Mensuration

EXERCISE 16.1

Question 1.
Find the area of a triangle whose base is 6 cm and corresponding height is 4 cm.
Solution:

Base of triangle = 6 cm
Height = 4 cm

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6 \times 4 \text{ cm}^2 = 6 \times 2 \text{ cm}^2 = 12 \text{ cm}^2$

Question 2.
Find the area of a triangle whose sides are
(i) 3 cm, 4 cm and 5 cm
(ii) 29 cm, 20 cm and 21 cm
(iii) 12 cm, 9.6 cm and 7.2 cm
Solution:

(*i*) Here $a = 3$ cm, $b = 4$ cm and $c = 5$ cm

s = semi perimeter = $\frac{a+b+c}{2}$

$= \frac{3+4+5}{2} \text{ cm} = \frac{12}{2} \text{ cm} = 6 \text{ cm}$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)} \text{ cm}^2 = \sqrt{6 \times 3 \times 2 \times 1} \text{ cm}^2$

$= \sqrt{6 \times 6} \text{ cm}^2 = 6 \text{ cm}^2$

(*ii*) $a = 29$ cm, $b = 20$ cm and $c = 21$ cm

$$s = \text{semi perimeter} = \frac{a+b+c}{2} = \frac{29+20+21}{2} \text{ cm}$$

$$= \frac{70}{2} \text{ cm} = 35 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{35(35-29)(35-20)(35-21)} \text{ cm}^2$$

$$= \sqrt{35 \times 6 \times 15 \times 14} \text{ cm}^2$$

$$= \sqrt{7 \times 5 \times 3 \times 2 \times 5 \times 3 \times 7 \times 2} \text{ cm}^2$$

$$= \sqrt{7 \times 7 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2} \text{ cm}^2$$

$$= 7 \times 5 \times 3 \times 2 = 210 \text{ cm}^2$$

(*iii*) $a = 12$ cm, $b = 9.6$ cm and $c = 7.2$ cm

$$s = \text{semi perimeter} = \frac{a+b+c}{2} = \frac{12+9.6+7.2}{2} \text{ cm}$$

$$= \frac{28.8}{2} \text{ cm} = 14.4 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{14.4 \times (14.4-12) \times (14.4-9.6) \times (14.4-7.2)} \text{ cm}^2$$

$$= \sqrt{14.4 \times 2.4 \times 4.8 \times 7.2} \text{ cm}^2$$

$$= \sqrt{6 \times 2.4 \times 2.4 \times 2 \times 2.4 \times 3 \times 2.4} \text{ cm}^2$$

$$= 2.4 \times 2.4 \sqrt{6 \times 6} \text{ cm}^2 = 2.4 \times 2.4 \times 6 \text{ cm}^2$$

$$= 34.56 \text{ cm}^2$$

Question 3.

Find the area of a triangle whose sides are 34 cm, 20 cm and 42 cm. Hence, find the length of the altitude corresponding to the shortest side.

Solution:

Given sides of triangle are 34 cm, 20 cm, and 42 cm

i.e. $a = 34$ cm, $b = 20$ cm, and $c = 42$ cm

$$s = \frac{a+b+c}{2} = \frac{34+20+42}{2} \text{ cm} = \frac{96}{2} \text{ cm} = 48 \text{ cm}$$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-34)(48-20)(48-42)}$ cm^2

$= \sqrt{48 \times 14 \times 28 \times 6}$ cm^2 $= \sqrt{8 \times 6 \times 14 \times 14 \times 2 \times 6}$ cm^2

$= 14 \times 6 \sqrt{8 \times 2}$ cm^2 $= 14 \times 6 \times \sqrt{16}$ cm^2

$= 14 \times 6 \times 4$ cm^2 $= 336$ cm^2

The shortest side of the triangle is 20 cm.

Let h cm be the corresponding altitude, then area of triangle $= \frac{1}{2} \times$ base $\times$ height

$$\Rightarrow \quad 336 = \frac{1}{2} \times 20 \times h \quad \Rightarrow \quad h = \frac{336 \times 2}{20} \text{ cm}$$

$$\Rightarrow \quad h = \frac{336}{10} \text{ cm} \quad \Rightarrow \quad h = 33.6 \text{ cm}$$

$\therefore$ The required altitude of the triangle= 33.6 cm

Question 4.
The sides of a triangular field are 975m, 1050 m and 1125 m. If this field is sold at the rate of Rs. 1000 per hectare, find its selling price. [1 hectare = 10000 m²].
Solution:

Given : Sides of a triangular field are 975 m, 1050 m, and 1125 m
i.e. $a = 975\ m,\ b = 1050\ m,\ c = 1125\ m$

$$s = \frac{a+b+c}{2} = \frac{975+1050+1125}{2}\text{ cm} = \frac{3150}{2}\text{ cm}$$
$= 1575$ cm

Area of triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{1575(1575-975)(1575-1050)(1575-1125)}\ \text{m}^2$

$= \sqrt{1575\times600\times525\times450}\ \text{m}^2$

$= \sqrt{525\times3\times150\times4\times525\times150\times3}\ \text{m}^2$

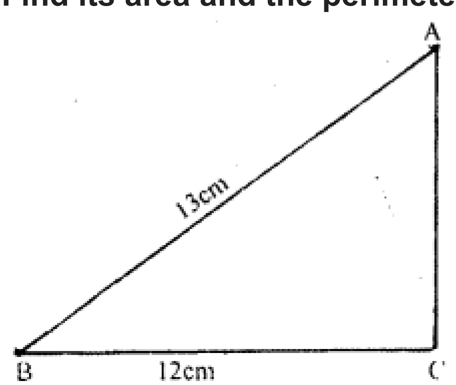
$= 525\times3\times150\ \sqrt{4}\ \text{m}^2 = 525\times450\times2\ \text{m}^2$

$= \dfrac{525\times900}{10000}$ hectare $\quad (\because\ 1\text{ hectare} = 10000\text{ m}^2)$

$= \dfrac{525\times90}{100}$ hectare $= \dfrac{4725}{100}$ hectare $=$ 47.25 hectare

Selling price of 1 hectare field = Rs. 1000
Selling price of 47.25 hectare field = Rs. 1000 × 47.25
= Rs. 47250

Question 5.
The base of a right angled triangle is 12 cm and its hypotenuse is 13 cm long. Find its area and the perimeter.



Solution:

Here ABC is a right angled triangle BC = 12 cm,
AB = 13 cm

By Pythagoras theorem,

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad (13)^2 = (AC)^2 + (12)^2$

$\Rightarrow \quad (AC)^2 = (13)^2 - (12)^2 \quad \Rightarrow \quad (AC)^2 = 169 - 144$

$\Rightarrow \quad (AC)^2 = 25 \quad \Rightarrow \quad AC = 25$

$\Rightarrow \quad AC = \sqrt{25} = 5$ cm

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12 \times 5\ \text{cm}^2 = 30\ \text{cm}^2$

Perimeter of $\Delta ABC = AB + BC + CA$

$= 13 + 12 + 5 = 30$ cm

Question 6.

Find the area of an equilateral triangle whose side is 8 m. Given your answer correct to two decimal places.

Solution:

Side of equilateral triangle = 8 m

Area of equilateral triangle $= \frac{\sqrt{3}}{4}\ (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 8 \times 8\ \text{m}^2 = \sqrt{3} \times 2 \times 8\ \text{m}^2 = 1.73 \times 16\text{m}^2$

$= 27.71\ \text{m}^2$

Question 7.
If the area of an equilateral triangle is 81√3 cm² find its. perimeter.
Solution:

Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

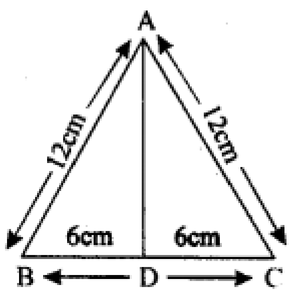
$\Rightarrow \quad 81\sqrt{3} = \frac{\sqrt{3}}{4}(\text{side})^2 \quad \Rightarrow \quad (\text{side})^2 = \frac{81\sqrt{3}\times 4}{\sqrt{3}}$

$\Rightarrow \quad (\text{side})^2 = 81 \times 4 \text{ cm}^2 \quad \Rightarrow \quad \text{side} = \sqrt{81\times 4} \text{ cm}$

$\Rightarrow \quad \text{side} = 9 \times 2 \text{ cm} \quad \Rightarrow \quad \text{side} = 18 \text{ cm}$

Perimeter of equilateral triangle $= 3 \times \text{side}$
$= 3 \times 18 \text{ cm} = 54 \text{ cm}$

Question 8.
If the perimeter of an equilateral triangle is 36 cm, calculate its area and height.
Solution:



Perimeter of an equilateral triangle $= 3 \times \text{side}$

$\Rightarrow 36 = 3 \times \text{side} \Rightarrow \text{side} = \frac{36}{3} \text{ cm} \Rightarrow \text{side} = 12 \text{ cm}$

i.e. $AB = BC = CA = 12 \text{ cm}$

Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (12)^2 \text{ cm}^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2$

$= \sqrt{3} \times 3 \times 12 \text{ cm}^2 = 1.73 \times 36 \text{ cm}^2 = 62.4 \text{ cm}^2$

In triangle ABD, By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad (12)^2 = AD^2 + (6)^2 \quad \left[\because BD = \frac{12}{2} = 6\text{cm}\right]$

$\Rightarrow \quad 144 = AD^2 + 36 \quad \Rightarrow \quad AD^2 = 144 - 36$

$\Rightarrow \quad AD^2 = 108 \quad \Rightarrow \quad AD = \sqrt{108} = 10.4$

Hence, required height = 10.4 cm

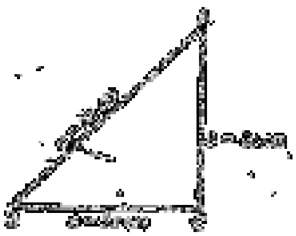
Question 9.

(i) If the length of the sides of a triangle are in the ratio 3:4:5 and its perimeter is 48 cm, find its area.

(ii) The sides of a triangular plot are in the ratio 3: 5:7 and its perimeter is 300 m. Find its area.

Solution:

[illegible]



[illegible]

Question 10.

ABC is a triangle in which AB = AC = 4 cm and ∠ A = 90°. Calculate the area of △ABC. Also find the length of perpendicular from A to BC.

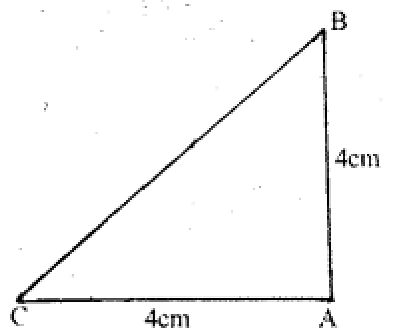
Solution:

Given that AB = AC = 4 cm

By Pythagoras theorem,

$BC^2 = AB^2 + AC^2 \Rightarrow BC^2 = (4)^2 + (4)^2$

$\Rightarrow BC^2 = 16 + 16 \Rightarrow BC = \sqrt{32} \Rightarrow BC = 4\sqrt{2}$ cm



Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 4 \times 4 = 2 \times 4 = 8 \text{ cm}^2$

Let length of perpendicular from A to BC = h cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times h$

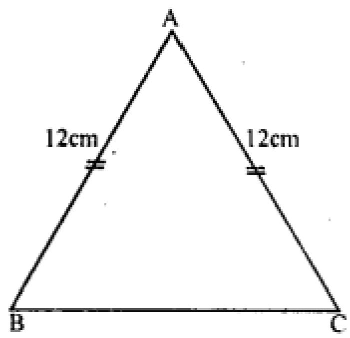
$\Rightarrow 8 = \frac{1}{2} \times 4\sqrt{2} \times h \Rightarrow h = \frac{8 \times 2}{4\sqrt{2}}$

$= \frac{2 \times 2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2 \times \sqrt{2}$

$= 2 \times 1.41 = 2.82$ cm

Question 11.
Find the area of an isosceles triangle whose equal sides are 12 cm each and the perimeter is 30 cm.
Solution:

In Δ ABC,
AB = AC = 12 cm



But perimeter = 30 cm
$\therefore$ BC = 30 − (12 + 12) = 30 − 24 = 6cm

Now $S = \frac{a+b+c}{2} = \frac{30}{2} = 15$

$\therefore$ Area of Δ ABC = $\sqrt{s(s-a)(s-b)(s-c)}$
(Using hero's formula)

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15\times3\times3\times9} = \sqrt{81\times15} = 9\sqrt{15}$ cm^2

$= 9 \times 3.873$ cm^2 = 34.857 cm^2

= 34.86 cm^2

Question 12.

Find the area of an isosceles triangle whose base is 6 cm and perimeter is 16 cm.

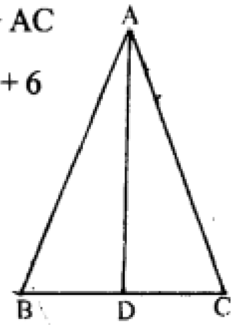
Solution:

Base = 6 cm, Perimeter = 16 cm

Here ABC is isosceles triangle in which

AB = AC = x (say)

and BC = 6 cm



Perimeter of ΔABC = AB + BC + AC

$\Rightarrow \quad 16 = x + 6 + x \quad \Rightarrow \quad 16 = 2x + 6$

$\Rightarrow \quad 16 - 6 = 2x \quad \Rightarrow \quad 10 = 2x$

$\Rightarrow \quad x = \frac{10}{2}$

$\Rightarrow \quad x = 5$

i.e. AB = AC = 5 cm and BD = $\frac{1}{2} \times 6 = 3$ cm.

In right angled ΔABD

$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad (5)^2 = AD^2 + (3)^2 \quad \Rightarrow \quad 25 = AD^2 + 9$

$\Rightarrow \quad AD^2 = 25 - 9 \quad \Rightarrow \quad AD^2 = 16 \quad \Rightarrow \quad AD = 4$ cm

Area of ΔABC = $\frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 6 \times 4 \text{ cm}^2 = 3 \times 4 \text{ cm}^2 = 12 \text{ cm}^2$

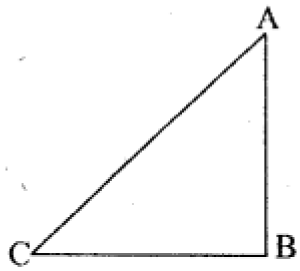
Question 13.

The sides of a right angled triangle containing the right angle are 5x cm and (3x – 1) cm. Calculate the length of the hypotenuse of the triangle if its area is 60 cm².

Solution:

Here ABC be a right angled triangle

AB = $5x$ cm and BC = $(3x - 1)$ cm



Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$60 = \frac{1}{2} \times 5x(3x-1) \Rightarrow 120 = 5x(3x-1)$

$\Rightarrow 120 = 15x^2 - 5x \Rightarrow 0 = 15x^2 - 5x - 120$

$\Rightarrow 15x^2 - 5x - 120 = 0 \Rightarrow 5(3x^2 - x - 24) = 0$

$\Rightarrow 3x^2 - x - 24 = 0 \Rightarrow 3x^2 - 9x + 8x - 24 = 0$

$\Rightarrow 3x(x-3) + 8(x-3) = 0 \Rightarrow (3x+8)(x-3) = 0$

Either, $3x + 8 = 0$ or, $x - 3 = 0$

$3x = -8$ or, $x = 3$

$x = \frac{-8}{3}$ or, $x = 3$

$\therefore x = 3$ $\left(\because x = \frac{-8}{3}, \text{ is not possible}\right)$

AB = 5×3 cm = 15 cm

BC = $(3 \times 3 - 1)$ cm = $(9 - 1)$ cm = 8 cm

In right angled ΔABC,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (15)^2 + (8)^2$

$\Rightarrow AC^2 = 225 + 64 \Rightarrow AC^2 = 289$

$\Rightarrow AC^2 = (17)^2 \Rightarrow AC = 17$ cm

Hence, hypotenuse of right angled triangle = 17 cm

Question 14.
In Δ ABC, ∠B = 90°, AB = (2A + 1) cm and BC = (A + 1). cm. If the area of the Δ ABC is 60 cm², find its perimeter.
Solution:

$AB = (2x + 1)$ cm, $BC = (x + 1)$ cm

Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$\Rightarrow 60 = \frac{1}{2} \times (2x + 1) \times (x + 1)$

$\Rightarrow 60 \times 2 = (2x + 1)(x + 1) \Rightarrow 120 = (2x + 1)(x + 1)$

$\Rightarrow 120 = 2x^2 + 2x + x + 1 \Rightarrow 120 = 2x^2 + 3x + 1$

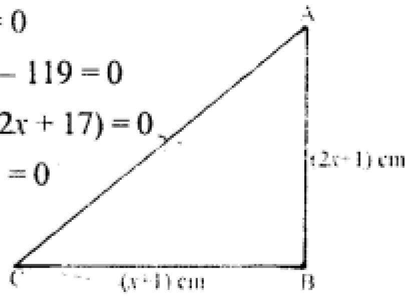
$\Rightarrow 0 = 2x^2 + 3x + 1 - 120 \Rightarrow 0 = 2x^2 + 3x - 119$

$\Rightarrow 2x^2 + 3x - 119 = 0$

$\Rightarrow 2x^2 + 17x - 14x - 119 = 0$

$\Rightarrow x(2x + 17) - 7(2x + 17) = 0$

$\Rightarrow (x - 7)(2x + 17) = 0$



Either $x - 7 = 0$

or $2x + 17 = 0$

$x = 7$ or

$x = \frac{-17}{2}$ (not possible)

$\therefore AB = (2x + 1)$ cm $= (2 \times 7 + 1)$ cm

$= (14 + 1)$ cm $= 15$ cm

$BC = (x + 1)$ cm $= 7 + 1 = 8$ cm

In right angled ΔABC,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (15)^2 + (8)^2$

$\Rightarrow AC^2 = 225 + 64 \Rightarrow AC^2 = 289$

$\Rightarrow AC^2 = (17)^2 \Rightarrow AC = 17$

Perimeter $= AB + BC + AC = (15 + 8 + 17)$ cm

$= 40$ cm

Question 15.

If the perimeter of a right angled triangle is 60 cm and its hypotenuse is 25 cm, find its area.

Solution:

Perimeter of right angled triangle = 60 cm
and hypotenuse = 25 cm

$\therefore$ Sum of two sides = 60 – 25 = 35 cm

Let base = x cm

$\therefore$ Then altitude = $(35 - x)$ cm

But $x^2 + (35 - x)^2 = (25)^2$ (By Pythagoras Theorem)

$x^2 + 1225 + x^2 - 70x = 625$

$2x^2 - 70x + 1225 - 625 = 0 \Rightarrow \Rightarrow 2x^2 - 70x + 600 = 0$

$\Rightarrow x^2 - 35x + 300 = 0 \Rightarrow x^2 - 15x - 20x + 300 = 0$

$\Rightarrow x(x - 15) - 20(x - 15) = 0 \Rightarrow (x - 15)(x - 20) = 0$

Either $x - 15 = 0$, then $x = 15$

or $x - 20 = 0$, then $x = 20$

$\therefore$ Sides are 15 cm and 20 cm

and area = $\frac{1}{2}$ base × altitude

$= \frac{1}{2} \times 15 \times 20 = 150 \text{ cm}^2$

Question P.Q.

In Δ ABC, $\angle$B = 90° and D is mid-point of AC. If AB = 20 cm and BD = 14.5 cm, find the area and the perimeter of Δ ABC.

Solution:

Here, in ΔABC

$\angle B = 90°$ and D is mid-point of AC

$BD = AD = DC$ (By Geometry)

$\therefore AC = 2BD \Rightarrow AC = 2 \times 14.5 \Rightarrow AC = 29$ cm.

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow (29)^2 = (20)^2 + BC^2$

$\Rightarrow 841 = 400 + BC^2$

$\Rightarrow BC^2 = 841 - 400$

$\Rightarrow BC^2 = 441$

$\Rightarrow BC = \sqrt{441} = 21$ cm

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 21 \times 20 \text{ cm}^2 = 21 \times 10 \text{ cm}^2 = 210 \text{ cm}^2$

Perimeter of $\Delta ABC = AB + BC + AC$

$= (20 + 21 + 29)$ cm = 70 cm

Question 16.

The perimeter of an isosceles triangle is 40 cm. The base is two third of the sum of equal sides. Find the length of each side.

Solution:

Perimeter of isosceles ΔABC = 40 cm.

Let each equal side = x

$\therefore$ base $= \frac{2}{3}(2x) = \frac{4}{3}x$

According to the sum,

$2x + \frac{4}{3}x = 40 \Rightarrow 6x + 4x = 120$

$\Rightarrow 10x = 120 \Rightarrow x = \frac{120}{10} = 12$

$\therefore$ length of each equal side = 12 cm.

Question 17.

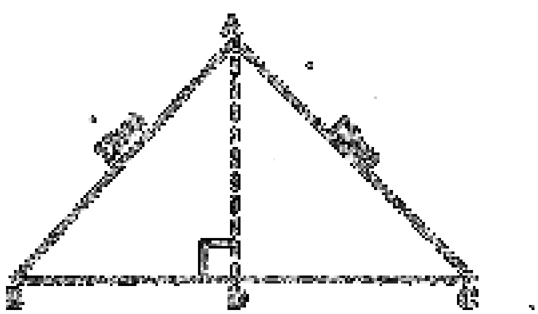
If the area of an isosceles triangle is 60 cm2 and the length of each of its equal sides is 13 cm, find its base.

Solution:

Area of isosceles triangle = 60 cm²

Length of each equal side = 13 cm

Let base BC = x cm



Draw AD $\perp$ BC which bisects BC at D

$\therefore BD = DC = \frac{x}{2}$ cm

Now in right ΔABD,

$AB^2 = BD^2 + AD^2 \Rightarrow (13)^2 = \left(\frac{x}{2}\right)^2 + AD^2$

(Pythagoras theorem)

$\Rightarrow 169 = \frac{x^2}{4} + AD^2 \Rightarrow AD^2 = 169 - \frac{x^2}{4}$ ...(i)

Area = 60 cm²

$\therefore AD = \frac{Area \times 2}{base} = \frac{60 \times 2}{x} = \frac{120}{x}$ ...(ii)

From (i) and (ii)

$169 - \frac{x^2}{4} = \left(\frac{120}{x}\right)^2$

$$\frac{676-x^2}{4} = \frac{14400}{x^2} \Rightarrow 676x^2 - x^4 = 57600$$

$x^4 - 676x^2 + 57600 = 0 \Rightarrow x^4 - 576x^2 - 100x^2 + 57600 = 0$

$x^2(x^2 - 576) - 100(x^2 - 576) = 0$

$\Rightarrow (x^2 - 576)(x^2 - 100) = 0$

Either $x^2 - 576 = 0$, then $x^2 = 576$

$\Rightarrow x^2 = (24)^2 \Rightarrow x = 24$

or $x^2 - 100 = 0$, then $x^2 = 100 = (10)^2$

$\therefore x = 10$

$\therefore$ Base = 10 cm or 24 cm.

Question 18.

The base of a triangular field is 3 times its height If the cost of cultivating the field at the rate of 25 per 100m2 is 60000, find its base and height.

Solution:

Cost of cultivating the field at the rate of ₹25 Per 100 m^2 is ₹60000

i.e. Cost of cultivating the field of ₹25 for 100 m^2

Cost of cultivating the field of ₹1 for $= \frac{100}{25}$ m^2

Cost of cultivating the field of ₹60000 for

$\frac{100}{25} \times 60000 \text{ m}^2 = 4 \times 60000 \text{ m}^2 = 240000 \text{ m}^2$

i.e. Area of field = 240000 m^2

$\Rightarrow \quad \frac{1}{2} \times \text{base} \times \text{height} = 240000 \text{ m}^2 \quad \ldots(1)$

Let height of triangular field = h m^2

Then, Base of triangular field = $3h$ m^2

Putting this value in equation (1), we get

$\frac{1}{2} \times 3h \times h = 240000 \Rightarrow \frac{1}{2} \times 3h^2 = 240000$

$\Rightarrow h^2 = \frac{240000 \times 2}{3} \Rightarrow h^2 = 80000 \times 2$

$\Rightarrow h^2 = 160000 \Rightarrow h = \sqrt{160000} = 400$

Hence, height of triangular field = 400 m

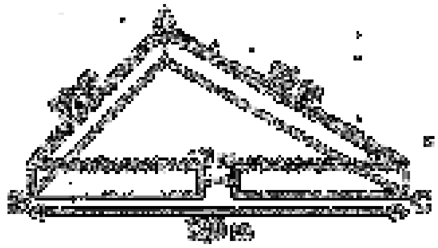
and Base of triangular field = 3 × 400 m

= 1200 m^2

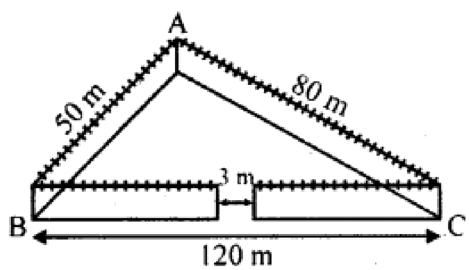
Question 19.

A triangular park ABC? has sides 120 m, 80 m and 50 m (as shown in the given figure). A gardner Dhania has to put a fence around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of □20 per metre leaving a space 3 m wide for a gate on one side.

Solution:



ΔABC is a triangular park whose sides are 120 m, 80 m and 50 m.



∴ Perimeter of ΔABC = 120 + 80 + 50 m
= 250 m

Portion at which a gate is build = 3 m

∴ Remaining perimeter = 250 – 3 = 247 m

Now, length of fence around it = 247 m

∴ Rate of fencing = ₹20 per m

∴ Total cost = ₹20 × 247 = ₹4940

and area of the park

$$s = \frac{a+b+c}{2} = \frac{250}{2} = 125$$

∴ Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{125(125-50)(125-80)(125-120)}$$

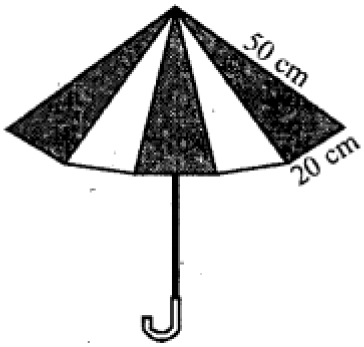
$$= \sqrt{125 \times 75 \times 45 \times 5}$$

$$= \sqrt{5 \times 5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 5 \times 5}$$

$$= 5 \times 5 \times 3 \times 5\sqrt{15}$$

$$= 375\sqrt{15} \text{ m}^2$$

Question 20.
An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (shown in the given figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?



Solution:

An umberalla is made by stitching 10 triangular piece of cloth of two different colours.

The measurement of each triangular is 20 cm, 50 cm, 50 cm.

$$\frac{s}{2} = \frac{20+50+50}{2} = \frac{120}{2} = 60$$

∴ Area of one trianlge

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{60(60-20)(60-50)(60-50)}$$

$$= \sqrt{60 \times 40 \times 10 \times 10} = \sqrt{240000}$$

$$= 100\sqrt{24} \text{ cm}^2$$

$$= 100 \times 2\sqrt{6} = 200\sqrt{6} \text{ cm}^2$$

Now, area of 5 triangular piece of first colour

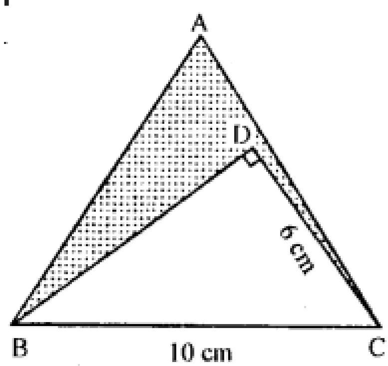
$$= 5 \times 200\sqrt{6} \text{ m}^2 = 1000\sqrt{6} \text{ cm}^2$$

and area of triangular of second colour

$$= 1000\sqrt{6} \text{ cm}^2$$

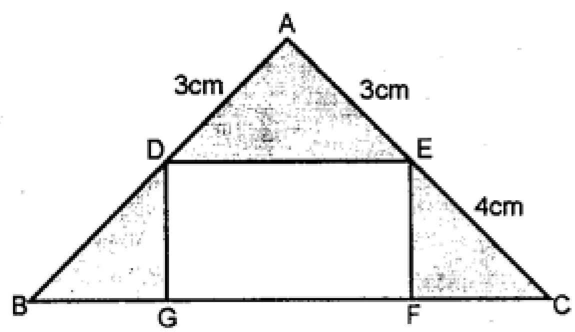
Question 21.

(a) In the figure (1) given below, ABC is an equilateral triangle with each side of length 10 cm. In Δ BCD, $\angle D = 90°$ and CD = 6 cm.
Find the area of the shaded region. Give your answer correct to one decimal place.



(1)

(b) In the figure given, ABC is an isosceles right angled triangle and DEFG is a rectangle. If AD = AE = 3 cm and DB = EC = 4 cm, find the area of the shaded region.



Solution:

(*a*) ABC is an equilateral triangle side of equilateral $\Delta = 10$ cm.

Area of equilateral ΔABC

$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (10)^2 \text{ cm}^2$

$= \frac{\sqrt{3}}{4} \times 100 \text{ cm}^2 = \sqrt{3} \times 25 \text{ cm}^2$

$= 25 \times \sqrt{3} \text{ cm}^2 = 25 \times 1.73 \text{ cm}^2 = 43.300 \text{ cm}^2$

In right angled ΔBDC

$\angle D = 90°$, BC = 10 cm, CD = 6 cm

By Pythagoras theorem,

$BD^2 + DC^2 = BC^2$

$\Rightarrow \quad BD^2 + (6)^2 = (10)^2 \quad \Rightarrow \quad BD^2 + 36 = 100$

$\Rightarrow \quad BD^2 = 100 - 36 \quad \Rightarrow \quad BD^2 = 64$

$\Rightarrow \quad BD = \sqrt{64} = 8$

Area of $\Delta BDC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BD \times DC$

$= \frac{1}{2} \times 8 \times 6 \text{ cm}^2 = 4 \times 6 \text{ cm}^2 = 24 \text{ cm}^2$

$\therefore$ Area of shaded portion

= Area of ΔABC – Area of ΔBDC

$= 43.300 \text{ cm}^2 - 24 \text{ cm}^2 = 19.300 \text{ cm}^2 = 19.3 \text{ cm}^2$

(*b*) AD = AE = 3 cm.

DB = EC = 4 cm.

Adding, AD + DB = AE + EC = (3 + 4) cm.

$\Rightarrow$ AB = AC = 7 cm.

$\because \angle A = 90°$

$\therefore$ Area of right $\Delta ADE = \frac{1}{2} AD \times AE$

$= \frac{1}{2} \times 3 \times 3 = \frac{9}{2} \text{ cm}^2$

$\because$ ΔBDG is an isosceles right triangle

$\therefore$ $DG^2 + BG^2 = BD^2 \Rightarrow DG^2 + DG^2 = (4)^2$

$\Rightarrow 2DG^2 = 16 \Rightarrow DG^2 = \frac{16}{2} = 8$

$\therefore$ $DG = \sqrt{8}$ cm

Now area of Δ BDG $= \frac{1}{2}$ BG $\times$ DG

$= \frac{1}{2} DG \times DG = \frac{1}{2}(\sqrt{8})^2 = \frac{1}{2} \times 8 = 4\, cm^2$.

Similarly area of isosceles right $\Delta EFC = 4\ cm^2$

Now area of shaded portion $= \frac{9}{2} + 4 + 4$

$= \frac{9+8+8}{2} = \frac{25}{2} cm^2 = 12{\cdot}5\ cm^2$

EXERCISE 16.2

Question 1.

(i) Find the area of quadrilateral whose one diagonal is 20 cm long and the perpendiculars to this diagonal from other vertices are of length 9 cm and 15 cm.

Solution:

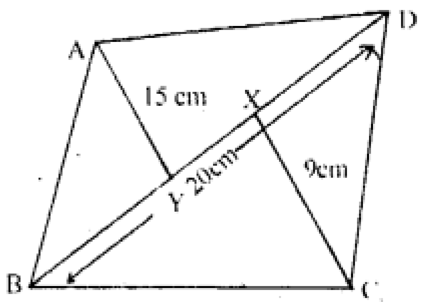
Let ABCD be quadrilateral in which AC = 20 cm

$\perp$ BY = 9 cm

$\perp$ DY = 15 cm

(*a*) Area of quadrilateral ABCD

= Area of ΔABC + Area of ΔACD



$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{base} \times \text{height}$

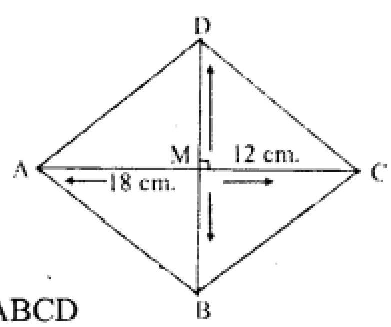
$= \frac{1}{2} \times AC \times BX + \frac{1}{2} \times AC \times DY$

$= \left(\frac{1}{2} \times 20 \times 9\right) + \left(\frac{1}{2} \times 20 \times 15\right) \text{ cm}^2$

$= (10 \times 9 + 10 \times 15) \text{ cm}^2 = (90 + 150) \text{ cm}^2 = 240 \text{ cm}^2$

(*ii*) Find the area of a quadrilateral whose diagonals are of length 18 cm and 12 cm, and they intersect each other at right angles.

Let ABCD be a quadrilateral in which diagonals AC and BD intersect each other at M at right angles AC = 18 cm and BD = 12 cm



Area of quadrilateral ABCD

$= \frac{1}{2} \times \text{diagonal } AC \times \text{diagonal } BD = \frac{1}{2} \times 18 \times 12 \text{cm}^2$

$= 9 \times 12 \text{ cm}^2 = 108 \text{ cm}^2$

Question 2.
Find the area of the quadrilateral field ABCD whose sides AB = 40 m, BC = 28 m, CD = 15 m, AD = 9 m and ∠A = 90°
Solution:

A quadrilateral ABCD in which AB = 40 m, BC = 28 m, CD = 15 m, AD = 9 m.

In ∠BAD ∠A = 90°

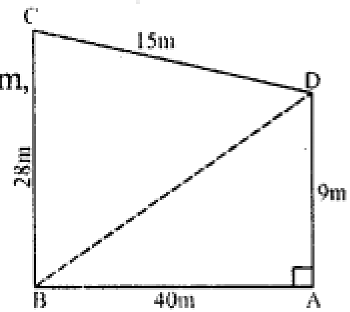
By Pythagoras theorem,

$BD^2 = BA^2 + AD^2$

$BD^2 = (40)^2 + (9)^2$

$BD^2 = 1600 + 81 = 1681$

$BD = 41$



Area of quadrilateral ABCD = Area of Δ BAD + Area of Δ BDC

$= \frac{1}{2} \times \text{Base} \times \text{height} + \text{Area of } \Delta\text{ BDC}$

$= \frac{1}{2} \times 40\text{ m} \times 9\text{ m} + \text{Area of } \Delta\text{ BDC}$

$= 20\text{ m} \times 9\text{ m} + \text{Area of } \Delta\text{ BDC}$

$= 180\text{ m}^2 + \text{Area of } \Delta\text{ BDC}$

Now to find area of Δ BDC

Let a = BD = 41 m, b = CD = 15 m, c = BC = 28 m

$S = \frac{a+b+c}{2} = \frac{41+15+28}{2} = 42\text{ m}$

Area of Δ BDC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-41)(42-15)(42-28)} = \sqrt{42 \times 1 \times 27 \times 14}$

$= \sqrt{2 \times 3 \times 7 \times 3 \times 3 \times 3 \times 2 \times 7}$

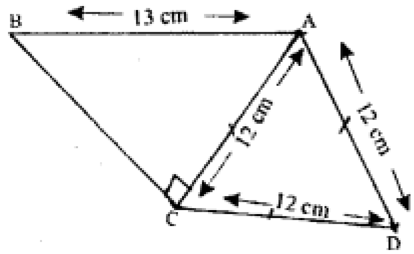
$= 2 \times 7 \times 3 \times 3 = 126\text{ m}^2$

Area of quadrilateral ABCD

$= 180\text{ m}^2 + \text{Area of } \Delta\text{ BDC}$

$= 180\text{ m}^2 + 126\text{ m}^2 = 306\text{ m}^2$

Question 3.
Find the area of the quadrilateral ABCD in which ∠BCA= 90°, AB = 13 cm and ACD is an equilateral triangle of side 12 cm.



