Coordinate Geometry

EXERCISE 19.1

Question 1.
Find the co-ordinates of points whose
(i) abscissa is 3 and ordinate -4.
(ii) abscissa is – $\frac{3}{2}$and ordinate 5.
(iii) whose abscissa is -1$\frac{2}{3}$ and ordinate -2$\frac{1}{4}$.
(iv) whose ordinate is 5 and abscissa is -2
(v) whose abscissa is -2 and lies on x-axis.
(vi) whose ordinate is $\frac{3}{2}$ and lies on y-axis.
Solution:

(*i*) The co-ordinates of point whose abscissa is 3 and ordinate –4 = (3, –4)

(*ii*) The co-ordinates of point whose abscissa is $\frac{-3}{2}$ and ordinate 5 = $\left(\frac{-3}{2}, 5\right)$

(*iii*) The co-ordinates of point whose abscissa is $-1\frac{2}{3}$ and ordinate $-2\frac{1}{4}$ = $\left(-1\frac{2}{3}, -2\frac{1}{4}\right)$

(*iv*) The co-ordinates of point whose ordinate is 5 and abscissa is –2 = (–2, 5)

(*v*) The co-ordinates of points whose abscissa is –2 and lies on *x*-axis = (–2, 0)

(*vi*) The co-ordinates of points whose ordinate is $\frac{3}{2}$ and lie on *y*-axis = $\left(0, \frac{3}{2}\right)$

Question 2.
In which quadrant or on which axis each of the following points lie?
(-3, 5), (4, -1) (2, 0), (2, 2), (-3, -6)
Solution:

Points (–3, 5) lies in II quadrant
(4, –1) in IV quadrant
(2, 0) on *x*-axis
(2, 2) in I quadrant
(3, –6) in III quadrant

Question 3.
Which of the following points lie on
(i) x-axis? (ii) y-axis?
A (0, 2), B (5, 6), C (23, 0), D (0, 23), E (0, -4), F (-6, 0), G (√3,0)

Solution:

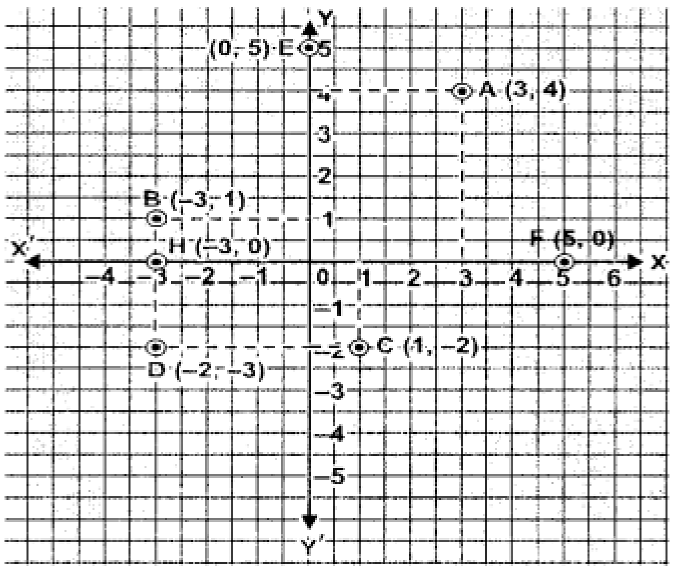
On x-axis C (23, 0), F (–6, 0), G ($\sqrt{3}$, 0)

On y-axis : A (0, 2), D (0, 23), E (0, –4)

Question 4.

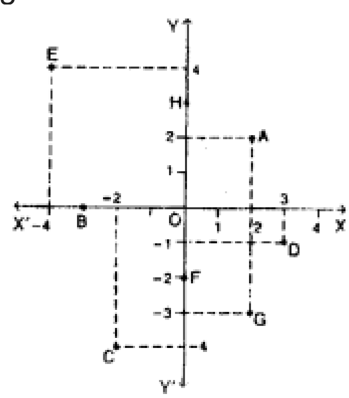
Plot the following points on the same graph paper :
A (3, 4), B (-3, 1), C (1, -2), D (-2, -3), E (0, 5), F (5, 0), G (0, -3), H (-3, 0).

Solution:



Question 5.

Write the co-ordinates of the points A, B, C, D, E, F, G and H shown in the adjacent figure.



Solution:

. Co-ordinates of the points

A (2, 2), B (– 3, 0), C (–2, –4), D (3, –1), E (–4, 4) F (0, –2), G (2, –3), H (0, 3)

Question 6.

In which quadrants are the points A, B, C and D of problem 3 located ?

Solution:

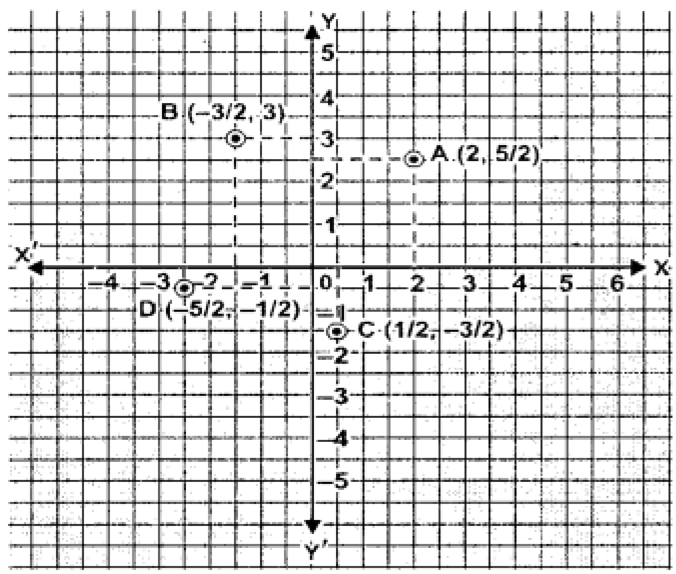
A Lies in the first quadrant, B lies on x-axis C lies in the third quadrant and D lies in the fourth quadrant.

Question 7.

Plot the following points on the same graph paper :

$$A\left(2, \frac{5}{2}\right), B\left(-\frac{3}{2}, 3\right), C\left(\frac{1}{2}, -\frac{3}{2}\right) \text{ and } D\left(-\frac{5}{2}, -\frac{1}{2}\right).$$

Solution:

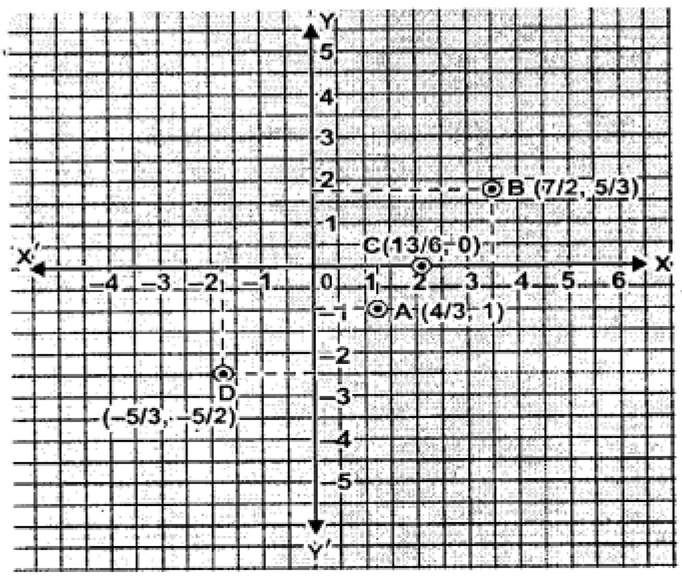


Question 8.

Plot the following points on the same graph paper.

$$A\left(\frac{4}{3} - 1\right), B\left(\frac{7}{2}, \frac{5}{3}\right), C\left(\frac{13}{6}, 0\right), D\left(-\frac{5}{3}, -\frac{5}{2}\right).$$

Solution:



Question 9.

Plot the following points and check whether they are collinear or not:

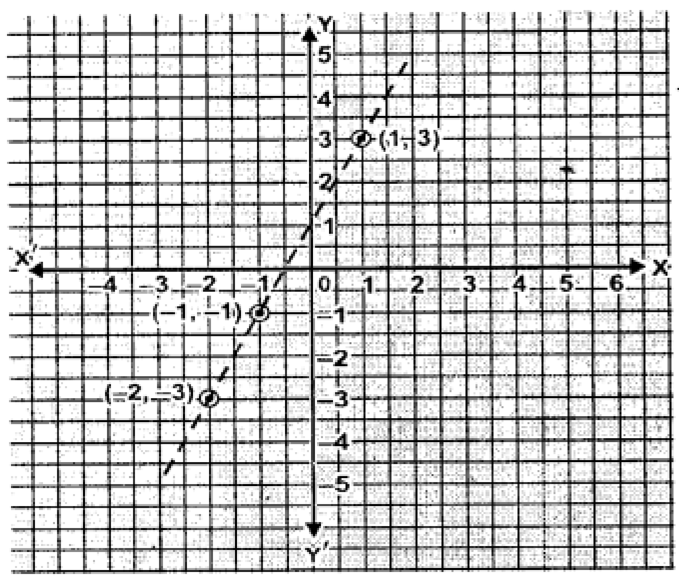
(i) (1,3), (-1,-1) and (-2,-3)

(ii) (1,2), (2,-1) and (-1, 4)

(iii) (0,1), (2, -2) and ($\frac{2}{3}$,0)

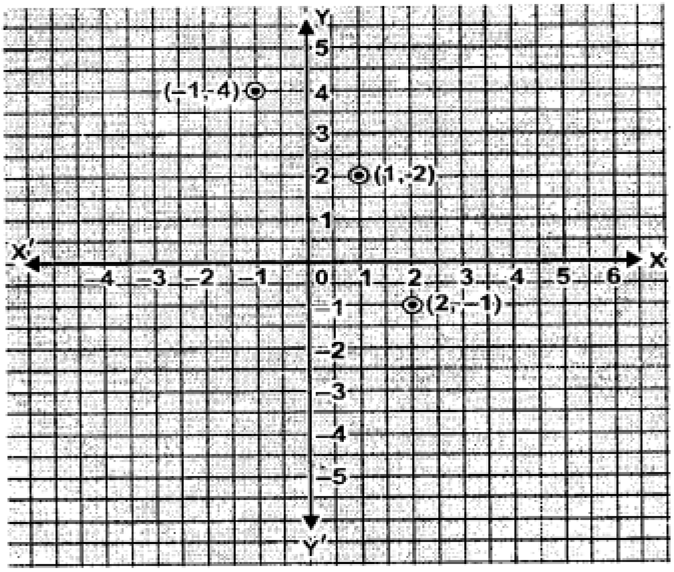
Solution:

(*i*) (1, 3), (–1, –1) and (–2, –3)



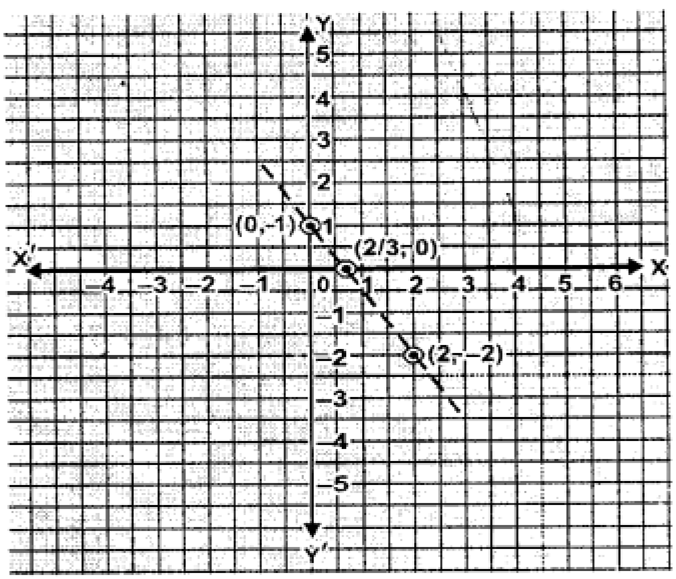
These points are collinear.

(*ii*) (1, 2) (2, –1) and (–1, 4)



These points are not collinear.

(*iii*) (0, 1), (2, −2) and $\left(\frac{2}{3}, 0\right)$

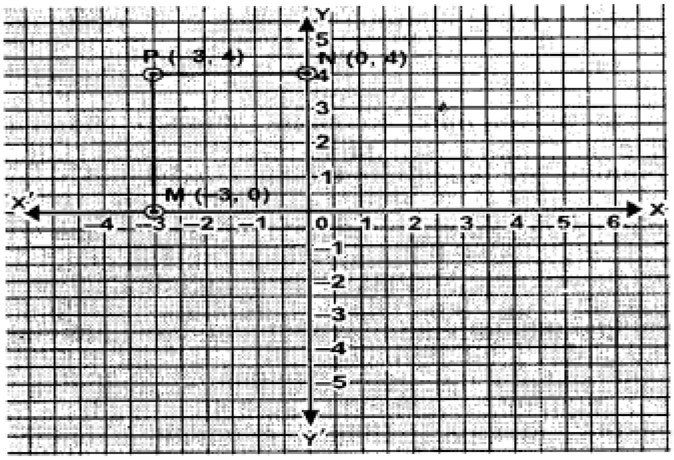


These points are collinear

Question 10.

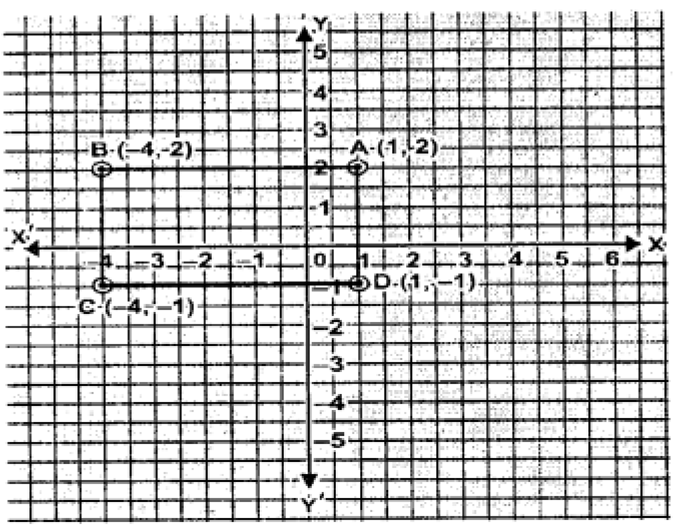
Plot the point P(-3, 4). Draw PM and PN perpendiculars to x-axis and y-axis respectively. State the co-ordinates of the points M and N.

Solution:



Co-ordinates of point M → (–3, 0)
Co-ordinates of point N → (0, 4)

Question 11.
Plot the points A (1,2), B (-4,2), C (-4, -1) and D (1, -1). What kind of quadrilateral is ABCD ? Also find the area of the quadrilateral ABCD.
Solution:
. Given points A(1, 2), B (–4, 2), C (–4, –1) and (1, -1)

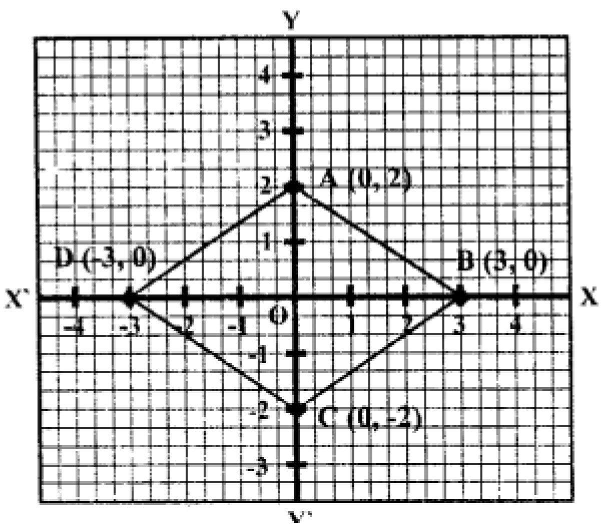


quadrilateral ABCD is rectangle.
Area of rectangle ABCD = AB × BC
= [1 – (–4)] × [2 –(–1)] sq. units
= 5 × 3 sq. units = 15 sq. units.

Question 12.
Plot the points (0,2), (3,0), (0, -2) and (-3,0) on a graph paper. Join these points (in order). Name the figure so obtained and find the area of the figure obtained.

Solution:

The given points A (0, 2), B (3, 0), C (0, –2) and D (–3, 0) have been plotted o[n] the graph and these points are joined in orde[r] we get a quadrilateral which is a rhombus a[s] shown in the graph



AC and BD are its diagonals AC = 4 units and BD = 6 units

$\therefore$ Its area $= \dfrac{d_1 \times d_2}{2}$

$= \dfrac{4 \times 6}{2}$ sq. units

$= 12$ sq. units

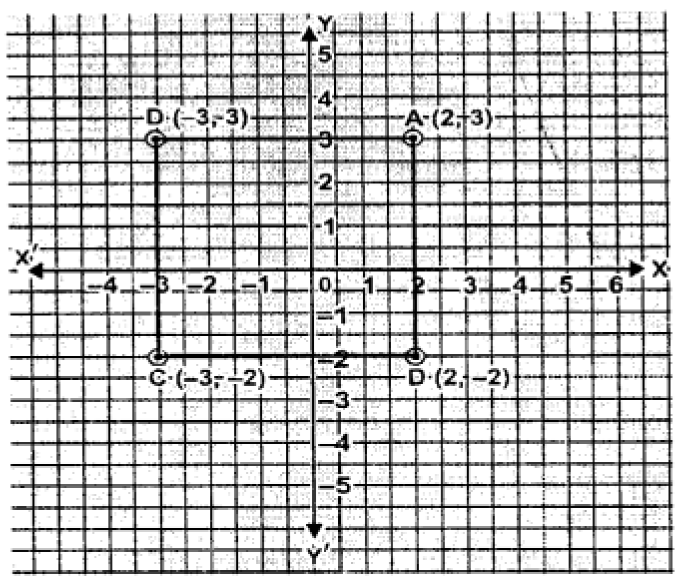
Question 13.

Three vertices of a square are A (2,3), B(-3, 3) and C (-3, -2). Plot these points on a graph paper and hence use it to find the co-ordinates of the fourth vertex. Also find the area of the square.

