

Chapter 3 Expansions

EXERCISE - 3.1

Solution - 1

(i) $(2x + 7y)^2$

It is in the form of $(a+b)^2 = a^2 + 2ab + b^2$

$$\therefore a = 2x ; b = 7y$$

$$\begin{aligned}\therefore (2x + 7y)^2 &= (2x)^2 + 2 \cdot 2x \cdot 7y + (7y)^2 \\ &= 4x^2 + 28xy + 49y^2.\end{aligned}$$

(ii) $(\frac{1}{2}x + \frac{2}{3}y)^2$

$$\Rightarrow (\frac{1}{2}x)^2 + 2 \cdot \frac{1}{2}x \cdot \frac{2}{3}y + (\frac{2}{3}y)^2$$

$$= \frac{x^2}{4} + \frac{2xy}{3} + \frac{4}{9}y^2$$

Solution - 2 :

(i) $(3x + \frac{1}{2x})^2$

It is in the form of $(a+b)^2 = a^2 + 2ab + b^2$

$$\therefore (3x)^2 + 2 \cdot 3x \cdot \frac{1}{2x} + (\frac{1}{2x})^2$$

$$\Rightarrow 9x^2 + 3 + \frac{1}{4x^2}$$

$$(ii) (3x^2y + 5z)^2$$

It is in the form of $(a+b)^2 = a^2 + 2ab + b^2$

$$\text{Here } a = 3x^2y \text{ ; } b = 5z.$$

$$\Rightarrow (3x^2y)^2 + 2 \cdot 3x^2y \cdot 5z + (5z)^2$$

$$\Rightarrow 9x^4y^2 + 30x^2yz + 25z^2.$$

Solution - 3

$$(i) \left(3x - \frac{1}{2x}\right)^2$$

It is in the form of $(a-b)^2 = a^2 - 2ab + b^2$

$$\text{Here } a = 3x \text{ ; } b = \frac{1}{2x}$$

$$\Rightarrow (3x)^2 - 2 \cdot 3x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2.$$

$$\rightarrow 3^2 \cdot x^2 - 3 + \frac{1}{2^2 x^2}$$

$$\Rightarrow 9x^2 - 3 + \frac{1}{4x^2} //$$

$$(ii) \left(\frac{1}{2}x - \frac{3}{2}y\right)^2$$

It is in the form of $(a-b)^2 = a^2 - 2ab + b^2$

$$\text{here } a = \frac{1}{2}x, \quad b = \frac{3}{2}y.$$

$$\therefore \Rightarrow \left(\frac{1}{2}x\right)^2 - 2 \cdot \frac{1}{2}x \cdot \frac{3}{2}y + \left(\frac{3}{2}y\right)^2$$

$$\Rightarrow \frac{x^2}{4} - \frac{3xy}{2} + \frac{9y^2}{4}$$

Solution-4:

$$(i) (x+3)(x+5)$$

$$\Rightarrow x(x+5) + 3(x+5)$$

$$\Rightarrow x^2 + 5x + 3x + 15$$

$$\Rightarrow x^2 + 8x + 15$$

$$(ii) (x+3)(x-5)$$

$$\Rightarrow x(x-5) + 3(x-5)$$

$$\Rightarrow x \cdot x - x \cdot 5 + 3 \cdot x - 3 \cdot 5$$

$$\Rightarrow x^2 - 5x + 3x - 15$$

$$\Rightarrow x^2 - 2x - 15$$

$$(ii) (x-7)(x+9)$$

$$\rightarrow x(x+9) - 7(x+9)$$

$$\rightarrow x \cdot x + 9 \cdot x - 7 \cdot x - 7 \cdot 9$$

$$\rightarrow x^2 + 9x - 7x - 63$$

$$\rightarrow x^2 + 2x - 63.$$

$$(iv) (x-2y)(x-3y)$$

$$\rightarrow x(x-3y) - 2y(x-3y)$$

$$\rightarrow x \cdot x - x \cdot 3y - 2y \cdot x + 2y \cdot 3y$$

$$\rightarrow x^2 - 3xy - 2xy + 6y^2$$

$$\rightarrow x^2 - 5xy + 6y^2$$

Solution - 5

$$(i) (x-2y-z)^2$$

It is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$.

Here $a=x$; $b=-2y$; $c=-z$.

$$\therefore \Rightarrow x^2 + (-2y)^2 + (-z)^2 + 2(x(-2y) + (-2y)(-z) + (-z) \cdot x)$$

$$\rightarrow x^2 + 4y^2 + z^2 + 2(-2xy + 2yz - zx)$$

$$\rightarrow x^2 + 4y^2 + z^2 + 4yz - 4xy - 2zx$$

$$(ii) (2x - 3y + 4z)^2$$

It is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$.

Here $a = 2x$; $b = -3y$; $c = 4z$.

$$\therefore \Rightarrow (2x)^2 + (-3y)^2 + (4z)^2 + 2(2x \cdot (-3y) + (-3y)(4z) + 4z \cdot 2x)$$

$$\Rightarrow 4x^2 + 9y^2 + 16z^2 + 2(-6xy - 12yz + 8xz)$$

$$\Rightarrow 4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16xz$$

Solution-6 :

$$(i) \left(2x + \frac{3}{x} - 1\right)^2$$

It is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Here $a = 2x$; $b = \frac{3}{x}$; $c = -1$

$$\therefore \Rightarrow (2x)^2 + \left(\frac{3}{x}\right)^2 + (-1)^2 + 2\left(2x \cdot \frac{3}{x} + \frac{3}{x}(-1) + (-1) \cdot 2x\right)$$

$$\Rightarrow 4x^2 + \frac{9}{x^2} + 1 + 2\left(6 - \frac{3}{x} - 2x\right)$$

$$\Rightarrow 4x^2 + \frac{9}{x^2} + 1 + 12 - \frac{6}{x} - 4x$$

$$\Rightarrow 4x^2 + \frac{9}{x^2} - \frac{6}{x} - 4x + 13$$

$$(ii) \left(\frac{2}{3}x - \frac{3}{2x} - 1\right)^2$$

It is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Here $a = \frac{2}{3}x$; $b = -\frac{3}{2x}$; $c = -1$

$$\therefore \Rightarrow \left[\left(\frac{2}{3}x\right)^2 + \left(-\frac{3}{2x}\right)^2 + (-1)^2 + 2\left[\frac{2}{3}x\left(-\frac{3}{2x}\right) + \left(-\frac{3}{2x}\right)(-1) + (-1)\left(\frac{2}{3}x\right)\right] \right]$$

$$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + 1 + 2\left[-1 + \frac{3}{2x} - \frac{2}{3}x\right]$$

$$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + 1 - 2 + \frac{6}{2x} - \frac{4x}{3}$$

$$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + \frac{3}{x} - \frac{4x}{3} - 1 = .$$

