7. Indices

EXERCISE - 7

1

Solution - (1)

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$\Rightarrow \left(\frac{3^4}{2^4}\right)^{-3/4}$

$\Rightarrow \left[\left(\frac{3}{2}\right)^4\right]^{-3/4}$

$\Rightarrow \left(\frac{3}{2}\right)^{4\times\frac{-3}{4}}$

$\Rightarrow \left(\frac{3}{2}\right)^{-3}$

$\Rightarrow \left(\frac{2}{3}\right)^{3}$

$\Rightarrow \frac{2^3}{3^3}$

$\Rightarrow \frac{2\times2\times2}{3\times3\times3}$

$\Rightarrow \frac{8}{27}$

(ii) $\left(1\frac{61}{64}\right)^{-2/3}$

$\Rightarrow \left(\frac{125}{64}\right)^{-2/3}$

$\Rightarrow \left(\frac{5^3}{4^3}\right)^{-2/3}$

$\Rightarrow \left[\left(\frac{5}{4}\right)^3\right]^{-2/3}$

$\Rightarrow \left(\frac{5}{4}\right)^{\cancel{3}\times\frac{-2}{\cancel{3}}}$

$\Rightarrow \left(\frac{5}{4}\right)^{-2}$

$\Rightarrow \left(\frac{4}{5}\right)^{2}$

$\Rightarrow \frac{4^2}{5^2}$

$\Rightarrow \frac{16}{25}$

Solution- 2

(i) $(2a^{-3}b^2)^3$

$\Rightarrow 2^3 a^{-3\times3} b^{2\times3}$

$\Rightarrow 8a^{-9}b^6$.

(ii) $$\frac{a^{-1}+b^{-1}}{(ab)^{-1}}$$

$$\Rightarrow \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}}$$

$$\Rightarrow \frac{\frac{b+a}{ab}}{\frac{1}{ab}}$$

$$\Rightarrow \frac{b+a}{\cancel{ab}} \times \frac{\cancel{ab}}{1}$$

$$\Rightarrow a+b$$

Solution- 3 :

(i) $$\frac{x^{-1}y^{-1}}{x^{-1}+y^{-1}}$$

$$\Rightarrow \frac{(xy)^{-1}}{\frac{1}{x}+\frac{1}{y}}$$

$$\Rightarrow \frac{\frac{1}{xy}}{\frac{y+x}{xy}}$$

$$\Rightarrow \frac{1}{x+y}.$$

(ii) $$\frac{(4\times10^{7})(6\times10^{-5})}{8\times10^{10}}$$

$$\Rightarrow \frac{\cancel{4}\times10^{7}\times \overset{3}{\cancel{6}}\times10^{-5}}{\underset{4}{\cancel{8}}\times10^{10}}$$

$$\Rightarrow \frac{3\times10^{7-5}}{10^{10}}$$

$$\Rightarrow \frac{3\times10^{2}}{10^{10}}$$

$$\Rightarrow \frac{3}{10^{10-2}}$$

$$\Rightarrow \frac{3}{10^{8}}$$

solution-4 :

(i) $$\frac{3a}{b^{-1}}+\frac{2b}{a^{-1}}$$

$$\Rightarrow \frac{3a}{\left(\frac{1}{b}\right)}+\frac{2b}{\left(\frac{1}{a}\right)}$$

$$\Rightarrow 3ab+2ab$$

$$\Rightarrow 5ab.$$

(ii) $5^0 \times 4^{-1} + 8^{1/3}$. $(a^0 = 1)$

$\Rightarrow 1 \times \frac{1}{4} + (2^3)^{1/3}$

$\Rightarrow \frac{1}{4} + 2$

$\Rightarrow \frac{1+8}{4}$

$\Rightarrow \frac{9}{4}$

Solution - 5

(i) $\left(\frac{8}{125}\right)^{-1/3}$

$\Rightarrow \left(\frac{2^3}{5^3}\right)^{-1/3}$

$\Rightarrow \left(\frac{2}{5}\right)^{\cancel{3} \times -1/\cancel{3}}$

$\Rightarrow \left(\frac{2}{5}\right)^{-1}$

$\Rightarrow \frac{5}{2}$

(ii) $(0.027)^{-1/3}$

$\Rightarrow \left(\frac{27}{1000}\right)^{-1/3}$

$\Rightarrow \left(\frac{3^3}{10^3}\right)^{-1/3}$

$\Rightarrow \left(\frac{3}{10}\right)^{\cancel{3}\times -1/\cancel{3}}$

$\Rightarrow \left(\frac{3}{10}\right)^{-1}$

$\Rightarrow \frac{10}{3}$

Solution 6 :

(i) $\left(\frac{-1}{27}\right)^{-2/3}$

$\Rightarrow -\left(\frac{1}{3^3}\right)^{-2/3}$

$\Rightarrow -\frac{1}{(3^3)^{-2/3}}$

$\Rightarrow -\frac{1}{3^{\cancel{3}\times -2/\cancel{3}}}$

$\Rightarrow -\frac{1}{3^{-2}}$

$\Rightarrow -3^2$

$\Rightarrow -9$ //

(ii) $(64)^{-2/3} \div 9^{-3/2}$

$\Rightarrow (4^3)^{-2/3} \div (3^2)^{-3/2}$

$\Rightarrow 4^{\cancel{3}\times -2/\cancel{3}} \div 3^{\cancel{2}\times -3/\cancel{2}}$

$$\Rightarrow \quad 4^{-2} \div 3^{-3}$$

$$\Rightarrow \quad \frac{4^{-2}}{3^{-3}}$$

$$\Rightarrow \quad \frac{\frac{1}{4^{2}}}{\frac{1}{3^{3}}}$$

$$\Rightarrow \quad \frac{3^{3}}{4^{2}}$$

$$\Rightarrow \quad \frac{27}{16}$$

Solution – 7

(i) $$\frac{(27)^{2n/3} \times 8^{-n/6}}{(18)^{-n/2}}$$

$$\Rightarrow \quad \frac{(3^{3})^{\frac{2n}{3}} \times (2^{3})^{-n/6}}{(2\times 9)^{-n/2}}$$