Solution:

Given three vertices of a square are

A (2, 3), B (–3, 3) and C(–3, –2)



From graph fourth vertices of square is D(2, –2)

Area of square ABCD = AB × AB

[∵ area of square = side × side]

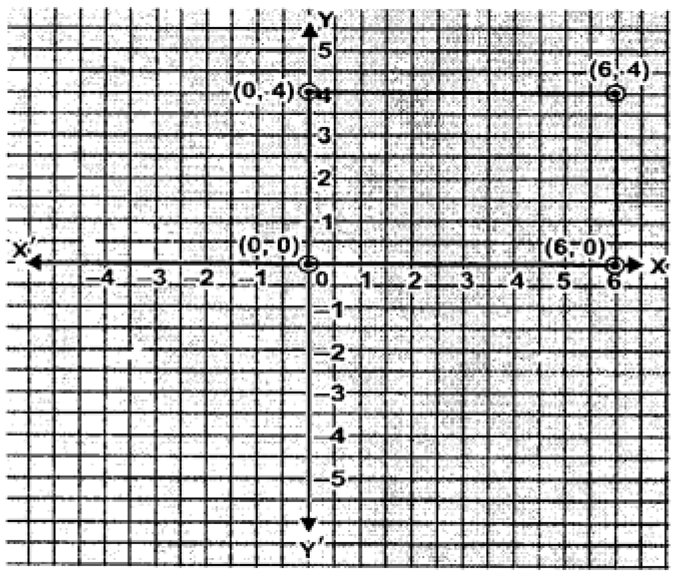
= 5 × 5 sq. units = 25 sq. units.

Question 14.

Write the co-ordinates of the vertices of a rectangle which is 6 units long and 4 units wide if the rectangle is in the first quadrant, its longer side lies on the x-axis and one vertex is at the origin.

Solution:

A rectangle which is 6 units long and 4 units wide and this rectangle is in the first quadrant.



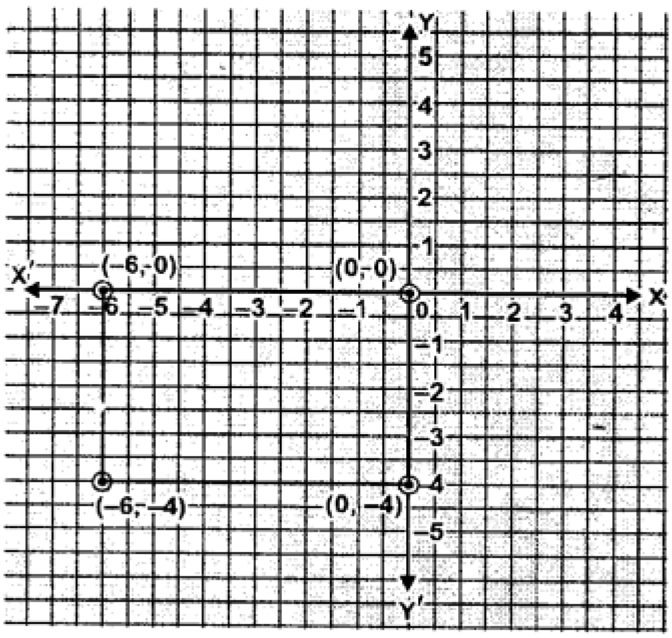
Co-ordinates of rectangle are (0, 0), (6, 0), (6, 4), (0, 4).

Question 15.

Repeat problem 12 assuming that the rectangle is in the third quadrant with all other conditions remaining the same.

Solution:

A rectangle which is 6 unit long and 4 units wide and this rectangle is in the third quadrant.

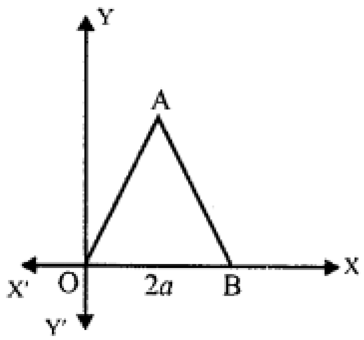


Co-ordinates of rectangle are (0, 0), (–6, 0), (–6, –4), (0, –4).

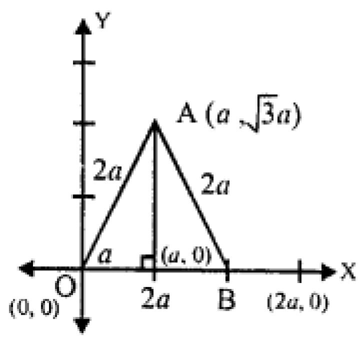
Question 16.

The adjoining figure shows an equilateral triangle OAB with each side = 2a units. Find the coordinates of the vertices.

Solution:



In the figure given,
OAB is an equilateral triangle and its each side is $2a$ units



Draw AD $\perp$ OB

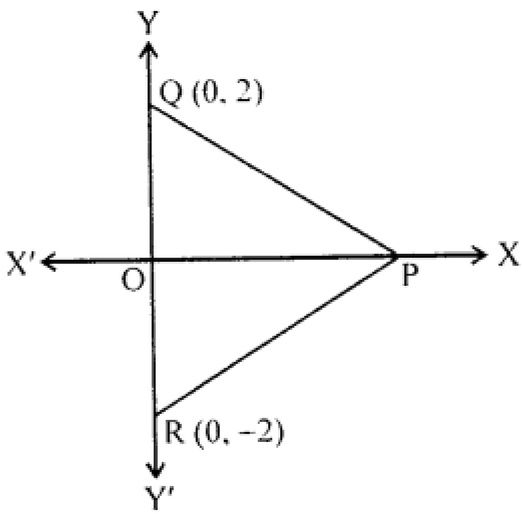
$$DA = \sqrt{OA^2 - OD^2} = \sqrt{(2a)^2 - a^2}$$

$$= \sqrt{4a^2 - a^2} = \sqrt{3}\, a$$

Co-ordinates of O (0, 0), of A $\left(a, \sqrt{3}a\right)$ and of B $(2a, 0)$.

Question 17.

In the given figure, APQR is equilateral. If the coordinates of the points Q and R are (0, 2) and (0, -2) respectively, find the coordinates of the point P.



Solution:

In the figure, PQR is an equilateral triangle in which Q (0, 2) and R (0, –2).

Let coordinates of P be $(x, 0)$ as it lies on x-axis.

$\because$ PQ = PR = QR = 2 + 2 = 4

In right ΔPQO

$OP^2 = PQ^2 - QO^2$

$= 4^2 - 2^2 = 16 - 4 = 12$

$\therefore$ OP = $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

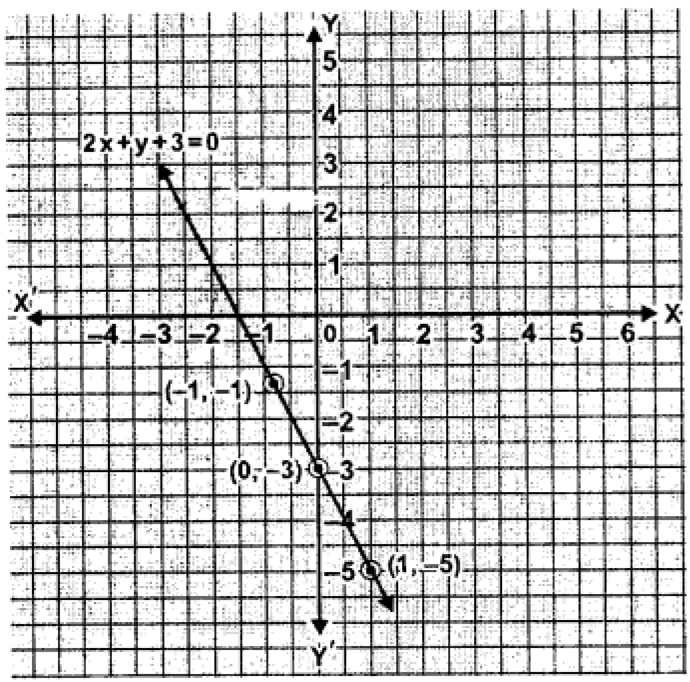
$\therefore$ Co-ordinates of P will be $(2\sqrt{3}, 0)$

EXERCISE 19.2

Question 1.
Draw the graphs of the following linear equations :
(i) 2x + + 3 = 0
(ii) x- 5y- 4 = 0
Solution:

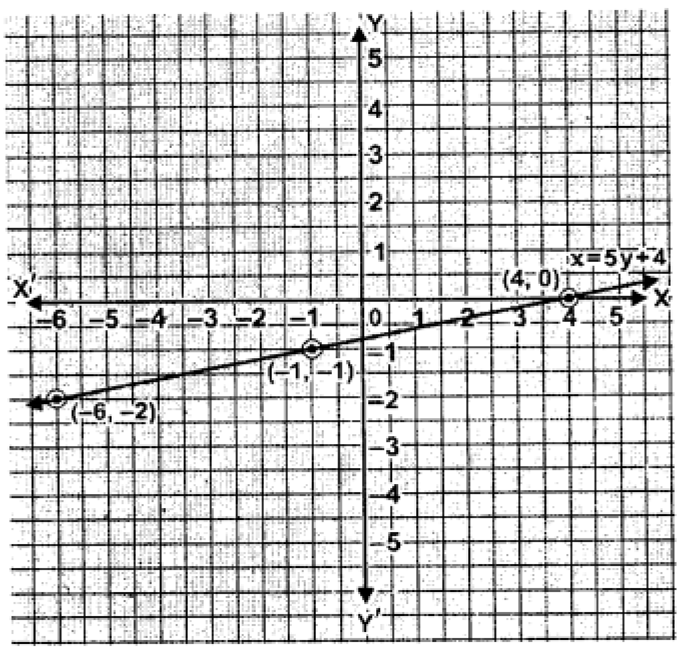
(*i*) $2x + y + 3 = 0 \quad \Rightarrow \quad y = -2x - 3$

x	0	1	–1
y	–3	–5	–1



(*ii*) $x - 5y - 4 = 0 \quad \Rightarrow \quad x = 5y + 4$

x	4	–1	–6
y	0	–1	–2

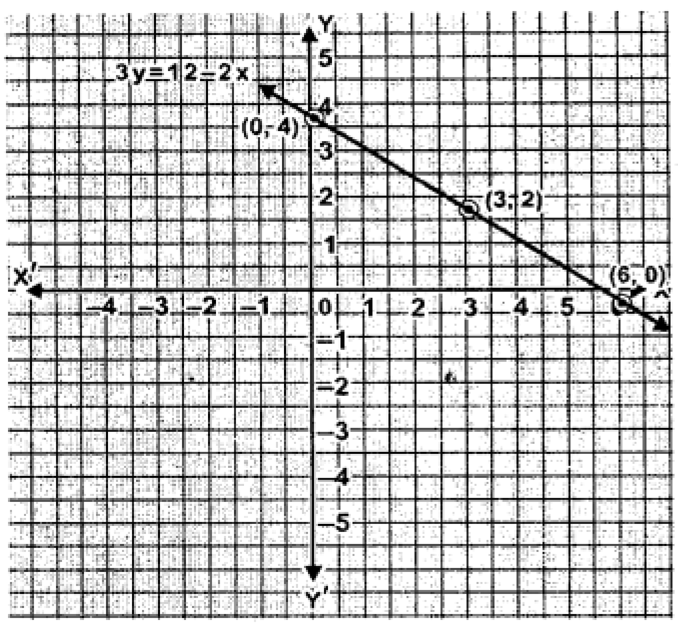


Question 2.
Draw the graph of 3y= 12 – 2x. Take 2cm = 1 unit on both axes.

Solution:

$$3y = 12 - 2x \quad \Rightarrow \quad y = \frac{12 - 2x}{3}$$

x	0	3	6
y	4	2	0



Question 3.

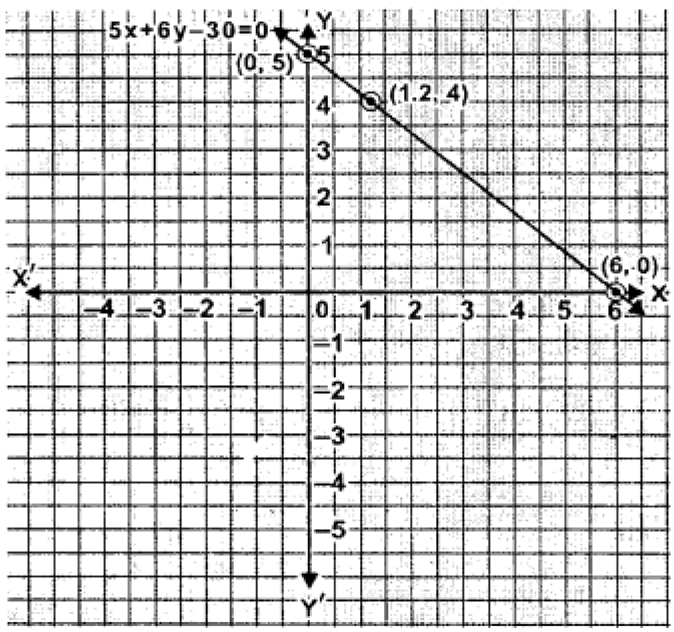
Draw the graph of 5x + 6y – 30 = 0 and use it to find the area of the triangle formed by the line and the co-ordinate axes.

Solution:

$$5x + 6y - 30 = 0 \quad \Rightarrow \quad 5x + 6y = 30$$

$$\Rightarrow \quad 5x = 30 - 6y \quad \Rightarrow \quad x = \frac{30 - 6y}{5}$$

x	6	1.2	0
y	0	4	5



Area of triangle formed by the line and coordinate axes

$$= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 6 \times 5 = 3 \times 5 = 15$$

square units.

Question 4.

Draw the graph of 4x- 3y + 12 = 0 and use it to find the area of the triangle formd by the line and the co-ordinate axes. Take 2 cm = 1 unit on both axes.

Solution:

$$4x - 3y + 12 = 0$$

$$\Rightarrow \quad 4x = 3y - 12 \quad \Rightarrow \quad x = \frac{3y - 12}{4}$$

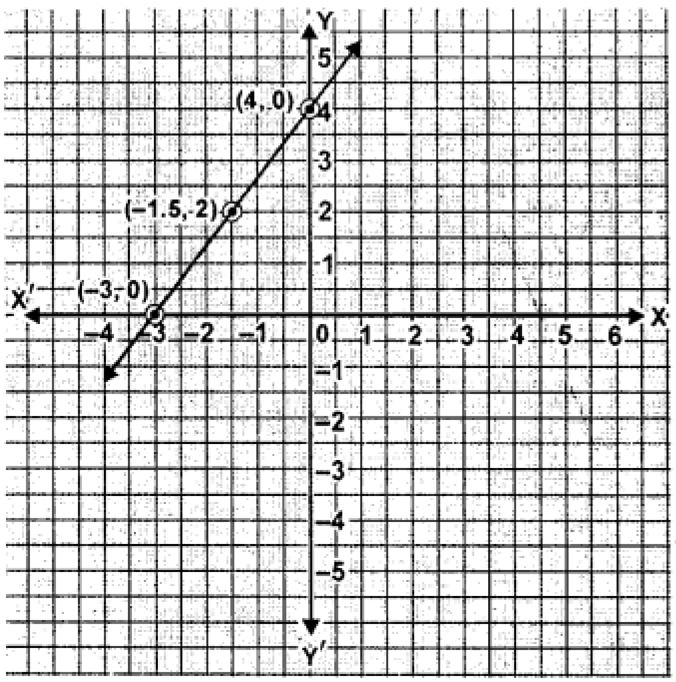
When $y = 0$, $x = \frac{3 \times 0 - 12}{4} = \frac{0 - 12}{4} = \frac{-12}{4} = -3$

$y = 2, x = \frac{3 \times 2 - 12}{4} = \frac{6 - 12}{4} = \frac{-6}{4} = -1.5$

$y = 4, x = \frac{3 \times 4 - 12}{4} = \frac{12 - 12}{4} = \frac{0}{4} = 0$

Table of values

x	–3	–1.5	0
y	0	2	4



Area of the triangle formed by the line and the

co-ordinate axes $= \frac{1}{2} \times |OA| \times |OB|$

$= \frac{1}{2} \times 3 \times 4 = \frac{1}{2} \times 4 \times 3 = 2 \times 3 = 6$ Sq. units.

Question 5.

Draw the graph of the equation y = 3x – 4. Find graphically.
(i) the value of y when x = -1
(ii) the value of x when y = 5.

Solution:

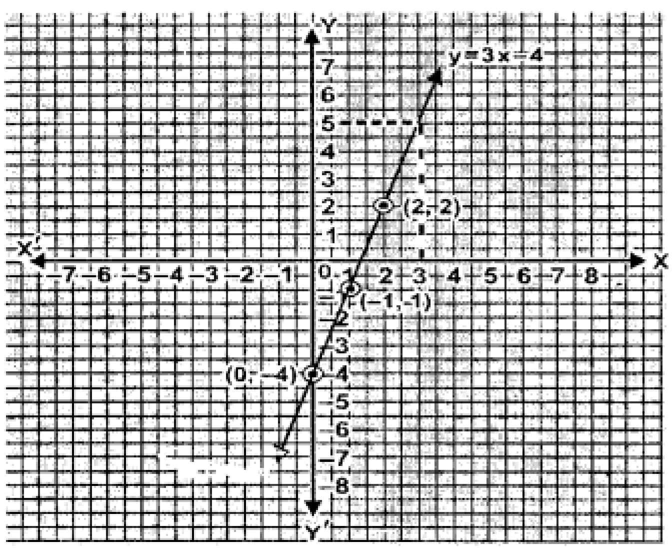
$y = 3x - 4$, when $x = 0$, $y = 3 \times 0 - 4 = 0 - 4 = -4$

$x = 1$, $y = 3 \times 1 - 4 = 3 - 4 = -1$

$x = 2$, $y = 3 \times 2 - = 6 - 4 = 2$

Table of values

x	0	1	2
y	–4	–1	2



(*i*) The value of $y = -7$, when $x = -1$

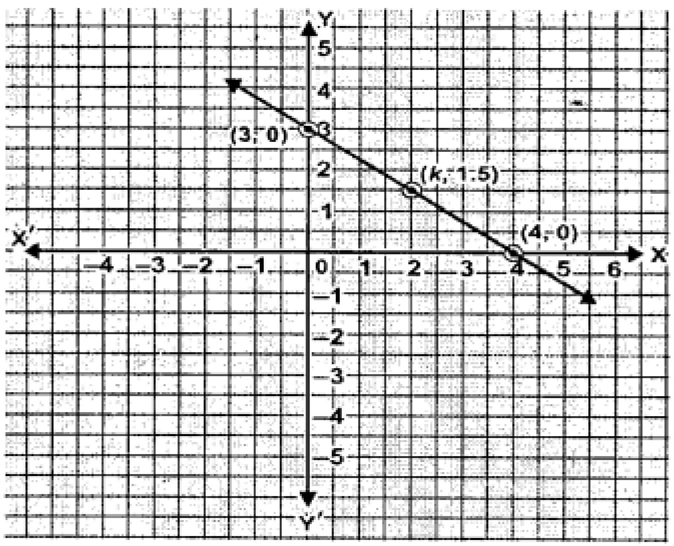
(*ii*) The value of $x = 3$, when $y = 5$.

