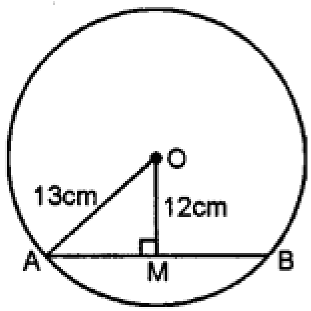
Circle

Question 1.

Calculate the length of a chord which is at a distance of 12 cm from the centre of a circle of radius 13 cm.

Solution:

AB is chord of a circle with centre O and OA is its radius OM ⊥ AB.



∴ OA = 13 cm, OM = 12 cm

Now in right ΔOAM,

$OA^2 = OM^2 + AM^2$

(By Pythagorus Axiom)

$\Rightarrow (13)^2 = (12)^2 + AM^2$

$\Rightarrow AM^2 = (13)^2 - (12)^2$

$\Rightarrow AM^2 = 169 - 144 = 25 = (5)^2$

$\Rightarrow AM = 5$ cm.

∵ OM ⊥ AB

∴ M is the mid-point of AB.

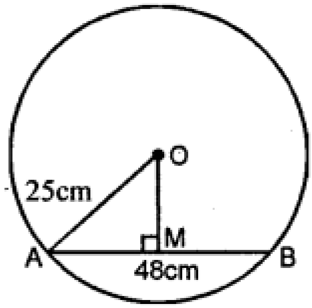
∴ AB = 2 AM = 2 × 5 = 10 cm

Question 2.

A chord of length 48 cm is drawn in a circle of radius 25 cm. Calculate its distance from the centre of the circle.

Solution:

AB is the chord of the circle with centre O and radius OA and $OM \perp AB$.



$\therefore$ AB = 48 cm,

OA = 25 cm

$\because OM \perp AB$

$\therefore$ M is the mid-point of AB

$\therefore AM = \frac{1}{2}AB = \frac{1}{2} \times 48 = 24$ cm.

Now in right ΔOAM,

$$OA^2 = OM^2 + AM^2$$

(By Pythagorus Axiom)

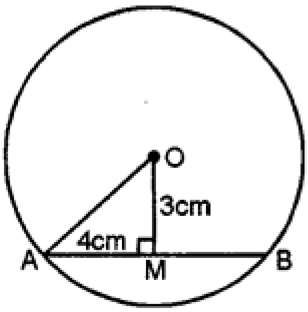
$\Rightarrow (25)^2 = OM^2 + (24)^2$

$\Rightarrow OM^2 = (25)^2 - (24)^2 = 625 - 576$

$= 49 = (7)^2$

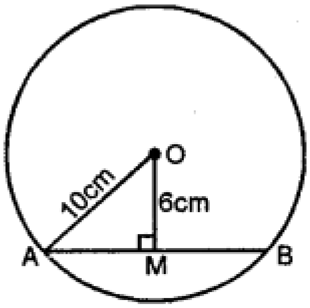
$\therefore$ OM = 7 cm

Question 3.
A chord of length 8 cm is at a distance of 3 cm from the centre of the circle. Calculate the radius of the circle.
Solution:
AB is the chord of a circle with centre O and radius OA and OM $\perp$ AB



$\therefore$ AB = 8 cm
OM = 3 cm
$\because$ OM $\perp$ AB
$\therefore$ M is the mid-point of AB
$\therefore$ $AM = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4$ cm.
Now in right ΔOAM,
$OA^2 = OM^2 + AM^2$
(By Pythagorus Axiom)
$= (3)^2 + (4)^2 = 9 + 16 = 25$
$= (5)^2$
$\therefore$ OA = 5 cm.

Question 4.
Calculate the length of the chord which is at a distance of 6 cm from the centre of a circle of diameter 20 cm.
Solution:
AB is the chord of the circle with centre O and radius OA and OM ⊥ AB



∴ Diameter of the circle = 20 cm

∴ Radius $= \frac{20}{2} = 10$ cm

∴ OA = 10 cm, OM = 6 cm

Now in right Δ OAM,

$OA^2 = AM^2 + OM^2$

(By Pythagorus Axiom)

$\Rightarrow (10)^2 = AM^2 + (6)^2$

$\Rightarrow AM^2 = 10^2 - 6^2$

$\Rightarrow AM^2 = 100 - 36 = 64 = (8)^2$

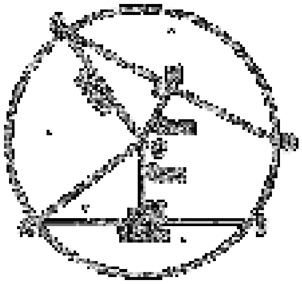
∴ AM = 8 cm

∵ OM ⊥ AB

∴ M is the mid-point of AB.

∴ AB = 2 AM = 2 × 8 = 16 cm.

Question 5.
A chord of length 16 cm is at a distance of 6 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 8 cm from the centre.
Solution:



$\therefore$ AB = 16 cm, OM = 6 cm

$\because$ OM $\perp$ AB

$\therefore$ $AM = \frac{1}{2}AB = \frac{1}{2} \times 16 = 8$ cm.

Now in right ΔOAM,

$OA^2 = AM^2 + OM^2$ (By Pythagorous Axiom)

$= (8)^2 + (6)^2$

$= 64 + 36 = 100 = (10)^2$

$\therefore$ OA = 10 cm.

Now CD is another chord of the same circle ON $\perp$ CD and OC is the radius.

$\therefore$ In right ΔONC

$OC^2 = ON^2 + NC^2$ (By Pythagorous Axioms)

$\Rightarrow (10)^2 = (8)^2 + (NC)^2$

$\Rightarrow 100 = 64 + NC^2$

$\Rightarrow NC^2 = 100 - 64 = 36 = (6)^2$

$\therefore$ NC = 6

But ON $\perp$ AB

$\therefore$ N is the mid-point of CD

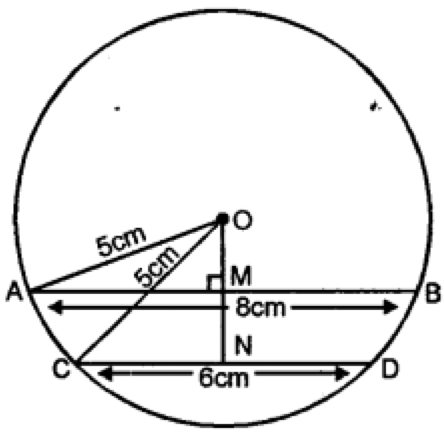
$\therefore$ CD = 2 NC = 2 $\times$ 6 = 12 cm

Question 6.

In a circle of radius 5 cm, AB and CD are two parallel chords of length 8 cm and 6 cm respectively. Calculate the distance between the chords if they are on :
(i) the same side of the centre.
(ii) the opposite sides of the centre.

