + BC.CD$

$AD^2 = AC^2 + CD [CD + BC]$

$AD^2 = AC^2 + CD.BD$

i.e $AD^2 = AC^2 + BD.CD$

Hence, the result.

Question P.Q.

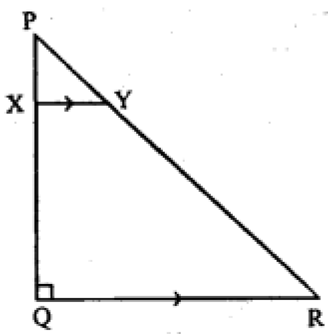
(a) In figure (i) given below, PQR is a right angled triangle, right angled at Q. XY is parallel to QR. PQ = 6 cm, PY = 4 cm and PX : OX = 1:2. Calculate the length of PR and QR.

(b) In figure (ii) given below, ABC is a right angled triangle, right angled at B.DE || BC.AB = 12 cm, AE = 5 cm and AD : DB = 1: 2. Calculate the perimeter of A ABC.

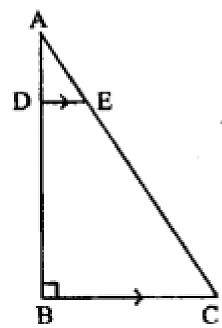
(c)In figure (iii) given below. ABCD is a rectangle, AB = 12 cm, BC – 8 cm and E is a point on BC such that CE = 5 cm. DE when produced meets AB produced at F.

(i) Calculate the length DE.

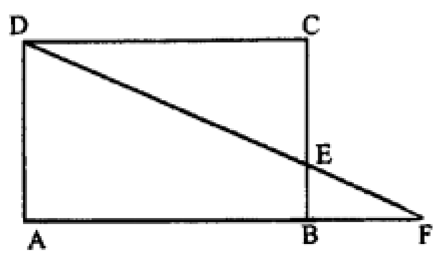
(ii) Prove that Δ DEC ~ AEBF and Hence, compute EF and BF.



(*i*)



(*ii*)



(*iii*)

Solution:

(*a*) **Given.** In right angled Δ PQR, XY || QR , PQ = 6 cm, PY = 4 cm and PX : QX = 1 : 2.

Required. The length of PR and QR.

Sol. PX : QX = 1 : 2 (given)

LetPX = x cm

then QX = $2x$ cm

$\therefore$ PQ = PX + QX

$$\Rightarrow \quad 6 = x + 2x \quad \Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = \frac{6}{3} = 2$$

$\therefore$ PX = 2 cm and QX = 2 × 2 cm = 4 cm

In right angled Δ PXY

$PY^2 = PX^2 + XY^2$ (By Pythagoras theorem)

$$\Rightarrow \quad (4)^2 = (2)^2 + XY^2 \quad \Rightarrow \quad XY^2 = (4)^2 - 4$$

$$\Rightarrow \quad XY^2 = 12 \quad \Rightarrow \quad XY = \sqrt{12} = 2\sqrt{3}$$

Also, XY || QR

$$\therefore \quad \frac{PX}{PQ} = \frac{XY}{QR} \quad \Rightarrow \quad \frac{2}{6} = \frac{2\sqrt{3}}{QR}$$

$$\Rightarrow \quad 2QR = 2\sqrt{3} \times 6$$

$$\Rightarrow \quad QR = \frac{2\sqrt{3} \times 6}{2}$$

$= 6\sqrt{3}$ cm.

Also $\frac{PX}{PQ}=\frac{PY}{PR}$

$\Rightarrow \quad \frac{2}{6}=\frac{4}{PR}$

$PR = \frac{6\times4}{2} = \frac{24}{2} = 12$ cm

Hence, PR = 12 cm and QR = $6\sqrt{3}$ cm **Ans.**

(b) Given. In right angled ΔABC,

$\angle B = 90°$, DE || BC, AB = 12 cm, AE = 5 cm and

AD : DB = 1 : 2

Required. The perimeter of ΔABC.