Solution:

Quadrilateral ABCD in which $\angle BCA = 90^\circ$

AB = 13 cm

ΔACD is equilateral in which AC = CD = AD = 12 cm

In right angled ΔABC

By Pythagoras theorem,

$AB^2 = AC^2 + BC^2 \Rightarrow (13)^2 = (12)^2 + BC^2$

$\Rightarrow BC^2 = (13)^2 - (12)^2 \Rightarrow BC^2 = 169 - 144$

$\Rightarrow BC^2 = 25 \Rightarrow BC = \sqrt{25} = 5$ cm

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{1}{2} \times AC \times BC + \frac{\sqrt{3}}{4} \times (12)^2 \text{ cm}^2$

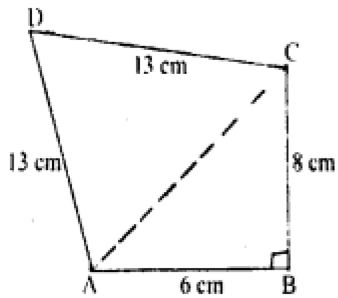
$= \frac{1}{2} \times 12 \times 5 + \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2$

$= 6 \times 5 + \sqrt{3} \times 3 \times 12 \text{ cm}^2 = 30 + 36\sqrt{3}$

$= 30 + 36 \times 1.732 = 30 + 62.28 = 92.28 \text{ cm}^2$

Question 4.

Find the area of quadrilateral ABCD in which ∠B = 90°, AB = 6 cm, BC = 8 cm 13 and CD = AD = 13 cm.



Solution:

A quadrilateral ABCD in which AB = 6 cm, BC = 8 cm, CD = 13 cm and AD = 13 cm

In ΔABC, $\angle B = 90°$

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (6)^2 + (8)^2$

$\Rightarrow AC^2 = 36 + 64 \Rightarrow AC^2 = 100$

$\Rightarrow AC^2 = (10)^2 \Rightarrow AC = 10$ cm

Area of quadrilateral ABCD

= Area of ΔABC + Area of ΔACD

$= \frac{1}{2} \times \text{Base} \times \text{height} \times \text{Area of } \Delta ACD$

$= \frac{1}{2} \times AB \times BC +$ Area of ΔACD

$= \frac{1}{2} \times 6 \times 8 \text{ cm}^2 +$ Area of ΔACD

$= 3 \times 8 \text{ cm}^2 +$ Area of ΔACD

$= 24 \text{ cm}^2 +$ Area of ΔACD ....(1)

Now, to find area of ΔACD

$\therefore$ let $a = AC = 10$ cm, $b = CD = 13$ cm, $c = AD = 13$ cm

$$s = \frac{a+b+c}{2} = \frac{10+13+13}{2} \text{ cm}$$

$$= \frac{10+26}{2} \text{ cm} = \frac{36}{2} \text{ cm} = 18 \text{ cm}$$

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{18(18-10)(18-13)(18-13)} = \sqrt{18 \times 8 \times 5 \times 5}$$

$$= \sqrt{6 \times 3 \times 8 \times 5 \times 5} = \sqrt{3 \times 2 \times 3 \times 2 \times 2 \times 2 \times 5 \times 5}$$

$= 3 \times 2 \times 2 \times 5 = 60 \text{ cm}^2$

$\therefore$ From (1)

Area of quadrilateral ABCD

$= 24 \text{ cm}^2 +$ Area of $\Delta ACD = 24 \text{ cm}^2 + 60 \text{ cm}^2$

$= 84 \text{ cm}^2$

Question 5.

The perimeter of a rectangular cardboard is 96 cm ; If its breadth is 18 cm, find the length and the area of the cardboard.

Solution:

Let ABCD be a rectangle

Let length be x cm and breadth = 18 cm

Perimeter = 96 cm

[illegible]

[illegible]

[illegible]

[illegible]

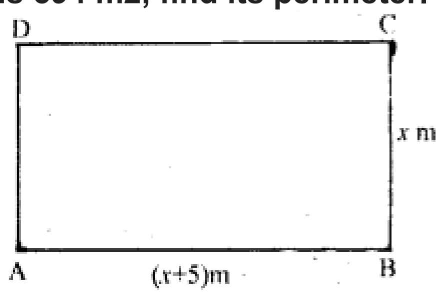
[illegible]

Area of the rectangular cardboard = $\ell \times b$

[illegible]

Question 6.

The length of a rectangular hall is 5 m more than its breadth, If the area of the hall is 594 m2, find its perimeter.



Solution:

Let ABCD be rectangular field.

Let Breadth = x m

then length = $(x + 5)$ m

Area of rectangular field = $\ell \times b$

$594 = x(x+5) \Rightarrow 594 = x^2 + 5x$

$\Rightarrow 0 = x^2 + 5x - 594 \Rightarrow x^2 + 5x - 594 = 0$

$\Rightarrow x^2 + 27x - 22x - 594 = 0$

$\Rightarrow x(x + 27) - 22(x + 27) = 0$

$\Rightarrow (x - 22)(x + 27) = 0$

Either $x - 22 = 0$ or $x + 27 = 0$

$x = 22$ m $x = -27$ (not possible)

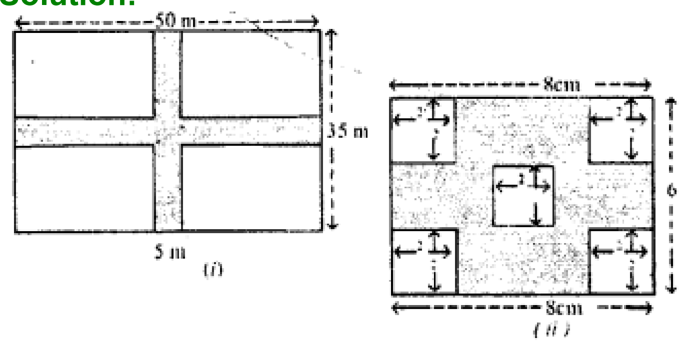
Breadth = 22 m

Length = $(x + 5)$ m = $(22 + 5)$ m = 27 m

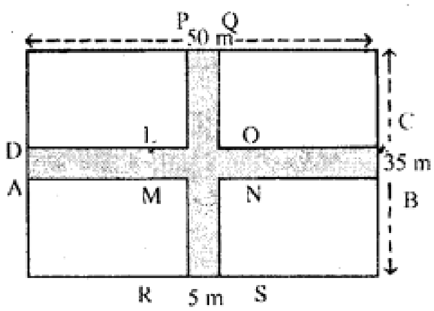
Perimeter = $2(\ell + b) = 2(27 + 22)$ m

$= 2 \times 49$ m = 98 m

Question 7.
(a) The diagram (i) given below shows two paths drawn inside a rectangular field 50 m long and 35 m wide. The width of each path is 5 metres. Find the area of the shaded portion.
(b) In the diagram (ii) given below, calculate the area of the shaded portion. All measurements are in centimetres.
Solution:

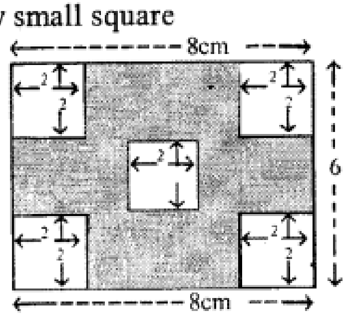


(*a*) Area of shaded portion = Area of rectangle ABCD + Area of rectangle PQRS – Area of square LMNO.



$= 50 \times 5\ m^2 + 5 \times 35\ m^2 - 5 \times 5\ m^2$
$= 250\ m^2 + 175\ m^2 - 25\ m^2 = 250\ m^2 + 150\ m^2 = 400\ m^2$

(*b*) Area of shaded portion = Area of ABCD – 5 × Area of any small square



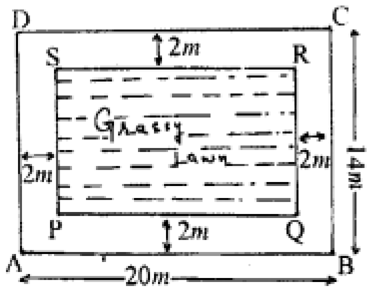
(*ii*)

$= \ell \times b - 5 \times \text{side} \times \text{side} = (8 \times 6 - 5 \times 2 \times 2)\ cm^2$
$= (48 - 20)\ cm^2 = 28\ cm^2$

Question 8.

A rectangular plot 20 m long and 14 m wide is to be covered with grass leaving 2 m all around. Find the area to be laid with grass.

Solution:



Let ABCD be the plot

Length of Plot = 20 m

Breadth of Plot = 14 m

Let PQRS be the grassy plot

Length of grassy Lawn = 20 m – 2 × 2 m

= 20 m – 4 m = 16 m

Breadth of grassy Lawn = 14 m – 2 × 2 m

= 14 m – 4 m = 10 m.

Area of grassy Lawn = Length × Breadth

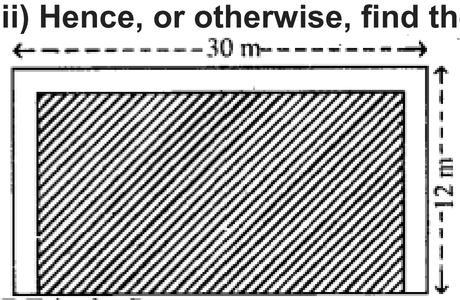
$= 16 \times 10 \text{ m}^2 = 160 \text{ m}^2$

Question 9.

The shaded region of the given diagram represents the lawn in front of a house. On three sides of the lawn there are flower beds of width 2 m.

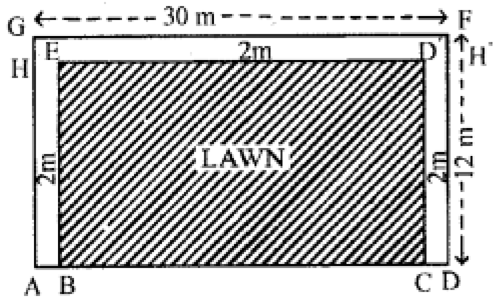
(i) Find the length and the breadth of the lawn.

(ii) Hence, or otherwise, find the area of the flower – beds.



Solution:

BCDE is the lawn



(*i*) Length of Lawn BCDE = BC

= AD – AB – CD = 30 m – 2 m – 2m

= 30 m – 4 m = 26 m

Breadth of Lawn BCDE

= BE = AG – GH = 12 m – 2 m = 10 m

(*ii*) Area of flower beds = Area of rectangle ADFG – Area of Lawn BCDE

= AD × AG – BC × BE = 30 × 12 m^2 – 26 × 10 m^2

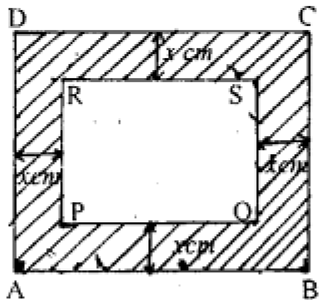
= 360 m^2 – 260 m^2 = 100 m^2

Question 10.

A foot path of uniform width runs all around the inside of a rectangular field 50 m long and 38m wide. If the area of the path is 492 m². Find its width.

Solution:

Here ABCD be a rectangular field having length = 50 m



and Breadth = 38 m

Area of rectangular field ABCD

$= \ell \times b = 50 \times 38\ m^2 = 1900\ m^2$

Let the width of foot path all around the inside of a rectangular field = x m

Then length of rectangular field PQRS

$= (50 - x - x)$ m $= (50 - 2x)$ m

Breadth of rectangular field PQRS

$= (38 - x - x)$ m $= (38 - 2x)$ m

Area of rectangular field PQRS $= \ell \times b$

$= (50 - 2x)(38 - 2x)$

Area of foot path = Area of rectangular field ABCD – Area of rectangular field PQRS

$\Rightarrow 492 = 1900 - (50 - 2x)(38 - 2x)$

$\Rightarrow 492 = 1900 - [50(38 - 2x) - 2x(38 - 2x)]$

$\Rightarrow 492 = 1900 - (1900 - 100x - 76x + 4x^2)$

$\Rightarrow 492 = 1900 - 1900 + 100x + 76x - 4x^2$

$\Rightarrow 492 = 176x - 4x^2 \Rightarrow 492 = 4(44x - x^2)$

$\Rightarrow \frac{492}{4} = 44x - x^2 \Rightarrow 123 = 44x - x^2$

$\Rightarrow x^2 - 44x + 123 = 0 \Rightarrow x^2 - 41x - 3x + 123 = 0$

$\Rightarrow x(x - 41) - 3(x - 41) \Rightarrow (x - 3)(x - 41) = 0$

Either $x - 3 = 0$ or $x - 41 = 0$

$x = 3$ m, $x = 41$ m (not possible)

Hence, width = 3 m

Question 11.

The cost of enclosing a rectangular garden with a fence all around at the rate of Rs. 15 per metre is Rs. 5400. If the length of the garden is 100 m And the area of the garden.

Solution:

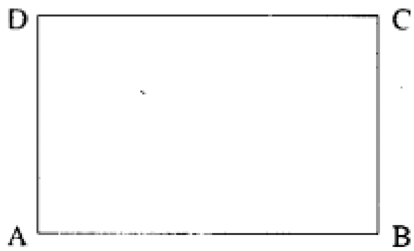
Here ABCD be a rectangular garden length of the garden = 100 m

Let Breadth of the garden = x m

Perimeter of the garden ABCD = $2(\ell + b)$

$= 2(100 + x)$ m $= (200 + 2x)$ m

Cost of 1 m to enclosing a rectangular garden = Rs. 15



Cost for $(200 + 2x)$ to enclosing a rectangular garden = Rs. $15(200 + 2x)$ = Rs. $3000 + 30x$

But given cost = Rs. 5400

Then, $3000 + 30x = 5400$

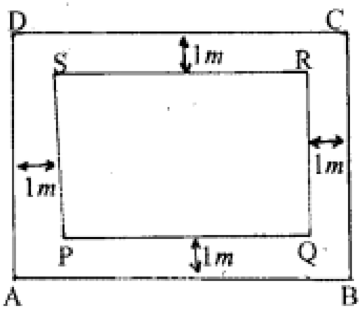
$$30x = 5400 - 3000 \Rightarrow x = \frac{2400}{30} = 80 \text{ m}$$

i.e. breadth of garden = 80 m.

Area of rectangular field = $\ell \times b$

$= 100 \times 80 \text{ m}^2 = 8000 \text{ m}^2$

Question 12.
A rectangular floor which measures 15 m x 8 m is to be laid with tiles measuring 50 cm x 25 cm find the number of tiles required further, if a carpet is laid on the floor so that a space of 1 m exists between its edges and the edges of the floor, what fraction of the floor is uncovered?
Solution:



Here ABCD is a rectangular field given, measurement of the field is 15 m × 8 m

∴ length of rectangular field = 15 m

and breadth of rectangular field = 8 m

Area of rectangular field = l × b = 15 m × 8 m = 120 m²

measurement of tiles = 50 cm × 25 cm

∴ length of tile = 50 cm = $\frac{50}{100}$ m = $\frac{1}{2}$ m

Breadth of tile = 25 cm = $\frac{25}{100}$ = $\frac{1}{4}$ m

Area of one tile = $\frac{1}{2} \times \frac{1}{4}$ m² = $\frac{1}{8}$ m²

Required no. of tiles

$= \frac{\text{Area of rectangular field}}{\text{Area of one tile}}$

$= \frac{(120)}{\left(\frac{1}{8}\right)} = \frac{120 \times 8}{1} = 960$ tiles.

Now, length of carpet = 15 m − 1 m − 1 m

= 15 m − 2 m = 13 m

Breadth of carpet = 8 m − 1 m − 1 m

= 8 m − 2 m = 6 m

Area of carpet = l × b = 13 × 6 m² = 78 m²

Area of floor which is uncovered by carpet = Area of floor − Area of carpet = 120 m² − 78 m² = 42 m²

Fraction $= \frac{\text{Area of floor which is uncovered by carpet}}{\text{area of floor}}$

$= \frac{42}{120} = \frac{7}{20}$

Question 13.

The width of a rectangular room is $\frac{3}{5}$ of its length x metres. If its perimeter isy metres, write an equation connecting.vandy. Find the floor area of the room if its perimeter is 32 m.

Solution:

Given that the length of rectangular room = x m

and width of rectangular room = $\frac{3}{5}$ of its length

$= \frac{3}{5} \times x \text{ m} = \frac{3x}{5} \text{ m}$

Perimeter = $2(\ell + b)$

$y = 2\left(x + \frac{3}{5}x\right)$ [given perimeter = y m]

$\Rightarrow \quad y = 2\left(\frac{5x+3x}{5}\right) \quad \Rightarrow \quad y = 2 \times \frac{8x}{5}$

$\Rightarrow \quad y = \frac{16x}{5} \quad \Rightarrow \quad 5y = 16x$

$\Rightarrow \quad 16x = 5y$ ....(1)

Which is required relation between x and y

Now, given perimeter = 32 m

i.e. the value of y = 32 m

Substituting the value of y in equation (1), we get

$\Rightarrow \quad 16x = 5 \times 32 \quad \Rightarrow \quad x = \frac{5 \times 32}{16}$

$\Rightarrow \quad x = \frac{5 \times 2}{1} \quad \Rightarrow \quad x = 10 \text{ m}$

Breadth (width) = $\frac{3}{5} \times x \text{ m} = \frac{3}{5} \times 10 \text{ m} = 3 \times 2 \text{ m} = 6 \text{ m}$

Floor area of the room $= \ell \times b = 10 \times 6 \text{ m}^2$

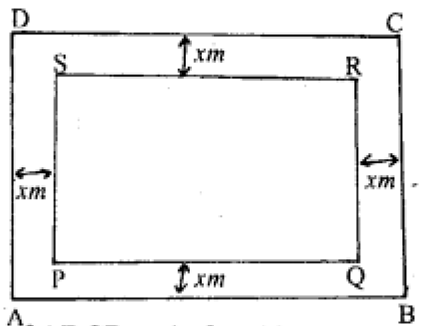
$= 60 \text{ m}^2$

Question 14.

A rectangular garden 10 m by 16 m is to be surrounded by a concrete walk of uniform width. Given that the area of the walk is 120 square metres, assuming the width of the walk to be x, form an equation in x and solve it to find the value of x.

Solution:

Here ABCD be a rectangular garden having length = 10 m and Breadth = 16 m



Then Area of ABCD = $\ell \times b = 10\text{ m} \times 16\text{ m} = 160\text{ m}^2$

Width of the walk to be = x m

Then, length of rectangular garden PQRS

$= (10 - x - x)\text{ m} = (10 - 2x)\text{ m}$

Breadth of rectangular garden PQRS = $(16 - x - x)$ m

$= (16 - 2x)$ m

Question 15.

A rectangular room is 6 m long, 4.8 m wide and 3.5 m high. Find the inner surface area of the four walls.

Solution:

Here length of rectangular room = 6 m

Breadth of rectangular room = 4.8 m

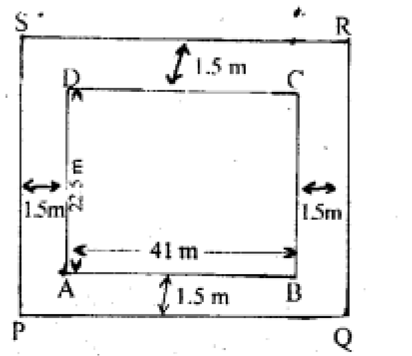
and height of rectangular room = 3.5 m

Then, inner surface area of four walls

$= 2(\ell + b) \times h = 2(6 + 4.8) \times 3.5\text{ m}^2$

$= 2 \times 10.8 \times 3.5\text{ m}^2 = 21.6 \times 3.5\text{ m}^2 = 75.6\text{ m}^2$

Question 16.
A rectangular plot of land measures 41 metres in length and 22.5 metres in width. A boundary wall 2 metres high is built all around the plot at a distance of 1.5 m from the plot. Find the inner surface area of the boundary wall.
Solution:
Length of rectangular plot = 41 metre



Breadth of rectangular plot = 22.5 metre

and height of boundary wall = 2 metre

But boundary wall is built at a distance of 1.5 m

Then new length = (41 + 1.5 + 1.5) m = (41 + 3.0) m = 44 m

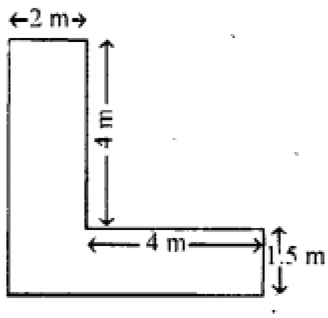
New Breadth = 22.5 m + 1.5 m + 1.5 m

= 22.5 m + 3.0 m = 25.5 m

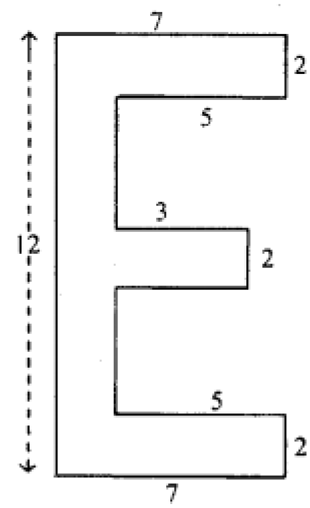
Now, The inner surface area of the boundary wall
$= 2(\ell + b) \times h = 2(44 + 25.5) \times 2 \text{ m}^2$

$= 2 \times 69.5 \times 2 \text{ m}^2 = 2 \times 139 \text{ m}^2 = 278 \text{ m}^2$

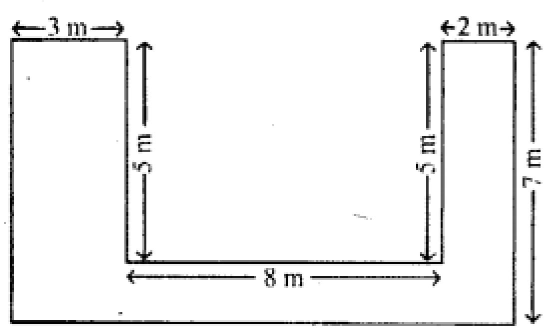
Question 17.
(a) Find the perimeter and area of the figure
(i) given below in which all corners are right angled.
(b) Find the perimeter and area of the figure
(ii) given below in which all corners are right angles.
(c) Find the area and perimeter of the figure
(iii) given below in which all corners are right angles and all measurement in centimetres.



(*i*)



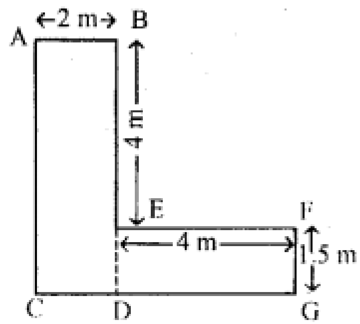
(*iii*)



(*ii*)

Solution:

(*a*) Given that



AB = 2 m, BE = 4 m, FE = 4 m and FG = 1.5 m

Now, BD = 4 m + 1.5 m = 5.5 m

Also AC = BD = 5.5 m

CG = (4 + 2) m = 6 m

Perimeter of figure (*i*)

= AC + CG + GF + FE + EB + BA

= 5.5 m + 6 m + 1.5 m + 4 m + 4 m + 2m = 23 m

Area of given fig. = Area of ABEDC + Area of FEDG

= Length × Breadth + Length × Breadth

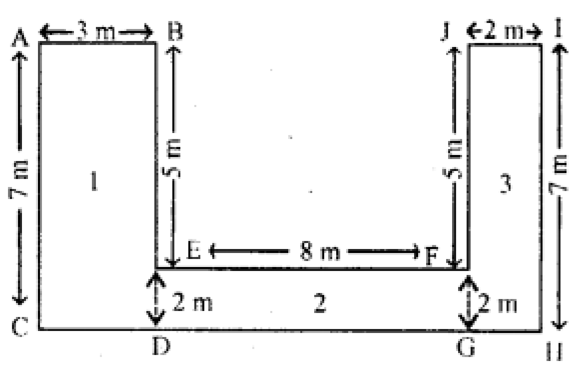
= 2m × 5.5 m + 4 m × 1.5 m

= 11 m^2 + 6.0 m^2 = 17 m^2

(*b*) In the figure (*ii*) AB = CD = 3 m

HI = AC = 7 m, JF = BE = 5 m, GF = DE = 2m

DG = EF = 8 m, GH = JI = 2 m



Also, CH = CD + DG + GH = 3 m + 8 m + 2 m = 13 m

Perimeter of given figure = AB + AC + CH + HI + IJ + JF + FE + BE

= 3 m + 7 m + 13 m + 7 m + 2 m + 5 m + 8 m + 5 m = 50 m

Area of given figure = Area of 1st figure + Area of 2nd figure + Area of 3rd figure

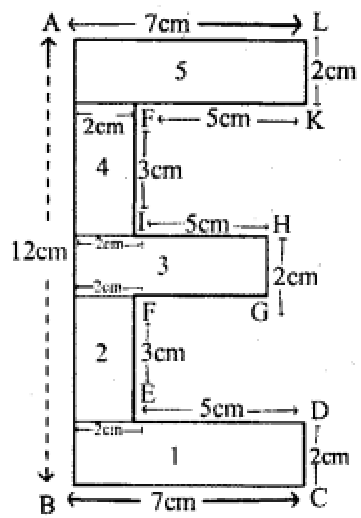
= 7 m × 3 m + 8 m × 2m + 7 m × 2m

= 21 m^2 + 16 m^2 + 14 m^2 = 51 m^2

(*c*) Here given and from, it is clear that

AB = 12 cm, AL = BC = 7 cm, JK = DE = 5 cm

HI = GF = 3 cm, LK = HG = CD = 2 cm



Now, Perimeter of given figure

= AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL + LA

= 12 cm + 7 cm + 2 cm + 5 cm + 3 cm + 3 cm

+ 2 cm + 3 cm + 3 cm + 5 cm + 2 cm + 7 cm

= 54 cm

Area of given figure = Area of 1st part + Area of 2nd part + Area of 3rd part + Area of 4th part + Area of 5th part

= 7 cm × 2 cm + 2 cm × 3 cm + (2 cm + 3 cm) × 2 cm + 2 cm × 3 cm + 7 cm × 2 cm

= 14 cm^2 + 6 cm^2 + 10 cm^2 + 6 cm^2 + 14 cm^2

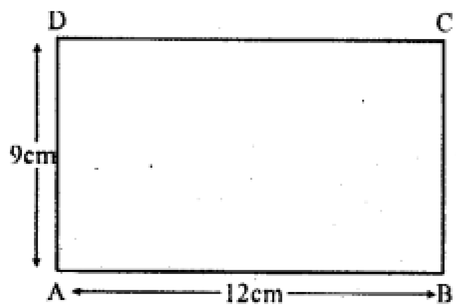
= 50 cm^2

Question 18.

The length and the breadth of a rectangle are 12 cm and 9 cm respectively. Find the height of a triangle whose base is 9 cm and whose area is one third that of rectangle.

Solution:

The given length of a rectangle = 12 cm
and Breadth of a rectangle = 9 cm
Area of rectangle = $\ell \times b = 12 \text{ cm} \times 9 \text{ cm} = 108 \text{ cm}^2$
By given condition,



Area of $\Delta ABC = \frac{1}{3} \times$ Area of rectangle

$= \frac{1}{3} \times 108 \text{ cm}^2 = 36 \text{ cm}^2$

Let height of $\Delta ABC = h$ cm

Now, we know that

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

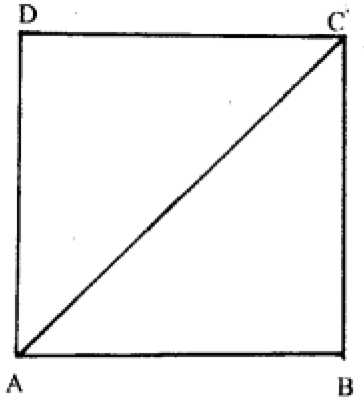
$36 \text{ cm}^2 = \frac{1}{2} \times 9 \text{ cm} \times h \text{ cm}$

$\Rightarrow \quad 36 = \frac{1}{2} \times 9 \times h \quad \Rightarrow \quad 36 \times 2 = 9 \times h$

$\Rightarrow \quad h = \frac{36 \times 2}{9} \quad \Rightarrow \quad h = 4 \times 2 \quad \Rightarrow \quad h = 8 \text{ cm}$

Hence, height of $\Delta ABC = 8$ cm

Question 19.
The area of a square plot is 484 mV Find the length of its one side and the length of its one diagonal.



Solution:

Let ABCD be any square plot whose area is 484 m^2, then sides of square is AB, BC, CD and AD

Now, Area of square = side × side

$\Rightarrow \quad 484 = (side)^2 \quad \Rightarrow \quad (side)^2 = 484$

$\Rightarrow \quad side = \sqrt{484} \quad \Rightarrow \quad side = 22$ m

i.e. AB = BC = 22 m

In Δ ABC, (By Pythagoras theorem)

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = (22)^2 + (22)^2 \quad \Rightarrow \quad AC^2 = 484 + 484$

$\Rightarrow \quad AC^2 = 968 \quad \Rightarrow \quad AC = \sqrt{968}$

$\Rightarrow \quad AC = \sqrt{484 \times 2} \quad \Rightarrow \quad AC = 22 \times \sqrt{2}$

$\Rightarrow \quad AC = 22 \times 1.414 \qquad (\because \sqrt{2} = 1.414)$

$\Rightarrow \quad AC = 31.11$m

Hence, length of side = 22 m

and length of diagonal = 31.11 m

Question 20.

A square has the perimeter 56 m. Find its area and the length of one diagonal correct upto two decimal places.

Solution:

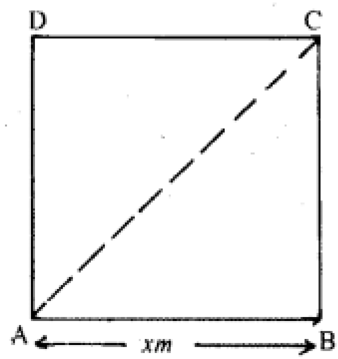
Here ABCD is a square let its side = x m

ınen, perimeter of square = 4 × side

$56 = 4x$

$\Rightarrow \quad 4x = 56 \quad \Rightarrow \quad x = \frac{56}{4} \quad \Rightarrow \quad x = 14$ m

In ΔABC, By Pythagoras theorem,



$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = (14)^2 + (14)^2 \quad \Rightarrow \quad AC^2 = 196 + 196$

$\Rightarrow \quad AC^2 = 392 \quad \Rightarrow \quad AC = \sqrt{392}$

$\Rightarrow \quad AC = \sqrt{196 \times 2} \quad \Rightarrow \quad AC = 14\sqrt{2}$

$\Rightarrow \quad AC = 14 \times 1.414 \qquad (\because \sqrt{2} = 1.414)$

$\Rightarrow \quad AC = 19.80$ m

Hence, side of square = 14 m and diagonal = 19.80 m **Ans.**

Question 21.
A wire when bent in the form of an equilateral triangle encloses an area of 36√3 cm2. Find the area enclosed by the same wire when bent to form:
(i) a square, and
(ii) a rectangle whose length is 2 cm more than its width.
Solution:

Area of equilateral triangle = $36\sqrt{3}$ cm²
Let side of equilateral triangle = x cm
Then, Area = $\frac{\sqrt{3}}{4}$ (side)²

$36\sqrt{3} = \frac{\sqrt{3}}{4} \times (\text{side})^2 \Rightarrow 36\sqrt{3} = \frac{\sqrt{3}}{4} x^2$

$\Rightarrow$ [illegible] $\Rightarrow x^2 = \frac{36\sqrt{3} \times 4}{\sqrt{3}}$

$\Rightarrow x^2 = 36 \times 4 \Rightarrow x = \sqrt{36 \times 4} \Rightarrow x = 6 \times 2$

$\Rightarrow x = 12$ cm

Perimeter of equilateral triangle = 3 × side
= 3 × 12 cm = 36 cm

(i) Perimeter of equilateral triangle = Perimeter of square

$\Rightarrow \quad 36 = 4 \times \text{side} \quad \Rightarrow \quad 4 \times \text{side} = 36$

$\Rightarrow \quad \text{side} = \frac{36}{4} \text{ cm} \quad \Rightarrow \quad \text{side} = 9 \text{ cm}$

i.e. side of square = 9 cm
Area of square = side × side = 9 × 9 cm²
= 81 cm²

(*ii*) Perimeter of triangle = Perimeter of rectangle ....(1)

But, given condition in rectangle,
The length is 2 cm more than its width
Let width of rectangle = x cm
Length of rectangle = $(x + 2)$ cm
Then, perimeter of rectangle = $2(\ell + b)$
$= 2[(x + 2) + x] = 2(2x + 2) = 4x + 4$
But, from equation (1),
$4x + 4$ = Perimeter of triangle

$\Rightarrow \quad 4x + 4 = 36 \quad \Rightarrow \quad 4x = 36 - 4 \quad \Rightarrow \quad 4x = 32$

$\Rightarrow \quad x = \frac{32}{4} \quad \Rightarrow \quad x = 8 \text{ cm}$

i.e. Length of rectangle = 8 cm + 2 cm = 10 cm
Breadth of rectangle = 8 cm
Area of rectangle = length × Breadth
= 10 cm × 8 cm = 80 cm²

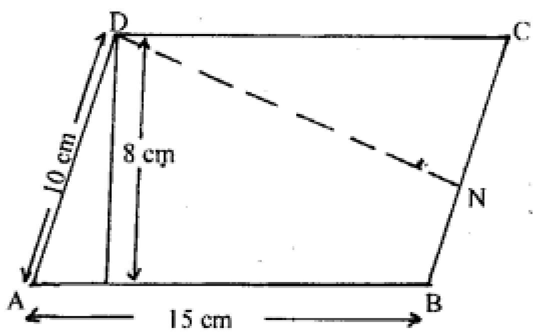
Question 22.

Two adjacent sides of a parallelogram are: 15 cm and 10 cm. If the distance between the longer sides is 8 cm, find the area of the parallelogram. Also find the distance between shorter sides.