Solution:

Two chords AB and CD of a circle with centre O and radius OA or OC



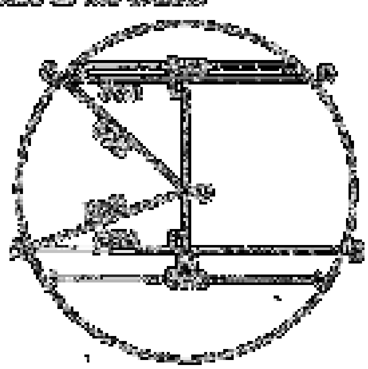
(i)

∴ OA = OC = 5 cm
AB = 8 cm
CD = 6 cm

OM and ON are perpendiculars from O to AB and CD respectively.

∴ M and N are the mid-points of AB and CD respectively.

In figure (i) chords are on the same side and in figure (ii) chords are on the opposite sides of the centre.



[illegible]

In right ΔOAM,

$OA^2 = AM^2 + OM^2$

(By Pythagorus Axiom)

$\Rightarrow (5)^2 = (4)^2 + OM^2$

$\left(\because AM = \frac{1}{2}AB\right)$

$\Rightarrow 25 = 16 + OM^2$

$\Rightarrow OM^2 = 25 - 16 = 9 = (3)^2$

$\therefore OM = 3$ cm.

Again in right ΔOCN,

$OC^2 = CN^2 + ON^2$

$\Rightarrow (5)^2 = (3)^2 + ON^2$

$\left(\because CN = \frac{1}{2}CD\right)$

$\Rightarrow 25 = 9 + ON^2$

$\Rightarrow ON^2 = 25 - 9 = 16 = (4)^2$

$\therefore ON = 4$

In fig. (*i*), distance $MN = ON - OM$

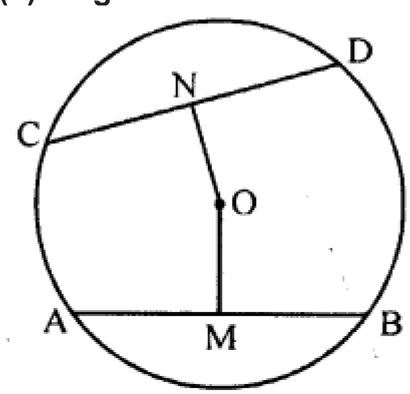
$= 4 - 3 = 1$ cm.

In fig. (*ii*)

$MN = OM + ON = 3 + 4 = 7$ cm

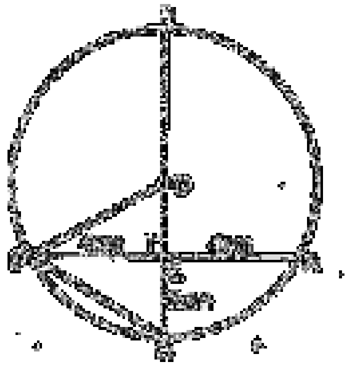
Question 7.

(a) In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle, OM is perpendicular to AB and ON is perpendicular to CD. AB = 24 cm, OM = 5 cm, ON = 12 cm. Find the:
(i) radius of the circle.
(ii) length of chord CD.



(b) In the figure (ii) given below, CD is the diameter which meets the chord AB in

E such that AE = BE = 4 cm. If CE = 3 cm, find the radius of the circle.



Solution:

(i) Given $AB = 24$ cm, $OM = 5$ cm, $OB = 12$ cm

$OM \perp AB$

M is mid point of AB.

$AM = 12$ cm

∴ Radius of circle $(r) = \sqrt{OM^2 + AM^2}$

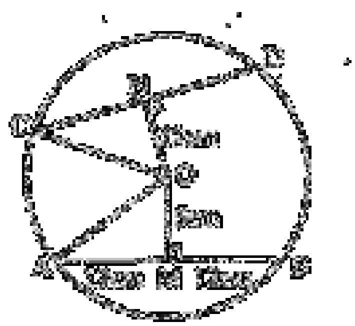
$= \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ cm

(ii) Again $OC^2 = OM^2 + CM^2$

$13^2 = 12^2 + CM^2$

$CM = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25}$

$CM = 5$ cm



As $OM \perp CD$, M is mid point of CD.

∴ [illegible]

(ii) $AE = 4$ cm, $EC = 3$ cm

Let radius $OE = OC = r$

∴ $OE = r - 3$ cm.

Now in right $\triangle OAE$

$OA^2 = AE^2 + OE^2$

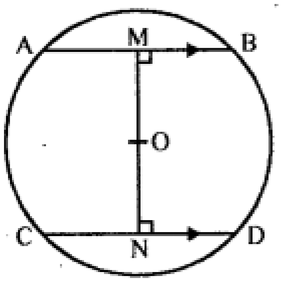
⇒ $r^2 = 4^2 + (r - 3)^2$

⇒ $r^2 = 16 + r^2 - 6r + 9$

⇒ $6r = 25$ ⇒ $r = \frac{25}{6} = 4\frac{1}{6}$ cm

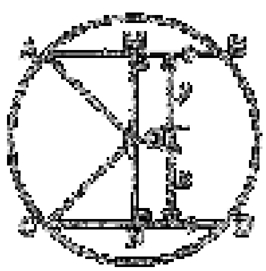
Question 8.

In the adjoining figure, AB and CD ate two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively.



Solution:

In the figure, chords AB ∥ CD
O is the centre of the circle
Radius of the circle = 15 cm,
Length of AB = 24 cm and CD = 18 cm
Join OA and OC



∵ AB = 24 cm and OM ⊥ AB

$\therefore AM = MB = \frac{24}{2} = 12$ cm

Similarly ON ⊥ CD

$\therefore CN = ND = \frac{18}{2} = 9$ cm

In right ΔAMO,

$AO^2 = AM^2 + OM^2$ (Pythagoras Theorem)

$\Rightarrow (15)^2 = (12)^2 + OM^2 \Rightarrow 225 = 144 + OM^2$

$\Rightarrow OM^2 = 225 - 144 = 81 = (9)^2$

$\therefore OM = 9$ cm

Similarly in right ΔCNO

$OC^2 = CN^2 + ON^2 \Rightarrow (15)^2 = (9)^2 + ON^2$

$\Rightarrow 225 = 81 + ON^2$

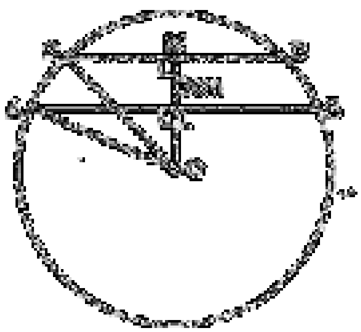
$\Rightarrow ON^2 = 225 - 81 = 144 = (12)^2$

$\therefore ON = 12$ cm

Now $MN = OM + ON = 9 + 12 = 21$ cm

Question 9.
AB and CD are two parallel chords of a circle of lengths 10 cm and 4 cm respectively. If the chords lie on the same side of the centre and the distance between them is 3 cm, find the diameter of the circle.
Solution:

AB and CD are two parallel chords and AB = 10 cm, CD = 4 cm and distance between AB and CD = 3 cm.



Let radius of circle OA = OC = r

OM ⊥ CD which intersects AB in L.