Sol. AD : DB = 1 : 2 (given)

let AD = x cm

then DB = $2x$ cm

$\therefore$ AB = AD + DB

$\Rightarrow \quad 12 = x + 2x \quad \Rightarrow \quad 3x = 12 \quad \Rightarrow \quad x = \frac{12}{3} = 4$

$\therefore$ AD = x = 4 cm and DB = $2x$ = 2 × 4 cm = 8 cm

In right angled ΔADE

$AE^2 = AD^2 + DE^2$ (By Pythagoras theorem)

$\Rightarrow \quad (5)^2 = (4)^2 + DE^2 \quad \Rightarrow \quad 25 = 16 + DE^2$

$\Rightarrow \quad DE^2 = 25 - 16 \quad \Rightarrow \quad DE^2 = 9$

$\Rightarrow \quad DE = \sqrt{9} = 3$ cm

Now, DE || BC (given)

$\therefore \quad \frac{AD}{AB}=\frac{DE}{BC}$

$\Rightarrow \quad \frac{4}{12}=\frac{3}{BC} \quad \Rightarrow \quad BC = \frac{12\times3}{4} = 3\times 3 = 9$

cm

Also, $\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \quad \frac{4}{12} = \frac{5}{AC} \quad \Rightarrow \quad AC = \frac{12 \times 5}{4} = 3 \times 5 = 15$ cm

Perimeter of $\Delta ABC = AB + BC + AC$
$= 12$ cm $+ 9$ cm $+ 15$ cm $= 36$ cm **Ans.**

(*c*) Given. ABCD is a rectangle, AB = 12 cm, BC = 8 cm, and E is a point on BC such that CE = 5 cm.

Required. (*i*) The length of DE.

(*ii*)To prove $\Delta DEC \sim \Delta EBF$ and Hence, find EF and BF.

(*i*) In right angled ΔCDE,

$DE^2 = CD^2 + CE^2$

$DE^2 = AB^2 + CE^2$ [CD = AB]

$\Rightarrow DE^2 = (12)^2 + (5)^2 \Rightarrow DE^2 = 144 + 25$

$\Rightarrow DE^2 = 169 \Rightarrow DE = \sqrt{169} = 13$ cm

Ans.

(*ii*)In ΔDEC and ΔEBF

$\angle DEC = \angle BEF$ (vertically opposite angles)

$\angle DCE = \angle EBF$ (each 90°)

$\therefore \Delta DCE \sim \Delta EBF$ (By A. A. axiom of similarity)

$$\therefore \frac{CE}{BE} = \frac{DE}{EF}$$

$$\Rightarrow \frac{5}{3} = \frac{13}{EF} \quad (\because BE = 8 \text{ cm} - 5 \text{ cm} = 3 \text{ cm})$$

$$\Rightarrow 5 \times EF = 13 \times 3$$

$$\Rightarrow EF = \frac{13 \times 3}{5} = \frac{39}{5} = 7.8 \text{ cm}$$

$$\text{Also, } \frac{CE}{BE} = \frac{DE}{BF} \Rightarrow \frac{5}{3} = \frac{12}{BF}$$

($\because$ BF = 8.5 cm – 5 cm = 3 cm also CD = AB = 12 cm)

$$\Rightarrow BF \times 5 = 12 \times 3 \Rightarrow BF = \frac{12 \times 3}{5} = \frac{36}{5} = 7.2\text{cm}$$

Hence, DE = 13cm, EF = 7.8cm and BF = 7.2cm.

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 7):

Question 1.

In a ΔABC, if AB = $6\sqrt{3}$ cm, BC = 6 cm and AC = 12 cm, then $\angle B$ is

(a) 120°

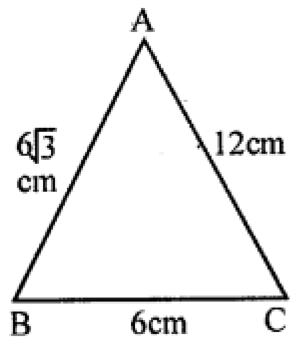
(b) 90°

(c) 60°

(d) 45°

Solution:

In ΔABC, $AB = 6\sqrt{3}$ cm, $BC = 6$ cm, AC = 12 cm



$\because AB^2 + BC^2 = (6\sqrt{3})^2 + (6)^2$

$= 108 + 36 = 144$

and $AC^2 = 12^2 = 144$

$\therefore \angle B = 90°$ (b)

(Converse of Pythagoras Theorem)

Question 2.

