

Theorems on Area

Question 1.

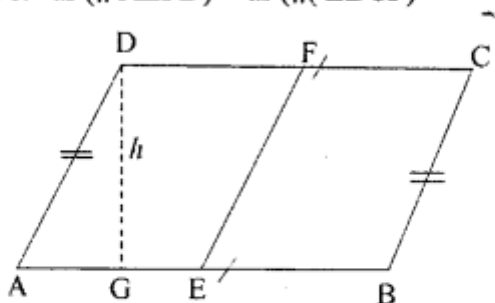
Prove that the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Solution:

Given. ABCD is a parallelogram in which E and F are mid-points of AB and CD respectively.

Joining EF.

To prove. ar (|| AEFD) = ar (|| EBCF)



Construction. $DG \perp AG$ and let $DG = h$ i.e. h is the Altitude on side AB.

Proof. ar (|| ABCD) = $AB \times h$

$$\text{ar (|| AEFD)} = AE \times h = \frac{1}{2} AB \times h \quad \dots (1)$$

(\because E is the mid-point of AB)

$$\text{ar (|| EBCF)} = EF \times h = \frac{1}{2} AB \times h \quad \dots (2)$$

(\because E is the mid-point of AB)

From (1) and (2)

$$\text{ar (|| gm ABFD)} = \text{ar (|| EBCF)}$$

Hence, the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms. **(Q.E.D.)**

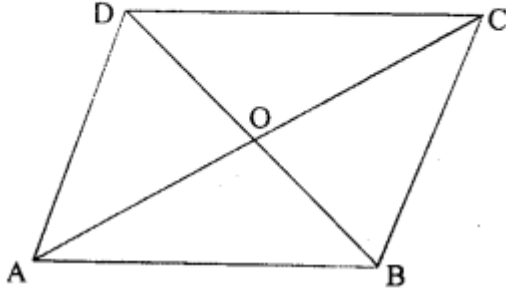
Question 2.

Prove that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution:

Given. In parallelogram ABCD the diagonals AC and BD are cut at point O.

To prove. $ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(\Delta AOD)$



Proof. We know that

In parallelogram ABCD the diagonals are bisecting each other.

$$\therefore AO = OC$$

In ΔACD , O is the mid-point of AC.

$\therefore DO$ is median

$$\therefore ar(\Delta AOD) = ar(\Delta COD) \quad \dots (1)$$

[Median of a triangle divides it into two triangles of equal areas]

Similarly, in ΔABC

$$ar(\Delta AOB) = ar(\Delta COB) \quad \dots (2)$$

In ΔADB

$$ar(\Delta AOD) = ar(\Delta AOB) \quad \dots (3)$$

and in ΔCDB

$$ar(\Delta COD) = ar(\Delta COB) \quad \dots (4)$$

From (1), (2), (3) and (4)

$$ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(\Delta AOD)$$

Hence, diagonals of a parallelogram divide it into four triangles of equal Area. **(Q.E.D.)**

Question 3.

(a) In the figure (1) given below, AD is median of ΔABC and P is any point on AD. Prove that

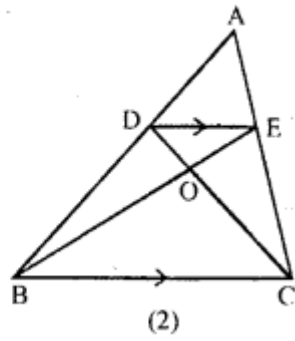
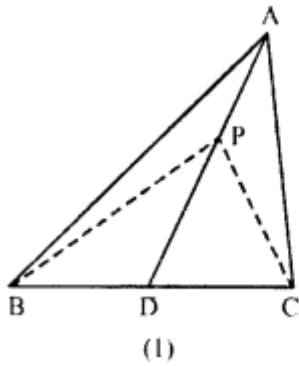
(i) area of $\Delta PBD =$ area of ΔPDC .

(ii) area of $\Delta ABP =$ area of ΔACP .

(b) In the figure (2) given below, $DE \parallel BC$. prove that (i) area of $\Delta ACD =$ area of Δ

ABE.

(ii) area of $\triangle OBD =$ area of $\triangle OCE$.

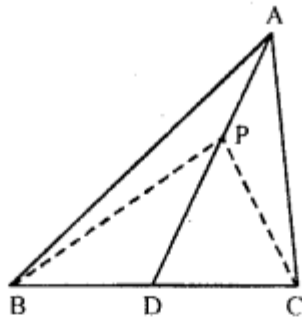


Solution:

(a) Given. A $\triangle ABC$ in which AD is median. P is any point on AD. Join PB and PC.

To prove. (i) $ar(\triangle PBD) = ar(\triangle PDC)$

(ii) $ar(\triangle ABP) = ar(\triangle ACP)$



(1)

Proof. Since, AD is median of $\triangle ABC$

$$\therefore ar(\triangle ABD) = ar(\triangle ADC) \quad \dots (1)$$

(Median of a triangle divides it into two triangles of equal areas)

Also PD is median of $\triangle BPD$

$$\text{Similarly } ar(\triangle PBD) = ar(\triangle PDC) \quad \dots (2)$$

Subtracting (2) from (1),

$$ar(\triangle ABD) - ar(\triangle PBD) = ar(\triangle ADC) - ar(\triangle PDC)$$

$$\text{or, } ar(\triangle ABP) = ar(\triangle ACP) \quad \text{(Q.E.D.)}$$

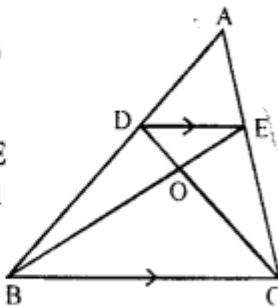
(b) **Given.** $\triangle ABC$ in which $DE \parallel BC$.

To prove.

$$(i) ar(\triangle ACD) = ar(\triangle ABE)$$

$$(ii) ar(\triangle OBD) = ar(\triangle OCE)$$

Proof. (i) $\triangle DEC$ and $\triangle BDE$ are on the same base DE and between the same \parallel line DE and BE.



(2)

$$ar(\triangle DEC) = ar(\triangle BDE)$$

Adding $ar(\triangle ADE)$ to both side,

$$ar(\triangle DEC) + ar(\triangle ADE) = ar(\triangle BDE) + ar(\triangle ADE)$$

$$\Rightarrow ar(\triangle ACD) = ar(\triangle ABE) \quad \text{(Q.E.D.)}$$

$$(ii) ar(\triangle DEC) = ar(\triangle BDE)$$

Subtracting $ar(\triangle DOE)$ from both side,

$$ar(\triangle DEC) - ar(\triangle DOE) = ar(\triangle BDE) - ar(\triangle DOE)$$

$$\Rightarrow ar(\triangle OBD) = ar(\triangle OCE) \quad \text{(Q.E.D.)}$$

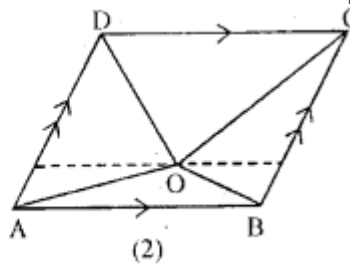
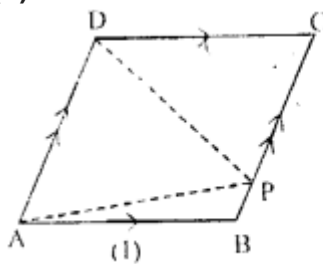
Question 4.

(a) In the figure (1) given below, ABCD is a parallelogram and P is any point in BC. Prove that: Area of $\triangle ABP$ + area of $\triangle DPC$ = Area of $\triangle APD$.

(b) In the figure (2) given below, O is any point inside a parallelogram ABCD. Prove that:

(i) area of $\triangle OAB$ + area of $\triangle OCD$ = $\frac{1}{2}$ area of || gm ABCD.

(ii) area of $\triangle OBC$ + area of $\triangle OAD$ = $\frac{1}{2}$ area of || gm ABCD



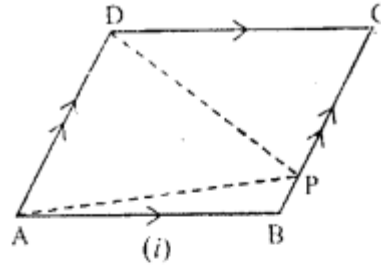
Solution:

(a) **Given.** ABCD is a parallelogram and P is any point in BC.

To prove.

$$ar(\Delta ABP) + ar(\Delta DPC) = ar(\Delta APD)$$

Proof. ΔAPD and $\parallel gm$ ABCD are on the same Base AD and between the same \parallel lines AD and BC,



$$ar(\Delta APD) = \frac{1}{2} ar(\parallel gm ABCD) \quad \dots (1)$$

In parallelogram ABCD

$$ar(\parallel gm ABCD) = ar(\Delta ABP) + ar(\Delta APD) + ar(\Delta DPC)$$

Dividing both sides by 2, we get

$$\frac{1}{2} ar(\parallel gm ABCD) = \frac{1}{2} ar(\Delta ABP) + \frac{1}{2} ar(\Delta APD) + \frac{1}{2} ar(\Delta DPC) \quad \dots (2)$$

From (1) and (2)

$$ar(\Delta APD) = \frac{1}{2} ar(\Delta ABP) + \frac{1}{2} ar(\Delta APD) + \frac{1}{2} ar(\Delta DPC)$$

$$ar(\Delta APD) - \frac{1}{2} ar(\Delta APD) = \frac{1}{2} ar(\Delta ABP) + \frac{1}{2} ar(\Delta DPC)$$

$$\Rightarrow \frac{1}{2} ar(\Delta APD) = \frac{1}{2} [ar(\Delta ABP) + ar(\Delta DPC)]$$

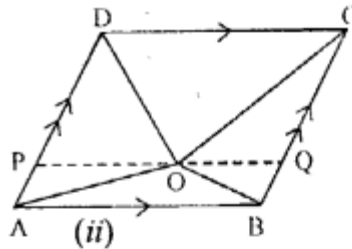
$$\Rightarrow ar(\Delta APD) = ar(\Delta ABP) + ar(\Delta DPC)$$

$$\therefore ar(\Delta ABP) + ar(\Delta DPC) = ar(\Delta APD)$$

(Q.E.D.)

(b) **Given.** \parallel gm ABCD
in which O is any point
inside it.

To prove. (i) ar
(Δ OAB) + ar (Δ OCD)
 $= \frac{1}{2} ar$ (\parallel gm ABCD)



(ii) ar (Δ OBC) + ar (Δ OAD) = $\frac{1}{2} ar$ (\parallel gm ABCD)

Construction. Draw $PQ \parallel AB$ through O. It meets
AD at P and BC at Q.

Proof. (i) $AB \parallel PQ$ and $AP \parallel BQ$

\therefore ABQP is a \parallel gm

Similarly PQCD is a \parallel gm

Now, Δ OAB and \parallel gm ABQP are on same base
AB and between same \parallel lines AB and PQ.

$$\therefore ar(\Delta OAB) = \frac{1}{2} ar(\parallel \text{ gm ABQP}) \quad \dots (1)$$

$$\text{Similarly, } ar(\Delta OCD) = \frac{1}{2} ar(\parallel \text{ gm PQCD}) \quad \dots (2)$$

Adding (1) and (2),

$$\begin{aligned} & ar(\Delta OAB) + ar(\Delta OCD) \\ &= \frac{1}{2} ar(\parallel \text{ gm ABQP}) + \frac{1}{2} ar(\parallel \text{ gm PQCD}) \\ &\Rightarrow ar(\Delta OAB) + ar(\Delta OCD) \\ &= \frac{1}{2} [ar(\parallel \text{ gm ABQP}) + ar(\parallel \text{ gm PQCD})] \end{aligned}$$

$$\Rightarrow ar(\Delta OAB) + ar(\Delta OCD) = \frac{1}{2} ar(\parallel \text{ gm ABCD})$$

(Q.E.D.)

$$(iii) \quad \because ar(\Delta OAB) + ar(\Delta OBC) + ar(\Delta OCD) + ar(\Delta OAD) = ar(\parallel gm ABCD)$$

$$\Rightarrow [ar(\Delta OAB) + ar(\Delta OCD)] + [ar(\Delta OBC) + ar(\Delta OAD)] = ar(\parallel gm ABCD)$$

$$\Rightarrow \frac{1}{2} ar(\parallel gm ABCD) + ar(\Delta OBC) + ar(\Delta OAD) = ar(\parallel gm ABCD)$$

$$\Rightarrow ar(\Delta OBC) + ar(\Delta OAD)$$

$$= ar(\parallel gm ABCD) - \frac{1}{2} ar(\parallel gm ABCD)$$

$$\Rightarrow ar(\Delta OBC) + ar(\Delta OAD) = \frac{1}{2} ar(\parallel gm ABCD)$$

(Q.E.D.)

Question 5.