Question 6.

The graph of a linear equation in x and y passes through (4, 0) and (0, 3). Find the value of k if the graph passes through (A, 1.5).

Solution:

Plot the points A (4, 0) and B (0, 3) on the graph paper. From graph it is clear that $k = 2$



Question 7.

Use the table given alongside to draw the graph of a straight line. Find, graphically the values of a and b.

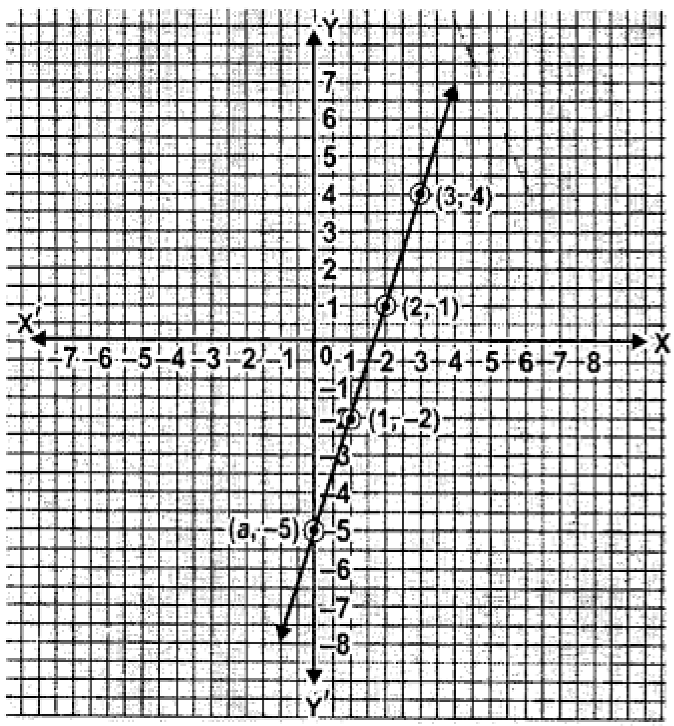
x	1	2	3	a
y	–2	b	4	–5

Solution:

Given table

x	1	2	3	a
y	–2	b	4	–5

Draw the graph on graph paper and it is clear that from graph, $b = 1$, $a = 0$



Hence, $a = 0$, $b = 1$

EXERCISE 19.3

Question 1.

Solve the following equations graphically: $3x - 2y = 4$, $5x - 2y = 0$

Solution:

Given equations are

$3x - 2y = 4$ and $5x - 2y = 0$

$\Rightarrow \quad 3x - 2y = 4 \quad \Rightarrow \quad -2y = 4 - 3x$

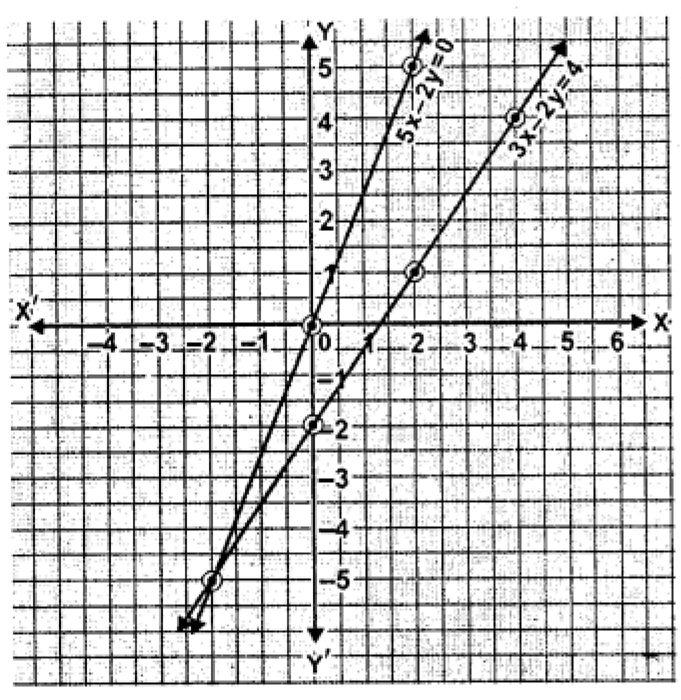
$$\Rightarrow \quad y = \frac{4 - 3x}{-2} = \frac{-4 + 3x}{2} \quad \Rightarrow \quad y = \frac{3x - 4}{2}$$

x	0	2	4
y	–2	1	4

and $5x - 2y = 0 \quad \Rightarrow \quad 5x = 2y \quad \Rightarrow \quad 2y = 5x$

$$\Rightarrow \quad y = \frac{5x}{2}$$

x	0	2	–2
y	0	5	–5



From graph, $y = -2$, $y = -5$.

Question 2.
Solve the following pair of equations graphically. Plot at least 3 points for each straight line $2x - 7y = 6$, $5x - 8y = -4$
Solution:

Given equations are

$2x - 7y = 6$ and $5x - 8y = -4$

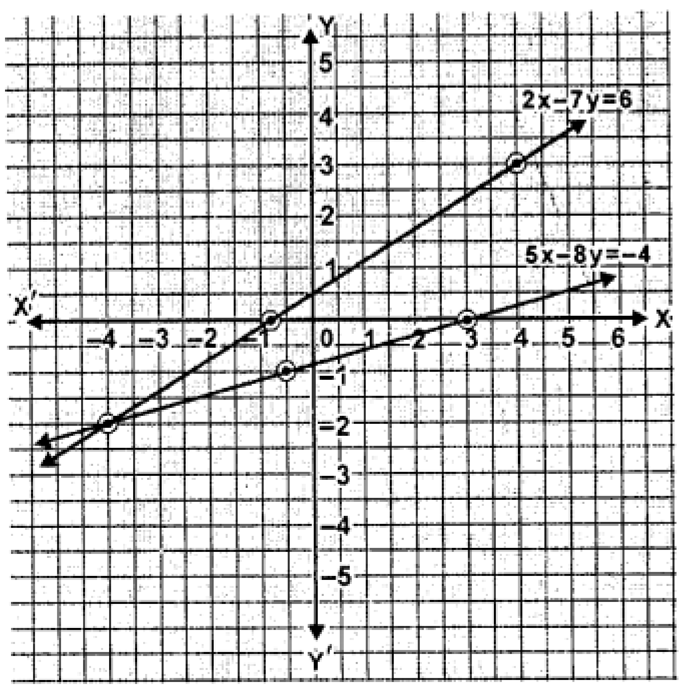
$2x = 6 + 7y \Rightarrow x = \frac{6 + 7y}{2}$

x	3	–0.5	–4
y	0	–1	–2

Also, $5x - 8y = -4 \Rightarrow 5x = -4 + 8y$

$\Rightarrow x = \frac{-4 + 8y}{5}$

x	–0.8	4	–4
y	0	3	–2



From graph, $x = -4$, $y = -2$

Question 3.
Using the same axes of co-ordinates and the same unit, solve graphically.
x+y = 0, 3x – 2y = 10
Solution:

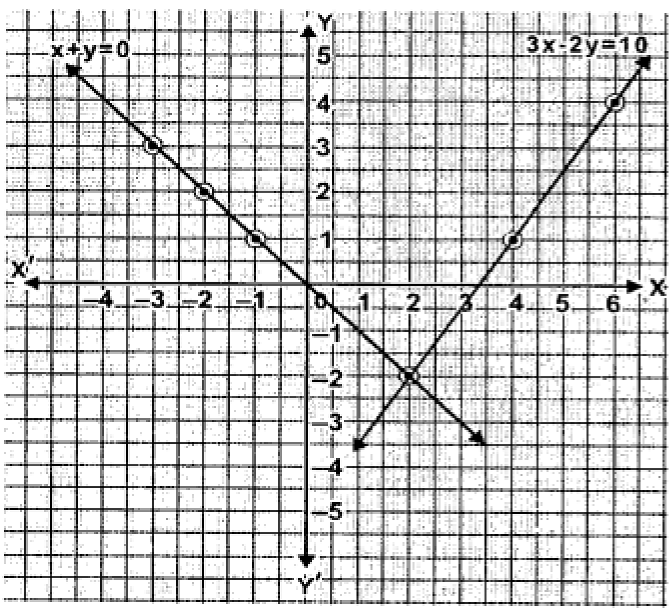
Given equations are $x + y = 0$ and $3x - 2y = 10$

$\Rightarrow \quad x + y = 0 \quad \Rightarrow \quad x = -y$

x	-1	-2	-3
y	1	2	3

$3x - 2y = 10 \quad \Rightarrow \quad x = \frac{10 + 2y}{3}$

x	4	4	6
y	1	-2	4



From graph $x = 2, \; y = -2$

Question 4.
Take 1 cm to represent 1 unit on each axis to draw the graphs of the equations 4x- 5y = -4 and 3x = 2y – 3 on the same graph sheet (same axes). Use your graph to find the solution of the above simultaneous equations.
Solution:

Given equations are,

$4x - 5y = -4$ and $3x = 2y - 3$

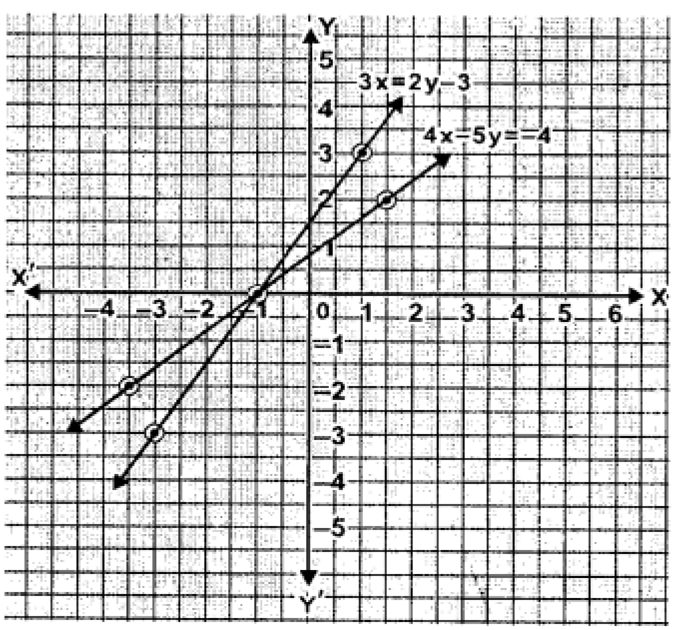
$\Rightarrow \quad 4x - 5y = -4 \quad \Rightarrow \quad 4x = -4 + 5y$

$\Rightarrow \quad x = \frac{-4 + 5y}{4} \quad \Rightarrow \quad x = \frac{5y - 4}{4}$

x	1.5	–1	–3.5
y	2	0	–2

$\Rightarrow \quad 3x = 2y - 3 \quad \Rightarrow \quad x = \frac{2y - 3}{3}$

x	–1	1	–3
y	0	3	–3



From graph, $x = -1$, $y = 0$

Question 5.
Solve the following simultaneous equations graphically, x + 3y = 8, 3x = 2 + 2y

Solution:

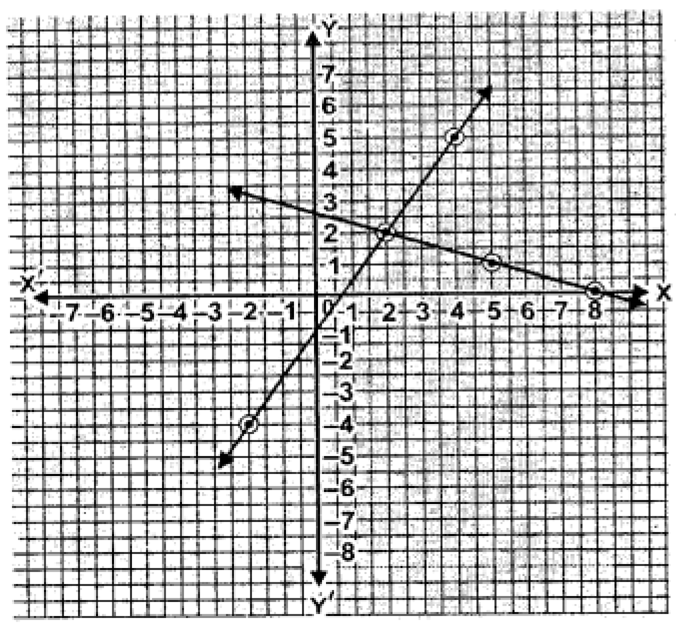
Given simultaneous equations are

$x + 3y = 8$ and $3x = 2 + 2y$, $x + 3y = 8, x = 8 - 3y$

x	8	5	2
y	0	1	2

and $3x = 2 + 2y \Rightarrow x = \frac{2 + 2y}{3}$

x	2	4	–2
y	2	5	– 4



From graph, $x = 2$, $y = 2$

Question 6.

Solve graphically the simultaneous equations 3y = 5 – x, 2x = y + 3 (Take 2cm = 1 unit on both axes).

Solution:

Given simultaneous equations are

$3y = 5 - x, \; 2x = y + 3$

$\Rightarrow \quad 3y = 5 - x$

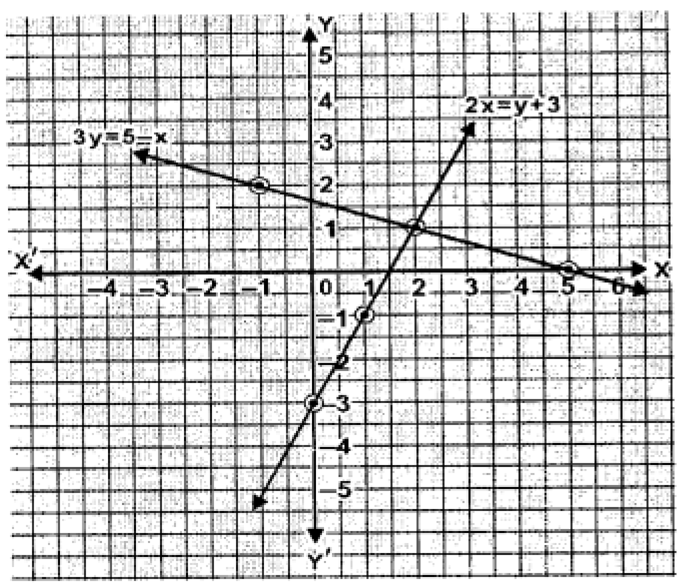
$\Rightarrow \quad x = 5 - 3y$

x	5	2	–1
y	0	1	2

and $2x = y + 3 \quad \Rightarrow \quad 2x - 3 = y$

$\Rightarrow \quad y = 2x - 3$

x	0	1	2
y	–3	–1	1



From graph, $x = 2, \; y = 1$.

Question 7.
Use graph paper for this question.
Take 2 cm = 1 unit on both axes.
(i) Draw the graphs of x +y + 3 = 0 and 3x-2y + 4 = 0. Plot only three points per line.

(ii) Write down the co-ordinates of the point of intersection of the lines.
(iii) Measure and record the distance of the point of intersection of the lines from the origin in cm.

(i) Given equations are,

$x + y + 3 = 0$, and $3x - 2y + 4 = 0$

Now $x + y + 3 = 0$

$\Rightarrow \quad x = -y - 3$

x	-3	-2	-1
y	0	-1	-2

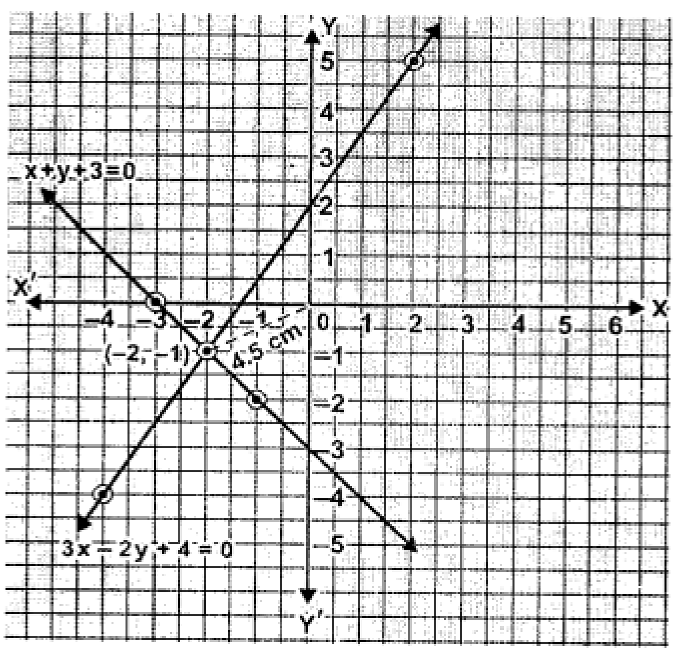
and $3x - 2y + 4 = 0$

$\Rightarrow \quad 3x = 2y - 4$

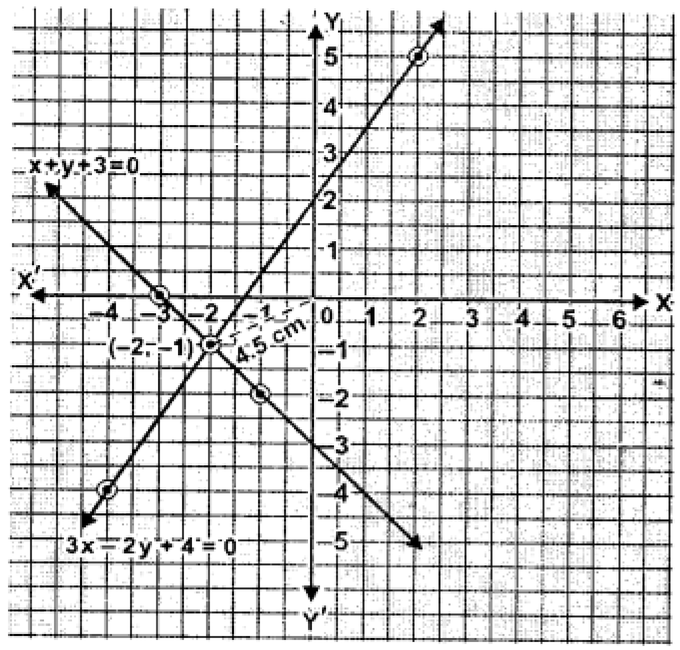
$\Rightarrow \quad x = \dfrac{2y - 4}{3}$

Solution:

x	−2	−4	2
y	−1	−4	5



(*ii*)



From graph the co-ordinates of point of the intersection of the lines = (–2, –1)

(*iii*) Distance of the point of intersection of the lines from the origin = 4.5 cm.

Question 8.

Solve the following simultaneous equations, graphically :
$2x - 3y + 2 = 4x + 1 = 3x - y + 2$

Solution:

Given equations are

$2x - 3y + 2 = 4x + 1 = 3x - y + 2$.