∴ Let OL = x, then OM = x + 3

Now in right △OAL,

$OA^2 = AL^2 + OL^2$

$\Rightarrow \quad r^2 = (5)^2 + x^2 = 25 + x^2 \quad ...(i)$

(∵ L is mid-point of AB)

Again in right △OCM,

$OC^2 = CM^2 + OM^2$

$\Rightarrow \quad r^2 = (2)^2 + (x + 3)^2$

(∵ M is mid-point of CD)

$\Rightarrow \quad r^2 = 4 + (x + 3)^2 \quad ...(ii)$

From (i) and (ii)

$25 + x^2 = 4 + (x + 3)^2$

$\Rightarrow 25 + x^2 = 4 + x^2 + 9 + 6x$

$\Rightarrow \quad 6x = 25 - 13 = 12$

$\Rightarrow \quad x = \frac{12}{6} = 2 \text{ cm}$

Substituting the value of x in (i)

$r^2 = 25 + x^2 = 25 + (2)^2 = 25 + 4$

$r^2 = 29$

$\therefore r = \sqrt{29}$ cm

∴ Diameter of the circle = 2r

$= 2 \times \sqrt{29}$ cm $= 2\sqrt{29}$ cm

Question 10.
ABC is an isosceles triangle inscribed in a circle. If AB = AC = 12√5 cm and BC = 24 cm, find the radius of the circle.
Solution:

$AB = AC = 12\sqrt{5}$ and $BC = 24$ cm.

Join OB and OC and OA.

Draw $AD \perp BC$ which will pass through centre O.

$\therefore$ OD bisects BC in D

$\therefore BD = DC = 12$ cm.

In right ΔABD

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow (12\sqrt{5})^2 = AD^2 + (12)^2$$

$$\Rightarrow 144 \times 5 = AD^2 + 144$$

$$\Rightarrow 720 - 144 = AD^2$$

$$\Rightarrow AD^2 = 576 \Rightarrow AD = \sqrt{576} = 24$$

Let radius of the circle $= OA = OB = OC = r$

$\therefore OD = AD - AO = 24 - r$

Now in right ΔOBD,

$$OB^2 = BD^2 + OD^2$$

$$\Rightarrow r^2 = (12)^2 + (24 - r)^2$$

$$\Rightarrow r^2 = 144 + 576 + r^2 - 48r$$

$$\Rightarrow 48r = 720$$

$$r = \frac{720}{48} = 15\text{cm.}$$

$\therefore$ Radius = 15 cm

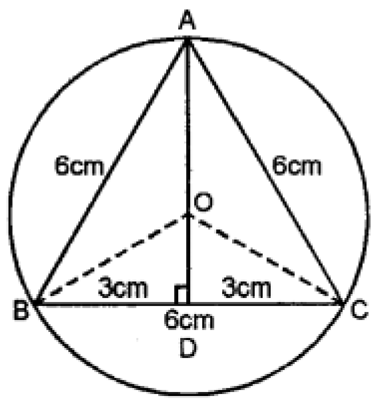
Question 11.

An equilateral triangle of side 6 cm is inscribed in a circle. Find the radius of the circle.

Solution:

ABC is an equilateral triangle inscribed in a circle with centre O. Join OB and OC.

From A, draw AD ⊥ BC which will pass through the centre O of the circle.



∵ Each side of ΔABC = 6 cm.

∴ $AD = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}$ cm.

$OD = AD - AO = 3\sqrt{3} - r.$

Now in right ΔOBD,

$OB^2 = BD^2 + OD^2$

$\Rightarrow \quad r^2 = (3)^2 + (3\sqrt{3} - r)^2$

$\Rightarrow \quad r^2 = 9 + 27 + r^2 - 6\sqrt{3}\,r$

(∵ D is mid-point of BC)

$6\sqrt{3}\,r = 36$

$r = \frac{36}{6\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ cm

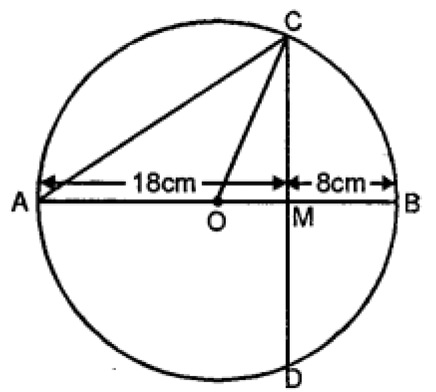
∴ Radius $= 2\sqrt{3}$ cm

Question 12.

AB is a diameter of a circle. M is a point in AB such that AM = 18 cm and MB = 8 cm. Find the length of the shortest chord through M.

Solution:

In a circle with centre O, AB is the diameter and M is a point on AB such that



AM = 18 cm and MB = 8 cm

$\therefore$ AB = AM + MB = 18 + 8 = 26 cm

$\therefore$ Radius of the circle $= \frac{26}{2} = 13$ cm

Let CD is the shortest chord drawn through M.

$\therefore$ CD $\perp$ AB.

Join OC.

OM = AM – AO = 18 – 13 = 5 cm

OC = OA = 13 cm.

Now in right ΔOMC,

$OC^2 = OM^2 + MC^2$

$\Rightarrow (13)^2 = (5)^2 + MC^2 \Rightarrow MC^2 = 13^2 - 5^2$

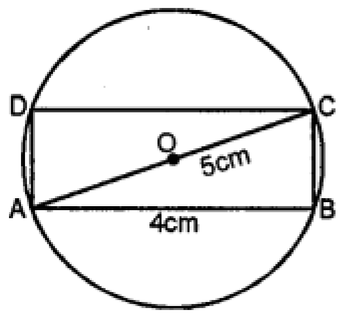
$\Rightarrow MC^2 = 169 - 25 = 144 = (12)^2$

$\therefore$ MC = 12

$\because$ M is mid-point of CD

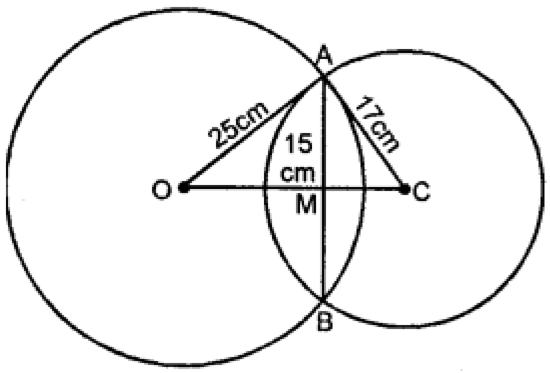
$\therefore$ CD = 2 × MC = 2 × 12 = 24 cm

Question 13.
A rectangle with one side of length 4 cm is inscribed in a circle of diameter 5 cm. Find the area of the rectangle.
Solution:
ABCD is a rectangle inscribed in a circle with centre O and diameter 5 cm.