If the sides of a rectangular plot are 15 m and 8 m, then the length of its diagonal is

(a) 17 m
(b) 23 m
(c) 21 m
(d) 17 cm

Solution:

Length of a rectangle (l) = 15 m

and breadth (b) = 8 m

$\therefore$ Diagonal $= \sqrt{l^2 + b^2}$

$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$

$= \sqrt{289} = 17$ m (a)

Question 3.

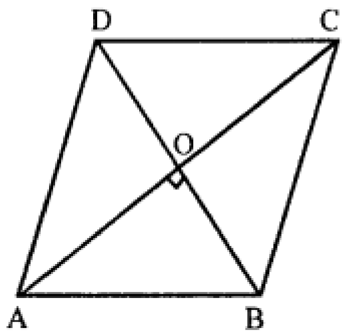
The lengths of the diagonals of a rhombus are 16 cm and 12 cm. The length of the side of the rhombus is

(a) 9 cm
(b) 10 cm

(c) 8 cm
(d) 20 cm

Solution:

Lengths of diagonals of rhombus are 16 cm and 12 cm



∵ Diagonals of rhombus bisect each other at right angles

Length of side

$$= \sqrt{\left(\frac{\text{First diagonal}}{2}\right)^2 + \left(\frac{\text{Second diagonal}}{2}\right)^2}$$

$$= \sqrt{\left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2}$$

$$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$$

$$= \sqrt{100} = 10 \text{ cm} \qquad \text{(b)}$$

Question 4.

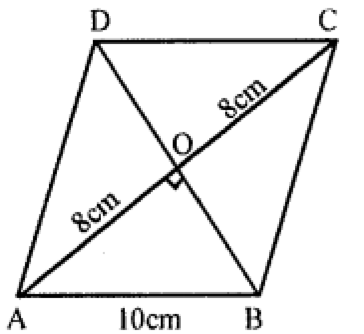
If a side of a rhombus is 10 cm and one of the diagonals is 16 cm, then the length of the other diagonals is

(a) 6 cm
(b) 12 cm
(c) 20 cm
(d) 12 cm

Solution:

One diagonal of rhombus = 16 cm

Side = 10 cm



$\because$ The diagonals of a rhombus bisect each other at right angles

$\therefore$ In right ΔAOB,

$AO = \frac{16}{2} = 8$ cm, AB = 10 cm

$\therefore AB^2 = AO^2 + BO^2$

$\Rightarrow 10^2 = 8^2 + BO^2 \Rightarrow 100 = 64 + BO^2$

$\Rightarrow BO^2 = 100 - 64 = 36 = (6)^2$

$\therefore$ BO = 6 cm

$\therefore$ Other diagonal BD = $6 \times 2 = 12$ cm (b)

Question 5.

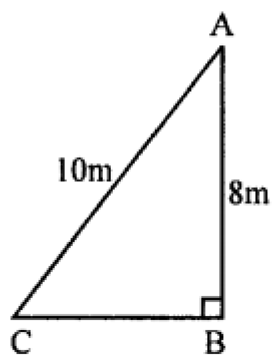
If a ladder 10 m long reaches a window 8 m above the ground, then the distance of the foot of the ladder from the base of the wall is

(a) 18 m
(b) 8 m
(c) 6 m
(d) 4 m

Solution:

Length of ladder = 10 m

Height of window = 8 m



∴ Distance of ladder from the base of wall

$= \sqrt{AC^2 - AB^2} = \sqrt{10^2 - 8^2}$

$= \sqrt{100 - 64} = \sqrt{36} = 6$ m (c)

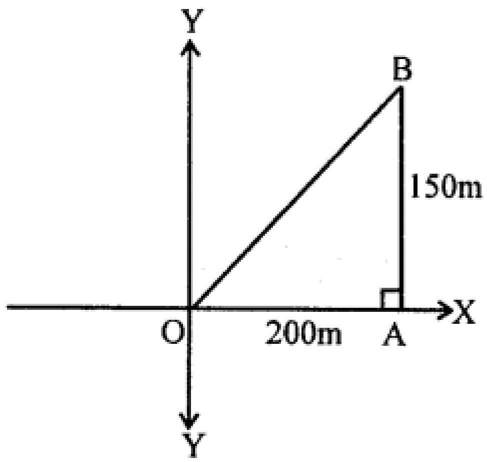
Question 6.