Taking First and Second terms

$2x - 3y + 2 = 4x + 1 \Rightarrow 2x - 4x - 3y + 2 = 1$

$\Rightarrow -2x - 3y + 2 = 1 \Rightarrow -2x = 1 - 2 + 3y$

$\Rightarrow -2x = -1 + 3y$

$\Rightarrow x = \dfrac{-1 + 3y}{-2} \Rightarrow x = -\dfrac{(3y-1)}{2}$

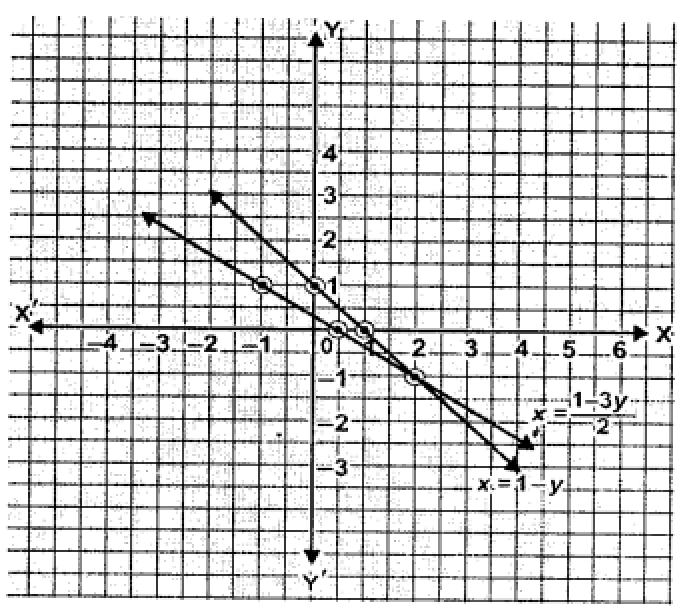
$\Rightarrow x = \dfrac{1 - 3y}{2}$

x	0.5	−1	2
y	0	1	−1

And $4x + 1 = 3x - y + 2 \Rightarrow 4x - 3x + y = 2 - 1$

$\Rightarrow x + y = 1 \Rightarrow x = 1 - y$

x	1	0	2
y	0	1	−1



From graph, $x = 2, \quad y = -1$

Question 9.

Use graph paper for this question.
(i) Draw the graphs of $3x - y – 2 = 0$ and $2x + y – 8 = 0$. Take 1 cm = 1 unit on both axes and plot three points per line.
(ii) Write down the co-ordinates of the point of intersection and the area of the traingle formed by the lines and the x-axis.

Solution:

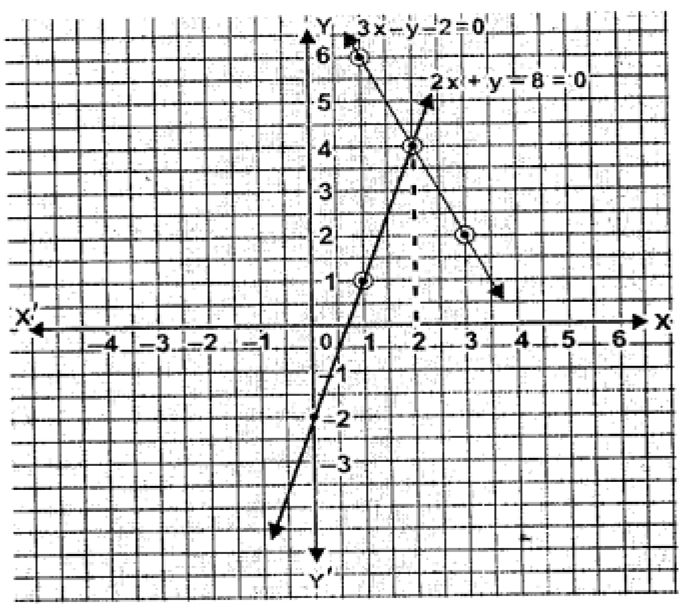
(*i*) Given equations are, $3x - y - 2 = 0$ and, $2x + y - 8 = 0$

$3x - y - 2 = 0 \Rightarrow 3x - 2 = y \Rightarrow y = 3x - 2$

x	0	1	2
y	–2	1	4

Also $2x + y - 8 = 0 \Rightarrow y = -2x + 8$

x	1	2	3
y	6	4	2



(*ii*) The co-ordinates of the point of intersection = (2, 4)

And area of the triangle formed by lines and the

x-axis $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \left(4 - \frac{2}{3}\right) \times 4$

$= \frac{1}{2} \times \frac{10}{3} \times 4 = \frac{10}{3} \times 2 = \frac{20}{3} = 6\frac{2}{3}$ sq. units

Question 10.

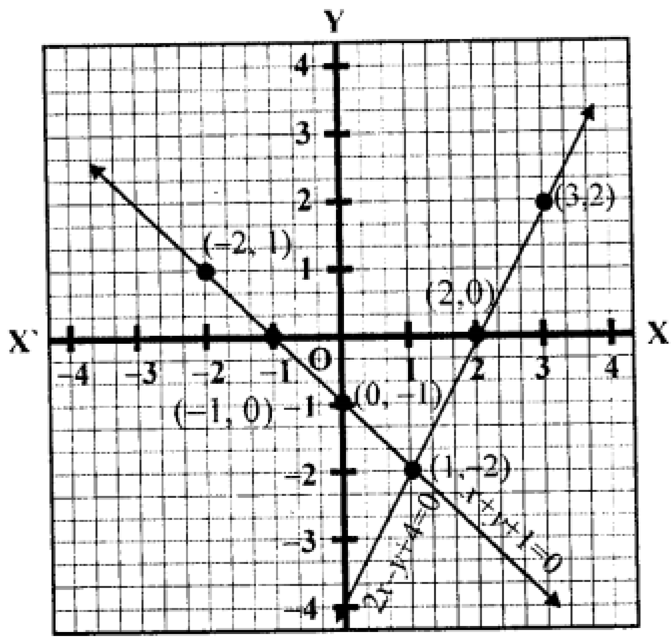
Solve the following system of linear equations graphically : 2x -y – 4 = 0, x + y + 1 = 0. Hence, find the area of the triangle formed by these lines and the y-axis.

Solution:

$$2x - y - 4 = 0 \Rightarrow 2x = y + 4 \Rightarrow x = \frac{y+4}{2}$$

Substituting some different values of y, we get the corresponding values of x as shown below:

x	2	3	1
y	0	2	–2



Plot the points (2, 0), (3, 2) and (1, –2) on the graph and join them to get a line.

Similarly in the equation, $x + y + 1 = 0$

$\Rightarrow \quad x = -(y + 1)$

Substituting the different values to y, we get the corresponding values of x, as

x	–1	–2	0
y	0	1	–1

Now plot the points (–1, 0), (–2, 1) and (0, –1) on the graph and join them to get another line which intersects the first line at (1, –2)

$\therefore \quad x = 1, y = -2$

Question 11.
Solve graphically the following equations: x + 2y = 4, 3x – 2y = 4
Take 2 cm = 1 unit on each axis. Write down the area of the triangle formed by the lines and the x-axis.
Solution:

Given equations are,

$x + 2y = 4$, and $3x - 2y = 4$

$\therefore \quad x + 2y = 4$

$\Rightarrow \quad x = 4 - 2y$

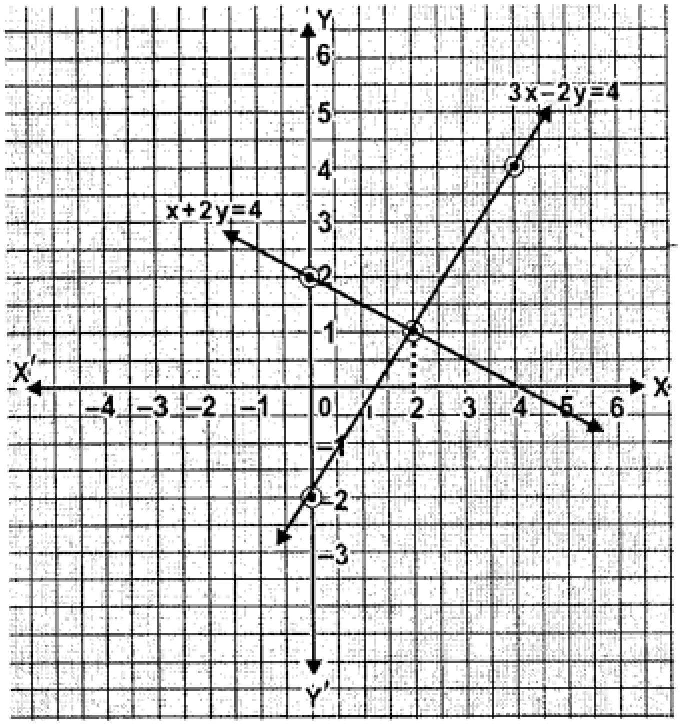
x	4	2	0
y	0	1	2

And $\quad 3x - 2y = 4$

$\Rightarrow \quad 3x = 4 + 2y$

$\Rightarrow \quad x = \frac{4 + 2y}{3}$

x	2	4	0
y	1	4	–2



From graph, $x = 2, \quad y = 1$

Now, Area of the triangle formed by the lines and

the x-axis $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times \left(4 - \frac{4}{3}\right) \times 1 \text{ sq. units}$$

$$= \frac{1}{2} \times \left(\frac{12-4}{3}\right) \times 1 \text{ sq. units}$$

$$= \frac{1}{2} \times \frac{8}{3} \times 1 \text{ sq. units}$$

$$= \frac{4}{3} \text{ sq. units.}$$

Question 12.
On graph paper, take 2 cm to represent one unit on both the axes, draw the lines :
$x + 3 = 0$, $y - 2 = 0$, $2x + 3y = 12$.
Write down the co-ordinates of the vertices of the triangle formed by these lines.
Solution:

Given equations of lines are,

$x + 3 = 0, y - 2 = 0, 2x + 3y = 12$

$x + 3 = 0 \Rightarrow x = -3$ .....(1)

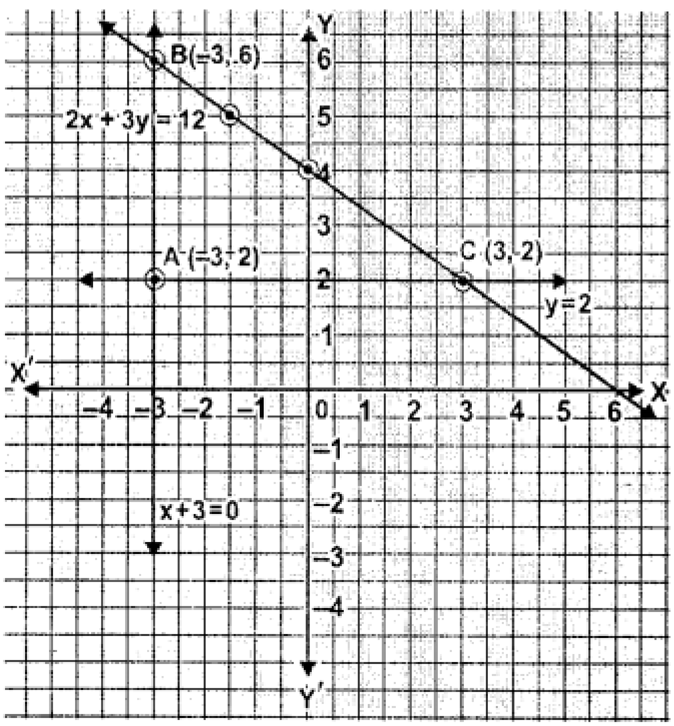
And $y - 2 = 0 \Rightarrow y = 2$ ....(2)

And $2x + 3y = 12$ ....(3)

$2x = 12 - 3y$

$$\Rightarrow x = \frac{12 - 3y}{2}$$

x	6	3	0
y	0	2	4



From graph vertices of the triangle formed by these lines are (−3, 2), (−3, 6) and (3, 2) **Ans.**

Question 13.

Find graphically the co-ordinates of the vertices of the triangle formed by the

lines y = 0, y – x and 2x + 3y= 10. Hence find the area of the triangle formed by these lines.

Solution:

Given equations of lines are

$y = 0, y = x, 2x + 3y = 10$

$$y = 0 \qquad .....(1)$$

and $$y = x \qquad .....(2)$$

Putting the different values of x

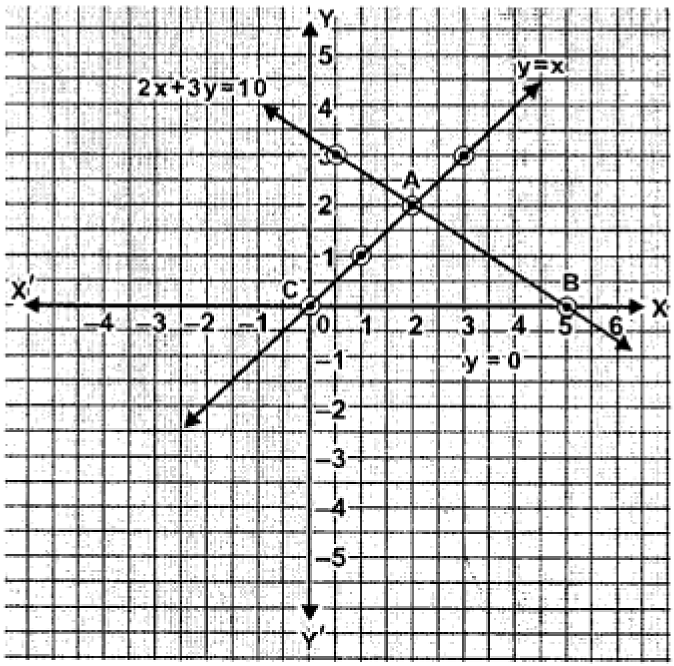
x	1	2	3
y	1	2	3

and $2x + 3y = 10$ .....(3)

$\Rightarrow \quad 2x = 10 - 3y$

$$\Rightarrow \quad x = \frac{10 - 3y}{2}$$

x	5	2	0.5
y	0	2	3



From graph, vertices of the triangle formed by these lines are (0, 0, (5, 0), (2, 2)

Hence Area of triangle = $\frac{1}{2}$ × base × height

= $\frac{1}{2}$ × 5 × 2 sq. units, = 5 sq. units.

EXERCISE 19.4

Question 1.

Find the distance between the following pairs of points :

(i) (2, 3), (4, 1)

(ii) (0, 0), (36, 15)

(iii) (a, b), (-a, -b)

Solution:

(*i*) Distance between (2, 3) and (4, 1)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{2^2 + (-2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

(*ii*) Distance (0, 0) and (36, 15)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$

$= \sqrt{1296+225} = \sqrt{1521} = 39$ cm

(*iii*) Distance between (a, b) and $(-a, -b)$

$= \sqrt{(-a-a)^2 + (-b-b)^2}$

$= \sqrt{(-a)^2 + (-b)^2}$

$= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2}$

$= \sqrt{4(a^2 + b^2)} = 2\sqrt{a^2 + b^2}$

Question 2.

A is a point on y-axis whose ordinate is 4 and B is a point on x-axis whose abscissa is -3. Find the length of the line segment AB.

Solution:

$\because$ A lies on y-axis,

$\therefore$ Abscissa = 0, and ordinate = 4

i.e., A (0, 4)

$\because$ B lies on x-axis

$\therefore$ Ordinate = 0, and abscissa = –3

i.e., B (–3, 0)

$$\therefore \text{AB} = \sqrt{(-3-0)^2 + (0-4)^2}$$

$$= \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units.}$$

Question 3.

Find the value of a, if the distance between the points A (-3, -14) and B (a, -5) is 9 units.

Solution:

Distance A (–3, –14) and B (a, –5)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(a+3)^2 + (-5+14)^2}$$

$$= \sqrt{(a+3)^2 + (9)^2}$$

$$= \sqrt{a^2 + 9 + 6a + 81}$$

$$\therefore \sqrt{(a^2 + 6a + 90)} = 9$$

Squaring both sides,

$a^2 + 6a + 90 = 81 \Rightarrow a^2 + 6a + 90 - 81 = 0$

$\Rightarrow a^2 + 6a + 9 = 0$

$= (a + 3)^2 = 0$

$\therefore a + 3 = 0 \Rightarrow a = -3$

Question 4.

(i) Find points on the x-axis which are at a distance of 5 units from the point (5, -4).

(ii) Find points on the y-axis are at a distance of 10 units from the point (8, 8) ?

(iii) Find points (or points) which are at a distance of √10 from the point (4, 3)

given that the ordinate of the point or points is twice the abscissa.

Solution:

(i) Let the points on x-axis be $(x, 0)$, then Distance between $(x, 0)$ and $(5, -4) = 5$ units

$$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$$

$$\Rightarrow \sqrt{(5 - x)^2 + (-4 - 0)^2} = 5$$

$$\sqrt{(5 - x)^2 + (-4)^2} = 5$$

Squaring both sides,

$(5 - x)^2 + 16 = 25$

$\Rightarrow 25 - 10x + x^2 + 16 - 25 = 0$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow x^2 - 2x - 8x + 16 = 0$

$\Rightarrow x(x - 2) - 8(x - 2) = 0 \Rightarrow (x - 2)(x - 8) = 0$

Either $x - 2 = 0$, then $x = 2$

or $x - 8 = 0$, then $x = 8$

$\therefore$ The points are $(2, 0)$ and $(8, 0)$

(ii) Let the co-ordinates of points or points be (x, y), which are at a distance of 10 units from the points $(8, 8)$

$$\therefore \sqrt{(8 - x)^2 + (8 - y)^2} = 10$$

Squaring both sides,

$(8 - x)^2 + (8 - y)^2 = 100$

$\Rightarrow 64 + x^2 - 16x + 64 + y^2 - 16y = 100$

$x^2 + y^2 - 16x - 16y + 128 = 100$

$\because$ Points are on y-axis

$\therefore$ $x = 0$

Hence $(0)^2 + y^2 - 16 \times 0 - 16y + 128 = 100$

$\Rightarrow y^2 - 16y + 128 - 100 = 0$

$\Rightarrow y^2 - 16y + 28 = 0$

$\Rightarrow y^2 - 14y - 2y + 28 = 0$

$\Rightarrow y(y - 14) - 2(y - 14) = 0$

$\Rightarrow (y - 14)(y - 2) = 0$

Either $y - 14 = 0$, then $y = 14$

or $y - 2 = 0$, then $y = 2$

$\therefore$ Points will be (0, 14) and (0, 2)

(*iii*) Let the abscissa of point $= x$

the ordinate $= 2x$

$\because$ point $(x, 2x)$ is at a distance of $\sqrt{10}$ from the point (4, 3), then

$$\sqrt{(x-4)^2 + (2x-3)^2} = \sqrt{10}$$

Squaring both sides,

$(x - 4)^2 + (2x - 3)^2 = 10$

$\Rightarrow x^2 - 8x + 16 + 4x^2 - 12x + 9 = 10$

$\Rightarrow 5x^2 - 20x + 25 - 10 = 0$

$\Rightarrow 5x^2 - 20x + 15 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$ (Dividing by 5)

$\Rightarrow x^2 - x - 3x + 3 = 0$

$\Rightarrow x(x - 1) - 3(x - 1) = 0$

$\Rightarrow (x - 1)(x - 3) = 0$

Either $x - 1 = 0$, then $x = 1$

or $x - 3 = 0$, then $x = 3$

$\therefore$ Points will be (1, 2) and (3, 6)

Question 5.

