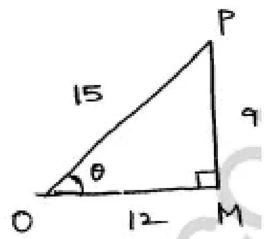
TRIGONOMETRIC RATIOS

EXERCISE - 16



1

ⓐ Sol: By pythagorus theorem

$OP^2 = OM^2 + PM^2$

$15^2 = 12^2 + PM^2$

$225 = 144 + PM^2$

$PM^2 = 225 - 144$

$PM^2 = 81$

$PM = \sqrt{81}$

$PM = 9.$

(i) Sol: $\sin\theta = \frac{PM}{OP}$

$= \frac{\cancel{9}^{3}}{\cancel{15}_{5}}$

$\sin\theta = \frac{3}{5}$

(ii) $\cos\theta = \frac{OM}{OP}$

$\cos\theta = \frac{12}{15}$

$\cos\theta = \frac{4}{5}$

(iii) $\tan\theta = \frac{PM}{OM}$

$= \frac{9}{12}$

$\tan\theta = \frac{3}{4}$

(iv) $\cot\theta = \frac{OM}{PM}$

$= \frac{12}{9}$

$\cot\theta = \frac{4}{3}$

(v) $\sec\theta = \frac{OP}{OM}$

$= \frac{15}{12}$

$\sec\theta = \frac{5}{4}$

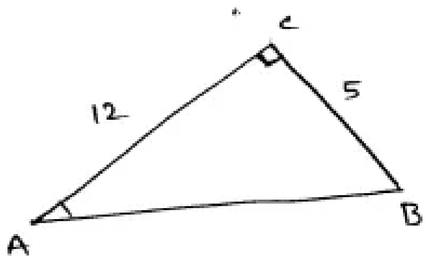
(vi) $\mathrm{cosec}\,\theta = \frac{OP}{PM}$

$= \frac{15}{9}$

$\mathrm{cosec}\,\theta = \frac{5}{3}$

(b) Sol

By pythagoras theorem



$AB^2 = AC^2 + BC^2$

$= 12^2 + 5^2$

$= 144 + 25$

$AB^2 = 169$

$AB = \sqrt{169}$

$AB = 13$

(i) $\sin A = \frac{BC}{AB}$

$\sin A = \frac{5}{13}$

(ii) $\cos A = \frac{AC}{AB}$

$\cos A = \frac{12}{13}$

(iii) $\sin^2 A + \cos^2 A = \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2$

$= \frac{25}{169} + \frac{144}{169}$

$= \frac{25+144}{169}$

$= \frac{\cancel{169}}{\cancel{169}}$

$= 1.$

$\therefore \sin^2 A + \cos^2 A = 1$

(iv) $\sec^2 A - \tan^2 A$

$\because \sec A = \frac{1}{\cos A}$

$= \frac{1}{\frac{12}{13}}$

$\sec A = \frac{13}{12}$

$\because \tan A = \frac{\sin A}{\cos A}$

$= \frac{\left(\frac{5}{\cancel{13}}\right)}{\left(\frac{12}{\cancel{13}}\right)}$

$$\text{Tan}A = \frac{5}{12}$$

$$\therefore \sec^2 A - \tan^2 A = \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2$$

$$= \frac{169}{144} - \frac{25}{144}$$

$$= \frac{169-25}{144}$$

$$= \frac{\cancel{144}}{\cancel{144}}$$

$$= 1.$$

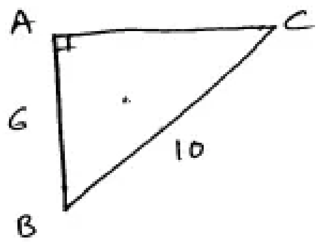
$$\therefore \sec^2 A - \text{Tan}^2 A = 1.$$

2

Sol.

(a)

By phytagorus theorem



$\because$ hypotenuse = BC

$$\therefore BC^2 = AB^2 + AC^2$$

$$10^2 = 6^2 + AC^2$$

$$100 = 36 + AC^2$$

$$AC^2 = 100 - 36$$

$$AC^2 = 64$$

$$AC = \sqrt{64}$$

$$AC = 8.$$

(i) $\sin B = \dfrac{AC}{BC}$

$= \dfrac{8}{10}$

$\sin B = \dfrac{4}{5}$

(ii) $\cos C = \dfrac{AC}{BC}$

$= \dfrac{8}{10}$

$\cos C = \dfrac{4}{5}$

(iii) $\sin B + \sin C$

$\because \sin C = \dfrac{AB}{BC}$

$= \dfrac{6}{10}$

$= \dfrac{3}{5}$

$\therefore \sin B = \dfrac{4}{5}$

$\therefore \sin B + \sin C = \dfrac{4}{5} + \dfrac{3}{5}$

$= \dfrac{4+3}{5}$

$\sin B + \sin C = \dfrac{7}{5}$

(iv) $\sin B \cdot \cos C + \sin C \cdot \cos B$

$\cos B = \dfrac{AB}{BC} = \dfrac{6}{10}$

$= \dfrac{3}{5}$

$\therefore \sin B \cdot \cos C + \sin C \cdot \cos B$

$\Rightarrow \frac{4}{5} \cdot \frac{4}{5} + \frac{3}{5} \cdot \frac{3}{5}$

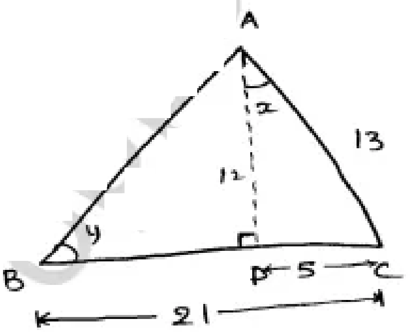
$\Rightarrow \frac{16}{25} + \frac{9}{25}$

$\Rightarrow \frac{16+9}{25}$

$\Rightarrow \frac{\cancel{25}}{\cancel{25}}$

$\Rightarrow 1.$

(b) From Figure



ΔADC

$\Rightarrow AD^2 + CD^2 = AC^2$

$AD^2 + 5^2 = 13^2$

$AD^2 + 25 = 169$

$AD^2 = 169 - 25$

$AD^2 = 144$

$AD = \sqrt{144}$

$AD = 12.$

From Figure

ΔABD

$\Rightarrow BD = BC - CD$

$= 21 - 5$

$BD = 16$

$\therefore AB^2 = BD^2 + AD^2$

$AB^2 = 16^2 + 12^2$

$AB^2 = 256 + 144$

$AB^2 = 400$

$AB = \sqrt{400}$

$AB = 20.$

$\therefore$ $AB = 20$, $AD = 12$;