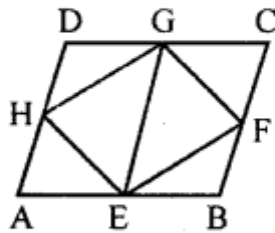
If E, F, G and H are mid-points of the sides AB, BC, CD and DA respectively of a parallelogram ABCD, prove that area of quad. EFGH = 1/2 area of $\parallel gm$ ABCD.

Solution:

Given : In parallelogram ABCD, E, F, G, H are the mid-points of its sides AB, BC, CD and DA respectively EF, FG, GH and HE are joined

To prove : Area of quad. EFGH = $\frac{1}{2}$ area $\parallel\text{gm}$ ABCD

Construction : Join EG



Proof : \because E and G are mid-points of AB and CD respectively

\therefore EG \parallel AD \parallel BC

\therefore AEGD and EBCG are parallelogram

Now $\parallel\text{gm}$ AEGD and Δ EHG are on the same base and between the parallel lines

$$\therefore \text{area } \Delta\text{EHG} = \frac{1}{2} \text{ area } \parallel\text{gm AEGD}$$

Similarly,

$$\text{area } \Delta\text{EFG} = \frac{1}{2} \text{ area } \parallel\text{gm EBCG}$$

Adding (i) and (ii),

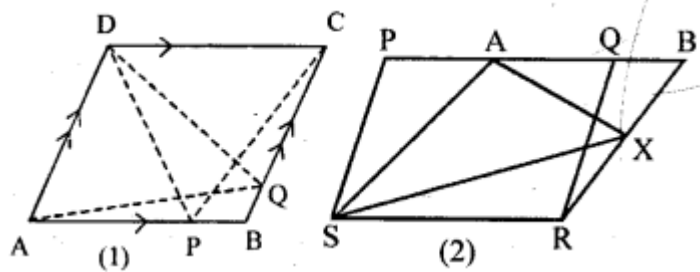
$$\text{area } \Delta\text{EHG} + \text{area } \Delta\text{EFG} = \frac{1}{2} \text{ area } \parallel\text{gm AEGD} + \text{area } \parallel\text{gm EBCG}$$

$$\Rightarrow \text{area quad. EFGH} = \frac{1}{2} \text{ area } \parallel\text{gm ABCD.}$$

Hence proved.

Question 6.

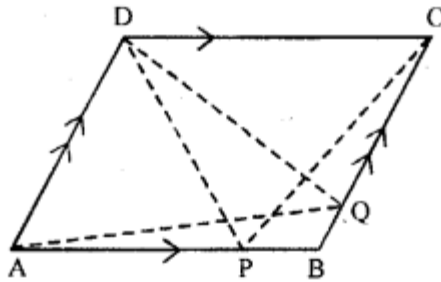
(a) In the figure (1) given below, ABCD is a parallelogram. P, Q are any two points on the sides AB and BC respectively. Prove that, area of Δ CPD = area of Δ AQD.



(b) In the figure (2) given below, PQRS and ABRS are parallelograms and X is any point on the side BR. Show that area of $\triangle AXS = \frac{1}{2}$ area of ||gm PQRS

Solution:

(a) Given. \parallel gm ABCD in which P is a point on AB and Q is a point on BC.



To prove. (i) $ar(\Delta CPD) = ar(\Delta AQD)$

Proof. ΔCPD and \parallel gm ABCD are on the same base CD and between the same parallels AB and CD.

$$\therefore ar(\Delta CPD) = \frac{1}{2} ar(\parallel \text{ gm ABCD}) \dots (1)$$

ΔAQD and \parallel gm ABCD are on the same base AD and between the same \parallel lines AD and BC,

$$\therefore ar(\Delta AQD) = \frac{1}{2} ar(\parallel \text{ gm ABCD}) \dots (2)$$

From (1) and (2),

$$ar(\Delta CPD) = ar(\Delta AQD)$$

Hence, area of $\Delta CPD =$ area of ΔAQD .

(Q.E.D.)

(b) Given : PQRS and ABRS are parallelogram on the same base SR. X is any point on BR. AX and SX are joined.

$$\text{To prove : area } \Delta AXS = \frac{1}{2} \text{ area } \parallel \text{ gm PQRS}$$

$\therefore \parallel$ gm PQRS and ABRS are on the same base SR and between the same parallels

$$\therefore \text{ area } \parallel \text{ gm PQRS} = \text{ area } \parallel \text{ gm ABRS} \dots (i)$$

$\therefore \Delta AXS$ and \parallel gm ABRS are on the same base AS and between the same parallels

$$\therefore \text{ area } \Delta AXS = \frac{1}{2} \text{ area } \parallel \text{ gm ABRS}$$

$$= \frac{1}{2} \text{ area } \parallel \text{ gm PQRS} \quad [\text{From } (i)]$$

Question 7.

D, E and F are mid-point of the sides BC, CA and AB respectively of a $\triangle ABC$. Prove that

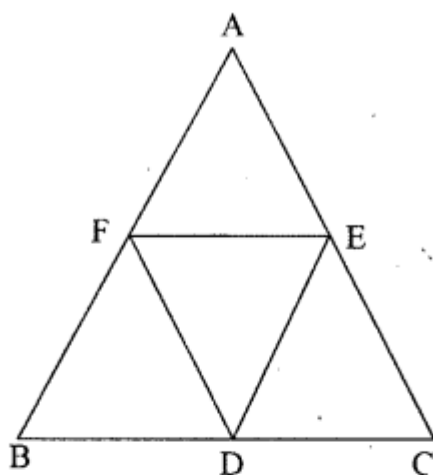
(i) FDCE is a parallelogram

(ii) area of $ADEF = \frac{1}{4}$ area of $\triangle ABC$

(iii) area of $\parallel gm$ FDCE = $\frac{1}{2}$ area of $\triangle ABC$.

Solution:

Given. D, E, F are mid-points of the sides BC, CA, and AB respectively of a ΔABC .



To prove. (i) FDCE is a parallelogram.

(ii) Area of $\Delta DEF = \frac{1}{4}$ area of ΔABC .

(iii) Area of $\parallel gm FDCE = \frac{1}{2}$ area of ΔABC .

Proof. \because F and E are mid-points of AB and AC respectively.

$$\therefore FE \parallel BC \text{ and } FE = \frac{1}{2} BC \quad \dots (1)$$

Also D is mid-point of BC

$$CD = \frac{1}{2} BC \quad \dots (2)$$

From (1) and (2)

$$FE \parallel BC \text{ and } FE = CD$$

$$\text{i.e. } FE \parallel CD \text{ and } FE = CD \quad \dots (3)$$

Similarly D and F are mid-points of BC and AB respectively.

$$\therefore DF \parallel EC \text{ is a } \parallel gm$$

(Q.E.D.)

(ii) Since FDCE is a $\parallel gm$

And DE is diagonal of $\parallel gm FDCE$

$$\therefore ar(\Delta DEF) = ar(\Delta DEC) \quad \dots (5)$$

[\because A diagonal of a parallelogram divides it into two triangles of equal areas]

Similarly, we show BDEF and DEAF are $\parallel gm$ and

$$ar(\triangle DEF) = ar(\triangle BDF) = ar(\triangle AFE) \quad \dots (6)$$

From (5) and (6)

$$ar(\triangle DEF) = ar(\triangle BDF) = ar(\triangle AFE) = ar(\triangle DEC)$$

$$\text{Now, } ar(\triangle ABC) = ar(\triangle BDF) + ar(\triangle DEF) + ar(\triangle DEC) + ar(\triangle AFE)$$

$$\Rightarrow ar(\triangle ABC) = ar(\triangle DEF) + ar(\triangle DEF) + ar(\triangle DEF) + ar(\triangle DEF)$$

$$\Rightarrow ar(\triangle ABC) = 4 ar(\triangle DEF)$$

$$\Rightarrow ar(\triangle DEF) = \frac{1}{4} ar(\triangle ABC)$$

$$\therefore \text{ area of } \triangle DEF = \frac{1}{4} \text{ area of } \triangle ABC \quad \dots (7)$$

(Q.E.D.)

$$(iii) \text{ Area of } \parallel \text{ gm FDCE} = ar(\triangle DEF) + ar(\triangle DEC)$$

$$= ar(\triangle DEF) + ar(\triangle DEF)$$

$$= 2ar(\triangle DEF) \quad [\text{From (5)}]$$

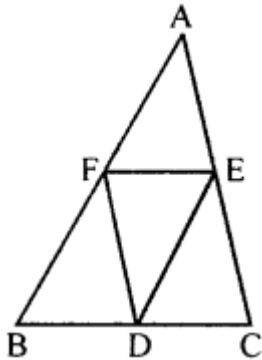
$$= 2 \left[\frac{1}{4} ar(\triangle ABC) \right] \quad [\text{From (7)}]$$

$$\therefore \text{ Area of } \parallel \text{ gm FDCE} = \frac{1}{2} \text{ area of } \triangle ABC$$

(Q.E.D.)

Question 8.

In the given figure, D, E and F are mid points of the sides BC, CA and AB respectively of $\triangle ABC$. Prove that BCEF is a trapezium and area of trap. BCEF = $\frac{3}{4}$ area of $\triangle ABC$.



Solution:

Given : In ΔABC , D, E and F are the mid-points of the sides BC, CA and AB respectively and are joined in order.

To prove : Area trapezium BCEF = $\frac{3}{4}$ area ΔABC .

Proof : \because D and E are the mid-points of BC and CA respectively.

$$\therefore DE \parallel AB \text{ and } \frac{1}{2} AB$$

$$\text{Similarly, } EF \parallel BC \text{ and } \frac{1}{2} BC$$

$$\text{and } FD \parallel AC \text{ and } \frac{1}{2} AC$$

\therefore BDEF, CEFD and AFDE are parallelograms which are equal in area.

ED, DE and EF are the diagonals of these ||gms which divide corresponding parallelogram into two triangles equal in area.

Now, area of trapezium BCEF has the equal triangles and ΔABC has 4 equal triangles.

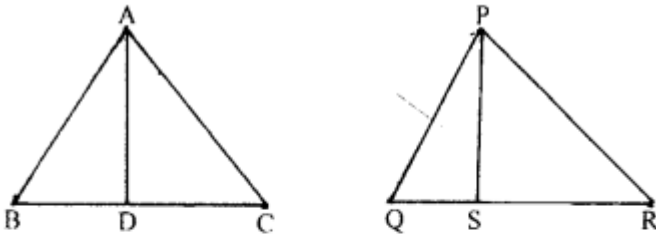
$$\therefore \text{ area of trap. BCEF} = \frac{3}{4} \text{ area } (\Delta ABC)$$

Question P.Q.

Prove that two triangles having equal areas and having one side of one of the triangles equal to one side of the other, have their corresponding altitudes equal.

Solution:

Given. Area of $\triangle ABC =$ area of $\triangle PQR$
 Also $BC = QR$



To prove. $AD = PS$, where AD and PS are Altitudes of $\triangle ABC$ and $\triangle PQR$ respectively.

Proof. Area of $\triangle ABC =$ Area of $\triangle PQR$ (1)

Now, area of $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{AD}$ (2)

Area of $\triangle PQR = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$\Rightarrow \text{ar}(\triangle PQR) = \frac{1}{2} \times QR \times PS$ (3)

From (1), (2) and (3),

$$\frac{1}{2} \times BC \times AD = \frac{1}{2} \times QR \times PS$$

$$\text{or } BC \times AD = QR \times PS$$

$$\text{or } QR \times AD = QR \times PS \quad [BC = QR \text{ (given)}]$$

$$\text{or } AD = PS$$

i.e. Altitude of $\triangle ABC =$ Altitude of $\triangle PQR$

(Q.E.D.)

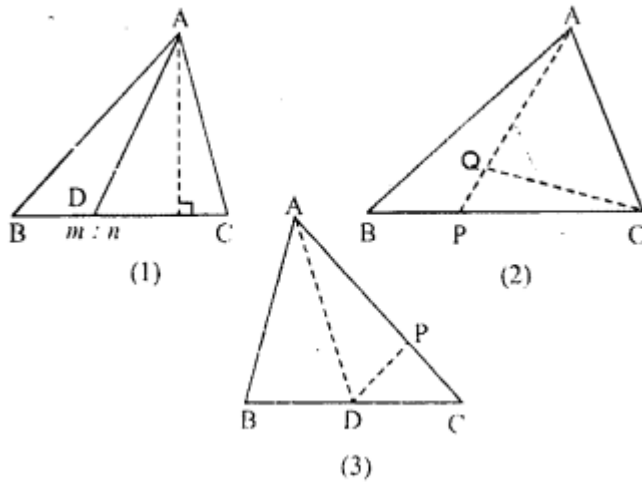
Question 9.

(a) In the figure (1) given below, the point D divides the side BC of $\triangle ABC$ in the ratio $m : n$. Prove that area of $\triangle ABD$: area of $\triangle ADC = m : n$.

(b) In the figure (2) given below, P is a point on the side BC of $\triangle ABC$ such that $PC = 2BP$, and Q is a point on AP such that $QA = 5PQ$, find area of $\triangle AQC$: area of $\triangle ABC$.