Solution:

Here ABCD is a parallelogram in which longer side AB = 15 cm and shorter side = 10 cm

Distance between longer side = DM = 8 cm (given)



Let DN is the distance between the shorter side

Now, Area of parallelogram ABCD = Base × height

$= AB \times DM = 15 \text{ cm} \times 8 \text{ cm} = 120 \text{ cm}^2$

Now, when base is AD

Then, Area of parallelogram = AD × DN

$\Rightarrow \quad 120 = 10 \times DN \quad \Rightarrow \quad 10 \times DN = 120$

$\Rightarrow \quad DN = \frac{120}{10} \quad \Rightarrow \quad DN = 12 \text{ cm}$

Hence, Area of parallelogram = 120 cm^2 and distance between shorter side = 12 cm

Question 23.

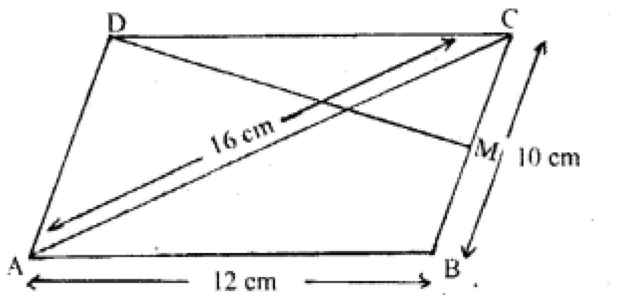
ABCD is a parallelogram with sides AB = 12 cm, BC = 10 cm and diagonal AC = 16 cm. Find the area of the parallelogram. Also find the distance between its shorter sides.

Solution:

Ans. Here ABCD be a parallelogram with sides AB = 12 cm, BC = 10 cm and AC = 16 cm

Now, for Area of ΔABC

$BC = a = 10$ cm, $AC = b = 16$ cm, $AB = c = 12$ cm



$$s = \frac{a+b+c}{2} = \frac{10+16+12}{2} = \frac{38}{2} \text{ cm} = 19 \text{ cm}.$$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{19 \times (19-10)(19-16)(19-12)}$

$= \sqrt{19 \times 9 \times 3 \times 7} = \sqrt{19 \times 3 \times 3 \times 3 \times 7}$

$= 3\sqrt{19 \times 3 \times 7} = 3\sqrt{19 \times 21} = 3\sqrt{399}$ cm^2

Area of parallelogram = 2 × Area of ΔABC

$= 2 \times 3\sqrt{399}$ cm^2 $= 6 \times \sqrt{399}$ cm^2 $= 6 \times 19.96$ cm^2

$= 119.8$ cm^2

Let DM be the distance between the shorter lines.

we take base = AD = BC = 10 cm

Area of parallelogram = AD × DM

($\because$ Area = base × height)

$\Rightarrow \quad 119.8 = 10 \times DM \quad \Rightarrow \quad 10 \times DM = 119.8$

$\Rightarrow \quad DM = \frac{119.8}{10} \quad \Rightarrow \quad DM = 11.98$ cm

Hence, distance between shorter lines

= 11.98 cm

Question 24.

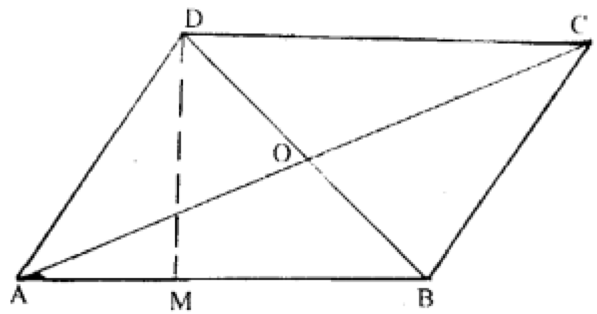
Diagonals AC and BD of a parallelogram ABCD intersect at O. Given that AB = 12 cm and perpendicular distance between AB and DC is 6 cm. Calculate the area of the triangle AOD.

Solution:

Ans.Here ABCD be a parallelogram

AC and BD be its diagonals which is intersect at O.

AB = 12 cm and DM = 6 cm



Area of parallelogram ABCD = AB × DM

($\because$ Base × height)

= 12 cm × 6 cm = 72 cm²

Area of ΔAOD $= \frac{1}{4} \times$ Area of parallelogram

ABCD $= \frac{1}{4} \times 72\ cm^2 = 18\ cm^2$

Question 25.

ABCD is a parallelogram with side AB = 10 cm. Its diagonals AC and BD are of length 12 cm and 16 cm respectively. Find the area of the parallelogram ABCD.

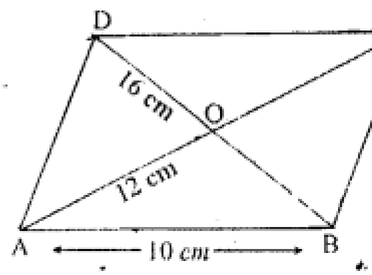
Solution:

Here ABCD be a parallelogram,

AB = 10 cm, AC = 12 cm

$AO = CO = \frac{12cm}{2} = 6\ cm$

BD = 16 cm



$BO = OD = \frac{16cm}{2} = 8\ cm$

In ΔAOB,

$a = 10$ cm, $b = AO = 6$ cm, $c = BO = 8$ cm

$$s = \frac{a+b+c}{2} = \frac{10\,cm + 6\,cm + 8\,cm}{2} = \frac{24cm}{2} = 12cm$$

Area of $\Delta AOB = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12 \times (12-10)(12-6)(12-8)}\ cm^2$

$= \sqrt{12 \times 2 \times 6 \times 4}$

$= \sqrt{12 \times 12 \times 4}\ cm^2 = 12 \times 2\ cm^2 = 24\ cm^2$

Area of parallelogram ABCD = 4 × Area of ΔAOB.

$= 4 \times 24\ cm^2 = 96\ cm^2$

Question 26.

The area of a parallelogram is p cm2 and its height is q cm. A second parallelogram has equal area but its base is ‘r’ cm more than that of the first. Obtain an expression in terms of p, q and r for the height h of the second parallelogram.

Solution:

Given area of first parallelogram = p cm^2

height of first parallelogram = q cm

Then, Area of parallelogram = Base × height

$$\Rightarrow \quad p = \text{Base} \times q \quad \Rightarrow \quad \text{Base} = \frac{p}{q}$$

Now, Base of second parallelogram

$$= \left(\frac{p}{q} + r\right) \text{ or } = \frac{p+qr}{q} \text{ cm}$$

Also, Area of second parallelogram = Area of first parallelogram.

Area of the second parallelogram = p cm^2

$$\Rightarrow \quad \text{Base} \times \text{height} = p \text{ cm}^2 \quad \Rightarrow \quad \left(\frac{P+qr}{q}\right) \times h = P$$

$$\Rightarrow \quad h = \frac{P \times q}{(P+qr)} \text{ cm} \quad \Rightarrow \quad h = \frac{Pq}{P+qr} \text{ cm}$$

Hence, Height of second parallelogram

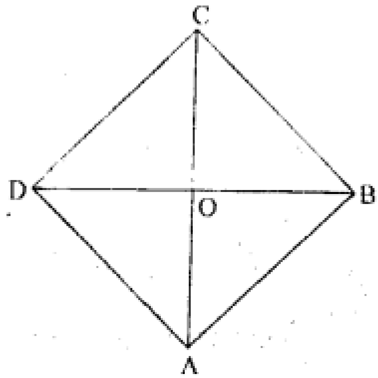
$$= \frac{pq}{P+qr} \text{ cm}$$

Question 27.

What is the area of a rhombus whose diagonals are 12 cm and 16 cm ?

Solution:

Here ABCD be a rhombus



BD = 12 cm and AC = 16 cm are diagonals
Then Area of rhombus ABCD

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 16\text{ cm} \times 12\text{ cm}$
$= 8\text{ cm} \times 12\text{ cm} = 96\text{ cm}^2$

Question 28.
The area of a rhombus is 98 cm². If one of its diagonal is 14 cm, what is the length of the other diagonal?
Solution:

Area of rhombus = 98 cm^2
one diagonal = 14 cm
We know that,

Area of rhombus $= \frac{1}{2} \times$ product of diagonals

$\Rightarrow \quad 98 = \frac{1}{2} \times$ one diagonal $\times$ other diagonal.

$\Rightarrow \quad 98 = \frac{1}{2} \times 14 \times$ other diagonal

$\Rightarrow \quad$ other diagonal $= \frac{98 \times 2}{14}$ cm

$\Rightarrow \quad$ other diagonal = 7 × 2 cm = 14 cm
Hence, other diagonal = 14 cm

Question 29.
The perimeter of a rhombus is 45 cm. If its height is 8 cm, calculate its area.

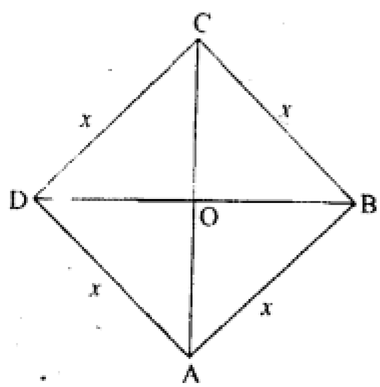
Solution:

Here ABCD be a rhombus

Let each side = x cm

Given perimeter = 45 cm

i.e. AB + BC + CD + AD = 45 cm



$\Rightarrow \quad x + x + x + x = 45 \quad \Rightarrow \quad 4x = 45$

$\Rightarrow \quad x = \frac{45}{4}$ cm

Given height = 8 cm

Then area of rhombus = base × height

$= \frac{45}{4} \times 8\ cm^2 = 45 \times 2\ cm^2 = 90\ cm^2$

Note : For base take here any side of rhombus.

Question 30.

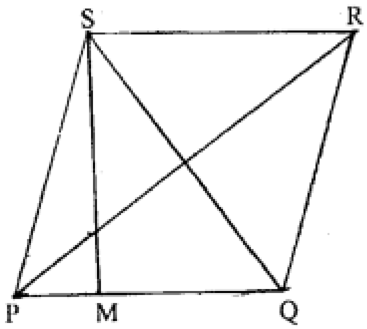
PQRS is a rhombus. If it is given that PQ = 3 cm and the height of the rhombus is 2.5 cm, calculate its area.

Solution:

Here given that PQRS is a rhombus

PQ = 3 cm (given) and height = 2.5 cm

Here PQ is base of rhombus PQRS.



Also SM = 2.5 cm

i.e. height of rhombus

Area of rhombus PQRS = base × height

$= 3 \text{ cm} \times 2.5 \text{ cm} = 7.5 \text{ cm}^2$

Question 31.

If the diagonals of a rhombus are 8 cm and 6 cm, find its perimeter.

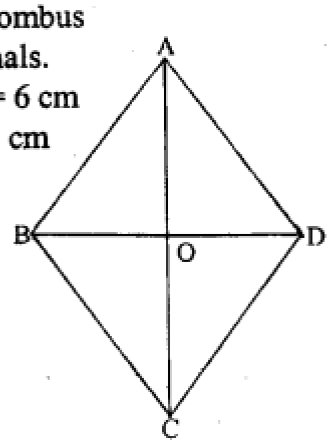
Solution:

Let ABCD be any rhombus

AC and BD are two diagonals.

Then, AC = 8 cm and BD = 6 cm

Here, AO = 4 cm & BO = 3 cm



In ΔABC

By Pythagoras theorem,

$\Rightarrow \quad AB^2 = AO^2 + BO^2$

$\Rightarrow \quad AB^2 = (4)^2 + (3)^2$

$\Rightarrow \quad AB^2 = 16 + 9$

$\Rightarrow \quad AB^2 = 25$

$\Rightarrow \quad AB = \sqrt{25} \quad \Rightarrow \quad AB = 5 \text{ cm}$

i.e. side of rhombus ABCD = 5 cm

Perimeter of rhombus = 4 × side

= 4 × 5 cm = 20 cm

Question 32.

If the sides of a rhombus are 5 cm each and one diagonal is 8 cm, calculate

(i) the length of the other diagonal, and

(ii) the area of the rhombus.

Solution:

Here ABCD be a rhombus

AB, BC, CD and AD are the sides of rhombus

Then, $AB = BC = CD = AD = 5$ cm

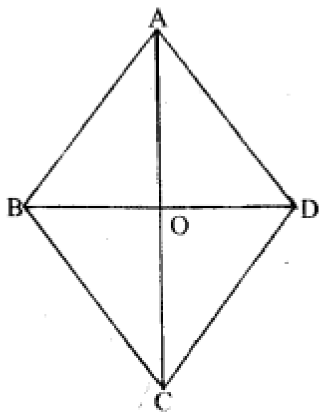
Also, $AC = 8$ cm

Then, $AO = 4$ cm

In ΔAOB,

By Pythagoras theorem,

$AB^2 = AO^2 + BO^2$



$\Rightarrow \quad (5)^2 = (4)^2 + BO^2 \quad \Rightarrow \quad 25 = 16 + BO^2$

$\Rightarrow \quad BO^2 = 25 - 16 \quad \Rightarrow \quad BO^2 = 9$

$\Rightarrow \quad BO = \sqrt{9} = 3$ cm

$\because \quad BD = 2 \times BO = 2 \times 3 \text{ cm} = 6$ cm

Hence, length of other diagonal = 6 cm

Area of rhombus $= \frac{1}{2} \times$ Product of diagonals

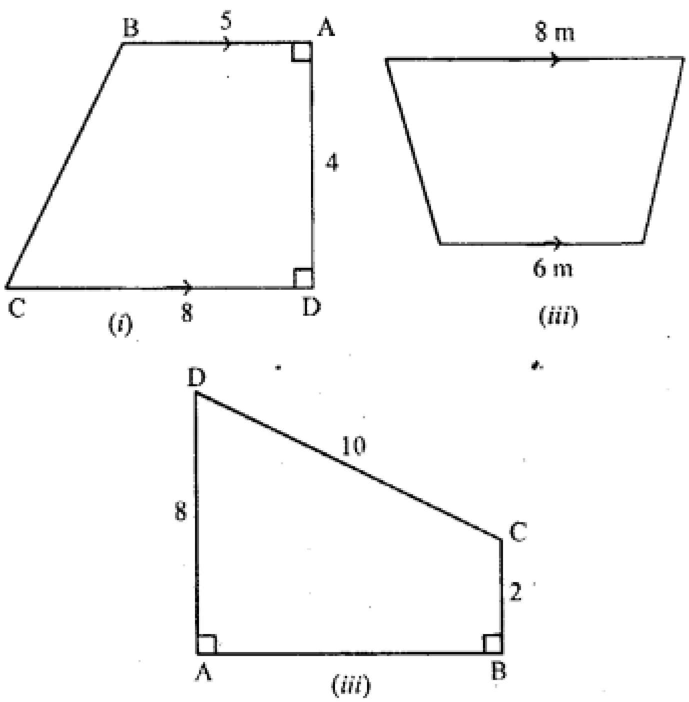
$= \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm} = 4 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$

Question 33.

(a) The diagram (t) given below is a trapezium. Find the length of BC and the area of the trapezium Assume AB = 5 cm, AD = 4 cm, CD = 8 cm

(b) The diagram (ii) given below is a trapezium Find (i) AB (ii) area of trapezium ABCD.

(c) The cross-section of a canal is shown in figure (iii) given below. If the canal is 8 m wide at the top and 6 m wide at the bottom and the area of the cross-section is 16.8 m², calculate its depth



Solution:

(*a*) Here ABCD is a trapezium

AB = 5 cm, AD = 4 cm and CD = 8 cm

Also Draw BN ⊥ CD

Then, BN = 4 cm,

CN = CD − ND

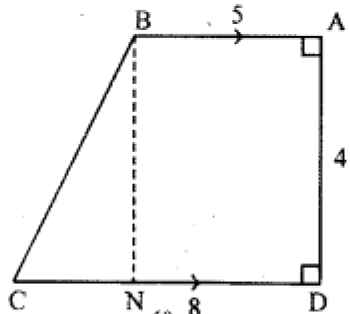
CN = CD − AO

CN = 8 cm − 5 cm = 3 cm

Now, in ΔBCN

By Pythagoras theorem,

$BC^2 = BN^2 + CN^2$



(*i*)

$\Rightarrow \quad BC^2 = (4)^2 + (3)^2$

$\Rightarrow \quad BC^2 = 16 + 9$

$\Rightarrow \quad BC^2 = 25 \quad \Rightarrow \quad BC = \sqrt{25} \quad \Rightarrow \quad BC = 5$ cm

Hence, length of BC = 5 cm

Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2}$ (AB + CD) × AD = $\frac{1}{2}$ (5 + 8) × 4 cm²

$= \frac{1}{2} \times 13 \times 4$ cm² = 13 × 2 cm² = 26 cm²

Hence, Area of trapezium = 26 cm²

(*b*) In diagram (*ii*) Given that

AD = 8 unit, BC = 2 unit, CD = 10 unit

Draw CN ⊥ AD

Then, AN = 2 units

DN = AD − DN

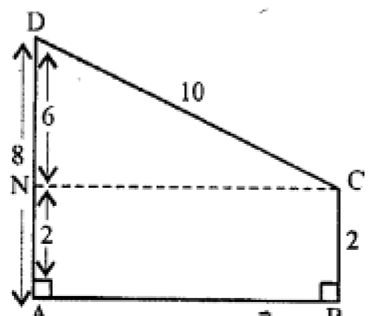
= 8 units − 6 units

= 2 units

In ΔCDN

By Pythagoras theorem

$CD^2 = DN^2 + NC^2$



(*ii*)

$\Rightarrow \quad (10)^2 = (6)^2 + NC^2$

$\Rightarrow \quad NC^2 = (10)^2 - (6)^2 \quad \Rightarrow \quad NC^2 = 100 - 36$

$\Rightarrow \quad NC^2 = 64 \quad \Rightarrow \quad NC = \sqrt{64}$

$\Rightarrow$ NC = 8 units

Also from figure NC = AB = 8 units

Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2}$ (BC + AD) × AB = $\frac{1}{2}$ (2 + 8) × 8 sq. units

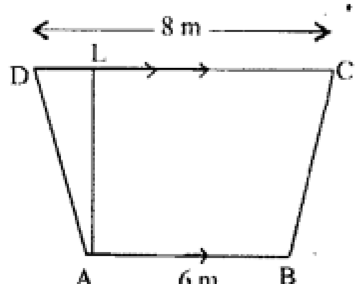
$= \frac{1}{2}$ × 10 × 8 sq. units = 5 × 8 sq. units

= 40 sq. units

(c) Let ABCD be the cross section of canal in the shape of trapezium.

AB = 6 m, DC = 8 m

Let AL be the depth of canal



(iii)

Area of cross–section

= 16.8 m²

$\frac{1}{2}$ × (sum of parallel sides) × depth = 72

$\Rightarrow$ $\frac{1}{2}$ × (AB + DC) × AL = 72

$\Rightarrow$ $\frac{1}{2}$ × (6 + 8) × AL = 16.8

$\Rightarrow$ $\frac{1}{2}$ × 14 × AL = 16.8 $\Rightarrow$ AL = $\frac{16.8 \times 2}{14}$ m

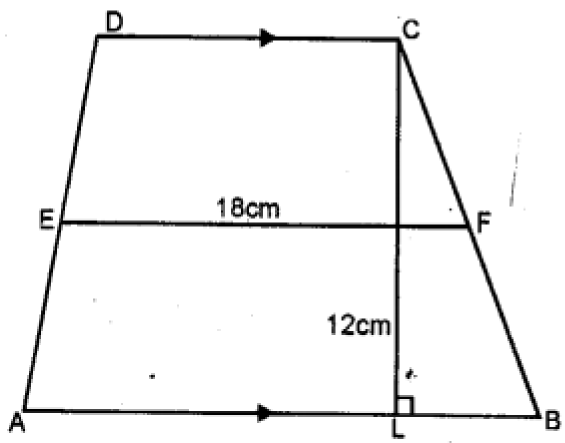
$\Rightarrow$ AL = $\frac{16.8 \times 1}{7}$ m $\Rightarrow$ AL = 2.4 m

Question 34.

The distance between parallel sides of a trapezium is 12 cm and the distance between mid-points of other sides is 18 cm. Find the area of the trapezium.

Solution:

Let ABCD be the triapezium in which AB || DC.
Height CL = 12 cm.



Let E and F be the mid-points of sides AD and BC respectivity, then EF = 18 cm.

∵ E and F are the mid-points of sides AD and BC

$$\therefore EF = \frac{1}{2}(AB + DC) = 18 \text{ cm.}$$

Now area of trap. ABCD

$$= \frac{1}{2}(AB + DC) \times \text{height} = 18 \text{ cm} \times 12 \text{ cm.}$$

$$= 216 \text{ cm}^2$$

Question 35.

The area of a trapezium is 540 cm². If the ratio of parallel sides is 7 : 5 and the distance between them is 18 cm, find the length of parallel sides.

Solution:

Area of trapezium = 540 cm^2

ratio of parallel sides = 7 : 5

Let one parallel side = $7x$ cm

Then other parallel side = $5x$ cm

distance between parallel sides = 18 cm

i.e. height = 18 cm

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$\Rightarrow \quad 540 = \frac{1}{2} \times (7x + 5x) \times 18$$

$$\Rightarrow \quad 540 = \frac{1}{2} \times 12x \times 18 \quad \Rightarrow \quad 540 = 6x \times 18$$

$$\Rightarrow \quad 540 = 108x \Rightarrow 108x = 540 \Rightarrow \quad x = \frac{540}{108} = 5$$

First parallel side = $7x = 7 \times 5 = 35$ cm

and second parallel side = $5x = 5 \times 5 = 25$ cm

Question 36.

The parallel sides of an isosceles trapezium are in the ratio 2 : 3. If its height is 4 cm and area is 60 cm2, find the perimeter.

Solution:

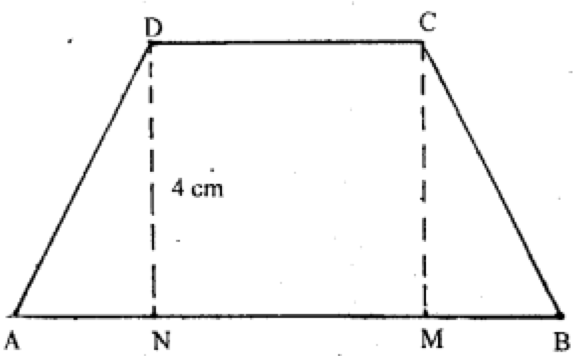
Here ABCD is an isosceles trapezium

Where BC = AD

Height = 4 cm

Let CD = $2x$

Then, AB = $3x$



Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × height

$\Rightarrow \quad 60 = \frac{1}{2} \times (2x + 3x) \times 4 \quad \Rightarrow \quad 60 = \frac{1}{2} \times 5x \times 4$

$\Rightarrow \quad 60 = 5x \times 2 \quad \Rightarrow \quad 60 = 10x \quad \Rightarrow \quad 10x = 60$

$\Rightarrow \quad x = \frac{60}{10} \quad \Rightarrow \quad x = 6$

i.e. CD = $2x = 2 \times 6$ cm = 12 cm

AB = $3x = 3 \times 6$ cm = 18 cm

Now, AN = BM

Also AN = AB – BN

$\Rightarrow \quad$ AN = AB – (MN + BM)

$\Rightarrow \quad$ AN = AB – (CD + BM) $\qquad (\because$ MN = CD)

$\Rightarrow \quad$ AN = AB – (CD + AN) $\qquad (\because$ BM = AN)

$\Rightarrow \quad$ AN = 18 – (12 + AN)

$\Rightarrow \quad$ AN = 18 – 12 – AN $\quad \Rightarrow \quad$ AN + AN = 6

$\Rightarrow \quad$ 2 AN = 6 $\quad \Rightarrow \quad$ AN = $\frac{6}{2}$ $\quad \Rightarrow \quad$ AN = 3

In ΔAND,

By Pythagoras theorem,

$AD^2 = DN^2 + AN^2$

$\Rightarrow \quad AD^2 = (4)^2 + (3)^2 \qquad$ [Height = DN = 4 cm]

$\Rightarrow \quad AD^2 = 16 + 9 \quad \Rightarrow \quad AD^2 = 25$

$\Rightarrow \quad AD = \sqrt{25} \quad \Rightarrow \quad$ AD = 5 cm

then AD = BC = 5 cm

Perimeter of trapezium = AB + BC + CD + AD

= 18 cm + 5 cm + 12 cm + 5 cm = 40 cm

Question 37.

The area of a parallelogram is 98 cm². If one altitude is half the corresponding base, determine the base and the altitude of the parallelogram.

Solution:

The given area of a parallelogram = 98 cm^2

given condition that one altitude is half the corresponding base

Let base = x cm

Then corresponding altitude = $\frac{x}{2}$ cm

Area of parallelogram = Base × Altitude

(where Base is corresponding base)

$\Rightarrow \quad 98 = x \text{ cm} \times \frac{x}{2} \text{ cm} \quad \Rightarrow \quad 98 = \frac{x^2}{2}$

$\Rightarrow \quad 98 \times 2 = x^2 \quad \Rightarrow \quad x^2 = 196 \quad \Rightarrow \quad x = \sqrt{196}$

$\Rightarrow \quad x = 14$ cm

i.e. Base = 14 cm

and Altitude = $\frac{14\text{cm}}{2} = 7$ cm

Question 38.

The length of a rectangular garden is 12m more than its breadth. The numerical value of its area is equal to 4 times the numerical value of its perimeter. Find the dimensions of the garden

Solution:

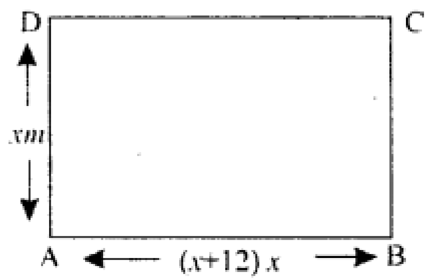
Let the breadth of rectangular garden = x m

Then length of rectangular garden = $(x + 12)$ m

Area = $\ell \times b = (x + 12) \times x \text{ m}^2 = (x^2 + 12x) \text{ m}^2$

Perimeter = $2(\ell + b) = 2[(x + 12) + x]$ m

$= 2[x + 12 + x] \text{ m} = 2(2x + 12) \text{ m} = 4x + 24 \text{ m}$



According to question,

Numerical value of Area = 4 × numerical value of perimeter

$\Rightarrow \quad x^2 + 12x = 4 \times (4x + 24)$

$\Rightarrow \quad x^2 + 12x = 16x + 96$

$\Rightarrow \quad x^2 + 12x - 16x - 96 = 0 \quad \Rightarrow \quad x^2 - 4x - 96 = 0$

$\Rightarrow \quad x^2 - 12x + 8x - 96 = 0$

$\Rightarrow \quad x(x - 12) + 8(x - 12) = 0$

$\Rightarrow \quad (x + 8)(x - 12) = 0$

Either $x + 8 = 0$ or $x - 12 = 0$

$x = -8$ (not possible) or $x = 12$

Hence, Breadth of rectangular garden = 12 m

Length of rectangular garden = 12 m + 12 m = 24 m

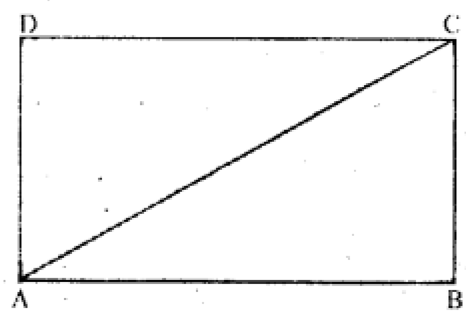
Question 39.

If the perimeter of a rectangular plot is 68 m and length of its diagonal is 26 m, find its area.

Solution:

Given that perimeter of a rectangular plot = 68 m and length of its diagonal = 26 m

Here, ABCD be the rectangular plot let length of rectangular plot = x m



and Breadth of rectangular plot = y m

Then, Perimeter = 2 (Length + Breadth)

$\Rightarrow \quad 68 = 2(x + y) \quad \Rightarrow \quad \frac{68}{2} = x + y$

$\Rightarrow \quad 34 = x + y \quad \Rightarrow \quad x + y = 34$

$\Rightarrow \quad x = (34 - y) \qquad \text{....(1)}$

Also, in ΔABC

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \quad \Rightarrow \quad (26)^2 = x^2 + y^2$

($\because$ AC = diagonal of rectangular plot)

$\Rightarrow \quad x^2 + y^2 = 676$

Substituting the value of x from (1), we get

$\Rightarrow (34 - y)^2 + y^2 = 676 \Rightarrow 1156 + y^2 - 68y + y^2 = 676$

$\Rightarrow \quad 2y^2 - 68y + 1156 - 676 = 0$

$\Rightarrow \quad 2y^2 - 68y - 480 = 0 \quad \Rightarrow \quad 2(y^2 - 34y - 240) = 0$

$\Rightarrow y^2 - 34y - 240 = 0 \quad \Rightarrow \quad y^2 - 24y - 10y - 240 = 0$

$\Rightarrow \quad y(y - 24) - 10(y - 24) = 0$

$\Rightarrow \quad (y - 10)(y - 24) = 0$

Either, $\quad y - 10 = 0 \quad$ or $\quad y - 24 = 0$

$y = 10$ m $\quad$ or $\quad y = 24$ m

Substituting the value of y in equation (1), we get

when $y = 10$ m, $x = (34 - 10)$ m $= 24$ m

Either $y = 24$ m

$x = (34 - 24)$ m $= 10$ m

the required Area in both the cases $= xy$

$= 24\text{m} \times 10\text{m}$ or $10\text{m} \times 24\text{m} = 240$ m or 240 m

Hence, area of the rectangular block = 240 m.

Question 40.

A rectangle has twice the area of a square. The length of the rectangle is 12 cm greater and the width is 8 cm greater than 2 side of a square. Find the perimeter of the square.

Solution:

Let the side of a square = x cm
Then length of rectangle = $(x + 12)$ cm
Breadth of rectangle = $(x + 8)$ cm
Area of square = side × side = x cm × x cm = x^2 cm^2
Area of rectangle = length × Breadth
$= (x + 12)$ cm × $(x + 8)$ cm $= (x + 12)(x + 8)$ cm^2
According to question
Area of rectangle = 2 × Area of square

$\Rightarrow \quad (x + 12)(x + 8) = 2 \times x^2$

$\Rightarrow \quad x(x + 8) + 12(x + 8) = 2x^2$

$\Rightarrow \quad x^2 + 8x + 12x + 96 = 2x^2$

$\Rightarrow \quad x^2 - 2x^2 + 8x + 12x + 96 = 0$

$\Rightarrow \quad -x^2 + 20x + 96 = 0 \quad \Rightarrow \quad -(x^2 - 20x - 96) = 0$

$\Rightarrow \quad x^2 - 20x - 96 = 0 \quad \Rightarrow \quad x^2 - 24x + 4x - 96 = 0$

$\Rightarrow \quad x(x - 24) + 4(x - 24) = 0$

$\Rightarrow \quad (x + 4)(x - 24) = 0$

Either $(x + 4) = 0$ or $x - 24 = 0$
$x = -4$ (not possible) $x = 24$ cm
$\therefore$ side of square = 24 cm
Perimeter of square = 4 × side
= 4 × 24 cm = 96 cm

Question 41.
The perimeter of a square is 48 cm. The area of a rectangle is 4 cm2 less than the area of the square. If the length of the rectangle is 4 cm greater than its breadth, find the perimeter of the rectangle.

Solution:

Perimeter of square = 48 cm

$\therefore$ Side = $\frac{\text{Perimeter}}{4} = \frac{48}{4} = 12$ cm

Area = $(\text{side})^2 = (12)^2 = 144$ cm^2

$\therefore$ Area of rectangle = $144 - 4 = 140$ cm^2

Let breadth of rectangle = x cm

Then length = $x + 4$ cm

$\therefore$ Area = $(x + 4) \times x$ cm^2

$\therefore$ $(x + 4)x = 140 \Rightarrow x^2 + 4x - 140 = 0$

$\Rightarrow x^2 + 14x - 10x - 140 = 0$

$\Rightarrow x(x + 14) - 10(x + 14) = 0 \Rightarrow (x + 14)(x - 10) = 0$

Either $x + 14 = 0$, then $x = -14$

or $x - 10 = 0$, then $x = 10$

$\therefore$ Breadth = 10 cm

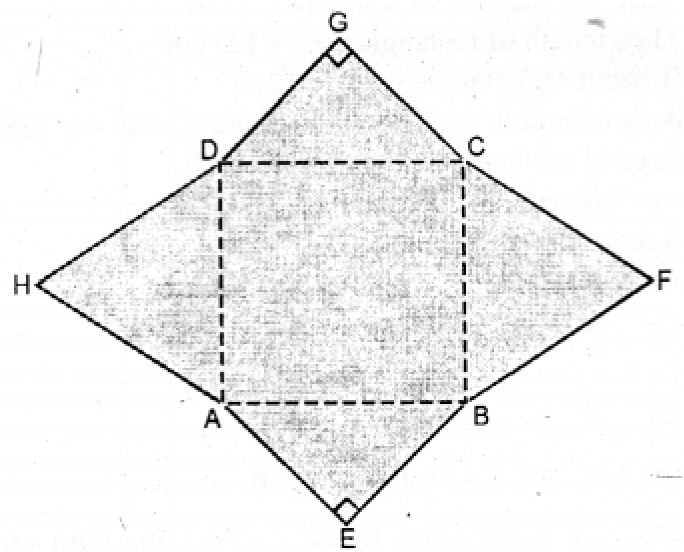
$\therefore$ Then length = $10 + 4 = 14$ cm

$\therefore$ Perimeter = $2(l + b) = 2(14 + 10) = 2 \times 24$ cm $= 48$ cm

Question 42.

In the adjoining figure, ABCD is a rectangle with sides AB = 10 cm and BC = 8 cm. HAD and BFC are equilateral triangles; AEB and DCG are right angled isosceles triangles. Find the area of the shaded region and the perimeter of the figure.

Solution:


G
D
C
H
F
A
B
E

ABCD is a rectangle and AB = 10 cm,
BC = 8 cm.

ΔHAD and ΔBFC and equilateral triangles whose each side is 8 cm.

ΔAEB and ΔDCG are right angled isosceles triangles whose each hypotenues = 10 cm.

Let AE = EB = x cm.

Now in ΔABE, $AE^2 + EB^2 = AB^2$

$\Rightarrow x^2 + x^2 (10)^2 \quad \Rightarrow 2x^2 = 100$

$\Rightarrow x^2 = \frac{100}{2} = 50$

$\therefore \quad x = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ cm.