$BD = 16$

$AC = 13$

$CD = 5$

(i) $\tan x = \frac{CD}{AD}$

$\tan x = \frac{5}{12}$

(ii) $\cos y = \frac{BD}{AB}$

$= \frac{16}{20}$

$\cos y = \frac{4}{5}$

(iii) $\operatorname{cosec}^2 y - \cot^2 y$

$\rightarrow$ $\operatorname{cosec} y = \frac{AB}{AD}$

$= \frac{20}{12}$

$\operatorname{cosec} y = \frac{5}{3}$

$\Rightarrow$ $\cot y = \frac{BD}{AD}$

$= \frac{16}{12}$

$\cot y = \frac{4}{3}$

$\therefore \operatorname{cosec}^2 y - \cot^2 y = \left(\frac{5}{3}\right)^2 - \left(\frac{4}{3}\right)^2$

$= \frac{25}{9} - \frac{16}{9}$

$= \frac{25-16}{9}$

$= \frac{9}{9}$

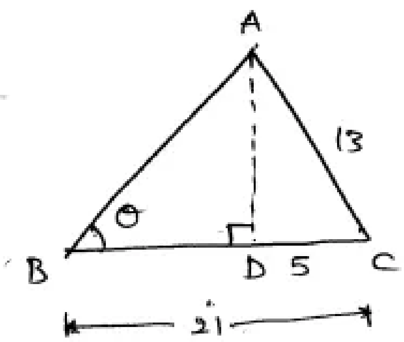
$\therefore \operatorname{cosec}^2 y - \cot^2 y = 1$

(3)

Sol:

(a)

From figure



$BD = BC - CD$

$BD = 21 - 5$

$BD = 16$

From ΔADC

$AD^2 + DC^2 = AC^2$

$AD^2 + 5^2 = 13^2$

$AD^2 + 25 = 169$

$AD^2 = 169 - 25$

$AD^2 = 144$

$AD = \sqrt{144}$

$AD = 12$.

$\therefore \quad \sec\theta = \frac{AB}{BD}$

from fig ΔABD

$$AB^2 = BD^2 + AD^2$$
$$= 16^2 + 12^2$$
$$= 256 + 144$$
$$= 400$$
$$AB^2 = 400$$
$$AB = \sqrt{400}$$
$$AB = 20$$

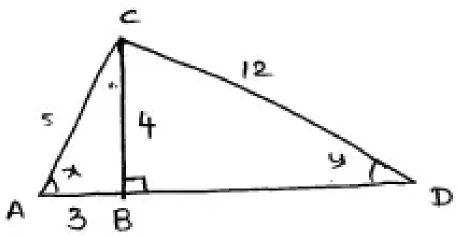
$\therefore \quad \sec\theta = \frac{20}{16}$

$$= \frac{5}{4}$$

(6)

Sol:

From figure



ΔABC

$$AC^2 = AB^2 + BC^2$$
$$= 3^2 + 4^2$$
$$= 9 + 16$$
$$AC^2 = 25$$
$$AC = \sqrt{25}$$
$$AC = 5$$

From figure

ΔBCD

$$CD^2 = BC^2 + BD^2$$
$$12^2 = 4^2 + BD^2$$

$144 = 16 + BD^2$

$BD^2 = 144 - 16$

$= 12\ 8$

$BD^2 = 128$

$BD = \sqrt{128}$

$= \sqrt{16 \times 4 \times 2}$

$= 4 \times 2\sqrt{2}$

$BD = 8\sqrt{2}$

$\therefore$

(i) $\sin x = \frac{BC}{AC}$

$\sin x = \frac{4}{5}$

(ii) $\cot x = \frac{AB}{BC}$

$= \frac{3}{4}$

(iii) $\cot^2 x - \operatorname{cosec}^2 x$

$\because \cot x = \frac{3}{4}$

$\operatorname{cosec} x = \frac{AC}{BC}$

$= \frac{5}{4}$

$\therefore \cot^2 x - \operatorname{cosec}^2 x = \left(\frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2$

$= \frac{9}{16} - \frac{25}{4}$

$$\therefore \cot^2 x - \operatorname{cosec}^2 x = \frac{9-25}{16}$$

$$= \frac{-16}{16}$$

$$= -1$$

(iv) $$\sec y = \frac{CD}{BD}$$

$$= \frac{12}{8\sqrt{2}}$$

$$\sec y = \frac{3}{2\sqrt{2}}$$

(v) $$\tan^2 y - \frac{1}{\cos^2 y}$$

$$\because \sec^2 y = \frac{1}{\cos^2 y}$$

$$= \left(\frac{3}{2\sqrt{2}}\right)^2$$

$$= \frac{9}{4\times 2}$$

$$\frac{1}{\cos^2 y} = \frac{9}{8}$$

$$\therefore \tan^2 y = \left(\frac{BC}{BD}\right)^2$$

$$= \left(\frac{4}{8\sqrt{2}}\right)^2$$

$$= \frac{\cancel{16}}{\cancel{64}_{4}\times 2}$$

$$\tan^2 y = \frac{1}{8}$$

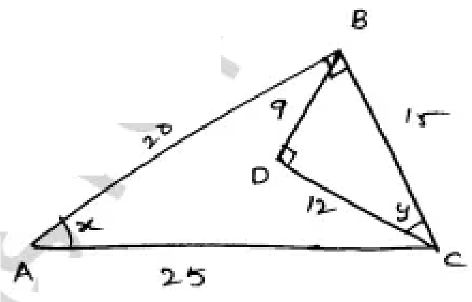
$$\therefore \tan^2 y - \frac{1}{\cos^2 y} = \frac{1}{8} - \frac{9}{8}$$

$$= \frac{1-9}{8}$$

$$= \frac{-8}{8}$$

$$= -1$$

(4)
Sol.
(9)



From figure

ΔBCD

$$\Rightarrow BC^2 = BD^2 + CD^2$$

$$BC^2 = 9^2 + 12^2$$

$$= 81 + 144$$

$$BC^2 = 225$$

$$BC = \sqrt{225}$$

$$BC = 15.$$

$\Delta ABC \Rightarrow$

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 15^2 = 25^2$$

$$AB^2 = 625 - 225$$

$$AB^2 = 400$$

$$AB = \sqrt{400} = 20.$$

(i) $2\sin y - \cos y$

$\because \sin y = \dfrac{BD}{BC}$

$= \dfrac{9}{15}$

$\therefore \sin y = \dfrac{3}{5}$

$\cos y = \dfrac{CD}{BC}$

$= \dfrac{12}{15}$

$\therefore \cos y = \dfrac{4}{5}$

$\therefore 2\sin y - \cos y = 2 \cdot \dfrac{3}{5} - \dfrac{4}{5}$

$= \dfrac{6}{5} - \dfrac{4}{5}$

$= \dfrac{6-4}{5}$

$= \dfrac{2}{5}$ //

(ii) $2\sin x - \cos x$

$\sin x = \dfrac{BC}{AC} = \dfrac{15}{25} = \dfrac{3}{5}$

$\cos x = \dfrac{AB}{AC} = \dfrac{20}{25} = \dfrac{4}{5}$

$\therefore 2\sin x - \cos x = 2 \cdot \dfrac{3}{5} - \dfrac{4}{5}$

$= \dfrac{6}{5} - \dfrac{4}{5}$

$= \dfrac{6-4}{5} = 2/5$

(iii) $1 - \sin x + \cos y$

$\because \sin x = \frac{3}{5}$

$\cos y = \frac{4}{5}$

$\therefore 1 - \sin x + \cos y = 1 - \frac{3}{5} + \frac{4}{5}$

$= \frac{5-3+4}{5}$

$= \frac{6}{5}$

(iv) $2\cos x - 3\sin y + 4\tan x$

$\because \sin x = \frac{3}{5}$

$\cos x = \frac{4}{5}$

$\sin y = \frac{3}{5}$

$\therefore \tan x = \frac{\sin x}{\cos x}$

$= \frac{\frac{3}{5}}{\frac{4}{5}}$

$\tan x = \frac{3}{4}$

$\therefore 2\cos x - 3\sin y + 4\tan x$

$\Rightarrow 2 \cdot \frac{4}{5} - 3 \cdot \frac{3}{5} + \cancel{4} \cdot \frac{3}{\cancel{4}}$