AB = 4 cm and AC = 5 cm.
In right ΔABC,
$AC^2 = AB^2 + BC^2$
$\Rightarrow (5)^2 = (4)^2 + BC^2 \Rightarrow BC^2 = 5^2 - 4^2$
$\Rightarrow BC^2 = 25 - 16 = 9 = (3)^2$
$\therefore$ BC = 3 cm.
$\therefore$ Area of rectangle ABCD = AB × BC
= 4 × 3 = 12 cm^2

Question 14.
The length of the common chord of two intersecting circles is 30 cm. If the radii of the two circles are 25 cm and 17 cm, find the distance between their centres.
Solution:
AB is the common chord of two circles with centre O and C. Join OA, CA and OC



$AB = 30$ cm

$OA = 25$ cm and $AC = 17$ cm

$\therefore$ OC is the perpendicular bisector of AB at M.

$\therefore AM = MB = 15$ cm.

In right ΔOAM,

$OA^2 = OM^2 + AM^2$

$\Rightarrow 25^2 = OM^2 + (15)^2$

$\Rightarrow OM^2 = 25^2 - 15^2$

$= 625 - 225 = 400 = (20)^2$

$\therefore OM = 20$ cm.

Again in ΔAMC,

$AC^2 = AM^2 + MC^2$

$\Rightarrow 17^2 = 15^2 + MC^2$

$\Rightarrow MC^2 = 17^2 - 15^2$

$\Rightarrow MC^2 = 289 - 225 = 64 = (8)^2$

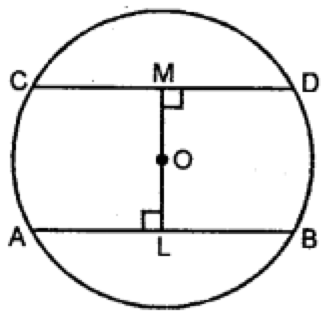
$\therefore MC = 8$ cm

Now $OC = OM + MC$,

$= 20 + 8 = 28$ cm.

Question 15.
The line joining the mid-points of two chords of a circle passes through its centre. Prove that the chords are parallel.
Solution:

Given : Two chords AB and CD where L and M are the mid-points of AB and CD respectively. LM passes through O, the centre of the circle.



To Prove : AB || CD.

Proof : $\because$ L is mid-point of AB.

$\therefore$ $OL \perp AB$

$\therefore \angle OLA = 90°$ ...(*i*)

Again M is mid point of CD

$\therefore$ $OM \perp CD$

$\therefore \angle OMD = 90°$ ...(*ii*)

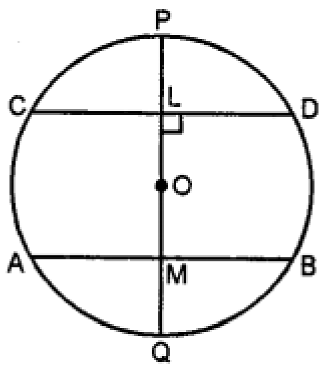
From (*i*) and (*ii*)

$\angle OLA = \angle OMD$

But these are alternate angles

$\therefore$ AB || CD Q.E.D.

Question 16.
If a diameter of a circle is perpendicular to one of two parallel chords of the circle, prove that it is perpendicular to the other and bisects it.
Solution:
Given : Chord AB || CD
and diameter PQ is perpendicular to AB



To Prove : PQ is perpendicular to CD.

Proof : ∵ Diameter PQ is perpendicular to AB.
∴ $\angle AMO = 90°$
∴ PQ bisects AB
∵ AB || CD (given)
∴ $\angle OLD = 90°$ (Alt. angles)
∴ OL or PQ is perpendicular to CD.
Hence PQ bisects CD. Q.E.D.

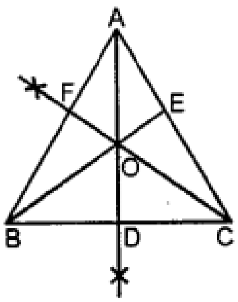
Question 17.

In an equilateral triangle, prove that the centroid and the circumcentre of the triangle coincide.

Solution:

Given : ΔABC in which $AB = BC = CA$.

To Prove : The centroid and the circumcentre coincide each other.



Construction : Draw perpendicular bisectors of AB and BC intersecting each other at O. Join AD, OB and OC.

Proof : $\because$ O lies on the perpendicular bisectors of AB and BC

$\therefore OA = OB = OC$

$\therefore$ O is the cimcumcentre of ΔABC.

$\because$ D is mid-point of BC.

$\therefore$ AD is the median of ΔABC.

Now in ΔABD and ΔACD,

$AB = AC$ (given)

$AD = AD$ (common)

$BD = BC$ ($\because$ D is mid-point of BC)

$\therefore \Delta ABD \cong \Delta ACD$

(SSS axiom of congruency)

$\therefore \angle ADB = \angle ADC$ (c.p.c.t)

But $\angle ADB + \angle ADC = 180°$

(Linear pair)

$\therefore \angle ADB = \angle ADC = 90°$

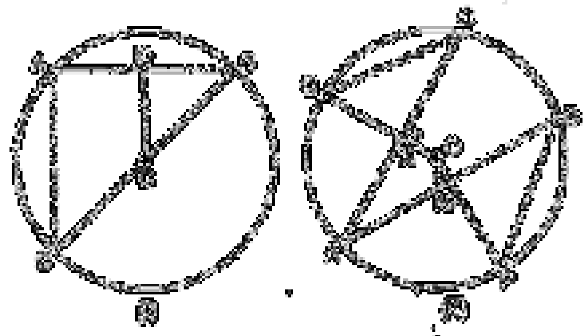
$\therefore$ AD is perpendicular on BC which passes through O.

Hence centroid and circumcentre of ΔABC coincide each other. Q.E.D.

Question 18.

(a) In the figure (i) given below, OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter, show that CA = 2 OD.

(b) In the figure (ii) given below, O is the centre of a circle. If AB and AC are chords of the circle such that AB = AC and OP ⊥ AB, OQ ⊥ AC, Prove that PB = QC.



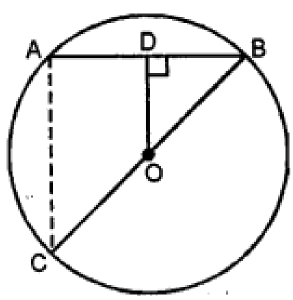
Solution:

(a) Given : OD is perpendicular to chord AB of the circle and BOC is the diameter.

CA is joined.

To Prove : CA = 2 OD.

Proof : ∵ $OD \perp AB$



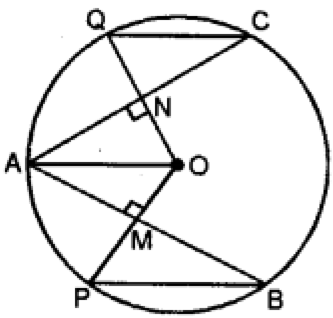
∴ D is mid point of AB and O is the mid point of BC.

∴ In ΔBAC,

$$OD \parallel CA \quad \text{and} \quad OD = \frac{1}{2} CA$$

$\Rightarrow CA = 2\ OD$ Q.E.D.

(*b*) **Given :** AB and AC are chords of a circle with centre O and AB = AC, $OP \perp AB$ and $OQ \perp AC$. BP and QC are joined.



