Quadratic Equations

Question 1.
Solve the following (1 to 12) equations:
(i) $x^2 - 11x + 30 = 0$
(ii) $4x^2 - 25 = 0$
Solution:

(i) $x^2 - 11x + 30 = 0$

$x^2 - 5x - 6x + 30 = 0 \quad \left\{ \begin{array}{l} \because 30 = -5 \times (-6) \\ -11 = -5 - 6 \end{array} \right\}$

$\Rightarrow x(x - 5) - 6(x - 5) = 0$

$\Rightarrow (x - 5)(x - 6) = 0$

Either, $x - 5 = 0$, then $x = 5$

or $x - 6 = 0$, then $x = 6$

$\therefore x = 5, 6$

(ii) $4x^2 - 25 = 0 \Rightarrow 4x^2 = 0 + 25$

$\Rightarrow x^2 = \dfrac{25}{4}$

$\therefore x = \pm\sqrt{\dfrac{25}{4}} = \pm\dfrac{5}{2}$

$\therefore x = \dfrac{5}{2}, \dfrac{-5}{2}$

Question 2.
(i) 2x² – 5x = 0
(ii) x² – 2x = 48
Solution:

(i) $2x^2 - 5x = 0$

$x(2x - 5) = 0$

Either, $x = 0$

or $2x - 5 = 0$, then $2x = 5$

$\Rightarrow x = \dfrac{5}{2}$

$\therefore x = 0, \dfrac{5}{2}$

(ii) $x^2 - 2x = 48$

$\Rightarrow x^2 - 2x - 48 = 0$

$\Rightarrow x^2 - 8x + 6x - 48 = 0 \qquad \left\{ \begin{array}{l} \because -48 = -8 \times 6 \\ -2 = -8 + 6 \end{array} \right\}$

$\Rightarrow x(x - 8) + 6(x - 8) = 0$

$\Rightarrow (x - 8)(x + 6) = 0$

Either, $x - 8 = 0$, then $x = 8$

or $x + 6 = 0$, then $x = -6$

$\therefore x = 8, -6$

Question 3.
(i) 6 + x = x²
(ii) 2x² + 3x + 1= 0
Solution:

(i) $6 + x = x^2$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0 \qquad \left\{ \begin{array}{l} \because -6 = -3 \times 2 \\ -1 = -3 + 2 \end{array} \right\}$

$\Rightarrow x(x - 3) + 2(x - 3) = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

Either, $x - 3 = 0$, then $x = 3$

or $x + 2, = 0$, then $x = -2$

$\therefore x = 3, -2$

(ii) $2x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 2x - x + 1 = 0$

$\Rightarrow 2x(x - 1) - 1(x - 1) = 0$

$\Rightarrow (x - 1)(2x - 1) = 0$

Either, $x - 1 = 0$, then $x = 1$

or $2x - 1 = 0$, then $2x = 1$

$\Rightarrow x = \dfrac{1}{2}$.

$\therefore x = 1, \dfrac{1}{2}$

Question 4.
(i) 3x² = 2x + 8
(ii) 4x² + 15 = 16x
Solution:

(i) $3x^2 = 2x + 8$

$\Rightarrow 3x^2 - 2x - 8 = 0$

$\Rightarrow 3x^2 - 6x + 4x - 8 = 0$
$\left\{\begin{array}{l} \because -8 \times 3 = -24 \\ -24 = -6 \times 4 \\ -2 = -6 + 4 \end{array}\right.$

$\Rightarrow 3x(x - 2) + 4(x - 2) = 0$

$\Rightarrow (x - 2)(3x + 4) = 0$

Either, $x - 2 = 0$, then $x = 2$

or $3x + 4 = 0$, then $3x = -4$

$\Rightarrow x = \dfrac{-4}{3}$

$\therefore x = 2, \dfrac{-4}{3}$

(ii) $4x^2 + 15 = 16x$

$\Rightarrow 4x^2 - 16x + 15 = 0$

$\Rightarrow 4x^2 - 6x - 10x + 15 = 0$

$\left\{\begin{array}{l} \because 4 \times 15 = 60 \\ -16 = -6 + (-10) \\ -16 = -6 - 10 \end{array}\right.$