(c) In the figure (3) given below, AD is a median of $\triangle ABC$ and P is a point in AC such that area of $\triangle ADP$: area of $\triangle ABD = 2:3$. Find

(i) $AP : PC$ (ii) area of $\triangle PDC$: area of $\triangle ABC$.



Solution:

Given. In $\triangle ABC$, D divides the side BC in the ratio $m : n$ i.e $BD : DC = m : n$

To prove. Area of $\triangle ABD : \text{Area of } \triangle ADC = m : n$

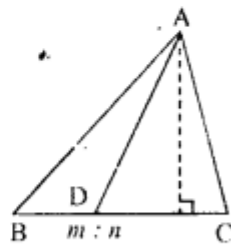
Proof. Area of $\triangle ABD = \frac{1}{2} \times \text{Base} \times \text{height}$

$$\Rightarrow \text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AE \quad \dots (1)$$

$$\text{Area of } (\triangle ACD) = \frac{1}{2} \times DC \times AE \quad \dots (2)$$

Dividing (1) by (2)

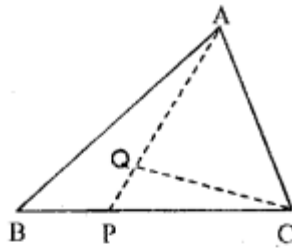
$$\frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle ACD)} = \frac{\frac{1}{2} \times BD \times AE}{\frac{1}{2} \times DC \times AE}$$



$$\Rightarrow \frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle ACD)} = \frac{BD}{DC} = \frac{m}{n} \quad (1)$$

[given $BD : DC = m : n$]
(Q.E.D.)

(b) **Given.** In $\triangle ABC$, P is a point on side BC such that $PC = 2BP$ and Q is a point on AP such that $QA = 5PQ$.



Required. Area of $\triangle AQC$:

Area of $\triangle ABC$

Sol. $PC = 2BP$ [given]

But $BC = BP + PC$

$$\Rightarrow BC = \frac{PC}{2} + PC \quad \left[\because BP = \frac{PC}{2} \right]$$

$$\Rightarrow BC = \frac{PC + 2PC}{2} \Rightarrow BC = \frac{3PC}{2}$$

$$\Rightarrow PC = \frac{2}{3}BC$$

$$\therefore \text{Area of } \triangle APC = \frac{2}{3} \text{Area of } \triangle ABC \quad \dots (1)$$

$$QA = 5PQ \quad \text{(given)}$$

$$\Rightarrow AQ = \frac{5}{6}AP \quad [\because AQ = AP - PQ]$$

$$\Rightarrow \text{Area of } \triangle AQC = \frac{5}{6} \text{Area of } \triangle APC$$

$$= \frac{5}{6} \times \left(\frac{2}{3} \text{Area of } \triangle ABC \right) \quad \text{[From (1)]}$$

$$= \frac{5}{9} \text{Area of } \triangle ABC$$

$$\therefore \frac{\text{Area of } \triangle AQC}{\text{Area of } \triangle ABC} = \frac{5}{9}$$

Hence, Area of $\triangle AQC$: Area of $\triangle ABC$
 $= 5 : 9$ **Ans.**

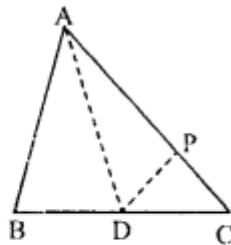
(c) **Given.** AD is a median of $\triangle ABC$. P is a point on AC such that

Area of $\triangle ADP$: area of $\triangle ABD$
= 2 : 3

Required. (i) AP : PC

(ii) Area of $\triangle PDC$: area of $\triangle ABC$

Sol. (i) Since AD is median of $\triangle ABC$



\therefore Area of $\triangle ABD$ = Area of $\triangle ADC$ = $\frac{1}{2}$ area
of $\triangle ABC$ (1)

[\because Median divides a triangle into two triangle of equal area]

Given. Area of $\triangle ADP$: Area of $\triangle ABD = 2 : 3$
(given)

$$\Rightarrow \text{Area of } \triangle ADP : \text{Area of } \triangle ADC = 2 : 3$$

$$\Rightarrow AP : AC = 2 : 3$$

$$\Rightarrow \frac{AP}{AC} = \frac{2}{3} \Rightarrow AP = \frac{2}{3} AC$$

$$\text{Now, } PC = AC - AP = AC - \frac{2}{3} AC = \frac{AC}{3} \dots (2)$$

$$\therefore \frac{AP}{PC} = \frac{\frac{2}{3} AC}{\frac{AC}{3}} = \frac{2}{1}$$

$$\Rightarrow AP : PC = 2 : 1$$

(ii) From (2) $PC = \frac{AC}{3}$

$$\Rightarrow \frac{PC}{AC} = \frac{1}{3}$$

Since the base AC, of $\triangle PDC$, $\triangle ADC$ lie along the same line, and these triangles have equal heights, therefore,

$$\frac{\text{Area of } \triangle PDC}{\text{Area of } \triangle ADC} = \frac{PC}{AC}$$

$$\Rightarrow \frac{\text{Area of } \triangle PDC}{\text{Area of } \triangle ADC} = \frac{1}{3}$$

$$\Rightarrow \frac{\text{Area of } \triangle PDC}{\frac{1}{2} \text{area of } \triangle ABC} = \frac{1}{3} \quad [\text{From (1)}]$$

$$\Rightarrow \frac{\text{Area of } \triangle PDC}{\text{Area of } \triangle ABC} = \frac{1}{3} \times \frac{1}{2}$$

$$\Rightarrow \frac{\text{Area of } \triangle PDC}{\text{Area of } \triangle ABC} = \frac{1}{6}$$

Hence, area of $\triangle PDC$: area of $\triangle ABC = 1 : 6$

Question 10.

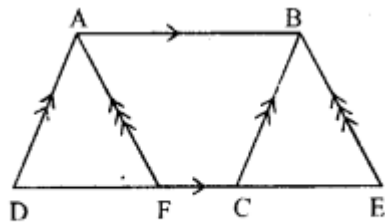
(a) In the figure (1) given below, area of parallelogram ABCD is 29 cm². Calculate the height of parallelogram ABEF if AB = 5.8 cm

(b) In the figure (2) given below, area of $\triangle ABD$ is 24 sq. units. If AB = 8 units, find the height of ABC.

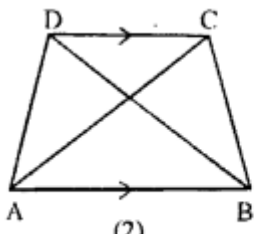
(c) In the figure (3) given below, E and F are mid points of sides AB and CD respectively of parallelogram ABCD. If the area of parallelogram ABCD is 36 cm².

(i) State the area of $\triangle APD$.

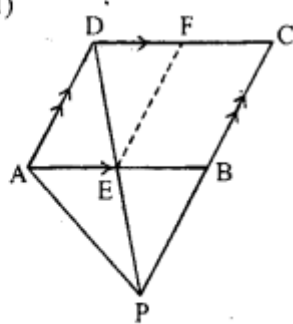
(ii) Name the parallelogram whose area is equal to the area of $\triangle APD$.



(1)



(2)



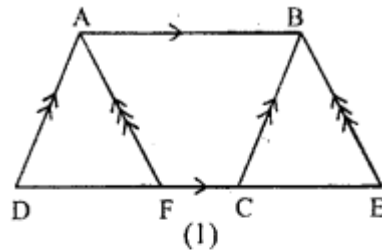
(3)

Solution:

Given. Area of \parallel gm ABCD = 29 cm^2

Required. Height of parallelogram ABEF if $AB = 5.8 \text{ cm}$.

Sol. Since \parallel gm ABCD and \parallel gm ABEF with equal Bases and between the same parallels so that their area are same.



$$\therefore \text{ar}(\parallel \text{ gm ABEF}) = \text{ar}(\parallel \text{ ABCD})$$

$$\Rightarrow \text{ar}(\parallel \text{ gm ABEF}) = 29 \text{ cm}^2 \quad \dots (1)$$

$$[\text{ar}(\parallel \text{ gm ABCD} = 29 \text{ cm}^2 \text{ given})]$$

Also $\text{ar}(\parallel \text{ gm ABEF}) = \text{Base} \times \text{height}$

$$\Rightarrow 29 = AB \times \text{height} \quad [\text{From (1)}]$$

$$\Rightarrow 29 = 5.8 \times \text{height} \quad [AB = 5.8 \text{ (given)}]$$

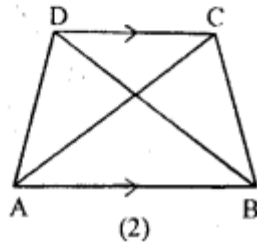
$$\Rightarrow \text{height} = \frac{29}{5.8} = \frac{29 \times 10}{58} = \frac{10}{2} = 5$$

\therefore Height of parallelogram ABEF = 5cm

(b) Given. Area of $\Delta ABD = 24 \text{ sq. units} = AB = 8 \text{ units}$.

Required. Height of ΔABC .

Sol. Area of $\Delta ABD = 24 \text{ sq. units}$... (1)



$$\therefore \text{Area of } \triangle ABD = \text{Area of } \triangle ABC \quad \dots (2)$$

(\because Triangles on the same base and between the same parallels are equal in area)

From (1) and (2),

$$\text{Area of } \triangle ABC = 24 \text{ sq. units.}$$

$$\Rightarrow \frac{1}{2} \times AB \times \text{height} = 24$$

$$\Rightarrow \frac{1}{2} \times 8 \times \text{height} = 24 \Rightarrow \text{height} = \frac{24 \times 2}{8}$$

$$\Rightarrow \text{height} = 3 \times 2 = 6$$

Hence, height of $\triangle ABC = 6$ Units.

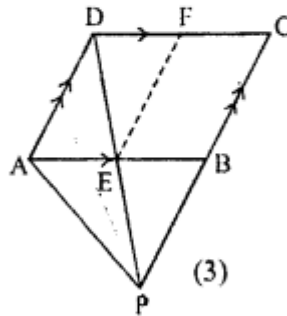
(c) **Given.** In \parallel gm ABCD, E and F are mid-point of sides AB and CD respectively.

$$ar(\parallel \text{ gm ABCD}) = 36 \text{ cm}^2$$

Required : (i) $ar(\triangle APD)$

(ii) name the \parallel gm whose area is equal to the area of $\triangle APD$.

Sol. $\triangle APD$ and \parallel gm ABCD are on the same base AD and between the same \parallel lines AD and BC.



$$\therefore ar(\triangle APD) = \frac{1}{2} ar(\parallel \text{ gm ABCD}) \quad \dots (1)$$

$$\text{But } ar(\parallel \text{ gm ABCD}) = 36 \text{ cm}^2 \quad \dots (2)$$

From (1) and (2),

$$ar(\triangle APD) = \frac{1}{2} \times 36 \text{ cm}^2$$

$$ar(\triangle APD) = 18 \text{ cm}^2$$

(ii) E and F are mid-points of AB and CD

In $\triangle CPD$, $EF \parallel PC$.

Also EF bisect the \parallel gm ABCD in two equal parts.

Now, $EF \parallel AD$ and $AE \parallel DF$

\therefore AEFD is a \parallel gm.

$$\therefore \text{ar}(\parallel \text{ gm AEFD}) = \frac{1}{2} \text{ar}(\parallel \text{ gm ABCD}) \quad \dots(3)$$

From (1) and (3),

$$\text{ar}(\triangle APD) = \text{ar}(\parallel \text{ gm AEFD})$$

\therefore AEFD is the required \parallel gm which is equal to the area of $\triangle APD$. (Q.E.D.)

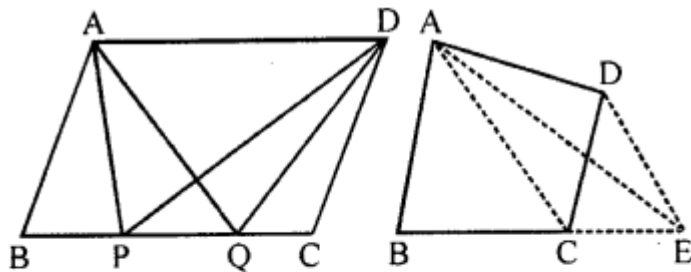
Question 11.

(a) In the figure (1) given below, ABCD is a parallelogram. Points P and Q on BC trisect BC into three equal parts. Prove that :

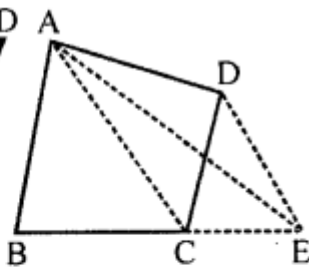
$$\text{area of } \triangle APQ = \text{area of } \triangle DPQ = \frac{1}{6} (\text{area of } \parallel \text{ gm ABCD})$$

(b) In the figure (2) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of $\triangle ABE$.