Find the point on the x-axis which, is equidistant from the points (2, -5) and (-2, 9).

Solution:

Using distance formula,
Let the required point on x-axis be $(x, 0)$
Then distance between $(x, 0)$ and $(2, -5)$ is equal to the distance between $(x, 0)$ and $(-2, 9)$

$$\therefore \sqrt{(2-x)^2+(-5-0)^2}$$

$$= \sqrt{(-2-x)^2+(9-0)^2}$$

Squaring both sides,
$(2-x)^2 + (-5)^2 = (-2-x)^2 + 9^2$
$4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$
$-4x + 29 = 85 + 4x \Rightarrow 4x + 4x = -85 + 29$

$$\Rightarrow 8x = -56 \Rightarrow x = \frac{-56}{5} = -7$$

$\therefore x = -7$
$\therefore$ Point $= (-7, 0)$

Question 6.
Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6, -1), (1, 3) and (x, 8) respectively.
Solution:
Using distance formula,
Points are P $(6, -1)$ Q $(1, 3)$ and R $(x, 8)$
and PQ = QR
$\therefore (1-6)^2 + (3+1)^2 = (x-1)^2 + (8-3)^2$
$(-5)^2 + (4)^2 = (x-1)^2 + (5)^2$
$25 + 16 = (x-1)^2 + 25$
$(x-1)^2 = 16 \Rightarrow x^2 - 2x + 1 = 16$
$\Rightarrow x^2 - 2x + 1 - 16 = 0$
$\Rightarrow x^2 - 2x - 15 = 0$
$\Rightarrow x^2 - 5x + 3x - 15 = 0$
$\Rightarrow x(x-5) + 3(x-5) = 0$
$\Rightarrow (x-5)(x+3) = 0$
Either $x - 5 = 0$, then $x = 5$
or $x + 3 = 0$, then $x = -3$
$\therefore x = 5, -3$

Question 7.

If Q (0, 1) is equidistant from P (5, -3) and R (x, 6) find the values of x.

Solution:

$\because$ Q (0, 1) is equidistant from P (5, –3) and R (x, 6) find the value of x

$\therefore$ QP = QR

$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow (5)^2 + (-4)^2 = x^2 + 5^2$

$25 + 16 = x^2 + 25 \Rightarrow x^2 = 16 = (\pm 4)^2$

$\therefore x = \pm 4$

$\therefore x = 4, -4$

Question 8.

Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

Solution:

$\because$ Points (x, y) is equidistant from the points (7, 1) and (3, 5)

$= (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$

$= x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$49 + 1 - 9 - 25 = -6x - 10y + 14x + 2y$

$50 - 34 = 8x - 8y$

$\Rightarrow 8x - 8y = 16$

$\Rightarrow x - y = 2$ (Dividing by 8)

Question 9.

The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from the points Q (2, -5) and U (-3, 6), then find the coordinates of P.

Solution:

$\because$ x-coordinates of a point P = twice its y-coordinate

Let coordinates of point P be $(2x, x)$

$\because$ P is equidistant from points Q (2, –5) and R (–3, 6)

$\therefore$ PQ = PR

Now,

$(2x - 2)^2 + (x + 5)^2 = (2x + 3)^2 + (x - 6)^2$

$\Rightarrow 4x^2 - 8x + 4 + x^2 + 10x + 25$

$= 4x^2 + 12x + 9 + x^2 - 12x + 36$

$2x + 29 = 45$

$2x = 45 - 29 = 16$

$x = \frac{16}{2} = 8$

$\therefore$ Coordinates of points P will be $(2 \times 8, 8)$

i.e., (16, 8)

Question 10.

If the points A (4,3) and B (x, 5) are on a circle with centre C (2, 3), find the value of x.

Solution:

Points A (4, 3) and B $(x, 5)$ are on the circle whose centre C (2, 3)

$\therefore$ AC = BC (radii of the same circle)

$\Rightarrow (4 - 2)^2 + (3 - 3)^2 = (x - 2)^2 + (5 - 3)^2$

$\Rightarrow (2)^2 + 0 = (x - 2)^2 + 2^2$

$4 = (x - 2)^2 + 4$

$\Rightarrow (x - 2)^2 = 4 - 4 = 0$

$\therefore x - 2 = 0 \Rightarrow x = 2$

$\therefore x = 2$

Question 11.

If a point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

Solution:

Points A (0, 2) is equidistant from B (3, p) and C (p, 5)

$\therefore$ AB = AC

$(3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 3^2 + (p-2)^2 = p^2 + 3^2$

$9 + p^2 - 4p + 4 = p^2 + 9$

$-4p + 4 = 0 \Rightarrow 4p = 4 \Rightarrow p = \frac{4}{4} = 1$

Question 12.

Using distance formula, show that (3, 3) is the centre of the circle passing through the points (6, 2), (0, 4) and (4, 6).

Solution:

To show O (3, 3) is the centre of a circle passing through the points A (6, 2), B (0, 4) and C (4, 6)

$\therefore$ OA = OB = OC

Now OA = $\sqrt{(6-3)^2 + (2-3)^2}$

$= \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

OB = $\sqrt{(0-3)^2 + (4-3)^2} = \sqrt{(-3)^2 + (1)^2}$

$= \sqrt{9+1} + \sqrt{10}$

and OC = $\sqrt{(4-3)^2 + (6-3)^2} = \sqrt{1^2 + 3^2}$

$= 1 + 9 = \sqrt{10}$

$\because$ OA = OB = OC

$\therefore$ O is the centre of the circle passing through the points A, B and C.

Question 13.

The centre of a circle is C (2α – 1, 3α + 1) and it passes through the point A (-3, -1). If a diameter of the circle is of length 20 units, find the value(s) of α.

Solution:

Centre of a circle is C [(2a – 1), (3a + 1)] and it passes through the points A (–3, –1) and length of diameter = 20 units

i.e., length of radius $= \frac{20}{2} = 10$ units

$\Rightarrow$ AC = 10

Now AC $= \sqrt{(2a-1+3)^2}$

$+ \sqrt{(2a-1+3)^2+(3a+1+1)^2}$

$= \sqrt{(2a+2)^2+(3a+2)^2}$

$\therefore \sqrt{(2a+2)^2+(3a+2)^2} = 10$

Squaring,

$(2a + 2)^2 + (3a + 2)^2 = 10^2$

$4a^2 + 8a + 4 + 9a^2 + 12a + 4 = 100$

$13a^2 + 20a + 8 - 100 = 0$

$\Rightarrow 13a^2 + 20a - 92 = 0$

$\Rightarrow 13a^2 - 26a + 46a - 92 = 0$

$\Rightarrow 13a(a - 2) + 46(a - 2) = 0$

$\Rightarrow (a - 2)(13a + 46) = 0$

Either $a - 2 = 0$, then $a = 2$

or $13a + 46 = 0$, then $13 = -46 \Rightarrow a = \frac{-46}{13}$

Hence $a = 2, \frac{-46}{13}$

Question 14.

Using distance formula, show that the points A (3, 1), B (6, 4) and C (8, 6) are coliinear.

Solution:

To show that the points A (3, 1), B (6, 4) and C (8, 6) are collinear, if sum of any two lines is equal to the third line

Now, AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-3)^2 + (4-1)^2} = \sqrt{3^2 + 3^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

BC = $\sqrt{(8-6)^2 + (6-4)^2} = \sqrt{2^2 + 2^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

AC = $\sqrt{(8-3)^2 + (6-1)^2} = \sqrt{5^2 + 5^2}$

$= \sqrt{25+25} = \sqrt{25 \times 2} = 5\sqrt{2}$

$\therefore$ AB + BC = $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$ = AC

$\therefore$ Points A, B and C are collinear.

Question 15.

Check whether the points (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Solution:

To check that points A (5, –2), B (6, 4), C (7, –2) are the vertices of an isosceles triangle ABC

Now, AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1^2 + 6^2}$

$= \sqrt{1+36} = \sqrt{37}$

BC = $\sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{1^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$

AC = $\sqrt{(7-5)^2 + (-2+2)^2}$

$= \sqrt{(2)^2 + 0^2} = \sqrt{4} = 2$

∴ Two sides AB = BC

∴ ΔABC is an isosceles triangle

∴ Whose vertices are A, B and C

Question 16.

Name the type of triangle formed by the points A (-5, 6), B (-4, -2) and (7, 5).

Solution:

Three points of a triangle are

A (–5, 6), B (–4, –2) and (7, 5)

Now, AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4+5)^2 + (-2-6)^2} = \sqrt{(1)^2 + (-8)^2}$

$= \sqrt{1+64} = \sqrt{65}$

BC = $\sqrt{(7+4)^2 + (5+2)^2} = \sqrt{11^2 + 7^2}$

$= \sqrt{121+49} = \sqrt{170}$

CA = $\sqrt{(7+5)^2 + (5-6)^2}$

$= \sqrt{12^2 + (-1)^2} = \sqrt{144+1} = \sqrt{145}$

∵ All the sides are different

∴ ΔABC is a scalene.

Question 17.

Show that the points (1, 1), (- 1, – 1) and (-√3,√3) form an equilateral triangle.

Solution:

Let the vertices of a ΔABC be $A(1, 1)$ $B(-1, -1)$ and $C(-\sqrt{3}, \sqrt{3})$

then $AB = \sqrt{[1-(-1)^2]+[1-(-1)]^2}$

$= \sqrt{(1+1)^2+(1+1)^2} = \sqrt{(2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times2} = 2\sqrt{2}$ units

$BC = \sqrt{[-\sqrt{3}-(-1)]^2+(\sqrt{3}-(-1))^2}$

$= \sqrt{(-\sqrt{3}+1)^2+(\sqrt{3}+1)^2}$

$= \sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}} = \sqrt{8}$

$= \sqrt{4\times2} = 2\sqrt{2}$ units.

$AC = \sqrt{[-\sqrt{3}-1]^2+(\sqrt{3}-1)^2}$

$= \sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}}$

$= \sqrt{8} = \sqrt{4\times2} = 2\sqrt{2}$ units

$\because AB = BC = AC = 2\sqrt{2}$ units

$\therefore \Delta ABC$ is an equilateral triangle.

Question 18.

Show that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

Solution:

Let points are A (7, 10), B (–2, 5) and C (3, –4)

Now AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{(-9)^2 + (-5)^2}$

$= \sqrt{81+25} = \sqrt{106}$

Similarly, BC = $\sqrt{(3+2)^2 + (-4-5)^2}$

$= (5)^2 + (-9)^2$

$= \sqrt{25+81} = \sqrt{106}$

and AC = $\sqrt{(3-7)^2 + (-4-10)^2}$

$= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$

We see that AB = BC = $\sqrt{106}$

∴ It is an isosceles triangle

and $AB^2 + BC^2 = (\sqrt{106})^2 + (\sqrt{106})^2$

$= 106 + 106 = 212$

and $AC^2 = (\sqrt{212})^2 = 212$

∴ $AB^2 + BC^2 = AC^2$

∴ It is an isosceles right triangle

Question 19.

The points A (0, 3), B (- 2, a) and C (- 1, 4) are the vertices of a right angled triangle at A, find the value of a.

Solution:

$\because$ A, B and C are the vertices of a right angled ΔABC, right angle at A.

$\therefore$ $AB^2 = (-2-0)^2 + (a-3)^2$

$= (-2)^2 + (a-3)^2 = 4 + (a-3)^2$

$AC^2 = (-1-0)^2 + (4-3)^2 = (-1)^2 + (1)^2$

$= 1 + 1 = 2$

$BC^2 = (-1+2)^2 + (4-a)^2$

$= (1)^2 + (4-a)^2$

$\therefore$ $AB^2 + AC^2 = BC^2$

(By pythagorus theorem)

$\Rightarrow 4 + (a-3)^2 + 2 = 1 + (4-a)^2$

$= 4 + a^2 - 6a + 9 + 2 = 1 + 16 - 8a + a^2$

$\Rightarrow a^2 - 6a + 15 = a^2 - 8a + 17$

$\Rightarrow 8a - 6a = 17 - 15$

$\Rightarrow 2a = 2 \Rightarrow a = 1$

Question 20.

Show that the points (0, – 1), (- 2, 3), (6, 7) and (8, 3), taken in order, are the vertices of a rectangle. Also find its area.

Solution:

Let A (0, – 1), B (– 2, 3), C (6, 7) and D (8, 3) are the vertices of *a* quadrilateral ABCD.

Now $AB = \sqrt{(-2-0)^2 + [3-(-1)]^2}$

$= \sqrt{(-2)^2 + (3+1)^2} = \sqrt{4+16}$

$= \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ units

$BC = \sqrt{[6-(-2)]^2 + (7-3)^2}$

$= \sqrt{(6+2)^2 + (4)^2}$

$= \sqrt{(8)^2 + (4)^2} = \sqrt{64+16}$

$= \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$ units

$CD = \sqrt{(8-6)^2 + (3-7)^2}$

$= \sqrt{(2)^2 + (-4)^2} = \sqrt{4+16}$

$= \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ units.

$AD = \sqrt{(8-0)^2 + [3-(-1)]^2}$

$= \sqrt{(8)^2 + (3+1)^2} = \sqrt{64+16} = \sqrt{80}$

$= \sqrt{16 \times 5} = 4\sqrt{5}$ units

$\because AB = CD$ and $BC = AD$

$\therefore$ ABCD is a rectangle.

Question 21.

If P (2, -1), Q (3, 4), R (-2, 3) and S (-3, -2) be four points in a plane, show that PQRS is a rhombus but not a square. Find the area of the rhombus.

Solution:

Four pionts are P (2, –1), Q (3, 4), R (–2, 3) and S, (–3, –2) are the vertices of a quad.

Now, PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{1^2 + 5^2}$

$= \sqrt{1+25} = \sqrt{26}$

QR = $\sqrt{(-2-3)^2 + (3-4)^2}$

$= \sqrt{(-5)^2 + (1)^2} = \sqrt{25+1} = \sqrt{26}$

RS = $\sqrt{(-3+2)^2 + (-2-3)^2}$

$= \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$

and SP = $\sqrt{(-3-2)^2 + (-2+1)^2}$

$= \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$

$\because$ PQ = QR = RS = SP

$\therefore$ PQRS is a square or a rhombus

Now, diagonal PR = $\sqrt{(-2-2)^2 + (3+1)^2}$

$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ cm

and QS = $\sqrt{(-3-3)^2+(-2-4)^2}$

$= \sqrt{(-6)^2+(-6)^2} = \sqrt{36+36}$

$= \sqrt{72} = 6\sqrt{2}$

$\therefore$ PQ = QS

$\therefore$ PQRS is a rhombus not a square

Now, area of rhombus PQRS

$= \frac{1}{2} \times PR \times QS$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

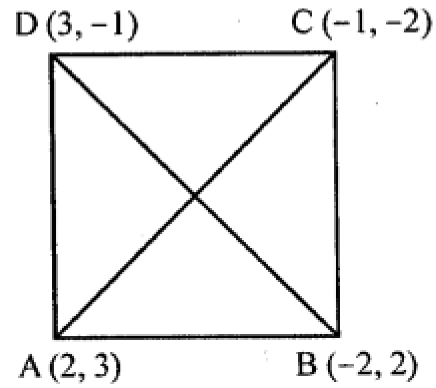
$= \frac{1}{2} \times 4 \times 6 \times 2 = 24$ sq. units

Question 22.

Prove that the points A (2, 3), B {-2, 2), C (-1, -2) aqd D (3, -1) are the vertices of a square ABCD.

Solution:

Points A (2, 3), B (–2, 2), C (–1, –2) and D (3, –1)



$\therefore$ AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-2)^2 + (2-3)^2}$

$= \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{16+1} = \sqrt{17}$

Similarly,

BC $= \sqrt{(-1+2)^2 + (-2-2)^2}$

$= \sqrt{(1)^2 + (-4)^2}$

$= \sqrt{1+16} = \sqrt{17}$

$CD = \sqrt{(3+1)^2 + (-1+2)^2}$

$= \sqrt{(4)^2 + (1)^2}$

$= \sqrt{16+1} = \sqrt{17}$

and $DA = \sqrt{(2-3)^2 + (3+1)^2}$

$= \sqrt{(-1)^2 + (4)^2}$

$= \sqrt{1+16} = \sqrt{17}$

$AC = \sqrt{(-1-2)^2 + (-2-3)^2}$

$= \sqrt{(-3)^2 + (-5)^2}$

$= \sqrt{9+25} = \sqrt{34}$

$BD = \sqrt{(3+2)^2 + (-1-2)^2}$

$= \sqrt{(5)^2 + (-3)^2}$

$= \sqrt{25+9} = \sqrt{34}$

$\because$ Sides AB, BC, CD and DA are equal and diagonals AC and BD are also equal

$\therefore$ ABCD is a square

Question 23.