Now area of ΔAEB = ΔGCD

$= \frac{1}{2} x \times x = \frac{1}{2} x^2 \text{ cm}^2 = \frac{1}{2} \times 50 = 25 \text{ cm}^2$

and area of ΔHAD = area of BFC

$= \frac{\sqrt{3}}{4} \times (8)^2 \text{ cm}^2 = \frac{\sqrt{3}}{4} \times 64 = 16\sqrt{3} \text{ cm}^2$

Area of shaded portion = Area of rect. ABCD + 2 area of ΔAEB + 2 area of ΔBFC

$= (10 \times 8 + 2 \times 25 + 2 \times 16\sqrt{3}) \text{ cm}^2$

$= (80 + 50 + 32\sqrt{3}) \text{ cm}^2 = (130 + 32\sqrt{3}) \text{ cm}^2$

Perimeter of the figure = AE + EB + BF + FC + CD + GD + DH + HA

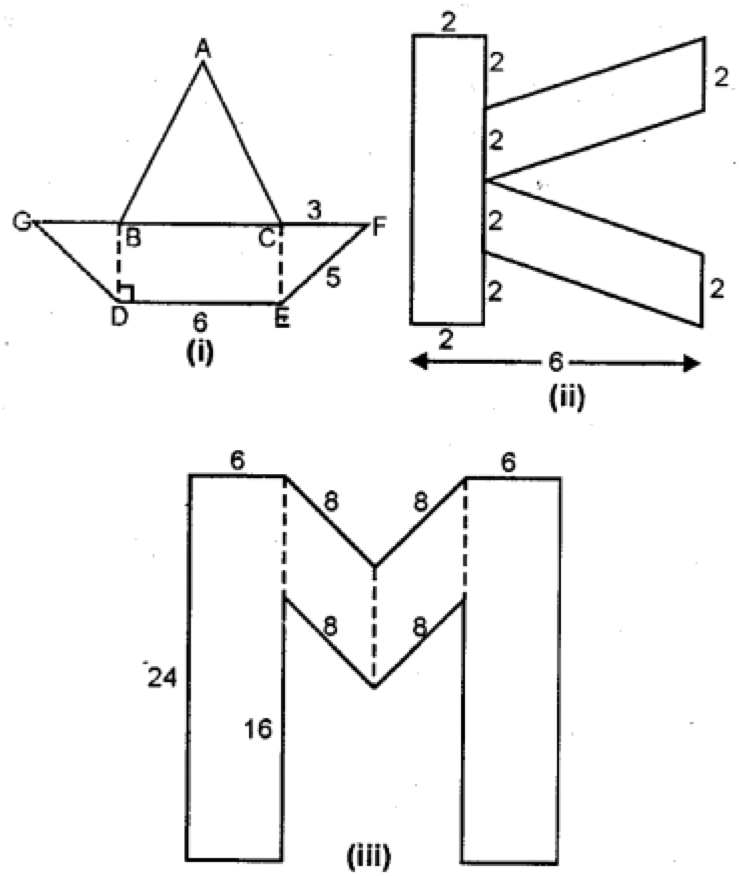
$= 4AE + 4BF = (4 \times 5\sqrt{2} + 4 \times 8)$ cm

$= (20\sqrt{2} + 32)$ cm. $= (32 + 20\sqrt{2})$ cm.

Question 43.

(a) Find the area enclosed by the figure (i) given below, where ABC is an equilateral triangle and DGFG is an isosceles trapezium.
All measurements are in centmetces.
(b) Find the area enclosed by the figure (ii) given below. AH measurements are in centimetres.
(c) In the figure (iii) given below, from a 24. cm x 24 cm piece of cardboard, a

block in the shape of letter M is cut off. Find the area of the cardboard left over, all measurements are in centimetres.



Solution:

(a) ΔABC is an equilateral triangle and DEFG is an isosceles trapezium in which

$$EF = GD = 5 \text{ cm.}$$

$$DE = 6 \text{ cm}$$

and $GF = GB + BC + CF = 3 + 6 + 3 = 12$ cm.

$$AB = AC = BC = 6 \text{ cm.}$$

Join BD and CE

In right Δ CEF, $CE^2 = EF^2 - CF^2$

$$= 5^2 - 3^2 = 25 - 9 = 16$$

$$\therefore \; CE = \sqrt{16} = 4 \text{ cm}$$

Now area of ΔABC

$$= \frac{\sqrt{3}}{4} \times (6)^2 = 36 \times \frac{\sqrt{3}}{4} \text{cm}^2 = 9\sqrt{3} \text{ cm}^2$$

and area of trap. DEFG

$$= \frac{1}{2} (DE + GF) \times CE = \frac{1}{2} (6 + 12) \times 4 \text{ cm}^2$$

$$= \frac{1}{2} \times 18 \times 4 = 36 \text{ cm}^2$$

$\therefore$ Area of the figure $= (9\sqrt{3} + 36)$ cm^2

$= 9 \times 1{\cdot}732 + 36 = 15{\cdot}59 + 36 \text{ cm}^2$

$= 51{\cdot}59 \text{ cm}^2$

(*b*) Length of rectangle $= 2 + 2 + 2 + 2 = 8$ cm.

and width $= 2$ cm.

$\therefore$ Area $= l + b = 8 \times 2 = 16 \text{ cm}^2$

Area of each trap $= \frac{1}{2}(2 + 2) \times (6 - 2)$

$= \frac{1}{2} \times 4 \times 4 = 8 \text{ cm}^2$

$\therefore$ Total area = area of react.

+ area of 2 tapezium

$= 16 + 8 + 8 = 32 \text{ cm}^2$

(*c*) Length of each rectangle $= 24$ cm.

width $= 6$ cm.

$\therefore$ Area of each rectangle $= l \times b = 24 \times 6$

$= 144 \text{ cm}^2$

Base of each parallelogram $= 8$ cm.

and height $= 6$ cm.

$\therefore$ Area of each parallelogram $= 8 \times 6$

$= 48 \text{ cm}^2$

Now area of the M-shaped figure $= 2 \times 144$

$+ 2 \times 48 = \text{cm}^2$

$= 288 + 96 = 384 \text{ cm}^2$

and area of the square cardboard $= 24 \times 24$

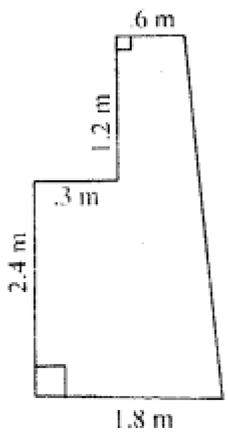
$= 576 \text{ cm}^2$

$\therefore$ Area of the removing cardboard

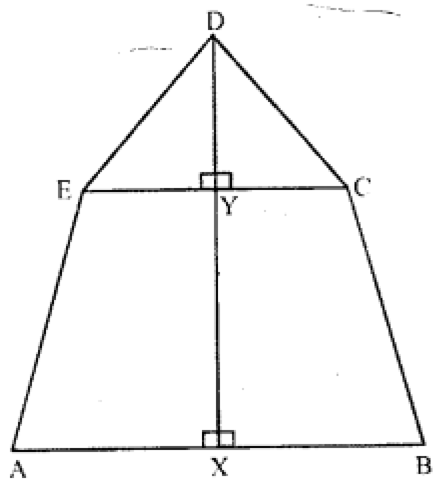
$= 576 - 384 = 192 \text{ cm}^2$

Question 44.

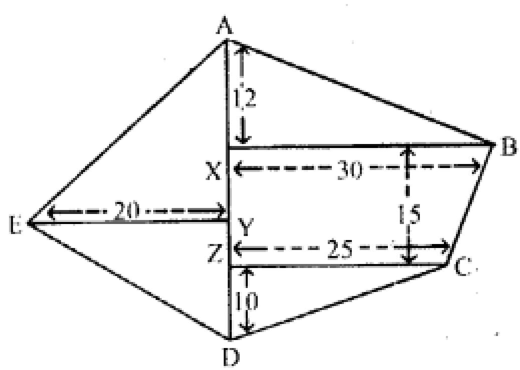
(a) The figure (i) given below shows the cross-section of the concrete structure with the measurements as given. Calculate the area of cross-section.
(b) The figure (ii) given below shows a field with the measurements given in metres. Find the area of the field.
(c) Calculate the area of the pentagon ABCDE shown in fig. (iii) below, given that AX = BX = 6 cm, EY = CY = 4 cm, DE = DC = 5cm,DX = 9cmand DX is perpendicular to EC and AB.



(*i*)



(*iii*)



(*ii*)

Solution:

(*a*) In figure (*i*)

AB = 1.8 m, CD = 0.6 m, DE = 1.2 m

EF = 0.3 m, AF = 2.4 m

Produce DE to meet AB in G then

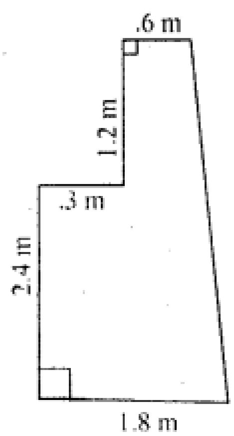
$\angle FEG = \angle GAF$ 90°

$\therefore$ AGEF is a rectangle

Area of given figure

= Area of rectangle AGEF

+ Area of trapezium GBCD



(*i*)

$= \ell \times b + \frac{1}{2}$ (sum of parallel sides × height)

$= AF \times AG + \frac{1}{2}(GB + CD) \times DG$

$= 2.4\text{ m} \times 0.3\text{ m} + \frac{1}{2}$

$[(AB - AG) + CD] \times (DE + EG)$

($\because$ AG = FE and using EG = AF)

$= 0.72\text{ m}^2 + \frac{1}{2}[(1.8\text{ m} - 0.3\text{ m}) + 0.6\text{ m}] \times (1.2\text{ m} + 2.4\text{ m})$

$= 0.72\text{ m}^2 + \frac{1}{2}[1.5\text{ m} + 0.6\text{ m}] \times 3.6\text{ m}$

$= 0.72\text{m}^2 + \frac{1}{2} \times 2.1\text{ m} \times 3.6\text{ m}$

$= 0.72\text{ m}^2 + 2.1\text{ m} \times 1.8\text{ m}$

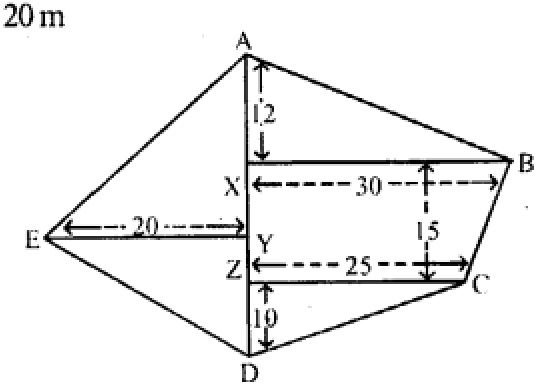
$= 0.72\text{ m}^2 + 3.78\text{ m}^2 = 4.50\text{ m}^2 = 4.5\text{ m}^2$

(*b*) ABCD is a pentagonal field in which

AX = 12 m, BX = 30 m, XZ = 15 m, CZ = 25 m

DZ = 10 m, AD = 12 m + 15 m + 10m = 37 m,

EY = 20 m



(*ii*)

Area of pentagonal field ABCDE = Area of ΔABX + Area of trapezium BCZX + Area of ΔCDZ + Area of ΔAED.

$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2}$ (sum of parallel sides) $\times \text{height} + \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times BX \times AX + \frac{1}{2}(BX + CZ) \times XZ + \frac{1}{2} \times CZ \times DZ + \frac{1}{2} \times AD \times EY$

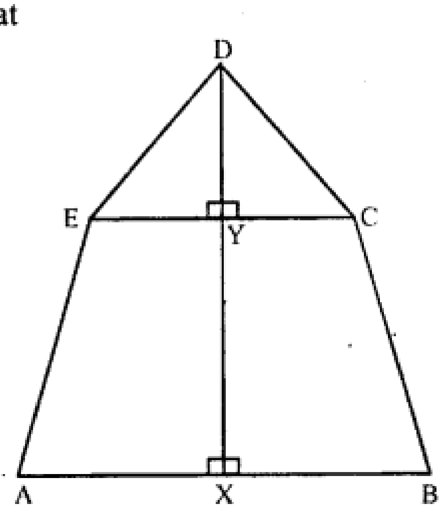
$= \frac{1}{2} \times 30\text{ m} \times 12\text{ m} + \frac{1}{2}(30\text{ m} + 25\text{ m}) \times 15\text{ m} + \frac{1}{2} \times 25\text{ m} \times 10\text{ m} + \frac{1}{2} \times 37\text{ m} \times 20\text{ m}$

$= 15\text{m} \times 12\text{ m} + 7.5\text{ m} \times 55\text{ m} + 25\text{ m} \times 5\text{ m} + 37\text{ m} \times 10\text{m}$

$= 180\text{ m}^2 + 412.5\text{ m}^2 + 125\text{ m}^2 + 370\text{ m}^2$

$= 1087.5\text{ m}^2$

(*c*) Here ABCDE is the pentagon
Given that



(*iii*)

AX = BX = 6 cm, EY = CY = 4 cm
DE = DC = 5 cm, DX = 9 cm

And DX $\perp$ to EC and AB

In ΔDEY

By Pythagoras theorem,

$DE^2 = DY^2 + EY^2 \Rightarrow (5)^2 = DY^2 + (4)^2$

$\Rightarrow 25 = DY^2 + 16 \Rightarrow DY^2 = 25 - 16 = 9$

$\Rightarrow DY = \sqrt{9} = 3$ cm

Area of pentagonal field ABCDE = Area of ΔDEY + Area of ΔDCY + Area of trapezium EYXA + Area of trapezium CYXB.

$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height} + \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$= \frac{1}{2} \times EY \times DY + \frac{1}{2} \times CY \times DY + \frac{1}{2} \times (EY + AX) \times (XY) + \frac{1}{2} \times (CY + BX) \times (XY)$

$= \frac{1}{2} \times 4\text{ cm} \times 3\text{ cm} + \frac{1}{2} \times 4\text{ cm} \times 3\text{ cm} + \frac{1}{2} (4\text{ cm} + 6\text{ cm}) \times (DX - DY) + \frac{1}{2} (4\text{ cm} + 6\text{ cm}) \times (DX - DY)$

$= 2\text{ cm} \times 3\text{ cm} + 2\text{ cm} \times 3\text{ cm} + \frac{1}{2} (10\text{ cm}) \times (9\text{ cm} - 3\text{ cm}) + \frac{1}{2} \times 10\text{ cm} \times (9\text{ cm} - 3\text{ cm})$

$= 6\text{ cm}^2 + 6\text{ cm}^2 + 5\text{ cm} \times 6\text{ cm} + 5\text{ cm} \times 6\text{ cm}$

$= 6\text{ cm}^2 + 6\text{ cm}^2 + 30\text{ cm}^2 + 30\text{ cm}^2$

$= 72\text{ cm}^2$

Question 45.

If the length and the breadth of a room are increased by 1 metre the area is increased by 21 square metres. If the length is increased by 1 metre and breadth is decreased by 1 metre the, area is decreased by 5 square metres. Find the

perimeter of the room.

Solution:

Let the length of room = x m

and breadth of room = y m

Area of room = $\ell \times b$

$= x \text{ m} \times y \text{ m} = xy \text{ m}^2$

Length is increased by 1m then new length becomes = $(x + 1)$ m

Breadth is increased by 1 m then new Breadth = $(y + 1)$ m

Then new Area becomes = new length × new breadth = $(x + 1)$ m $(y + 1)$ m

$(x + 1)\ (y + 1)$ m²

According to question,

$xy = (x + 1)(y + 1) - 21$

$\Rightarrow \quad xy = x(y + 1) + 1(y + 1) - 21$

$\Rightarrow \quad xy = xy + x + y + 1 - 21$

$\Rightarrow \quad 0 = x + y + 1 - 21 \quad \Rightarrow \quad 0 = x + y - 20$

$\Rightarrow \quad x + y - 20 = 0 \quad \Rightarrow \quad x + y = 20 \qquad ....(1)$

Again, length is increased by 1 metre then new length becomes = $(x + 1)$ metre

Breadth is decreased by 1 metre new Breadth becomes = $(y - 1)$ metre

New Area = new length × new breadth = $(x + 1)$ $(y - 1)$ m²

Again, According to question

$xy = (x+1)(y-1)+5$

$\Rightarrow xy = x(y-1) + 1(y-1) + 5$

$\Rightarrow xy = xy - x + y - 1 + 5$

$\Rightarrow 0 = -x + y + 4$

$\Rightarrow x - y = 4 \quad \text{...(2)}$

From (1) and (2)

$$x + y = 20 \quad \text{....(1)}$$
$$x - y = 4 \quad \text{....(2)}$$

Adding , $2x = 24$

$\Rightarrow \quad x = \frac{24}{2} = 12 \text{ m}$

Substituting the value of x in equation (1) , we get

$12 + y = 20 \Rightarrow y = 20 - 12 \Rightarrow y = 8 \text{ m}$

length of room = 12 m, Breadth of room = 8 m

Perimeter = $2(\ell + b)$

$= 2(12 \text{ m} + 8 \text{ m}) = 2 \times 20 \text{ m} = 40\text{m}$

Question 46.

A triangle and a parallelogram have the same base and same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Solution:

Sides of the triangle = 26 cm, 28 cm, and 30 cm

$$\therefore s = \frac{26+28+30}{2} = \frac{84}{2} = 42$$

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

$$= \sqrt{42 \times 16 \times 14 \times 12}$$

$$= \sqrt{7 \times 6 \times 4 \times 4 \times 7 \times 2 \times 6 \times 2}$$

$$= 2 \times 4 \times 6 \times 7 = 336 \text{ cm}$$

$\therefore$ Area of parallelogram = 336 cm^2

Base = 28 cm

$$\therefore \text{Height} = \frac{\text{Area}}{\text{Base}} = \frac{336}{28} = 12 \text{ cm Ans.}$$

Question 47.

A rectangle of area 105 cm² has its length equal to x cm. Write down its breadth in terms of x. Given that its perimeter is 44 cm, write down an equation in x and solve it to determine the dimensions of the rectangle.

Solution:

Area of rectangle = 105 cm^2

given length of rectangle = x cm

Then, Area = length × Breadth

$\Rightarrow \quad 105 = x \times \text{Breadth}$

$\Rightarrow \quad \text{Breadth} = \frac{105}{x} \text{ cm}$

Given perimeter of rectangle = 44 cm

$2(\ell + b) = 44 \quad \Rightarrow \quad 2\left(x + \frac{105}{x}\right) = 44$

$\Rightarrow \quad \frac{x^2 + 105}{x} = 22 \quad \Rightarrow \quad x^2 + 105 = 22x$

$\Rightarrow \quad x^2 - 22x + 105 = 0 \Rightarrow \quad x^2 - 15x - 7x + 105 = 0$

$\Rightarrow \quad x(x - 15) - 7(x - 15) = 0$

$\Rightarrow \quad (x - 7)(x - 15) = 0$

Either $x - 7 = 0$ or $x - 15 = 0$

$x = 7$ cm or $x = 15$ cm

When $x = 7$, Breadth = $\frac{105}{7} = 15$ cm

When $x = 15$, Breadth = $\frac{105}{15} = 7$ cm

Hence, required dimensions of rectangle = 15 cm, 7 cm

Question 48.

The perimeter of a rectangular plot is 180 m and its area is 1800 m². Take the length of plot as x m. Use the perimeter 180 m to write the value of the breadth in terms of x. Use the value of the length, breadth and the area to,write an equation in x. Solve the equation to calculate the length and breadth of the plot.

Solution:

Given perimeter of a rectangle plot = 180 m

and Area of a rectangle plot = 1800 m^2

Taking length of rectangle = x m

Perimeter = 2 (length + breadth)

$\Rightarrow$ 180 = 2 (x + Breadth)

$\Rightarrow \frac{180}{2} = x + \text{Breadth} \Rightarrow 90 = x + \text{Breadth}$

$\Rightarrow x + \text{Breadth} = 90$

$\Rightarrow$ Breadth = 990 – x) m

Area of rectangle = Length × Breadth

1800 = x m × (90 – x) m

$\Rightarrow x(90 - x) = 1800$

$\Rightarrow 90x - x^2 = 1800 \Rightarrow -(x^2 - 90x) = 1800$

$\Rightarrow x^2 - 90x = -1800 \Rightarrow x^2 - 90x + 1800 = 0$

$\Rightarrow x^2 - 60x - 30x + 1800 = 0$

$\Rightarrow x(x - 60) - 30(x - 60) = 0$

$\Rightarrow (x - 30)(x - 60) = 0$

$\Rightarrow$ Either $x - 30 = 0$ or $x - 60 = 0$

x = 30 m or x = 60 m

When x = 30 m then

Breadth = (90 – 30) m = 60 m

When x = 60 m then

Breadth = (90 – 60) m = 30 m

Hence, required length of rectangle = 60m

and breadth of rectangle = 30 m

EXERCISE 16.3

Question 1.

Find the length of the diameter of a circle whose circumference is 44 cm.

Solution:

Let radius of the circle = r

then circumference = $2\pi r$

$\therefore\ 2\pi r = 44 \Rightarrow = \frac{2 \times 22}{7} r = 44$

$\Rightarrow\ r = \frac{44 \times 7}{2 \times 22} = 7\text{cm}$

$\therefore$ Diameter = $2r = 2 \times 7 = 14$ cm.

Question 2.
Find the radius and area of a circle if its circumference is 18π cm.
Solution:

Let r be the radius of the circle

$\therefore$ Circumference = $2\pi r$

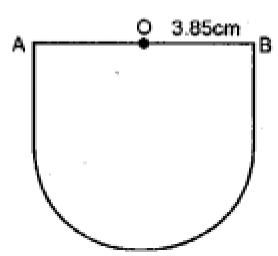
$\therefore\ 2\pi r = 18\pi \Rightarrow 2r = 18 \Rightarrow r = \frac{18}{2} = 9\text{cm}$

Area = $\pi r^2 = \pi \times 9 \times 9 = 81\pi\ \text{cm}^2$

Question 3.
Find the perimeter of a semicircular plate of radius 3.85 cm.
Solution:

Radius of semicircular plate = 3·85 cm

$\therefore$ Length of semicircular plate = πr



$\therefore$ Perimeter = $\pi r + 2r = r(\pi + 2)$

$= 3{\cdot}85\left(\frac{22}{7} + 2\right) = 3{\cdot}85 \times \frac{36}{7}$

$= 0{\cdot}55 \times 36 = 19{\cdot}80 = 19{\cdot}8\text{cm}$.

Question 4.
Find the radius and circumference of a circle whose area is 144π cm².

Solution:

Area of the circle = 144π cm^2

Let radius = r

$\therefore \pi r^2 = 144\pi \Rightarrow r^2 = 144$

$\Rightarrow r = \sqrt{144} = 12$ cm

Circumference = $2\pi r = 2 \times 12 \times \pi$

= 24π cm.

Question 5.

A sheet is 11 cm long and 2 cm wide. Circular pieces 0.5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared.

Solution:

Length of sheet = 11 cm

Width = 2 cm

First of all, we have to cut the sheet is squares of side 0·5 cm.

$\therefore$ No. of squares $= \frac{11}{0\cdot5} \times \frac{2}{0\cdot5}$

$= \frac{11 \times 10}{5} \times \frac{2 \times 10}{5} \Rightarrow 22 \times 4 = 88$

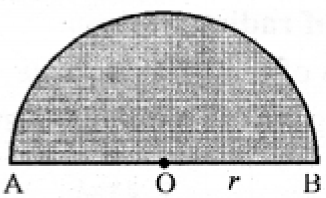
$\therefore$ No. of discs will be equal to number of squares cut out = 88

Question 6.

If the area of a semicircular region is 77cm^2, find its perimeter.

Solution:

Area of semicircular region = 77cm



Let r be the radius of the region

Then area = $\frac{1}{2}\pi r^2$

$\therefore \frac{1}{2}\pi r^2 = 77$

$\Rightarrow \frac{1}{2} \times \frac{22}{7}(r)^2 = 77$

$\Rightarrow r^2 = \frac{77 \times 2 \times 7}{22} = 49 = (7)^2$

$r = 7$ cm

Now, perimeter of the region

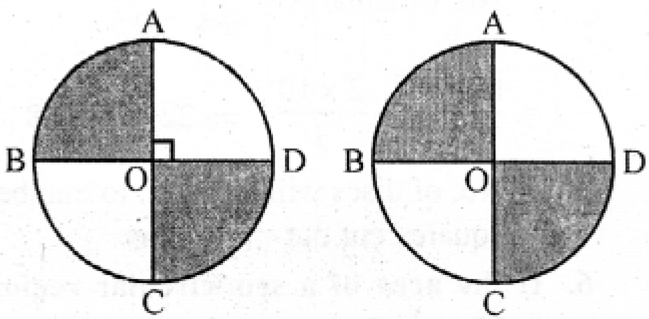
$= \pi r + 2r$

$= \frac{22}{7} \times 7 + 2 \times 7$

$= 22 + 14 = 36$cm

Question 7.

(a) In the figure (i) given below, AC and BD are two perpendicular diameters of a circle ABCD. Given that the ara of the shaded portion is 308 cm2, calculate
(i) the length of AC and
(ii) the circumference of the circle.
(b) In the figure (ii) given below, AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16 cm, calculate the area and perimeter of the shaded part. (Take π = 3.14)



(i) (ii)

Solution:

(a) Area of shaded portion

= Area of semicircle = 308 cm^2

Let r be the radius of the circle, then

$$\frac{1}{2}\pi r^2 = 308 \Rightarrow \frac{1}{2} \times \frac{22}{7} r^2 = 308$$

$$\Rightarrow r^2 = \frac{308 \times 2 \times 7}{22} \Rightarrow r^2 = 196 = (14)^2$$

$\therefore$ $r = 14$ cm

(i) Now AC = $2r = 2 \times 14 = 28$ cm

(ii) Circumference of the circle = $2\pi r$

$$= 28 \times \frac{22}{7} \text{ cm} = 4 \times 22 = 88 \text{ cm}$$

(b) Diameters of circle = 16 cm

$\therefore$ Radius = $\frac{16}{2} = 8$ cm

Area of shaded part

Area = 2 × area of one quadrant

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times 3.14 \times 8 \times 8 = 100.48 \text{ cm}^2$$

Perimeter of shaded part = $\frac{1}{2}$ of circumference + $4r$

$$= \frac{1}{2} \times 2\pi r + 4r = \pi r + 4r = r(\pi + 4)$$

$$= 8(3.14 + 4) = 8 \times 7.14 = 57.12 \text{ cm}$$

Question 8.

A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 minute 28 seconds with a uniform speed of 1.1 m/sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

Solution:

Diameter of wheel = 77 cm.

$$\therefore \quad \text{radius} = \frac{77}{2} \text{ cm}$$

$$\therefore \text{Circumference} = 2\pi r = 2 \times \frac{22}{7} \times \frac{77}{2} = 242 \text{ cm.}$$

Length of rope $= 1\frac{28}{60}$ minutes at the speed of 1·1 m/sec.

$= 88 \times 1{\cdot}1 = 96{\cdot}8 = 96{\cdot}8$ m

$= 96{\cdot}8 \times 100$ cm $= 9680$ cm.

$$\therefore \text{No. of revolutions} = \frac{9680}{242} = 40$$

Question 9.

The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Give your answer correct to the nearest km.

Solution:

Diameter of wheel = 84 cm

$$\therefore \text{Radius} = \frac{84}{2} = 42 \text{ cm}$$

Circumference of the wheel

$$= 2\pi r = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm.}$$

Distance covered in 5 reductions

$= 264 \times 5 = 1320$ cm.

Time = 1 second

$$\therefore \text{speed of the wheel} = \frac{1320}{1} \times \frac{60 \times 60}{100 \times 1000} \text{ km/hr}$$

$= 47{\cdot}52$ km/hr. $= 48$ km/hr.

Question 10.

The circumference of a circle is 123.2 cm. Calculate :

(i) the radius of the circle in cm.

(ii) the area of the circle in cm^2, correct to the nearest cm^2.

(iii) the effect on the area of the circle if the radius is doubled.

Solution:

Circumference of a circle = 123·2 cm.

Let radius = r

$$\therefore\ 2\pi r = 123{\cdot}2 \Rightarrow \frac{2 \times 22}{7} r = \frac{1232}{10}$$

$$\Rightarrow \quad r = \frac{1232 \times 7}{10 \times 2 \times 22} = 19{\cdot}6\,\text{cm}$$

(*i*) ∴ Radius = 15·6 cm

(*ii*) Area of the circle = πr^2

$$= \frac{22}{7} \times 19{\cdot}6 \times 19{\cdot}6\ \text{cm}^2$$

$$= 1207{\cdot}36\ \text{cm}^2 = 1207\ \text{cm}^2$$

(*iii*) If radius is doubled *i.e.* = 19·6 × 2

= 39·2 cm

Then area of the circle = πr^2

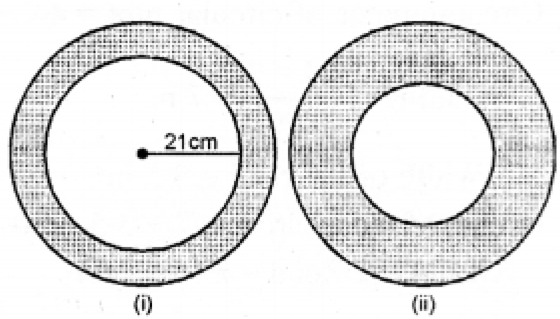
$$= \frac{22}{7} \times 39{\cdot}2 \times 39{\cdot}2\ \text{cm}^3 = 4829{\cdot}44\ \text{cm}^2$$

$$\text{Effect on area} = \frac{4829.44}{1207} = 4 \text{ times}$$

Question 11.

(a) In the figure (i) given below, the area enclosed between the concentric circles is 770 cm^2. Given that the radius of the outer circle is 21 cm, calculate the radius of the inner circle.

(b) In the figure (ii) given below, the area enclosed between the circumferences of two concentric circles is 346.5 cm^2. The circumference of the inner circle is 88 cm. Calculate the radius of the outer circle.



Solution:

(*i*) Radius of the outer circle (R) = 21 cm

Let radius of inner circle = r cm

$\therefore$ Area of the ring = $\pi (R^2 - r^2)$

$$= \frac{22}{7}(21^2 - r^2) = \frac{22}{7}(441 - r^2)$$

But area of the ring = 770 cm^2.

$$\therefore \quad \frac{22}{7}(441 - r^2) = 770$$

$$441 - r^2 = \frac{770 \times 7}{22} = 245$$

$$\Rightarrow r^2 = 441 - 245 = 196 \Rightarrow \quad r = \sqrt{196} = 14$$

$\therefore$ radius of inner circle = 14 cm

(*ii*) Area of the ring = 346·5 cm^2

Circumference of inner circle = 88 cm.

$$\therefore \quad \text{radius} = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

Let radius of outer circle = R

$\therefore$ Area of ring = $\pi (R^2 - r^2)$

$$= \frac{22}{7}(R^2 - 14^2)\text{cm}^2 = \frac{22}{7}(R^2 - 196) \text{ cm}^2$$

$$\therefore \frac{22}{7}(R^2 - 196) = 346{\cdot}5$$

$$R^2 - 196 = \frac{346{\cdot}5 \times 7}{22} = 110{\cdot}25$$

$$R^2 = 110{\cdot}25 + 196 = 306{\cdot}25$$

$$\therefore \quad R = \sqrt{306{\cdot}25} = 17{\cdot}5$$

$\therefore$ Radius of outer circle = 17·5 cm

Question 12.

A road 3.5 m wide surrounds a circular plot whose circumference is 44 m. Find the cost of paving the road at □50 per m^2.

Solution:

Circumference of circular plot = 44 m

$\therefore$ Radius $= \frac{44 \times 7}{22 \times 2} = 7$ m

Width of the road = 3·5 m

$\therefore$ Radius of outer circle = 7 + 3·5 = 10·5 m

Area of the Road $= \pi (R^2 - r^2)$

$= \frac{22}{7} (10{\cdot}5^2 - 7^2)$ m^2

$= \frac{22}{7} (10{\cdot}5 + 7)(10{\cdot}5 - 7)$ m^2

$= \frac{22}{7} \times 17{\cdot}5 \times 3{\cdot}5 = 192{\cdot}5$ m^2.

Rate of paving the road = ₹50 per m^2.

$\therefore$ Total cost = ₹192·5 × 50

= ₹9625

Question 13.

The sum of diameters of two circles is 14 cm and the difference of their circumferences is 8 cm. Find the circumference of the two circles.

Solution:

Sum of the diameters of two circles = 14 cm

Let R and r be the radii of two circles

$2R + 2r = 14$

$R + r = 7$ ...(*i*)

(Dividing by 2)

Difference of their circumferences = 8 cm

$\Rightarrow 2\pi R - 2\pi r = 8$

$\Rightarrow 2\pi(R - r) = 8 \Rightarrow \frac{2 \times 22}{7}(R - r) = 8$

$\Rightarrow R - r = \frac{8 \times 7}{2 \times 22} = \frac{14}{11}$ ...(*ii*)

Adding (*i*) and (*ii*),

$2R = 7 + \frac{14}{11} = \frac{77 + 14}{11} = \frac{91}{11}$

$R = \frac{91}{11 \times 2} = \frac{91}{22}$

From (*i*)

$R + r = 7$

$\Rightarrow \frac{91}{22} + r = 7 \Rightarrow r = 7 - \frac{91}{22}$

$\Rightarrow r = \frac{154 - 91}{22} = \frac{63}{22}$

Now, circumference of first circle

$= 2\pi R = 2 \times \frac{22}{7} \times \frac{91}{22}$ cm

$= 26$ cm

and the circumference of second circle

$= 2\pi R = 2 \times \frac{22}{7} \times \frac{63}{22} = 18$ cm

Question 14.

Find the circumference of the circle whose area is equal to the sum of the areas

of three circles with radius 2 cm, 3 cm and 6 cm.

Solution:

Radius of first circle = 2 cm

$\therefore$ Area $= \pi r^2 = \pi (2)^2 = 4\pi$ cm^2

Radius of second circle = 3 cm

$\therefore$ Area $= \pi r^2 = \pi (3)^2 = 9\pi$ cm^2

Radius of third circle = 6 cm

$\therefore$ Area $= \pi r^2 = \pi (6)^2 = 36\pi$ cm^2

Total area of the three circles

$= 4\pi + 9\pi + 36\pi = 49\pi$ cm^2.

or Area of the given circle $= 49\pi$ cm^2.

$$\therefore \text{ radius} = \sqrt{\frac{49\pi}{\pi}} = \sqrt{49} = 7\text{cm}$$

$$\text{and circumference} = 2\pi r = 2 \times \frac{22}{7} \times 7$$

$= 44$ cm

Question 15.

A copper wire when bent in the form of a square encloses an area of 121 cm2. If the same wire is bent into the form of a circle, find the area of the circle.

Solution:

Area of the square $= 121$ cm^2

$\therefore$ side $= \sqrt{121} = 11$cm

Perimeter $= 4a = 4 \times 11 = 44$ cm

Now, circumference of the circle = 44 cm

$$\therefore \text{ radius} = \frac{44 \times 7}{2 \times 22} = 7\text{cm}$$

$$\text{and area of the circle} = \pi r^2 = \frac{22}{7}(7)^2$$

$$= \frac{22}{7} \times 7 \times 7$$

$= 154$ cm^2

Question 16.

A copper wire when bent into an equilateral triangle has area 121√3 cm2. If the same wire is bent into the form of a circle, find the area enclosed by the wire.

Solution:

Area of the equilateral triangle

$= 121\sqrt{3}\ cm^2$

Let side of the triangle $= a$

$\therefore$ area $= \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 121\sqrt{3}$

$\Rightarrow a^2 = \frac{121 \times \sqrt{3} \times 4}{\sqrt{3}}$

$\Rightarrow a^2 = 484$

$\Rightarrow a = \sqrt{484} = 22$ cm

Length of the wire = 66 cm

$\therefore$ radius of the circle $= \frac{66}{2\pi} = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm.