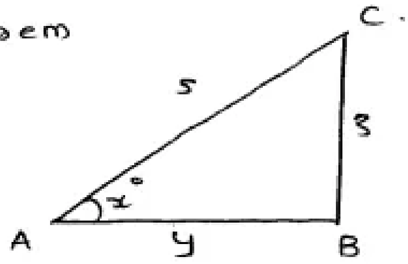
$\Rightarrow \frac{8}{5} - \frac{9}{5} + 3$

$\Rightarrow \frac{8-9+15}{5}$

$\Rightarrow \frac{14}{5}$

(b) Sol) (ii)

By pythagorus theorem



$AC^2 = BC^2 + AB^2$

$5^2 = 3^2 + AB^2$

$25 = 9 + AB^2$

$AB^2 = 25 - 9$

$AB^2 = 16$

$AB = \sqrt{16}$

$AB = 4$

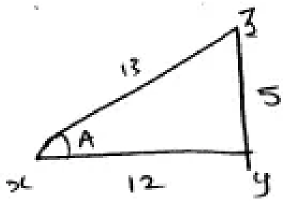
$\therefore AB = y = 4.$

(i)

$\sin x° = \frac{BC}{AC}$

$\sin x° = \frac{3}{5}$

5 Sol:



Given $\tan A = \frac{yz}{xy} = \frac{5}{12}$

By pythagorus theorem

$xy^2 + yz^2 = xz^2$

$12^2 + 5^2 = xz^2$

$\therefore 144 + 25 = xz^2$

$xz^2 = 169$

$xz = 13.$

(i) $\cos A = \frac{xy}{xz}$

$= \frac{12}{13}$

(ii) $\operatorname{cosec} A - \cot A$

$\operatorname{cosec} A = \frac{xz}{yz}$

$= \frac{13}{5}$

$\cot A = \frac{xy}{yz}$

$= \frac{12}{5}$

$\therefore \operatorname{cosec} A - \cot A = \frac{13}{5} - \frac{12}{5}$

$= \frac{13-12}{5}$

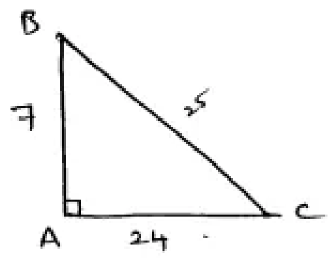
$= \frac{1}{5}$

6

sol:

(a) Given $AB = 7$

$BC - AC = 1$



By pythagorus theorem

$BC^2 = AB^2 + AC^2$

$\because BC = 1 + AC$

$\therefore (1+AC)^2 = 7^2 + AC^2$

$1 + \cancel{AC^2} + 2AC = 49 + \cancel{AC^2}$

$2AC = 49 - 1$

$2AC = 48$

$AC = 24$

(i) $\sin C = \frac{AB}{BC}$

$\because$ from given $BC - AC = 1$

$BC - 24 = 1$

$BC = 1 + 24$

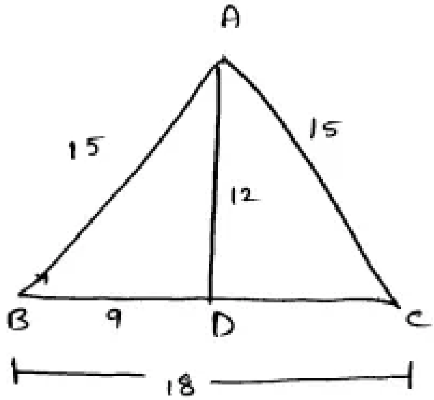
$BC = 25$

$\sin C = \frac{7}{25}$

(ii) $\tan B = \frac{AC}{AB}$

$= \frac{24}{7}$

7
Sol:



(i) $\cos \angle ABC = \frac{BD}{BA}$

$= \frac{\cancel{9}^{3}}{\cancel{15}_{5}}$

$= \frac{3}{5}$

(ii) $\sin\angle ACB = \dfrac{AD}{AC}$

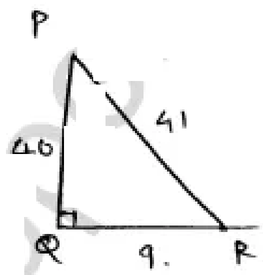
$= \dfrac{12}{15}$

$= \dfrac{4}{5}$

6 (b)

Given $PQ = 40$

$PR + QR = 50$



By pythagorous theorem

$PR^2 = PQ^2 + QR^2$

$(50 - QR)^2 = PQ^2 + QR^2$

$2500 + \cancel{QR^2} - 100\,QR = 40^2 + \cancel{QR^2}$

$2500 - 1600 = 100\,QR$

$100\,QR = 900$

$QR = \dfrac{90\cancel{0}}{\cancel{100}}$

$QR = 9.$

$\therefore$ Given $PR + QR = 50$

$PR + 9 = 50$

$PR = 50 - 9$

$PR = 41$

(i) $\sin P = \frac{QR}{PR}$

$= \frac{9}{41}$

(ii) $\cos P = \frac{PQ}{PR}$

$= \frac{40}{41}$

(iii). $\tan R = \frac{\sin R}{\cos R}$.

$\because \sin R = \frac{PQ}{PR}$

$= \frac{40}{41}$

$\cos R = \frac{QR}{PR}$

$= \frac{9}{41}$

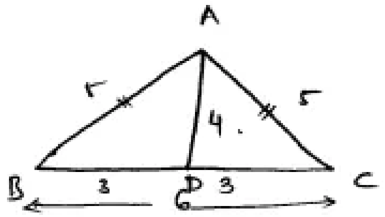
$\therefore \tan R = \frac{\frac{40}{\cancel{41}}}{\frac{9}{\cancel{41}}}$

$= \frac{40}{9}$

$\therefore \operatorname{Tan} R = \frac{40}{9}$.