$$\Rightarrow \quad \frac{3^{\cancel{3}\times\frac{2n}{\cancel{3}}} \times 2^{\cancel{3}\times\frac{-n}{\cancel{6}\,2}}}{2^{-n/2} \times 3^{\cancel{2}\times\frac{-n}{\cancel{2}}}}$$

$$\Rightarrow \quad \frac{3^{2n} \times 2^{-n/2}}{2^{-n/2} \times 3^{-n}}$$

$$\Rightarrow \frac{3^{2n}}{2^{-n/2}} \times \frac{1}{2^{n/2}} \times 3^{n}$$

$$\Rightarrow \frac{3^{2n+n}}{2^{-n/2} \cdot 2^{n/2}}$$

$$\Rightarrow \frac{3^{3n}}{2^{-n/2} \cdot 2^{n/2}}$$

$$\Rightarrow \frac{3^{3n} \times \cancel{2^{n/2}}}{\cancel{2^{n/2}}}$$

$$\Rightarrow 3^{3n} \; //$$

(ii) $$\frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$$

$$\Rightarrow \frac{5^{1} \cdot (5^{2})^{n+1} - 5^{2} \cdot 5^{2n}}{5^{1} \cdot 5^{2n+3} \quad (5^{2})^{n+1}}$$

$$\Rightarrow \frac{5^{1+2n+2} - 5^{2+2n}}{5^{1+2n+3} - 5^{2n+2}}$$

$$\Rightarrow \frac{5^{2n+3} - 5^{2+2n}}{5^{2n+4} - 5^{2+2n}}$$

$$\Rightarrow \frac{5^{2n} \cdot 5^3 - 5^2 \cdot 5^{2n}}{5^{2n} \cdot 5^4 - 5^2 \cdot 5^{2n}}$$

$$\Rightarrow \frac{\cancel{5^{2n}}\left[5^3 - 5^2\right]}{\cancel{5^{2n}}\left[5^4 - 5^2\right]}$$

$$\Rightarrow \frac{125 - 25}{625 - 25}$$

$$\Rightarrow \frac{100}{600}$$

$$\Rightarrow \frac{1}{6}$$

Solution - 8

(i) $\left(8^{-4/3} \div 2^{-2}\right)^{\frac{1}{2}}$

$$\Rightarrow \left(\left(2^3\right)^{-4/\cancel{3}} \div 2^{-2}\right)^{1/2}$$

$$\Rightarrow \left(\frac{2^{-4}}{2^{-2}}\right)^{1/2}$$

$$\Rightarrow \left(2^{-4+2}\right)^{1/2}$$

$$\Rightarrow \left(2^{-\cancel{2}}\right)^{1/\cancel{2}}$$

$$\Rightarrow 2^{-1}$$

$$\Rightarrow 1/2 \;//$$

(ii) $\left(\frac{27}{8}\right)^{2/3} - \left(\frac{1}{4}\right)^{-2} + 5^0$

$\Rightarrow \left(\frac{3^3}{2^3}\right)^{\frac{2}{3}} - \left(\frac{1}{2^2}\right)^{-2} + 1$

$\Rightarrow \left(\frac{3}{2}\right)^{\cancel{3}\times\frac{2}{\cancel{3}}} - \frac{1}{2^{2\times-2}} + 1$

$\Rightarrow \left(\frac{3}{2}\right)^{2} - \frac{1}{2^{-4}} + 1$

$\Rightarrow \frac{9}{4} - 2^4 + 1$

$\Rightarrow \frac{9}{4} - 16 + 1$

$\Rightarrow \frac{9}{4} - 15$

$\Rightarrow \frac{9 - 60}{4}$

$\Rightarrow \frac{-51}{4}$

Solution – 9

(i) $(3x^2)^{-3} \times (x^9)^{2/3}$

$\Rightarrow \frac{1}{(3x^2)^3} \times x^{\overset{3}{\cancel{9}}\times 2/\cancel{3}}$

$$\Rightarrow \quad \frac{1}{3^{3} \cdot x^{2\times 3}} \times x^{3\times 2}$$

$$\Rightarrow \quad \frac{1}{27 \cdot \cancel{x^{6}}} \times \cancel{x^{6}}$$

$$\Rightarrow \quad \frac{1}{27}.$$

(ii) $(8x^{4})^{1/3} \div x^{1/3}.$

$$\Rightarrow \quad (2^{3} \cdot x^{4})^{1/3} \div x^{1/3}$$

$$\Rightarrow \quad \frac{2^{\cancel{3}\cdot\frac{1}{\cancel{3}}} \cdot x^{4\cdot 1/3}}{x^{1/3}}$$

$$\Rightarrow \quad \frac{2 \cdot x^{4/3}}{x^{1/3}}.$$

$$\Rightarrow \quad 2 \cdot x^{4\cdot 1/3} - x^{1/3}$$

$$\Rightarrow \quad x^{1/3} \left[2x^{4} - 1 \right] \; //$$

Solution-10.

(i) $(3^{2})^{0} + 3^{-4} \times 3^{6} + \left(\frac{1}{3}\right)^{-2}.$

$$= \quad 3^{0} + 3^{-4+6} + \frac{1}{3^{-2}}$$

$$\Rightarrow \quad 1 + 3^{+2} + 3^{2}.$$

$$\Rightarrow \quad 1 + \frac{\cancel{}}{3^{2}} + 3^{2}$$

$\Rightarrow \quad 1 + 9 + 9$

$\Rightarrow \quad 19$ //.