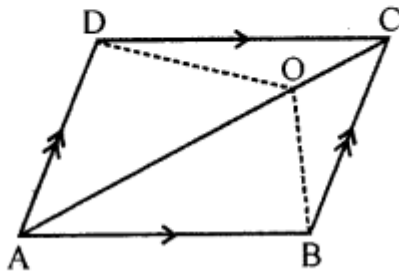
(c) In the figure (3) given below, ABCD is a parallelogram. O is any point on the diagonal AC of the parallelogram. Show that the area of $\triangle AOB$ is equal to the area of $\triangle AOD$.



(1)



(2)



(3)

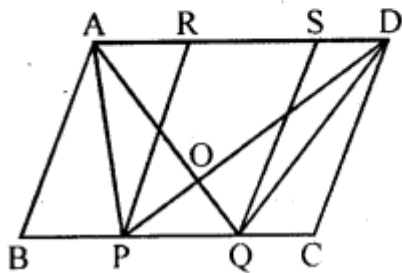
Solution:

(a) **Given** : In $\parallel\text{gm}$ ABCD, points P and Q trisect BC into three equal parts.

To prove : $\text{area } (\triangle APQ) = \text{area } (\triangle DPQ) =$

$$\frac{1}{6} \text{ area } \parallel\text{gm ABCD}.$$

Construction : Through P and Q, draw PR and QS parallel to AB and CD.



Proof : Now, ΔAPD and ΔAQD lie on the same base AD and between the same parallel AD and BC.

$$\therefore \text{ar}(\Delta APD) = \text{ar}(\Delta AQD)$$

$$\text{ar}(\Delta APD) - \text{ar}(\Delta AOD) = \text{ar}(\Delta AQD) - \text{ar}(\Delta AOD)$$

[on subtracting $\text{ar}(\Delta AOD)$ from both sides]

$$\Rightarrow \text{ar}(\Delta APO) = \text{ar}(\Delta OQD) \quad \dots(i)$$

$$\Rightarrow \text{ar}(\Delta APO) + \text{ar}(\Delta OPQ) = \text{ar}(\Delta OQD) + \text{ar}(\Delta OPQ)$$

[on adding $\text{ar}(\Delta OPQ)$ on both sides]

$$\text{ar}(\Delta APQ) = \text{ar}(\Delta DPQ) \quad \dots(ii)$$

Again, ΔAPQ and parallelogram PQSR are on the same base PQ and between same parallels PQ and AD.

$$\therefore \text{ar}(\Delta APQ) = \frac{1}{2} \text{ar}(\text{||gm PQRS}) \quad \dots(iii)$$

Now,

$$\frac{\text{ar}(\text{|| gm ABCD})}{\text{ar}(\text{|| gm PQRS})} = \frac{BC \times \text{height}}{PQ \times \text{height}} = \frac{3PQ \times \text{height}}{PQ \times \text{height}} = 3$$

$$\text{ar}(\text{||gm PQRS}) = \frac{1}{3} \text{ar}(\text{||gm ABCD}) \quad \dots(iv)$$

Using (ii), (iii) and (iv), we get

$$\text{ar}(\Delta APQ) = \text{ar}(\Delta DPQ)$$

$$= \frac{1}{2} \text{ar}(\text{||gm PQRS})$$

$$= \frac{1}{2} \times \frac{1}{3} \text{ar}(\text{||gm ABCD})$$

$$\Rightarrow \text{ar}(\Delta APQ) = \text{ar}(\Delta DPQ) = \frac{1}{6} \text{ar}(\text{||gm ABCD})$$

Hence proved.

(b) **Given :** In the given figure, $DE \parallel AC$ the diagonal of quadrilateral $ABCD$ which meets at E on producing BC . AC, AE are joined.

To prove : Area of quadrilateral $ABCD = \text{area } \triangle ABE$.

Proof : $\triangle ACE$ and $\triangle ADE$ are on the same base AC and between the same parallel lines.

$$\therefore \text{area } \triangle ACE = \text{area } \triangle ADC$$

Adding area $\triangle ABC$ to both sides

$$\text{area } \triangle ACE + \text{area } \triangle ABC$$

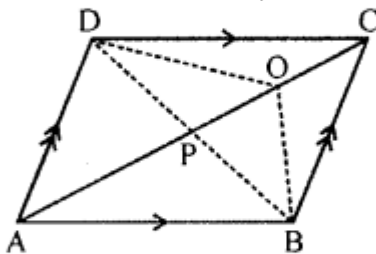
$$= \text{area } \triangle ADC + \text{area } \triangle ABC$$

$$\Rightarrow \text{area } \triangle ABE = \text{area quad. } ABCD$$

(c) **Given :** In $\parallel\text{gm } ABCD$, O is any point on diagonal AC .

To prove : $\text{area } \triangle AOB = \text{area } \triangle AOD$

Construction : Join BD which meets AC at P .



Proof : In $\triangle ABD$, AP is median

(\because Diagonals of a $\parallel\text{gm}$ bisect each other)

$$\therefore \text{area } \triangle ABP = \text{area } \triangle ADP \quad \dots(i)$$

$$\text{Similarly, area } \triangle PBO = \text{area } \triangle PDO \quad \dots(ii)$$

Adding, (i) and (ii), we get

$$\text{area } \triangle ABO = \text{area } \triangle ADO \quad \dots(iii)$$

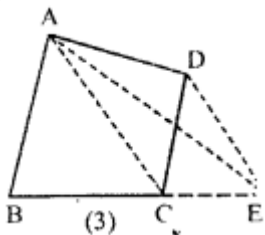
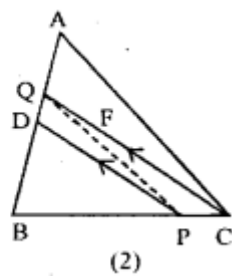
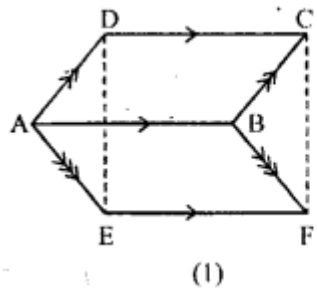
$$\Rightarrow \text{area } \triangle AOB = \text{area } \triangle AOD$$

Question P.Q.

(a) In the figure (1) given below, two parallelograms $ABCD$ and $AEFB$ are drawn on opposite sides of AB , prove that: area of $\parallel\text{gm } ABCD + \text{area of } \parallel\text{gm } AEFB = \text{area of } \parallel\text{gm } EFCD$.

(b) In the figure (2) given below, D is mid-point of the side AB of $\triangle ABC$. P is any point on BC , CQ is drawn parallel to PD to meet AB in Q . Show that area of $\triangle BPQ = \frac{1}{2}$ area of $\triangle ABC$.

(c) In the figure (3) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of $\triangle ABE$.



Solution:

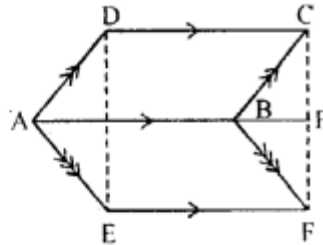
(a) **Given.** ABCD and AEFB are two || gm on opposite sides of AB.

To prove. $ar(\text{|| gm ABCD}) + ar(\text{|| gm AEFB}) + ar(\text{|| gm EFCD})$

Construction.

Produced AB to meet CE at P.

Proof. || gm PQDC and || gm ABCD are on the same base CD and between same || lines.



$$\therefore ar(\text{|| gm PQDC}) = ar(\text{|| gm ABCD}) \quad \dots (1)$$

Again || gm PQEF and || gm AEFB are on the same base EF and between same || lines

$$\therefore ar(\text{|| gm PQEF}) = ar(\text{|| gm AEFB}) \quad \dots (2)$$

Adding (1) and (2)

$$ar(\text{|| gm PQDC}) + ar(\text{|| gm PQEF}) = ar(\text{|| gm ABCD}) + ar(\text{|| gm AEFB})$$

$$\Rightarrow ar(\text{|| gm EFCD}) = ar(\text{|| gm ABCD}) + ar(\text{|| gm AEFB})$$

Hence, area of || gm ABCD + area of || gm AEFB = area of || gm EFCD. **(Q.E.D.)**

(b) **Given.** A ΔABC , in which D is mid-point of the side AB. P is any point on BC, CQ || PD to meet AB in Q.

To prove. $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$

Const. Join CD.

Proof.

\therefore CD is median of ΔABC

$$\therefore ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$$

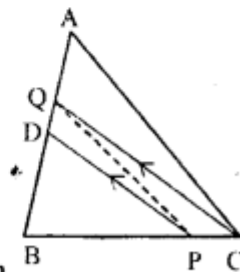
(\because Median divides a triangle into two triangles of equal area)

Now, ΔDPQ and ΔDPC are on the same Base DP and between the same parallel lines DP and QC.

$$\therefore ar(\Delta DPQ) = ar(\Delta DPC) \quad \dots (2)$$

From (1),

$$ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$$



$$\text{or } ar(\Delta BPD) + ar(\Delta DPC) = \frac{1}{2} ar(\Delta ABC)$$

$$\text{or } ar(\Delta BPD) + ar(\Delta DPQ) = \frac{1}{2} ar(\Delta ABC)$$

[From (2)]

$$\text{or } ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$$

Hence, area of $\Delta BPQ = \frac{1}{2}$ area of ΔABC .

(Q.E.D.)

(c) **Given.** ABCD is a || gm.
DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E.

To prove. Area of quad.

ABCD = area of ΔABE

Proof. $DE \parallel BC$ (given)

$\therefore \Delta ACE$ and ΔACD on the same base AC and between the same || lines AC and DE.

\therefore area of $\Delta ACE =$ area of ΔACD

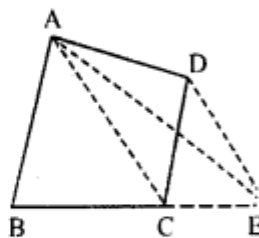
Adding both sides area of ΔABC .

area of $\Delta ACE +$ area of $\Delta ABC =$ area of $\Delta ACD +$ area of ΔABC .

or area of $\Delta ABE =$ area of quad. ABCD

Hence, area of quad. ABCD = area of ΔABE .

(Q.E.D.)

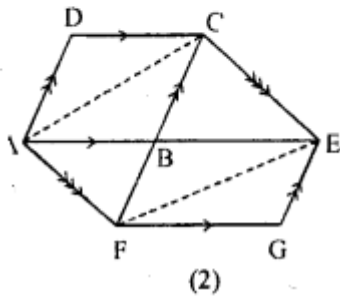
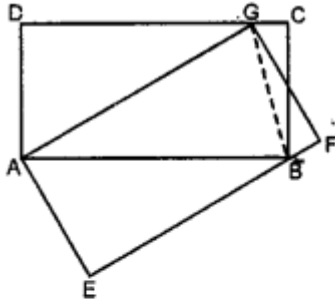


Question 12.

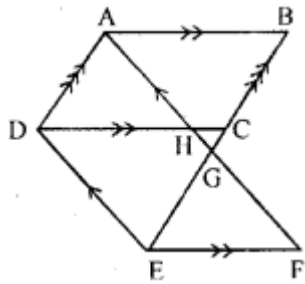
(a) In the figure given, ABCD and AEFG are two parallelograms.

Prove that area of || gm ABCD = area of || gm AEFG.

(b) In the fig. (2) given below, the side AB of the parallelogram ABCD is produced to E. A st. line At through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is Completed prove that area of || gm BFGE = Area of || gm ABCD.



(c) In the figure (3) given below $AB \parallel DC \parallel EF$, $AD \parallel BE$ and $DE \parallel AF$. Prove the area of $DEFH$ is equal to the area of $ABCD$.

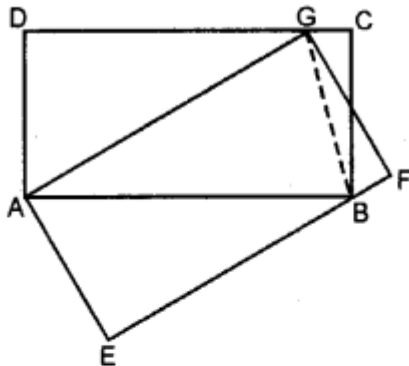


Solution:

(a) **Given.** ABCD and AEFG are two parallelograms as shown in the figure.

To prove. Area ABCD = area AEFG

Construction. Join BG.



Proof. $\therefore \Delta ABG$ and \parallel gm ABCD are on the same base AB and between the same parallels

$$\therefore \text{area } \Delta ABG = \frac{1}{2} (\text{area } \parallel \text{ gm ABCD})$$

...(i)

Similarly ΔABG and \parallel gm AEFG are on the same base AG and between the same parallels.

$$\therefore \text{area } (\Delta ABG) = \frac{1}{2} (\text{area } \parallel \text{ gm AEFG})$$

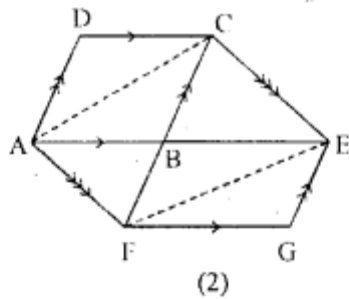
from (i) and (ii)

$$\frac{1}{2} (\text{area } \parallel \text{ gm ABCD}) = \frac{1}{2} (\text{area } \parallel \text{ gm AEFG})$$

$$\therefore \text{area } \parallel \text{ gm ABCD} = \text{area } \parallel \text{ gm AEFG}.$$

Hence proved.