A girl walks 200 m towards East and then she walks ISO m towards North. The distance of the girl from the starting point is

(a) 350 m
(b) 250 m
(c) 300 m
(d) 225 m

Solution:

A girl walks 200 m towards East and then 150 m towards North



Distance of girls from the starting point (OB)

$= \sqrt{OA^2 + AB^2} = \sqrt{(200)^2 + (150)^2}$

$= \sqrt{40,000 + 22500} = \sqrt{62500} = 250$ m (b)

Question 7.

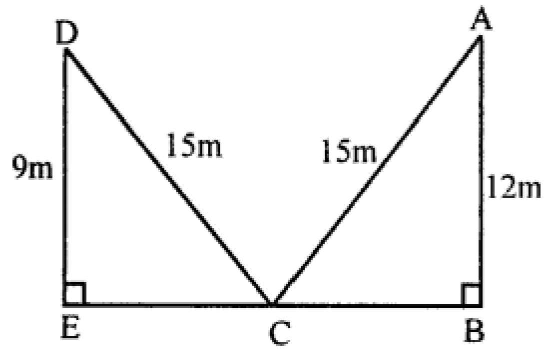
A ladder reaches a window 12 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 9 m high. If the length of the ladder is 15 m, then the width of the street is

(a) 30 m
(b) 24 m
(c) 21 m
(d) 18 m

Solution:

Height of window = 12 m

Length of ladder = 15 m



In right ΔABC

$AC^2 = AB^2 + BC^2 \Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2 = 15^2 - 12^2 = 225 - 144 = 81 = (9)^2$

$\therefore$ BC = 9 m

Similarly in right ΔCDE

$EC^2 = DC^2 - DE^2 = 15^2 - 9^2$

$= 225 - 81 = 144 = (12)^2$

$\therefore$ EC = 12 m

$\therefore$ Width of street EB = EC + CB

= 9 + 12 = 21 m (c)

Chapter Test

Question 1.

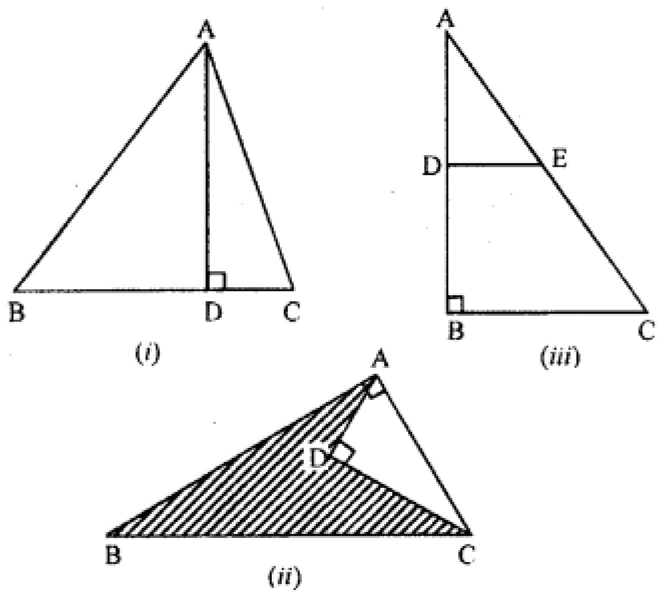
(a) In fig. (i) given below, AD ⊥ BC, AB = 25 cm, AC = 17 cm and AD = 15 cm. Find the length of BC.

(b) In figure (ii) given below, ∠BAC = 90°, ∠ADC = 90°, AD = 6 cm, CD = 8 cm and BC = 26 cm. Find :

(i) AC (ii) AB (iii) area of the shaded region.

(c) In figure (iii) given below, triangle ABC is right angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are mid-points of the sides AB and AC respectively, calculate

(i) the length of BC (ii) the area of Δ ADE.



Solution:

(*a*) **Given.** In ΔABC , $AD \perp BC$, AB = 25 cm, AC = 17 cm and AD = 15 cm

Required. The length of BC.