To Prove : PB = QC.

Proof : $\because$ OP $\perp$ AB (given)

$\therefore$ M is mid-point of AB

$\therefore$ AM = MB $\Rightarrow$ MB = $\frac{1}{2}$AB

Similarly OQ $\perp$ AC

$\therefore$ AN = NC $\Rightarrow$ NC = $\frac{1}{2}$AC.

But AB = AC

$\therefore$ MB = NC

$\because$ Chord AB = Chord AC

$\therefore$ OM = ON

But OP = OQ (radii of the same circle)

$\therefore$ MP = NQ

Now in ΔMPB and ΔNQC,

MB = NC (proved)

MP = NQ (proved)

$\angle$PMB = $\angle$QNC (each 90°)

$\therefore$ ΔMPB $\cong$ ΔNQC

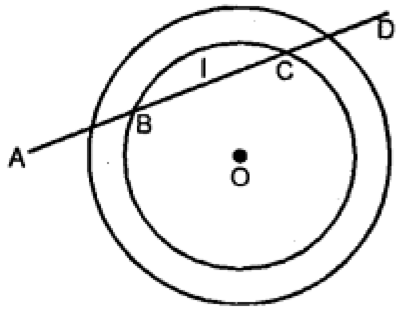
(SAS axiom of congruency)

$\therefore$ PB = QC (c.p.c.t) Q.E.D

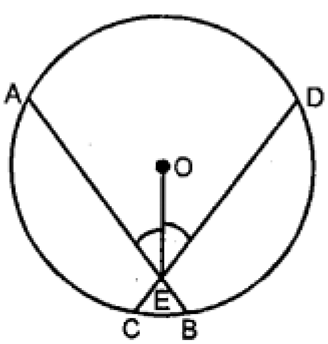
Question 19.

(a) In the figure (i) given below, a line l intersects two concentric circles at the points A, B, C and D. Prove that AB = CD.

(b) In the figure (it) given below, chords AB and CD of a circle with centre O intersect at E. If OE bisects ∠AED, Prove that AB = CD.



(i)



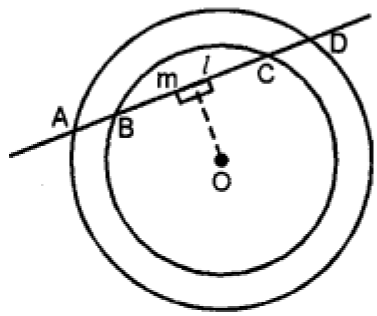
(ii)

Solution:

(a) **Given :** A line l intersects two concentric circles with centre O.

To Prove : AB = CD

Construction : Draw OM $\perp$ l.



Proof : $\because$ OM $\perp$ BC.

$\therefore$ BM = MC ...(i)

Again OM $\perp$ AD

$\therefore$ AM = MD. ...(ii)

Substracting (i) from (ii)

AM – BM = MD – MC

$\Rightarrow$ AB = CD

Q.E.D.

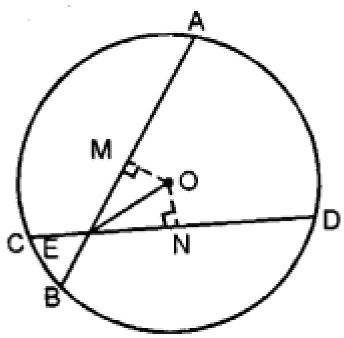
(b) **Given :** Two chords AB and CD intersect each other at E inside the circle with centre O. OE bisects $\angle$AED *i.e.* $\angle$OEA = $\angle$OED.

To Prove : AB = CD

Construction : From O, draw OM ⊥ AB and ON ⊥ CD.

Proof : In ΔOME and ΔONE

∠M = ∠N (each 90°)



OE = OE (common)

∠OEM = ∠OEN (given)

∴ ΔOME ≅ ΔONE

(ASS axiom of congruency)

∴ OM = ON

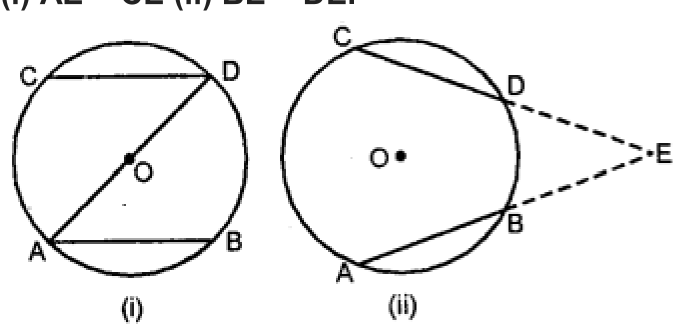
∴ AB = CD

(chords which are equidistant from the centre are equal)

Q.E.D.

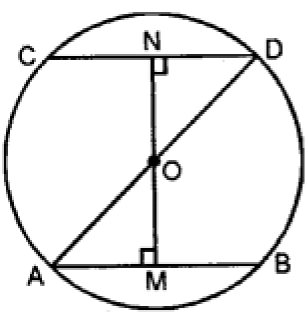
Question 20.

(a) In the figure (i) given below, AD is a diameter of a circle with centre O. If AB || CD, prove that AB = CD.
(b) In the figure (ii) given below, AB and CD are equal chords of a circle with centre O. If AB and CD meet at E (outside the circle) Prove that :
(i) AE = CE (ii) BE = DE.



Solution:

(*a*) **Given :** AD is the diameter of a circle with centre O and chords AB and CD are parallel.



To Prove : AB = CD.

Construction : From O, draw OM ⊥ AB and ON ⊥ CD

Proof : In ΔOMA and ΔOND,

∠AOM = ∠DON

(Vertically opposite angles)

OA = OD (radii of the same circle)

and ∠M = ∠N (each 90°)

∴ ΔOMA ≅ ΔOND

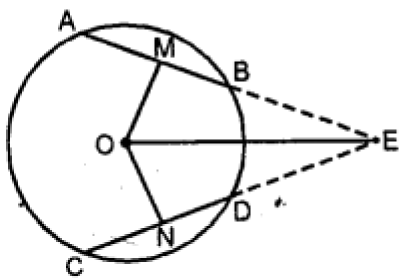
(AAS axiom of congruency)

∴ OM = ON (c.p.c.t)

But OM ⊥ AB and ON ⊥ CD

∴ AB = CD

(chords which are equidistant from the centre are equal) (*b*) **Given :** Chord AB = chord CD of circle with centre O. and meet at E on producing them.



To Prove : (*i*) AE = CE

(*ii*) BE = DE

Construction : From O, draw OM ⊥ AB and ON ⊥ CD. Join OE

In right ΔOME and ΔONE

Hyp. OE = OE (Common)

Side OM = ON

(Equal chords are equidistant from the centre)

∴ $\Delta OME \cong \Delta ONE$

(R.H.S. axiom of congruency)

∴ ME = NE (c.p.c.t) ...(*i*)

∵ $OM \perp AB$ and $ON \perp CD$

∴ M is mid-point of AB and N is mid point of CD.