Hence area of the circle $= \pi r^2$

$= \frac{22}{7} \times \left(\frac{21}{2}\right)^2$

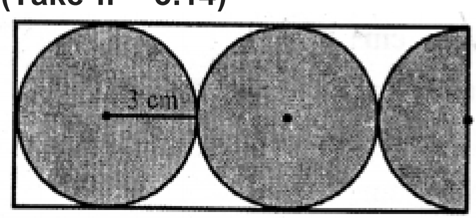
$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\ cm^2$

$= \frac{693}{2} = 346{\cdot}5 cm^2$

Question 17.

(a) Find the circumference of the circle whose area is 16 times the area of the circle with diameter 7 cm.

(b) In the given figure, find the area of the unshaded portion within the rectangle. (Take π = 3.14)



Solution:

(a) Diameter of the circle = 7 cm.

$\therefore$ Radius $= \frac{7}{2}$ cm

and area $= \pi r^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2$$

Now, area of the bigger circle

$$= \frac{77}{2} \times 16 = 616 \text{ cm}^2$$

Let radius = r

$\therefore$ $\pi r^2 = 616$

$\Rightarrow$ $\frac{22}{7} r^2 = 616$

$\Rightarrow$ $r^2 = \frac{616 \times 7}{22}$

$\Rightarrow\ r^2 = 196\ cm^2$

$\Rightarrow\quad r = \sqrt{196} = 14$ cm.

$\therefore$ Circumference

$= 2\pi r = \frac{2 \times 22}{7} \times 14$

$= 88$ cm

(b) In the figure radius of each circle = 3 cm.

$\therefore$ Diameter = 2 × 3 cm = 6 cm

$\therefore$ Length of rectangle (l) = 6 + 6 + 3 = 15 cm

and breadth (b) = 6 cm

$\therefore$ Area of rectangle = length × breadth

$= 15 \times 6 = 90\ cm^2$

and area of $2\frac{1}{2}$ circles $= \frac{5}{2}\pi r^2$

$= \frac{5}{2} \times 3.14 \times 3 \times 3\ cm^2$

$= 5 \times 1.57 \times 9\ cm^2$

$= 70.65\ cm^2$

$\therefore$ Area of unshaded portion

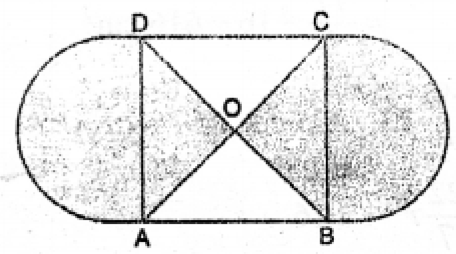
$= 90\ cm^2 - 70.65\ cm^2$

$= 19.35\ cm^2$

Question 18.

In the adjoining figure, A6CD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semicircle are drawn with AD and BC as diameters. Find the area of the shaded region. Take π = $\frac{22}{7}$.

Solution:



We have, side = 21 cm

Area of square = Side^2 = 21^2 = 441 cm^2

We know,

$\angle AOD + \angle COD + \angle AOB + \angle BOC = 441$ cm^2

$x + x + x + x = 441$ cm^2

$4x = 441$ cm^2

$x = \frac{441}{4} = 110.25$ cm^2

In this question, we have to find the area of shaded portion in square ABCD which is $\angle AOD$ and $\angle BOC$

$\therefore\ \angle AOD + \angle BOC = 110.25 + 110.25$ cm^2

= 220.5 cm^2

Now,

Area of two semcircle = πr^2

$= \frac{22}{7} \times 10.5 \times 10.5 = 346.50$ cm^2

$\Rightarrow$ Area of shaded portion = 220.5 + 346.5 cm^2

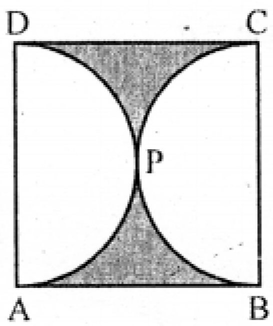
= 567 cm^2

Question 19.

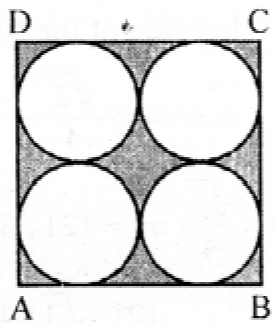
(a) In the figure (i) given below, ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area and the perimeter of the shaded region.

(b) In the figure (ii) given below, ABCD is a square of side 14 cm. Find the area of the shaded region.

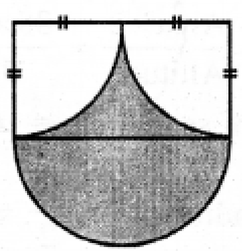
(c) In the figure (iii) given below, the diameter of the semicircle is equal to 14 cm. Calculate the area of the shaded region. Take π = $\frac{22}{7}$.



(*i*)



(*ii*)


(*iii*)

Solution:

(a) ABCD is a square whose each side (a) = 14 cm

APD and BPC are semi-circle with diameter 14 cm each

Radius of each semi circle (a) $= \frac{14}{2} = 7$ cm

(i) Area of square $= a^2 = (14)^2 = 196$ cm^2 and area of two semicircles

$= 2 \times \frac{1}{2} \pi r^2 = \pi r^2$

$= \frac{22}{7} \times 7 \times 7 = 154$ cm^2

$\therefore$ Area of shaded portion $= 196 - 154 = 42$ cm^2

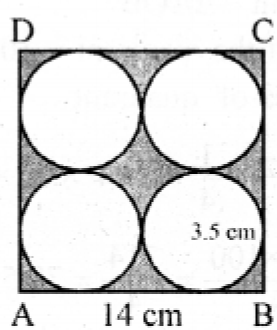
(ii) Length of arcs of two semicircles $= 2\pi r$

$= 2 \times \frac{22}{7} \times 7 = 44$ cm

Perimeter of shaded portion $= 44 + 14 + 14$ cm $= 72$ cm

(b) ABCD is a square whose each side (a) = 14 cm

Four circles are drawn which touch each other and also touch the sides of the square as shown



$\therefore$ Radius of each circle $(r) = \frac{7}{2} = 3.5$ cm

(*i*) Area of square ABCD $= a^2 = (14)^2$ cm^2
$= 196$ cm^2
and area of 4 circles $= 4 \times \pi r^2$

$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ cm$^2 = 154$ cm^2

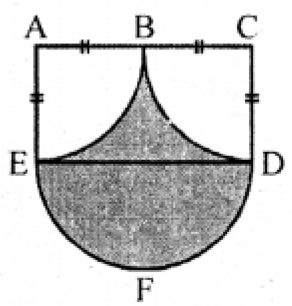
$\therefore$ Area of shaded portion $= 196 - 154$
$= 42$ cm^2

(*ii*) Perimeter of 4 circles $= 2 \times 2\pi r$

$= 4 \times 2 \times \frac{22}{7} \times \frac{7}{2} = 88$ cm

$\therefore$ Perimeter of shaded portion
= Perimeter of 4 circles + Perimeter of square
$= 88 + 4 \times 14 = 88 + 56 = 144$ cm

(c) Area of a rectangle ACDE = ED × AE
$= 14 \times 7 = 98$ cm^2



Area of semicircle DEF $= \frac{\pi r^2}{2}$

$= \frac{22 \times 7 \times 7}{7 \times 2} = 77$ cm^2

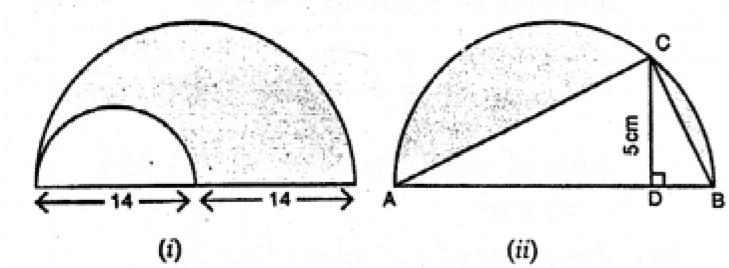
Area of shaded region $= 77 + (98 - 2 \times \frac{1}{4}$

$\times \frac{22}{7} \times 7 \times 7)$ cm^2

$= 77 + 21 = 98$ cm^2

Question 20.

(a) Find the area and the perimeter of the shaded region in figure (i) given below. The dimensions are in centimetres.
(b) In the figure (ii) given below, area of ∆ABC = 35 cm2. Find the area of the shaded region.



Solution:

(a) There are two semicircle, smaller is inside the larger radius of larger semicircles (R) 14 cm and radius of smaller circle

$$(r) = \frac{14}{2} = 7 \text{ cm}$$

(*i*) ∴ Area of shaded portion

= Area of larger semicircle – Area of smaller circle

$$= \frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi(R^2 - r^2) = \frac{1}{2} \times \frac{22}{7}[14^2 - 7^2] \text{ cm}^2$$

$$= \frac{11}{7}[14 + 7][14 - 7] \text{ cm}^2$$

$$= \frac{11}{7} \times 21 \times 7 \text{ cm}^2 = 231 \text{ cm}^2$$

(*ii*) Perimeter of shaded portion

= Circumference of larger semicircle + circumference of smaller semicircle + Radius of larger semicircle

$= \pi R + \pi r + R$

$$= \frac{22}{7} \times 14 + \frac{22}{7} \times 7 + 14 \text{ cm}$$

$= 44 + 22 + 14 = 80$ cm

(b) Area of ΔABC which is formed in a semicircle = 3.5 cm

Altitude CD = 5 cm

$\therefore$ Base AB $= \frac{\text{Area} \times 2}{\text{Altitude}} = \frac{35 \times 2}{5}$ cm = 14 cm

$\therefore$ Diameter of semicircle = 14 cm

and then radius (R) $= \frac{14}{2} = 7$ cm

Area of semicircle $= \frac{1}{2}\pi R^2 = \frac{1}{2} \times \frac{22}{7} \times 7$

$\times 7 \text{ cm}^2 = 77 \text{ cm}^2$

Area of shaded portion

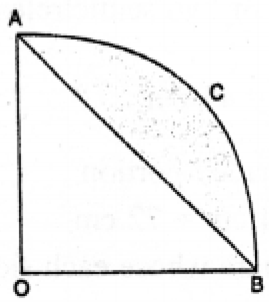
= Area of semicircle – Area of triangle

$= 77 \text{ cm}^2 - 35 \text{ cm}^2 = 42 \text{ cm}^2$

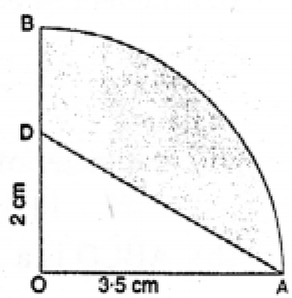
Question 21.

(a) In the figure (i) given below, AOBC is a quadrant of a circle of radius 10 m. Calculate the area of the shaded portion. Take π = 3.14 and give your answer correct to two significant figures.

(b) In the figure, (ii) given below, OAB is a quadrant of a cirlce. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion.



(i)



(ii)

Solution:

(a) In the figure, shaded portion
= quadrant – ΔAOB
Radius of the quadrant = 10 m
Now Area of quadrant

$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3{\cdot}14 \times 10 \times 10$

$= \frac{3{\cdot}14 \times 100}{4} = \frac{314}{4} = 78{\cdot}5 \text{m}^2$

and area of Δ AOB $= \frac{1}{2}$ AO × OB

$= \frac{1}{2} \times 10 \times 10 = 50 \text{ m}^2$

∴ Area of shaded portion
= 78·5 – 50 = 28·5 m^2

(*b*) In the figure (*ii*) radius of quadrant = 3·5 cm

(*i*) ∴ Area of quadrant $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625 \text{ cm}^2$

(*ii*) Area of ΔAOD $= \frac{1}{2} \times$ AO × OD

$= \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2$

∴ Area of shaded portion = Area of quadrant – Area of ΔAOD
$= 9.625 - 3.6 \text{ cm}^2 = 6.125 \text{ cm}^2$

Question 22.
A student takes a rectangular piece of paper 30 cm long and 21 cm wide. Find the area of the biggest circle that can be cut out from the paper. Also find the area of the paper left after cutting out the circle. (Take π = $\frac{22}{7}$)
Solution:

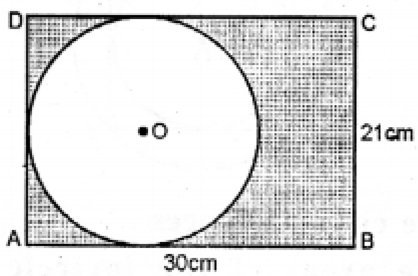
Length of rectangle = 30 cm.

and width = 21 cm.

$\therefore$ Area of rectangle = $l \times b$

$= 30 \times 21 = 630 \text{ cm}^2$

Radius of the biggest circle $= \frac{21}{2}$ cm



(*i*) $\therefore$ Area of the circle = πr^2

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} \text{cm}^2 = 346 \cdot 5 \text{ cm}^2$

Area of remaining part = $630 - 346 \cdot 5$

$= 283 \cdot 5 \text{ cm}^2$

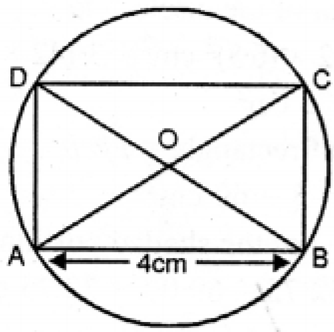
Question 23.

A rectangle with one side 4 cm is inscribed in a circle of radius 2.5 cm. Find the area of the rectangle.

Solution:

In rectagle ABCD, AB = 4 cm,

AC = diameter of circle = 2·5 cm × 2 = 5·0 cm



$\therefore$ $BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{(5)^2 - (4)^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ cm

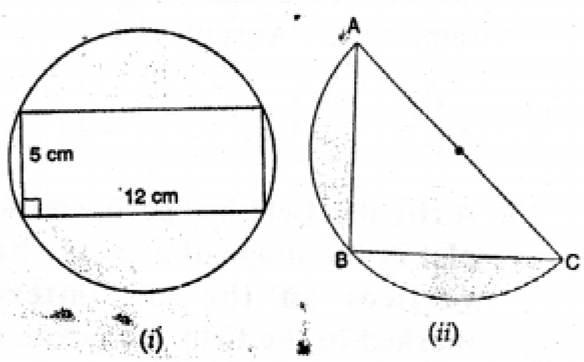
$\therefore$ Area of rectangle = AB × BC

= 4 × 3 = 12 cm^2

Question 24.

(a) In the figure (i) given below, calculate the area of the shaded region correct to two decimal places. (Take π = 3. 142).

(b) In the figure (ii) given below, ABC is an isosceles right angled triangle with ∠ABC = 90°. A semicircle is drawn with AC as diameter. If AB = BC = 7 cm, find the area of the shaded region. Take π = $\frac{22}{7}$.



Solution:

(a) In the figure, ABCD is a rectangle inscribed in a circle whose length = 12 cm and width = 5 cm.

$\therefore\ AC = \sqrt{AB^2 + BC^2} = \sqrt{(12)^2 + (5)^2}$

$= \sqrt{144 + 25} = \sqrt{169} = 13$ cm

$\therefore$ Diameter of the circle = AC = 13 cm.

$\therefore$ radius $= \frac{13}{2}$ cm.$= 6{\cdot}5$ cm.

$\therefore$ Area of the circle $= \pi r^2$

$= 3{\cdot}142 \times (6{\cdot}5)^2\ cm^2 = 3{\cdot}142 \times 42{\cdot}25$

$= 132{\cdot}75\ cm^2$

Area of rectangle $= l \times b$

$= 12 \times 5 = 60\ cm^2$

$\therefore$ Area of the shaded portion

$= 132{\cdot}75 - 60{\cdot}00 = 72{\cdot}75\ cm^2$

(b) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 7 \times 7\ cm^2 = \frac{49}{2} cm^2$

$= AC^2 = AB^2 + BC^2 = 49 + 49$

$\Rightarrow AC = 7\sqrt{2}$

So, radius of the semi-circle $= \frac{7\sqrt{2}}{2}$ cm

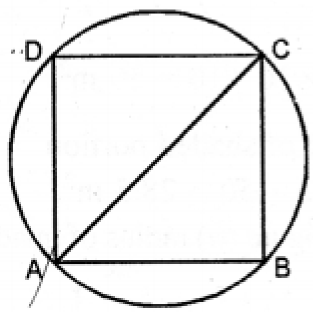
Area of the semi-circle $= \frac{\pi}{2} \times \left(\frac{7\sqrt{2}}{2}\right)^2 cm^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{98}{4} cm^2 = \frac{77}{2} cm^2$

So, area of the shaded region = Area of the semi-circle – Area of ΔABC

$= \left(\frac{77}{2} - \frac{49}{2}\right) cm^2 = \frac{28}{2} = 14\ cm^2$

Question 25.
A circular field has perimeter 660 m. A plot in the shape of a square having its vertices on the circumference is marked in the field. Calculate the area of the square field.
Solution:
Perimeter of circular field = 660 m.



$\therefore$ Radius of the field $= \frac{660}{2\pi}$

$= \frac{660 \times 7}{2 \times 22} = 105\,m$.

ABCD is a square which is inscribed in the circle whose diagonal is AC, which is the diameter of the circular field.

$\therefore$ Let a be the side of the square

$\therefore AC = \sqrt{2}\,a \Rightarrow a = \frac{AC}{\sqrt{2}} = \frac{105 \times 2}{\sqrt{2}}$

$a = \frac{105 \times 2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

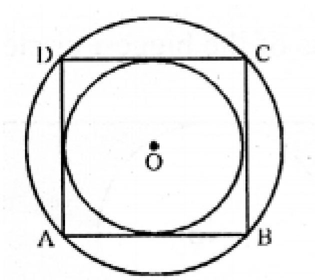
$= \frac{105 \times 2 \times \sqrt{2}}{2} = 105\sqrt{2}$ m.

$\therefore$ Area of the square $= a^2 = (105\sqrt{2})^2$

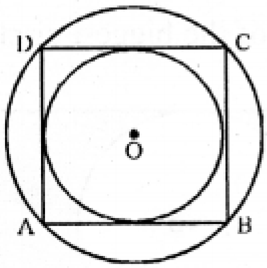
$= 105\sqrt{2} \times 105\sqrt{2}\ m^2$

$= 22050\ m^2$

Question 26.
In the adjoining figure, ABCD is a square. Find the ratio between



(i) the circumferences
(ii) the areas of the incircle and the circumcircle of the square.
Solution:



Let the side of the square = $2a$

$\therefore$ Area = $(2a)^2 = 4a^2$

and diagonal of AC = $\sqrt{2}$AB

(*i*) The radius of the circumcircle = $\frac{1}{2}$ AC

$$= \frac{1}{2}\left(\sqrt{2} \times AB\right)$$

$$= \frac{\sqrt{2}}{2} \times 2a = \sqrt{2} \cdot a$$

$\therefore$ Circumference = $2\pi r = 2 \times \pi \times \sqrt{2}a = 2\sqrt{2}\pi a$

The radius of the incircle = AB = $\frac{1}{2} \times 2a = a$

$\therefore$ Circumference = $2\pi r = 2\pi a$

Ratio between the circumference incircle and circum circle

$$= 2\pi a : 2\sqrt{2}\pi a = 1 : \sqrt{2}$$

(*ii*) Area of incircle = $\pi r^2 = \pi a^2$

Area of circumcircle = $\pi R^2 = \pi\left(\sqrt{2a}\right)^2$

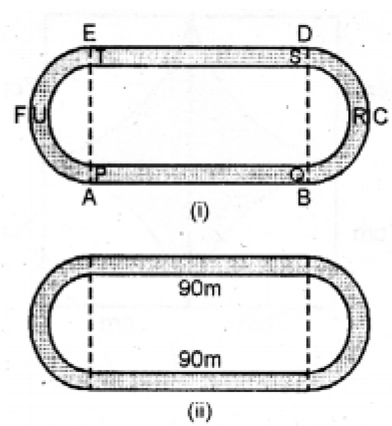
$$= \pi 2a^2 = 2\pi a^2$$

$\therefore$ Ratio = $\pi a^2 : 2\pi a^2$ = **1 : 2**

Question 27.

(a) The figure (i) given below shows a running track surrounding a grassed enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semicircular region at each end.

PQ = 200 m ; PT = 70 m.



(i) Calculate the area of the grassed enclosure in m2.
(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF of the track.
(b) In the figure (ii) given below, the inside perimeter of a practice running track with semi-circular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.
Solution:

(*a*) Length of PQ = 200 m
and width PT = 70 m

(*i*) ∴ Area of rectangle PQST = $l \times b$
$= 200 \times 70 = 14000 \text{ m}^2$

Radius of each semi-circular part on either side of the rectangle $= \frac{70}{2} = 35$ m

∴ Area of both semi-circular parts

$= 2 \times \frac{1}{2} \pi r^2 = \frac{22}{7} \times 35 \times 35 \text{ m}^2$

$= 3850 \text{ m}^2$

∴ Total area of grassed enclosure
$= 14000 + 3850 = 17850 \text{ m}^2$

(*ii*) Width of track around the enclosure = 7 m
∴ Outer length = 200 m
and width = 70 + 7 × 2
= 70 + 14 = 84 m

outer radius $= \frac{84}{2} = 42$ m

∴ Circumference of both semi-circular part

$= 2 \times \pi r = 2 \times \frac{22}{7} \times 42 = 264$ m.

Outer perimeter = 264 + 200 × 2 m
= 264 + 400 = 664 m

(*b*) Inside perimeter = 312 m
Total length of the parallel sides
= 90 + 90 = 180 m
∴ Circumference of two semi-circles
= 312 – 180 = 132 m

$\therefore$ Radius of each semi-circle

$$= \frac{132}{2\pi} = \frac{66}{3 \cdot 14} \text{ m} = 21 \cdot 02 \text{ m}$$

$$\text{Diameter} = \frac{66}{\pi} \times 2 = \frac{132}{\pi} = \frac{132}{3 \cdot 14} \text{ m}$$

$$= \frac{132 \times 100}{314} = 42 \cdot 04 \text{ m}$$

Width of track = 2 *m*

$\therefore$ Outer diameter = 42·04 + 4

= 46·04 m

$$\text{radius} = \frac{46 \cdot 04}{2} = 23 \cdot 02 \text{ m}$$

Now area of two semi-circles

$$= 2 \times \frac{1}{2} \times \pi R^2$$

$= \pi R^2 = 3 \cdot 14 \times (23 \cdot 02)^2 \text{ m}^2$

$= 3 \cdot 14 \times 23 \cdot 02 \times 23 \cdot 02 \text{ m}^2$

$= 1663 \cdot 95 \text{ m}^2$

and area of rectangle = 90 × 46·04

$= 4143 \cdot 6 \text{ m}^2$

Total area $= 1663 \cdot 95 + 4143 \cdot 60 = 5807 \cdot 55 \text{m}^2$

and area of two inner circles $= 2 \times \frac{1}{2} \pi r^2$

$= 3 \cdot 14 \times 21 \cdot 02 \times 21 \cdot 02 \text{ m}^2$

$= 1387 \cdot 38 \text{ m}^2$

and area of inner rectangle

$= 90 \times 42 \cdot 04 \text{ m}^2$

$= 3783 \cdot 6 \text{ m}^2$

Total inner area = 3783·60 + 1387·38

$= 5170 \cdot 98 \text{ m}^2$

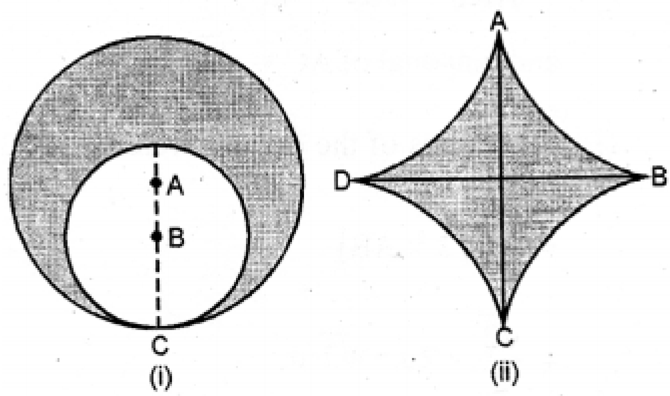
$\therefore$ Area of path = 5807·55 – 5170·98

$= 636 \cdot 57 \text{ m}^2$

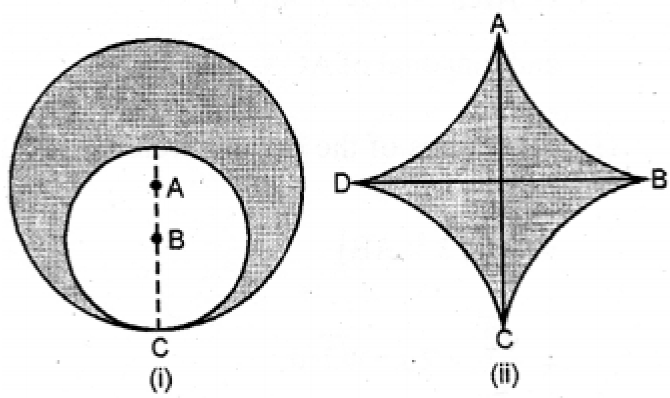
Question 28.

(a) In the figure (i) given below, two circles with centres A and B touch each other

at the point C. If AC = 8 cm and AB = 3 cm, find the area of the shaded region.
(b) The quadrants shown in the figure (ii) given below are each of radius 7 cm. Calculate the area of the shaded portion.



Solution:



(a) AC = 8 cm,

BC = AC – AB = 8 – 3 = 5 cm.

Area of big circle of radius AC = πR^2

$= \frac{22}{7} \times 8 \times 8 \text{cm}^2 = 64 \times \frac{22}{7} \text{cm}^2$

and area of smaller circle

$= \pi r^2 = \frac{22}{7} \times 5 \times 5 = \frac{25 \times 22}{7} \text{cm}^2$

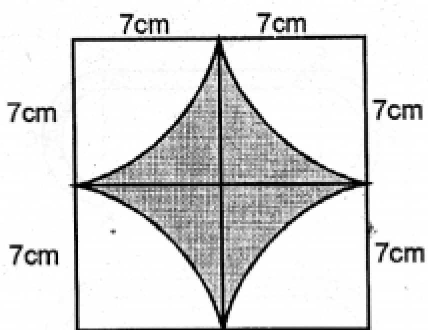
∴ Area of shaded portion

$= \frac{64 \times 22}{7} - \frac{25 \times 22}{7}$

$= \frac{22}{7}(64 - 25) \text{cm}^2 = \frac{22}{7} \times 39 \text{cm}^2$

$= 122{\cdot}57 \text{ cm}^2$

(b) Radius of each quadrant = 7 cm



(ii)

Area of shaded region

= Area of square – 4 area of the quadrant

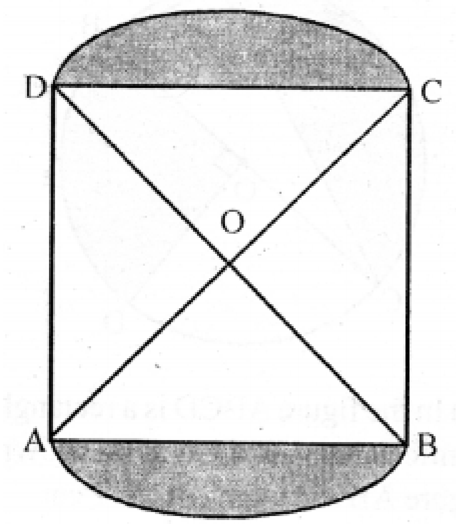
$= (\text{side})^2 - 4 \times \frac{1}{4} \pi r^2$

$= (14)^2 - \frac{22}{7} \times 7 \times 7 \text{cm}^2$

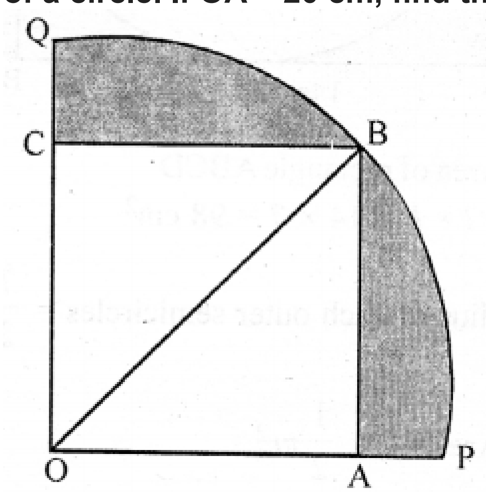
$= 196 - 154 = 42 \text{ cm}^2$

Question 29.

(a) In the figure (i) given below, two circular flower beds have been shown on the two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.



(b) In the figure (ii) given below, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)



Solution:

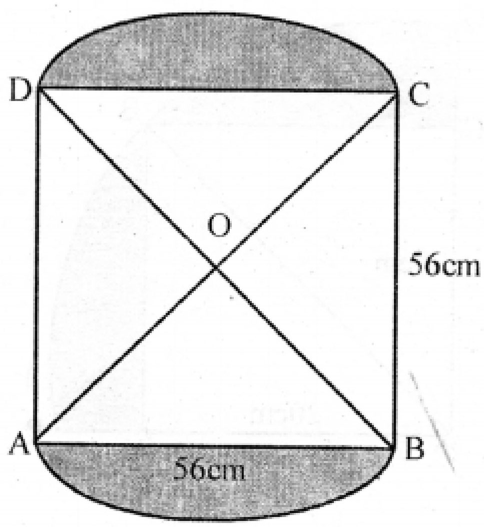
(a) Side of square lawn ABCD (a) = 56 cm.

$\therefore$ Area = $a^2 = (56)^2$

= 3136 cm^2

Length of the diagonal of the square = $\sqrt{2}\, a$

= $\sqrt{2} \times 56$ cm



$\therefore$ Radius of each quadrant = $\dfrac{\sqrt{2} \times 56}{2}$

$= 28\sqrt{2}$ cm

Area of each segment

$= \dfrac{1}{4}\pi r^2 -$ area ΔOBC

$= \dfrac{1}{4} \times \dfrac{22}{7} \times 28\sqrt{2} \times 28\sqrt{2} - \dfrac{1}{2} \times$
$28\sqrt{2} \times 28\sqrt{2}$

$= 28\sqrt{2} \times 28\sqrt{2}\left(\dfrac{1}{4} \times \dfrac{22}{7} - \dfrac{1}{2}\right)$

$= 784 \times 2\left(\dfrac{11}{14} - \dfrac{1}{2}\right)$

$= 784 \times 2 \times \dfrac{4}{14} = 448$ cm^2

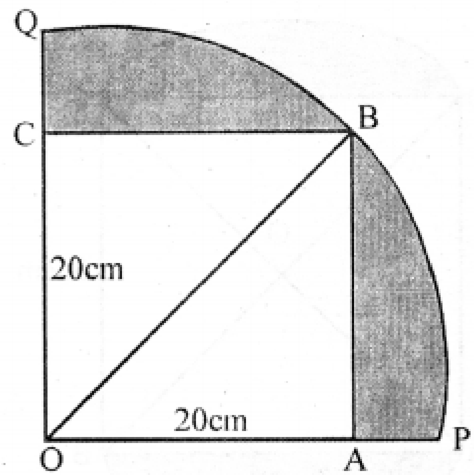
$\therefore$ Area of two segments $= 448 \times 2 = 896$ cm^2

$\therefore$ Total area of the lawn and beds

$= 3136 + 896 = 4032 \text{ cm}^2$

(b) In the figure OPBQ is a quadrant and OABC is a square which is inscribed in it side of square = 20 cm

OB is joined



$\therefore$ OB = $\sqrt{2}\,a = \sqrt{2} \times 20$ cm

$\therefore$ Radius of quadrant = OB = $20\sqrt{2}$ cm

Now, area of quadrant = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$= \frac{1}{4} \times 3.14 \times 800$ cm

$= 314 \times 2 = 628 \text{ cm}^2$

Area of square = $a^2 = (20)^2$

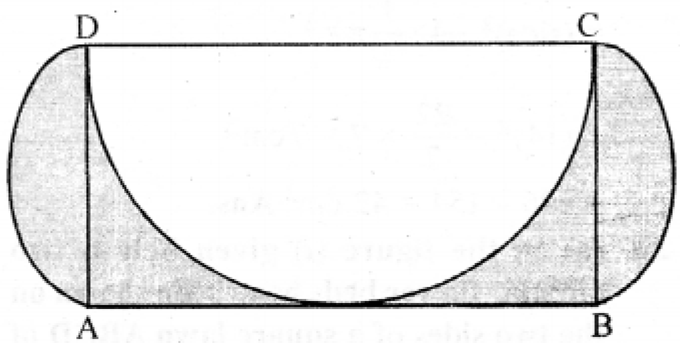
$= 400 \text{ cm}^2$

$\therefore$ Area of shaded portion = 628 – 400

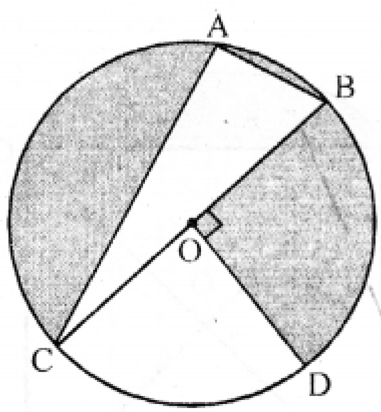
$= 228 \text{ cm}^2$

Question 30.

(a) In the figure (i) given below, ABCD is a rectangle, AB = 14 cm and BC = 7 cm. Taking DC, BC and AD as diameters, three semicircles are drawn as shown in the figure. Find the area of the shaded portion.

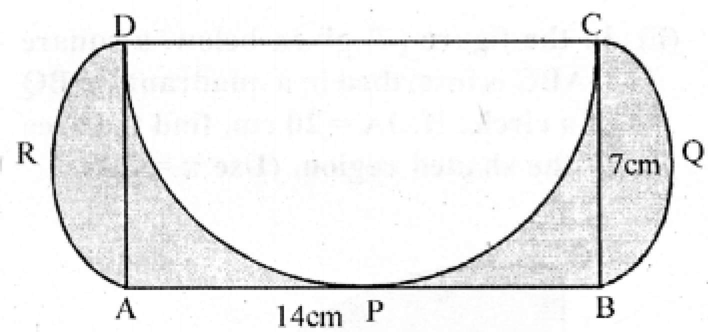


(b) In the figure (ii) given below, O is the centre of a circle with AC = 24 cm, AB = 7 cm and ∠BOD = 90°. Find the area of the shaded region. (Use π = 3.14).