Solution-7

$$(i) (x+2)^3$$

It is in the form of $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Here $a = x$; $b = 2$

$$\therefore \Rightarrow x^3 + 3x^2 \cdot 2 + 3x \cdot 2^2 + 2^3$$

$$\Rightarrow x^3 + 6x^2 + 3x \cdot 4 + 8$$

$$\Rightarrow x^3 + 6x^2 + 12x + 8$$

$$(ii) (2a+b)^3$$

$$\Rightarrow (2a)^3 + 3 \cdot (2a)^2 \cdot b + 3 \cdot 2a \cdot b^2 + b^3$$

$$\Rightarrow 8a^3 + 3 \cdot 4a^2 \cdot b + 6ab^2 + b^3$$

$$\Rightarrow 8a^3 + 12a^2b + 6ab^2 + b^3$$

Solution - 8:

$$(i) \left(3x + \frac{1}{x}\right)^3$$

It is in the form of $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$a = 3x; b = \frac{1}{x}$$

$$\therefore (3x)^3 + 3 \cdot (3x)^2 \cdot \frac{1}{x} + 3 \cdot 3x \cdot \left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$$

$$\Rightarrow 27x^3 + 3 \cdot 9x^2 \cdot \frac{1}{x} + 9x \cdot \frac{1}{x^2} + \frac{1}{x^3}$$

$$\Rightarrow 27x^3 + 27x + \frac{9}{x} + \frac{1}{x^3} //$$

$$(ii) (2x-1)^3$$

It is in the form of $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$\text{Here } a = 2x, b = 1$$

$$\therefore (2x)^3 - 3(2x)^2 \cdot 1 + 3(2x)(1)^2 - (1)^3$$

$$\Rightarrow 8x^3 - 3 \cdot 4x^2 + 6x - 1$$

$$\Rightarrow 8x^3 - 12x^2 + 6x - 1 //$$

Solution-9:

(i) $(5x-3y)^3$

It is in the form of $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$a = 5x; \quad b = 3y$$

$$\therefore (5x)^3 - 3(5x)^2 \cdot 3y + 3 \cdot 5x \cdot (3y)^2 - (3y)^3$$

$$\Rightarrow 125x^3 - 3 \cdot 25x^2 \cdot 3y + 3 \cdot 5x \cdot 9y^2 - 27y^3$$

$$\Rightarrow 125x^3 - 225x^2y + 135y^2x - 27y^3$$

(ii) $(2x - \frac{1}{3y})^3$

$$\Rightarrow (2x)^3 - 3(2x)^2 \cdot \frac{1}{3y} + 3 \cdot 2x \cdot \left(\frac{1}{3y}\right)^2 - \left(\frac{1}{3y}\right)^3$$

$$\Rightarrow 8x^3 - \cancel{3} \cdot 4x^2 \cdot \frac{1}{\cancel{3}y} + \cancel{3} \cdot 2x \cdot \frac{1}{9y^2} - \frac{1}{27y^3}$$

$$\Rightarrow 8x^3 - \frac{4x^2}{y} + \frac{2x}{3y^2} - \frac{1}{27y^3} //$$

Solution-10

(i) $(a+b)^2 + (a-b)^2$

$$\Rightarrow a^2 + 2ab + b^2 + a^2 - 2ab + b^2$$

$$\Rightarrow 2a^2 + 2b^2$$

$$\Rightarrow 2(a^2 + b^2)$$

$$(ii) (a+b)^2 - (a-b)^2$$

$$\rightarrow (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$

$$\rightarrow a^2 + 2ab + b^2 - a^2 + 2ab - b^2$$

$$\rightarrow 2ab + 2ab$$

$$\rightarrow 4ab$$

Solution - 11

$$(i) \left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2$$

$$\rightarrow \left(a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right) + \left(a^2 - 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$$

$$\rightarrow a^2 + 2 + \frac{1}{a^2} + a^2 - 2 + \frac{1}{a^2}$$

$$\rightarrow 2a^2 + \frac{2}{a^2}$$

$$\rightarrow 2 \left(a^2 + \frac{1}{a^2}\right)$$

$$(ii) \left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2$$

$$\rightarrow \left(a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right) - \left(a^2 - 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$$

$$\rightarrow a^2 + 2 + \frac{1}{a^2} - a^2 + 2 - \frac{1}{a^2}$$

$$\rightarrow 2 + 2$$

$$\rightarrow 4$$

Solution-12 :

(i) $(3x-1)^2 - (3x-2)(3x+1)$

$\Rightarrow (3x)^2 - 2 \cdot 3x \cdot 1 + 1^2 - 3x(3x+1) + 2(3x+1)$

$\Rightarrow 9x^2 - 6x + 1 - 9x^2 - 3x + 6x + 2$

$\Rightarrow 3x + 3$

$\Rightarrow 3(x+1)$

(ii) $(4x+3y)^2 - (4x-3y)^2 - 48$

$\Rightarrow (4x)^2 + 2 \cdot 3y \cdot 4x + (3y)^2 - ((4x)^2 - 2 \cdot 4x \cdot 3y + (3y)^2) - 48$

$\Rightarrow 16x^2 + 24xy + 9y^2 - 16x^2 + 24xy - 9y^2 - 48$

$\Rightarrow 48xy - 48$

$\Rightarrow 48(xy - 1)$

Solution-13 :

(i) $(7p+9q)(7p-9q)$

$\Rightarrow 7p(7p-9q) + 9q(7p-9q)$

$\Rightarrow 49p^2 - 63pq + 63pq - 81q^2$

$\Rightarrow 49p^2 - 81q^2$

$$(ii) \left(2x - \frac{3}{x}\right) \left(2x + \frac{3}{x}\right)$$

$$\rightarrow (2x)^2 - \left(\frac{3}{x}\right)^2$$

\rightarrow Since it is in the form of $(a+b)(a-b) = a^2 - b^2$

$$\therefore \rightarrow 4x^2 - \frac{9}{x^2}$$

Solution-14:

$$(i) (2x - y + 3) (2x - y - 3)$$

$$\rightarrow ((2x - y) + 3) ((2x - y) - 3)$$

It is in the form of $(a+b)(a-b) = a^2 - b^2$.

$$\therefore (2x - y)^2 - 3^2$$

$$\rightarrow (2x)^2 - 2 \cdot 2x \cdot y + y^2 - 9$$

$$\rightarrow 4x^2 - 4xy + y^2 - 9.$$

$$(ii) (3x + y - 5) (3x - y - 5)$$

$$\rightarrow (3x + (y - 5)) (3x - (y - 5))$$

$$\rightarrow [(3x - 5) + y] [(3x - 5) - y]$$

\rightarrow It is in the form of $(a+b)(a-b) = a^2 - b^2$

$$\therefore a = 3x - 5 ; b = y.$$

$$\therefore (3x - 5)^2 - y^2$$

$$\rightarrow (3x)^2 - 2 \cdot 3x \cdot 5 + 5^2 - y^2$$

$$\rightarrow 9x^2 - 30x + 25 - y^2 //$$

Solution - 15

$$(i) \left(x + \frac{2}{x} - 3\right) \left(x - \frac{2}{x} - 3\right)$$

$$\Rightarrow \left((x-3) + \frac{2}{x}\right) \left((x-3) - \frac{2}{x}\right)$$