$8\ \underline{\underline{\text{Po}}}$

(a)



Given $AB = AC = 5$ cm

$BC = 6$

(i) $\sin C = \dfrac{AD}{AC}$

$= \dfrac{4}{5}$

(ii) $\tan B = \dfrac{AD}{BD}$

$= \dfrac{4}{3}$

(iii) $\tan C - \cot B$

$\because \tan C = \dfrac{AD}{DC} = \dfrac{4}{3}$

$\cot B = \dfrac{BD}{AD} = \dfrac{3}{4}$

$\therefore \Rightarrow \tan C - \cot B = \dfrac{4}{3} - \dfrac{3}{4}$

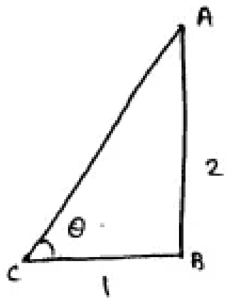
$= \dfrac{16-9}{12}$

$= \dfrac{7}{12}$

(b)

Given $AB = 2$

$BC = 1$



$\therefore \sin\theta = \dfrac{AB}{AC}$

$\tan\theta = \dfrac{AB}{BC}$

$\therefore$ By pythagorous theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 2^2 + 1^2$

$AC^2 = 4+1$

$AC^2 = 5$

$AC = \sqrt{5}$

$\therefore \sin^2\theta = \left(\dfrac{AB}{AC}\right)^2 = \left(\dfrac{2}{\sqrt{5}}\right)^2 = \dfrac{4}{5}$

$\tan^2\theta = \left(\dfrac{AB}{BC}\right)^2 = \left(\dfrac{2}{1}\right)^2 = 4$

$\therefore \sin^2\theta + \tan^2\theta = \dfrac{4}{5} + 4$

$= \dfrac{4+20}{5}$

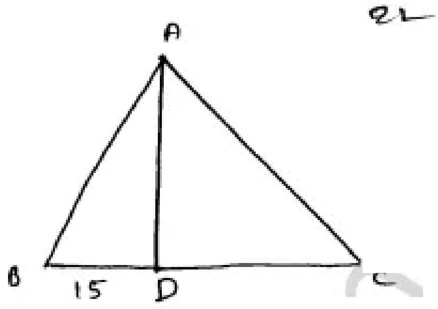
$= \dfrac{24}{5}$

$\sin^2\theta + \tan^2\theta = 4\dfrac{4}{5}$

(c) Given $BD = 15$

$\sin B = \frac{4}{5}$

$\tan C = 1$



$\because \sin B = \frac{4}{5} \times \frac{5}{5}$

$\sin B = \frac{AD}{AB} = \frac{20}{25}$

(i) $\because AD = 20$

$\because \tan C \Rightarrow \frac{\sin C}{\cos C} = 1$

$\Rightarrow \frac{AD}{DC} = 1$

$\therefore AD = DC$

$\therefore DC = 20$

In ΔACD ; By pythagorus theorem

$AC^2 = AD^2 + DC^2$

$= 20^2 + 20^2$

$= 400 + 400$

$AC^2 = 800$

$AC = \sqrt{800}$

$= \sqrt{400 \times 2}$

$AC = 20\sqrt{2}$

(ii) $\tan^2 B - \frac{1}{\cos^2 B} = -1$.

$\therefore$ LHS $\Rightarrow$ $\tan^2 B - \frac{1}{\cos^2 B}$

$$\tan^2 B = \left(\frac{AD}{BD}\right)^2 = \left(\frac{20}{15}\right)^2 = \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9}$$

$$\cos^2 B = \left(\frac{BD}{AB}\right)^2 = \left(\frac{15}{25}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$\therefore \tan^2 B - \frac{1}{\cos^2 B} \Rightarrow \frac{16}{9} - \frac{1}{\left(\frac{9}{25}\right)}$$

$$\Rightarrow \frac{16}{9} - \frac{25}{9}$$

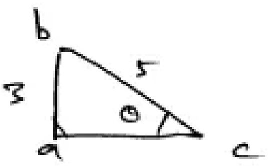
$$\Rightarrow \frac{16-25}{9}$$

$$\Rightarrow -1.$$

$\therefore$ LHS = RHS

Hence proved.

9/18 (i) Given



$\sin\theta = \frac{3}{5}$

$\therefore$ From pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$5^2 = 3^2 + ac^2$$

$$25 = 9 + ac^2$$

$$ac^2 = 25 - 9$$

$$ac^2 = 16$$

$$ac = \sqrt{16}$$

$$ac = 4$$

$$\therefore \cos\theta = \frac{ac}{bc}$$

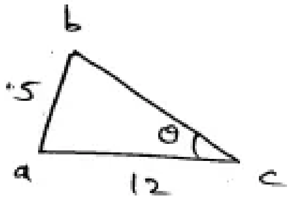
$$= \frac{4}{5}$$

(ii) $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{3}{8}}{\frac{4}{8}}$$

$$= \frac{3}{4}$$

(10) Given that, $\tan\theta = \frac{5}{12}$



Sol: By pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$= 5^2 + 12^2$$

$$= 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$\therefore bc = 13.$$

$$\sin\theta = \frac{ab}{bc}$$

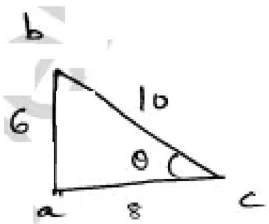
$$= \frac{5}{13}$$

$$\cos\theta = \frac{ac}{bc}$$

$$= \frac{12}{13}$$

(11) Sol:

Given $\sin\theta = \frac{6}{10}$



by pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$10^2 = 6^2 + ac^2$$

$$100 = 36 + ac^2$$

$$ac^2 = 100 - 36$$

$$ac^2 = 64$$

$$ac = \sqrt{64}$$

$$ac = 8$$

$$\therefore \cos\theta = \frac{ac}{bc} = \frac{8}{10}$$

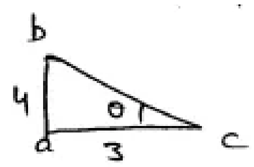
$$\tan\theta = \frac{ab}{ac} = \frac{6}{8}$$

$$\therefore \cos\theta + \tan\theta = \frac{8}{10} + \frac{6}{8} = \frac{64 + 60}{80}$$

$$= \frac{124}{80} \Rightarrow \frac{31}{20} = 1\frac{11}{20}$$

(12)
Sol

Given $\tan\theta = \frac{4}{3}$



by pythagorous theorem

$$ab^2 + ac^2 = bc^2$$

$$4^2 + 3^2 = bc^2$$

$$16 + 9 = bc^2$$

$$bc^2 = 25$$

$$bc = \sqrt{25}$$

$$bc = 5.$$

$$\therefore \sin\theta = \frac{ab}{bc} = \frac{4}{5}$$

$$\cos\theta = \frac{ac}{bc} = \frac{3}{5}$$

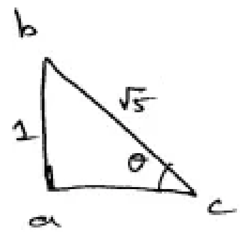
$$\therefore \sin\theta + \cos\theta = \frac{4}{5} + \frac{3}{5}$$