(ii) $9^{5/2} - 3.(5)^{0} - \left(\frac{1}{81}\right)^{-1/2}$

$\Rightarrow \quad 9^{5/2} - 3(1) - \left(\frac{1}{9^{2}}\right)^{-1/2}$

$\Rightarrow \quad 9^{5/2} - 3 - \frac{1}{9^{2\times -1/2}}$

$\Rightarrow \quad 3^{2\times 5/2} - 3 - \frac{1}{9^{-1}}$

$\Rightarrow \quad 3^{5} - 3 - 9$

$\Rightarrow \quad 3^{5} - 3 - 3^{2}$

$\Rightarrow \quad 3\,(3^{4} - 1 - 3)$

$\Rightarrow \quad 3\,(81 - 1 - 3)$

$\Rightarrow \quad 3\,(77)$

$\Rightarrow \quad 231$

Solution – 11

(i) $16^{3/4} + 2\left(\frac{1}{2}\right)^{-1} \cdot 3^{0}$

$\Rightarrow (2^4)^{3/4} + 2\left(\frac{1}{2^{-1}}\right) \cdot 1$

$\Rightarrow 2^{\cancel{4} \times \frac{3}{\cancel{4}}} + 2 \cdot 2$

$\Rightarrow 2^3 + 4$

$\Rightarrow 8 + 4$

$\Rightarrow 12$

(ii) $(81)^{3/4} - \left(\frac{1}{32}\right)^{-2/5} + (8)^{1/3}\left(\frac{1}{2}\right)^{-1} \cdot (2)^{0}$

$\Rightarrow (3^{\cancel{4}})^{3/\cancel{4}} - \left(\frac{1}{2^5}\right)^{-2/5} + (2^{\cancel{3}})^{1/\cancel{3}}\left(\frac{1}{2^{-1}}\right) 1$

$\Rightarrow 3^3 - \frac{1}{2^{\cancel{5} \times -2/\cancel{5}}} + 2(2^1)1$

$\Rightarrow 3^3 - \frac{1}{2^{-2}} + 2(2)$

$\Rightarrow 27 - 2^2 + 4$

$\Rightarrow 27 - 4 + 4$

$\Rightarrow 27.$

Solution-12

(i) $$\left(\frac{64}{125}\right)^{-2/3} \div \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^{0}$$

$$\Rightarrow \left(\frac{4^3}{5^3}\right)^{-2/3} \div \frac{1}{\left(\frac{4^4}{5^4}\right)^{1/4}} + 1$$

$$\Rightarrow \left(\frac{4}{5}\right)^{\cancel{3} \times \frac{-2}{\cancel{3}}} \div \frac{1}{\left(\frac{4}{5}\right)^{\cancel{4} \times \frac{1}{\cancel{4}}}} + 1$$

$$\Rightarrow \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)} + 1$$

$$\Rightarrow \frac{\left(\frac{5}{4}\right)^{2}}{\frac{1}{\left(\frac{4}{5}\right)}} + 1$$

$$\Rightarrow \left(\frac{5}{4}\right)^{\cancel{2}} \times \left(\cancel{\frac{4}{5}}\right) + 1$$

$$\Rightarrow \frac{5}{4} + 1$$

$$\Rightarrow \frac{5+4}{4}$$

$$\Rightarrow \frac{9}{4}$$

(ii) $$\frac{5^{n+3} - 6\times 5^{n+1}}{9\times 5^{n} - 2^{2}\times 5^{n}}$$

$$\Rightarrow \frac{5^{n}\cdot 5^{3} - 6\times 5^{n}\cdot 5}{9\times 5^{n} - 2^{2}\times 5^{n}}$$

$$\Rightarrow \frac{\cancel{5^{n}}\,[5^{3} - 6\times 5]}{\cancel{5^{n}}\,[9-4]}$$

$$\Rightarrow \frac{125-30}{5}$$

$$\Rightarrow \frac{95}{5}$$

$$\Rightarrow 19.$$

Solution- 13 :

(i) $$\left[(64)^{2/3}\cdot 2^{-2} \div 8^{0}\right]^{-1/2}$$

$$\Rightarrow \left((4^{\cancel{3}})^{2/\cancel{3}}\cdot \frac{1}{2^{2}} \div 1\right)^{-1/2}$$

$$\Rightarrow \left(\frac{4^{2}}{2^{2}}\right)^{-1/2}$$

$$\Rightarrow \left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)^{\cancel{2}\times -1/\cancel{2}}$$

$$\Rightarrow 2^{-1}$$

$$\Rightarrow \frac{1}{2} \quad //.$$

(ii) $3^{n} \times 9^{n+1} \div 3^{n-1} \times 9^{n-1}$

$\Rightarrow 3^{n} \times 3^{2(n+1)} \div 3^{n-1} \times 3^{2(n-1)}$

$\Rightarrow 3^{n} \times 3^{2n+2} \div 3^{n-1} \times 3^{2n-2}$

$$\Rightarrow \frac{3^{n+2n+2}}{3^{n-1+2n-2}}$$

$$\Rightarrow \frac{3^{3n+2}}{3^{3n-3}}$$

$$\Rightarrow \frac{\cancel{3^{3n}} \cdot 3^{2}}{\cancel{3^{3n}} \cdot 3^{-3}}$$

$\Rightarrow 3^{2} \times 3^{+3}$

$\Rightarrow 3^{2+3}$

$\Rightarrow 3^{5}$

$\Rightarrow 243$

Solution- 14

(i) $$\frac{\sqrt{2^{2}} \times \sqrt[4]{256}}{\sqrt[3]{64}} - \left(\frac{1}{2}\right)^{-2}$$

$$\Rightarrow \frac{(2^{2})^{1/2} \times (4^{4})^{1/4}}{(4^{3})^{1/3}} - \frac{1}{2^{-2}}$$