It is in the form of $(a+b)(a-b) = a^2 - b^2$

$$\therefore a = x-3 ; \quad b = \frac{2}{x}$$

$$\Rightarrow (x-3)^2 - \left(\frac{2}{x}\right)^2$$

$$\Rightarrow x^2 - 2 \cdot x \cdot 3 + 3^2 - \frac{4}{x^2}$$

$$\Rightarrow x^2 - 6x + 9 - \frac{4}{x^2}$$

$$(ii) \underline{(5-2x)(5+2x)(25+4x^2)}$$

It is in the form of $(a+b)(a-b) = a^2 - b^2$

$$\therefore (5^2 - (2x)^2)(25+4x^2)$$

$$\Rightarrow (25 - 4x^2)(25+4x^2)$$

$$\Rightarrow (25)^2 - (4x^2)^2$$

$$\Rightarrow 625 - 16x^4$$

$$(i) (x+2y+3)(2x+y+7)$$

$$\Rightarrow x(2x+y+7) + 2y(2x+y+7) + 3(2x+y+7)$$

$$\Rightarrow 2xy + x^2 + 7x + 4y^2 + 2xy + 14y + 6y + 6x + 21$$

$$\Rightarrow x^2 + 4y^2 + 4xy + 10x + 20y + 21$$

$$(ii) (2x+y+5)(2x+y-9)$$

$$\Rightarrow 2x(2x+y-9) + y(2x+y-9) + 5(2x+y-9)$$

$$\Rightarrow 4x^2 + 2xy - 18x + 2xy + y^2 - 9y + 10x + 5y - 45$$

$$\Rightarrow 4x^2 + y^2 + 4xy - 8x - 4y - 45$$

$$(iii) (x-2y-5)(x-2y+3)$$

$$\Rightarrow x(x-2y+3) - 2y(x-2y+3) - 5(x-2y+3)$$

$$\Rightarrow x^2 - 2xy + 3x - 2xy + 4y^2 - 6y - 5x + 10y - 15$$

$$\Rightarrow x^2 + 4y^2 - 4xy - 2x - 4y - 15$$

$$(iv) (3x-4y-2)(3x-4y-6)$$

$$\Rightarrow 3x(3x-4y-6) - 4y(3x-4y-6) - 2(3x-4y-6)$$

$$\Rightarrow 9x^2 - 12xy - 18x - 12xy + 16y^2 + 24y - 6x + 8y + 12$$

$$\Rightarrow 9x^2 + 16y^2 - 24xy - 6x - 24x + 32y + 12 //$$

Solution-17

$$(i) (2p+3q)(4p^2-6pq+9q^2)$$

$$(2p+3q)(2p)^2-2p \cdot 3q+(3q)^2$$

It is in the form of $(a+b)(a^2-ab+b^2)$ is a^3+b^3

$$\therefore \text{Here } a=2p ; b=3q$$

$$\therefore (2p)^3+(3q)^3$$

$$\Rightarrow 8p^3+27q^3 //$$

$$(ii) \left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$$

It is in the form of $(a+b)(a^2-ab+b^2)$ is a^3+b^3

$$\therefore \text{Here } a=x ; b=\frac{1}{x}$$

$$\therefore x^3+\frac{1}{x^3}$$

Solution-18:

$$(i) (3p-4q) (9p^2+12pq+16q^2)$$

$$(3p-4q) ((3p)^2+3p \cdot 4q+(4q)^2)$$

It is in the form of $(a-b)(a^2+ab+b^2)$ is a^3-b^3

$$\therefore \text{Here } 3p = a ; b = 4q$$

$$\therefore (3p)^3 - (4q)^3$$

$$\Rightarrow 27p^3 - 64q^3 //$$

$$(ii) \left(x - \frac{3}{x}\right) \left(x^2 + 3 + \frac{9}{x^2}\right)$$

$$\therefore \left(x - \frac{3}{x}\right) \left(x^2 + x \cdot \frac{3}{x} + \left(\frac{3}{x}\right)^2\right)$$

It is in the form of $(a-b)(a^2+ab+b^2)$ is a^3-b^3 .

$$\therefore \text{Here } a = x ; b = \frac{3}{x}$$

$$\therefore x^3 - \left(\frac{3}{x}\right)^3$$

$$\Rightarrow x^3 - \frac{27}{x^3} //$$

Solution-19

$$\text{Given } (2x + 3y + 4z) (4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)$$

$$\Rightarrow (2x + 3y + 4z) ((2x)^2 + (3y)^2 + (4z)^2 - 2x \cdot 3y - 3y \cdot 4z - 4z \cdot 2x)$$

\therefore It is in the form of

$$(a+b+c) (a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$$

$$\therefore \text{Here } a = 2x; b = 3y; c = 4z$$

$$\therefore (2x)^3 + (3y)^3 + (4z)^3 - 3 \cdot 2x \cdot 3y \cdot 4z$$

$$\Rightarrow 8x^3 + 27y^3 + 64z^3 - 72xyz$$

Solution-20

$$(i) (x+1)(x+2)(x+3)$$

$$[x(x+2) + 1](x+2)(x+3)$$

$$\Rightarrow (x^2 + 2x + 1)(x+2)(x+3)$$

$$\Rightarrow (x^2 + 3x + 2)(x+3)$$

$$\Rightarrow (x^2 + 3x + 2)(x) + (x^2 + 3x + 2) \cdot 3$$

$$\Rightarrow x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$$

$$\Rightarrow x^3 + 6x^2 + 11x + 6$$

$$(ii) (x-2)(x-3)(x+4)$$

$$\rightarrow [x(x-3) - 2(x-3)](x+4)$$

$$\rightarrow (x^2 - 3x - 2x + 6)(x+4)$$

$$\rightarrow (x^2 - 5x + 6)(x+4)$$

$$\rightarrow (x^2 - 5x + 6)x + (x^2 - 5x + 6)4$$

$$\rightarrow x^3 - 5x^2 + 6x + 4x^2 - 20x + 24$$

$$\rightarrow x^3 - x^2 - 14x + 24$$

Solution-21:

$$\text{Given } (x-3)(x+7)(x-4)$$

$$[x(x+7) - 3(x+7)](x-4)$$

$$(x^2 + 7x - 3x - 21)(x-4)$$

$$(x^2 + 4x - 21)(x-4)$$

$$(x^2 + 4x - 21)x - 4(x^2 + 4x - 21)$$

$$x^3 + 4x^2 - 21x - 4x^2 - 16x + 84$$

$$x^3 - 37x + 84$$

\therefore The coefficient of x^2 is 0

The coefficient of x is -37

Solution- 22

$$\text{Given } a^2 + 4a + x = (a+2)^2$$

$$\therefore a^2 + 4a + x = a^2 + 2 \cdot a \cdot 2 + 2^2$$

$$a^2 + 4a + x = a^2 + 4a + 4$$

$$\therefore x = a^2 + 4a + 4 - a^2 - 4a$$

$$\boxed{x = 4}$$

Solution- 23

$$(i) (101)^2$$

$$\Rightarrow (100+1)^2$$

$$\Rightarrow (100)^2 + 2 \cdot 100 \cdot 1 + 1^2$$

$$\Rightarrow 10000 + 200 + 1$$

$$\Rightarrow \underline{10201}$$

$$(ii) (1003)^2$$

$$\Rightarrow (1000+3)^2$$

$$\Rightarrow (1000)^2 + 2 \cdot 1000 \cdot 3 + 3^2$$

$$\Rightarrow 1000000 + 6000 + 9$$

$$\Rightarrow \underline{1006009}$$

$$\begin{aligned}
 & \text{(iii)} \quad (10.2)^2 \\
 & \quad (10 + 0.2)^2 \\
 & \quad (10)^2 + 2 \times 10 \times 0.2 + (0.2)^2 \\
 & \quad 100 + 4 + 0.04 \\
 & \quad 104.04
 \end{aligned}$$