$$= \frac{4+3}{5}$$

$$= \frac{7}{5}$$

(13)
Sol

Given $\operatorname{cosec}\theta = \sqrt{5}$



$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{bc}{ab} = \frac{\sqrt{5}}{1}.$$

$$\therefore bc = \sqrt{5} \; ; \; ab = 1$$

By pythagorus theorem ;

$$bc^2 = ab^2 + ac^2$$

$$(\sqrt{5})^2 = 1^2 + ac^2$$

$$5 = 1 + ac^2$$

$$ac^2 = 5 - 1$$

$$ac^2 = 4$$

$$ac = \sqrt{4}$$

$$ac = 2.$$

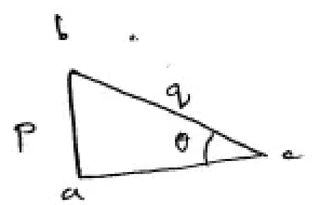
$$\therefore \cot\theta - \cos\theta = \frac{ac}{ab} - \frac{ac}{bc}$$

$$= \frac{2}{1} - \frac{2}{\sqrt{5}}$$

$$= \frac{2\sqrt{5} - 2}{\sqrt{5}}$$

$$= \frac{2(\sqrt{5} - 1)}{\sqrt{5}}$$

Solution - 14 :-



Given $\sin\theta = \frac{p}{q}$

by pythagorus theorem

$$q^2 = p^2 + ac^2$$

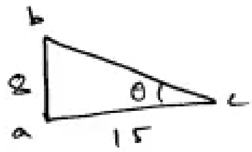
$$ac^2 = q^2 - p^2$$

$$ac = \sqrt{q^2 - p^2}$$

$$\cos\theta = \frac{ac}{bc} = \frac{\sqrt{-p^2 + q^2}}{q}$$

$$\therefore \cos\theta + \sin\theta = \frac{\sqrt{q^2 - p^2}}{q} + \frac{p}{q} = \frac{p + \sqrt{q^2 - p^2}}{q}.$$

Solution- 15



Given $\tan\theta = \frac{8}{15}$

by pythagorus theorem

$bc^2 = ab^2 + ac^2$

$bc^2 = 8^2 + 15^2$

$bc^2 = 64 + 225$

$bc^2 = 289$

$bc = \sqrt{289}$

$bc = 17.$

$\therefore ab = 8 \;;\; ac = 15 \;;\; bc = 17$

$\sec\theta = \frac{bc}{ac} = \frac{17}{15}$

$\operatorname{cosec}\theta = \frac{bc}{ab} = \frac{17}{8}$

$\therefore \sec\theta + \operatorname{cosec}\theta = \frac{17}{15} + \frac{17}{8}$

$= \frac{17\times 8 + 17\times 15}{120}$

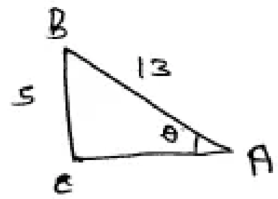
$= \frac{391}{120}$

$= 3\frac{31}{120}$

Solution-16 :

Given $13 \sin A = 5$

$\sin A = \frac{5}{13}$



by pythagorus theorem

$BA^2 = CB^2 + AC^2$

$\Rightarrow 13^2 = 5^2 + AC^2$

$$169 = 25 + AC^2$$

$$AC^2 = 169 - 25$$

$$AC^2 = 144$$

$$AC = \sqrt{144}$$

$$AC = 12$$

$$\therefore CB = 5 \; ; \; BA = 13 \; ; \; AC = 12$$

$$\sin A = \frac{5}{13}$$

$$\cos A = \frac{12}{13}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{\left(\frac{5}{\cancel{13}}\right)}{\left(\frac{12}{\cancel{13}}\right)}$$

$$\Rightarrow \frac{5}{12}$$

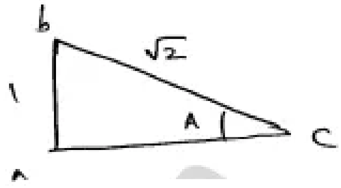
$$\therefore \frac{5\sin A - 2\cos A}{\tan A} = \frac{5\left(\frac{5}{13}\right) - 2\left(\frac{12}{13}\right)}{\left(\frac{5}{12}\right)}$$

$$= \frac{\frac{25}{13} - \frac{24}{13}}{\frac{5}{12}}$$

$$= \frac{\frac{25-24}{13}}{\frac{5}{12}}$$

$$= \frac{1}{13} \times \frac{12}{5} \Rightarrow \frac{12}{65}$$

Solution -17



Given $\quad \operatorname{cosec} A = \sqrt{2}$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \sqrt{2}$$

$$\therefore \sin A = \frac{1}{\sqrt{2}}$$

By pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$(\sqrt{2})^2 = 1^2 + ac^2$$

$$2 = 1 + ac^2$$

$$ac^2 = 2 - 1$$

$$ac^2 = 1$$

$$ac = \sqrt{1}$$

$$ac = 1$$

$\therefore ac = 1 \; ; \; bc = \sqrt{2} \; ; \; ab = 1$

$$\therefore \sin A = \frac{1}{\sqrt{2}}$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\left(\frac{1}{\cancel{\sqrt{2}}}\right)}{\left(\frac{1}{\cancel{\sqrt{2}}}\right)} = 1$$

$$\cot A = \frac{1}{\tan A}$$

$$= \frac{1}{1}$$

$$\cot A = 1$$

$$\therefore \frac{2\sin^2 A + 3\cot^2 A}{\tan^2 A - \cos^2 A}$$

$$\Rightarrow \frac{2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + 3\,(1)^2}{(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \frac{\cancel{2} \cdot \frac{1}{\cancel{2}} + 3 \cdot (1)}{1 - \frac{1}{2}}$$

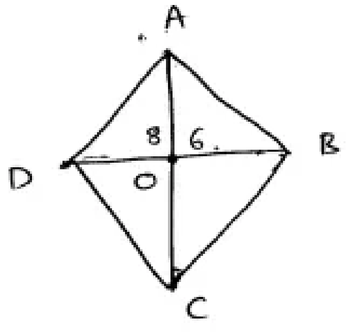
$$\Rightarrow \frac{1+3}{\left(\frac{2-1}{2}\right)}$$

$$\Rightarrow \frac{4 \times 2}{1}$$

$$\Rightarrow 8.$$

Solution 18 :-

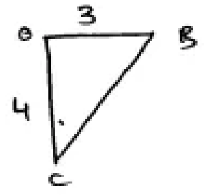
Given ABCD is a rhombus



$AC = 8$; $BD = 6$

From figure

ΔOBC ;