∴ $MB = \frac{1}{2}AB$ and $ND = \frac{1}{2}CD$

But AB = CD ...(*ii*)

∴ MB = ND

∴ Subtracting, (*ii*) from (*i*)

ME – MB = NE – ND

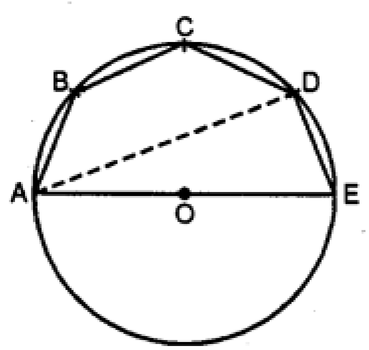
⇒ BE = DE

But AB = CD (given)

∴ Adding, we get

AB + BE = CD + DE

⇒ AE = CE (Hence proved)



EXERCISE 15.2

Question 1.

If arcs APB and CQD of a circle are congruent, then find the ratio of AB: CD.

Solution:

$\overparen{APB} = \overparen{CQD}$

(given)

$\therefore$ AB = CD

($\because$ If two arcs are congruent, then their corresponding chords are equal)

$\therefore$ Ratio of AB and CD $= \frac{AB}{CD} = \frac{AB}{AB} = \frac{1}{1}$

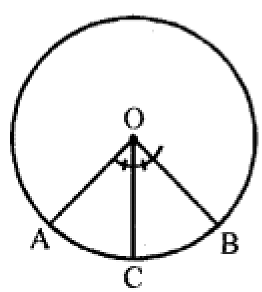
$\Rightarrow$ AB : CD = 1 : 1

Question 2.

A and B are points on a circle with centre O. C is a point on the circle such that OC bisects ∠AOB, prove that OC bisects the arc AB.

Solution:

Given : In a given circle with centre O, A and B are two points on the circle. C is another point on the circle such that $\angle AOC = \angle BOC$



To prove : arc AC = arc BC

Proof : $\because$ OC is the bisector of $\angle AOB$

or $\angle AOC = \angle BOC$

But these are the angle subtended by the arc AC and BC.

$\therefore$ arc AC = arc BC. Q.E.D.

Question 3.

Prove that the angle subtended at the centre of a circle is bisected by the radius

passing through the mid-point of the arc.

Solution:

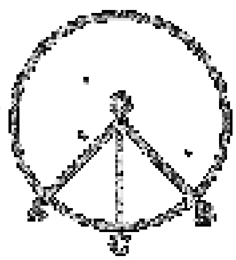
Given : AB is the arc of the circle with centre O and C is the mid-point of arc AB.

To prove : OC bisects the ∠AOB

i.e., ∠AOC = ∠BOC

Proof : ∵ C is the mid-point of arc AB.

∴ arc AC = arc BC



But arc AC and arc BC subtend ∠AOC and ∠BOC at the centre

∴ ∠AOC = ∠BOC

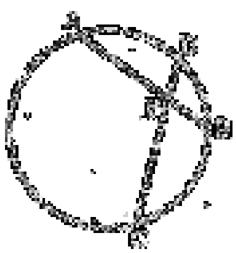
Hence OC bisects the ∠AOB.

Q.E.D.

Question 4.

In the given figure, two chords AB and CD of a circle intersect at P. If AB = CD, prove that arc AD = arc CB.

Solution:



Given : Two chords AB and CD of a circle intersect at P and AB = CD.

To prove : arc AD = arc CB

Proof : AB = CD (given)

∴ minor arc AB = minor arc CD

Subtracting arc BD from both sides

arc AB – arc BD = arc CD – arc BD

⇒ arc AD = arc CD Q.E.D.

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 6) :

Question 1.

If P and Q are any two points on a circle, then the line segment PQ is called a

(a) radius of the circle
(b) diameter of the circle
(c) chord of the circle
(d) secant of the circle

Solution:

chord of the circle **(c)**

Question 2.

If P is a point in the interior of a circle with centre O and radius r, then

(a) $OP = r$
(b) $OP > r$
(c) $OP \geq r$
(d) $OP < r$

Solution:

$OP > r$ **(b)**

Question 3.

The circumference of a circle must be

(a) a positive real number
(b) a whole number
(c) a natural number
(d) an integer

Solution:

a positive real number **(a)**

Question 4.

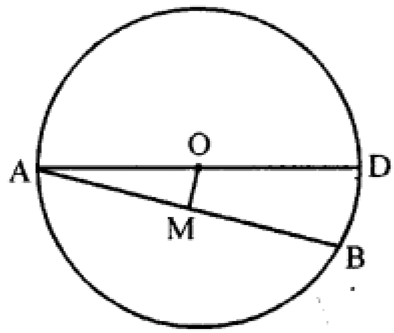
AD is a diameter of a circle and AB is a chord. If AD = 34 cm and AB = 30 cm, then the distance of AB from the centre of circle is

(a) 17 cm
(b) 15 cm
(c) 4 cm
(d) 8 cm

Solution:

AD is the diameter of the circle whose length is AD = 34 cm

AB is the chord of the circle whose length is AB = 30 cm



Distance of the chord from the centre is OM

Since the line through the centre of the chord of the circle is the perpendicular bisector, we have $\angle OMA = 90°$

and AM = BM

Thus, ΔAMO is a right angled triangle

Now, by applying Pythagorean Theorem,

$OA^2 = AM^2 + OM^2$ ...(*i*)

Since the diameter AD = 34 cm, radius of the circle is 17 cm

$\therefore$ OA = 17 cm

Since AM = BM and AB = 30 cm

$\therefore$ We have AM = BM = 15 cm

We have, $OA^2 = AM^2 + OM^2$

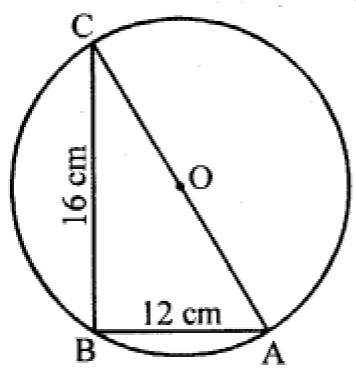
$17^2 = 15^2 + OM^2$

$OM^2 = 289 - 225$

$OM^2 - 64$

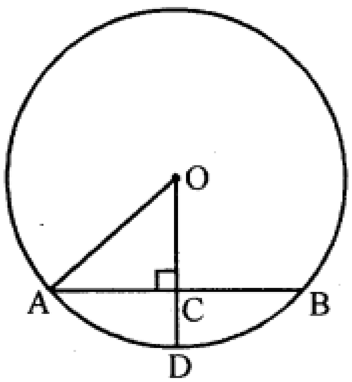
$OM = \sqrt{64} = 8$ cm (d)

Question 5.
If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is
(a) 6 cm
(b) 8 cm
(c) 10 cm
(d) 12 cm
Solution:
Give that AB = 12 cm and BC = 16 cm and
$\angle ABC = 90^\circ$