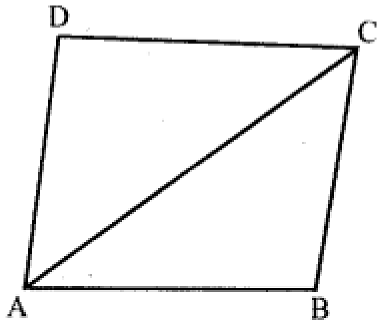
Name the type of quadrilateral formedby the following points and give reasons for your answer :
(i) (-1, -2), (1, 0), (-1, 2), (-3, 0)
(ii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution:

(*i*) A (–1, –2), B (1, 0), C (–1, 2),
and D (–3, 0)

Now AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (1 + 1)^2 + (0 + 2)^2 = (2)^2 + (2)^2$
$= 4 + 4 = 8$



Similarly,
$BC^2 = (-1 - 1)^2 + (2 - 0)^2 = (-2)^2 + (2)^2$
$= 4 + 4 = 8$
$CD^2 = (-3 + 1)^2 + (0 - 2)^2 = (-2)^2 + (-2)^2$
$= 4 + 4 = 8$
$DA^2 = (-1 + 3)^2 + (-2 + 0)^2 = (2)^2 + (-2)^2$
$= 4 + 4 = 8$
Diagonal $AC^2 = (-1 + 1)^2 + (2 + 2)^2$
$= (0)^2 + (4)^2 = 0 + 16 = 16$
and $BD^2 = (-3 - 1)^2 + (0 - 0)^2 = (-4)^2 + 0 = 16$

$\because$ The sides are equal and diagonal are also equal

$\therefore$ The quadrilateral ABCD is a square (*ii*) Points are A (4, 5), B (7, 6), C (4, 3), D (1, 2)

Now $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (7 - 4)^2 + (6 - 5)^2$

$= (3)^2 + (1)^2$

$= 9 + 1 = 10$

Similarly $BC^2 = (4 - 7)^2 + (3 - 6)^2$

$= (3)^2 + (-3)^2 = 9 + 9 = 18$

$CD^2 = (1 - 4)^2 + (2 - 3)^2$

$= (-3)^2 + (-1)^2$

$= 9 + 1 = 10$

$DA^2 = (4 - 1)^2 + (5 - 2)^2$

$= (3)^2 + (3)^2$

$= 9 + 9 = 18$

$\therefore$ Here AB = CD and BC = DA

Diagonal $AC^2 = (4 - 4)^2 + (3 - 5)^2$

$= (0)^2 + (-2)^2$

$= 0 + 4 = 4$

and $BD^2 = (1 - 7)^2 + (2 - 6)^2$

$= (-6)^2 + (-4)^2$

$= 36 + 16 = 52$

$\because$ Opposite sides are equal and diagonals are not equal

$\therefore$ It is a parallelogram

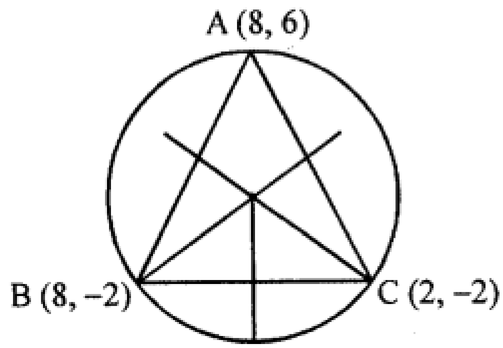
Question 24.

Find the coordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, -2) and (2, -2). Also, find its circumradius.

Solution:

O is the circumference of ΔABC

Whose vertices are A (8, 6), B (8, –2), C (2, –2)



Let coordinates of centre O (x, y)

$\because OA = OB = OC \Rightarrow OA^2 = OB^2 = OC^2$

$\therefore OA^2 = OB^2$

$\Rightarrow (x-8)^2 + (y-6)^2 = (x-8)^2 + (y+2)^2$

$x^2 - 16x + 64 + y^2 - 12y + 36$

$= x^2 - 16x + 64 + y^2 + 4y + 4$

$36 - 4 = 4y + 12y \Rightarrow 16y = 32$

$\Rightarrow y = \frac{32}{16} = 2$

Similarly $OB^2 = OC^2$

$(x-8)^2 + (y+2)^2 = (x-2)^2 + (y+2)^2$

$x^2 - 16x + 64 = x^2 - 4x + 4$

$16x - 4x = 64 - 4$

$\Rightarrow 12x = 60 \Rightarrow x = \frac{60}{12} = 5$

$\therefore$ Coordinates of O are (5, 2)

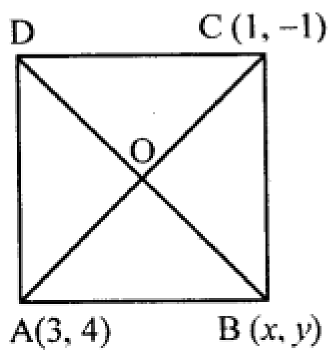
$\therefore OA = \sqrt{(x-8)^2 + (y-6)^2}$

$= \sqrt{(5-8)^2 + (2--6)^2} = \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ units

Question 25.
If two opposite vertices of a square are (3, 4) and (1, -1), find the coordinates of the other two vertices.
Solution:

Sol. Length of hypotenuse of square = $\sqrt{2}$ × side of the square



$\therefore$ AC = $\sqrt{2}$ AB

$\Rightarrow AC^2 = 2AB^2$ (squaring both sides)

$\Rightarrow (3-1)^2 + [4-(-1)]^2 = 2[(x-3)^2 + (y-4)^2]$

$$\Rightarrow 29 = 2\left[(x-3)^2 + \left(\frac{23-4x}{10} - 4\right)^2\right]$$

[From (*i*)]

$$\Rightarrow 29 = 2\left[(x-3)^2 + \frac{(-4x-17)^2}{100}\right]$$

$\Rightarrow 2900 = 2[100(x^2 - 6x + 9) + (16x^2 + 289 + 136x)]$

$\Rightarrow 2900 = 2[116x^2 - 464x + 1189]$

$\Rightarrow 116x^2 - 464x + 1189 = 1450$

$\Rightarrow 116x^2 - 464x - 261 = 0$

$$\Rightarrow x = \frac{464 \pm \sqrt{(-464)^2 - 4 \times 116 \times (-261)}}{2 \times 116}$$

$$\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$\Rightarrow x = \frac{464 \pm \sqrt{336400}}{2 \times 116} \Rightarrow x = \frac{464 \pm 580}{232}$$

$$\Rightarrow x = \frac{464 + 580}{232} \text{ or } x = \frac{464 - 580}{232}$$

$$x = \frac{1044}{232} = \frac{9}{2} \text{ or } x = \frac{-116}{232} = \frac{-1}{2}$$

When $x = \frac{9}{2}$

$$y = \frac{23 - 4 \times \frac{9}{2}}{10} = \frac{46 - 36}{20} = \frac{10}{20} = \frac{1}{2}$$

When $x = \frac{-1}{2}$

$$y = \frac{23 - 4 \times \left(\frac{-1}{2}\right)}{10} = \frac{25}{10} = \frac{5}{2}$$

Thus, the coordinates of the remaining vertices of square are $\left(\frac{9}{2}, \frac{1}{2}\right)$ and $\left(\frac{-1}{2}, \frac{5}{2}\right)$

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 16):

Question 1.

Point (-3, 5) lies in the

(a) first quadrant

(b) second quadrant

(c) third quadrant

(d) fourth quadrant

Solution:

Point (-3, 5) lies in second quadrant, **(b)**

Question 2.

Point (0, -7) lies

(a) on the x-axis

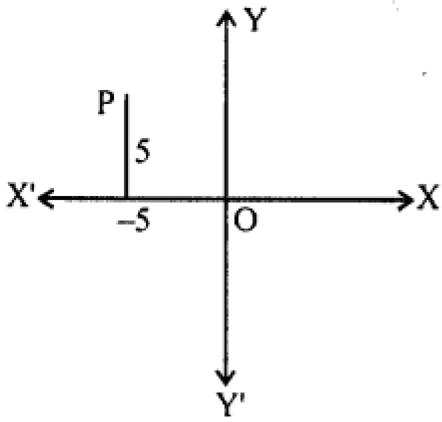
(b) in the second quadrant
(c) on the y-axis
(d) the fourth quadrant
Solution:
Point (0, -7) lies on y-axis (as x = 0) **(c)**

Question 3.
Abscissa of a point is positive in
I and II quadrants
I and IV quadrants
I quadrant only
II quadrant only
Solution:
Abscissa of a point is positive in first and fourth quadrants. **(b)**

Question 4.
The point which lies ony-axis at a distance of 5 units in the negative direction of y- axis is
(a) (0, 5)
(b) (5, 0)
(c) (0, -5)
(d) (-5, 0)
Solution:
(0, -5) is the required point. **(c)**

Question 5.
If the perpendicular distance of a point P from the x-axis is 5 units and the foot of perpendicular lies on the negative direction of x-axis, then the point P has
(a) x-coordinate = -5
(b) y-coordinate = 5 only
(c) y-coordinate = -5 only
(d) y-coordinate = 5 or -5
Solution:
Perpendicular distance for a point P on x- axis in negative direction.

It will has y = 5 and x = -5 **(d)**



Question 6.
The points whose abscissa and ordinate have different signs will lie in
(a) I and II quadrants
(b) II and III quadrants
(c) I and III quadrants
(d) II and IV quadrants
Solution:
Point which has abscissa and ordinate having different signs will lie in second and fourth quadrants. **(d)**

Question 7.
The points (-5, 2) and (2, -5) lie in
(a) same quadrant
(b) II and III quadrants respectively
(c) II and IV quadrants respectively
(d) IV and II quadrants respectively
Solution:
Points (-5, 2) and (2, -5) lie in second and fourth quadrants respectively. **(b)**

Question 8.
If P (-1,1), Q (3, -4), R (1, -1), S (-2, -3) and T (-4, 4) are plotted on the graph paper, then point(s) in the fourth quadrant are
(a) P and T
(b) Q and R
(c) S only
(d) P and R
Solution:
Points P (-1, 1), Q (3, -4), R (1, -1), S (-2, -3) and T (-4, 4) are plotted on graph The points in 4th quadrant are Q and R **(b)**

Question 9.
On plotting the points O (0, 0), A (3, 0), B (3, 4), C (0, 4) and joining OA, AB, BC and CO which of the following figure is obtained?
(a) Square
(b) Rectangle
(c) Trapezium
(d) Rhombus
Solution:
On plotting the points O (0, 0), A (3, 0), B (3, 4), C (0, 4)
OA, AB, BC and CO are joined
The figure so formed will a rectangle **(b)**

Question 10.
Which of the following points lie on the graph of the equation :
3x-5y + 7 = 0?
(a) (1, -2)
(b) (2, 1)
(c) (-1, 2)
(d) (1, 2)
Solution:
$3x - 5y + 7 = 0$
Let (1, –2), subtracting the value of $x = 1, y = -2$, then
$3 \times 1 - 5(-2) + 7 = 3 + 10 + 7 = 17 \neq 0$
Similarly subsituting the value of $x = 2, y = 1$
then $3 \times 2 - 5 \times 1 + 7 = 6 - 5 + 7 \neq 0$
(–1, 2)
$3 \times (-1) - (5 \times 2) + 7$
$\Rightarrow -3 - 10 + 7 \neq 0$
and (1, 2)
$3 \times 1 - 5 \times 2 + 7 = 0$
$3 - 10 + 7 = 10 - 10 = 0$
$\therefore$ (1, 2) lies on $3x - 5y + 7 = 0$ (d)

Question 11.
The pair of equation x – a and y = b graphically represents lines which are
(a) parallel
(b) intersecting at (b, a)
(c) coincident
(d) intersecting at (a, b)
Solution:

x = a, y = 6
Which are intersecting at (a, b) **(d)**

Question 12.
The distance of the point P (2, 3) from the x>axis is
(a) 2 units
(b) 3 units
(c) 1 unit
(d) 5 units
Solution:
The distance of the point P (2, 3) from x- axis is 3 units (as y = 3). **(b)**

Question 13.
The distance of the point P (-4, 3) from the y-axis is
(a) 5 units
(b) -4 units
(c) 4 units
(d) 3 units
Solution:
The distance of the point P (-4, 3) from y- axis will be 4 units. **(c)**

Question 14.
The distance of the point P (-6, 8) from the origin is
(a) 8 units
(b) $2\sqrt{7}$ units
(c) 10 units
(d) 6 units
Solution:
The distance of point P (–6, 8) from origin

is $\sqrt{(6)^2+(8)^2} = \sqrt{36+64}$

$= \sqrt{100} = 10$ units (c)

Question 15.
The distance between the points A (0, 6) and B (0, -2) is
(a) 6 units
(b) 8 units
(c) 4 units
(d) 2 units

Solution:

1. $AB = \sqrt{(0-0)^2 + (6+2)^2} = \sqrt{0^2 + 8^2}$

$= \sqrt{8^2} = 8$ units (b)

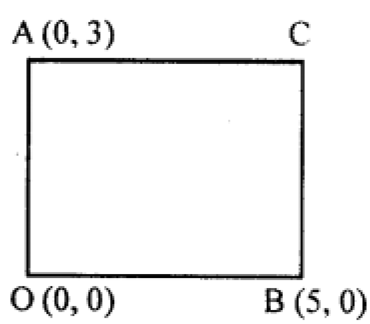
Question 16.
The distance between the points (0, 5) and (-5, 0) is
(a) 5 units
(b) 5$\sqrt{2}$units
(c) 2 $\sqrt{7}$ units
(d) 10 units
Solution:
The distance between the points (0, 5) and (-5, 0) is

$= \sqrt{(-5-0)^2 + (0-5)^2} = \sqrt{(-5)^2 + (-5)^2}$

$= \sqrt{25+25} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ (b)

Question 17.
AOBC is a rectangle whose three vertices are A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is
(a) 5 units
(b) 3 units
(c) $\sqrt{34}$ units
(d) 4 units
Solution:

Length of its diagonal



$AB = \sqrt{(5-0)^2 + (0-3)^2} = \sqrt{(5)^2 + (-3)^2}$

$= \sqrt{25+9} = \sqrt{34}$ units (c)

Question 18.
If the distance between the points (2, -2) and (-1, x) is S units, then one of the value of x is
(a) -2
(b) 2
(c) -1
(d) 1
Solution:
Distance between $(2, -2)$ and $(-1, x) = 5$ units

$$\therefore \sqrt{(2+1)^2 + (-2-x)^2} = 5$$
$$\Rightarrow \sqrt{3^2 + (-2-x)^2} = 5$$

Squaring,
$\Rightarrow 3^2 + 4 + x^2 + 4x = 25$
$\Rightarrow x^2 + 4x + 13 - 25 = 0$
$\Rightarrow x^2 + 4x - 12 = 0$
$\Rightarrow x^2 + 6x - 2x - 12 = 0$
$\Rightarrow x(x + 6) - 2(x + 6) = 0$
$\Rightarrow (x + 6)(x - 2) = 0$
$\therefore$ Either $x + 6 = 0$, then $x = -6$
or $x - 2 = 0$, then $x = 2$

One value of $x = 2$ (b)

Question 19.
The distance between the points (4, p) and (1, 0) is 5 units, then the value of p is
(a) 4 only
(b) -4 only
(c) ±4
(d) 0
Solution:
Distance between $(4, p)$ and $(1, 0)$ is 5 units

$$\therefore \sqrt{(4-1)^2 + (p-0)^2} = 5$$
$$\sqrt{3^2 + p^2} = 5 \Rightarrow 9 + p^2 = 25 \quad \text{(squaring)}$$
$$p^2 = 25 - 9 = 16$$
$$\therefore p = \pm 4$$ (c)

Question 20.
The points (-4, 0), (4, 0) and (0, 3) are the vertices of a
(a) right triangle
(b) isosceles triangle
(c) equilateral triangle
(d) scalene triangle
Solution:

Points A (–4, 0), B (4, 0), C (0, 3) are the vertices of a triangle

Now, AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4+4)^2 + (0)^2} = \sqrt{(8)^2} = 8$ units

BC = $\sqrt{(0-4)^2 + (3-0)^2}$

$= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

CA = $\sqrt{0+4)^2 + (3-0)^2}$

$= \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$\because$ Two sides are equal in length $(\because BC = CA)$
$\therefore$ It is an isosceles triangle. (b)

Question 21.
The area of a square whose vertices are A (0, -2), B (3, 1), C (0, 4) and D (-3, 1) is
(a) 18 sq. units
(b) 15 sq. units
(c) $\sqrt{18}$ sq. units
(d) $\sqrt{15}$ sq. units
Solution:

Vertices of a square are

A (0, –2), B (3, 1), C (0, 4) and D (–3, 1)

$\therefore$ AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

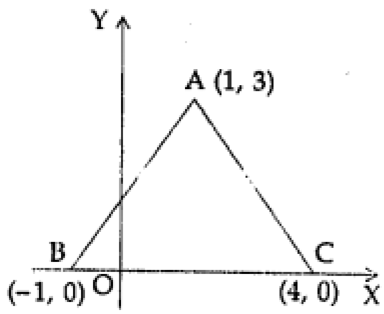
$= \sqrt{(3-0)^2 + (1+2)^2} = \sqrt{3^2 + 3^2}$

$= \sqrt{9+9} = \sqrt{18}$

$\therefore$ Area of square = $(\text{side})^2$

$= (\sqrt{18})^2 = 18$ sq. units (a)

Question 22.
In the given figure, the area of the triangle ABC is
(a) 15 sq. units
(b) 10 sq. units
(c) 7.5 sq. units
(d) 2.5 sq. units
Solution:



Vertices of a ΔABC are A (1, 3), B (–1, 0), C (4, 0)

$\therefore\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1-1)^2 + (0-3)^2}$

$= \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

$BC = \sqrt{(4+1)^2 + (0+0)^2} = \sqrt{5^2 + 0}$

$= \sqrt{5^2} = 5$ units

$\because$ Coordinates of A are (1, 3)

$\therefore$ Distance from A is x-axis = 3 units

$\therefore\ \text{Area} = \frac{1}{2} BC \times 3 = \frac{1}{2} \times 5 \times 3$

$= \frac{15}{2} = 7.5$ sq. units (c)

Question 23.

The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is
(a) 5 units
(b) 12 units
(c) 11 units
(d) 7 + $\sqrt{5}$ units

Solution:

Vertices of a ΔABC are A (0, 4), B (0, 0), C (3, 0)

$\therefore$ $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-0)^2 + (0-4)^2}$

$= \sqrt{0^2 + (-4)^2}$

$= \sqrt{0+16} = \sqrt{16} = 4$ units

$BC = \sqrt{(3-0)^2 + (0-0)^2}$

$= \sqrt{3^2 + 0^2}$

$= \sqrt{9+0} = \sqrt{9} = 3$ units

and $CA = \sqrt{(3-0)^2 + (0-4)^2}$

$= \sqrt{3^2 + (-4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ units

$\therefore$ Perimeter of $\Delta ABC = AB + BC + CA$

$= 4 + 3 + 5 = 12$ units (b)

Question 24.

If A is a point on the .y-axis whose ordinate is 5 and B is the point (-3, 1), then the length of AB is

(a) 8 units
(b) 5 units
(c) 3 units
(d) 25 units

Solution:

A is a point an y-axis whose ordinate is 4 and B is a point (–3, 1), then length of coordinates of A will be (0, 5)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-3-0)^2 + (1-5)^2}$$

$$= \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units} \qquad \text{(b)}$$

Question 25.