By pythagorus theorem

$BC^2 = OC^2 + OB^2$

$= 3^2 + 4^2$

$= 9 + 16 = 25$

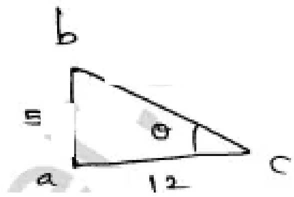
$BC = \sqrt{25} = 5$

$$\therefore \sin \angle OCB = \frac{OB}{BC}$$

$$= \frac{3}{5}.$$

<u>Solution-19 :-</u>

Given $\tan\theta = \frac{5}{12}$



By pythagorean theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = 5^2 + 12^2$$

$$bc^2 = 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$bc = 13$$

$\therefore$ $bc = 13$; $ab = 5$; $ac = 12$

$$\therefore \sin\theta = \frac{ab}{bc} = \frac{5}{13}$$

$$\cos\theta = \frac{ac}{bc} = \frac{12}{13}$$

$$\therefore \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$

$$= \frac{\frac{12+5}{\cancel{13}}}{\frac{12-5}{\cancel{13}}} = \frac{17}{7} = 2\frac{3}{7}$$

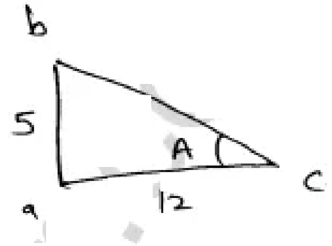
Solution - 20 :

Given $5\cos A - 12\sin A = 0$

$$5\cos A = 12\sin A$$

$$\frac{5}{12} = \frac{\sin A}{\cos A}$$

$$\frac{5}{12} = \tan A$$



By pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = 5^2 + 12^2$$

$$bc^2 = 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$bc = 13$$

$\therefore ab = 5$; $bc = 13$; $ac = 12$

$$\sin A = \frac{ab}{bc} = \frac{5}{13}$$

$$\cos A = \frac{ac}{bc} = \frac{12}{13}$$

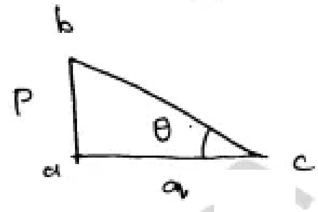
$$\therefore \frac{\sin A + \cos A}{2\cos A - \sin A} = \frac{\frac{5}{13} + \frac{12}{13}}{2 \cdot \frac{12}{13} - \frac{5}{13}}$$

$$= \frac{(5+12)/\cancel{13}}{(24-5)/\cancel{13}}$$

$$= \frac{17}{19}$$

२५

Solution - 21



Given $\tan\theta = \frac{p}{q}$

by pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = p^2 + q^2$$

$$bc = \sqrt{p^2+q^2}$$

$$\therefore \sin\theta = \frac{ab}{bc} = \frac{p}{\sqrt{p^2+q^2}}$$

$$\cos\theta = \frac{ac}{bc} = \frac{q}{\sqrt{p^2+q^2}}$$

$$\therefore \frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \frac{p \cdot \frac{p}{\sqrt{p^2+q^2}} - q \cdot \frac{q}{\sqrt{p^2+q^2}}}{p \cdot \frac{p}{\sqrt{p^2+q^2}} + q \cdot \frac{q}{\sqrt{p^2+q^2}}}$$

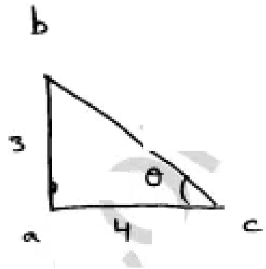
$$= \frac{\frac{p^2-q^2}{\cancel{\sqrt{p^2+q^2}}}}{\frac{p^2+q^2}{\cancel{\sqrt{p^2+q^2}}}}$$

$$= \frac{p^2-q^2}{p^2+q^2}$$

Solution - 22 :

Given $3\cot\theta = 4$

$\cot\theta = \frac{4}{3}$



By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$bc^2 = 3^2 + 4^2$

$bc^2 = 9 + 16$

$bc^2 = 25$

$bc = \sqrt{25}$

$bc = 5$

$\therefore \sin\theta = \frac{ab}{bc} = \frac{3}{5}$

$\cos\theta = \frac{ac}{bc} = \frac{4}{5}$

$$\therefore \frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{5\cdot\frac{3}{5} - 3\cdot\frac{4}{5}}{5\cdot\frac{3}{5} + 3\cdot\frac{4}{5}}$$

$$= \frac{(15 - 12)/5}{(15 + 12)/5}$$

$$= \frac{3}{27}$$

$$= \frac{1}{9}$$

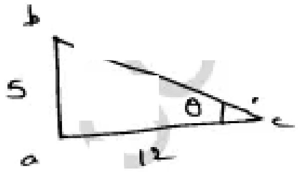
Solution - 23 (i)

Given $5\cos\theta - 12\sin\theta = 0$

$$5\cos\theta = 12\sin\theta$$

$$\frac{5}{12} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{5}{12} = \tan\theta$$



By pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = 5^2 + 12^2$$

$$bc^2 = 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$bc = 13.$$

$\therefore \sin\theta = \frac{ab}{bc} = \frac{5}{13}$

$\cos\theta = \frac{ac}{bc} = \frac{12}{13}$

$$\therefore \frac{\sin\theta + \cos\theta}{2\cos\theta - \sin\theta} = \frac{\frac{5}{13} + \frac{12}{13}}{2\cdot\frac{12}{13} - \frac{5}{13}}$$

$$= \frac{\frac{5+12}{\cancel{13}}}{\frac{24-5}{\cancel{13}}}$$

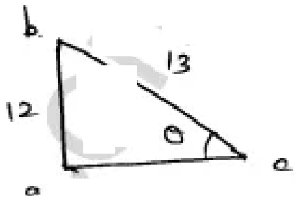
$$= \frac{17}{16} \,//$$

Solution-23 (ii) :-

Given $\operatorname{cosec}\theta = \frac{13}{12}$

$\Rightarrow \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{13}{12}$

$\therefore \sin\theta = \frac{12}{13}$



By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$13^2 = 12^2 + ac^2$

$169 = 144 + ac^2$

$169 - 144 = ac^2$

$25 = ac^2$

$ac = \sqrt{25}$

$ac = 5$

$\therefore \sin\theta = \frac{12}{13}$

$\cos\theta = \frac{ac}{bc} = \frac{5}{13}$

$\therefore \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2 \cdot \frac{12}{13} - 3 \cdot \frac{5}{13}}{4 \cdot \frac{12}{13} - 9 \cdot \frac{5}{13}}$

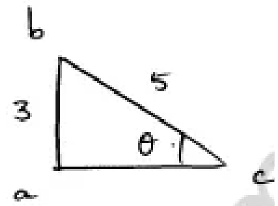
$= \frac{\frac{24 - 15}{13}}{\frac{48 - 45}{13}}$

$= \frac{9}{3} = 3$

Solution - 24 :-

Given $5\sin\theta = 3$

$\sin\theta = \frac{3}{5}$



By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$5^2 = 3^2 + ac^2$

$25 = 9 + ac^2$

$ac^2 = 25 - 9$

$ac^2 = 16$

$ac = \sqrt{16}$

$ac = 4.$

$\cos\theta = \frac{ac}{bc} = \frac{4}{5}$

$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/\cancel{5}}{4/\cancel{5}} = \frac{3}{4}..$

$$\therefore \frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5-3}{\cancel{4}}}{\frac{5+3}{\cancel{4}}}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}.$$