$\Rightarrow 2x(2x - 3) - 5(2x - 3) = 0$

$\Rightarrow (2x - 3)(2x - 5) = 0$

Either, $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \dfrac{3}{2}$

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \dfrac{5}{2}$

$\therefore x = \dfrac{3}{2}, \dfrac{5}{2}$

Question 5.
(i) x (2x + 5) = 25
(ii) (x +3) (x – 3) = 40
Solution:

$(i)\ x(2x + 5) = 25$

$\Rightarrow 2x^2 + 5x - 25 = 0$

$\Rightarrow 2x^2 + 10x - 5x - 25 = 0$

$$\left[\begin{array}{l} \because -25 \times 2 = -50 \\ -50 = 10 \times (-5) \\ 5 = 10 - 5 \end{array}\right]$$

$\Rightarrow 2x(x + 5) - 5(x + 5) = 0$

$\Rightarrow (x + 5)(2x - 5) = 0$

Either, $x + 5 = 0$, then $x = -5$

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \dfrac{5}{2}$

$\therefore x = -5,\ \dfrac{5}{2}$

$(ii)\ (x + 3)(x - 3) = 40$

$\Rightarrow x^2 - 9 = 40 \Rightarrow x^2 - 9 - 40 = 0$

$\Rightarrow x^2 - 49 = 0 \Rightarrow (x)^2 - (7)^2 = 0$

$\Rightarrow (x + 7)(x - 7) = 0$

Either, $x + 7 = 0$, then $x = -7$

or $x - 7 = 0$, then $x = 7$

$\therefore x = 7, -7$

Question 6.
(i) $(2x + 3)(x - 4) = 6$
(ii) $(3x + 1)(2x + 3) = 3$
Solution:

(i) $(2x + 3)(x - 4) = 6$

$\Rightarrow 2x^2 - 8x + 3x - 12 - 6 = 0$

$\Rightarrow 2x^2 - 5x - 18 = 0$

$\Rightarrow 2x^2 - 9x + 4x - 18 = 0$

$$\begin{bmatrix} \because -18 \times 2 = -36 \\ \therefore -36 = -9 \times 4 \\ -5 = -9 + 4 \end{bmatrix}$$

$\Rightarrow x(2x - 9) + 2(2x - 9) = 0$

$\Rightarrow (2x - 9)(x + 2) = 0$

Either, $2x - 9 = 0$, then $2x = 9 \Rightarrow x = \dfrac{9}{2}$

or $x + 2 = 0$, then $x = -2$

$\therefore x = \dfrac{9}{2}, -2$

(ii) $(3x + 1)(2x + 3) = 3$

$\Rightarrow 6x^2 + 9x + 2x + 3 - 3 = 0$

$\Rightarrow 6x^2 + 11x = 0$

$\Rightarrow x(6x + 11) = 0$

Either, $x = 0$,

or $6x + 11 = 0$, then $6x = -11 \Rightarrow x = \dfrac{-11}{6}$

$\therefore x = 0, \dfrac{-11}{6}$

Question 7.
(i) 4x² + 4x + 1 = 0
(ii) (x – 4)² + 5²= 132
Solution:

(i) $4x^2 + 4x + 1 = 0$
$\Rightarrow 4x^2 + 2x + 2x + 1 = 0$
$\Rightarrow 2x(2x + 1) + 1(2x + 1) = 0$
$\Rightarrow (2x + 1)(2x + 1) = 0$

Either, $2x + 1 = 0$, then $x = \dfrac{-1}{2}$

$\therefore x = \dfrac{-1}{2}, \dfrac{-1}{2}$

(ii) $(x - 4)^2 + 5^2 = 13^2$
$\Rightarrow x^2 - 8x + 16 + 25 = 169$
$x^2 - 8x + 16 + 25 - 169 = 0$
$x^2 - 8x - 128 = 0$