The point A (9, 0), B (9, 6), C (-9, 6) and D (-9, 0) are the vertices of a
(a) rectangle
(b) square
(c) rhombus
(d) trapezium

Solution:

A (9, 0), B (9, 6), C (–9, 6) and D (–9, 0)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9-9)^2 + (6-0)^2}$

$= \sqrt{0^2 + 6^2} = \sqrt{0+36} = \sqrt{36} = 6$ units

$BC = \sqrt{(-9-9)^2 + (6-6)^2}$

$= \sqrt{(-18)^2 + 0^2} = \sqrt{18^2 + 0^2}$

$= \sqrt{324} = 18$ units

$CD = \sqrt{[-9-(-9)]^2 + (0-6)^2}$

$= \sqrt{(9-9)^2 + (-6)^2}$

$= \sqrt{(0)^2 + 6^2} = \sqrt{36} = \sqrt{36}$

$= 6$ units

$DA = \sqrt{(-9-9)^2 + (0-0)^2}$

$= \sqrt{(-18)^2 + (0)^2}$

$= \sqrt{324+0} = \sqrt{324} = 18$ units

$\because$ AB = CD and BC = DA

and these are opposite sides

$\therefore$ ABCD is a rectangle. (a)

Chapter Test

Question 1.

Three vertices of a rectangle are A (2, -1), B (2, 7) and C(4, 7). Plot these points on a graph and hence use it to find the co-ordinates of the fourth vertex D Also find the co-ordinates of

(i) the mid-point of BC

(ii) the mid point of CD

(iii) the point of intersection of the diagonals. What is the area of the rectangle ?

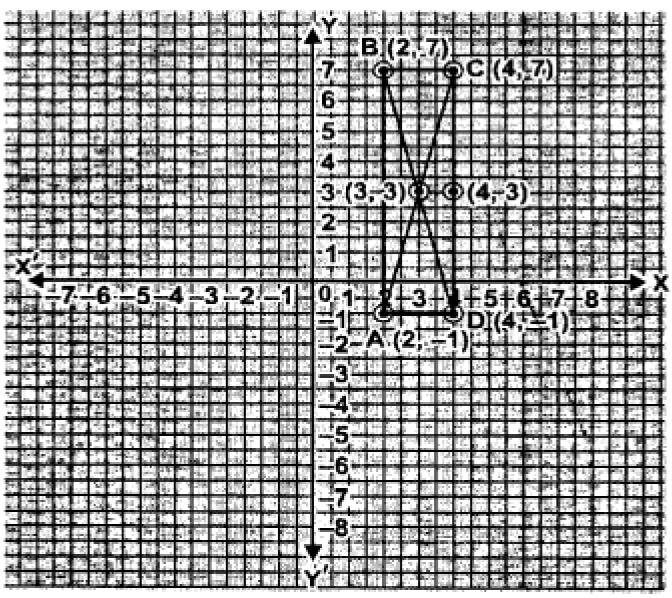
Solution:

Given three vertices of a rectangle are
A (2, –1), B (2, 7) and C (4, 7)
From graph the co-ordinates of the fourth vertex D (4, –1)

(*i*) mid-point of BC is (3, 7)

(*ii*) mid-point of CD is (4, 3)

(*iii*) The point of intersection of the diagonals (3, 3). Area of rectangle ABCD = AB × BC = 8 × 2 sq. units = 16 sq. units.

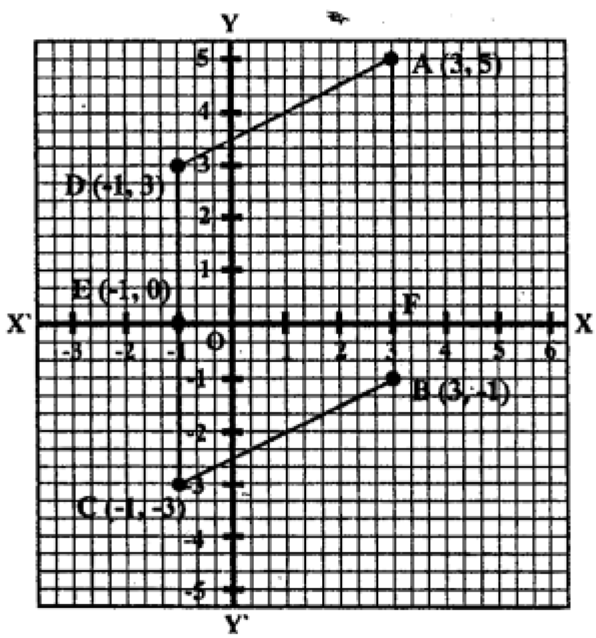


Question 2.

Three vertices of a parallelogram are A (3, 5), B (3, -1) and C (-1, -3). Plot these points on a graph paper and hence use it to find the coordinates of the fourth vertex D. Also find the coordinates of the mid-point of the side CD. What is the area of the parallelogram?

Solution:

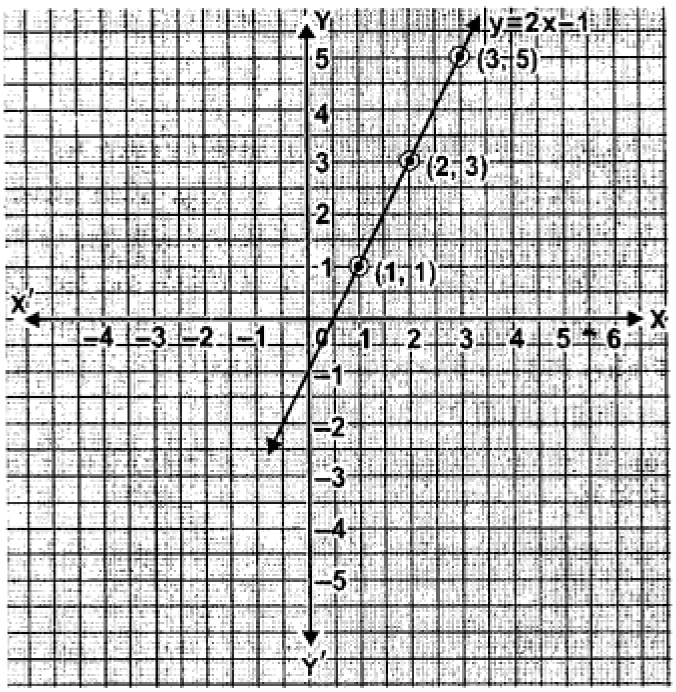
The vertices A, B and C of parallelogram are A (3, 5), B (3, –1) and C (–1, –3)
D is the fourth vertex of the parallelogram which is (–1, 3)
E is the mid-piont of CD whose coordinates are (–1, 0)
Now area of the parallelogram ABCD
= Base × Height = AB × EF = 6 × 4 = 24 sq. units



Question 3.
Draw the graphs of the following linear equations.
(i) y = 2x – 1
(ii) 2x + 3y = 6
(iii) 2x – 3y = 4.
Also find slope and y-intercept of these lines.
Solution:

(*i*) $y = 2x - 1$

x	1	2	3
y	1	3	5



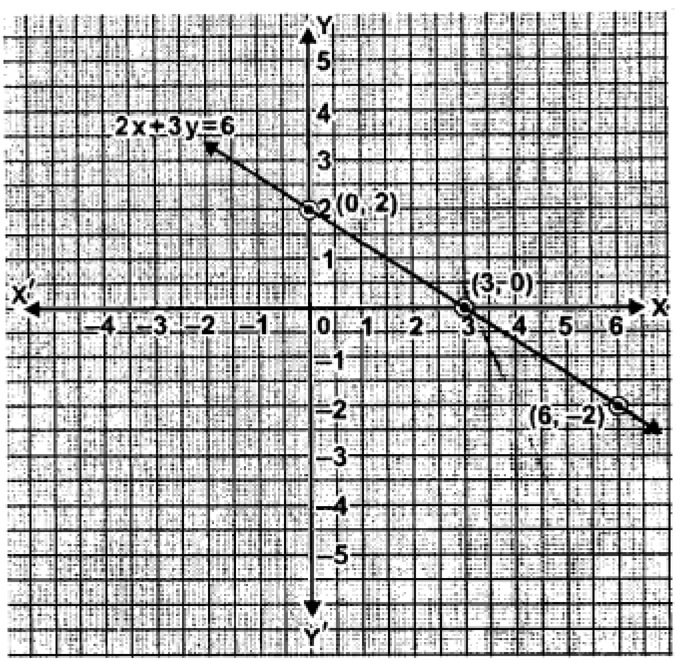
$m = 2$ and $c = -1$

(*ii*) $2x + 3y = 6$ or $3y = 6 - 2x$

or $\quad y = \dfrac{6 - 2x}{3}$

$$= \frac{-2x}{3} + 2$$

x	0	3	6
y	2	0	–2



$m = \frac{-2}{3}$ and $c = 2.$

(iii) $2x - 3y = 4$

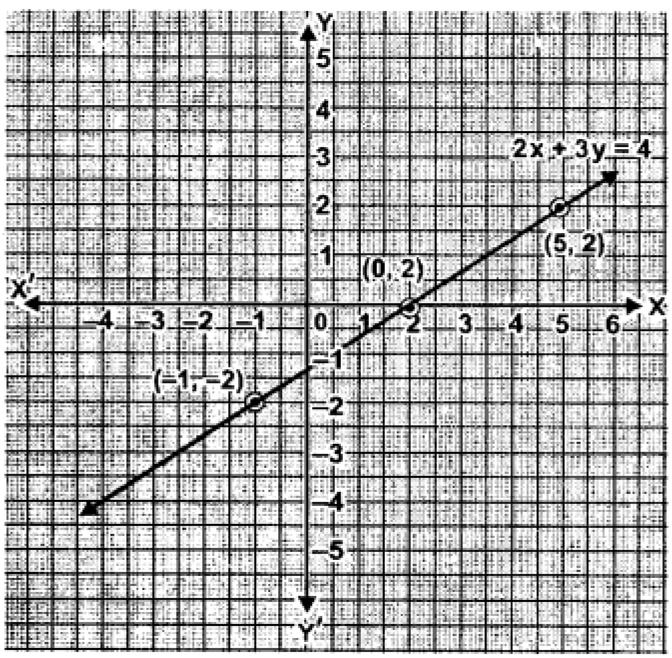
or $-3y = 4 - 2x$

or $3y = 2x - 4$

or $y = \frac{2x - 4}{3}$

$= \frac{2}{3}x - \frac{4}{3}$

x	2	5	–1
y	0	2	–2



$m = \frac{2}{3}$ and $c = \frac{-4}{3}$

Question 4.

Draw the graph of the equation 3x – 4y = 12. From the graph, find :

(i) the value of y when x = -4
(ii) the value of x when y = 3.
Solution:

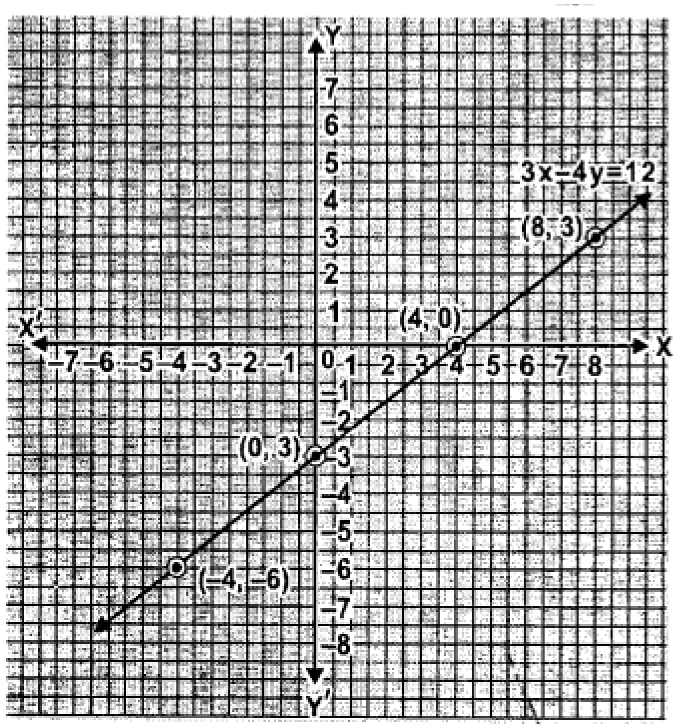
Given equation is $3x - 4y = 12$

or $3x = 12 + 4y$ or $x = \dfrac{12 + 4y}{3}$ or x

$= \dfrac{4y + 12}{3}$ or $x = \dfrac{4}{3}y + \dfrac{12}{3}$

or $x = \dfrac{4}{3}y + 4$

x	4	8	0
y	0	3	–3



(*i*) when $x = -4$ then value of $y = -6$
(*ii*) when $y = 3$ then value of $x = 8$.

Question 5.
Solve graphically, the simultaneous equations: $2x - 3y = 7$; $x + 6y = 11$.
Solution:

$2x - 3y = 7, x + 6y = 11$

$2x - 3y = 7 \Rightarrow 2x = 3y + 7$

$$\Rightarrow x = \frac{3y+7}{2}$$

Giving some different value to y, we get corresponding value of x.

x	5	2	–1
y	1	–1	–3

Plot the points (5, 1), (2, –1) and (–1, –3) on the graph and join them to get a line.

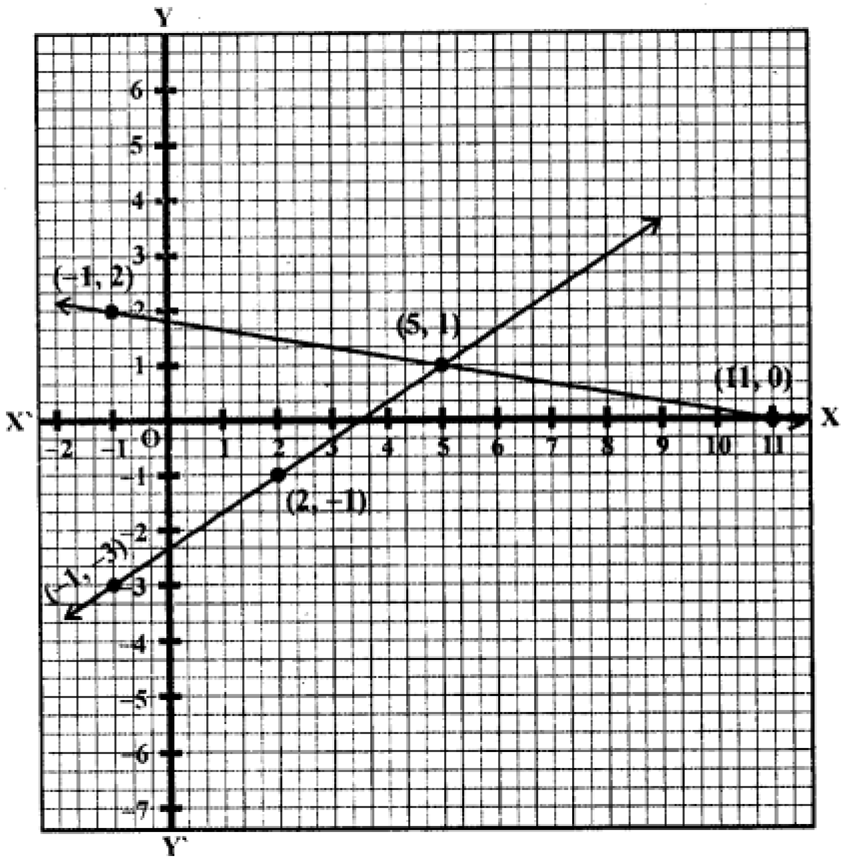
Similarly in

$x + 6y = 11 \Rightarrow x = 11 - 6y$

x	5	–1	11
y	1	2	0

Now plot the points (5, 1), (–1, 2) and (11, 0) and join them to get another line.

We see that there two lines intersect at (5, 1)
Hence $x = 5, y = 1$



Question 6.
Solve the following system of equations graphically: x – 2y – 4 = 0, 2x + .y – 3 = 0.
Solution:

$x - 2y - 4 = 0$ and $2x + y - 3 = 0$

$x - 2y - 4 = 0 \Rightarrow x = 2y + 4$

Giving some different value to y, we get corresponding values of x

x	4	2	0
y	0	–1	–2

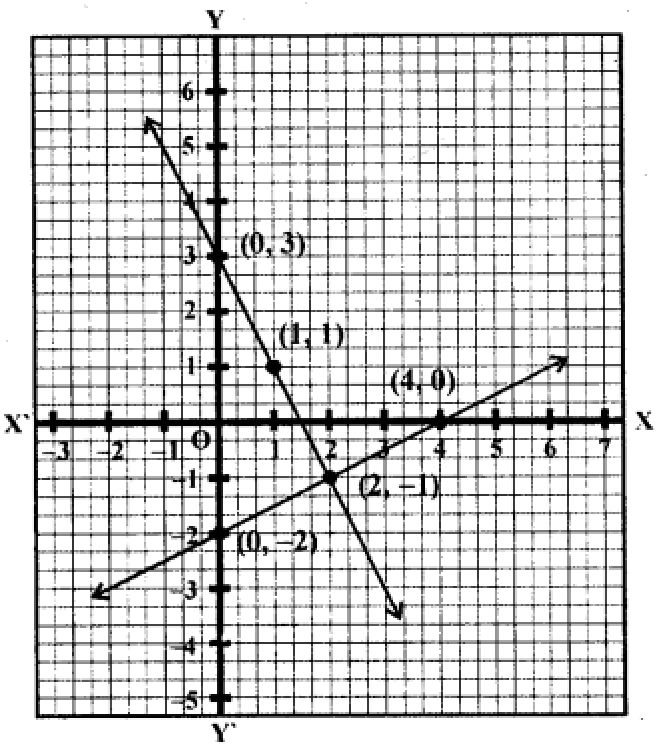
Plot the points (4, 0), (2, –1) and (0, –2) on the graph and join them to get a line.

Similarly in $2x + y - 3 = 0 \Rightarrow y = 3 - 2x$

x	0	1	2
y	3	1	–1

Plot the points (0, 3), (1, 1) and (2, –1) and join them to get another line.

We see that these lines intersect each other at $x = 2$, $y = -1$



Question 7.

Using a scale of l cm to 1 unit for both the axes, draw the graphs of the following equations : 6y = 5x:+ 10,y = 5;c-15. From the graph, find

(i) the coordinates of the point where the two lines intersect.

(ii) the area of the triangle between the lines and the x-axis.