Solution - 25 :-

Given $\sin\theta = \cos\theta$

Then $2\tan^2\theta + \sin^2\theta - 1$

$\Rightarrow 2\left(\frac{\sin\theta}{\cos\theta}\right)^2 + \sin^2\theta - 1$

$\Rightarrow 2\left(\frac{\cancel{\sin\theta}}{\cancel{\sin\theta}}\right)^2 + \sin^2\theta - 1$

$\Rightarrow 2(1)^2 + \sin^2\theta - 1$

$\Rightarrow 2 + \sin^2\theta - 1$

$\Rightarrow \sin^2\theta + 1$

from given $\sin\theta = \cos\theta$

so $\Rightarrow \theta = 45$

$\therefore \sin 45 = \cos 45$

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore 2\tan^2\theta + \sin^2\theta - 1 \Rightarrow \sin^2\theta + 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + 1$

$\Rightarrow \frac{1}{2} + 1$

$\Rightarrow \frac{3}{2}$

$\therefore 2\tan^2\theta + \sin^2\theta - 1 = \frac{3}{2}$

Solution - 26 (i)

LHS $\Rightarrow \cos\theta \cdot \tan\theta$ $\quad \left(\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right)$

$\Rightarrow \cancel{\cos\theta} \cdot \dfrac{\sin\theta}{\cancel{\cos\theta}}$

$\Rightarrow \sin\theta$

RHS $= \sin\theta$

$\therefore$ LHS $=$ RHS.

Solution - 26 (ii)

LHS $\Rightarrow \sin\theta \cdot \cot\theta$ $\quad \because \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \cancel{\sin\theta} \cdot \dfrac{\cos\theta}{\cancel{\sin\theta}}$

$\Rightarrow \cos\theta$

$\therefore$ LHS $=$ RHS.

Solution - 26 (iii)

LHS $\Rightarrow \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$

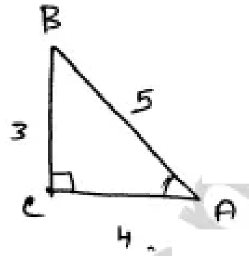
$\Rightarrow \dfrac{\sin^2\theta + \cos\theta \cdot \cos\theta}{\cos\theta}$

$\Rightarrow \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}$ $\quad \left(\because \sin^2\theta + \cos^2\theta = 1\right)$

$\Rightarrow \dfrac{1}{\cos\theta}$

$\therefore$ LHS $=$ RHS.

Solution- 27 :-



Given $\angle C = 90°$

$\tan A = \frac{3}{4}$.

By pythagorus theorem

$AB^2 = BC^2 + AC^2$

$AB^2 = 3^2 + 4^2$

$AB^2 = 9 + 16$

$AB^2 = 25$

$AB = \sqrt{25}$

$AB = 5$

$\therefore \sin A = \frac{BC}{AB} = \frac{3}{5}$

$\cos A = \frac{AC}{AB} = \frac{4}{5}$

$\cos B = \frac{BC}{AB} = \frac{3}{5}$

$\sin B = \frac{AC}{AB} = \frac{4}{5}$

$\therefore \sin A \cos B + \cos A \sin B \Rightarrow \frac{3}{5} \cdot \frac{3}{5} + \frac{4}{5} \cdot \frac{4}{5}$

$\Rightarrow \frac{9}{25} + \frac{16}{25}$

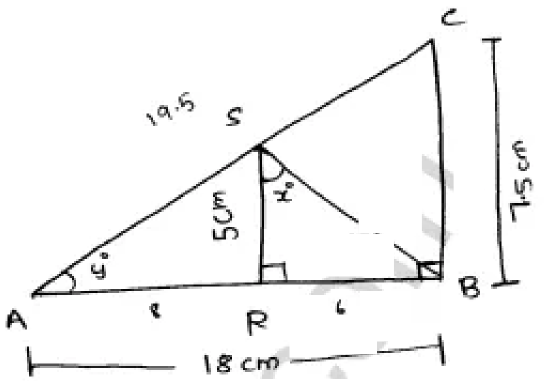
$\Rightarrow \frac{9+16}{25}$

$\Rightarrow \frac{25}{25}$

$= 1$

$\therefore$ LHS = RHS.

Solution - 28 :- (a)



Given

In ΔABC, ΔBRS

AB = 18cm

BC = 7.5 cm.

RS = 5 cm

$\angle BSR = x^\circ$

$\angle SAB = y^\circ$

From Hint ; AR = 12 cm

RB = 6 cm.

AC = 19.5

(i) $\tan x^\circ = \frac{RB}{SR} = \frac{6}{5}$

(ii) $\sin y = \frac{BC}{AC}$

$= \frac{7.5}{19.5} \times \frac{10}{10}$

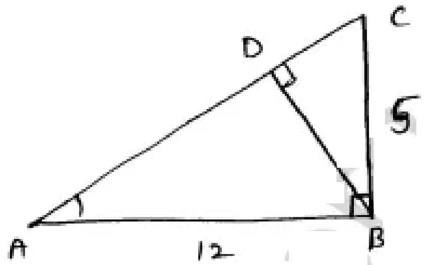
$= \frac{\cancel{75}\ 15\ 5}{\cancel{195}\ \cancel{39}\ 13}$

$= \frac{5}{13}$

$\therefore \sin y = \frac{5}{13}$

Solution - 28 (b) :-

(i) From fig



By pythagorus theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

$AC = \sqrt{169}$

$AC = 13.$

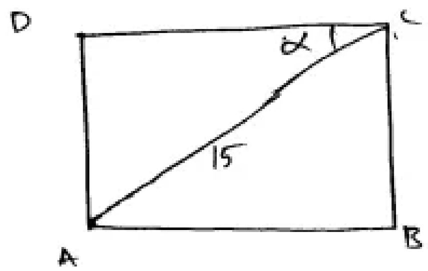
$\therefore \cos \angle CBD = \frac{AB}{AC}$

$= \frac{12}{13}$

(ii) $\cot \angle ABD = \frac{BC}{AC}$

$= \frac{5}{13}.$

Solution - 24 :-



Given ABCD is rectangle

$AC = 15$

$\angle ACD = \alpha$; $\cot\alpha = \frac{3}{2}$

$\cot\alpha = \frac{CD}{AD} = \frac{3}{2}$

$\therefore CD = \frac{3}{2} AD$

From ΔACD

$$AC^2 = AD^2 + CD^2$$

$$15^2 = AD^2 + \left(\frac{3}{2}AD\right)^2$$

$$225 = AD^2 + \frac{9}{4}AD^2$$

$$225 = \frac{4AD^2 + 9AD^2}{4}$$

$$225 \times 4 = 13AD^2$$

$$AD^2 = \frac{225 \times 4}{13}$$

$$AD = \sqrt{\frac{225 \times 4}{13}}$$

$$AD = \frac{15 \times 2}{\sqrt{13}}$$

$$AD = \frac{30}{\sqrt{13}}$$

$$\therefore CD = \frac{3}{2}AD$$

$$= \frac{3}{\cancel{2}} \cdot \frac{\overset{15}{\cancel{30}}}{\sqrt{3}}$$

$$= \frac{45}{\sqrt{13}}$$

$\therefore$ Area of ΔACD + Area of ΔABC = Area of rectangle ABCD.