(b) **Given.** A \parallel gm ABCD in which AB is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F, and \parallel gm BFGE is completed.



To prove. area of \parallel gm BFGE = area of \parallel gm ABCD

Construction : Join AC and EF.

Proof. \because ΔAFC and ΔAFE are on the same base AF and between parallel lines AC and EF.

$$\therefore \text{ar}(\Delta AFC) = \text{ar}(\Delta AFE) \quad \dots\dots (1)$$

subtracting both sides $\text{ar}(\Delta ABF)$ $\text{ar}(\Delta AFC) - \text{ar}(\Delta ABF) = \text{ar}(\Delta AFE) - \text{ar}(\Delta ABF)$

$$\text{or } \text{ar}(\Delta ABC) = \text{ar}(\Delta BEF)$$

Multiplying both sides by 2,

$$2 \text{ar}(\Delta ABC) = 2 \text{ar}(\Delta BEF)$$

$$\text{or } \text{ar}(\parallel \text{ gm ABCD}) = \text{ar}(\parallel \text{ gm BFGE})$$

(\because diagonals of \parallel gm divides it into two triangles of equal areas.)

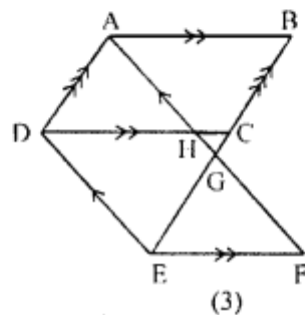
Hence, area of \parallel gm BFGE = area of \parallel gm ABCD.

(Q.E.D.)

(c) **Given.** $DC \parallel EF$, $AD \parallel BE$ and $DE \parallel AF$

To prove. $\text{ar}(\text{DEFH}) = \text{ar}(\text{ABCD})$

Proof. $DE \parallel AF$ and $AD \parallel BE$



\therefore ADEG is a \parallel gm. (given)

Now, \parallel gm ABCD and \parallel gm ADEG are on the same base AD and between the same \parallel lines AD and BE.

$\therefore ar(\parallel \text{ gm ABCD}) = ar(\parallel \text{ gm ADEG}) \dots (1)$

Again DEFG is a \parallel gm

(\because DE \parallel AF and DC \parallel EF (given))

$\therefore \parallel$ gm DEFH and \parallel gm ADEG are on the same base DE and between the same \parallel lines DE and AF.

$\therefore ar(\parallel \text{ gm DEFH}) = ar(\parallel \text{ gm ADEG}) \dots (2)$

From (1) and (2),

$ar(\parallel \text{ gm ABCD}) = ar(\parallel \text{ gm DEFH})$

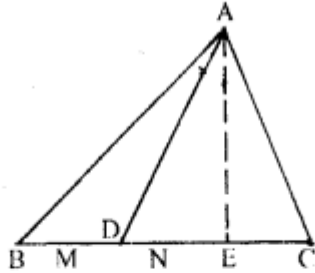
or $ar(\text{ABCD}) = ar(\text{DEFH})$ (Q.E.D.)

Question 13.

Any point D is taken on the side BC of, a $\triangle ABC$ and AD is produced to E such that AD=DE, prove that area of $\triangle BCE =$ area of $\triangle ABC$.

Solution:

Given. In $\triangle ABC$, D is taken on the side BC. AD produced to E such that $AD = DE$.



To prove. Area of $\triangle BCE =$ area of $\triangle ABC$

Proof. In $\triangle ABE$,

$AD = DE$ (Given)

\therefore BD is a median of $\triangle ABE$

$\Rightarrow ar(\triangle ABD) = ar(\triangle BED)$ (1)

[median divides a triangle into two triangles of equal area]

Similarly, in $\triangle ACE$

CD is median of $\triangle ACE$.

$\Rightarrow ar(\triangle ACD) = ar(\triangle CED)$ (2)

Adding (1) and (2),

$ar(\triangle ABD) + ar(\triangle ACD) = ar(\triangle BED) + ar(\triangle CED)$

or, $ar(\triangle ABC) = ar(\triangle BCE)$

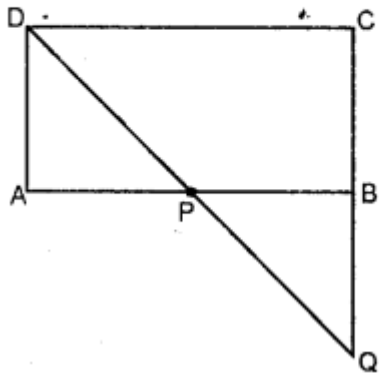
Hence, area of $\triangle BCE =$ area of $\triangle ABC$. (Q.E.D.)

Question 14.

ABCD is a rectangle and P is mid-point of AB. DP is produced to meet CB at Q. Prove that area of rectangle $\triangle BCD =$ area of $\triangle DQC$.

Solution:

Given. ABCD is a rectangle P is mid-point of AB DP is joined and produced meeting CB produced at Q.



To prove. Area rectangle ABCD
= area (Δ DQC)

Proof. In Δ APD and Δ BQP,

$$AP = BP \quad (\because P \text{ is mid-point of } AB)$$

$$\angle DAP = \angle QBP \quad (\text{each } 90^\circ)$$

$$\angle APD = \angle BPQ \text{ (vertically opposite angles)}$$

$$\therefore \Delta APD \cong \Delta BQP \quad (\text{ASA postulate})$$

$$\therefore \text{area } \Delta APD = \text{area } \Delta BQP$$

$$\text{Now area ABCD} = \text{area } \Delta APD$$

$$+ \text{area PBCD}$$

$$= \text{area } \Delta BQP + \text{area PBCD}$$

$$= \text{area } \Delta DQC$$

Hence proved

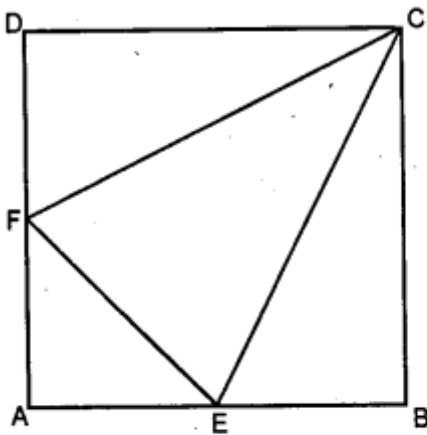
Question P.Q.

ABCD is a square, E and F are mid-points of the sides AB and AD respectively

Prove that area of Δ CEF = $\frac{3}{8}$ (area of square ABCD).

Solution:

Given. ABCD is a square. E and F are the mid-points of sides AB and AD respectively EF, EC and FC are joined.



To prove. area $\triangle CEF = \frac{3}{8}$
(area of square ABCD)

Proof. Let side of square = a
then area of square ABCD = a^2

Now area of $\triangle AEF = \frac{1}{2} AE \times AF$

$$= \frac{1}{2} \times \frac{a}{2} \times \frac{a}{2} = \frac{a^2}{8}$$

$$\text{area of } \triangle EBC = \frac{1}{2} \times EB \times BC$$

$$= \frac{1}{2} \times \frac{a}{2} \times a = \frac{a^2}{4}$$

$$\text{and area of } \triangle CDF = \frac{1}{2} \times CD \times DF$$

$$= \frac{1}{2} \times a \times \frac{a}{2} = \frac{a^2}{4}$$

\therefore Area of $\triangle CEF = \text{area of sq. ABCD} - (\text{area of } \triangle AEF) + \text{area of } \triangle EBC + \text{area of } \triangle CDF)$

$$= a^2 - \left(\frac{a^2}{8} + \frac{a^2}{4} + \frac{a^2}{4} \right)$$

$$= a^2 - \left(\frac{a^2 + 2a^2 + 2a^2}{8} \right) = a^2 - \frac{5a^2}{8}$$

$$= \frac{3}{8}a^2 = \frac{3}{8} \text{ area of sq. ABCD.}$$

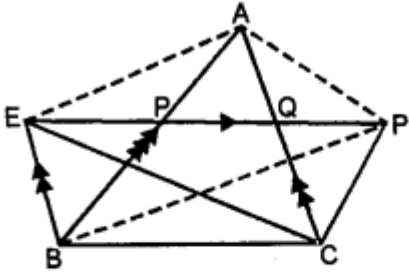
Question P.Q.

A line PQ is drawn parallel to the side BC of $\triangle ABC$. BE is drawn parallel to CA to meet QP (produced) at E and CF is drawn parallel to BA to meet PQ (produced) at F. Prove that

area of $\triangle ABE = \text{area of } \triangle ACF$.

Solution:

Given. A line PQ is drawn parallel to side BC of $\triangle ABC$.



BE \parallel CA and CF \parallel BA drawn which meet PQ produced both sides at E and F respectively
AE, AF and BF are joined.

To prove. area of $\triangle ABE$ = area of $\triangle ACF$

Proof. $\therefore \triangle ABE$ and $\triangle CBE$ are on the same base BE and between the same parallels

$$\therefore \text{area } \triangle ABE = \text{area } \triangle CBE \quad \dots(i)$$

Again $\therefore \triangle ACF$ and $\triangle BCF$ are on the same base CF and between the same parallels

$$\therefore \text{area } \triangle ACF = \text{area } \triangle BCF \quad \dots(ii)$$

But $\triangle CBE$ and $\triangle BCF$ are on the same base BC

and between the same parallels:

$$\therefore \text{area } \triangle CBE = \text{area } \triangle BCF \quad \dots(iii)$$

\therefore from (i), (ii) and (iii)

$$\text{area } \triangle ABE = \text{area } \triangle ACF$$

Hence proved.

Question 15.

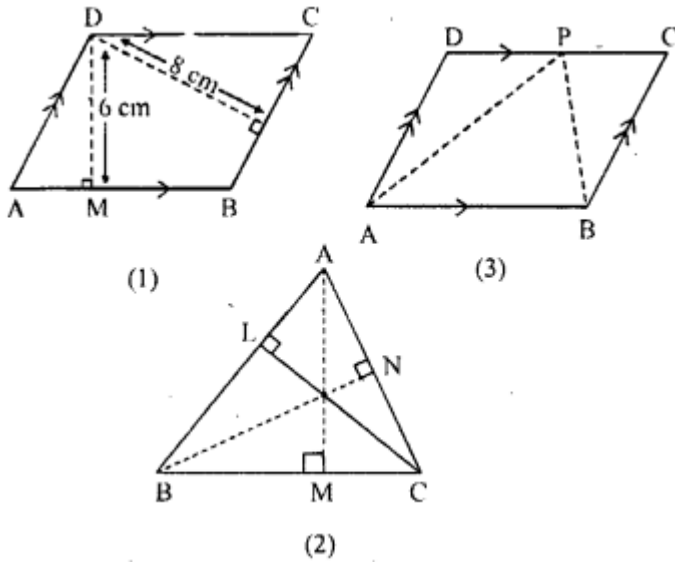
(a) In the figure (1) given below, the perimeter of parallelogram is 42 cm. Calculate the lengths of the sides of the parallelogram.

(b) In the figure (2) given below, the perimeter of $\triangle ABC$ is 37 cm. If the lengths of the altitudes AM, BN and CL are $5x$, $6x$, and $4x$ respectively, Calculate the lengths of the sides of $\triangle ABC$.

(c) In the fig. (3) given below, ABCD is a parallelogram. P is a point on DC such that area of $\triangle DAP = 25 \text{ cm}^2$ and area of $\triangle BCP = 15 \text{ cm}^2$. Find

(i) area of $\parallel \text{gm ABCD}$

(ii) DP : PC.

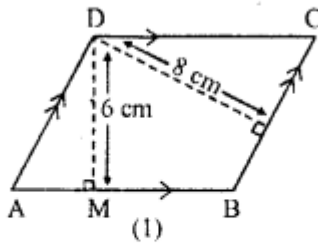


Solution:

(a) Given. Perimeter of \parallel gm ABCD = 42 cm.

Required. Lengths of the sides of \parallel gm ABCD.