Solution - 24 :

$$\begin{aligned}
 & \text{(i)} \quad 99^2 \\
 & \rightarrow (100 - 1)^2 \\
 & \rightarrow (100)^2 - 2 \cdot 100 \cdot 1 + 1^2 \\
 & \rightarrow 10000 - 200 + 1 \\
 & \rightarrow 9801
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii)} \quad (997)^2 \rightarrow (1000 - 3)^2 \\
 & \rightarrow 1000^2 - 2 \cdot 1000 \cdot 3 + 3^2 \\
 & \rightarrow 1000000 - 6000 + 9 \\
 & \rightarrow 994009 \\
 & \quad =
 \end{aligned}$$

In this we used the $(a-b)^2$ formulae
 i.e., $a^2 - 2ab + b^2$.

$$(ii) (9.8)^2$$

$$\Rightarrow (10 - 0.2)^2$$

$$\Rightarrow 10^2 - 2 \times 10 \times 0.2 + (0.2)^2$$

$$\Rightarrow 100 - 4 + 0.4$$

$$\Rightarrow 96.04$$

Solution - 25 :

$$(i) (103)^3$$

$$\Rightarrow (100 + 3)^3$$

\therefore It is in the form of

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\therefore \text{Here } a = 100, b = 3$$

$$\therefore \Rightarrow (100)^3 + 3 \cdot (100)^2 \cdot 3 + 3 \cdot 100 \cdot 3^2 + 3^3$$

$$\Rightarrow 1000000 + 90000 + 2700 + 27$$

$$\Rightarrow 1092727$$

$$(ii) 99^3$$

$$\Rightarrow (100 - 1)^3$$

$$\Rightarrow 100^3 - 3 \cdot 100^2 \cdot 1 + 3 \cdot 100 \cdot 1^2 - 1^3$$

$$\Rightarrow 1000000 - 30000 + 300 - 1$$

$$\Rightarrow 970299$$

$$(iii) \cdot (10.1)^3$$

$$\Rightarrow (10 + 0.1)^3$$

$$\Rightarrow 10^3 + 3 \cdot 10^2 \cdot (0.1) + 3 \cdot 10 \cdot (0.1)^2 + (0.1)^3$$

$$\Rightarrow 1000 + 30 + 3 + 0.01$$

$$\Rightarrow 1030.301$$

Solution - 26 :

$$\text{Given } 2a - b + c = 0$$

$$\Rightarrow (2a + c) = b$$

Squaring on both sides

$$\Rightarrow (2a + c)^2 = b^2$$

$$\Rightarrow (2a)^2 + 2 \cdot 2a \cdot c + c^2 = b^2$$

$$\Rightarrow 4a^2 + 4ac + c^2 = b^2$$

$$\Rightarrow 4a^2 - b^2 + c^2 + 4ac = 0$$

∴ Hence proved.

Solution-27

Given $a+b+2c=0$

$$a+b = -2c \quad \dots (i)$$

Cubing on both sides

$$(a+b)^3 = (-2c)^3$$

$$a^3 + b^3 + 3a^2b + 3ab^2 = -8c^3$$

$$a^3 + b^3 + 3ab(a+b) = -8c^3 \quad (\dots \text{ from (i)})$$

$$a^3 + b^3 + 3ab(-2c) = -8c^3$$

$$a^3 + b^3 - 6abc = -8c^3$$

$$a^3 + b^3 + 8c^3 = 6abc$$

Hence proved.

Solution-28 :

Given $a+b+c=0$

$$a+b = -c \quad \dots (i)$$

Cubing on both sides

$$(a+b)^3 = (-c)^3$$

$$\Rightarrow a^3 + b^3 + 3a^2b + 3ab^2 = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(a+b) = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(-c) = -c^3$$

$$\Rightarrow a^3 + b^3 - 3abc = -c^3$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow \frac{a^3 + b^3 + c^3}{abc} = 3$$

$$\Rightarrow \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$$

$$\Rightarrow \frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3 //$$

Solution - 29 :

(*) Given $x + y = 4$.

Cubing on both sides :

$$(x + y)^3 = 4^3$$

$$\Rightarrow x^3 + 3x^2y + 3xy^2 + y^3 = 64$$

$$\Rightarrow x^3 + 3xy(x + y) + y^3 = 64$$

$$\Rightarrow x^3 + 3xy(4) + y^3 = 64$$

$$\Rightarrow x^3 + 12xy + y^3 = 64$$

$$\Rightarrow x^3 + y^3 + 12xy - 64 = 0 //$$

Solution - 30 :

$$(i) \quad (27)^3 + (-17)^3 + (-10)^3$$

\therefore If $a+b+c=0$; then $a^3+b^3+c^3=3abc$

$$\therefore \text{Here } a=27$$

$$b=-17$$

$$c=-10$$

$$\therefore 27-17-10=0$$

$$\therefore a^3+b^3+c^3=3abc$$

$$= 3 \cdot 27 \cdot (-17) \cdot (-10)$$

$$= 13770$$

$$(ii) \quad (-28)^3 + 15^3 + 13^3$$

\therefore If $a+b+c=0$; then $a^3+b^3+c^3=3abc$

$$\rightarrow -28+15+13=0$$

$$\therefore \Rightarrow a^3+b^3+c^3=3abc$$

$$= 3(-28)(15)(13)$$

$$= -16380$$

Solution - 31

Given
$$\frac{86 \times 86 \times 86 + 14 \times 14 \times 14}{86 \times 86 - 86 \times 14 + 14 \times 14}$$

\therefore It is in the form of $\frac{a^3 + b^3}{a^2 - ab + b^2} = (a + b)$

$$\begin{aligned} \therefore \frac{(86)^3 + (14)^3}{86^2 - 86 \cdot 14 + 14^2} &= 86 + 14 \\ &= 100 // \end{aligned}$$

EXERCISE - 3.2

26

Solution-1 :

Given $x - y = 8$... (i)
 $xy = 5$... (ii)

Squaring on both sides in equ (i)

$$\therefore (x - y)^2 = 8^2$$

$$x^2 - 2xy + y^2 = 64$$

$$x^2 - 2(5) + y^2 = 64$$

\therefore from eq (ii)

$$x^2 - 10 + y^2 = 64$$

$$x^2 + y^2 = 64 + 10$$

$$x^2 + y^2 = 74 //$$

Solution-2 :

Given $x + y = 10$... (i)
 $xy = 21$... (ii)

Squaring on both sides in equ (i)

$$(x + y)^2 = 10^2$$

$$x^2 + 2xy + y^2 = 100$$

$$x^2 + 2(21) + y^2 = 100$$

$$x^2 + 42 + y^2 = 100$$

$$x^2 + y^2 = 100 - 42$$

$$x^2 + y^2 = 58$$

$$2(x^2 + y^2) = 2 \times 58$$

$$= 116 //$$

Solution-3 :