$$\Rightarrow \quad \frac{2\times\cancel{4}}{\cancel{4}} - 2^2$$

$$\Rightarrow \quad 2 - 4$$

$$\Rightarrow \quad -2.$$

(ii) $$\frac{3^{-6/7} \times 4^{-3/7} \times 9^{3/7} \times 2^{4/7}}{2^2 + 2^0 + 2^{-2}}$$

$$\Rightarrow \quad \frac{3^{-6/7} \times 3^{2\cdot\frac{3}{7}} \times 2^{2\times\frac{-3}{7}} \times 2^{6/7}}{4 + 1 + \frac{1}{2^2}}$$

$$\Rightarrow \quad \frac{3^{-6/7} \times 3^{6/7} \times 2^{-6/7} \times 2^{-6/7}}{4 + 1 + \frac{1}{4}}$$

$$\Rightarrow \quad \frac{3^{-\frac{6}{7}+\frac{6}{7}} \times 2^{-\frac{6}{7}+\frac{6}{7}}}{\frac{16+4+1}{4}}$$

$$\Rightarrow \quad \frac{3^0 \times 2^0}{\left(\frac{21}{4}\right)}$$

$$\Rightarrow \quad \frac{1}{\left(\frac{21}{4}\right)}$$

$$\Rightarrow \quad \frac{4}{21} \quad //$$

Solution - 15

(i) $$\frac{(32)^{2/5} \times (4)^{-1/2} \times (8)^{1/3}}{2^{-2} \div (64)^{-1/3}}$$

$$\Rightarrow \frac{(2^{\cancel{5}})^{-2/\cancel{5}} \times (2^{\cancel{2}})^{-1/\cancel{2}} \times (2^{\cancel{3}})^{1/\cancel{3}}}{\frac{1}{2^2} \div (4^{\cancel{3}})^{-1/\cancel{3}}}$$

$$\Rightarrow \frac{2^{-2} \times 2^{-1} \times 2^{1}}{\frac{1}{2^2} \div (4^{-1})}$$

$$\Rightarrow \frac{2^{-1-2+1}}{\frac{\cancel{\left(\frac{1}{2^2}\right)}}{\cancel{\left(\frac{1}{2^2}\right)}}}$$

$$\Rightarrow 2^{-2}$$

$$\Rightarrow \frac{1}{2^2}$$

$$\Rightarrow \frac{1}{4}$$

(iii) $\dfrac{5^{2(x+6)} \times 25^{-7+2x}}{(125)^{2x}}$

$\Rightarrow \dfrac{5^{2x+12} \times 5^{2(-7+2x)}}{(5^3)^{2x}}$

$\Rightarrow \dfrac{5^{2x+12} \times 5^{-14+4x}}{5^{6x}}$

$\Rightarrow \dfrac{5^{2x+12-14+4x}}{5^{6x}}$

$\Rightarrow \dfrac{5^{6x-2}}{5^{6x}}$

$\Rightarrow \dfrac{\cancel{5^{6x}} \cdot 5^{-2}}{\cancel{5^{6x}}}$

$\Rightarrow 5^{-2}$

$\Rightarrow \dfrac{1}{5^2}$

$\Rightarrow \dfrac{1}{25}$

Solution- 16

(i) $$\frac{7^{2n+3} - 49^{n+2}}{\left((343)^{n+1}\right)^{2/3}}$$

$$\Rightarrow \frac{7^{2n+3} - 7^{2(n+2)}}{\left(7^{\cancel{3}(n+1)}\right)^{2/\cancel{3}}}$$

$$\Rightarrow \frac{7^{2n+3} - 7^{2n+4}}{7^{2(n+1)}}$$

$$\Rightarrow \frac{7^{2n+3} - 7^{2n+4}}{7^{2n+2}}$$

$$\Rightarrow \frac{7^{2n}.7^{3} - 7^{2n}.7^{4}}{7^{2n}.7^{2}}$$

$$\Rightarrow \frac{\cancel{7^{2n}}\left[7^{3} - 7^{4}\right]}{\cancel{7^{2n}}.7^{2}}$$

$$\Rightarrow \frac{343 - 2401}{49}$$

$$\Rightarrow -\frac{2058}{49}$$

$$\Rightarrow -42$$ //

(ii) $(27)^{4/3} + (32)^{0.8} + (0.8)^{-1}$.

$\Rightarrow \quad (3^{\cancel{3}})^{4/\cancel{3}} + (2^5)^{\frac{\cancel{8}^{4}}{\cancel{10}_{5}}} + \left(\frac{\cancel{8}^{4}}{\cancel{10}_{5}}\right)^{-1}$.

$\Rightarrow \quad 3^4 + 2^{\cancel{5} \times \frac{4}{\cancel{5}}} + \left(\frac{4}{5}\right)^{-1}$

$\Rightarrow \quad 3^4 + 2^4 + \frac{5}{4}$

$\Rightarrow \quad 81 + 16 + \frac{5}{4}$

$\Rightarrow \quad 97 + \frac{5}{4}$

$\Rightarrow \quad \frac{388+5}{4}$

$\Rightarrow \quad \frac{393}{4}$

solution- 17

(i) $(\sqrt{32} - \sqrt{5})^{1/3} \cdot (\sqrt{32} + \sqrt{5})^{1/3}$.