Sol. Let $AB = P$



Then, perimeter of \parallel gm = $2 (AB + BC)$

$$\Rightarrow 42 = 2 (P + BC)$$

$$\Rightarrow \frac{42}{2} = P + BC$$

$$\Rightarrow 21 = P + BC$$

$$\Rightarrow BC = 21 - P$$

Area of \parallel gm ABCD = AB \times DM

$$= P \times 6 = 6P \quad \dots\dots(1)$$

(Taking base AB and height DM)

Again, area of \parallel gm ABCD = BC \times DN

(Taking Base BC and height DN)

$$= (21 - P) \times 8 = 8(21 - P) \quad \dots\dots(2)$$

From (1) and (2),

$$6P = 8(21 - P) \Rightarrow 6P = 168 - 8P$$

$$\Rightarrow 6P + 8P = 168 \Rightarrow 14P = 168$$

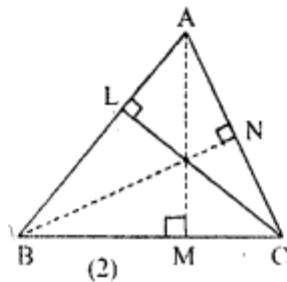
$$\Rightarrow P = \frac{168}{14} = 12$$

Hence, sides of \parallel gm, AB = 12 cm and BC = (21 - 12) cm = 9 cm.

(b) Given. The perimeter of $\Delta ABC = 37$ cm. Length of the Altitudes AM, BN, and CL are $5x$, $6x$, and $4x$ respectively.

Required. Lengths of BC, CA, and AB.

Sol. Let BC = P and CA = Q



Then perimeter of ΔABC
 $= AB + BC + CA$

$$\Rightarrow 37 = AB + P + Q$$

$$\Rightarrow AB = 37 - P - Q$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AM = \frac{1}{2} CA \times BN$$

$$= \frac{1}{2} \times AB \times CL$$

$$\text{i.e. } \frac{1}{2} \times P \times 5x = \frac{1}{2} \times Q \times 6x$$

$$= \frac{1}{2} (37 - P - Q) \times 4x \Rightarrow \frac{5P}{2} = 3Q = 2(37 - P - Q)$$

Taking first two parts

$$\frac{5P}{2} = 3Q \Rightarrow 5P = 6Q \Rightarrow 5P - 6Q = 0 \dots (1)$$

Taking second and third parts

$$3Q = 2(37 - P - Q) \Rightarrow 3Q = 74 - 2P - 2Q$$

$$\Rightarrow 3Q + 2Q + 2P = 74 \Rightarrow 2P + 5Q = 74 \dots (2)$$

Multiplying equation (1) by (5) & (2) by (6), we get

$$\begin{aligned} 25P - 30Q &= 0 \\ 12P + 30Q &= 444 \end{aligned}$$

Adding,
$$\frac{37P}{\quad} = 444$$

$$\Rightarrow P = \frac{444}{37} = 12$$

Substituting the value of P in equation (1), we get

$$5 \times 12 - 6Q = 0 \Rightarrow 60 - 6Q = 0 \Rightarrow 60 = 6Q$$

$$\Rightarrow Q = \frac{60}{6} = 10$$

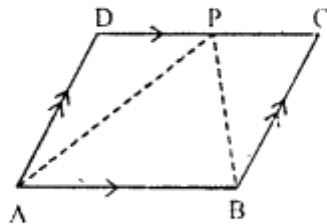
Hence, $BC = P = 12$ cm, $CA = Q = 10$ cm and $AB = 37 - P - Q = 37 - 12 - 10 = 15$ cm.

(c) **Given.** ABCD is a || gm. P is a point on DC such that $ar(\Delta DAP) = 25 \text{ cm}^2$ and $ar(\Delta BCP) = 15 \text{ cm}^2$

Required. (i) $ar(\text{|| gm ABCD})$ (ii) $DP : PC$

Sol. (i) $ar(\Delta APB) = \frac{1}{2} ar(\text{|| gm ABCD})$

(\because Area of a triangle is half that of a || gm on the same base and between the same parallels)



(3)

$$\begin{aligned} \text{Then } \frac{1}{2} ar(\text{|| gm ABCD}) &= ar(\Delta DAP) + ar(\Delta BCP) \\ &= 25\text{cm}^2 + 15\text{cm}^2 = 40\text{cm}^2 \end{aligned}$$

$$\Rightarrow ar(\text{|| gm ABCD}) = 2 \times 40 \text{ cm}^2 = 80 \text{ cm}^2$$

(ii) Since ΔADP and ΔBCP are on the same base CD and between same || lines CD and AB.

$$\therefore \frac{ar(\Delta DAP)}{ar(\Delta BCP)} = \frac{DP}{PC}$$

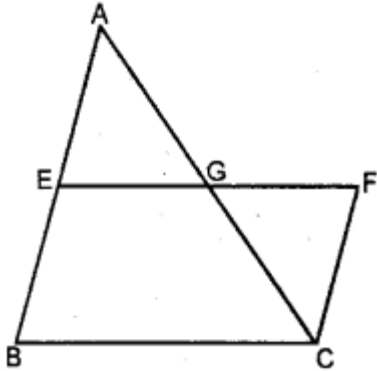
$$\Rightarrow \frac{25}{15} = \frac{DP}{PC} \Rightarrow \frac{DP}{PC} = \frac{25}{15} = \frac{5}{3}$$

$$\Rightarrow DP : PC = 5 : 3$$

Question 16.

In the adjoining figure, E is mid-point of the side AB of a triangle ABC and EBCF is a parallelogram. If the area of $\triangle ABC$ is 25 sq. units, find the area of \parallel gm EBCF.

Solution:



Let EF, side of \parallel gm BCEF meets AC at G.

\therefore E is mid point and $EF \parallel BC$

\therefore G is mid point of AC.

$\Rightarrow AG = GC$

Now in $\triangle AEG$ and $\triangle CFG$,

$\angle EAG, \angle GCF$ (Alternate angles)

$\angle EGA = \angle CGF$

(vertically opposite angles)

$AG = GC$ (proved)

$\therefore \triangle AEG \cong \triangle CFG$

$\Rightarrow \text{area } \triangle AEG = \text{area } \triangle CFG.$

Now

$$\begin{aligned} \text{area } \parallel \text{ gm EBCF} &= \text{area BCGE} + \text{area } \triangle CFG \\ &= \text{area BCGE} + \text{area } \triangle AEG = \text{area } \triangle ABC \end{aligned}$$

But $\text{area } \triangle ABC = 25 \text{ sq. units.}$

$\therefore \text{area } \parallel \text{ gm EBCF} = 25 \text{ sq. units}$

Question 17.

(a) In the figure (1) given below, $BC \parallel AE$ and $CD \parallel BE$. Prove that: area of $\triangle ABC =$ area of $\triangle EBD$.

(b) In the figure (2) given below, ABC is right angled triangle at A. AGFB is a square on the side AB and BCDE is a square on the hypotenuse BC. If $AN \perp ED$, prove that:

(i) $\triangle BCF \cong \triangle ABE.$

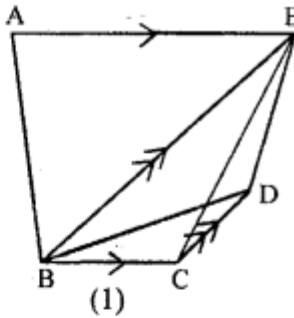
(ii) area of square ABFG = area of rectangle BENM.

(a) Given. $BC \parallel AE$ and $CD \parallel BE$

To prove. Area of $\triangle ABC$
= area of $\triangle EBD$.

Construction. Join CE

Proof. $\triangle ABC$ and $\triangle EBC$
are on the same Base BC
and between the same \parallel
lines AE and BC.



$$\therefore ar(\triangle ABC) = ar(\triangle EBC) \quad \dots (1)$$

$\because \triangle EBC$ and $\triangle EBD$ are on the same base BE
and between same \parallel lines BE and CD.

$$\therefore ar(\triangle EBC) = ar(\triangle EBD) \quad \dots (2)$$

From (1) and (2)

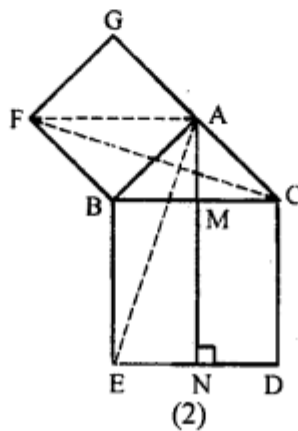
$$ar(\triangle ABC) = ar(\triangle EBD)$$

Hence, area of $\triangle ABC$ = area of $\triangle EBD$ (Q.E.D.)

(b) Given. A right angled $\triangle ABC$ in which
 $\angle A = 90^\circ$. Squares AGFB and BCDE
are drawn on the side AB and hypotenuse BC
of $\triangle ABC$. $AN \perp ED$ meeting BC at M.

To Prove: (i) $\triangle BCF \cong \triangle ABE$

(ii) area of square ABFG = area of rectangle BENM



Solution:

Proof: (i) $\angle FBC = \angle FBA + \angle ABC$

$$\Rightarrow \angle FBC = 90^\circ + \angle ABC \quad \dots (1)$$

$$\angle ABE = \angle EAC + \angle ABC$$

$$\angle ABE = 90^\circ + \angle ABC \quad \dots (2)$$

From (1) and (2),

$$\angle FBC = \angle ABE \quad \dots (3)$$

Now, in $\triangle BCF$ and $\triangle ABE$

$$BF = AB$$

$$\angle FBC = \angle ABE \quad \text{[From (3)]}$$

$$BC = BE$$

$$\therefore \triangle BCF \cong \triangle ABE$$

(By S.A.S. axiom of congruency)

(ii) $\triangle BCF \cong \triangle ABE$ (Proved in part (i) above)

$$ar(\triangle BCF) = ar(\triangle ABE) \quad \dots (4)$$

$$\angle BAG + \angle BAC = 90^\circ + 90^\circ$$

$$\Rightarrow \angle BAG + \angle BAC = 180^\circ$$

\therefore GAC is a straight line.

Now, $\triangle BCF$ and square AGFB are on the same base BF and between the same \parallel lines BF and GC.

$$\therefore ar(\triangle BCF) = \frac{1}{2} ar(\text{square AGFB}) \quad \dots (5)$$

Again, $\triangle ABE$ and rectangle BENM are on the same base BE and between the same \parallel lines BE and AN.

$$\therefore ar(\triangle ABE) = \frac{1}{2} ar(\text{Rectangle BENM}) \quad \dots (6)$$

From (4), (5) and (6)

$$\frac{1}{2} ar(\text{square AGFB}) = \frac{1}{2} ar(\text{Rectangle BENM})$$

$$\Rightarrow ar(\text{square AGFB}) = ar(\text{Rectangle BENM})$$

Hence, area of square AGFB = area of Rectangle BENM.
(Q.E.D.)

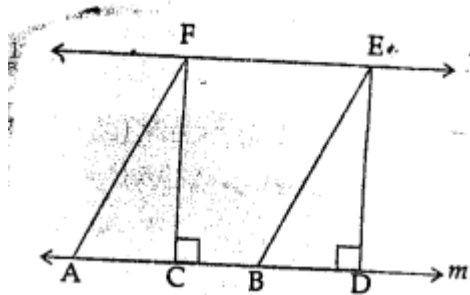
Multiple Choice Questions

Choose the correct answer from the given four options (1 to 8):

Question 1.

In the given figure, if $l \parallel m$, $AF \parallel BE$, $FC \perp m$ and $ED \perp m$, then the correct statement is

- (a) area of $\parallel\text{ABEF} = \text{area of rect. CDEF}$
- (b) area of $\parallel\text{ABEF} = \text{area of quad. CBEF}$
- (c) area of $\parallel\text{ABEF} = 2 \text{ area of } \triangle\text{ACF}$
- (d) area of $\parallel\text{ABEF} = 2 \text{ area of } \triangle\text{EBD}$



Solution:

In the given figure,

$l \parallel m$, $AF \parallel BE$, $FC \perp m$ and $ED \perp m$

$\therefore \parallel\text{gm ABEF}$ and rectangle CDEF are on the same base EF and between the same parallel

$\therefore \text{area } \parallel\text{gm ABEF} = \text{area rect. CDEF}$ (a)

Question 2.

Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

- (a) 1 : 2
- (b) 1 : 1
- (c) 2 : 1
- (d) 3 : 1

Solution:

A triangle and a parallelogram are on the same base and between same parallel, then

\therefore They are equal in area

\therefore Their ratio 1:1 (b)

Question 3.

If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of area of the triangle to the area of parallelogram is

- (a) 1 : 3
- (b) 1 : 2
- (c) 3 : 1
- (d) 1 : 4

Solution:

A triangle and a parallelogram are on the same base and between same parallel, then area of

triangle = $\frac{1}{2}$ area $\parallel\text{gm}$

\therefore Their ratio 1 : 2 (b)

Question 4.

A median of a triangle divides it into two

- (a) triangles of equal area
- (b) congruent triangles
- (c) right triangles
- (d) isosceles triangles

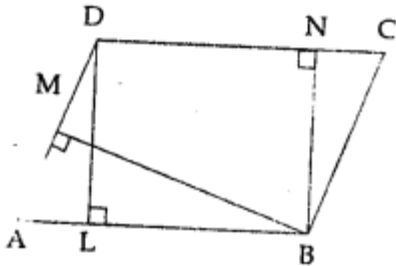
Solution:

A median of a triangle divides it into two triangles equal in area. (a)

Question 5.