Solution:

(a) In the figure ABCD is a rectangle. Three semicircles are drawn as shown in the figure AB = 14 cm, BC = 7 cm



$\therefore$ Area of rectangle ABCD

$= l \times b = 14 \times 7 = 98 \text{ cm}^2$

The radius of each outer semicircles $= \frac{7}{2}$ cm

$\therefore$ Area $= 2 \times \frac{1}{2}\pi r^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}\ cm^2$

$= 38.5\ cm^2$

Area of semicircle drawn on CD as diameter

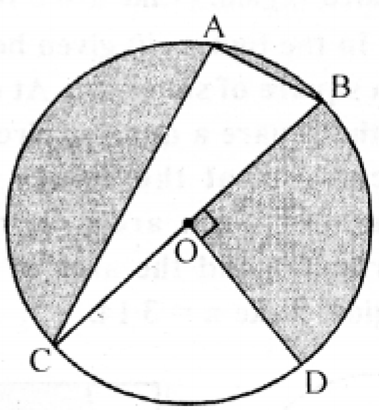
$= \frac{1}{2}\pi R^2 = \frac{1}{2} \times \frac{22}{7} \times (7)^2$

$= \frac{11}{7} \times 7 \times 7 = 77\ cm^2$

$\therefore$ Area of shaded region

$= (98 + 38.5 - 77)\ cm^2 = 59.5\ cm^2$

(b) In the given figure, AC = 24 cm, AB = 7 cm $\angle BOD = 90°$



In ΔABC,

$BC^2 = AC^2 + AB^2$ (Pythagoras Theorem)

$= 24^2 + 7^2 = \sqrt{576 + 49}$

$= \sqrt{625} = 25$ cm

$\therefore$ Radius of the circle $= \frac{25}{2}$ cm

Now area of $\Delta ABC = \frac{1}{2}$ AB × AC

$= \frac{1}{2} \times 7 \times 24$

$= 84\ cm^2$

and area of quadrant COD

$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\ cm^2$

$= \frac{1962.5}{16} = 122.66\ cm^2$

Area of circle $= \pi r^2 = 3.14 \times \frac{25}{2} \times \frac{25}{2}\ cm^2$

$= \frac{1962.5}{4} = 490.63\ cm^2$

$\therefore$ Area of shaded portion = Area of circle – (Area of ΔABC + area of quad. COD)

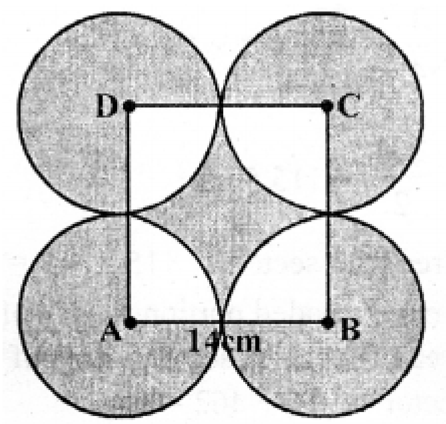
$= 490.63 - (84 + 122.66)\ cm^2$

$= 490.63 - 206.66\ cm^2$

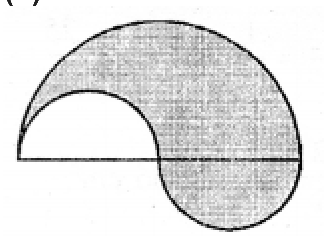
$= 283.97\ cm^2$

Question 31.

(a) In the figure given below ABCD is a square of side 14 cm. A, B, C and D are centres of the equal circle which touch externally in pairs. Find the area of the shaded region.



(b) In the figure (ii) given below, the boundary of the shaded region in the given diagram consists of three semi circular arcs, the smaller being equal. If the diameter of the larger one is 10 cm, calculate.
(i) the length of the boundary.
(ii) the area of the shaded region. (Take π to be 3.14)



Solution:

(a) Side of square ABCD = 14 cm

Radius of each circle drawn from A, B, C and D and touching externally in pairs

$= \frac{14}{2} = 7$ cm

Now area of square $= a^2 = 14 \times 14$

$= 196$ cm^2

Area of 4 sectors of 90° each $= 4 \times \pi \times r^2$

$= 4 \times \frac{22}{7} \times 7 \times 7 \times \frac{1}{4}$

$= 154$cm

Area of each sector of 270° angle $= \frac{3}{4} \pi r^2$

$= \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$ cm^2

$= \frac{231}{2} = 115.5$ cm^2

Area of 4 sectors $= 115.5 \times 4 = 462$ cm^2

$\therefore$ Area of shaded portion = Area of square + area of 4 bigger sector – area of 4 smaller sector $= 196 + 462 - 154$

$= 658 - 154 = 504$ cm^2

(*b*) Radius of big semi-circle $= \frac{10}{2} = 5$cm

and radius of each smaller circle $= \frac{5}{2}$cm

(*i*) Length of the boundary

= Circumference of bigger semi-circle + 2 circumference of smaller semi-circles

$= \pi R + \pi r + \pi r$

$= 3 \cdot 14\ (R + 2r) = 3 \cdot 14 \left(5 + 2 \times \frac{5}{2}\right)$

$= 3 \cdot 14 \times 10 = 31 \cdot 4$ cm.

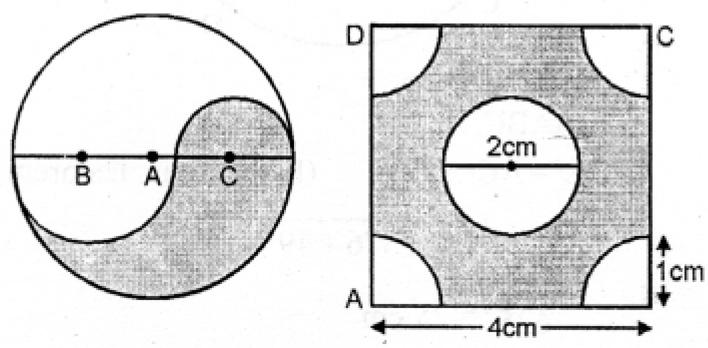
(*ii*) Area of shaded region = Area of bigger semi-circle + area of one smaller semi-circle – area of other smaller semi-circle

$= \text{area of bigger semi-circle} = \frac{1}{2} \pi R^2$

$= \frac{3 \cdot 14}{2} \times 5 \times 5 = 1 \cdot 57 \times 25 = 39 \cdot 25$ cm

Question 32.

(a) In the figure (i) given below, the points A, B and C are centres of arcs of circles of radii 5 cm, 3 cni and 2 cm respectively. Find the perimeter and the area of the shaded region. (Take π = 3.14).
(b) In the figure (ii) given below, ABCD is a square of side 4 cm. At each corner of the square a quarter circle of radius 1 cm, and at the centre a circle of diameter 2 cm are drawn. Find the perimeter and the area of the shaded region. Take π = 3.14.



(i) (ii)

Solution:

(a) Radius of bigger circle = 5 cm.

Radius of small circle (r_1) = 3 cm

and radius of smaller circle (r_2) = 2 cm

(*i*) Perimeter of the shaded region

= Circumference of bigger semi-circle

+ circumference of small semi-circle

+ circumference of smaller semi-circle

$= \pi R + \pi r_1 + \pi r_2 = \pi (R + r_1 + r_2)$

$= \pi (5 + 3 + 2) = 3.14 \times 10 = 31.4 \text{ cm}^2$

(*ii*) Area of the shaded region

= Area of bigger semi-circle + area of smaller semi-circle – area of small semicircle

$$= \frac{1}{2}\pi R^2 + \frac{1}{2}\pi r_2^2 - \frac{1}{2}\pi r_1^2$$

$$= \frac{1}{2}\pi (R^2 + r_2^2 - r_1^2)$$

$$= \frac{1}{2}\pi (5^2 + 2^2 - 3^2)$$

$$= \frac{1}{2}\pi (25 + 4 - 9) = \frac{1}{2}\pi \times 20 \text{ cm}^2$$

$= 10 \times 3.14 = 31.4 \text{ cm}^2$

(*b*) Side of square ABCD = 4 cm

Radius of each quadrant circle = 1 cm.

and radius of circle in the square

$$= \frac{2}{2} = 1 \text{ cm}$$

(*i*) Perimeter of shaded region = Circumference of four quadrants + Circumference of circle

$+ 4 \times \frac{1}{2}$ side of square.

$$= 4 \times \frac{1}{4}(2\pi r) + (2\pi r) + 4 \times 2 \text{ cm}$$

$= 2\pi r + 2\pi r + 8$ cm

$= 4\pi r + 8 = 4 \times 3{\cdot}14 \times 1 \text{ cm} + 8 \text{ cm}$

$= 12{\cdot}56 \text{ cm} + 8 \text{ cm} = 20{\cdot}56 \text{ cm}$

(*ii*) Area of shaded regions

= Area of square – area of 4 quadrants – area of circle

$$= (\text{side})^2 - 4 \times \frac{1}{4}\pi r^2 - \pi r^2$$

$= (4)^2 - \pi r^2 - \pi r^2 = 16 - 2\pi r^2 \text{cm}^2$

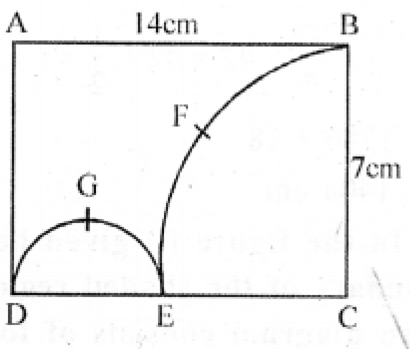
$= 16 - 2 \times 3{\cdot}14 \times (1)^2$

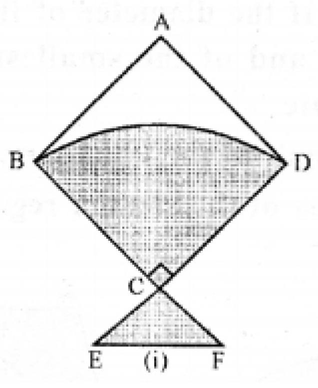
$= 16 - 6{\cdot}28 \text{ cm}^2 = 9{\cdot}72 \text{ cm}^2$

Question 33.

(a) In the figure given below, ABCD is a rectangle. AB = 14 cm, BC = 7 cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle. (Take π = 22/7)

(b) The figure (ii) given below shows a kite, in which BCD is in the shape of a quadrant of circle of radius 42 cm. ABCD is a square and Δ CEF is an isosceles right angled triangle whose equal sides are 6 cm long. Find the area of the shaded region.





Solution:

(a) Area of remaining piece.

Area of rectangle ABCD – area of semicircle DGE – area of quarter BFEC

$$= 14 \times 7 - \frac{1}{2} \times \pi\left(\frac{7}{2}\right)^2 - \frac{1}{4}\pi \times 7^2$$

$$= 14 \times 7 - \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= 98 - \frac{77}{4} - \frac{154}{4}$$

$$= 98 - 19.25 - 38.5$$

$$= 98 - 57.75 = 40.25 \text{ cm}^2$$

(b) In the figure, ABCD is a square whose side = radius of the quadrant = 42 cm

ΔCEF is an isosceles right-triangle whose, each equal side = 6cm.

Now, the area of the shaded portion = Area of the quadrant + area of isosceles right triangle

$$= \frac{1}{4}\pi r^2 + \frac{1}{2} EC \times FC$$

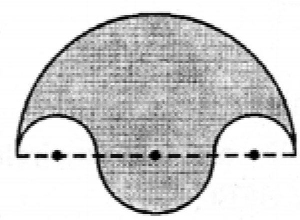
$$= \frac{1}{4} \times \frac{22}{7} \times 42 \times 42 + \frac{1}{2} \times 6 \times 6$$

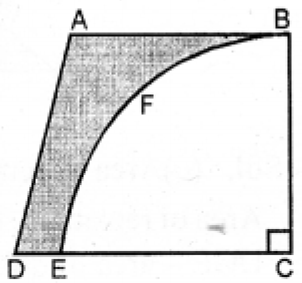
$$= 1386 + 18$$

$$= 1404 \text{ cm}^2$$

Question 34.

(a) In the figure (i) given below, the boundary of the shaded region in the given diagram consists of four semi circular arcs, the smallest two being equal. If the diameter of the largest is 14 cm and of the smallest is 3.5 cm, calculate
(i) the length of the boundary.
(ii) the area of the shaded region.



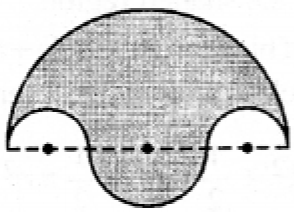


(ii)

(b) In the figure (ii) given below, a piece of cardboard, in the shape of a trapezium ABCD, and AB || DC and ∠BCD = 90°, quarter circle BFEC is removed. Given AB = BC = 3.5 cm and DE = 2 cm. Calculate the area of the remaining piece of the cardboard.

Solution:

(*a*) (*i*) Length of boundary = Circumference of bigger semi-circle



+ Circumference of small semi-circle

+ 2 × circumference of the smaller semi-circles

$$= \pi R + \pi r_1 + 2 \times \pi r_2 = \pi (R + r_1) + 2\pi r_2$$

$$= \frac{22}{7}(7 + 3{\cdot}5) + 2 \times \frac{22}{7} \times \frac{3{\cdot}5}{2}$$

$$= \frac{22}{7} \times 10{\cdot}5 + 11 = 33 + 11 = 44 \text{ cm.}$$

(*ii*) Area of shaded region = Area of bigger semicircle + area of small semicircle – 2 × area of smaller semicircles.

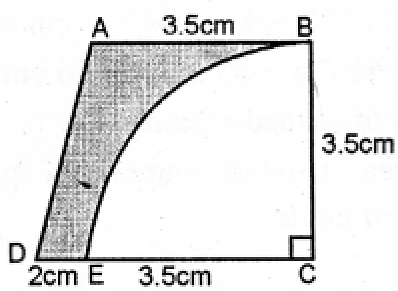
$$= \frac{1}{2}\pi(7)^2 + \frac{1}{2}\pi(3{\cdot}5)^2 - 2 \times \frac{1}{2}\pi(1{\cdot}75)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$$

$$- \frac{22}{7}(1{\cdot}75) \times (1{\cdot}75)$$

$= 77{\cdot}0 + 19{\cdot}25 - 9{\cdot}625 = 86{\cdot}625$ cm^2 **Ans.**

(*b*) ABCD is a trapezium in which



AB || DC and $\angle C = 90°$

AB = BC = 3·5 cm, DE = 2 cm

Radius of quadrant = 3·5 cm.

$$\text{Area of trapezium} = \frac{1}{2}(AB + DC) \times BC$$

$$= \frac{1}{2}(3·5 + 3·5 + 2) \times 3·5 \text{ cm}^2$$

$$= \frac{1}{2}(9 \times 3·5) = 4·5 \times 3·5 = 15·75 \text{ cm}^2$$

$$\text{Area of quadrant} = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 3·5 \times 3·5 \text{ cm}^2$$

$$= 9·625 \text{ cm}^2$$

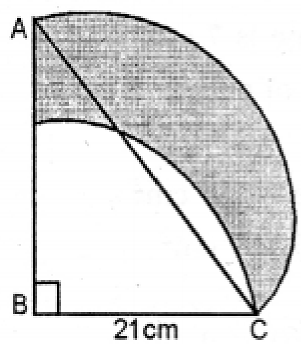
∴ Area of shaded portions

$$= 15·75 - 9·625 = 6·125 \text{ cm}^2$$

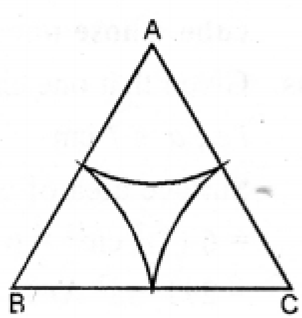
Question 35.

(a) In the figure (i) given below, ABC is a right angled triangle, ∠B = 90°, AB = 28 cm and BC = 21 cm. With AC as diameter a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region correct to two decimal places.

(b) In the figure (ii) given below, ABC is an equilateral triangle of side 8 cm. A, B and C are the centres of circular arcs of equal radius. Find the area of the shaded region correct upto 2 decimal places. (Take π = 3.142 and √3 = 1.732).



(i)



(ii)

Solution:

(a) In right Δ ABC, $\angle B = 90°$

$\therefore AC^2 = AB^2 + BC^2$

$= (28)^2 + (21)^2$

$= 784 + 441 = 1225$

$\therefore AC = \sqrt{1225} = 35$ cm.

Radius of semi-circle (R) $= \frac{35}{2}$

and radius of quadrant (r) = 21 cm

Area of shaded region

= Area of Δ ABC + area of semi-circle

– area of quadrant

$= \frac{1}{2} \times 28 \times 21 + \frac{1}{2}\pi R^2 - \frac{1}{4} r^2 \text{ cm}^2$

$= 294 \text{ cm} + \frac{1}{2} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$

$- \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$

$= 294 + \frac{1925}{4} - \frac{693}{2}$

$= 294 + 481{\cdot}25 - 346{\cdot}5 \text{ cm}^2$

$= 775.25 - 346.50 = 428{\cdot}75 \text{ cm}^2$

(*b*) Δ ABC is an equilateral triangle of side 8 cm. At A, B and C as centre three circular arcs of equal radius.

$\therefore$ Radius = $\frac{8}{4}$ = 4 cm

Now area of Δ ABC,

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(8 \times 8)\text{ cm}^2$$

$$= \frac{\sqrt{3}}{4} \times 64 = 16\sqrt{3}\text{cm}^2$$

$= 16\ (1{\cdot}732) = 27{\cdot}712\text{ cm}^2$

Area of 3 equal sectors of 60° whose radius = 4 cm

$$= 3 \times \pi r^2 \times \frac{60°}{360°}$$

$$= 3 \times 3.142 \times 4 \times 4 \times \frac{1}{6}\text{ cm}^2$$

$= 3{\cdot}142 \times 8 = 25{\cdot}136\text{ cm}^2$

$\therefore$ Area of shaded region

$= 27{\cdot}712 - 25{\cdot}136$

$= 2{\cdot}576\text{ cm}^2 = 2{\cdot}58\text{ cm}^2$

Question 36.

A circle is inscribed in a regular hexagon of side 2√3 cm. Find

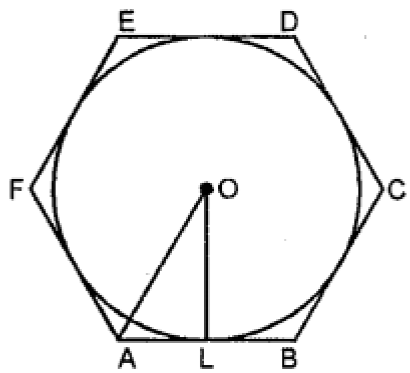
(i) the circumference of the inscribed circle

(ii) the area of the inscribed circle

Solution:

ABCDEF is a regular hexagon of side 2√3 cm. and a circle is inscribed in it with centre

O.



Radius of inscribed circle

$= \frac{\sqrt{3}}{2} \times \text{side of regular hexagon}$

$= \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3 \text{ cm}$

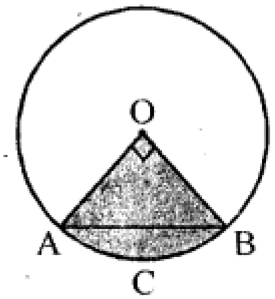
(*i*) $\therefore$ Circumference of the circle $= 2\pi r$

$= 2\pi \times 3 = \frac{6 \times 22}{7} \text{ cm} = \frac{132}{7} \text{ cm.}$

(*ii*) Area of the circle $= \pi r^2 = \pi \times 3 \times 3$

$= \frac{9 \times 22}{7} = \frac{198}{7} \text{ cm}^2$

Question 37.
In the figure (i) given below, a chord AB of a circle of radius 10 cm subtends a right angle at the centre O. Find the area of the sector OACB and of the major segment. Take π = 3.14.
Solution:



Radius of circle = 10 cm.

Angle at the centre subtended by a chord AB = 90°

$\therefore$ Area of sector OACB $= \pi r^2 \times \frac{90°}{360}$

$= 3\cdot14 \times 10 \times 10 \times \frac{90°}{360°}$

$= 314 \times \frac{1}{4} = 78\cdot5\,\text{cm}^2$

Area of $\Delta OAB = \frac{1}{2} \times 10 \times 10 = 50\ \text{cm}^2$

Area of minor segment

= Area of sector ΔACB – Area of ΔOAB

$= 78\cdot5 - 50 = 28\cdot5\ \text{cm}^2$

Area of circle $= \pi r^2$

$= 3\cdot14 \times 10 \times 10 = 314\ \text{cm}^2$

So Area of Major segment

= Area of circle – Area of minor segment

$= 314 - 28\cdot5 = 285\cdot5\ \text{cm}^2$

EXERCISE 16.4

Question 1.
Find the surface area and volume of a cube whose one edge is 7 cm.

Solution:

Given that one edge of cube = 7 cm

i.e., $a = 7$ cm

Surface area of cube = $6a^2$ cm^2

$= 6(7)^2$ cm^2 $= 6 \times 7 \times 7$ cm^2

$= 294$ cm^2

Volume of cube = $(a)^3$ cm^3

$= (7)^3$ cm^3 $= 7 \times 7 \times 7$ cm^3

$= 343$ cm^3

Question 2.

Find the surface area and the volume of a rectangular solid measuring 5 m by 4 m by 3 m. Also find the length of a diagonal.

Solution:

Ans. Given that in rectangular solid,

$l = 5$ m, $b = 4$ m and $h = 3$m

Surface area of rectangular solid = $2(lb + bh + lh)$ sq. m

$= 2(5 \times 4 + 4 \times 3 + 5 \times 3)$ sq. m

$= 2(20 + 12 + 15)$ sq. m $= 2 \times 47$ sq. m $= 94$ sq. m

Volume of rectangular solid = $l \times b \times h$ m^3

$= 5 \times 4 \times 3$ m^3 $= 60$ m^3

Length of Diagonal = $\sqrt{l^2 + b^2 + h^2}$ m

$= \sqrt{(5)^2 + (4)^2 + (3)^2}$ m

$= \sqrt{25 + 16 + 9}$ m

$= \sqrt{50}$ m $= \sqrt{25 \times 2}$ m

$= 5\sqrt{2}$ m $= 5 \times 1.414$ m

$= 7.07$ m

Hence, length of diagonal = 7.07 m

Question 3.

The length and breadth of a rectangular solid are respectively 25 cm and 20 cm. If the volume is 7000 cm3, find its height.

Solution:

Given that length of rectangular solid = 25 cm

Breadth of rectangular solid = 20 cm

Also volume of rectangular solid = 7000 cm^3

Let the height of rectangular solid = h cm

Then, volume = $l \times b \times h$

$\Rightarrow \quad 7000 = 25 \times 20 \times h$

$\Rightarrow \quad 25 \times 20 \times h = 7000$

$\Rightarrow \quad h = \frac{7000}{25 \times 20}$ cm $\quad \Rightarrow \quad h = \frac{700}{25 \times 2}$ cm

$\Rightarrow \quad h = \frac{350}{25}$ cm $\quad \Rightarrow \quad \frac{70}{5} = 14$ cm

Hence, height of rectangular solid = 14 cm

Question 4.

A class room is 10 m long, 6 m broad and 4 m high. How many students can it accommodate if one student needs 1.5 m2 of floor area ? How many cubic metres of air will each student have ?

Solution:

Length of class room (l) = 10 m

Breadth of class room (b) = 6 m

Height of class room (h) = 4 m

Floor area of class room = $l \times b$ = 10 m × 6m

= 60 m^2

one student needs 1.5 m^2 floor area

then number of students = $\frac{60m^2}{1.5m^2}$

$= \frac{60 \times 10}{15} = \frac{600}{15} = 40$ Students

Volume of class room = $l \times b \times h$

= 10 m × 6 m × 4 m = 240 m^3.

Cubic metres of air for each student

$= \frac{\text{Volume of classroom}}{\text{Number of students}}$

$= \frac{240}{40} m^3 = 6 m^3$.

Question 5.

(a) The volume of a cuboid is 1440 cm^3. Its height is 10 cm and the cross-section is a square. Find the side of the square.

(b) The perimeter of one face of a cube is 20 cm. Find the surface area and the volume of the cube.

Solution:

(a) Given that volume of cuboid = 1440 cm^3

height of cuboid = 10 cm

Volume of cuboid = Area of square × height

$\Rightarrow$ 1440 cm^3 = Area of square × 10 cm

$$\Rightarrow \text{Area of square} = \frac{1440 cm^3}{10 cm}$$

$\Rightarrow$ Area of square = 144 cm^2

$\Rightarrow$ side × side = 144 cm^2 $\Rightarrow$ side = $\sqrt{144}$ cm

$\Rightarrow$ side = 12 cm

Hence, side of square = 12 cm

(b) Given that perimeter of one face of a cube = 20 cm

We know that perimeter of one face of a cube = 4 × side

i.e. 20 = 4 × side $\Rightarrow$ 4 × side = 20

$$\Rightarrow \text{side} = \frac{20}{4} \Rightarrow \text{side} = 5 \text{ cm}$$

Area of one face = side × side = 5 cm × 5 cm = 25 cm^2

Area of 6 faces = 6 × 25 cm^2 = 150 cm^2

Volume of cube = side × side × side = 5 cm × 5 cm × 5 cm = 125 cm^3

Question 6.

Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden box covered with coloured papers with pictures of Santa Claus. She must know the exact quantity of paper to buy for this purpose. If the box has length 80 cm, breadth 40 cm and height 20 cm respectively, then how many square sheets of paper of side 40 cm would she require ?

Solution:

Length of box $(l) = 80$ cm

Breadth $(b) = 40$ cm, Height $(h) = 20$ cm

$\therefore$ Surface area of the box $= 2(lb + bh + hl)$

$= 2[80 \times 40 + 40 \times 20 + 20 \times 80]$ cm^2

$= 2[3200 + 800 + 1600]$ cm^2

$= 2 \times 5600 = 11200$ cm^2

Area of square sheet $= (\text{side})^2 = (40)^2$

$= 1600$ cm^2

$$\therefore \text{No. of sheets} = \frac{\text{Area of box}}{\text{Area of one sheet}}$$

$$= \frac{11200}{1600} = 7$$

Question 7.

The volume of a cuboid is 3600 cm^3 and its height is 12 cm. The cross-section is a rectangle whose length and breadth are in the ratio 4 :3. Find the perimeter of the cross-section.

Solution:

Given that volume of a cuboid = 3600 cm^3

Height of cuboid = 12 cm

Volume of cuboid = Area of rectangle × height

$\Rightarrow$ 3600 = Area of rectangle × 12

$\Rightarrow$ Area of rectangle × 12 = 3600

$\Rightarrow$ Area of rectangle = $\frac{3600}{12}$ cm^2

$\Rightarrow$ Area of rectangle = 300 cm^2 ....(1)

Now given that ratio of length and breadth of rectangle = 4 : 3

Let length of rectangle = $4x$

and Breadth of rectangle = $3x$

Area of rectangle = length × Breadth

Area of rectangle = $4x \times 3x$ cm^2

Area of rectangle = $12x^2$ cm^2 ....(2)

From (1) and (2), we get

$$12x^2 = 300 \Rightarrow x^2 = \frac{300}{12} \Rightarrow x^2 = 25$$

$$\Rightarrow x = \sqrt{25} \Rightarrow x = 5$$

$\therefore$ Length of rectangle = 4 × 5 cm = 20 cm

$\therefore$ Breadth of rectangle = 3 × 5 cm = 15 cm

$\therefore$ Perimeter of the cross section = $2(l + b)$

= 2 (20 + 15) cm = 2 × 35 cm = 70 cm

Question 8.

The volume of a cube is 729 cm^3. Find its surface area and the length of a diagonal.

Solution:

Given that volume of a cube = 729 cm^3

$\Rightarrow$ side × side × side = 729 cm^3

$\Rightarrow$ $(\text{side})^3$ = 729 cm^3

$\Rightarrow$ side = $\sqrt[3]{729}$ cm = $\sqrt[3]{9 \times 9 \times 9}$ cm

side = 9 cm

Surface Area of cube = 6 $(\text{side})^2$

= $6 \times (9)^2$ cm^2 = 6 × 9 × 9 cm^2 = 486 cm^2

Length of a diagonal = $\sqrt{3}$ × side

= $\sqrt{3}$ × 9 cm = 1.73 × 9 cm = 15.57 cm

Question 9.
The length of the longest rod which can be kept inside a rectangular box is 17 cm. If the inner length and breadth of the box are 12 cm and 8 cm respectively, find its inner height.
Solution:

Let the inner height = h m

Length of longest rod inside a rectangular box = 17 cm

Which same as diagonal of rectangular box

i.e. $17 = \sqrt{\ell^2 + b^2 + h^2}$

$\Rightarrow 17 = \sqrt{(12)^2 + (8)^2 + h^2}$

Squaring both sides, we get

$\Rightarrow (17)^2 = (12)^2 + (8)^2 + h^2 \Rightarrow 289 = 144 + 64 + h^2$

$\Rightarrow 289 = 208 + h^2 \Rightarrow h^2 + 208 = 289$

$\Rightarrow h^2 = 289 - 208 \Rightarrow h^2 = 81$

$\Rightarrow h = \sqrt{81} = 9$

Hence, inner height of rectangular box = 9 cm

Question 10.
A closed rectangular box has inner dimensions 90 cm by 80 cm by 70 cm. Calculate its capacity and the area of tin-foil needed to line its inner surface.
Solution:

Given that

Inner length of rectangular box = 90 cm

Inner breadth of rectangular box = 80 cm

Inner height of rectangular box = 70 cm

Capacity of rectangular box = Volume of rectangular box = $l \times b \times h$

$= 90 \text{ cm} \times 80 \text{ cm} \times 70 \text{ cm} = 504000 \text{ cm}^3$

Required area of tin foil

$= 2(lb + bh + lh) = 2(90 \times 80 + 80 \times 70 + 90 \times 70) \text{ cm}^2$

$= 2(7200 + 5600 + 6300) \text{ cm}^2 = 2 \times 19100 \text{ cm}^2 = 38200 \text{ cm}^2$

Question 11.
The internal measurements of a box are 20 cm long, 16 cm wide and 24 cm high. How many 4 cm cubes could be put into the box ?

Solution:

Volume of box = 20 cm × 16 cm × 24 cm

Volume of cubes = 4 cm × 4 cm × 4 cm

$$\text{No. of cubes put into the box} = \frac{\text{Volume of box}}{\text{Volume of cubes}}$$

$$= \frac{20\text{cm} \times 16\text{cm} \times 24\text{cm}}{4\text{cm} \times 4\text{cm} \times 4\text{cm}} = 5 \times 4 \times 6 = 120$$

Hence, 120 cubes put into the box.

Question 12.

The internal measurements of a box are 10 cm long, 8 cm wide and 7 cm high. How many cubes of side 2 cm can be put into the box ?

Solution:

Internal measurements of box are given that

Length = 10 cm, Breadth = 8 cm and height = 7 cm

3 Number of cubes of side 2 cm can be put in box.

(Because height of box is 7 cm, only 3 cubes can be put height wise)

Question 13.

A certain quantity of wood costs Rs. 250 per m3. A solid cubical block of such wood is bought for Rs. 182.25. Calculate the volume of the block and use the method of factors to find the length of one edge of the block.

Solution:

Cost of Rs. 250 for 1 m^3 wood

Cost of Rs. 1 for $\frac{1}{250}$ m^3 wood

Cost of Rs. 182.25 for $\frac{182.25}{250}$ m^3 wood

i.e. quantity of wood $= \frac{182.25}{250}$ m^3

$$= \frac{18225}{250 \times 100} m^3 = \frac{18225}{25 \times 1000} m^3$$

$$= \frac{729}{1000} m^3 = 0.729 m^3$$

i.e. Volume of given block $= 0.729 m^3$

Let length of one edge of the block $= x$ m

then, $(x)^3 = 0.729 m^3$

taking cube root on both sides,

$$x = \sqrt[3]{0.729} \text{ m} = \sqrt[3]{\frac{729}{1000}} \text{ m}$$

$$= \sqrt[3]{\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5}} \text{ m}$$

$$= \frac{3 \times 3}{2 \times 5} \text{ m} = \frac{9}{10} \text{ m} = 0.9 \text{ m}$$

Hence, length of one edge of 0.9 m.

3	729	2	1000
3	243	2	500
3	81	2	250
3	27	5	125
3	9	5	25
3	3	5	5
	1		1

Question 14.

A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base of the vessel are 15 cm x 12 cm, find the rise in the water level in centimetres correct to 2 decimal places, assuming that no water over flows.

Solution:

Given that edge of cube = 11 cm

Volume of cube = $(\text{edge})^3$

$= (11 \text{ cm})^3 = 11 \text{ cm} \times 11 \text{ cm} \times 11 \text{ cm}$

$= 11 \text{ cm} \times 11 \text{ cm} \times 11 \text{ cm} = 1331 \text{ cm}^3$

Given dimensions of the base of the vessel are $15 \text{ cm} \times 12 \text{ cm}$

Let the rise in the water level = h cm

Then, volume of cube = volume of vessel.

$1331 \text{ cm}^3 = 15 \text{ cm} \times 12 \text{ cm} \times h \text{ cm}$

$\Rightarrow 15 \times 12 \times h = 1331$

$$\Rightarrow h = \frac{1331}{15 \times 12} \text{ cm} = \frac{1331}{180} \text{ cm} = 7.39 \text{ cm}$$

Hence, the rise in the water level = 7.39 cm

Question 15.

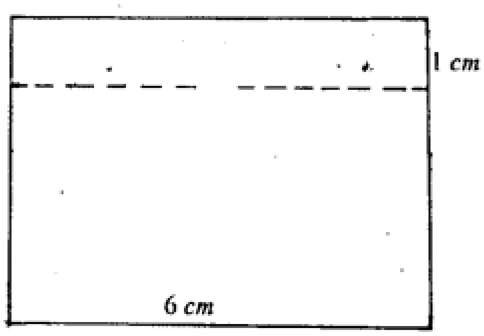
A rectangular container, whose base is a square of side 6 cm, stands on a horizontal table and holds water upto 1 cm from the top. When a cube is placed in the water and is completely submerged, the water rises to the top and 2 cm3 of water over flows.. Calculate the volume of the cube.

Solution:

Base of rectangular container is a square

$\therefore \quad l = 6 \text{ cm}, b = 6 \text{ cm}$

When a cube is placed in it, water rises to top *i.e.* through height 1 cm and also 2 cm^3 of water overflows.



$\therefore$ Volume of cube = Volume of water displaced

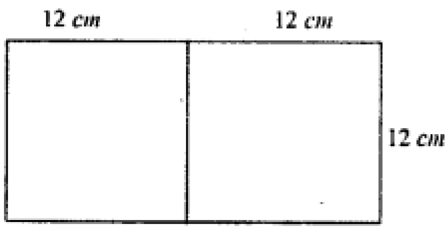
$= 6 \times 6 \times 1 + 2 = 36 + 2 = 38 \text{ cm}^3$

Question 16.