$\Rightarrow \left[(\sqrt{2^5} - \sqrt{5}) \cdot (\sqrt{2^5} + \sqrt{5})\right]^{1/3}$

$\Rightarrow \left[(\sqrt{2^5})^2 - (\sqrt{5})^2\right]^{1/3}$

$\Rightarrow \quad (2^5 - 5)^{1/3}$

$$\Rightarrow \quad (32-5)^{1/3}$$

$$\Rightarrow \quad (27)^{1/3}$$

$$\Rightarrow \quad (3^3)^{1/3}$$

$$\Rightarrow \quad \underline{\underline{3}}$$

(ii) $$\left(x^{1/3} - x^{-1/3}\right)\left(x^{2/3} + 1 + x^{-2/3}\right)$$

$$\Rightarrow \quad \left(x^{1/3} - \frac{1}{x^{1/3}}\right)\left(x^{2/3} + 1 + \frac{1}{x^{2/3}}\right)$$

$\because$ It is in the form of

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3$$

$\therefore$ Here $a = x^{1/3}$; $b = \frac{1}{x^{1/3}}$

$$\therefore \quad (x^{1/3})^3 - \left(\frac{1}{x^{1/3}}\right)^3$$

$$\Rightarrow \quad x^{\cancel{3}/\cancel{3}} - \frac{1}{x^{\cancel{3}/\cancel{3}}}$$

$$\Rightarrow \quad x - \frac{1}{x} \;//$$

Solution – 18 :

(i) $\left(\dfrac{x^m}{x^n}\right)^l \cdot \left(\dfrac{x^n}{x^l}\right)^m \cdot \left(\dfrac{x^l}{x^m}\right)^n$

$\Rightarrow \quad \left(x^{m-n}\right)^l \cdot \left(x^{n-l}\right)^m \cdot \left(x^{l-m}\right)^n$

$\Rightarrow \quad x^{ml-nl} \cdot x^{mn-ml} \cdot x^{nl-nm}$

$\Rightarrow \quad x^{ml-nl+mn-ml+nl-nm}$

$\Rightarrow \quad x^0$

$\Rightarrow \quad 1$

(ii) $\left(\dfrac{x^{a+b}}{x^c}\right)^{a-b} \cdot \left(\dfrac{x^{b+c}}{x^a}\right)^{b-c} \cdot \left(\dfrac{x^{c+a}}{x^b}\right)^{c-a}$

$\Rightarrow \quad \dfrac{x^{(a+b)(a-b)}}{x^{c(a-b)}} \cdot \dfrac{x^{(b+c)(b-c)}}{x^{a(b-c)}} \cdot \dfrac{x^{(c+a)(c-a)}}{x^{b(c-a)}}$

$\Rightarrow \quad \dfrac{x^{a^2-b^2}}{x^{ac-bc}} \cdot \dfrac{x^{b^2-c^2}}{x^{ab-ac}} \cdot \dfrac{x^{c^2-a^2}}{x^{bc-ab}}$

$\Rightarrow \quad \dfrac{x^{a^2-b^2+b^2-c^2+c^2-a^2}}{x^{ac-bc+ab-ac+bc-ab}}$

$\Rightarrow \quad \dfrac{x^0}{x^0}$

$\Rightarrow \quad 1$

Solution – 19 :

(i) $\sqrt[lm]{\frac{x^l}{x^m}} \cdot \sqrt[mn]{\frac{x^m}{x^n}} \cdot \sqrt[nl]{\frac{x^n}{x^l}}$

$\Rightarrow \sqrt[lm]{x^{l-m}} \cdot \sqrt[mn]{x^{m-n}} \cdot \sqrt[nl]{x^{n-l}}$

$\Rightarrow (x^{l-m})^{\frac{1}{lm}} \cdot (x^{m-n})^{\frac{1}{nm}} \cdot (x^{n-l})^{\frac{1}{nl}}$

$\Rightarrow x^{\frac{l-m}{lm}} \cdot x^{\frac{m-n}{nm}} \cdot x^{\frac{n-l}{nl}}$

$\Rightarrow x^{\frac{l-m}{lm}+\frac{m-n}{nm}+\frac{n-l}{nl}}$

$\Rightarrow x^{\frac{n(l-m)+l(m-n)+m(n-l)}{lmn}}$

$\Rightarrow x^{\frac{nl-nm+lm-ln+mn-lm}{lmn}}$

$\Rightarrow x^0$

$\Rightarrow 1$

(ii) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

$\Rightarrow x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ac+a^2)}$

$\Rightarrow x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3}$

$\Rightarrow \quad x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$\Rightarrow \quad x^0$

$\Rightarrow \quad 1$

(iii) $\left(\dfrac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \cdot \left(\dfrac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \cdot \left(\dfrac{x^c}{x^{-a}}\right)^{c^2-ac+a^2}$

$\Rightarrow \quad x^{(a-(-b))(a^2-ab+b^2)} \cdot x^{(b-(-c))(b^2-bc+c^2)} \cdot x^{(c-(-a))(c^2-ac+a^2)}$

$\Rightarrow \quad x^{(a+b)(a^2-ab+b^2)} \cdot x^{(b+c)(b^2-cb+c^2)} \cdot x^{(c+a)(c^2-ac+a^2)}$

$\Rightarrow \quad x^{a^3+b^3} \cdot x^{b^3+c^3} \cdot x^{c^3+a^3}$

$\Rightarrow \quad x^{a^3+b^3+b^3+c^3+c^3+a^3}$

$\Rightarrow \quad x^{2a^3+2b^3+2c^3}$

$\Rightarrow \quad x^{2(a^3+b^3+c^3)}$

Solution - 20 :

(i) $(a^{-1}+b^{-1}) \div (a^{-2}-b^{-2})$

$\Rightarrow \quad \left(\dfrac{1}{a}+\dfrac{1}{b}\right) \div \left(\dfrac{1}{a^2}-\dfrac{1}{b^2}\right)$