Solution:

$6y = 5x + 10, y = 5x - 15$

$$6y = 5x + 10 \Rightarrow y = \frac{5x+10}{6}$$

Giving some different values to x, we get corresponding values of y

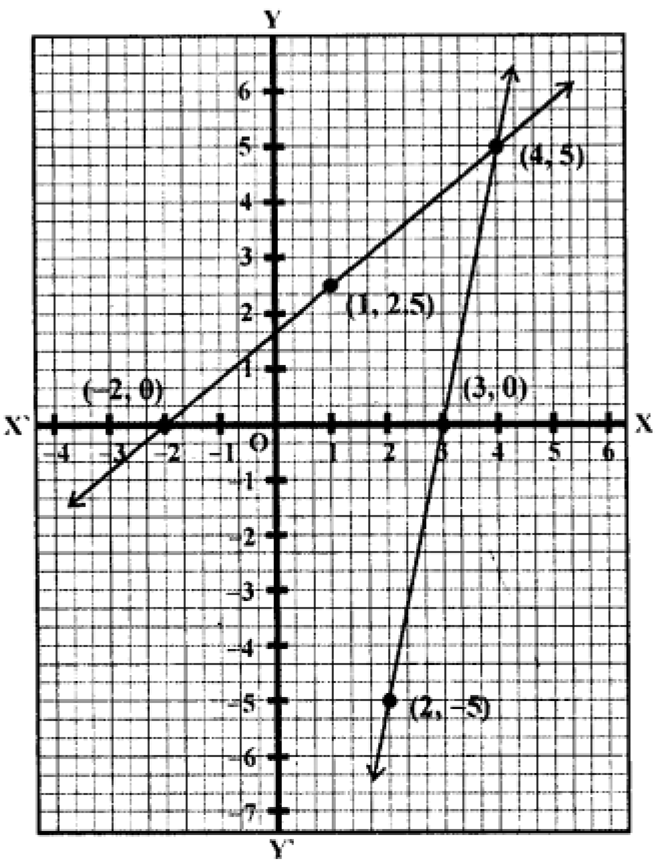
x	1	–2	4
y	2.5	0	5

Plot the points (1, 2.5), (–2, 0) and (4, 5) on the graph and join them to get a line.

Similarly in $y = 5x - 15$

x	2	3	4
y	–5	0	5

Plot the points (2, –5), (3, 0) and (4, 5) on the graph and join them to get a line.
We see that three two lines intersect each other at



Question 8.
Find, graphically, the coordinates of the vertices of the triangle formed by the lines : 8y – 3x + 7 = 0, 2x-y + 4 = 0 and 5x + 4y = 29.
Solution:

$8y - 3x + 7 = 0 \Rightarrow 8y = 3x - 7$

$\Rightarrow y = \dfrac{3x-7}{8}$

Giving some different values to x, we get corresponding values of y

x	1	5	–3
y	$\dfrac{-1}{2}$	1	–2

Plot the points $\left(1, \dfrac{-1}{2}\right)$, (5, 1), (–3, –2) on the graph and join them to get a line

$2x - y + 4 = 0 \Rightarrow 2x = y - 4$

$\Rightarrow x = \dfrac{y-4}{2}$

Giving some different values to y, we get corresponding value of x

x	–2	–1	0
y	0	2	4

Plot the point on the graph and join them to get a line

and $5x + 4y = 29 \Rightarrow 5x = 29 - 4y$

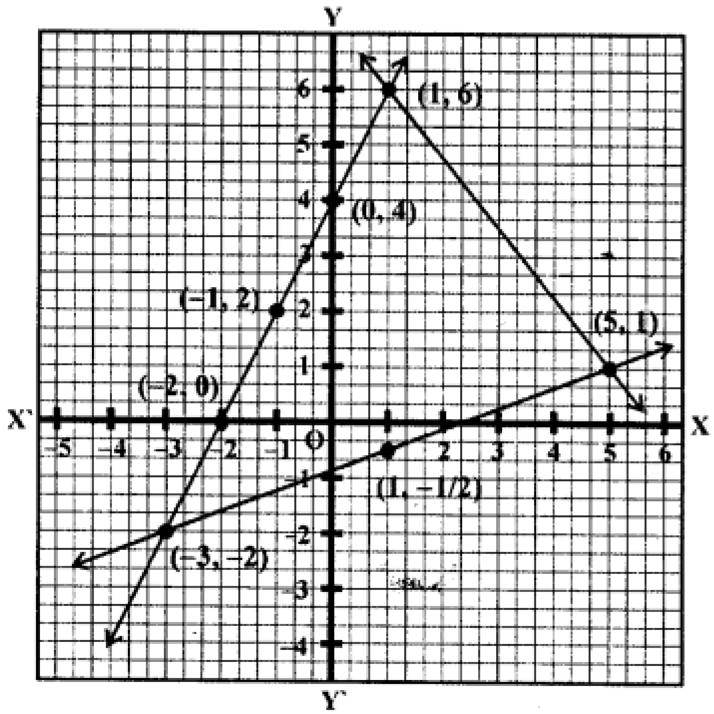
$\Rightarrow x = \dfrac{29-4y}{5}$

x	5	1	–4
y	1	6	9

Plot the points on the graph and join them to get another line.

We see that there three lines intersect each other at (–3, –2), (1, 5) and (1, 6) respectively

Therefore vertices of (–3, –2, (1, 5), (1, 6).



Question 9.
Find graphically the coordinates of the vertices of the triangle formed by the lines y-2 = 0,2y + x = 0 and y + 1 = 3(x – 2). Hence, find the area of the triangle formed by these lines.
Solution:

$y - 2 = 0$

$y = 2$, which is parallel to x-axis

x	0	1	3
y	2	2	2

$2y + x = 0 \Rightarrow x = -2y$

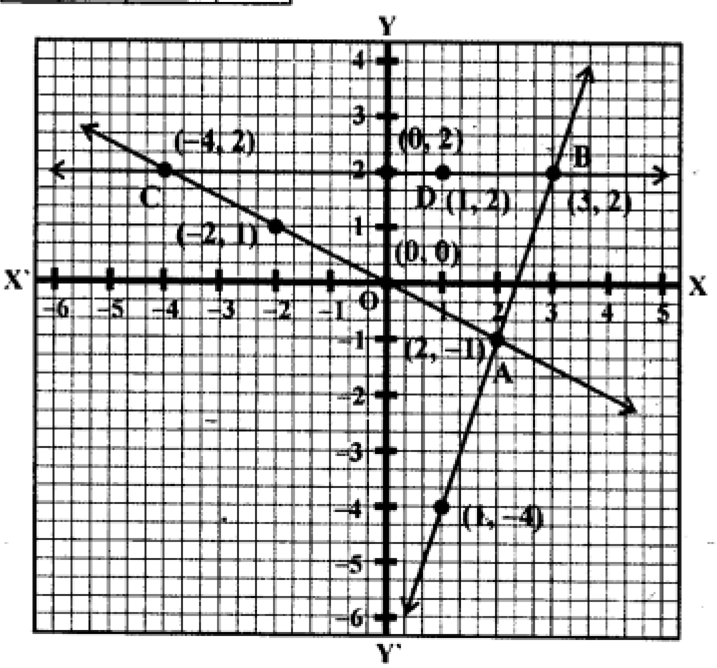
x	0	–2	–4
y	0	1	2

Plot the points (0, 0), (–2, 1) and (–4, 2) on the graph and join them to get a line.

$y + 1 = 3(x - 2) = 3x - 6$

Giving some different values to x, we get corresponding the value of y

x	1	2	3
y	–4	–1	2



Plot the points (1, –4), (2, –1) and (3, 2) on the graph and join them to get another line. Now we see that three lines intersect each other.

Coordinates of the vertices of the triangle are (2, –1), (3, 2), (–4, 2) and

$\therefore$ Area of triangle $= \dfrac{BC \times AB}{2}$

$= \dfrac{7 \times 3}{2} = \dfrac{21}{2} = 10.5 \text{ cm}^2$

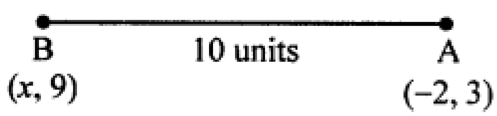
Question 10.
A line segment is of length 10 units and one of its end is (-2,3). If the ordinate of the other end is 9, find the abscissa of the other end.
Solution:

Ordinates of the point on the other end $(y) = 9$

Let abscissa $(x) = x$

Then distance between the two ends (–2, 3) and $(x, 9) = \sqrt{(x+2)^2 + (9-3)^2}$

$\therefore \sqrt{(x+2)^2 + (6)^2} = 10$



$\Rightarrow x^2 + 4x + 4 + 36 = 100$

$\Rightarrow x^2 + 4x = 100 - 36 - 4 = 60$

$\Rightarrow x^2 + 4x - 60 = 0$

$\Rightarrow x^2 + 10x - 6x - 60 = 0$

$\Rightarrow x(x + 10) - 6(x + 10) = 0$

$\Rightarrow (x + 10)(x - 6) = 0$

Either $x + 10 = 0$, then $x = -10$

or $x - 6 = 0$, then $x = 6$

$\therefore$ Abscissa will be –10 or 6

Question 11.
A (-4, -1), B (-1, 2) and C (a, 5) are the vertices of an isosceles triangle. Find the value of a, given that AB is the unequal side.
Solution:

A (–4, –1), B (–1, 2) and C (α, 5) are vertices of an isosceles triangle. AB is the unequal side.

∴ AC = BC

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\alpha + 4)^2 + (5 + 1)^2} = \sqrt{(\alpha + 4)^2 + 6^2}$$

and $BC = \sqrt{(\alpha + 1)^2 + (5 - 2)^2}$

$$= \sqrt{(\alpha + 1)^2 + 3^2}$$

$$\therefore \sqrt{(\alpha + 4)^2 + 6^2} = \sqrt{(\alpha + 1)^2 + 3^2}$$

Squaring both sides,

$(\alpha + 4)^2 + 36 = (\alpha + 1)^2 + 9$

$\alpha^2 + 8\alpha + 16 + 36 = \alpha^2 + 2\alpha + 1 + 9$

$8\alpha - 2\alpha = 1 + 9 - 16 - 36$

$$6\alpha = -42 \Rightarrow \alpha = \frac{-42}{6} = -7$$

∴ $\alpha = -7$

Question 12.
If A (-3, 2), B (a, p) and C (-1, 4) are the vertices of an isosceles triangle, prove that α + β = 1, given AB = BC.
Solution:

A (–3, 2), B (α, β) and C (–1, 4) are the value of an isosceles triangle AB = BC

Now, AB = $\sqrt{(\alpha+3)^2+(\beta-2)^2}$

and BC = $\sqrt{(\alpha+1)^2+(\beta-4)^2}$

$\because$ AB = BC

$\therefore \sqrt{(\alpha+3)^2+(\beta-2)^2} = \sqrt{(\alpha+1)^2+(\beta-4)^2}$

Squaring both sides,

$(\alpha + 3)^2 + (\beta - 2)^2 = (\alpha + 1) + (\beta - 4)^2$

$\Rightarrow \alpha^2 + 6\alpha + 9 + \beta^2 - 4\beta + 4 = \alpha^2 + 2\alpha + 1 + \beta^2 - 8\beta + 16$

$6\alpha - 2\alpha - 4\beta + 8\beta = 16 - 9 - 4 + 1$

$4\alpha + 4\beta = 4 \Rightarrow \alpha + \beta = 1$ (dividing by 4)

Hence $\alpha + \beta = 1$

A (–3, 2), B (α, β) and C (–1, 4) are the value of an isosceles triangle AB = BC

Now, AB = $\sqrt{(\alpha+3)^2+(\beta-2)^2}$

and BC = $\sqrt{(\alpha+1)^2+(\beta-4)^2}$

$\because$ AB = BC

$\therefore \sqrt{(\alpha+3)^2+(\beta-2)^2} = \sqrt{(\alpha+1)^2+(\beta-4)^2}$

Squaring both sides,

$(\alpha + 3)^2 + (\beta - 2)^2 = (\alpha + 1) + (\beta - 4)^2$

$\Rightarrow \alpha^2 + 6\alpha + 9 + \beta^2 - 4\beta + 4 = \alpha^2 + 2\alpha + 1 + \beta^2 - 8\beta + 16$

$6\alpha - 2\alpha - 4\beta + 8\beta = 16 - 9 - 4 + 1$

$4\alpha + 4\beta = 4 \Rightarrow \alpha + \beta = 1$ (dividing by 4)

Hence $\alpha + \beta = 1$

Question 13.

Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

Solution:

Let points A (3, 0), B (6, 4) and (–1, 3) are the vertices of a right angled.

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$BC = \sqrt{(-1-6)^2 + (3-4)^2}$

$= \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50}$

$= \sqrt{25 \times 2} = 5\sqrt{2}$

$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

$\therefore\ AB^2 + AC^2 = 5^2 + 5^2$

$= 25 + 25 = 50$

$= BC^2$

$\therefore\ \Delta ABC$ is a right angled triangle.

Question 14.
(i) Show that the points (2, 1), (0,3), (-2, 1) and (0, -1), taken in order, are the vertices of a square. Also find the area of the square.
(ii) Show that the points (-3, 2), (-5, -5), (2, -3) and (4, 4), taken in order, are the vertices of rhombus. Also find its area. Do the given points form a square?
Solution:

(*i*) Let points A (2, 1), B (0, 3), C (–2, 1) and D (0, –1) taking in order, are the vertices of the square

Now, AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-2)^2 + (3-1)^2} = \sqrt{2^2 + 2^2}$

$= \sqrt{4+4} = \sqrt{8}$

BC = $\sqrt{(-2-0)^2 + (1-3)^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

CD = $\sqrt{(0-2)^2 + (-1-1)^2}$

$= \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$

CA = $\sqrt{(2-0)^2 + (1+1)^2} = \sqrt{2^2 + 2^2}$

$= \sqrt{4+4} = \sqrt{8}$

DA = $\sqrt{(2-0)^2 + (1+1)^2} = \sqrt{2^2 + 2^2}$

$= \sqrt{4+4} = \sqrt{8}$

$\because$ AB = BC = CD = DA

$\therefore$ ABCD is a square with side $\sqrt{8}$

Area = $(\text{side})^2 = (\sqrt{8})^2 = 8$ sq. units

(*ii*) Let the given points are A (–3, 2), B (–5, –5), C (2, –3) and D (4, 4)

$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$

$= \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$

$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{7^2 + 2^2}$

$= \sqrt{49+4} = \sqrt{53}$

$BC = \sqrt{(2+5)^2 + (-3+5)^2}$

$= \sqrt{7^2 + 2^2} = \sqrt{49+4} = \sqrt{53}$

$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{2^2 + 7^2}$

$= \sqrt{4+49} = \sqrt{53}$

$DA = \sqrt{(-3-4)^2 + (2-4)^2}$

$= \sqrt{(-7)^2 + (-2)^2} = \sqrt{4+49} = \sqrt{53}$

$\because$ AB = BC = CD = DA

ABCD is a square or rhombus

Now diagonal AC = $\sqrt{(2+3)^2+(-3-2)^2}$

$= \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

and BD = $\sqrt{(4+5)^2+(4+5)^2}$

$= \sqrt{9^2+9^2} = \sqrt{81+81} = \sqrt{162}$

$\because$ AC $\neq$ BD

$\therefore$ ABCD is a rhombus not a square

$\therefore$ Area = $\frac{\text{Product of diagonal}}{2}$

$= \frac{\sqrt{50}\times\sqrt{162}}{2} = \frac{\sqrt{8100}}{2}$

$= \frac{90}{2} = 45$ sq. units

Question 15.

The ends of a diagonal of a square have co-ordinates (-2, p) and (p, 2). Find p if the area of the square is 40 sq. units.

Solution:

Ends of a diagonal of a square are $(-2, p)$ and $(p, 2)$

Area of square = 40 sq. units

$\therefore$ Side = $\sqrt{40}$ units = $2\sqrt{10}$ units

and diagonal = $\sqrt{2}$ × side

$= \sqrt{2} \times \sqrt{40} = \sqrt{80} = 4\sqrt{5}$ unit

Diagonal = AC = $\sqrt{(x_2 - x_1)^2 \times (y_2 - y_1)^2}$

$= \sqrt{(p+2)^2 + (2-p)^2} = 4\sqrt{5}$

Squaring both side,

$(p + 2)^2 + (2 - p)^2 = 16 \times 5 = 80$

$\Rightarrow p^2 + 4p + 4 + 4 - 4p + p^2 = 80$

$\Rightarrow 2p^2 + 8 = 80 \Rightarrow 2p^2 = 80 - 8 = 72$

$\Rightarrow p^2 = \frac{72}{2} = 36 = (\pm 6)^2$

$\therefore p = \pm 6$

$\therefore p = 6, -6$

Question 16.

What type of quadrilateral do the points A (2, -2), B (7, 3), C (11, -1) and D (6, -6), taken in the order, form?

Solution:

Vertices of a quadrilateral ABCD are A (2, –2), B (7, 3), C (11, –1), D (6, –6) taken on order.

$$AB = \sqrt{(x_2 - x_1)^2 \times (y_2 - y_1)^2}$$

$$= \sqrt{(7-2)^2 + (3+2)^2}$$

$$= \sqrt{5^2 + 5^2}$$

$$= \sqrt{25+25} = \sqrt{50}$$

$$BC = \sqrt{(11-7)^2 + (-1-3)^2}$$

$$= \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32}$$

$$CD = \sqrt{(6-11)^2 + (-6+1)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50}$$

$$DA = \sqrt{(6-2)^2 + (-6+2)^2}$$

$$= \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32}$$

$\therefore$ AB = CD and BC = DA

$\therefore$ ABCD is a rectangle

($\because$ Opposite sides are equal)

Question 17.

Find the coordinates of the centre of the circle passing through the three given points A (5, 1), B (-3, -7) and C (7, -1).

Solution:

Let coordinates of the centre of the circle be (x, y)

Points A (5, 1), B (–3, –7) and C (7, –1) are on the circle

$\therefore$ OA = OB = OC

Now, OA = $\sqrt{(x_2 - x_1)^2 \times (y_2 - y_1)^2}$

$= \sqrt{(x-5)^2 + (y-1)^2}$

OB = $\sqrt{(x+3)^2 + (y+7)^2}$

OC = $\sqrt{(x-7)^2 + (y+1)^2}$

$OA^2 = OB^2$ and $OA^2 = OC^2$

$\therefore (x-5)^2 + (y-1)^2 = (x+3)^2 + (y+7)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 6x + 9 + y^2 + 14y + 49$

$\Rightarrow 6x + 14y + 10x + 2y = -9 - 49 + 25 + 1$

$\Rightarrow 16x + 16y = -32$

$\Rightarrow x + y = -2$

$\Rightarrow x = -2 - y$ ...(*i*)

Now $OA^2 = OC^2$

$(x-5)^2 + (y-1)^2 = (x-7)^2 + (y+1)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 - 14x + 49 + y^2 + 1 + 2y$

$\Rightarrow -10x + 14x - 2y - 2y = 49 + 1 - 25 - 1$

$\Rightarrow 4x - 4y = 24$

$\Rightarrow x - y = 6$ ...(*ii*)

(Taking 4 common)

Now substitute the value of (*i*) in (*ii*), we get

$\Rightarrow (-2 - y) - y = 6$

$\Rightarrow -2 - y - y = 6$

$\Rightarrow -2y = 6 + 2 \Rightarrow y = \dfrac{-8}{2} \Rightarrow y = -4$

Now put the value of $y = -4$ in equation (i)

$x = -2 - y = -2 - (-4)$

$= -2 + 4 = 2$

$\therefore$ The coordinates of the centre of the circle are $(2, -4)$