Given $2a + 3b = 7$ — (i)

$ab = 2$ — (ii)

Squaring on both sides in equ(i)

$$(2a + 3b)^2 = 7^2$$

$$(2a)^2 + 2 \cdot 2a \cdot 3b + (3b)^2 = 49$$

$$4a^2 + 12ab + 9b^2 = 49$$

$$4a^2 + 12(2) + 9b^2 = 49$$

$$4a^2 + 24 + 9b^2 = 49$$

$$4a^2 + 9b^2 = 49 - 24$$

$$4a^2 + 9b^2 = 25$$

Solution-4 :

Given $3x - 4y = 16$ — (i)

$xy = 4$ — (ii)

Squaring on both sides in equ(i)

$$(3x - 4y)^2 = 16^2$$

$$(3x)^2 - 2 \cdot 3x \cdot 4y + (4y)^2 = 256$$

$$9x^2 - 24xy + 16y^2 = 256$$

$$9x^2 - 24(4) + 16y^2 = 256$$

$$9x^2 - 96 + 16y^2 = 256$$

$$9x^2 + 16y^2 = 256 + 96$$

$$9x^2 + 16y^2 = 352$$

Solution- 5:

Given $x + y = 8$ (i)
 $x - y = 2$ (ii)

Since we know that

$$2(x^2 + y^2) = (x + y)^2 + (x - y)^2$$

from (i) \rightarrow squaring on both sides.

$$(x + y)^2 = 8^2$$

$$= 64$$

from (ii) \rightarrow squaring on both sides

$$(x - y)^2 = 2^2$$

$$= 4$$

$$\therefore 2(x^2 + y^2) = (x + y)^2 + (x - y)^2$$

$$= 64 + 4$$

$$= 68 //$$

Solution- 6

Given $a^2 + b^2 = 13$ — (i)
 $ab = 6$ — (ii)

(i) $a + b$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$= a^2 + b^2 + 2ab$$

$$= 13 + 2(6)$$

$$= 13 + 12$$

$$(a+b)^2 = 25$$

$$a+b = \sqrt{25}$$

$$a+b = 5 \quad \text{1.}$$

(ii) a-b :

$$\begin{aligned} \therefore (a-b)^2 &= a^2 - 2ab + b^2 \\ &= a^2 + b^2 - 2ab \\ &= 13 - 2(6) \\ &= 13 - 12 \end{aligned}$$

$$(a-b)^2 = 1$$

$$(a-b) = \sqrt{1}$$

$$\therefore a-b = 1 \quad \text{2.}$$

Solution-7 :

Given $a+b = 4$... (i)

$ab = -12$... (ii)

(i) a-b :

from (i) & (ii)

$$ab = -12$$

$$a(4-a) = -12$$

$$4a - a^2 = -12$$

$$a^2 - 12 - 4a = 0$$

$$a^2 - 4a - 12 = 0$$

$$a^2 - 6a + 2a - 12 = 0$$

$$a(a-6) + 2(a-6) = 0$$

$$(a-6)(a+2) = 0$$

$$\therefore a-6=0 \quad ; \quad a+2=0$$

$$\therefore a=6 \quad \checkmark$$

$$a=-2.$$

$$\therefore a+b=4$$

$$6+b=4$$

$$b=4-6$$

$$b=-2.$$

$$\therefore a+b=4$$

$$-2+b=4$$

$$b=4+2$$

$$b=6.$$

$$\therefore (a, b) = (6, -2)$$

$$(i) \quad a-b \Rightarrow 6 - (-2) = 8$$

$$(ii) \quad a^2 - b^2 = (a+b)(a-b)$$

$$= (4)(8)$$

$$= 32.$$

=

Solution - 8

Given $P - q = 9$ --- (i)

$Pq = 36$ --- (ii)

from (i) & (ii)

$$P = q + 9$$

$$\therefore Pq = 36$$

$$(q+9)q = 36$$

$$q^2 + 9q = 36$$

$$q^2 + 9q - 36 = 0$$

$$q^2 + 12q - 3q - 36 = 0$$

$$q(q+12) - 3(q+12) = 0$$

$$(q+12)(q-3) = 0$$

$$\therefore q+12 = 0$$

$$q = -12$$

$$q-3 = 0$$

$$q = 3$$

$$\therefore P - q = 9$$

$$P - (-12) = 9$$

$$P = 9 - 12$$

$$P = -3$$

$$P - q = 9$$

$$P - 3 = 9$$

$$P = 9 + 3$$

$$P = 12 \checkmark$$

$$\boxed{P = 12 ; q = 3}$$

$$(i) \quad P+q$$

$$\rightarrow 12+3$$

$$= 15 //$$

$$(ii) \quad P^2 - q^2$$

$$\rightarrow (P+q)(P-q)$$

$$\rightarrow 15 \cdot 9$$

$$\rightarrow 135 //$$

Solution - 9:

$$\text{Given } x+y = 6 \quad \text{--- (i)}$$

$$x-y = 4 \quad \text{--- (ii)}$$

from (i) \rightarrow squaring on both sides

$$(x+y)^2 = 6^2$$

$$x^2 + y^2 + 2xy = 36$$

$$x^2 + y^2 = 36 - 2xy \quad \text{--- (iii)}$$

from (ii) \rightarrow squaring on both sides

$$(x-y)^2 = 4^2$$

$$x^2 - 2xy + y^2 = 16$$

$$x^2 + y^2 = 16 + 2xy \quad \text{--- (iv)}$$

∴ equate equ (iii) + (iv)

$$36 - 2xy = 16 + 2xy$$

$$36 - 16 = 2xy + 2xy$$

$$20 = 4xy$$

$$4xy = 20$$

$$xy = \frac{20}{4}$$

$$xy = 5 //$$

(i) $x^2 + y^2$

$$\begin{aligned} \Rightarrow \text{from equ (iii)} \quad x^2 + y^2 &= 36 - 2xy \\ &= 36 - 2(5) \\ &= 36 - 10 \\ &= 26 \end{aligned}$$

(ii) $xy = 5 //$

solution - 10 :

84

Given $x - 3 = \frac{1}{x}$.