$$\therefore CD = \frac{45}{\sqrt{13}} = AB$$

$$AD = \frac{30}{\sqrt{13}} = BC$$

$\therefore$ Area of $\square ABCD$ = $CD \times AD$ $= \frac{45}{\sqrt{13}} \times \frac{30}{\sqrt{13}}$

$$\text{Area} = \frac{45 \times 30}{\sqrt{13} \cdot \sqrt{13}}$$

$$= \frac{1350}{13}$$

$$= 103\frac{11}{13}\ cm^2.$$

$$\therefore \text{Perimeter} = 2(AB + BC)$$

$$= 2\left(\frac{45}{\sqrt{13}} + \frac{30}{\sqrt{13}}\right)$$

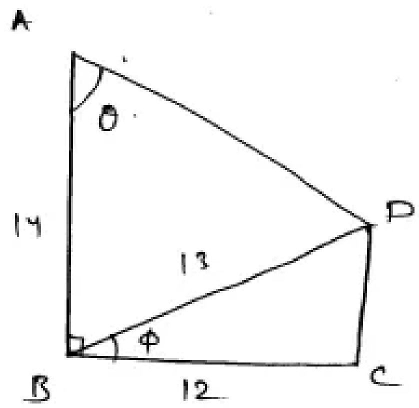
$$= 2\left(\frac{45+30}{\sqrt{13}}\right)$$

$$= 2 \times \frac{75}{\sqrt{13}}$$

$$= \frac{150}{\sqrt{13}}$$

Solution - 30 :-

(a)



From ΔBCD

$\Rightarrow$ By pythagorus theorem

$BD^2 = BC^2 + CD^2$

$13^2 = 12^2 + CD^2$

$169 = 144 + CD^2$

$CD^2 = 169 - 144$

$CD^2 = 25$

$CD = \sqrt{25} = 5$

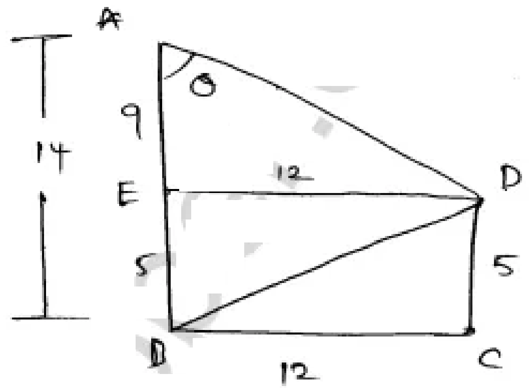
(i) $\sin\phi = \frac{CD}{BD}$

$= \frac{5}{13}$

(ii) $\tan\theta = \frac{DE}{AE}$

$= \frac{12}{9} = \frac{4}{3}$

Solution - 30 (b) :-



$\tan\theta = \frac{4}{3}$

$\therefore \sin\theta = \frac{DE}{AD}$

$AD = \frac{12}{\sin\theta}$.

(or) $\cos\theta = \frac{AE}{AD}$

$AD = \frac{9}{\cos\theta}$.

Solution - 31 :- (i)

LHS $\Rightarrow (\sin A + \cos A)^2 + (\sin A - \cos A)^2$

$\Rightarrow \sin^2 A + \cos^2 A + \cancel{2\sin A\cos A} + \sin^2 A + \cos^2 A - \cancel{2\sin A\cos A}$.

$\left(\because (a+b)^2 = a^2+b^2+2ab \atop (a-b)^2 = a^2+b^2-2ab \right)$

$\Rightarrow 2\sin^2 A + 2\cos^2 A$

$\Rightarrow 2(\sin^2 A + \cos^2 A)$

$\Rightarrow 2(1) = 2$

$\therefore$ LHS = RHS.

Solution - 31 (ii) :-

LHS $\Rightarrow \cot^2 A - \dfrac{1}{\sin^2 A} + 1$

$\Rightarrow \dfrac{\cos^2 A}{\sin^2 A} - \dfrac{1}{\sin^2 A} + \cancel{\text{or}}\ 1$

$\Rightarrow \dfrac{\cos^2 A - 1 + \sin^2 A}{\sin^2 A}$

$\Rightarrow \dfrac{(\cos^2 A + \sin^2 A) - 1}{\sin^2 A}$

$\Rightarrow \dfrac{1-1}{\sin^2 A}$

$\Rightarrow 0 = \text{RHS}.$

Solution - 31 (iii) :-

LHS $\Rightarrow \dfrac{1}{1+\tan^2 A} + \dfrac{1}{1+\cot^2 A}$

$\Rightarrow \dfrac{1}{1+\dfrac{\sin^2 A}{\cos^2 A}} + \dfrac{1}{1+\dfrac{\cos^2 A}{\sin^2 A}}$

$\Rightarrow \dfrac{1}{\dfrac{\cos^2 + \sin^2 A}{\cos^2 A}} + \dfrac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin^2 A}}$

$= \dfrac{\cos^2 A}{(1)} + \dfrac{\sin^2 A}{(1)}$

$\Rightarrow \cos^2 A + \sin^2 A$

$\Rightarrow 1 = \text{RHS}$ //.

Solution - 32 :

Given $\sqrt{\dfrac{1-\sin^2\theta}{1-\cos^2\theta}}$ $\quad (\because \sin^2\theta+\cos^2\theta=1)$

$\Rightarrow \sqrt{\dfrac{\cos^2\theta}{\sin^2\theta}}$

$\Rightarrow \sqrt{\cot^2\theta}$

$\Rightarrow \cot\theta$.

Solution - 33 :-

Given $\sin\theta + \csc\theta = 2$

Squaring on both sides.

$(\sin\theta + \csc\theta)^2 = 2^2$

$\sin^2\theta + \csc^2\theta + 2\cdot\sin\theta\cdot\csc\theta = 4.$

$\sin^2\theta + \csc^2\theta + 2\cancel{\sin\theta}\dfrac{1}{\cancel{\sin\theta}} = 4$

$\sin^2\theta + \csc^2\theta = 4-2$

$\sin^2\theta + \csc^2\theta = 2$

Solution - 34 :-

Given $x = a\cos\theta + b\sin\theta$.

$y = a\sin\theta - b\cos\theta$.

Squaring on b.s.

$x^2 = (a\cos\theta + b\sin\theta)^2$.

$$x^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta \quad ---(1)$$

$\because$ $y = a\sin\theta - b\cos\theta$

Squaring on b.s.

$$y^2 = (a\sin\theta - b\cos\theta)^2$$

$$y^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta \quad ---(2)$$

① + ②

$$\Rightarrow x^2+y^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$

$$= a^2(1) + b^2(1)$$

$$= a^2+b^2$$

$\therefore$ LHS = RHS

Hence proved.