Sol. In right angled ΔABD,

$AB^2 = AD^2 + BD^2$

(By Pythagoras theorem)

$\therefore \quad BD^2 = AB^2 - AD^2$

$= (25)^2 - (15)^2$

$= 625 - 225 = 400$

$\Rightarrow \quad BD = \sqrt{400} = 20$ cm.

Now, in right angled Δ ADC

$AC^2 = AD^2 + DC^2$ (By Pythagoras theorem)

$\therefore \quad DC^2 = AC^2 - AD^2$

$\Rightarrow \quad DC^2 = (17)^2 - (15)^2$

$\Rightarrow \quad DC^2 = 289 - 225 = 64$

$DC = \sqrt{64} = 8$cm

Hence, BC = BD + DC = 20 cm + 8 cm = 28 cm.

(*b*) **Given.** In ΔABC,

$\angle BAC = 90°$, $\angle ADC = 90°$ AD = 6cm, CD = 8 cm and BC = 26 cm.

Required. (*i*) AC (*ii*) AB

(*iii*) area of the shaded region

Sol. In right angled ΔADC

$AC^2 = AD^2 + DC^2$ (By Pythagoras theorem)

$= (6)^2 + (8)^2$

$= 36 + 64 = 100$

$\therefore \quad = \sqrt{100} = 10$ cm **Ans.**

In right angled ΔABC

$BC^2 = AB^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow (26)^2 = AB^2 + (10)^2$

$\Rightarrow AB^2 = (26)^2 - (10)^2$

$\Rightarrow AB^2 = 676 - 100 = 576$

$\Rightarrow AB^2 = 576$

$\Rightarrow AB = \sqrt{576} = 24$ cm

Now, Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 24 \times 10 \text{ cm}^2 = 12 \times 10 \text{ cm}^2 = 120 \text{ cm}^2$

Area of $\Delta ADC = \frac{1}{2} \times AD \times DE$

$= \frac{1}{2} \times 6 \times 8 \text{ cm}^2 = 3 \times 8 \text{ cm}^2 = 24 \text{ cm}^2$

Now, Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 24 \times 10 \text{ cm}^2 = 12 \times 10 \text{ cm}^2 = 120 \text{ cm}^2$

Area of $\Delta ADC = \frac{1}{2} \times AD \times DC$

$= \frac{1}{2} \times 6 \times 8 \text{ cm}^2 = 3 \times 8 \text{ cm}^2 = 24 \text{ cm}^2$

Hence, area of shaded region =

Area of ΔABC – Area of ΔADC

$= 120 \text{ cm}^2 - 24 \text{ cm}^2 = 96 \text{ cm}^2$

(*c*) **Given.** In right angled ΔABC, AB = 9 cm, AC = 15 cm, and D, E are mid-points of the sides AB and AC respectively.

Required. (*i*) length of BC

(*ii*) the area of ΔADE

Sol. In right angled Δ ADE,

(By Pythagoras theorem)

$AE^2 = AD^2 + DE^2$

$$\Rightarrow \left(\frac{AC}{2}\right)^2 = \left(\frac{AB}{2}\right)^2 + DE^2$$

($\because$ D and E are mid-points of AB and AC respectively.)

$$\Rightarrow \left(\frac{15}{2}\right)^2 = \left(\frac{9}{2}\right)^2 + DE^2$$

$$\Rightarrow DE^2 = \frac{225}{4} - \frac{81}{4} \Rightarrow DE^2 = \frac{144}{4} = 36$$

$$\Rightarrow DE = \sqrt{36} = 6 \text{ cm}$$

Since D and E are mid-points of AB and AC respectively.