(a) Two cubes, each with 12 cm edge, are joined end to end. Find the surface area of the resulting cuboid,

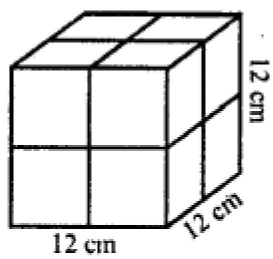
(b) A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube ? Also, find the ratio between the surface area of the original cube and the sum of the surface areas of the new cubes.
Solution:

(a) On joining two cubes end to end a cuboid is formed whose dimensions are
$l = 12 + 12 = 24$ cm, $b = 12$ cm, $h = 12$ cm



Total surface area of cuboid
$= 2(lb + bh + hl)$
$= 2(24\text{ cm} \times 12\text{ cm} + 12\text{ cm} \times 12\text{ cm} + 12\text{ cm} \times 24\text{ cm})$
$= 2(288 + 144 + 288)\text{ cm}^2 = 2 \times 720\text{ cm}^2 = 1440\text{ cm}^2$

(b) Side of a cube = 12 cm
$\therefore$ Volume $= (\text{Side})^3 = (12)^3$
$= 1728\text{ cm}^3$



Cutting it into 8 equal cubes, then

Volume of each cube $= \frac{1728}{8} = 216\text{ cm}^3$

$\therefore$ Side $= \sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6}$ cm $= 6$ cm

Now surface area of original cube $= 6 \times (\text{side})^2$
$= 6 \times (12)^2 = 6 \times 144\text{ cm}^2 = 864\text{ cm}^2$
and surface area of one smaller cube
$= 6 \times (6)^2 = 6 \times 36 = 216\text{ cm}^2$
and surface area of 8 cube $= 216 \times 8\text{ cm}^2 = 1728\text{ cm}^2$
Now ratio between their areas $= 864 : 1728 = 1 : 2$

Question 17.
A cube of a metal of 6 cm edge is melted and cast into a cuboid whose base is 9 cm x g cm. Find the height of the cuboid.
Solution:

Given that edge of melted cube = 6 cm

Volume of melted cube = $6\text{ cm} \times 6\text{ cm} \times 6\text{ cm} = 216\text{ cm}^3$

Given that dimension of cuboid

Length = 9 cm, Breadth = 8 cm

Let height = h cm

Volume of cuboid = $l \times b \times h$

$= 9\text{ cm} \times 8\text{ cm} \times h\text{ cm} = 72h\text{ cm}^3$

Now, volume of cuboid = Volume of melted metal cube

$$\Rightarrow \quad 72h = 216 \quad \Rightarrow \quad h = \frac{216}{72}\text{ cm} \quad \Rightarrow \quad h = 3\text{ cm}$$

Hence, height of cuboid = 3 cm.

Question 18.

The area of a playground is 4800 m2. Find the cost of covering it with gravel 1 cm deep, if the gravel costs Rs. 260 per cubic metre.

Solution:

Area of playground = 4800 m^2

i.e. $l \times b = 4800\text{ m}^2$

Depth of level = 1cm, *i.e.* $h = 1\text{ cm} = \frac{1}{100}\text{ m}$

Volume of gravel = $l \times b \times h = 4800 \times \frac{1}{100}\text{ m}^3 = 48\text{ m}^3$

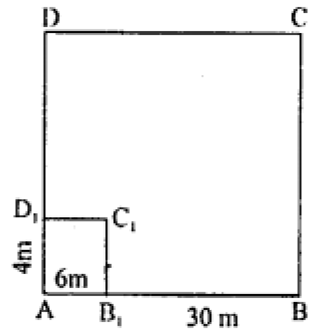
Cost = Rs. 260 per cubic meter

$\therefore$ Total cost = Rs. 260 × 48 = Rs. 12480

Question 19.

A field is 30 m long and 18 m broad. A pit 6 m long, 4m wide and 3 m deep is dug out from the middle of the field and the earth removed is evenly spread over the remaining area of the field. Find the rise in the level of the remaining part of the field in centimetres correct to two decimal places.

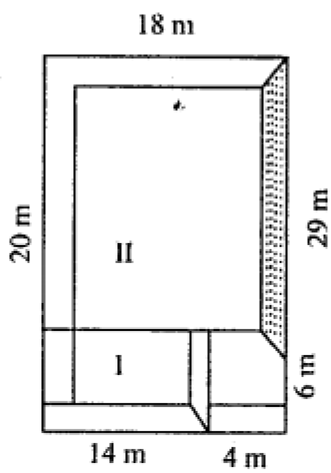
Solution:



Let ABCD be a field.
Let $ABCD_1$ be the part of the field where a pit is dug.
Volume of the earth dug out
$= 6\text{ m} \times 4\text{ m} \times 3\text{ m} = 72\text{ m}^3$
Let h m be the level raised over the field uniformly.
Divide the raised level of the field into parts I and II
Volume of part I $= 14\text{ m} \times 6\text{ m} \times h\text{ m} = 84h\text{ m}^3$
Volume of part II $= 24\text{ m} \times 18\text{ m} \times h\text{ m} = 432h\text{ m}^3$
Total volume of part I and II
$= [(84h) + (432h)]\text{ m}^3$
$= (516h)\text{ m}^3$
Hence, $516h\text{ m}^3$ = Volume of the earth dug out
$\Rightarrow \quad 516h = 72$

$\Rightarrow \quad h = \frac{72}{516}\text{ m} = 0.1395\text{ m}$

$= 0.1395 \times 100\text{ cm} = 13.95\text{ cm}$
Hence, the level has been raised by 13.95 cm (correct up) to 2 decimal places.



Question 20.
A rectangular plot is 24 m long and 20 m wide. A cubical pit of edge 4 m is dug at each of the four corners of the field and the soil removed is evenly spread over the remaining part of the plot. By what height does the remaining plot get raised?

Solution:

Length of the plot $(l) = 24\ m$

and width $(b) = 20\ m$

$\therefore$ Area of the plot $= l \times b = 24\ m \times 20\ m = 480\ m^2$

Side of cubical pit = 4m

$\therefore$ Volume of each pit $= (4)^3 = 64\ m^3$

and volume of 4 pits at the corners

$= 4 \times 64 = 256\ cm^3$

and area of the surface of 4 pits

$= 4 \times (a)^2 = 4 \times (4)^2 = 64\ m^2$

Area of remaining plot $= 480 - 64 = 416\ m^2$

$\therefore$ Hieght of the soil spread over the remaining plot

$$= \frac{256}{416} m = \frac{8}{13} m$$

Question 21.

The inner dimensions of a closed wooden box are 2 m, 1.2 m and .75 m. The thickness of the wood is 2.5 cm. Find the cost of wood required to make the box if 1 m^3 of wood costs Rs. 5400.

Solution:

Inner dimensions of wooden box are

2 m, 1.2 m, 0.75 m

Thickness of the wood = 2.5 cm

$$= \frac{25}{10}\ cm = \frac{25}{10} \times \frac{1}{100}\ m$$

$$= \frac{1}{10} \times \frac{1}{4}\ m = \frac{1}{40}\ m = 0.025\ m.$$

External dimensions of wooden box are

$(2 + 2 \times 0.025), (1.2 + 2 \times 0.025), (0.75 + 2 \times 0.025)$

$= (2 + 0.05), (1.2 + 0.05), (0.75 + 0.5) = 2.05, 1.25, 0.80$

Volume of solid = External volume of box – Internal volume of box

$= 2.05 \times 1.25 \times 0.80\ m^3 - 2 \times 1.2 \times 0.75\ m^3$

$= 2.05 - 1.80 = 0.25\ m^3$

Cost = Rs. 5400 for 1 m^3

Total cost = Rs. 5400×0.25 = Rs. $5400 \times \frac{25}{100}$

= Rs. 54×25 = Rs. 1350

Question 22.

A cubical wooden box of internal edge 1 mis made of 5 cm thick wood. The box is open at the top. If the wood costs Rs. 9600 per cubic metre, find the cost of the wood required to make the box.

Solution:

Internal edge of cbical wooden box = 1 m.

Thickness of wood = 5 cm.

$\therefore$ External length = 1 m + 10 cm = 1·1 m.

breadth = 1 m + 10 m = 1·1 m

and height = 1 m + 5 cm = 1·05 m.

Now the volume of the wood used = outer volume – inner volume

$= 1 \cdot 1 \times 1 \cdot 1 \times 1 \cdot 05 \text{ m}^3 - 1 \times 1 \times 1 \text{ m}^3$

$= 1 \cdot 2705 - 1 \cdot 0000 = 0 \cdot 2705 \text{ m}^3$

Cost of 1 m^3 = Rs. 9600

$\therefore$ Cost of 0·2705 m^3 = Rs. 9600 × 0·2705

= Rs. 2596·80

Question 23.

A square brass plate of side x cm is 1mm thick and weighs 4725 g. If one. cc of brass weighs 8.4 gm, find the value of x.

Solution:

Side of square brass plate = x cm

i.e. $l = x$ cm, $b = x$ cm

Thickness of plate = 1 mm = $\frac{1}{10}$ cm

Volume of the plate = $l \times b \times h$

$= x \times x \times \frac{1}{10}$ cm^3 = $\frac{x^2}{10}$ cm^3 ....(1)

Now, 8.4 gm weight brass having volume = 1 cc

1 gm weight brass having volume = $\frac{1}{8.4}$ cc

4725 gm weight brass having volume = $4725 \times \frac{1}{8.4}$

cc = 562.5 cc

i.e. Volume of plate = 562.5 cc = 562.5 cm^3 ....(2)

From (1) and (2),

$\frac{x^2}{10} = 562.5 \Rightarrow x^2 = 562.5 \times 10 \Rightarrow x^2 = 5625$

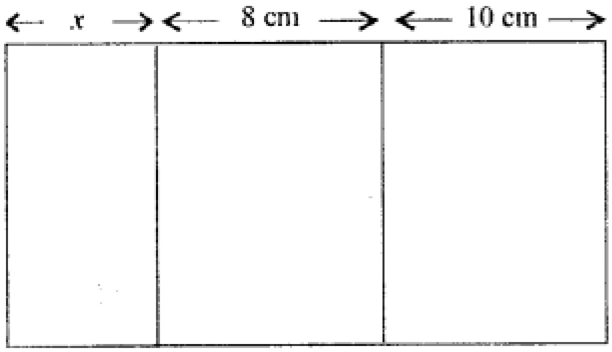
$\Rightarrow x = \sqrt{5625} \Rightarrow x = 75$ cm

Hence, the value of $x = 75$ cm

Question 24.

Three cubes whose edges are x cm, 8 cm and 10 cm respectively are melted and recast into a single cube of edge 12 cm. Find x.

Solution:



Edges of three cubes are x cm, 8 cm, 10 cm

Volume of these cubes are $(x)^3$ $(8)^3$ and $(10)^3$

i.e. x^3, 512 cm^3 and 1000 cm^3.

Edge of new cube formed = 12 cm

Volume of new cube = $(12)^3 = 1728$ cm^3

According to question,

$x^3 + 512 + 1000 = 1728$

$\Rightarrow \quad x^3 + 1512 = 1728 \quad \Rightarrow \quad x^3 = 216$

$\Rightarrow \quad x^3 = 6 \times 6 \times 6 \quad \Rightarrow \quad x^3 = 6 \times 6 \times 6$

$\Rightarrow \quad x = 6$ cm

Question 25.

The area of cross-section of a pipe is 3.5 cm2 and water is flowing out of pipe at the rate of 40 cm/s. How much water is delivered by the pipe in one minute ?

Solution:

Area of cross-section of pipe = 3.5 cm^2

Speed of water = 40 cm/sec.

Length of water column in 1 sec = 40 cm

$\therefore$ Volume of water flowing in 1 second

= Area of cross-section × length

$= 3.5 \times 40 = 35 \times 4 = 140$ cm^3

volume of water flowing in 1 minute *i.e.* 60 sec.

$= 140 \times 60$ cm^3

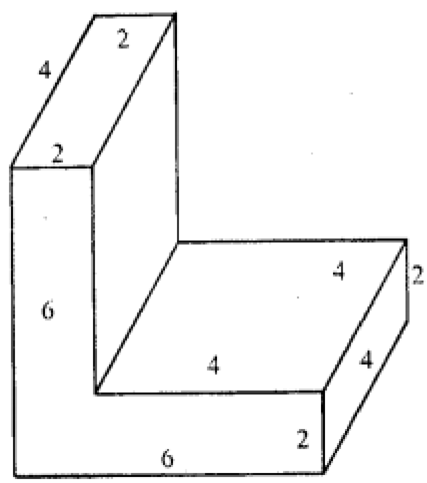
But 1 litre = 1000 cm^3

$\therefore$ Volume $= \frac{140 \times 60}{1000}$ litres

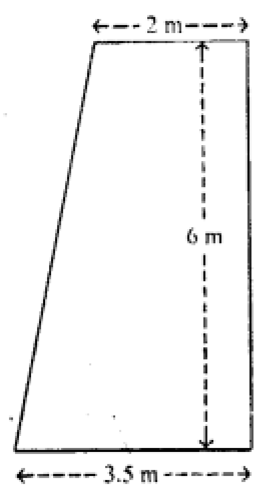
$= \frac{14 \times 6}{10}$ litres $= \frac{84}{10}$ litres $= 8.4$ litres.

Question 26.

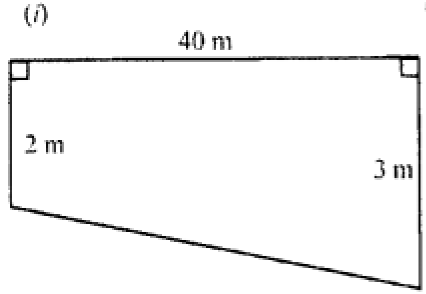
(a) The figure (i) given below shows a solid of uniform cross-section. Find the volume of the solid. All measurements are in cm and all angles in the figure are right angles.
(b) The figure (ii) given below shows the cross section of a concrete wall to be constructed. It is 2 m wide at the top, 3.5 m wide at the bottom and its height is 6 m, and its length is 400 m. Calculate (i) The cross-sectional area, and (ii) volume of concrete in the wall.
(c) The figure (iii) given below show the cross section of a swimming pool 10 m broad, 2 m deep at one end and 3 m deep at the other end. Calculate the volume of water it will hold when full, given that its length is 40 m.



(i)



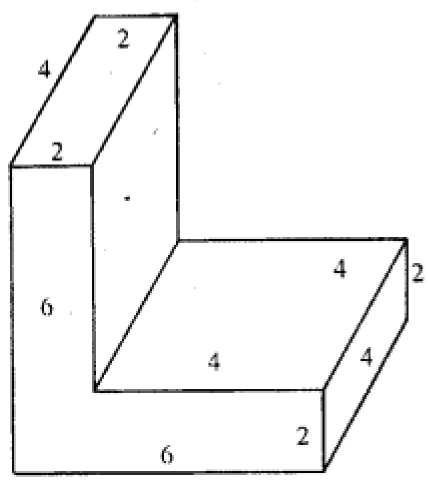
(ii)



(iii)

Solution:

(*a*) The given figure can be divided into two cuboids of dimensions. 4 cm, 4 cm, 2 cm and 4 cm, 2 cm, 6 cm respectively.



(*i*)

Hence, volume of solid = 4 cm × 4 cm × 2 cm + 4 cm × 2 cm × 6 cm = 32 cm³ + 48 cm³ = 80 cm³

(*b*) From figure (*ii*) It is clear that it is trapezium with parallel sides 2 m and 3.5 m.

(*i*) Area of cross section

$= \frac{1}{2}$ (sum of || sides) × height

$= \frac{1}{2}(2\text{ m} + 3.5\text{ m}) \times 6\text{ m} = \frac{1}{2} \times 5.5\text{ m} \times 6\text{ m}$

$= 5.5\text{ m} \times 3\text{ m} = 16.5\text{ m}^2$

(*ii*) Volume of concrete in the wall = Area of cross section × length

$= 16.5\text{ m}^2 \times 400\text{ m} = 16.5 \times 400\text{ m}^3 = 165 \times 40\text{ m}^3$

$= 6600\text{ m}^3$

(*c*) From figure (*iii*) It is clear that it is trapezium with parallel sides 2 m and 3 m.

Area of cross section $= \frac{1}{2}$ (sum of || sides) × height

$= \frac{1}{2}(2\text{ m} + 3\text{ m}) \times 10\text{ m} = \frac{1}{2} \times 5\text{ m} \times 10\text{ m}$

$= 5\text{ m} \times 5\text{ m} = 25\text{ m}^2$

Volume of water it full hold

when full = area of cross section × height

$= 25\text{ m}^2 \times 40\text{ m} = 1000\text{ m}^3$

Question 27.

A swimming pool is 50 metres long and 15 metres wide. Its shallow and deep ends arc $1\frac{1}{2}$ metres and $14\frac{1}{2}$metres deep respectively. If the bottom of the pool slopes uniformly, find the amount of water required to fill the pool.

Solution:

Given swimming pool length = 50 m and Width = 15 m

Its shallow and deep ends are $1\frac{1}{2}$ m and $5\frac{1}{2}$ m deep respectively

Area of cross section of swimming pool = $\frac{1}{2}$ (sum of || sides) × width

$$= \frac{1}{2} \times (1\frac{1}{2} m + 4\frac{1}{2} m) \times 15 m$$

$$= \frac{1}{2} \times \left(\frac{3}{2} m + \frac{9}{2} m\right) \times 15 m$$

$$= \frac{1}{2} \times \left(\frac{3+9}{2}\right) m \times 15 m = \frac{1}{2} \times \frac{12}{2} \times 15 m^2$$

$$= \frac{1}{2} \times 6 \times 15 m^2 = 3 \times 15 m^2 = 45 m^2$$

Amount of water required to fill pool
= Area of cross section × length
$= 45 m^2 \times 50 m = 2250 m^3$.

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 24):

Question 1.

Area of a triangle is 30 cm^2. If its base is 10 cm, then its height is

(a) 5 cm
(b) 6 cm
(c) 7 cm
(d) 8 cm

Solution:

Area of a triangle = 30 cm^2
Base = 10 cm

$$\therefore \text{Height} = \frac{\text{Area} \times 2}{\text{Base}} = \frac{30 \times 2}{10} = 6 \text{ cm} \quad \text{(b)}$$

Question 2.

If the perimeter of a square is 80 cm, then its area is

(a) 800 cm^2
(b) 600 cm^2
(c) 400 cm^2
(d) 200 cm^2
Solution:
Perimeter of a square = 80 cm

$$\therefore \text{Side} = \frac{P}{4} = \frac{80}{4} = 20 \text{ cm}$$

$$\therefore \text{Area} = (\text{side})^2 = 20 \times 20 = 400 \text{ cm}^2 \qquad \text{(c)}$$

Question 3.
Area of a parallelogram is 48 cm^2. If its height is 6 cm then its base is
(a) 8 cm
(b) 4 cm
(c) 16 cm
(d) None of these
Solution:
Area of parallelogram = 48 cm^2
Height = 6 cm

$$\therefore \text{Base} = \frac{\text{Area}}{\text{Height}} = \frac{48}{6} = 8 \text{ cm} \qquad \text{(a)}$$

Question 4.
If d is the diameter of a circle, then its area is

(a) πd^2 (b) $\frac{\pi d^2}{2}$

(c) $\frac{\pi d^2}{4}$ (d) $2\pi d^2$

Solution:
Diameter of circle = d

$$\therefore \text{area} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$$

$$= \frac{\pi d^2}{4} \qquad \text{(c)}$$

Question 5.
If the area of a trapezium is 64 cm^2 and the distance between parallel sides is 8

cm, then sum of its parallel sides is
(a) 8 cm
(b) 4 cm
(c) 32 cm
(d) 16 cm
Solution:

Area of trapezium = 64 cm^2
Distance between parallel (h) = 8 cm

$$\therefore \text{ Sum of its parallel sides} = \frac{\text{Area} \times 2}{h}$$

$$= \frac{64 \times 2}{8} = 16 \text{ cm} \qquad \text{(d)}$$

Question 6.
Area of a rhombus whose diagonals are 8 cm and 6 cm is
(a) 48 cm^2
(b) 24 cm^2
(c) 12 cm^2
(d) 96 cm^2
Solution:

$$\text{Area of rhombus} = \frac{d_1 \times d_2}{2}$$

$$= \frac{8 \times 6}{2} = 24 \text{ cm}^2 \qquad \text{(b)}$$

Question 7.
If the lengths of diagonals of a rhombus is doubled, then area of rhombus will be
(a) doubled
(b) tripled
(c) four times
(d) remains same
Solution:

Let d_1, d_2 be the diagonals of a rhombus

$$\text{Then area} = \frac{d_1 \times d_2}{2}$$

If diagonals are doubled, then

$$\text{Area} = \frac{2d_1 \times 2d_2}{2} = 4\frac{d_1\ d_2}{2}$$

= four times (c)

Question 8.

If the length of a diagonal of a quadrilateral is 10 cm and lengths of the perpendiculars on it from opposite vertices are 4 cm and 6 cm, then area of quadrilateral is

(a) 100 cm^2
(b) 200 cm^2
(c) 50 cm^2
(d) None of these

Solution:

Length of diagonal of a quadrilateral = 10cm

Length of perpendicular on it from opposite vertices are 4 cm and 6 cm

$\therefore$ Area $= \frac{1}{2} \times 10 \times (4 + 6)$ cm^2

$= 5 \times 10 = 50$ cm^2 (c)

Question 9.

Area of a rhombus is 90 cm2. If the length of one diagonal is 10 cm then the length of other diagonal is

(a) 18 cm
(b) 9 cm
(c) 36 cm
(d) 4.5 cm

Solution:

Area of rhombus = 90 cm^2

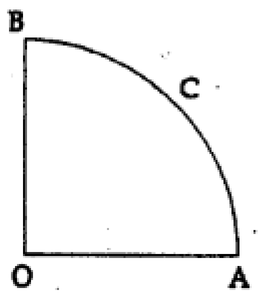
One diagonal (d_1) = 10 cm

Then $d_2 = \frac{\text{Area} \times 2}{d_1} = \frac{90 \times 2}{10} = 18$ cm (a)

Question 10.

In the given figure, OACB is a quadrant of a circle of radius 7 cm. The perimeter of the quadrant is

(a) 11 cm
(b) 18 cm
(c) 25 cm
(d) 36 cm



Solution:

OACB is a quadrant of a circle with radius 7 cm

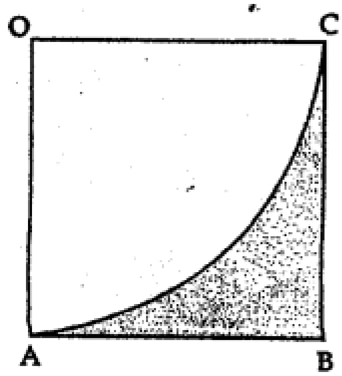
$\therefore$ Perimeter of the quadrant $= r + r + \frac{1}{4} \times 2\pi r$

$= 2r + \frac{1}{2}\pi r = 2 \times 7 + \frac{1}{2} \times \frac{22}{7} \times 7$

$= 14 + 11 = 25$ cm (c)

Question 11.

In the given figure, OABC is a square of side 7 cm. OAC is a quadrant of a circle with O as centre. The area of the shaded region is



(a) 10.5 cm²
(b) 38.5 cm
(c) 49 cm²
(d) 11.5 cm²

Solution:

OABC is a square with side 7 cm. OAC is a quadrant.

Area of shaded portion = area of square – area of quadrant

$= (7)^2 - \frac{1}{4}\pi r^2$

$= (7)^2 - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= 49 - \frac{77}{2} = 49 - 38.5 = 10.5 \text{ cm}^2$ (a)

Question 12.

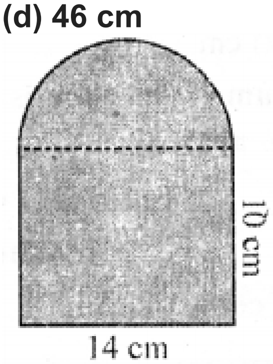
The given figure shows a rectangle and a semicircle. The perimeter of the shaded region is

(a) 70 cm

(b) 56 cm

(c) 78 cm

(d) 46 cm

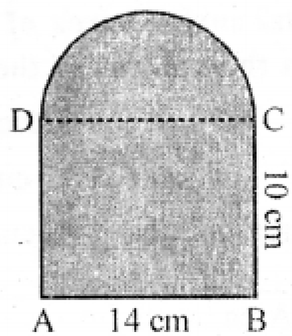


Solution:

In the figure, ABCD is a rectangle and a semicircle is drawn on its side CD as diameter = 14 cm

Length of rectangle = 14 cm

and breadth = 10 cm



∴ Perimeter of the shaded region

$= DA + AB + BC + \pi r$

(Radius of semicircle $= \frac{14}{2} = 7$ cm)

$= 10 + 14 + 10 + \frac{22}{7} \times \frac{14}{2} = 56$ cm (b)

Question 13.

The area of the shaded region shown in Q. 12 (above is

(a) 140 cm^2

(b) 77 cm^2

(c) 294 cm^2

(d) 217 cm^2

Solution:

Area of shaded portion of the figure of question (12)

= Area of rectangle + area of semicircle

$= l \times b + \frac{1}{2}\pi r^2 = 14 \times 10 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 14 \times 10 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$

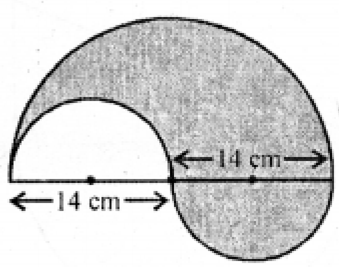
(Radius of semicircle = 7 cm)

$= 140 + 77 = 217 \text{ cm}^2$ (d)

Question 14.

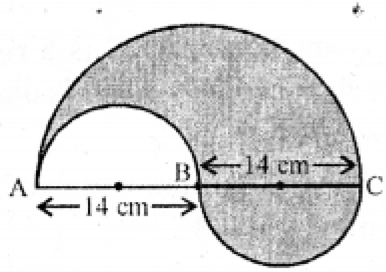
In the given figure, the boundary of the shaded region consists of semicircular arcs. The area of the shaded region is equal to

(a) 616 cm²
(b) 385 cm²
(c) 231 cm²
(d) 308 cm²



Solution:

Area of shaded portion



= Area of semicircle of 14 cm radius – area of semicircle of 7 cm radius + area of semicircle of 7 cm radius

= Area of semicircle of radius 14 cm

$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= 308 \text{ cm}^2$ (d)

Question 15.

The perimeter of the shaded region shown in Q. 14 (above) is
(a) 44 cm
(b) 88 cm
(c) 66 cm
(d) 132 cm

Solution:

Perimeter of shaded portion of the figure given in Q. 14

= Perimeter of bigger semicircle + perimeter of two small semicircles

$= \pi r + 2 \times \pi r$

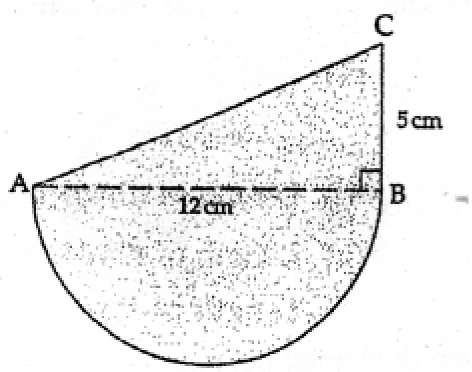
$= \frac{22}{7} \times 14 + 2 \times \frac{22}{7} \times 7 \text{ cm}$

$= 44 + 44 = 88 \text{ cm}$ (b)

Question 16.

In the given figure, ABC is a right angled triangle at B. A semicircle is drawn on AB as diameter. If AB = 12 cm and BC = 5 cm, then the area of the shaded region is
(a) $(60 + 18\pi)$ cm^2
(b) $(30 + 36\pi)$ cm^2
(c) $(30+18\pi)$ cm^2

(d) $(30 + 9\pi)\ cm^2$



Solution:

In the given figure, ABC is a right angled triangle right angle at B, AB is diameter = 12 cm, BC = 5 cm

Area of shaded portion = area of semicircle + area of right ΔABC

$$= \frac{1}{2}\pi r^2 + \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2}\pi(6)^2 + \frac{1}{2} \times 12 \times 5\ cm^2$$

$$= (18\pi + 30)\ cm^2 \qquad (c)$$

Question 17.

The perimeter of the shaded region shown in Q. 16 (above) is

(a) $(30 + 6\pi)$ cm

(b) $(30 + 12\pi)$ cm

(c) $(18 + 12\pi)$ cm

(d) $(18 + 6\pi)$ cm

Solution:

Perimeter of the shaded region in Q. 16

$$= \frac{1}{2} \times 2\pi r + 5 \text{ cm} + AC$$

But $AC = \sqrt{AB^2 + BC^2}$ (Pythagoras Theorem)

$$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25} \text{ cm}$$

$$= \sqrt{169} = 13 \text{ cm}$$

$\therefore$ Perimeter $= \pi \times 6 + 13$ cm

$= (18 + 6\pi)$ cm (d)

Question 18.

If the volume of a cube is 729 m^3, then its surface area is

(a) 486 cm^2

(b) 324 cm^2

(c) 162 cm^2

(d) None of these

Solution:

Volume of cube $= 729 \text{ m}^3$

$\therefore$ Side $= \sqrt[3]{\text{Volume}} = \sqrt[3]{729} = \sqrt[3]{9^3}$

$$= 9^{3 \times \frac{1}{3}} = 9 \text{ m}$$

$\therefore$ Surface area $= 6(9)^2 = 6 \times 81$

$= 486 \text{ m}^2$ (a)

Question 19.

If the total surface area of a cube is 96 cm^2, then the volume of the cube is

(a) 8 cm^3

(b) 512 cm^3

(c) 64 cm^3

(d) 27 cm^3

Solution:

Surface area of cube = 96 cm^2

$\therefore$ Side = $\sqrt{\frac{\text{Area}}{6}}$

$\sqrt{\frac{96}{6}} = \sqrt{16} = 4$ cm

$\therefore$ Volume = (side)^3 = (4)^3 cm^3

= 4 × 4 × 4 = 64 cm^3 (c)

Question 20.

The length of the longest pole that can be put in a room of dimensions (10 m x 10 m x 5 m) is

(a) 15 m
(b) 16 m
(c) 10 m
(d) 12 m

Solution:

Longest pole in a room = $\sqrt{l^2 + b^2 + h^2}$

$= \sqrt{10^2 + 10^2 + 5^2} = \sqrt{100 + 100 + 25}$

$= \sqrt{225} = 15$ m (a)

Question 21.

The lateral surface area of a cube is 256 m^2. The volume of the cube is

(a) 512 m^3
(b) 64 m^3
(c) 216 m^3
(d) 256 m^3

Solution:

Lateral surface area of a cube = 256 m^2

$\therefore$ Side = $\sqrt{\frac{\text{Area}}{4}} = \sqrt{\frac{256}{4}} = \sqrt{64} = 8$ m

$\therefore$ Volume of cube = (side)^3 = (8)^3 m^3

= 8 × 8 × 8 = 512 m^3

Question 22.

If the perimeter of one face of a cube is 40 cm, then the sum of lengths of its edge

is
(a) 80 cm
(b) 120 cm
(c) 160 cm
(d) 240 cm

Solution:

Perimeter of one face of cube = 40 cm

$\therefore$ Side = $\frac{40}{4}$ = 10 cm

No. of edges = 12

$\therefore$ Sum of its edges = 12 × 10 = 120 cm (b)

Question 23.

A cuboid container has the capacity to hold 50 small boxes. If all the dimensions of the container are doubled, then it can hold (small boxes of same size)
(a) 100 boxes
(b) 200 boxes
(c) 400 boxes
(d) 800 boxes

Solution:

In a cuboid containes, number of boxes = 50

If the dimensions of the container are doubled then its volume 2 × 2 × 2 = 8 times

$\therefore$ Number of boxes in it will be = 50 × 8
= 400 boxes (c)

Question 24.

The number of planks of dimensions (4 m x 50 cm x 20 cm) that can be stored in a pit which is 16 m long, 12 m wide and 4 m deep is
(a) 1900
(b) 1920
(c) 1800
(d) 1840

Solution:

Size of one planks = 4 m × 50 cm × 20 cm

$= 4\text{ m} \times \frac{1}{2} \times \frac{1}{5}\text{ m}^3$

$= \frac{2}{5}\text{ m}^3$

Volume of the pit = 16 × 12 × 4 m

$= 768\text{ m}^3$

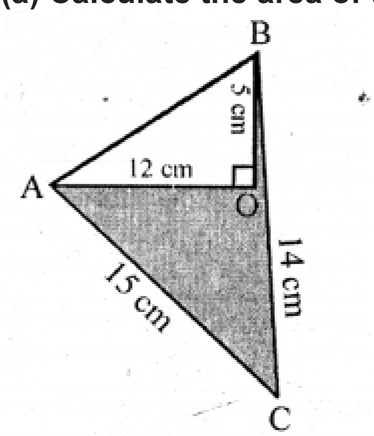
$\therefore$ Number of planks kept in it $= 768 \div \frac{2}{5}$

$= 768 \times \frac{5}{2} = 1920$ (b)

Chapter Test

Question 1.

(a) Calculate the area of the shaded region.



(b) If the sides of a square are lengthened by 3 cm, the area becomes 121 cm2. Find the perimeter of the original square.

Solution:

(a) In the figure,

$OA \perp BC$

$AC = 15$ cm, $AO = 12$ cm, $BO = 5$ cm, $BC = 14$ cm

$\therefore OC = BC - BO = 14 - 5 = 9$ cm

Area of right ΔAOC

$= \frac{1}{2}$ base × altitude

$= \frac{1}{2} \times 9 \times 12 \text{ cm}^2 = 54 \text{ cm}^2$

(b) Let the side of original square = x cm

Then length of given square = $(x + 3)$ cm

Area = side × side

$\Rightarrow 121 = (x + 3)(x + 3)$

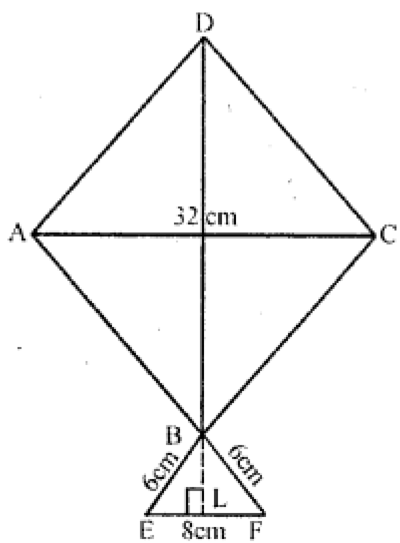
$\Rightarrow (11)^2 = (x + 3)^2$

$\Rightarrow 11 = x + 3 \Rightarrow x + 3 = 11$

$\Rightarrow x = 11 - 3$ cm $\Rightarrow x = 8$ cm

Question P.Q.

The given figure shows a kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and side 6cm each. How much paper is used in making the kite ? Ignore the wastage of the paper is making the kite.



Solution:

Length of each diagonal of the square ABCD

= 32 cm

i.e. AC = BD = 32 cm

Base EF of isosceles ΔBEF = 8 cm

and each side = 6 cm

Draw $BL \perp EF$

Area of square $= \frac{(\text{Diagonal})^2}{2}$

$= \frac{(32)^2}{2} = \frac{1024}{2}$ cm² = 512 cm²

In ΔBEL, $\angle L = 90°$

$BL^2 = BE^2 - EL^2$

(Pythagoras Theorem)

$= (6)^2 - (4)^2 = 36 - 16 = 20$

$\therefore$ BL $= \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ cm

$\therefore$ Area of isosceles triangle $= \frac{1}{2}$ Base × alt.