$\Rightarrow \quad \left(\dfrac{b+a}{ab}\right) \div \left(\dfrac{b^2-a^2}{a^2b^2}\right)$

$$\Rightarrow \frac{\frac{(b+a)}{ab}}{\frac{(b^2-a^2)}{a^2b^2}}$$

$$\Rightarrow \frac{(b+a)}{ab} \times \frac{(ab)^2}{(b^2-a^2)}$$

$$\Rightarrow \frac{\cancel{b+a}}{\cancel{ab}} \cdot \frac{(ab)^{\cancel{2}}}{(\cancel{b+a})(b-a)}$$

$$\Rightarrow \frac{ab}{b-a}.$$

<u>Solution - 21</u>

(i) $(a+b)^{-1}(a^{-1}+b^{-1}) = \frac{1}{ab}$

$$\text{LHS} \Rightarrow (a+b)^{-1}(a^{-1}+b^{-1})$$

$$\Rightarrow \frac{1}{a+b}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$\Rightarrow \frac{1}{a+b}\left(\frac{b+a}{ab}\right)$$

$$\Rightarrow \frac{1}{\cancel{a+b}}\left(\frac{\cancel{a+b}}{ab}\right)$$

$$\Rightarrow \frac{1}{ab}$$

$$= \text{RHS}.$$

(ii) $$\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}} = xyz.$$

LHS $\Rightarrow$ $$\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$$

$$\Rightarrow \frac{x+y+z}{\frac{1}{xy}+\frac{1}{yz}+\frac{1}{xz}}$$

$$\Rightarrow \frac{x+y+z}{\frac{z+x+y}{xyz}}$$

$$\Rightarrow \frac{\cancel{x+y+z}}{\frac{\cancel{(x+y+z)}}{xyz}}$$

$$\Rightarrow xyz$$

$\Rightarrow$ RHS.

Solution - 20

(ii) $\dfrac{1}{1+a^{m-n}} + \dfrac{1}{a^{n-m}+1}$

$\Rightarrow \dfrac{1}{1+a^{m-n}} + \dfrac{1}{1+a^{-(m-n)}}$

$\Rightarrow \dfrac{1}{1+a^{m-n}} + \dfrac{1}{1+\frac{1}{a^{m-n}}}$

$\Rightarrow \dfrac{1}{1+a^{m-n}} + \dfrac{1}{\frac{a^{m-n}+1}{a^{m-n}}}$

$\Rightarrow \dfrac{1}{1+a^{m-n}} + \dfrac{a^{m-n}}{a^{m-n}+1}$

$\Rightarrow \dfrac{1+\cancel{a^{m-n}}}{1+\cancel{a^{m-n}}}$

$\Rightarrow 1.$

Solution – 22 :

(ii) Given $a = c^z$; $b = a^x$; $c = b^y$.

$\Rightarrow$ $a = c^z$

$a = (b^y)^z$ $\quad (\because c = b^y)$

$a = b^{yz}$

$a = (a^x)^{yz}$ $\quad (\because b = a^x)$

$a^1 = a^{xyz}$

$\therefore$ Bases are equal ; so exponents are also equal.

$\therefore$ $xyz = 1$

Hence proved.

Solution – 23 .

Given $a = xy^{p-1}$; $b = xy^{q-1}$; $c = xy^{r-1}$

LHS $\Rightarrow$ $a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$

$\Rightarrow$ $(xy^{p-1})^{q-r} \cdot (xy)^{q-1(r-p)} \cdot (xy^{r-1})^{p-q}$

$\Rightarrow$ $xy^{(p-1)(q-r)} \cdot xy^{(q-1)\cdot(r-p)} \cdot xy^{(r-1)(p-q)}$

$\Rightarrow$ $xy^{pq-pr-q+r} \cdot xy^{qr-qp-r+p} \cdot xy^{rp-rq-p+q}$

$\Rightarrow \quad xy^{pr-pr-q+r+qr-qp-r+p+rp-rq-p+q}$

$\Rightarrow \quad xy^{0}$

$\Rightarrow \quad 1$

$= \quad RHS$

$\therefore$ Hence proved.

Solution – 24:

Given $2^x = 3^y = 6^{-z}$

let $2^x = 3^y = 6^{-z} = k$

$\Rightarrow \quad 2 = k^{1/x}$

$3 = k^{1/y}$

$6 = k^{-1/z}$

$\frac{1}{6} = k^{1/z}$

$\Rightarrow \quad \frac{1}{2 \times 3} = k^{1/z}$

$\frac{1}{k^{1/x} \cdot k^{1/y}} = k^{1/z}$

$1 = k^{\frac{1}{z}} \cdot k^{\frac{1}{x} \cdot \frac{1}{y}}$

$k^0 = k^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$

$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ //

Solution - 25

Given $\quad 2^x = 3^y = 12^z$

let $\quad 2^x = 3^y = 12^z = k$

$$\Rightarrow \quad 2 = k^{1/x}$$

$$3 = k^{1/y}$$

$$12 = k^{1/z}.$$

$$\Rightarrow \quad 2^2 \cdot 3 = k^{1/z}$$

$$\Rightarrow \quad (k^{1/x})^2 \cdot k^{1/y} = k^{1/z}$$

$$\Rightarrow \quad k^{\frac{2}{x}} \cdot k^{\frac{1}{y}} = k^{\frac{1}{z}}$$

$$\Rightarrow \quad \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$$

$$\Rightarrow \quad \frac{2y + x}{xy} = \frac{1}{z}$$

(or)
$$\frac{2}{x} = \frac{1}{z} - \frac{1}{y}$$

$$\frac{2}{x} = \frac{y-z}{yz}$$

$$x = \frac{2yz}{y-z}$$

$\therefore$ Hence proved.

Solution - 26 :

(i) $(3x^2)^0$

$\Rightarrow 1$

(ii) $(xy)^{-2}$

$\Rightarrow \frac{1}{(xy)^2}$

$\Rightarrow \frac{1}{x^2y^2}$

(iii) $(-27a^9)^{2/3}$

$- (3^3a^9)^{2/3}$

$- (3 \cdot a^3)^{\cancel{3} \times 2/\cancel{3}}$

$- (3a^3)^2$

$- 9^2 a^{3\times 2}$

$- 9a^6$

Solution - 27

Given $a = 3$; $b = -2$

(i) $a^a + b^b$

$\Rightarrow 3^3 + (-2)^{-2}$

$\Rightarrow 3^3 + \frac{1}{(-2)^2}$

$\Rightarrow \quad 27 + \frac{1}{4}$

$\Rightarrow \quad \frac{108+1}{4}$

$\Rightarrow \quad \frac{109}{4}$.