$DE \parallel BC$ and $DE = \frac{1}{2}BC$

$$\Rightarrow BC = 2DE = 2 \times 6 \text{ cm} = 12 \text{ cm}$$

(*ii*) Area of $\Delta ADE = \frac{1}{2} \times AD \times DE$

$$= \frac{1}{2} \times \left(\frac{AB}{2}\right) \times DE = \frac{1}{2} \times \frac{9}{2} \times 6 \text{ cm}^2$$

$$= \frac{9}{2} \times 3 \text{ cm}^2 = \frac{27}{2} \text{ cm}^2 = 13.5 \text{ cm}^2 \text{ Ans.}$$

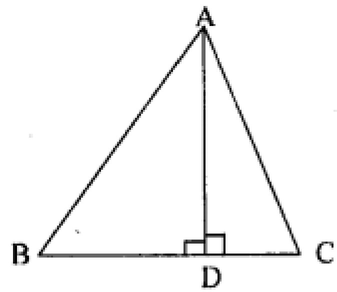
Question 2.

If in Δ ABC, AB > AC and ADI BC, prove that $AB^2 - AC^2 = BD^2 - CD^2$.

Solution:

Given. In ΔABC, $AB > AC$ and $AD \perp BC$

To prove. $AB^2 - AC^2 = BD^2 - CD^2$



Proof. In right angled ΔABD

$AB^2 = AD^2 + BD^2$ ...... (1)

(By Pythagoras theorem)

In right angled ΔACD

$AC^2 = AD^2 + CD^2$ ..... (2)

Subtracting (2) from (1), we get

$AB^2 - AC^2 = (AD^2 + BD^2) - (AD^2 + CD^2)$

$= AD^2 + BD^2 - AD^2 - CD^2$

$= BD^2 - CD^2$

$\therefore \quad AB^2 - AC^2 = BD^2 - CD^2$

Hence, the result.

Question 3.

In a right angled triangle ABC, right angled at C, P and Q are the points on the sides CA and CB respectively which divide these sides in the ratio 2:1. Prove that

(i) $9AQ^2 = 9AC^2 + 4BC^2$

(ii) $9BP^2 = 9BC^2 + 4AC^2$

(iii) $9(AQ^2 + BP^2) = 13AB^2$.

Solution:

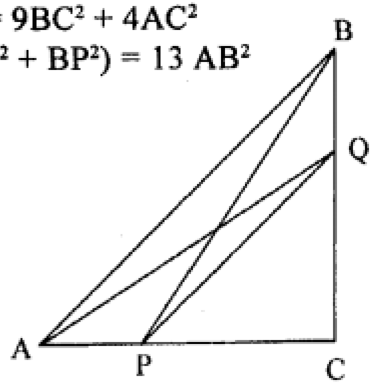
A right angled Δ ABC in which $\angle$ C

90°. P and Q are points on the side CA and CB respectively such that CP : AP = 2 : 1 and CQ : BQ = 2 : 1

To prove. (*i*) $9AQ^2 = 9AC^2 + 4BC^2$

(*ii*) $9BP^2 = 9BC^2 + 4AC^2$

(*iii*) $9(AQ^2 + BP^2) = 13AB^2$



Construction. Join AQ and BP.

Proof. (*i*) In right angled ΔACQ

$AQ^2 = AC^2 + QC^2$

(By Pythagoras theorem)

$9AQ^2 = 9AC^2 + 9QC^2$

(Multiplying both sides by 9)

$= 9AC^2 + (3QC)^2 = 9AC^2 + (2BC)^2$

$$\left[\because BQ:CQ:1:2 \Rightarrow \frac{QC}{BC} = \frac{QC}{BQ+CQ} = \frac{2}{3} \Rightarrow 3QC = 2BC\right]$$

$= 9AC^2 + 4BC^2$

$\therefore 9AQ^2 = 9AC^2 + 4BC^2$ ..... (1)

(*ii*) In right angled ΔBPC

$BP^2 = BC^2 + CP^2$ (By Pythagoras theorem)

$9BP^2 = 9BC^2 + 9CP^2$

($\because$ Multiplying both side by 9)

$= 9BC^2 + (3CP)^2 = 9BC^2 + (2AC)^2$

$$\left[\because AP:CP = 1:2 \frac{CP}{AC} = \frac{CP}{AP+CP} = \frac{2}{3} 3CP = 2AC\right]$$

$= 9BC^2 + 4AC^2$

$\therefore\ 9BP^2 = 9BC^2 + 4AC^2$ .....(2)