$$x - \frac{1}{x} = 3.$$

Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = 3^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 9$$

$$x^2 - 2 + \frac{1}{x^2} = 9$$

$$x^2 + \frac{1}{x^2} = 9 + 2$$

$$x^2 + \frac{1}{x^2} = 11 //$$

solution - 11

Given $x + y = 8$... (1)

$$xy = 3\frac{3}{4} = \frac{15}{4} \text{ --- (2)}$$

from eq (1) $\Rightarrow y = 8 - x$

from eq (2) $\Rightarrow xy = \frac{15}{4}$

$$x(8 - x) = \frac{15}{4}$$

$$8x - x^2 = \frac{15}{4}$$

$$4(8x - x^2) = 15$$

$$32x - 4x^2 = 15$$

$$4x^2 - 32x + 15 = 0$$

$$4x^2 - 32x + 15 = 0$$

$$4x^2 - 30x - 2x + 15 = 0$$

$$2x(2x-15) - 1(2x-15) = 0$$

$$(2x-15)(2x-1) = 0$$

$$\therefore 2x-15 = 0$$

$$2x = 15$$

$$x = \frac{15}{2}$$

$$\therefore x+y = 8$$

$$\frac{15}{2} + y = 8$$

$$y = 8 - \frac{15}{2}$$

$$y = \frac{16-15}{2}$$

$$y = \frac{1}{2}$$

$$\therefore x = \frac{15}{2}; y = \frac{1}{2}$$

$$4 \times 15 = 60$$

30 2

$$2x-1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x+y = 8$$

$$\frac{1}{2} + y = 8$$

$$y = 8 - \frac{1}{2}$$

$$y = \frac{16-1}{2}$$

$$y = \frac{15}{2}$$

$$(i) x-y$$

$$\Rightarrow \frac{15}{2} - \frac{1}{2}$$

$$\Rightarrow \frac{15-1}{2}$$

$$\Rightarrow \frac{14}{2}$$

$$\Rightarrow 7 //$$

$$(ii) 3(x^2 + y^2)$$

$$\Rightarrow 3 \left[\left(\frac{15}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \right]$$

$$\Rightarrow 3 \left[\frac{225}{4} + \frac{1}{4} \right]$$

$$\Rightarrow 3 \left[\frac{225+1}{4} \right]$$

$$\Rightarrow 3 \left(\frac{226}{4} \right)$$

$$\Rightarrow \frac{3 \times 113}{2}$$

$$\Rightarrow \frac{339}{2} //$$

$$(iii) 5(x^2 + y^2) + 4(x - y)$$

$$5 \left(\left(\frac{15}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \right) + 4(7)$$

$$\Rightarrow 5 \left(\frac{225}{4} + \frac{1}{4} \right) + 4(7)$$

$$\Rightarrow 5 \left(\frac{113}{2} \right) + 28$$

$$\Rightarrow \frac{565}{2} + 28$$

$$\Rightarrow \frac{565 + 56}{2}$$

$$\Rightarrow \frac{621}{2} //$$

Solution-12

$$\text{Given } x^2 + y^2 = 34$$

$$xy = 10\frac{1}{2} = \frac{21}{2}$$

$$\begin{aligned}\therefore (x+y)^2 &= x^2 + y^2 + 2xy \\ &= 34 + 2 \cdot \frac{21}{2} \\ &= 34 + 21 \\ &= 55\end{aligned}$$

$$\begin{aligned}\therefore (x-y)^2 &= x^2 + y^2 - 2xy \\ &= 34 - 2 \cdot \frac{21}{2} \\ &= 34 - 21 \\ &= 13\end{aligned}$$

$$\therefore 2(x+y)^2 + (x-y)^2$$

$$\Rightarrow 2(55) + 13$$

$$\Rightarrow 110 + 13$$

$$\Rightarrow 123$$

Solution-13

$$\text{Given } a - b = 3 \quad \text{--- (i)}$$

$$ab = 4 \quad \text{--- (ii)}$$

$$\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

from (i) squaring on both sides

$$(a-b)^2 = 3^2$$

$$a^2 + b^2 - 2ab = 9$$

$$a^2 + b^2 - 2(4) = 9$$

$$a^2 + b^2 - 8 = 9$$

$$a^2 + b^2 = 9 + 8$$

$$a^2 + b^2 = 17$$

$$\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= 3 \cdot (a^2 + b^2 + ab)$$

$$= 3(17 + 4)$$

$$= 3(21)$$

$$= 63$$

solution - 14:

$$\text{Given } 2a - 3b = 3 \quad \text{--- (1)}$$

$$ab = 2 \quad \text{--- (2)}$$

from (1) squaring on both sides

$$(2a - 3b)^2 = 3^2$$

$$(2a)^2 - 2 \cdot 2a \cdot 3b + (3b)^2 = 9$$

$$4a^2 - 12ab + 9b^2 = 9$$

$$4a^2 + 9b^2 - 12(2) = 9$$

$$4a^2 + 9b^2 - 24 = 9$$

$$4a^2 + 9b^2 = 9 + 24$$

$$= 33$$

$$\begin{aligned}
 \therefore (2a)^3 - (3b)^3 &\Rightarrow 8a^3 - 27b^3 \\
 &\Rightarrow (2a - 3b) ((2a)^2 + 2a \cdot 3b + (3b)^2) \\
 &\Rightarrow 3 (4a^2 + 6ab + 9b^2) \\
 &\Rightarrow 3 (4a^2 + 9b^2 + 6ab) \\
 &\Rightarrow 3 (33 + 6(2)) \\
 &\Rightarrow 3 (33 + 12) \\
 &\Rightarrow 3 (45) \\
 &\Rightarrow 135 //
 \end{aligned}$$

Solution-15 :

Given $x + \frac{1}{x} = 4$.

(i) Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 4^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 16$$

$$x^2 + 2 + \frac{1}{x^2} = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2$$

$$x^2 + \frac{1}{x^2} = 14.$$

$$(ii) \quad x^4 + \frac{1}{x^4}$$

$$\therefore \text{ we know that } x^2 + \frac{1}{x^2} = 14.$$

\therefore Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 14^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 196$$

$$x^4 + 2 + \frac{1}{x^4} = 196$$

$$x^4 + \frac{1}{x^4} = 196 - 2$$

$$x^4 + \frac{1}{x^4} = 194.$$

$$(iii) \quad x^3 + \frac{1}{x^3}$$

$$\therefore \text{ we know } x + \frac{1}{x} = 4$$

Cubing on both sides

$$\left(x + \frac{1}{x}\right)^3 = 4^3$$

$$x^3 + 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$$

$$x^3 + 3 \left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$$

$$x^3 + \frac{1}{x^3} + 3(4) = 64$$

$$x^3 + \frac{1}{x^3} = 64 - 12$$

$$= 52.$$

$$(iv) \quad x - \frac{1}{x}$$

$$\left(x - \frac{1}{x}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 14 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 12$$

$$x - \frac{1}{x} = \sqrt{12}$$

$$= \sqrt{4 \times 3}$$

$$= 2\sqrt{3}$$

∴ from (i)

$$x^2 + \frac{1}{x^2} = 14$$

Solution - 16 :

$$\text{Given } x - \frac{1}{x} = 5$$

Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = 5^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 25$$

$$x^2 + \frac{1}{x^2} = 25 + 2$$

$$x^2 + \frac{1}{x^2} = 27$$

Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 27^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 529$$

$$x^4 + 2 + \frac{1}{x^4} = 529$$

$$x^4 + \frac{1}{x^4} = 529 - 2$$

$$x^4 + \frac{1}{x^4} = 527 //$$

Solution-17

Given $x - \frac{1}{x} = \sqrt{5}$

(i) Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = (\sqrt{5})^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 5$$

$$x^2 - 2 + \frac{1}{x^2} = 5$$

$$x^2 + \frac{1}{x^2} = 5 + 2$$

$$x^2 + \frac{1}{x^2} = 7$$

(ii) $x + \frac{1}{x} =$ "