In the given figure, area of parallelogram ABCD is

- (a) $AB \times BM$
- (b) $BC \times BN$
- (c) $DC \times DL$
- (d) $AD \times DL$



Solution:

In the given figure,

Area of ||gm ABCD = $AB \times DL$ or $DC \times DL$ ($\because AB = DC$) (c)

Question 6.

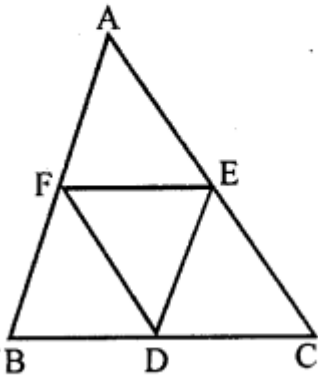
The mid-points of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to

- (a) $\frac{1}{2}$ area of $\triangle ABC$
- (b) $\frac{1}{3}$ area of $\triangle ABC$
- (c) $\frac{1}{4}$ area of $\triangle ABC$
- (d) area of $\triangle ABC$

Solution:

The mid-points of the sides of a triangle along with any of vertices as the fourth point makes

a parallelogram of area equal to $\frac{1}{2}$ the area of ΔABC



i.e., area $\parallel\text{gm DEAF} = \frac{1}{2}$ area ΔABC (a)

Question 7.

In the given figure, ABCD is a trapezium with parallel sides $AB = a$ cm and $DC = b$ cm. E and F are mid-points of the non parallel sides. The ratio of area of ABEF and area of EFCD is

- (a) $a : b$
- (b) $(3a + b) : (a + 3b)$
- (c) $(a + 3b) : (3a + b)$
- (d) $(2a + b) : (3a + b)$

Solution:

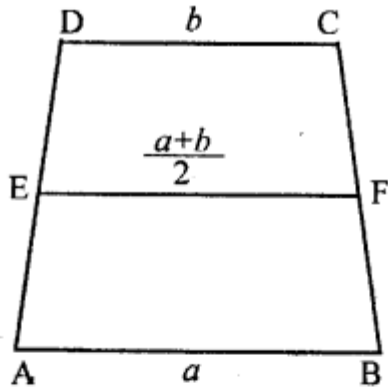
In the figure, ABCD is a trapezium in which
 $AB \parallel DC$

$AB = a, DC = b$

E and F are mid points on DA and CB
respectively

Let h be the height $(\because EF \parallel AB \parallel DC)$

$$\therefore EF = \frac{1}{2}(a + b)$$



Area of trapezium ABFE

$$= \left[\frac{1}{2} \frac{(a+b)}{2} \times \frac{h}{2} \right]$$

$$= \frac{h}{4} \left(\frac{2a+a+b}{2} \right)$$

$$= \frac{h}{8} (3a+b)$$

and area of trap. EFCD

$$= \frac{1}{2} [EF + DC] \times \frac{h}{2}$$

$$= \frac{h}{4} \left[\frac{a+b}{2} + b \right] = \frac{h}{4} \left[\frac{a+b+2b}{2} \right]$$

$$= \frac{h}{4} [a+3b]$$

$$\therefore \text{Ratio} = \frac{h}{8} (3a+b) : \frac{h}{8} (a+3b)$$

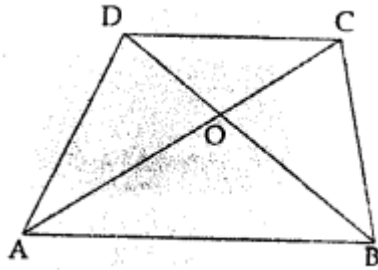
$$= (3a+b) : (a+3b) \quad (b)$$

Question 8.

In the given figure, $AB \parallel DC$ and $AB \neq DC$. If the diagonals AC and BD of the trapezium $ABCD$ intersect at O , then which of the following statements is not true?

(a) area of $\triangle ABC$ = area of $\triangle ABD$

- (b) area of $\triangle ACD =$ area of $\triangle BCD$
- (c) area of $\triangle OAB =$ area of $\triangle OCD$
- (d) area of $\triangle OAD =$ area of $\triangle OBC$



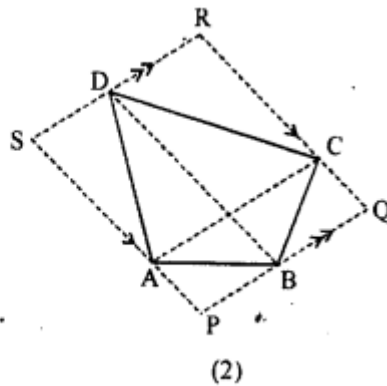
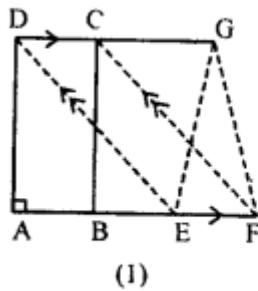
Solution:

In the trapezium ABCD, $AB \parallel DC$
 $AB \neq DC$
 The diagonals BD and AC intersect each other at O
 Only statement area of $\triangle OAB$ is not equal to area $\triangle COD$
 Other all statements are true
 Only (b) is not true. (b)

Chapter test

Question 1.

- (a) In the figure (1) given below, ABCD is a rectangle (not drawn to scale) with side $AB = 4$ cm and $AD = 6$ cm. Find :
 - (i) the area of parallelogram DEFC
 - (ii) area of $\triangle EFG$.
- (b) In the figure (2) given below, PQRS is a parallelogram formed by drawing lines parallel to the diagonals of a quadrilateral ABCD through its corners. Prove that area of $\parallel gm$ PQRS = 2 x area of quad. ABCD.



Solution:

(a) **Given.** ABCD is a rectangle AB = 4 cm and AD = 6 cm.

Required. (i) The area of || gm DEFC.

(ii) area of ΔEFG

(i) Since AB = 4cm and AD = 6 cm (given)

\therefore Area of rectangle ABCD = AB \times AD

$$= 4 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$$

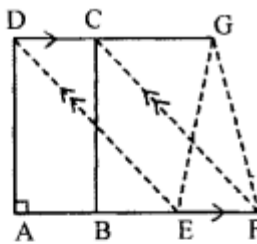
Now, area of rectangle ABCD

= area of || gm DEFC

(\because Both are on the same Base and between the same parallel lines)

\Rightarrow Area of || gm DEFC

$$= 24 \text{ cm}^2$$



(1)

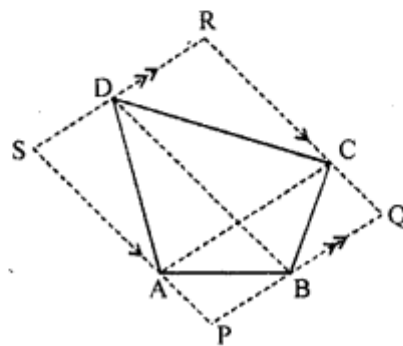
(ii) Area of $\Delta EFG = \frac{1}{2}$ (area of || gm DEFC)

(\because Both are on the same base and between the same parallel lines)

$$\therefore \text{Area of } \Delta EFG = \frac{1}{2} \times 24 \text{ cm}^2 = 12 \text{ cm}^2 \text{ Ans.}$$

(b) **Given.** PQRS is a || gm formed by drawing lines parallel to the diagonals of quadrilateral ABCD through its corners.

To prove. Area of || gm PQRS = 2. area of quad ABCD



(2)

Proof. $ar(\Delta ACD) = \frac{1}{2} ar(\parallel gm\ ACRS)$

[\therefore both are on same base AC and between the same \parallel AC and SR]

$\Rightarrow ar(\parallel gm\ ACRS) = 2ar(\Delta ACD) \dots\dots (1)$

Similarly,

$ar(\Delta ABC) = \frac{1}{2} ar(\parallel gm\ \Delta APQC)$

$\Rightarrow ar(\parallel gm\ APQC) = 2ar(\Delta ABC) \dots\dots (2)$

Adding (1) from (2),

$ar(\parallel gm\ ACRS) + ar(\parallel gm\ APQC) = 2ar(\Delta ACD) + 2ar(\Delta ABC)$

$\Rightarrow (\parallel gm\ PQRS) = 2[ar(\Delta ACD) + ar(\Delta ABC)]$

$\Rightarrow ar(\parallel gm\ PQRS) = 2ar(\text{quad. } ABCD)$

Hence, area of $\parallel gm\ PQRS = 2 \cdot \text{area of quad. } ABCD.$ (Q.E.D.)

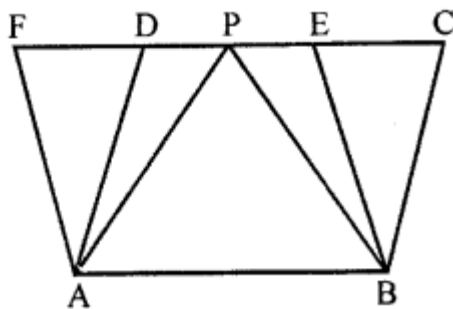
Question P.Q.

In the adjoining figure, ABCD and ABEF are parallelogram and P is any point on DC. If area of $\parallel gm\ ABCD = 90\text{ cm}^2$, find:

(i) area of $\parallel gm\ ABEF$

(ii) area of ΔABP .

(iii) area of ΔBEF .



Solution:

In the given figure,

ABCD and ABEF are parallelogram P is an point on DC

$$\text{Area of } \parallel\text{gm ABCD} = 90 \text{ cm}^2$$

$\parallel\text{gm ABCD}$ and ABEF are on the same base AB are between the same parallels

$$(i) \therefore \text{Area of } \parallel\text{gm ABEF} = \text{area of } \parallel\text{gm ABCD} = 90 \text{ cm}^2$$

(ii) $\therefore \triangle ABP$ and $\parallel\text{gm ABCD}$ are on the same base AB and between the same parallels

$$\therefore \text{Area } \triangle ABP = \frac{1}{2} \text{ area } \parallel\text{gm ABCD}$$

$$= \frac{1}{2} \times 90 \text{ cm}^2 = 45 \text{ cm}^2$$

(iii) $\therefore \triangle BEF$ and $\parallel\text{gm ABEF}$ are on the same base EF and between the same parallels

$$\therefore \text{Area } \triangle BEF = \frac{1}{2} \text{ area } \parallel\text{gm ABEF}$$

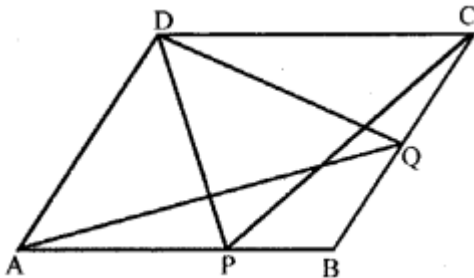
$$= \frac{1}{2} \times 90 = 45 \text{ cm}^2$$

Question 2.

In the parallelogram ABCD, P is a point on the side AB and Q is a point on the side BC. Prove that

(i) area of $\triangle CPD$ = area of $\triangle AQD$

(ii) area of $\triangle ADQ$ = area of $\triangle APD$ + area of $\triangle CPB$.



Solution:

Given. \parallel gm ABCD in which P is a point on AB and Q is a point on BC.

To prove. (i) $ar(\Delta CPD) = ar(\Delta AQD)$

(ii) $ar(\Delta ADQ) = ar(\Delta APD) + ar(\Delta CPB)$

Proof. ΔCPD and \parallel gm ABCD are on the same base CD and between the same parallels lines AB and CD.

$$\therefore ar(\Delta CPD) = \frac{1}{2} ar(\parallel \text{ gm ABCD}) \quad \dots (1)$$

ΔADQ and \parallel gm ABCD are on the same base AD and between the same \parallel lines AD and BC,

$$ar(\Delta ADQ) = \frac{1}{2} (\parallel \text{ gm ABCD}) \quad \dots (2)$$

From (1) and (2),

$$ar(\Delta CPD) = ar(\Delta ADQ)$$

$$\text{or } ar(\Delta CPD) = ar(\Delta ADQ) \quad \text{(Q.E.D.)}$$

$$(ii) \quad ar(\Delta ADQ) = \frac{1}{2} ar(\parallel \text{ gm ABCD})$$

(Proved in part (i) above)

$$\Rightarrow 2ar(\Delta ADQ) = ar(\parallel \text{ gm ABCD})$$

$$\Rightarrow ar(\Delta ADQ) + ar(\Delta ADQ) = ar(\parallel \text{ gm ABCD})$$

..... (3)

$$\text{But } ar(\Delta ADQ) = ar(\Delta CPD) \quad \dots (4)$$

(Proved in part (i) above)

From (3) and (4),

$$ar(\Delta ADQ) + ar(\Delta CPD) = ar(\parallel \text{ gm ABCD})$$

$$\Rightarrow ar(\Delta ADQ) + ar(\Delta CPD)$$

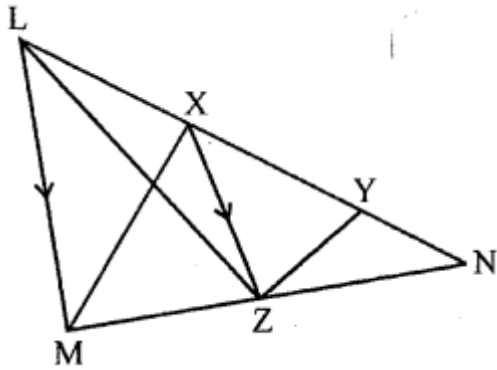
$$= ar(\Delta APD) + ar(\Delta CPD) + ar(\Delta CPB)$$

$$\Rightarrow ar(\Delta ADQ) = ar(\Delta APD) + ar(\Delta CPB)$$

(Q.E.D.)