Every angle inscribed in a semicircle is a right angle.
Since the inscribed angle
$\angle ABC = 90^\circ$, the arc ABC is a semicircle
Thus, AC is the diameter of the circle passing through the centre.
Now, by Pythagoras Theorem,
$AC^2 = AB^2 + BC^2$
$= 12^2 + 16^2$
$= 14 + 256 = 400$
$AC = \sqrt{400} = 20$ cm
$\therefore$ Diameter of the circle is 20 cm
Thus, the radius of the circle passing through A, B and C is 10 cm. (c)

Question 6.
In the given figure, O is the centre of the circle. If OA = 5 cm, AB = 8 cm and OD ⊥ AB, then length of CD is equal to
(a) 2 cm
(b) 3 cm
(c) 4 cm
(d) 5 cm
Solution:



$OC = \sqrt{AO^2 - AC^2}$

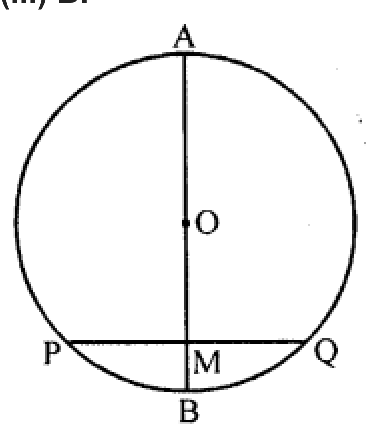
$= \sqrt{25-16} = \sqrt{9}$ cm = 3 cm

Since, OD = OA = 5 cm

$\therefore$ CD = OD – OC = 5 – 3 cm = 2 cm (a)

Chapter Test

Question 1.
In the given figure, a chord PQ of a circle with centre O and radius 15 cm is bisected at M by a diameter AB. If OM = 9 cm, find the lengths of :
(i) PQ
(ii) AP
(iii) BP

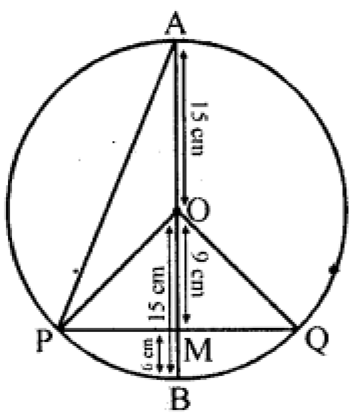


Solution:

Given, radius = 15 cm

$\Rightarrow$ OA = OB = OP = OQ = 15 cm

Also, OM = 9 cm



$\therefore$ MB = OB – OM = 15 – 9 = 6 cm

AM = OA + OM = 15 + 9 cm = 24 cm

In ΔOMP, by using Pythagoras Theoream,

$OP^2 = OM^2 + PM^2$

$15^2 = 9^2 + PM^2$

$= PM^2 = 225 - 81$

$PM = \sqrt{144} = 12$ cm

Also, In ΔOMQ,

by using Pythagoras Theorem,

$OQ^2 = OM^2 + QM^2$

$15^2 = OM^2 + QM^2$

$15^2 = 9^2 + QM^2 \Rightarrow QM^2 = 225 - 81$

$QM = \sqrt{144} = 12$ cm

$\therefore$ PQ = PM + QM

(As radius is bisected at M)

$\Rightarrow$ PQ = 12 + 12 cm = 24 cm

(*ii*) Now in ΔAPM

$AP^2 = AM^2 + OM^2$

$AP^2 = 24^2 + 12^2$

$AP^2 = 576 + 144$

$AP = \sqrt{720} = 12\sqrt{5}$ cm

(*iii*) Now in ΔBMP

$BP^2 = BM^2 + PM^2$

$BP^2 = 6^2 + 12^2$

$BP^2 = 36 + 144$

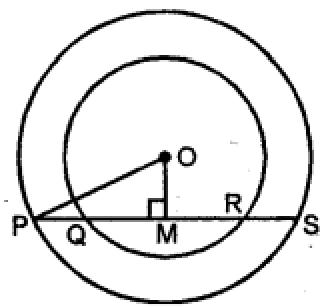
$BP = \sqrt{180} = 6\sqrt{5}$ cm

Question 2.

The radii of two concentric circles are 17 cm and 10 cm ; a line PQRS cuts the larger circle at P and S and the smaller circle at Q and R. If QR = 12 cm, calculate PQ.

Solution:

A line PQRS intersects the outer circle at P and S and inner circle at Q and R. Radius of outer circle OP = 17 cm and radius of inner circle OQ = 10 cm.



QR = 12 cm

From O, draw OM ⊥ PS

$$\therefore \quad QM = \frac{1}{2}QR = \frac{1}{2} \times 12 = 6\,\text{cm}$$

In right ΔOQM,

$$OQ^2 = OM^2 + QM^2$$

$$\Rightarrow \quad (10)^2 = OM^2 + (6)^2$$

$$\Rightarrow \quad OM^2 = 10^2 - 6^2$$

$$= 100 - 36 = 64 = (8)^2$$

$$\therefore \quad OM = 8\text{ cm}$$

Now in right ΔOPM,

$$OP^2 = OM^2 + PM^2$$

$$\Rightarrow \quad (17)^2 = (8)^2 + PM^2$$

$$\Rightarrow \quad PM^2 = (17)^2 - (8)^2$$

$$= 289 - 64 = 225 = (15)^2$$

$$\therefore \; PM = 15\text{ cm}$$

$$\therefore \; PQ = PM - QM = 15 - 6 = 9\text{ cm}$$

Question 3.

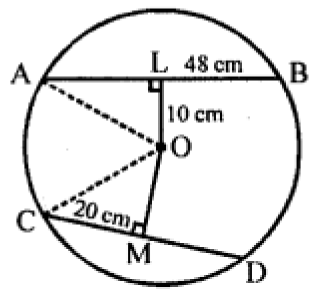
A chord of length 48 cm is at a distance of 10 cm from the centre of a circle. If another chord of length 20 cm is drawn in the same circle, find its distance from the centre of the circle.

Solution:

O is the centre of the circle

Length of chord AB = 48 cm

and chord CD = 20 cm



$OL \perp AB$ and $OM \perp CD$ are drawn

$\therefore AL = LB = \dfrac{48}{2} = 24$ cm

and $CM = MD = \dfrac{20}{2} = 10$ cm

OL = 10 cm

Now in right ΔAOL

$OA^2 = AL^2 + OL^2$ (Pythagoras Theorem)

$\Rightarrow OA^2 = (24)^2 + (10)^2 = 576 + 100$

$= 676 = (26)^2$

$\therefore OA = 26$ cm

But OC = OA (radii of the same circle)

$\therefore OC = 26$ cm

Now in right ΔOCM

$OC^2 = OM^2 + CM^2$

$(26)^2 = OM^2 + (10)^2$

$676 = OM^2 + 100 \Rightarrow OM^2 = 676 - 100$

$\Rightarrow OM^2 = 576 = (24)^2$

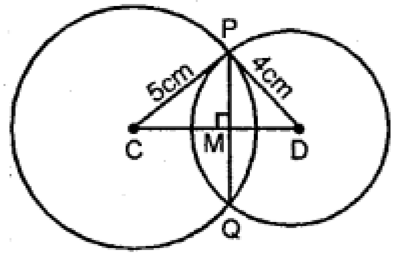
$\therefore OM = 24$ cm

Question 4.