$$\rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} + 2$$

$$= 7 + 2$$

$$= 9$$

[∵ from (i)]

$$\left(x + \frac{1}{x}\right)^2 = 9$$

$$x + \frac{1}{x} = \sqrt{9}$$

$$x + \frac{1}{x} = 3 \quad //$$

$$(iii) \quad x^3 + \frac{1}{x^3}$$

$$\Rightarrow \left(x + \frac{1}{x}\right) \left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right)$$

$$\Rightarrow \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$\Rightarrow 3 \cdot (7 - 1)$$

$$\Rightarrow 3 \times 6$$

$$\Rightarrow 18 \quad //$$

Solution - 18

$$\text{Given } x + \frac{1}{x} = 6$$

Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 6^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 36$$

$$x^2 + 2 + \frac{1}{x^2} = 36$$

$$x^2 + \frac{1}{x^2} = 36 - 2$$

$$= 34 \quad //$$

$$(i) \quad x - \frac{1}{x}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$$
$$= x^2 + \frac{1}{x^2} - 2$$

$$= 34 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 32$$

$$x - \frac{1}{x} = \sqrt{32}$$

$$x - \frac{1}{x} = \sqrt{16 \times 2}$$

$$x - \frac{1}{x} = 4\sqrt{2}$$

$$(ii) \quad x^2 - \frac{1}{x^2}$$

$$\Rightarrow x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$= 6 \cdot 4\sqrt{2}$$

$$\Rightarrow 24\sqrt{2}$$

Solution - 19 :

Given. $x + \frac{1}{x} = 2$.

Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 2^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 4$$

$$x^2 + 2 + \frac{1}{x^2} = 4$$

$$x^2 + \frac{1}{x^2} = 4 - 2$$

$$x^2 + \frac{1}{x^2} = 2 \quad \dots (i)$$

$$\therefore x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= 2 \cdot \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= 2(2 - 1)$$

$$= 2(1)$$

$$= 2 \quad \dots (ii)$$

$$\text{from (i)} \Rightarrow x^2 + \frac{1}{x^2} = 2$$

Squaring on both sides

$$= \left(x^2 + \frac{1}{x^2}\right)^2 = 2^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 4$$

$$x^4 + 2 + \frac{1}{x^4} = 4$$

$$x^4 + \frac{1}{x^4} = 4 - 2$$

$$= 2 \quad \dots (iii)$$

From eqs (i), (ii) & (iii)

$$x^2 + \frac{1}{x^2} = 2$$

$$x^3 + \frac{1}{x^3} = 2$$

$$x^4 + \frac{1}{x^4} = 2$$

$$\therefore x^2 + \frac{1}{x^2} = x^3 + \frac{1}{x^3} = x^4 + \frac{1}{x^4}$$

Hence proved.

Solution-20:

Given $x - \frac{2}{x} = 3$.

Cubing on both sides

$$\left(x - \frac{2}{x}\right)^3 = 3^3$$

$$x^3 - 3 \cdot x \cdot \frac{2}{x} \left(x - \frac{2}{x}\right) - \left(\frac{2}{x}\right)^3 = 27$$

$$x^3 - 6 \left(x - \frac{2}{x}\right) - \frac{8}{x^3} = 27$$

$$x^3 - \frac{8}{x^3} - 6(3) = 27$$

$$x^3 - \frac{8}{x^3} = 27 + 18$$

$$= 45 //$$

Solution - 21

Given $a + 2b = 5$

Cubing on both sides

$$(a + 2b)^3 = 5^3$$

$$a^3 + 3a \cdot 2b(a + 2b) + (2b)^3 = 125$$

$$a^3 + 6ab(5) + 8b^3 = 125$$

$$a^3 + 30ab + 8b^3 = 125$$

$$\therefore a^3 + 8b^3 + 30ab = 125$$

Solution - 22 :

Given $a + \frac{1}{a} = P$

Cubing on both sides

$$\left(a + \frac{1}{a}\right)^3 = P^3$$

$$a^3 + 3a \cdot \frac{1}{a} \left(a + \frac{1}{a}\right) + \frac{1}{a^3} = P^3$$

$$a^3 + 3(P) + \frac{1}{a^3} = P^3$$

$$a^3 + \frac{1}{a^3} = P^3 - 3P$$

$$a^2 + \frac{1}{a^3} = P(P^2 - 3)$$

Solution-23 :

$$\text{Given } x^2 + \frac{1}{x^2} = 27$$

$$\therefore \left(x - \frac{1}{x}\right)^2 = x^2 - 2x \cdot \frac{1}{x} + \frac{1}{x^2}$$
$$= x^2 - 2 + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 27 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 25$$

$$x - \frac{1}{x} = \sqrt{25}$$

$$x - \frac{1}{x} = 5 //$$

Solution-24 :

$$\text{Given } x^2 + \frac{1}{x^2} = 27$$

$$\text{Take } \left(x - \frac{1}{x}\right)^2 = x^2 - 2x \cdot \frac{1}{x} + \frac{1}{x^2}$$
$$= x^2 - 2 + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 27 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 25$$

$$x - \frac{1}{x} = \sqrt{25}$$

$$x - \frac{1}{x} = 5$$

$$3x^3 + 5x - \frac{3}{x^3} - \frac{5}{x}$$

$$\Rightarrow 3x^3 - \frac{3}{x^3} + 5x - \frac{5}{x}$$

$$\Rightarrow 3\left(x^3 - \frac{1}{x^3}\right) + 5\left(x - \frac{1}{x}\right)$$

$$\Rightarrow 3\left(x - \frac{1}{x}\right)\left(x^2 + x \cdot \frac{1}{x} + \frac{1}{x^2}\right) + 5\left(x - \frac{1}{x}\right)$$

$$\Rightarrow 3\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right) + 5\left(x - \frac{1}{x}\right)$$

$$\Rightarrow 3(5)(27+1) + 5(5)$$

$$\Rightarrow 15(28) + 25$$

$$\Rightarrow 420 + 25$$

$$\Rightarrow 445.$$

Solution - 25 :

$$\text{Given } x^2 + \frac{1}{25x^2} = 8\frac{3}{5}$$

$$x^2 + \left(\frac{1}{5x}\right)^2 = \frac{43}{5}$$

\therefore let us consider

$$\left(x + \frac{1}{5x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{5x} + \left(\frac{1}{5x}\right)^2 =$$

$$= x^2 + \frac{2}{5} + \frac{1}{25x^2}$$

$$= x^2 + \frac{1}{25x^2} + \frac{2}{5}$$

$$= \frac{43}{5} + \frac{2}{5}$$

$$\left(x + \frac{1}{5x}\right)^2 = \frac{43+2}{5}$$

$$\left(x + \frac{1}{5x}\right)^2 = \frac{47}{5}$$

$$x + \frac{1}{5x} = \sqrt{\frac{47}{5}}$$

Solution - 26 :

$$\text{Given } x^2 + \frac{1}{4x^2} = 8$$

$$x^2 + \left(\frac{1}{2x}\right)^2 = 8$$

let us consider

$$\left(x + \frac{1}{2x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2$$

$$= x^2 + 1 + \frac{1}{4x^2}$$

$$= x^2 + \frac{1}{4x^2} + 1$$

$$= 8 + 1$$

$$\left(x + \frac{1}{2x}\right)^2 = 9$$

$$x + \frac{1}{2x} = \sqrt{9}$$

$$x + \frac{1}{2x} = 3$$

$$x^3 + \left(\frac{1}{2x}\right)^3$$

$$\Rightarrow x^3 + \frac{1}{8x^3} = \left(x + \frac{1}{2x}\right) \left(x^2 - x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2\right)$$

$$\begin{aligned}
 x^3 + \frac{1}{8x^3} &= \left(x + \frac{1}{2x}\right) \left(x^2 + \frac{1}{4x^2} - 1\right) \\
 &= 3(8-1) \\
 &= 3(7) \\
 &= \underline{21}
 \end{aligned}$$