Question 3.

In the adjoining figure, X and Y are points on the side LN of triangle LMN. Through X, a line is drawn parallel to LM to meet MN at Z. Prove that area of ΔLZY = area of quad. MZYX.



Solution:

Given : In the figure,

X and Y are points on side LN of $\triangle LMN$.
Through X, a line $XZ \parallel LM$ is drawn which meets MN at Z.

To prove : area of $\triangle LZY$ = area of quad. MZYX

Construction : Join MX, ZY and LZ

Proof : $\because LM \parallel XZ$

and $\triangle LZX$ and $\triangle MZX$ are on the same base XZ and between the same parallels

$$\therefore \text{area } \triangle LZX = \text{area } \triangle MZX$$

Adding area $\triangle XZY$ to both sides

$$\text{area } \triangle LZX + \text{area } \triangle XZY$$

$$= \text{area } \triangle MZX + \text{area } \triangle XZY$$

$$\Rightarrow \text{area } \triangle LZY = \text{area quadrilateral MZYX}$$

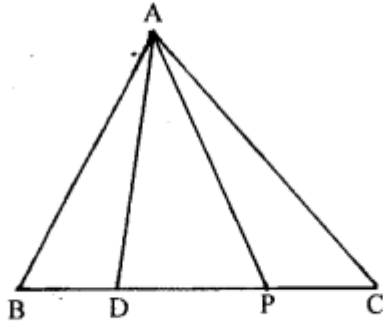
Question P.Q.

If D is a point on the base BC of a triangle ABC such that $2BD = DC$, prove that area of $\triangle ABD = \frac{1}{3}$ area of $\triangle ABC$.

Solution:

Given. ΔABC in which base BC. D is a point on BC such that $2BD = DC$.

To prove. $ar(\Delta ABD) = \frac{1}{3} ar(ABC)$



Construction. Let P is the mid-point of DC join AD = DC

$$\Rightarrow BD = \frac{1}{2} DC$$

i.e. $BD = DP$ (P is mid-point of DC)

\therefore D is mid-point of BP.

In ΔABP , AD is median of BP

(D is mid-point of BP)

$$\therefore ar(\Delta ABD) = ar(\Delta ADP) \quad \dots (1)$$

Again in ΔADC , AP is the median of DC.

(P is mid-point of DC)

$$\therefore ar(\Delta ADP) = ar(\Delta APC) \quad \dots (2)$$

From (1) and (2),

$$\therefore ar(\Delta ABD) = ar(\Delta ADP) = ar(\Delta APC)$$

$\therefore \Delta ABC$ is divided into three equal triangles

and each Δ will be of $\frac{1}{3} \Delta ABC$.

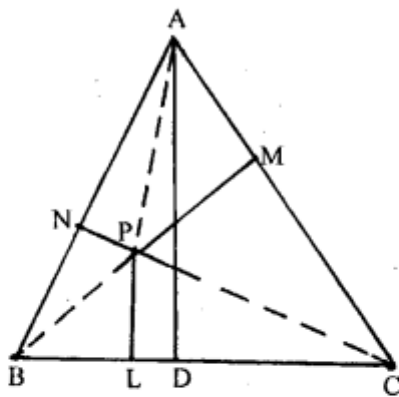
$$\therefore ar(\Delta ABD) = \frac{1}{3} ar(\Delta ABC) \quad \text{(Q.E.D.)}$$

Question 4.

Perpendiculars are drawn from a point within an equilateral triangle to the three sides. Prove that the sum of the three perpendiculars is equal to the altitude of the triangle.

Solution:

ABC is an equilateral triangle. *i.e.* $AB = BC = CA$. P is any point within an equilateral triangle to the three sides.



PN, PM, and PL are perpendicular on side AB, AC and BC respectively. AD is any altitude from point A on side BC.

To prove. $AD = NP + LP + MP$

Construction. Join PA, PB and PC.

Proof. Area of $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$\text{ar} (\triangle ABC) = \frac{1}{2} \times BC \times AD \quad \dots (1)$$

$$\text{Now, area of } \triangle APB = \frac{1}{2} \times AB \times NP \quad \dots (2)$$

$$\text{area of } \triangle APC = \frac{1}{2} \times AC \times MP \quad \dots (3)$$

$$\text{area of } \triangle BPC = \frac{1}{2} \times BC \times LP \quad \dots (4)$$

Adding (2), (3) and (4)

$$\begin{aligned} & ar(\triangle APB) + ar(\triangle APC) + ar(\triangle BPC) \\ &= \frac{1}{2} \times AB \times NP + \frac{1}{2} \times AC \times MP + \frac{1}{2} \times BC \times LP \\ ar(\triangle ABC) &= \frac{1}{2} [AB \times NP + AC \times MP + BC \times LP] \\ &= \frac{1}{2} [BC \times NP + BC \times MP + BC \times LP] \\ & \quad (\because AB = AC = CA) \end{aligned}$$

$$ar(\triangle ABC) = \frac{1}{2} \times BC [NP + MP + LP] \quad \dots (5)$$

From (4) and (5),

$$\frac{1}{2} \times BC \times AD = \frac{1}{2} \times BC \times (NP + LP + MP)$$

$$\Rightarrow AD = NP + LP + MP$$

$$\Rightarrow NP + LP + MP = AD$$

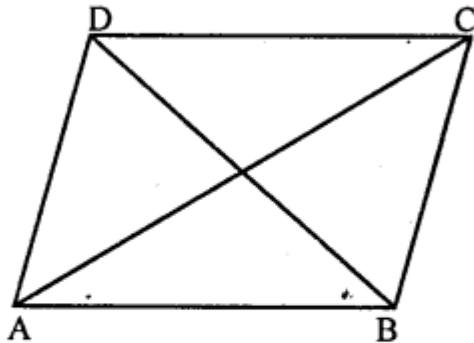
i.e. sum of three perpendiculars is equal to the altitude of the triangle.

Question 5.

If each diagonal of a quadrilateral divides it into two triangles of equal areas, then prove that the quadrilateral is a parallelogram.

Solution:

Given : In quadrilateral ABCD, diagonal AC bisects the quadrilateral ABCD in two triangles of equal area i.e.



$$\text{ar} (\Delta ABC) = \text{ar} (\Delta ADC)$$

To prove : ABCD is a parallelogram.

Proof : Join BD.

Proof : \because Diagonals of quad. ABCD divides the quad. into two triangles of equal area.

$$\therefore \text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$$

$$= \frac{1}{2} \text{ar} (ABCD)$$

But, these are on the same base AB

\therefore Their heights are equal

$$\therefore DC \parallel AB \quad \dots(i)$$

Similarly, we can prove that :

$$\text{ar} (\Delta ABC) = \text{ar} (\Delta BDC)$$

$$\therefore BC \parallel AD \quad \dots(ii)$$

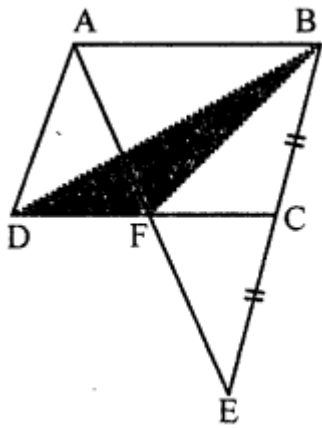
From (i) and (ii)

ABCD is a parallelogram.

Hence proved.

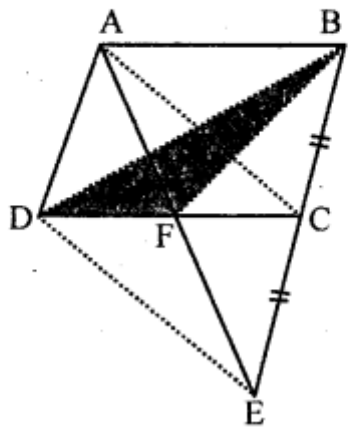
Question 6.

In the given figure, ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F. If area of $\Delta DFB = 3 \text{ cm}^2$, find the area of parallelogram ABCD.



Solution:

In the figure, ABCD is a parallelogram
BC is produced to E such that $CE = BC$



Join BD and AE

which intersects DC at F

Join BF, AC and DE

$$\therefore \text{Area of } \triangle DFB = 3 \text{ cm}^2$$

Find the area of $\parallel\text{gm ABCD}$

Solution : \because In $\triangle ABE$, C is mid-point of BE
and $CD \parallel AB$

\therefore F is mid-point of AE and CD

\therefore ABED is a $\parallel\text{gm}$

(\because Diagonals AE and CD bisect each other
at F)

\therefore BD is the diagonal of $\parallel\text{gm ABCD}$

$$\triangle BCD = \frac{1}{2} \parallel\text{gm ABCD}$$

\therefore F is mid-point of DC

$$\therefore \triangle DFB = \frac{1}{2} \triangle BCD$$

$$\Rightarrow \triangle DFB = \frac{1}{2} \times \frac{1}{2} (\parallel\text{gm ABCD})$$

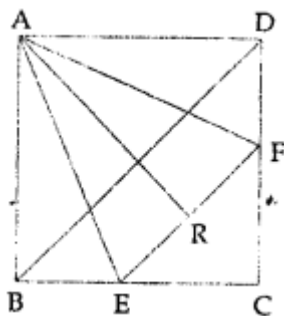
$$\Rightarrow \triangle DFB = \frac{1}{4} (\parallel\text{gm ABCD})$$

$$\therefore \text{area } \parallel\text{gm ABCD} = 4 \text{ area } \triangle DFB \\ = 4 \times 3 = 12 \text{ cm}^2$$

Question 7.

In the given figure, ABCD is a square. E and F are mid-points of sides BC and CD respectively. If R is mid-point of EF, prove that: area of $\triangle AER$ = area of $\triangle AFR$.

Solution:



Given : In square ABCD, BD is diagonals E and F are mid-point of BC and CD respectively. R is mid-point of EF.

To prove : area (ΔAER) = area (ΔAFR)

Proof : In ΔABE and ΔADF

$AB = AD$ (Sides of a square)

$\angle B = \angle D$ (Each 90°)

$BE = CE$ (E is mid-point of BC)

$\therefore \Delta ABE \cong \Delta ADF$ (SAS axiom)

$\therefore AE = AF$ (c.p.c.t.)

Again in ΔAER and ΔAFR

$AE = AF$ (Produced)

$AR = AR$ (Common)

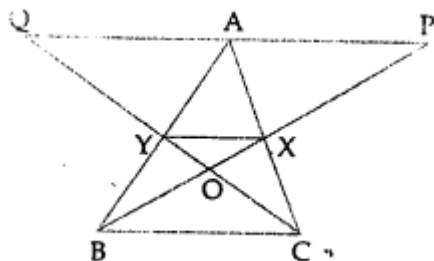
$ER = FR$ (R is mid-point of EF)

$\therefore \Delta AER \cong \Delta AFR$ (SSS axiom)

$\therefore \text{area}(\Delta AER) = \text{area}(\Delta AFR)$

Question 8.

In the given figure, X and Y are mid-points of the sides AC and AB respectively of ΔABC . $QP \parallel BC$ and CYQ and BXP are straight lines. Prove that area of $\Delta ABP =$ area of ΔACQ .



Solution:

Given : In the given figure,
X and Y are the mid-points of the sides AC
and AB respectively of ΔABC

$QP \parallel BC$

CYQ and BXP are straight lines

To prove : $\text{area}(\Delta ABP) = \text{area}(\Delta ACQ)$

Proof : \because X and Y are the mid-points of sides
AC and AB respectively

$\therefore YX \parallel BC$

But $QP \parallel BC$

$\therefore QP \parallel BC \parallel YX$

In ΔBAP , Y is mid of AB and $YX \parallel QP$

\therefore X is mid-point of BP

$$\therefore YX = \frac{1}{2} AP \quad \dots(i)$$

Similarly we can prove in ΔAQC

$$YX = \frac{1}{2} QA \quad \dots(ii)$$

From (i) and (ii),

$$QA = AP$$

Now ΔABP and ΔACQ are on the equal base
and between the same parallel lines

$\therefore \text{area}(\Delta ABP) = \text{area}(\Delta ACQ)$