$x^2 - 16x + 8x - 128 = 0 \left\{ \begin{array}{l} \because -128 = -16 \times 8 \\ -8 = -16 + 8 \end{array} \right\}$

$x(x - 16) + 8(x - 16) = 0$
$(x - 16)(x + 8) = 0$
Either, $x - 16 = 0$, then $x = 16$
or $x + 8 = 0$, then $x = -8$
$\therefore x = 16, -8$

Question 8.
(i) $21x^2 = 4(2x + 1)$
(ii) $\frac{2}{3}x^2 - \frac{1}{3}x - 1 = 0$
Solution:

(i) $21x^2 = 4(2x + 1)$

$\Rightarrow 21x^2 = 8x + 4$

$\Rightarrow 21x^2 - 8x - 4 = 0$

$\Rightarrow 21x^2 - 14x + 6x - 4 = 0$ $\left\{ \begin{array}{l} \because 21 \times (-4) = -84 \\ \therefore -84 = -14 \times 6 \\ -8 = -14 + 6 \end{array} \right.$

$\Rightarrow 7x(3x - 2) + 2(3x - 2) = 0$

$\Rightarrow (3x - 2)(7x + 2) = 0$

Either, $3x - 2 = 0$, then $3x = 2 \Rightarrow x = \dfrac{2}{3}$

or $7x + 2 = 0$, then $7x = -2 \Rightarrow x = \dfrac{-2}{7}$

$\therefore x = \dfrac{2}{3}, \dfrac{-2}{7}$

(ii) $\dfrac{2}{3}x^2 - \dfrac{1}{3}x - 1 = 0$

$\Rightarrow 2x^2 - x - 3 = 0$

$\Rightarrow 2x^2 - 3x + 2x - 3 = 0$

$\Rightarrow x(2x - 3) + 1(2x - 3) = 0$

$\Rightarrow (2x - 3)(x + 1) = 0$

Either, $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \dfrac{3}{2}$

or $x + 1 = 0$, then $x = -1$

$\therefore x = \dfrac{3}{2}, -1$

Question 9.

(i) $6x + 29 = \dfrac{5}{x}$

(ii) $x + \dfrac{1}{x} = 2\dfrac{1}{2}$

Solution:

Question 10.

(i) $3x - \dfrac{8}{x} = 2$

(ii) $\dfrac{x}{3} + \dfrac{9}{x} = 4$

Solution:

Question 11.

(i) $\dfrac{x-1}{x+1} = \dfrac{2x-5}{3x-7}$

(ii) $\dfrac{1}{x+2} + \dfrac{1}{x} = \dfrac{3}{4}$

Solution:

(i) $\dfrac{x-1}{x+1} = \dfrac{2x-5}{3x-7}$

By cross multiplication,

$(x-1)(3x-7) = (x+1)(2x-5)$

$\Rightarrow 3x^2 - 7x - 3x + 7 = 2x^2 - 5x + 2x - 5$

$3x^2 - 10x + 7 = 2x^2 - 3x - 5 = 0$

$3x^2 - 10x + 7 - 2x^2 + 3x + 5 = 0$

$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x - 4) - 3(x - 4) = 0$

$$\left\{ \begin{array}{l} \because 12 = -4 \times (-3) \\ -7 = -4 - 3 \end{array} \right\}$$

$\Rightarrow (x - 4)(x - 3) = 0$

Either, $x - 4 = 0$, then $x = 4$

or $x - 3 = 0$, then $x = 3$

$\therefore x = 3, 4$

(ii) $\dfrac{1}{x+2} + \dfrac{1}{x} = \dfrac{3}{4}$

$\dfrac{x + x + 2}{x(x+2)} = \dfrac{3}{4} \Rightarrow \dfrac{2x+2}{x(x+2)} = \dfrac{3}{4}$