Solution-27 :

Given $a^2 - 3a + 1 = 0$

dividing each term by a , we get

$$\frac{a^2}{a} - \frac{3a}{a} + \frac{1}{a} = 0$$

$$a - 3 + \frac{1}{a} = 0$$

$$a + \frac{1}{a} = 3$$

Now

$$(i) \quad \left(a + \frac{1}{a}\right)^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

$$a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$$

$$= 3^2 - 2$$

$$= 9 - 2$$

$$= \underline{7} //$$

$$(ii) a^3 + \frac{1}{a^2}$$

$$\rightarrow \left(a + \frac{1}{a}\right) \left(a^2 - a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$$

$$\rightarrow \left(a + \frac{1}{a}\right) \left(a^2 + \frac{1}{a^2} - 1\right)$$

$$\rightarrow (3)(7-1)$$

$$\rightarrow 3 \times 6$$

$$\rightarrow 18$$

Solution - 28

$$\text{Given } a = \frac{1}{a-5}$$

$$a(a-5) = 1$$

$$a^2 - 5a = 1$$

$$a^2 - 5a - 1 = 0$$

(i) Dividing each term by a , we get

$$\frac{a^2}{a} - \frac{5a}{a} - \frac{1}{a} = 0$$

$$a - 5 - \frac{1}{a} = 0$$

$$\therefore a - \frac{1}{a} = 5$$

(ii) Now $(a + \frac{1}{a})$,

$$\therefore a - \frac{1}{a} = 5$$

\therefore Squaring on both sides

$$(a - \frac{1}{a})^2 = 5^2$$

$$a^2 - 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2} = 25$$

$$a^2 + \frac{1}{a^2} = 25 - 2$$

$$a^2 + \frac{1}{a^2} = 23$$

$$\therefore (a + \frac{1}{a})^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}$$

$$= a^2 + \frac{1}{a^2} + 2$$

$$= 23 + 2$$

$$= 25 //$$

$$\therefore (a + \frac{1}{a})^2 = 25$$

$$a + \frac{1}{a} = \sqrt{25}$$

$$= 5 //$$

$$(ii) a^2 - \frac{1}{a^2} = (a + \frac{1}{a})(a - \frac{1}{a})$$

$$= 5 \cdot 5$$

$$= 25 //$$

Solution-29

$$\text{Given } \left(x + \frac{1}{x}\right)^2 = 3$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 3$$

$$x^2 + \frac{1}{x^2} = 3 - 2$$

$$x^2 + \frac{1}{x^2} = 1$$

$$\begin{aligned} \therefore x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right) \left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right) \\ &= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right) \\ &= \sqrt{3} (1 - 1) \\ &= \sqrt{3} (0) \\ &= 0 // \end{aligned}$$

Solution-30

$$\text{Given } x = 5 - 2\sqrt{6}$$

Squaring on both sides

$$x^2 = 5$$

Solution - 31

Given $a + b + c = 12$

Squaring on both sides

$$(a + b + c)^2 = 12^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$$

\therefore from given $ab + bc + ca = 22$

$$\therefore a^2 + b^2 + c^2 + 2(22) = 144$$

$$a^2 + b^2 + c^2 + 44 = 144$$

$$a^2 + b^2 + c^2 = 144 - 44$$

$$a^2 + b^2 + c^2 = 100$$

Solution - 32 :

Given $a + b + c = 12$

Squaring on both sides

$$(a + b + c)^2 = 12^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$$

$\therefore a^2 + b^2 + c^2 = 100$

$$\therefore 100 + 2(ab + bc + ca) = 144$$

$$2(ab + bc + ca) = 144 - 100$$

$$2(ab + bc + ca) = 44$$

$$ab + bc + ca = \frac{44}{2} = 22$$

Solution - 33

Given $a^2 + b^2 + c^2 = 125$

$ab + bc + ca = 50$

$$\begin{aligned}\therefore (a+b+c)^2 &= a^2 + b^2 + c^2 + 2(ab + bc + ca) \\ &= 125 + 2(50) \\ &= 125 + 100 \\ &= 225\end{aligned}$$

$(a+b+c)^2 = 225$

$a+b+c = \sqrt{225}$

$a+b+c = 15$

sdution- 34 :

Given $a+b-c = 5$

$a^2 + b^2 + c^2 = 29$

$$(a+b-c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$$

$$5^2 = 29 + 2(ab - bc - ca)$$

$$25 = 29 + 2(ab - bc - ca)$$

$$25 - 29 = 2(ab - bc - ca)$$

$$-4 = 2(ab - bc - ca)$$

$$ab - bc - ca = \frac{-4}{2} = -2$$

solution - 35

$$\text{Given } a-b=7$$

$$a^2+b^2=85$$

$$(a-b)^2 = a^2+b^2 - 2ab$$

$$7^2 = 85 - 2ab$$

$$49 = 85 - 2ab$$

$$2ab = 85 - 49$$

$$2ab = 36$$

$$ab = \frac{36}{2}$$

$$ab = 18$$

$$\therefore a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$= 7(a^2+b^2+ab)$$

$$= 7(85+18)$$

$$= 7(103)$$

$$= 721$$

Solution -36

Given, the number is x .

$$\therefore x = y - 3$$

$$\therefore x - y = -3$$

$$y - x = 3$$

and $x^2 + y^2 = 29$

$$\therefore y - x = 3$$

squaring on both sides

$$(y - x)^2 = 3^2$$

$$y^2 + x^2 - 2xy = 9$$

$$29 - 2xy = 9$$

$$29 - 9 = 2xy$$

$$2xy = 20$$

$$xy = \frac{20}{2}$$

$$xy = 10$$

Solution - 37 :

Given , sum of two numbers = 8
product of two numbers = 15

let, numbers be x and y

$$\therefore x + y = 8$$

$$xy = 15$$

$$\therefore x + y = 8$$

squaring on both sides

$$(x + y)^2 = 8^2$$

$$x^2 + y^2 + 2xy = 64$$

$$x^2 + y^2 + 2(15) = 64$$

$$x^2 + y^2 = 64 - 30$$

$$x^2 + y^2 = 34$$

$$\therefore \underline{x^3 + y^3}$$

$$\Rightarrow (x + y)(x^2 - xy + y^2)$$

$$\Rightarrow 8(x^2 + y^2 - xy)$$

$$\Rightarrow 8(34 - 15)$$

$$\Rightarrow 8(26)$$

$$\Rightarrow 208$$