$= \frac{1}{2} \times 8 \times 2\sqrt{5}$ cm²

$= 8\sqrt{5}$ cm²

$\therefore$ Total area of the kite $= (512 + 8\sqrt{5})$ cm²

= 512 + 8 (2.236) cm²

= 512 + 17.89 = 529.89 cm²

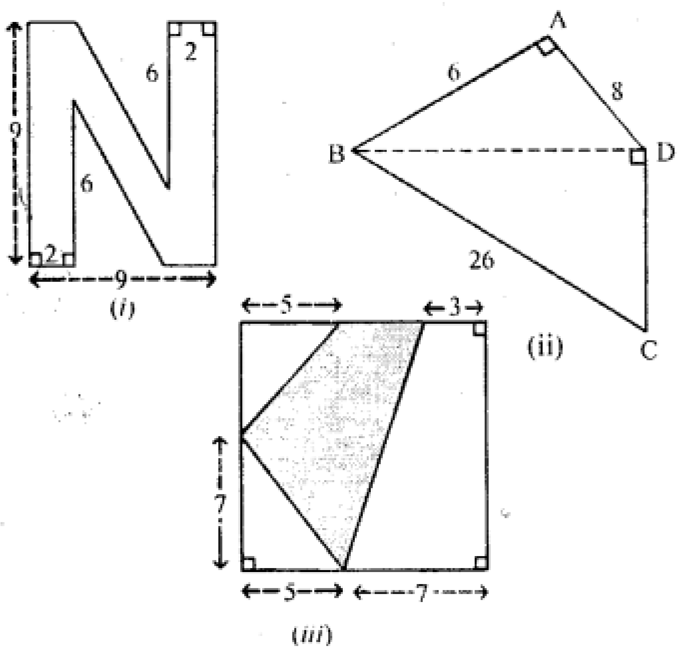
Question 2.

(a) Find the area enclosed by the figure (i) given below. All measurements are in centimetres:

(b) Find the area of the quadrilateral ABCD shown in figure (ii) given below. All measurements are in centimetres.

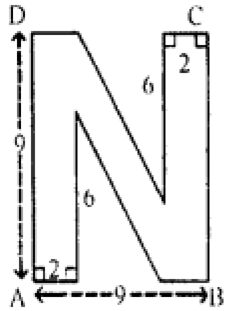
(c) Calculate the area of the shaded region shown in figure (iii) given below. All

measurements are in metres.



Solution:

(*a*) Area of figure (*i*) = Area of ABCD – Area of both triangles



$= (9 \times 9) - \left(\frac{1}{2} \times 5 \times 6\right) \times 2\ cm^2$

$= (81 - 15 \times 2)$

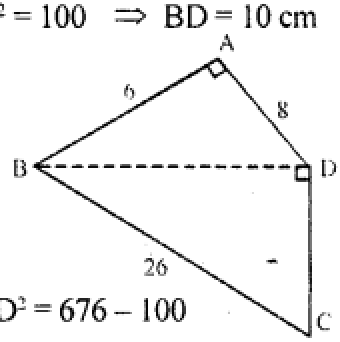
$= (81 - 30)\ cm^2 = 51\ cm^2$

(*b*) In ΔABD

By Pythagoras theorem,

$\Rightarrow \quad BD^2 = AB^2 + AD^2 \quad \Rightarrow \quad BD^2 = (6)^2 + (8)^2$

$\Rightarrow \quad BD^2 = 36 + 64 \quad \Rightarrow BD^2 = 100 \quad \Rightarrow \quad BD = 10\ cm$



In ΔBCD

By Pythagoras theorem

$BC^2 = BD^2 + CD^2$

$\Rightarrow (26)^2 = (10)^2 + (CD)^2$

$\Rightarrow 676 = 100 + CD^2$

$\Rightarrow \quad CD^2 + 100 = 676 \Rightarrow \quad CD^2 = 676 - 100$

$\Rightarrow CD^2 = 576 \Rightarrow \quad CD = \sqrt{576}\ cm \quad \Rightarrow CD = 24\ cm$

Area of given fig. = Area of ΔABD + Area of ΔBCD

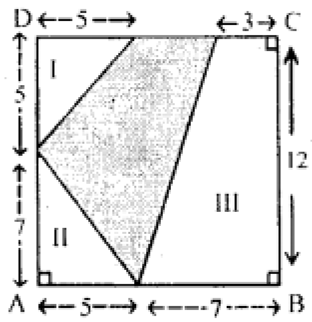
$= \frac{1}{2} \times \text{Base} \times \text{height} + \frac{1}{2} \times \text{Base} \times \text{height}$

$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times CD \times BD$

$= \left[\frac{1}{2} \times 6 \times 8 + \frac{1}{2} \times 24 \times 10\right] cm^2 = [3 \times 8 + 12 \times 10]\ cm^2$

$= (24 + 120)\ cm^2 = 144\ cm^2$

(c) Area of figure (*iii*) = Area of ABCD – (Area of 1st part + Area of 2nd part + Area of 3rd part)



$= (AB \times BC) - [(\frac{1}{2} \times \text{Base} \times \text{height})$

$+ (\frac{1}{2} \times \text{base} \times \text{height}) + \frac{1}{2}\ (\text{sum of} \parallel \text{side} \times \text{height}]$

$= (12 \times 12)\ m^2 - \left[\frac{1}{2} \times 5 \times 5 + \frac{1}{2} \times 5 \times 7 + \frac{1}{2}(7+3) \times 12\right] m^2$

$= 144\ m^2 - \left[\frac{25}{2} + \frac{35}{2} + 10 \times 6\right] m^2$

$= 144\ m^2 - \left(\frac{60}{2} + 60\right) m^2 = 144\ m^2 - (30 + 60)\ m^2$

$= 144\ m^2 - 90\ m^2 = 54\ m^2$

Hence, required area of given figure $= 54\ m^2$

Question 3.

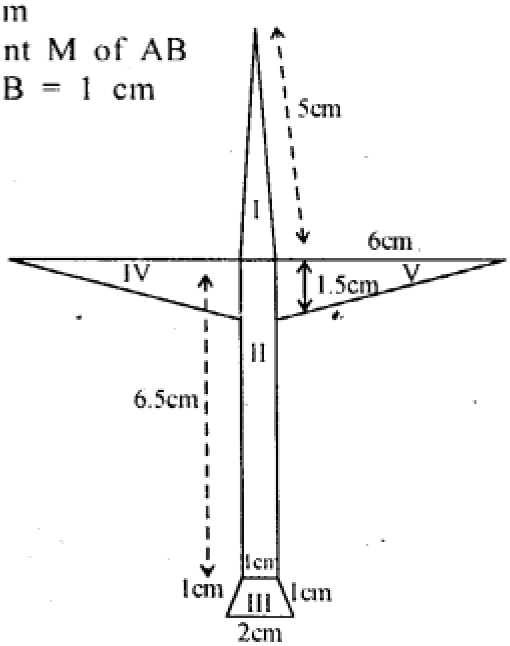
Asifa cut an aeroplane from a coloured chart paper (as shown in the adjoining figure). Find the total area of the chart paper used, correct to 1 decimal place.

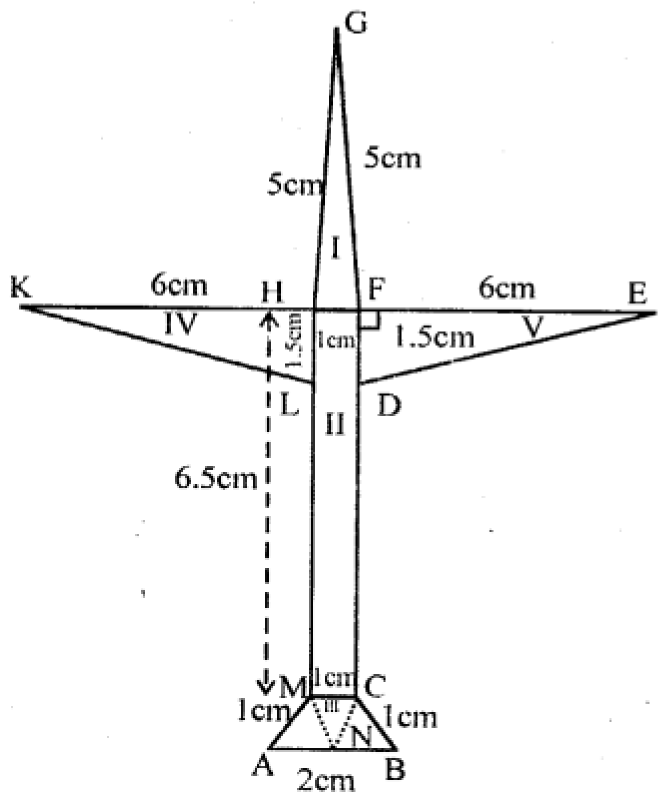
Solution:

The whole figure of an aeroplane consists 5 figures, three triangles, one rectangle and one trapezium

Take midpoint M of AB

$\therefore$ AM = MB = 1 cm





Join MN and CN

Their Δ AMN, Δ NCB and Δ MNC are equilateral traingles having 1 cm side each

Now area of Δ GHF

$$s = \frac{a+b+c}{2}$$

$$s = \frac{5+5+1}{2} = \frac{11}{2}$$

$$\therefore \text{ area } = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$$

$$= \sqrt{\frac{11}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{9}{2}} = \sqrt{\frac{11\times9}{16}} = \frac{3}{4}\sqrt{11} \text{ cm}^2$$

$$= \frac{3}{4} \times 3.316 = 3 \times 0.829 = 2.487$$

$$= 2.48 \text{ cm}^2$$

Area of rectangle II (MCFH) $= l \times b$

$= 6.5 \times 1 = 6.5\ cm^2$

area of Δ III + IV $= 2 \times \frac{1}{2} \times 6 \times 1.5$

$= 9.0\ cm^2$

area of 3 equilateral Δ s formed trapezium III

$= 3 \times \frac{\sqrt{3}}{4} \times (1)^2\ cm^2$

$= \frac{3}{4} \times 1.732\ cm^2$

$= 3 \times 0.433 = 1.299 = 1.3\ cm^2$

$\therefore$ Total area $= 2.48 + 6.50 + 9.00 + 1.30\ cm^2$

$= 19.28\ cm^2 = 19.3\ cm^2$

Question 4.

If the area of a circle is 78.5 cm², find its circumference. (Take π = 3.14)

Solution:

Area of a circle $= 78.5\ cm^2$

Let r be the radius

$\therefore r^2 = \frac{\text{Area}}{\pi} = \frac{78.50}{3.14} = 25 = (5)^2$

$\therefore r = 5$ cm

Now circumference $= 2\pi r$

$= 2 \times 3.14 \times 5$ cm $= 31.4$ cm

Question 5.

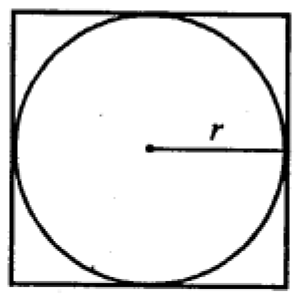
From a square cardboard, a circle of biggest area was cut out. If the area of the circle is 154 cm2, calculate the original area of the cardboard.

Solution:

Area of circle cut out from the square board = 154 cm^2

Let r be the radius

$\therefore \pi r^2 = 154 \quad \Rightarrow \frac{22}{7} r^2 = 154$



$\Rightarrow r^2 = \frac{154 \times 7}{22} = 49 = (7)^2$

$\Rightarrow r = 7$ cm

Now side of the square = 7 × 2 = 14 cm

$\therefore$ Area of the original cardboard

$= a^2 = (14)^2 = 196$ cm^2

Question 6.

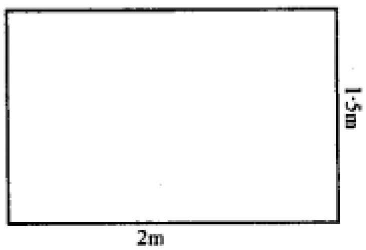
(a) From a sheet of paper of dimensions = 2m x 1.5m, how many circles can you cut of radius 5cm. Also find the area of the paper wasted. Take π = 3.14.

(b) If the diameter of a semicircular protractor is 14cm, then find its perimeter.

Solution:

Length of sheet of paper = 2m = 200cm

Breadth of sheet = 1·5 m = 150 cm



Area = $l \times b = 200 \times 150 \text{ cm}^2 = 30000 \text{ cm}^2$

Radius of circle = 5cm.

∴ No. of circles in lengthwise

$$= \frac{200}{5 \times 2} = 20$$

and widthwise $= \frac{150}{10} = 15$

∴ No. of circles = 20 × 15 = 300

Area of one circle = πr^2

$= 3{\cdot}14 \times 5 \times 5 \text{ cm}^2$

Area of 300 circles

$$= 300 \times \frac{314}{100} \times 25 \text{ cm}^2 = 23550 \text{ cm}^2$$

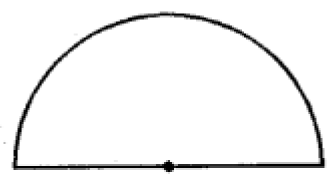
∴ Area of the remaining portion

= Area of square – area of 300 circles

$= (30000 - 23550) \text{ cm}^2$

$= \mathbf{6450 \text{ cm}^2}$

(*b*)Diameter of semicircular protractor = 14cm.



$\therefore$ Its perimeter $= \frac{1}{2}\pi d + d$

$= \frac{1}{2} \times \frac{22}{7} \times 14 + 14 = 22 + 14 = \mathbf{36\ cm}$

Question 7.

A road 3.5 m wide surrounds a circular park whose circumference is 88 m. Find the cost of paving the road at the rate of Rs. 60 per square metre.

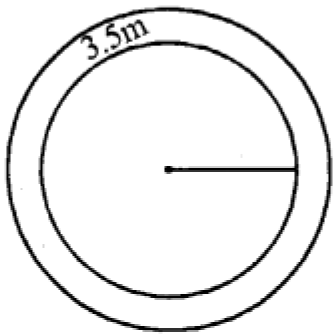
Solution:

Width of the road = 3.5 m

Circumference of the circular park = 88 m

Let r be the radius of the park

$\therefore 2\pi r = 88$



$\Rightarrow 2 \times \frac{22}{7} r = 88$

$\Rightarrow r = \frac{88 \times 7}{2 \times 22} = 14$ m

outer radius (R) = 14 + 3.5 = 17.5 m

Now area of the path

$= \frac{22}{7} \times (17.5 + 14)(17.5 - 14)$

$= \pi (R^2 - r^2) = \frac{22}{7} [(17.5)^2 - (14)^2]\ m^2$

$= \frac{22}{7} (17.5 + 14)(17.5 - 14)$

$= \frac{22}{7} \times 31.5 \times 3.5\ m^2 = 346.5\ m^2$

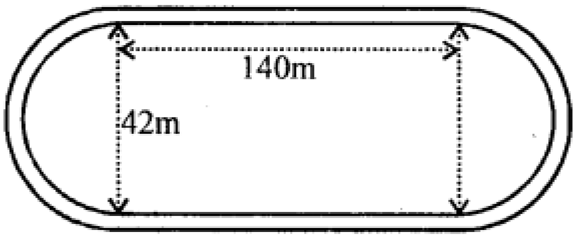
Rate of paving the road = Rs. 60 per m^2

$\therefore$ Total cost = Rs. 60 × 346.5

= Rs. 20790

Question 8.

The adjoining sketch shows a running tract 3.5 m wide all around which consists of two straight paths and two semicircular rings. Find the area of the track.



Solution:

Width of track = 3.5 m.

Inner length of rectangular base = 140 m

and width = 42 m

Outer length = 140 + 2 × 3.5 = 140 + 7
= 147 m

and width = 42 + 2 × 3.5 = 42 + 7 = 49 m

Radius of inner semicircle (r) = $\frac{42}{2}$ = 21m

and outer radius (R) = 21 + 3.5 = 24.5 m

Now area of track = $2(140 \times 3.5) + 2 \times \frac{1}{2}\pi$

$(R^2 - r^2)$

$= 2(490) + \frac{22}{7}\ [(24.5)^2 - (21)^2]$

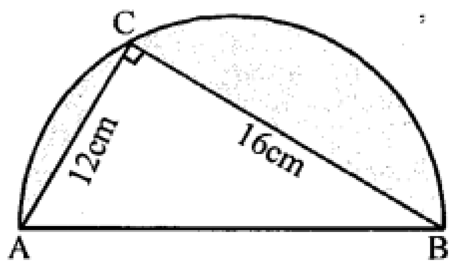
$= 980 + \frac{22}{7}\ (24.5 + 21)\ (24.5 - 21)$

$= 980 + \frac{22}{7} \times 45.5 \times 3.5\ m^2$

$= 980 + 500.5 = 1480.5\ m^2$

Question 9.

In the adjoining figure, O is the centre of a circular arc and AOB is a line segment.Find the perimeter and the area of the shaded region correct to one decimal place. (Take π = 3.142)



Solution:

In a semicircle, $\angle ACB = 90^\circ$

$\therefore$ ΔABC is a right angled triangle

Now $AB^2 = AC^2 + BC^2$

(Pythagoras Theorem)

$= 12^2 + 16^2$

$= 144 + 256 = 400$

$= (20)^2$

$\therefore$ AB = 20 cm

$\therefore$ Radius of semicircle $= \frac{20}{2} = 10$ cm

(i) Area of shaded portion

= Area of semicircle − area of ΔABC

$= \frac{1}{2}\pi r^2 - \frac{AC \times BC}{2}$

$= \frac{1}{2} \times 3.142\ (10)^2 - \frac{12 \times 16}{2}$

$= \frac{314.2}{2} - 96 = 157.1 - 96.0$

$= 61.1\ cm^2$

(ii) Perimeter of shaded portion

= circumference of semicircle + AC + BC

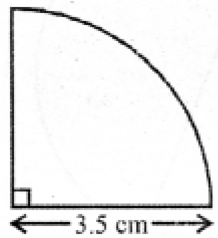
$= \pi r + 12 + 16 = 3.142 \times 10 + 28$

$= 31.42 + 28 = 59.42$ cm = 59.4 cm

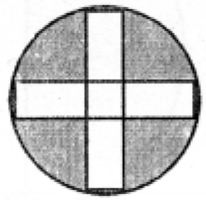
Question 10.

(a) In the figure (1) given below, the radius is 3.5 cm. Find the perimeter of the quarter of the circle.
(b) In the figure (ii) given below, there are five squares each of side 2 cm.
(i) Find the radius of the circle.
(ii) Find the area of the shaded region. (Take π= 3.14).

Solution:



(i)



(ii)

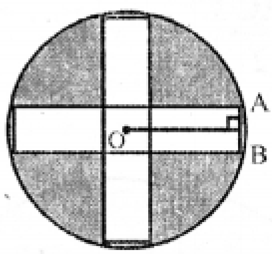
(a) Radius of quadrant = 3·5 cm

$\therefore$ Perimeter $= 2r + \frac{1}{4}\times 2\pi r$

$= 2r + \frac{1}{2}\times \pi r = 2\times 3{\cdot}5 + \frac{1}{2}\times\frac{22}{7}\times 3{\cdot}5\,\text{cm}$

$= 7 + 5{\cdot}5 = 12{\cdot}5$ cm.

(b) In the figure,



OB = 2 + 1 = 3 cm

AB = 1 cm

$\therefore$ OA $= \sqrt{OB^2 + AB^2}$

$= \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

$\therefore$ Radius of the circle $= \sqrt{10}$ cm

$\therefore$ Area of the circle $= \pi r^2$

$= 3.14 \times \left(\sqrt{10}\right)^2 \text{cm}^2$

$= 3.14 \times 10 \text{ cm}^2 = 31.4 \text{ cm}^2$

Area of 5 square of side 2 cm each

$= (2)^2 \times 5 = 4 \times 5 = 20 \text{ cm}^2$

$\therefore$ Area of shaded portion = 31.4 – 20

$= 11.4 \text{ cm}^2$

Question 11.

(a) In the figure (i) given below, a piece of cardboard in the shape of a quadrant of a circle of radius 7 cm is bounded by perpendicular radii OX and OY. Points A and B lie on OX and OY respectively such that OA = 3 cm and OB = 4 cm. The triangular part OAB is removed. Calculate the area and the perimeter of the remaining piece.

(*a*) Radius of quadrant = 7 cm

OA = 3 cm, OB = 4 cm

$\therefore$ AX = 7 – 3 = 4 cm and

BY = 7 – 4 = 3 cm

$\therefore AB^2 = OA^2 + OB^2 = (3)^2 + (4)^2$

$= 9 + 16 = 25$

$\Rightarrow AB = \sqrt{25} = 5\,\text{cm}$

Now (*i*) Area of shaded portion

$= \frac{1}{4}\pi r^2 - \frac{1}{2} OA \times OB$

$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 - \frac{1}{2} \times 3 \times 4$

$= \frac{1}{4} \times \frac{22}{7} \times 49 - 6\,\text{cm}^2$

$= \frac{77}{2} - 6 = \frac{65}{2}\,\text{cm}^2 = 32{\cdot}5\ \text{cm}^2$

(*ii*) Perimeter of shaded portion

(b) In the figure (ii) given below, ABCD is a square. Points A, B, C and D are centres of quadrants of circles of the same radius. If the area of the shaded portion is $21\frac{3}{7}$

cm2, find the radius of the quadrants. Take π = $\frac{22}{7}$.

Solution:

$$= \frac{1}{4}(2\pi r) + AX + BY + AB$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 + 4 + 3 + 5$$

$= 11 + 12 = 23$ cm.

(*b*) ABCD is a square and with centres A, B, C and D quadrants are drawn.

Let side of square $= a$

$\therefore$ Radius of each quadrant $= \frac{a}{2}$

$\therefore$ Area of shaded portion

$$= a^2 - 4 \times \left[\frac{1}{4}\pi\left(\frac{a}{2}\right)^2\right]$$

$$= a^2 - 4 \times \frac{1}{4}\pi\frac{a^2}{4} = a^2 - \frac{22}{7} \times \frac{a^2}{4}$$

$$= a^2 - \frac{11a^2}{14} = \frac{3a^2}{14}$$

But area of shaded portion

$$= 21\frac{3}{7} = \frac{150}{7} \text{cm}^2$$

$$\therefore \quad \frac{3}{14}a^2 = \frac{150}{7} \Rightarrow a^2 = \frac{150}{7} \times \frac{14}{3}$$

$$\Rightarrow \quad a^2 = 100 = (10)^2$$

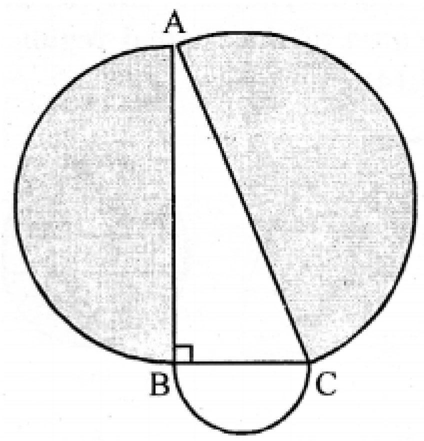
$$\therefore \quad a = 10$$

$\therefore$ Radius of each quadrant

$$= \frac{a}{2} = \frac{10}{2} = 5 \text{cm}$$

Question 12.

In the adjoining figure, ABC is a right angled triangle right angled at B. Semicircle are drawn on AB, BC and CA as diameter. Show that the sum of areas of semi circles drawn on AB and BC as diameter is equal to the area of the semicircle drawn on CA as diameter.



Solution:

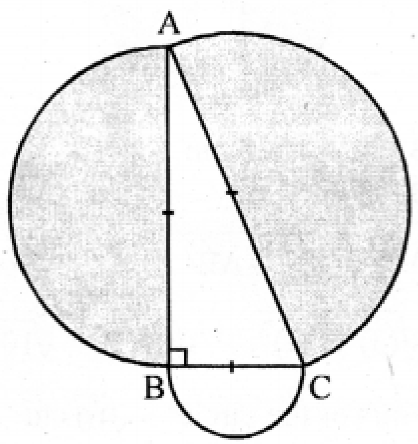
Δ ABC is a right angled triangle right angled at B

$\therefore\ AC^2 = AB^2 + BC^2$ ...(i)

(Pythagoras theorem)

Now area of semicircle on AC as diameter

$$= \frac{1}{2}\pi \left(\frac{AC}{2}\right)^2$$



$$= \frac{1}{2}\pi \times \frac{AC^2}{4} = \frac{\pi AC^2}{8}$$

Area of semicircle on AB as diameter

$$= \frac{1}{2}\pi \left(\frac{AB}{2}\right)^2 = \frac{1}{2}\pi \frac{AB^2}{4}$$

$$= \frac{\pi AB^2}{8}$$

and area of semicircle on BC as diameter

$$= \frac{1}{2}\pi\left(\frac{BC}{2}\right)^2 = \frac{1}{2}\pi\frac{BC^2}{4}$$

$$= \frac{\pi BC^2}{8}$$

$$\therefore \frac{\pi AB^2}{8} + \frac{\pi BC^2}{8} = \frac{\pi}{8}(AB^2 + BC^2)$$

$$= \frac{\pi}{8}(AC^2) \qquad \text{[from (i)]}$$

$$= \frac{\pi AC^2}{8}$$

Hence proved.

Question 13.
The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in 15 minutes.
Solution:
Radius of hand = 14 cm
∴ Area swept in 15 minutes

$$= \pi r^2 \times \frac{15}{60} = \frac{22}{7} \times 14 \times 14 \times \frac{1}{4}\,\text{cm}^2$$

$$= 154\ \text{cm}^2$$

Question 14.
Find the radius of a circle if a 90° arc has a length of 3.5 n cm. Hence, find the area of sector formed by this arc.

Solution:

Length of arc of the sector of a circle $= 3 \cdot 5\pi$ cm

and angle at the centre = 90°

$\therefore$ Radius of the arc $= \frac{3 \cdot 5\pi}{2\pi} \times \frac{360}{90}$

$= \frac{3 \cdot 5 \times 4}{2} = 7\,cm$

and area of the sector $= \pi r^2 \times \frac{90°}{360°}$

$= \frac{22}{7} \times 7 \times 7 \times \frac{1}{4} cm^2 = \frac{77}{2} = 38 \cdot 5 cm^2$

Question 15.

A cube whose each edge is 28 cm long has a circle of maximum radius on each of its face painted red. Find the total area of the unpainted surface of the cube.

Solution:

Edge of cube = 28 cm

$\therefore$ Surface area $= 6a^2 = 6 \times (28)^2\ cm^2$

$= 6 \times 28 \times 28 = 4704\ cm^2$

Now diameter of each circle = 28 cm

$\therefore$ Radius $= \frac{28}{2} = 14\,cm$

$\therefore$ Area of each circle

$= \pi r^2 = \frac{22}{7} \times 14 \times 14\,cm^2 = 616\ cm^2$

and area of such 6 circles drawn on 6 faces of cube

$= 616 \times 6 = 3696\ cm^2$

$\therefore$ Area of remaining portion of the cube

$= 4704 - 3696 = 1008\ cm^2$

Question 16.

Can a pole 6.5 m long fit into the body of a truck with internal dimensions of 3.5m,

3 m and 4m?

Solution:

No,

Because length of pole = 6.5 m

But internal dimensions of truck are 3.5 m, 3 m and 4 m all of these dimensions are less than that of 6.5 m. So that pole cannot fit into the body of truck with given dimensions.

Question 17.

A car has a petrol tank 40 cm long, 28 cm wide and 25 cm deep. If the fuel consumption of the car averages 13.5 km per litre, how far can the car travel with a full tank of petrol ?

Solution:

Capacity of car tank = 40cm × 28cm × 25cm

$$= (40 \times 28 \times 25)\ cm^3 = \frac{40 \times 28 \times 25}{1000} \text{ litre}$$

$(\because\ 1000cm^3 = 1 \text{ litre})$

Average of car = 13.5 km per litre

Then, distance travelled by car

$$= \frac{40 \times 28 \times 25}{1000} \times 13.5 \text{ km}$$

$$= \frac{(40 \times 25) \times 28}{1000} \times \frac{135}{10} \text{ km}$$

$$= \frac{1 \times 28}{1} \times \frac{135}{10} \text{ km} = \frac{14 \times 135}{5} \text{ km}$$

$= 14 \times 27$ km = 378 km

Hence, The car can travel 378 km with a full tank of petrol. **Ans.**

Question 18.

An aquarium took 96 minutes to completely fill with water. Water was filling the aquarium at a rate of 25 litres every 2 minutes. Given that the aquarium was 2 m long and 80 cm wide, compute the height of the aquarium.

Solution:

Water fill in 2 minutes = 25 litres

Water fill in 1 minutes = $\frac{25}{2}$ litres

Water fill in 96 minutes = $\frac{25}{2} \times 96$ litres

= 25 × 48 litres = 1200 litres

i.e. Capacity of aquarium = 1200 litres .....(1)

But, Length of aquarium = 2m = 2 × 100 cm = 200 cm

Breadth of aquarium = 80 cm

Let height of aquarium = h cm

Then, capacity of aquarium = $200 \times 80 \times h$ cm^3

$= \frac{200 \times 80 \times h}{1000}$ litre $= \frac{1}{5} \times 80 \times h$ litre

= 16 h litre ..... (2)

From (1) and (2)

$16h = 1200 \Rightarrow h = \frac{1200}{16}$ cm $\Rightarrow h =$ 75 cm

Hence, height of aquarium = 75 cm

Question 19.

The lateral surface area of a cubiod is 224 cm2. Its height is 7 cm and the base is a square. Find :

(i) a side of the square, and

(ii) the volume of the cubiod.

Solution:

Given that lateral surface Area of a cubiod = 224 cm^2

Height of cubiod = 7 cm

Also base is square

Let length of cubiod = x cm

Then Breadth of cubiod = x cm

($\because$ Base is square so length and breadth are same)

Lateral Surface Area = $2(l + b) \times h$

$\Rightarrow \quad 224 = 2(x + x) \times 7$

$\Rightarrow \quad 224 = 2 \times 2x \times 7 \quad \Rightarrow \quad 224 = 28x$

$\Rightarrow \quad 28x = 224 \quad \Rightarrow \quad x = \frac{224}{28}$ cm $= 8$ cm

(*i*) Hence, side of the square = 8 cm

(*ii*) volume of the cuboid = = $\ell \times b \times h$

$= 8 \times 8 \times 7$ cm^3 = 448 cm^3

Question 20.

If the volume of a cube is V m³, its surface area is S m2 and the length of a diagonal is d metres, prove that 6√3 V = S d.

Solution:

Volume of cube = (V) = $(\text{Side})^3$

Let a be the side of the cube, then

$$V = a^3 \text{ and } S = 6a^2$$

Diagonal (d) = $\sqrt{3}.\, a$.

Now $\quad Sd = 6a^2 \times \sqrt{3}\, a = 6\sqrt{3}\, a^3$

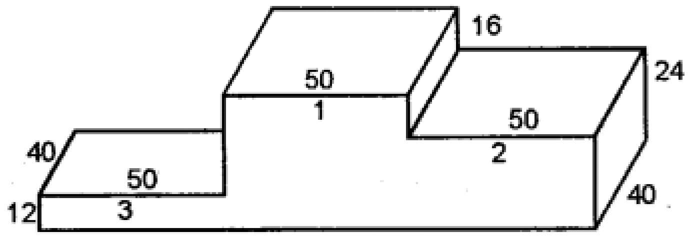
$\quad = 6\sqrt{3}\, V \qquad (\because V = a^3)$

Hence $6\sqrt{3}\, V = Sd$.

Question 21.

The adjoining figure shows a victory stand, each face is rectangular. All measurement are in centimetres. Find its volume and surface area (the bottom of the stand is open).

Solution:



In the figure, it has three parts as indicated by 3, 1 and 2.

$\therefore$ Volume of part (3) = $50 \times 40 \times 12$ cm^3
= 24000 cm^3

Volume of part (1) = $50 \times 40 \times (16 + 24)$ cm^3
= $50 \times 40 \times 40$ cm^3 = 80000 cm^3

and volume of part (2) = $50 \times 40 \times 24$ cm^3
= 48000 cm^3

$\therefore$ Total volume = (24000 + 8000 + 48000) cm^3 = 153000 cm^3

Now total surface area = Area of front and back
+ area of vertical faces + area of top faces
= $2 (50 \times 12 + 50 \times 40 + 50 \times 24)$ cm^2
+ $(12 \times 40 + 28 \times 40 + 16 \times 40 + 24 \times 40)$ cm^2
+ $3 (50 \times 40)$ cm^2
= 2 (600 + 2000 + 1200) cm^2 + (480 + 1120 + 640 + 960) cm^2 + 3×2000 cm^2
= 2 (3800) + 3200 + 6000 cm^2
= 7600 + 3200 + 6000 = 16800 cm^2

Question 22.

The external dimensions of an open rectangular wooden box are 98 cm by 84 cm by 77 cm. If the wood is 2 cm thick all around, find :
(i) the capacity of the box
(ii) the volume of the wood used in making the box, and
(iii) the weight of the box in kilograms correct to one decimal place, given that 1 cm3 of wood weighs 0.8 gm.

Solution:

Given that external dimensions of open rectangular wooden box = 98 cm, 84 cm, and 77 cm.

Thickness = 2 cm

Then internal dimensions of open rectangular wooden box $(98 - 2 \times 2)$ cm, $(84 - 2 \times 2)$ cm and

$(77 - 2)$ cm
$= (98 - 4)$ cm, $(84 - 4)$ cm, 75 cm = 94 cm, 80 cm, 75 cm

(*i*) Capacity of the box = 94cm × 80cm × 75cm

= 564000 cm^3

(*ii*) Internal volume of box = 564000 cm^3

External volume of box = 98cm × 84cm × 77 cm

= 633864 cm^3

Volume of wood used in making the

Box = 633864 cm^3 – 564000 cm^3 = 69864 cm^3

(*iii*) Weight of 1 cm^3 wood = 0.8 gm

Weight of 69864 cm^3 wood = 0. 8 × 69864 gm

$$= \frac{0.8 \times 69864}{1000} \text{ kg} = \frac{55891.2}{1000} \text{ kg}$$

= 55.9 kg (Correct to one decimal)

Question 23.
A cuboidal block of metal has dimensions 36 cm by 32 cm by 0.25 m. It is melted and recast into cubes with an edge of 4 cm.
(i) How many such cubes can be made ?
(ii) What is the cost of silver coating the surfaces of the cubes at the rate of Rs. 1.25 per square centimetre ?
Solution:

Given, dimensions of cuboidal block are 36 cm, 32 cm, 0.25 m.

Volume of cuboidal block = 36 cm × 32 cm × 0.25 m

= 36 cm × 32 cm × (0.25 ×100) cm = (36 × 32 × 25) cm^3

Volume of cube having edge is 4 cm

= 4 cm × 4 cm × 4 cm = 64 cm^3

(*i*) Number of cubes

$$= \frac{\text{volume of cuboidal block}}{\text{volume of one cube}}$$

$$= \frac{36 \times 32 \times 25}{64} = \frac{36 \times 25}{2} = 18 \times 25 = 450$$

(*ii*) Total surface area of one cube

= 6 $(a)^2$ = 6 $(4)^2$ cm^2 = 6 × 4 × 4 cm^2 = 96 cm^2

Total surface area of 450 cube = 450 × 96 cm^2 = 43200 cm^2

Cost of silver coating the surface for 1 cm^2 = Rs. 1.25

cost of silver coating the surface for 43200 cm^2

= 43200 × 1.25 = Rs. 54000

Question 24.

Three cubes of silver with edges 3 cm, 4 cm and 5 cm are melted and recast into a single cube. Find the cost of coating the surface of the new cube with gold at the rate of Rs. 3.50 per square centimetre.

Solution:

Volume of First cube = $(\text{edge})^3$

= $(3\text{ cm})^3$ = 3 × 3 × 3 cm^3 = 27 cm^3