By cross multiplication,

$\Rightarrow 3x(x + 2) = 4(2x + 2)$

$\Rightarrow 3x^2 + 6x = 8x + 8$

$\Rightarrow 3x^2 + 6x - 8x - 8 = 0$

$\Rightarrow 3x^2 - 2x - 8 = 0$

$\Rightarrow 3x^2 - 6x + 4x - 8 = 0$

$$\left\{ \begin{array}{l} \because 3 \times (-8) = -24 \\ \therefore -24 = -6 \times 4 \\ -2 = -6 + 4 \end{array} \right\}$$

$\Rightarrow 3x(x - 2) + 4(x - 2) = 0$

$\Rightarrow (x - 2)(3x + 4) = 0$

Either, $x - 2 = 0$, then $x = 2$

or $3x + 4 = 0$, then $3x = -4 \Rightarrow x = \dfrac{-4}{3}$

Hence, $x = 2, \dfrac{-4}{3}$

Question 12.

(i) $\dfrac{8}{x+3} - \dfrac{3}{2-x} = 2$

(ii) $\dfrac{x}{x+1} + \dfrac{x+1}{x} = 2\dfrac{1}{6}$

Solution:

(i) $\dfrac{8}{x+3} - \dfrac{3}{2-x} = 2$

$\Rightarrow \dfrac{8(2-x)-3(x+3)}{(x+3)(2-x)} = \dfrac{2}{1}$

$\Rightarrow \dfrac{16-8x-3x-9}{2x-x^2+6-3x} = \dfrac{2}{1}$

$\Rightarrow \dfrac{7-11x}{-x^2-x+6} = \dfrac{2}{1}$

$\Rightarrow 7 - 11x = -2x^2 - 2x + 12$

$\hspace{3cm}$ (By cross multiplication)

$\Rightarrow 2x^2 + 2x - 12 + 7 - 11x = 0$

$\Rightarrow 2x^2 - 9x - 5 = 0$

$\Rightarrow 2x^2 - 10x + x - 5 = 0$

$$\left\{ \begin{array}{l} \because 2 \times (-5) = -10 \\ \therefore -10 = -10 \times 1 \\ -9 = -10 + 1 \end{array} \right\}$$

$\Rightarrow 2x(x - 5) + 1(x - 5) = 0$

$\Rightarrow (x - 5)(2x + 1) = 0$

Either, $x - 5 = 0$, then $x = 5$

or $2x + 1 = 0$, then $2x = -1 \Rightarrow x = \dfrac{-1}{2}$

or $2x + 1 = 0$, then $2x = -1 \Rightarrow x = \dfrac{-1}{2}$

$\therefore x = 5, \dfrac{-1}{2}$

(ii) $\dfrac{x}{x+1} + \dfrac{x+1}{x} = 2\dfrac{1}{6}$

$\dfrac{x^2 + (x+1)^2}{x(x+1)} = \dfrac{13}{6}$

$\Rightarrow \dfrac{x^2 + x^2 + 2x + 1}{x^2 + x} = \dfrac{13}{6}$

$\Rightarrow \dfrac{2x^2 + 2x + 1}{x^2 + x} = \dfrac{13}{6}$

$13x^2 + 13x = 12x^2 + 12x + 6$

$\Rightarrow 13x^2 + 13x - 12x^2 - 12x - 6 = 0$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x^2 + 3x - 2x - 6 = 0 \qquad \left\{ \begin{array}{l} \because -6 = 3 \times (-2) \\ \quad 1 = 3 - 2 \end{array} \right\}$

$\Rightarrow x(x + 3) - 2(x + 3) = 0$

$\Rightarrow (x + 3)(x - 2) = 0$

Either, $x + 3 = 0$, then $x = -3$

or $x - 2 = 0$, then $x = 2$

$\therefore x = 2, -3$

Multiple Choice Questions

Choose the correct Solution from the given four options (1 to 5):

Question 1.

Which of the following is not a quadratic equation :

(a) $2x^2 = 3x - 5$

(b) $(2x-1)(x-1) = 2x^2 - 7x + 2$

(c) $(2x - 1)(x + 2) = (x - 1)(x + 1)$

(d) $(x + 1), = x, + 2x + 2$

Solution:

$(2x - 1)(x - 1) = 2x^2 - 7x + 2$ is not a quadratic equation. **(b)**

Question 2.