(a) In the figure (i) given below, two circles with centres C, D intersect in points P, Q. If length of common chord is 6 cm and CP = 5 cm, DP = 4 cm, calculate the distance CD correct to two decimal places.

(*a*) Two circles with centre C and D intersect each other at P and Q. PQ is the common chord = 6 cm. The line joining the centres C and D bisects the chord PQ at M.

(b) In the figure (ii) given below, P is a point of intersection of two circles with centres C and D. If the st. line APB is parallel to CD, Prove that AB = 2 CD.



$\therefore \quad PM = MQ = \frac{6}{2} = 3\,cm$

Now in right ΔCPM,

$CP^2 = CM^2 + PM^2$

$\Rightarrow (5)^2 = CM^2 + (3)^2 \Rightarrow 25 = CM^2 + 9$

$\Rightarrow CM^2 = 25 - 9 = 16 = (4)^2$

$\therefore \quad CM = 4\ cm$

and in right ΔPDM,

$PD^2 = PM^2 + MD^2$

$\Rightarrow (4)^2 = (3)^2 + MD^2 \Rightarrow 16 = 9 + MD^2$

$\Rightarrow MD^2 = 16 - 9 = 7$

$\therefore \quad MD = \sqrt{7} = 2{\cdot}65\,cm$

$\therefore \quad CD = CM + MD = 4 + 2{\cdot}65$

$= 6{\cdot}65\ cm$

Solution:

(*b*) **Given :** Two circles with centre C and D intersect each other at P and Q. A straight line APB is drawn parallel to CD.

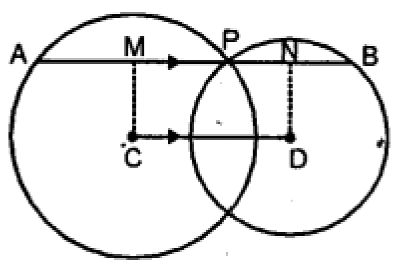
To Prove : AB = 2 CD.

Construction : Draw CM and DN perpendicular to AB from C and D.

Proof : $\because$ CM $\perp$ AP

$\therefore$ AM = MP or AP = 2 MP

and DN $\perp$ PB



$\therefore$ BN = PN or PB = 2 PN

Adding

AP + PB = 2 MP + 2 PN

$\Rightarrow$ AB = 2 (MP + PN) = 2 MN

$\Rightarrow$ AB = 2 CD. Q.E.D.

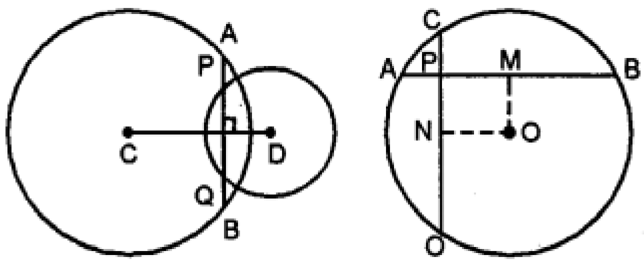
Question 5.

(a) In the figure (i) given below, C and D are centres of two intersecting circles. The line APQB is perpendicular to the line of centres CD.Provethat:

(i) AP=QB

(ii) AQ = BP.

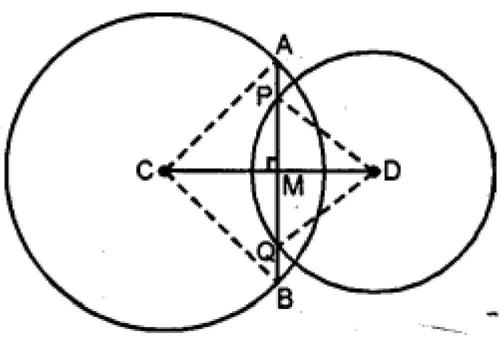
(b) In the figure (ii) given below, two equal chords AB and CD of a circle with centre O intersect at right angles at P. If M and N are mid-points of the chords AB and CD respectively, Prove that NOMP is a square.



(i) (ii)

Solution:

(*a*) **Given :** Two circles with centres C and D intersect each other. A line APQB is drawn perpendicular to CD at M.



To Prove : (*i*) AP = QB (*ii*) AQ = BP.

Construction : Join AC and BC, DP and DQ.

Proof : (*i*) In right ΔACM and ΔBCM

Hyp. AC = BC (radii of same circle)

Side CM = CM (common)

∴ ΔACM ≅ ΔBCM

(R.H.S. axiom of congruency)

$\therefore \quad AM = BM$ ...(i)

Again in right $\triangle OPM$ and $\triangle OQM$

Hyp. $PO = QO$

(radii of the same circle)

Side $OM = OM$ (common)

$\therefore \triangle OPM \cong \triangle OQM$

(R.H.S. axiom of congruency)

$\therefore \quad PM = QM$ ...(ii)

Subtracting (ii) from (i),

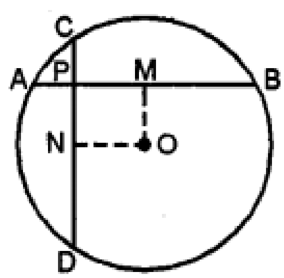
$AM - PM = BM - QM \quad \Rightarrow \quad AP = QB$

(ii) Adding PQ both sides,

$AP + PQ = PQ + QB$

$\Rightarrow \quad AQ = PB$ Q.E.D.

(*b*) **Given :** Two chords AB and CD intersect each other at P at right angle in the circle. M and N are mid-points of the chord AB and CD.



To Prove : NOMP is a square.

Proof : ∵ M and N are the mid-points of AB and CD respectively.

$\therefore$ $OM \perp AB$ and $ON \perp CD$

and $\quad OM = ON$

(∵ Equal chords are at equal distance from the centre)

$\because \quad AB \perp CD$

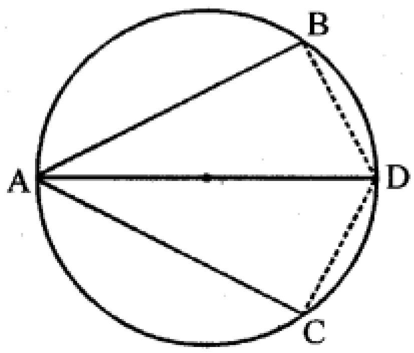
$\therefore \quad OM \perp ON$

Hence NOMP is a square.

Question 6.

In the given figure, AD is diameter of a circle. If the chord AB and AC are equidistant from its centre O, prove that AD bisects ∠BAC and ∠BDC.

Solution:



Given : AB and AC are equidistant from its centre O

So, AB = AC

In ΔABD and ΔACD

AB = AC (given)

∠B = ∠C (∵ Angle in a semicircle is 90°)

AD = AD (common)

∴ ΔABD ≅ ΔACD (SSS rule of congruency)

∴ AD bisects ∠BAC and ∠BDC