(*iii*) Adding (1) and (2),

$9AQ^2 + 9BP^2 = 9AC^2 + 4BC^2 + 9BC^2 + 4AC^2$

$= 13AC^2 + 13BC^2 = 13\ (AC^2 + BC^2) = 13\ AB^2$

[In right angled $\Delta ABC = AB^2 = AC^2 + BC^2$]

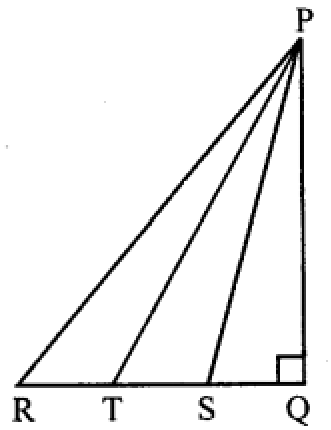
$\therefore\ 9AQ + 9BP^2 = 13\ AB^2$

Hence, the result.

Question 4.

In the given figure, △PQR is right angled at Q and points S and T trisect side QR. Prove that 8PT² – 3PR² + 5PS².

Solution:



In the ΔPQR, $\angle Q = 90°$

T and S are points on RQ such that these trisect it

i.e., $RT = TS = SQ$

To prove : $8PT^2 = 3PR^2 + 5PS^2$

Proof : Let $RT = TS = SQ = x$

In right ΔPRQ

$PR^2 = RQ^2 + PQ^2 = (3x)^2 + PQ^2 = 9x^2 + PQ^2$

Similarly in right PTS,

$PT^2 = TQ^2 + PQ^2 = (2x)^2 + PQ^2 = 4x^2 + PQ^2$

and in PSQ,

$PS^2 = SQ^2 + PQ^2 = x^2 + PQ^2$

$8PT^2 = 8(4x^2 + PQ^2) = 32x^2 + 8PQ^2$

$3PR^2 = 3(9x^2 + PQ^2) = 27x^2 + 3PQ^2$

$5PS^2 = 5(x^2 + PQ^2) = 5x^2 + 5PQ^2$

$LHS = 8PT^2 = 32x^2 + 8PQ^2$

$RHS = 3PR^2 + 5PS^2 = 27x^2 + 3PQ^2 + 5x^2 + 5PQ^2$

$= 32x^2 + 8PQ^2$

$\therefore$ LHS = RHS

Hence proved.

Question 5.

In a quadrilateral ABCD, $\angle B = 90°$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90°$.

Solution:

In quadrilateral ABCD, $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$

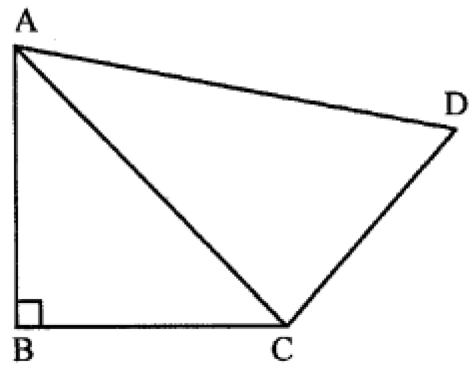
To prove : $\angle ACD = 90°$

Proof : In $\angle ABC$, $\angle B = 90°$

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(*i*)

(Pythagoras Theorem)

But $AD^2 = AB^2 + BC^2 + CD^2$ (Given)



$\Rightarrow$ $AD^2 = AC^2 + CD^2$ [From (*i*)]

$\therefore$ In ΔACD,

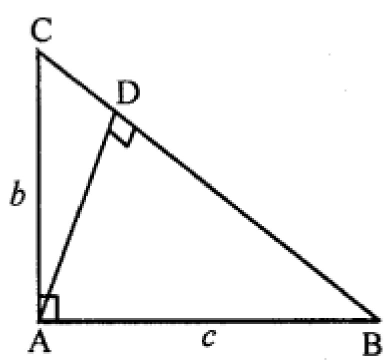
$\angle ACD = 90°$

(Converse of Pythagoras Theorem)

Question 6.

In the given figure, find the length of AD in terms of b and c.