If 2 is a root of the quadratic equation $2x^2 - kx + 1 = 0$, then the value of k is

(a) 9

(b) –9

(c) $\dfrac{9}{2}$

(d) $-\dfrac{9}{2}$

Solution:

$\because$ 2 is root of $2x^2 - kx + 1 = 0$

$\therefore$ $2(2)^2 - k(2) + 1 = 0$

$8 - 2k + 1 = 0 = 9 = 2k$

$\Rightarrow k = \dfrac{9}{2}$ (c)

Question 3.

If -3 is a root of the quadratic equation $kx^2 + 2x - 3 = 0$, then the value of k is

(a) 1

(b) -1

(c) $\dfrac{1}{9}$

(d) $\dfrac{1}{-9}$

Solution:

-3 is a root of quadratic equation.

$kx^2 + 2x - 3 = 0$

$\Rightarrow k(-3)^2 + 2(-3) - 3 = 0$

$\Rightarrow 9k - 6 - 3 = 0 \Rightarrow 9k = 9$

$\Rightarrow k = \dfrac{9}{9} = 1$ (a)

Question 4.
Which of the following quadratic equations has -1 as a root?
(a) $x^2 + 5x + 6 = 0$
(b) $2x^2 - 3x + 1 = 0$
(c) $2x^2 + x - 3 = 0$
(d) $2x^2 - x - 3 = 0$
Solution:

$\because -1$ is a root of the quadratic equation.

$\therefore$ It will satisfy it

$2x^2 - x - 3 = 0$

$\Rightarrow 2(-1)^2 - (-1) - 3 = 0$

$\Rightarrow 2 + 1 - 3 = 0$

$\Rightarrow 3 - 3 = 0$ (d)

Question 5.
The root of the quadratic equation $x^2 - 3x - 4 = 0$ are
(a) -4, 1
(b) 4, -1
(c) 4, 1
(d) -4, -1
Solution:

$x^2 - 3x - 4 = 0$

$\Rightarrow x^2 - 4x + x - 4 = 0 \Rightarrow x(x - 4) + 1(x - 4)$

$\Rightarrow (x - 4)(x + 1) = 0$

If $x - 4 = 0$, then $x = 4$

$x + 1 = 0$, then $x = -1$

$\therefore x = 4, -1$ (b)

Chapter Test

Solve the following (1 to 3) equations:
Question 1.
(i) x(2x+ 5) = 3
(ii) 3x² – 4x – 4 = 0
Solution:

(i) $x(2x + 5) = 3$

$$\Rightarrow 2x^2 + 5x - 3 = 0 \qquad \left\{ \begin{array}{l} \because 2 \times (-3) = -6 \\ \therefore -6 = 6 \times (-1) \\ 5 = 6 - 1 \end{array} \right\}$$

$$\Rightarrow 2x^2 + 6x - x - 3 = 0$$
$$\Rightarrow 2x(x + 3) - 1(x + 3) = 0$$
$$\Rightarrow (x + 3)(2x - 1) = 0$$

Either, $x + 3 = 0$, then $x = -3$

or $2x - 1 = 0$, then $2x = 1 \Rightarrow x = \dfrac{1}{2}$

$\therefore x = -3, \dfrac{1}{2}$

(ii) $3x^2 - 4x - 4 = 0$
$$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$$

$$\left\{ \begin{array}{l} \because 3 \times (-4) = -12 \\ \therefore -12 = -6 \times 2 \\ -4 = -6 + 2 \end{array} \right\}$$

$$\Rightarrow 3x(x - 2) + 2(x - 2) = 0$$
$$\Rightarrow (x - 2)(3x + 2) = 0$$

Either, $x - 2 = 0$, then $x = 2$

or $3x + 2 = 0$, then $3x = -2 \Rightarrow x = \dfrac{-2}{3}$

$\therefore x = 2, \dfrac{-2}{3}$

Question 2.