(ii) $\quad a^b + b^a$

$\Rightarrow \quad 3^{-2} + (-2)^3$

$\Rightarrow \quad \frac{1}{3^2} - 8$

$\Rightarrow \quad \frac{1}{9} - 8$

$\Rightarrow \quad \frac{1-72}{9}$

$\Rightarrow \quad \frac{-71}{9}$

Solution - 28

Given $\quad x = 10^3 \times 0.0099 \quad ; y = 10^{-2} \times 110$

$\Rightarrow \quad \sqrt{\frac{x}{y}} \Rightarrow \sqrt{\frac{10^3 \times 0.0099}{10^{-2} \times 110}}$

$= \sqrt{\frac{10^{3+2} \times 0.0099}{110}}$

$\Rightarrow \sqrt{\frac{10^5 \times 0.0099}{110}}$

$\Rightarrow \sqrt{\frac{99\cancel{0}}{11\cancel{0}}}$

$\Rightarrow \sqrt{9}$

$\Rightarrow 3$

Solution – 29:

Given $x = 9$, $y = 2$, $z = 8$

$$x^{1/2} \cdot y^{-1} \cdot z^{2/3}$$

$\Rightarrow 9^{1/2} \cdot 2^{-1} \cdot 8^{2/3}$

$\Rightarrow (3^{\cancel{2}})^{1/\cancel{2}} \cdot \left(\frac{1}{2}\right) (2^{\cancel{3}})^{2/\cancel{3}}$

$\Rightarrow 3 \frac{1}{2} \cdot 2^2$

$\Rightarrow 3 \cdot \frac{1}{\cancel{2}} \cdot \cancel{4}^2$

$\Rightarrow 6$

Solution – 30:

Given $x^4 y^2 z^3 = 49392$

$$x^4 y^2 z^3 = 2^4 \cdot 3^2 \cdot 7^4$$

$\because x, y, z$ are different primes

$\therefore x = 2$; $y = 3$; $z = 7$

2	49392
2	24696
2	12348
2	6174
3	3087
3	1029
7	343
7	49
	7

Solution – 31

Given $\sqrt[3]{a^6 b^{-4}} = a^x . b^{2y}$

$$(a^6 b^{-4})^{1/3} = a^x b^{2y}$$

$$a^{6/3} . b^{-4/3} = a^x b^{2y}.$$

$\therefore$ $x = \frac{6}{3}$; $2y = \frac{-4}{3}$

$x = 2$ $\qquad y = \frac{-\cancel{4}^{2}}{3 \times \cancel{2}}$

$y = -2/3$

Solution – 32

Given $(p+q)^{-1} (p^{-1} + q^{-1}) = p^a . q^b.$

$$\frac{1}{p+q} \left(\frac{1}{p} + \frac{1}{q}\right) = p^a . q^b.$$

$$\frac{1}{\cancel{p+q}} \left(\frac{\cancel{q+p}}{qp}\right) = p^a . q^b$$

$$\frac{1}{qp} = p^a q^b$$

$$(qp)^{-1} = p^a . q^b$$

$$p^{-1} . q^{-1} = p^a . q^b.$$

$a = -1$

$b = -1$

$\therefore$ LHS $\Rightarrow a + b + 2$

$\Rightarrow -1 + 2 - 1$

$\Rightarrow 0$

$= \text{RHS}$

Solution- 33

Given $\left(\dfrac{p^{-1}q^{2}}{p^{2}q^{-4}}\right)^{7} \div \left(\dfrac{p^{3}q^{-5}}{p^{-2}q^{3}}\right)^{-5} = p^{x}q^{y}$

$\Rightarrow \left(\dfrac{p^{-7}\cdot q^{2\times 7}}{p^{2\times 7}\ q^{-4\times 7}}\right) \div \left(\dfrac{p^{3\times 5}\cdot q^{-5\times -5}}{p^{-2\times -5}q^{3\times -5}}\right) = p^{x}q^{y}$

$\Rightarrow \left(\dfrac{p^{-7}\ q^{14}}{p^{14}\cdot q^{-28}}\right) \div \left(\dfrac{p^{15}\cdot q^{25}}{p^{10}\ q^{-15}}\right) = p^{x}\cdot q^{y}$

$\Rightarrow \left(p^{-7-14}\cdot q^{14+28}\right) \div \left(p^{15-10}, q^{25+15}\right) = p^{x}\cdot q^{y}$

$\Rightarrow \left(p^{-21}\cdot q^{42}\right) \div \left(p^{5}\cdot q^{40}\right) = p^{x}\cdot q^{y}$