Solution:



In the given figure,
ABC is a triangle, $\angle A = 90°$
$AB = c$, $AC = b$
$AD \perp BC$
To find : AD in terms of b and c

Solution : Area of $\Delta ABC = \frac{1}{2} AB \times AC =$

$\frac{1}{2} bc$ ...(*i*)

and $\Delta ABC = \frac{1}{2} BC \times AD$ ...(*ii*)

But $BC = \sqrt{AB^2 + AC^2} = \sqrt{c^2 + b^2}$

$= \sqrt{b^2 + c^2}$ ...(*iii*)

From (*i*) and (*ii*),

$$\frac{1}{2} BC \times AD = \frac{1}{2} bc \Rightarrow BC \times AD = b.c$$

$\Rightarrow \sqrt{b^2 + c^2} \times AD = bc$ [from (*iii*)]

Hence $AD = \dfrac{bc}{\sqrt{b^2 + c^2}}$

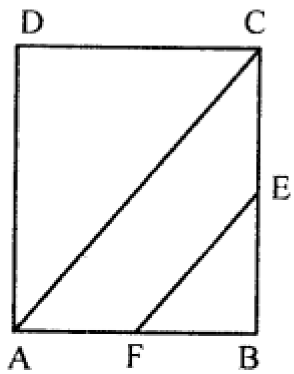
Question 7.
ABCD is a square, F is mid-point of AB and BE is one-third of BC. If area of ΔFBE is 108 cm^2, find the length of AC.
Solution:

Given : In square ABCD. F is mid piont of

AB and BE = $\frac{1}{3}$BC

Area of ΔFBE = 108 cm^2

AC and EF are joined



To find : AC

Solution : Let each side of square is = a

FB = $\frac{1}{2}$AB (F is mid point of AB)

$= \frac{1}{2}a$

and BE = $\frac{1}{3}$BC = $\frac{1}{3}a$

Now in square ABCD

AC = $\sqrt{2}$ × Side = $\sqrt{2}\,a$

and area ΔFBE = $\frac{1}{2}$FB × BE

$= \frac{1}{2} \times \frac{1}{2}a \times \frac{1}{3}a = \frac{1}{12}a^2$

$$\therefore \frac{1}{12}a^2 = 108 \Rightarrow a^2 = 12 \times 108 = 1296$$

$$\Rightarrow a = \sqrt{1296} = 36$$

$$\therefore AC = \sqrt{2}a = \sqrt{2} \times 36 = 36\sqrt{2} \text{ cm}$$

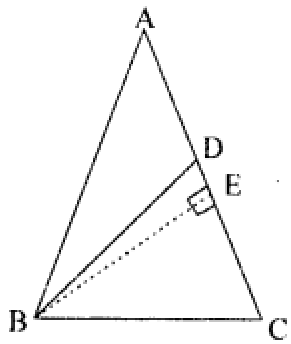
Question 8.

In a triangle ABC, AB = AC and D is a point on side AC such that $BC^2 = AC \times CD$, Prove that BD = BC.

Solution:

Given. In a triangle ABC, AB = AC and D is point on side AC such that $BC^2 = AC \times CD$

To prove. BD = BC



Construction. Draw $BE \perp AC$

Proof. In right angled ΔBCE

$BC^2 = BE^2 + EC^2$ (By Pythagoras theorem)

$= BE^2 + (AC - AE)^2$

$= BE^2 + AC^2 + AE^2 - 2AC.AE$

$= (BE^2 + AE^2) + AC^2 - 2AC.AE$

$= AB^2 + AC^2 - 2AC.AE$

(In rt. $\angle$ed ΔABC, $AB^2 = BE^2 + AE^2$)

$= AC^2 + AC^2 - 2AC.AE$ (given AB = AC)

$= 2AC^2 - 2AC.AE = 2AC(AC - AE)$

$= 2AC.EC$

But $BC^2 = AC \times CD$ (given)

$\Rightarrow AC \times CD = 2AC.EC \Rightarrow CD = 2EC$

$\therefore$ E is mid-points of CD $\Rightarrow$ EC = DE

Now, in ΔBED and ΔBEC

EC = DE (above proved)

BE = BE (common)

$\angle BED = \angle BEC$ (each 90°)

$\therefore \Delta BED \cong \Delta BEC$

(By S.A.S. axiom of congruency)

$\therefore$ BD = BC (c.p.c.t.)

Hence, the result.