(i) $4x^2 - 2x + \frac{1}{4} = 0$

(ii) $2x^2 + 7x + 6 = 0$

Solution:

(i) $4x^2 - 2x + \dfrac{1}{4} = 0$

$\Rightarrow 16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow (4x - 1)(4x - 1) = 0 \Rightarrow (4x - 1)^2 = 0$

$\Rightarrow 4x - 1 = 0 \Rightarrow 4x = 1$

$\therefore x = \dfrac{1}{4}, \dfrac{1}{4}$

(ii) $2x^2 + 7x + 6 = 0$

$\Rightarrow 2x^2 + 4x + 3x + 6 = 0 \qquad \left. \begin{aligned} &\because 2 \times 6 = 12 \\ &\therefore 12 = 3 \times 4 \\ &\quad 7 = 3 + 4 \end{aligned} \right\}$

$\Rightarrow 2x(x + 2) + 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x + 3) = 0$

Either, $x + 2 = 0$, then $x = -2$

or $2x + 3 = 0$, then $2x = -3 \Rightarrow x = \dfrac{-3}{2}$

$\therefore x = -2, \dfrac{-3}{2}$

Question 3.

(i) $\dfrac{x-1}{x-2} + \dfrac{x-3}{x-4} = 3\dfrac{1}{3}$

(ii) $\dfrac{6}{x} - \dfrac{2}{x-1} = \dfrac{1}{x-2}$

Solution:

(i) $\dfrac{x-1}{x-2} + \dfrac{x-3}{x-4} = 3\dfrac{1}{3}$

$\dfrac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \dfrac{10}{3}$

$\Rightarrow \dfrac{x^2 - 5x + 4 + x^2 - 5x + 6}{x^2 - 6x + 8} = \dfrac{10}{3}$

$\Rightarrow \dfrac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \dfrac{10}{3}$

$\Rightarrow 10x^2 - 60x + 80 = 6x^2 - 30x + 30$

$\Rightarrow 10x^2 - 60x + 80 - 6x^2 + 30x - 30 = 0$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow 2x^2 - 15x + 25 = 0$

$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$

$$\begin{cases} \because 2 \times 25 = 50 \\ \therefore 50 = -10 \times (-5) \\ -15 = -10 - 5 \end{cases}$$

$\Rightarrow 2x(x - 5) - 5(x - 5) = 0$

$\Rightarrow (x - 5)(2x - 5) = 0$

Either, $x - 5 = 0$, then $x = 5$

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \dfrac{5}{2}$

$\therefore x = 5, \dfrac{5}{2}$

(ii) $\dfrac{6}{x} - \dfrac{2}{x-1} = \dfrac{1}{x-2}$

$\dfrac{6x - 6 - 2x}{x(x-1)} = \dfrac{1}{x-2}$

$\Rightarrow \dfrac{4x - 6}{x^2 - x} = \dfrac{1}{x-2}$

$\Rightarrow (4x - 6)(x - 2) = x^2 - x$

(By cross multiplication)

$\Rightarrow 4x^2 - 8x - 6x + 12 = x^2 - x$

$\Rightarrow 4x^2 - 14x + 12 - x^2 + x = 0$

$\Rightarrow 3x^2 - 13x + 12 = 0$

$\Rightarrow 3x^2 - 4x - 9x + 12 = 0$

$$\left\{\begin{array}{l} \because 3 \times 12 = 36 \\ \therefore 36 = (-4) \times (-9) \\ \quad -13 = -4 - 9 \end{array}\right\}$$

$\Rightarrow x(3x - 4) - 3(3x - 4) = 0$

$\Rightarrow (3x - 4)(x - 3) = 0$

Either, $3x - 4 = 0$, then $3x = 4 \Rightarrow x = \dfrac{4}{3}$

or $x - 3 = 0$, then $x = 3$

$\therefore x = 3, \dfrac{4}{3}$