$\Rightarrow \left(\dfrac{p^{-21}\cdot q^{42}}{p^{5}\cdot q^{40}}\right) = p^{x}\cdot q^{y}$

$$\Rightarrow \left(p^{-21-5} \cdot q^{42-20}\right) = p^x q^y$$

$$\Rightarrow \left(p^{-26} \cdot q^{2}\right) = p^x \cdot q^y$$

$$\therefore x = -26 \; ; \; y = 2$$

$$\therefore x + y = -26 + 2$$
$$= -24.$$

Solution - 34

(i) $$5^{2x+3} = 1$$

$$5^{2x+3} = 5^0 \qquad (\because 5^0 = 1)$$

$$\therefore 2x + 3 = 0$$
$$2x = -3$$
$$x = -3/2$$

(ii) $$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$(13)^{\sqrt{x}} = 256 - 81 - 6$$

$$(13)^{\sqrt{x}} = 169$$

$$(13)^{\sqrt{x}} = (13)^2$$

$\Rightarrow (13)^{\sqrt{x}} = 13^2$

$\therefore \sqrt{x} = 2$

$x = 2^{1/2}$

(iii) $\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$

$\left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \frac{5^3}{3^3}$

$\left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{5}{3}\right)^3$

$\left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{3}{5}\right)^{-3}$

$\therefore \frac{x+1}{2} = -3$

$x+1 = -6$

$x = -6-1$

$x = -7$

(iv) $$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$\left[\left(2^2\right)^{1/3}\right]^{\frac{4x+1}{2}} = \frac{1}{32}$$

$$\left(2^{\frac{2}{3}}\right)^{\frac{4x+1}{2}} = \frac{1}{2^5}$$

$$2^{\frac{4x+1}{3}} = 2^{-5}$$

$$\Rightarrow \quad \frac{4x+1}{3} = -5$$

$$4x+1 = -15$$

$$4x = -15-1$$

$$4x = -16$$

$$x = \frac{-16}{4}$$

$$x = -4$$

Solution – 35 :

(i) $$\sqrt{\frac{p}{q}} = \left(\frac{q}{p}\right)^{1-2x}$$

$$\left(\frac{p}{q}\right)^{1/2} = \left(\frac{p}{q}\right)^{-(1-2x)}$$

$$\left(\frac{p}{q}\right)^{1/2} = \left(\frac{p}{q}\right)^{-1+2x}$$

$$\frac{1}{2} = -1 + 2x$$

$$-1 + 2x = \frac{1}{2}$$

$$2x = \frac{1}{2} + 1$$

$$2x = \frac{1+2}{2}$$

$$2x = \frac{3}{2}$$

$$x = \frac{3}{2\times 2}$$

$$x = \frac{3}{4}.$$

(ii) $$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{x}.$$

$$2^{2(x-1)} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^{x}.$$

$$2^{2x-2} \times \frac{1}{2^{3-2x}} = \frac{1}{2^{3x}}.$$

$$2^{2x-2} \times 2^{2x-3} = 2^{-3x}$$

$$\therefore 2x-2+2x-3 = -3x$$

$$4x-5 = -3x$$

$$4x+3x = 5$$

$$7x = 5$$

$$x = 5/7 \quad //.$$

Solution - 36

Given $5^{3x} = 125$

$10^{y} = 0.001$

$\Rightarrow 5^{3x} = 125$

$5^{3x} = 5^{3}$

$3x = 3$

$x = \frac{3}{3}$

$x = 1.$

$\because 10^{y} = 0.001$

$10^{y} = \left(\frac{1}{1000}\right)$

$10^{y} = \left(\frac{1}{10^{3}}\right)$

$10^{y} = 10^{-3}$

$y = -3.$

$\therefore x = 1 \; ; \; y = -3.$

Solution - 37

Given $$\frac{9^n 3^2 \cdot 3^n - 27^n}{3^{3m} \cdot 2^3} = \frac{1}{27}$$

$$\frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\frac{3^{3n+2} - 3^{3n}}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot (3^2 - 1)}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} (9-1)}{2^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot \cancel{8}}{3^{3m} \cdot \cancel{8}} = \frac{1}{3^3}$$

$$\Rightarrow 3^{3n} \cdot 3^3 = 3^{3m}$$

$$3^{3(n+1)} = 3^{3m}$$

$$\therefore \quad m = n+1$$

<u>Solution - 38</u>

Given $\quad 3^{4x} = (81)^{-1}$

$$3^{4x} = (3^4)^{-1}$$

$$3^{4x} = 3^{-4}.$$

$$\therefore \quad 4x = -4$$

$$x = \frac{-4}{4}$$

$$x = -1$$

and $\quad 10^{1/y} = 0.0001$

$$10^{1/y} = \frac{1}{10000}$$

$$10^{1/y} = \frac{1}{10^4}$$

$$10^{1/y} = 10^{-4}.$$

$$\therefore \quad \frac{1}{y} = -4$$

$$y = -\frac{1}{4}$$

$\Rightarrow \quad 2^{-x} \cdot (16)^{y}$

$\Rightarrow \quad 2^{-(-1)} \cdot (16)^{-1/4}$

$\Rightarrow \quad 2^{1} \cdot (2^{4})^{-1/4}$

$\Rightarrow \quad 2^{1} \cdot 2^{-1}$

$\Rightarrow \quad \cancel{2} \cdot \frac{1}{\cancel{2}}$

$\Rightarrow \quad \underline{1}$ //

Solution - 39 :

Given $\quad 3^{x+1} = 9^{x-2}$

$3^{x+1} = 3^{2(x-2)}$

$3^{x+1} = 3^{2x-4}$

$x+1 = 2x-4$

$2x - x = 1+4$

$x = 5$

$\Rightarrow \quad 2^{1+x} = 2^{1+5}$

$= 2^{6}$

$= 64.$

Solution- 40

(i) $$3(2^x+1) - 2^{x+2} + 5 = 0$$

$$3 \cdot 2^x + 3 - 2^x \cdot 2^2 + 5 = 0$$

$$3 \cdot 2^x + 3 - 2^x \cdot 4 + 5 = 0.$$

$$3 \cdot 2^x - 4 2^x + 8 = 0$$

$$-2^x + 8 = 0$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

(ii) $$3^x = 9.3^y$$

$$3^x = 3^2 \cdot 3^y$$

$$3^x = 3^{2+y}$$

$$x = 2+y$$

and $$8 \cdot 2^y = 4^x$$

$$2^3 \cdot 2^y = 2^{2x}.$$

$$3+y = 2x.$$

$$x = \frac{3+y}{2} \quad //$$