

Logarithms

Exercise 9.1

Question 1.

Convert the following to logarithmic form:

(i) $5^2 = 25$

(ii) $a^5 = 64$

(iii) $7^x = 100$

(iv) $9^0 = 1$

(v) $6^1 = 6$

(vi) $3^{-2} = \frac{1}{9}$

(vii) $10^{-2} = 0.01$

(viii) $(81)^{\frac{3}{4}} = 27$

Solution:

(i) $5^2 = 25 \Rightarrow \log_5 25 = 2$

(ii) $a^5 = 64 \Rightarrow \log_a 64 = 5$

(iii) $7^x = 100 \Rightarrow \log_7 100 = x$

(iv) $9^0 = 1 \Rightarrow \log_9 1 = 0$

(v) $6^1 = 6 \Rightarrow \log_6 6 = 1$

(vi) $3^{-2} = \frac{1}{9} \Rightarrow \log_3 \frac{1}{9} = -2$

(vii) $10^{-2} = 0.01 \Rightarrow \log_{10} 0.01 = -2$

(viii) $(81)^{\frac{3}{4}} = 27 \Rightarrow \log_{81} 27 = \frac{3}{4}$

Question 2.

Convert the following into exponential form:

(i) $\log_2 32 = 5$

(ii) $\log_3 81 = 4$

(iii) $\log_3 \frac{1}{3} = -1$

(iv) $\log_3 4 = \frac{2}{3}$

(v) $\log_8 32 = \frac{5}{3}$

(vi) $\log_{10} (0.001) = -3$

(vii) $\log_2 0.25 = -2$

(viii) $\log_a \left(\frac{1}{a}\right) = -1$

Solution:

$$(i) \log_2 32 = 5 \Rightarrow 2^5 = 32$$

$$(ii) \log_3 81 = 4 \Rightarrow 3^4 = 81$$

$$(iii) \log_3 \frac{1}{3} = -1 \Rightarrow 3^{-1} = \frac{1}{3}$$

$$(iv) \log_8 4 = \frac{2}{3} \Rightarrow (8)^{\frac{2}{3}} = 4$$

$$(v) \log_8 32 = \frac{5}{3} \Rightarrow (8)^{\frac{5}{3}} = 32$$

$$(vi) \log_{10} (0.001) = -3 \Rightarrow 10^{-3} = 0.001$$

$$(vii) \log_2 0.25 = -2 \Rightarrow 2^{-2} = 0.25$$

$$(viii) \log_a \frac{1}{a} = -1 \Rightarrow a^{-1} = \frac{1}{a}$$

Question 3.

By converting to exponential form, find the values of:

(i) $\log_2 16$

(ii) $\log_5 125$

(iii) $\log_4 8$

(iv) $\log_9 27$

(v) $\log_{10} (.01)$

(vi) $\log_7 \frac{1}{7}$

(vii) $\log_5 256$

(viii) $\log_2 0.25$

Solution:

$$(i) \text{ Let, } \log_2 16 = x$$

$$\Rightarrow (2)^x = 16$$

$$\Rightarrow (2)^x = 2 \times 2 \times 2 \times 2$$

$$\Rightarrow (2)^x = (2)^4$$

$$\therefore x = 4$$

$$(ii) \text{ Let, } \log_5 125 = x \Rightarrow (5)^x = 125$$

$$\Rightarrow (5)^x = 5 \times 5 \times 5 \Rightarrow (5)^x = (5)^3$$

$$\therefore x = 3$$

$$(iii) \text{ Let, } \log_4 8 = x \Rightarrow (4)^x = 8$$

$$\Rightarrow (2 \times 2)^x = 2 \times 2 \times 2 \Rightarrow (2)^{2x} = (2)^3$$

$$\Rightarrow 2x = 3$$

$$\therefore x = \frac{3}{2}$$

$$(iv) \log_9 27 = x$$

$$\Rightarrow (9)^x = 27$$

$$\Rightarrow (3 \times 3)^x = 3 \times 3 \times 3 \Rightarrow (3)^{2x} = (3)^3$$

$$\Rightarrow 2x = 3$$

$$\therefore x = \frac{3}{2}$$

$$(v) \log_{10} (.01) = x \Rightarrow (10)^x = .01$$

$$\Rightarrow (10)^x = \frac{1}{100} \Rightarrow (10)^x = \frac{1}{10} \times \frac{1}{10}$$

$$\Rightarrow (10)^x = \frac{1}{(10)^2} \Rightarrow (10)^x = (10)^{-2}$$

$$\therefore x = -2$$

$$(vi) \log_7 \frac{1}{7} = x \Rightarrow (7)^x = \frac{1}{7}$$

$$\Rightarrow (7)^x = (7)^{-1}$$

$$\therefore x = -1$$

$$(vii) \text{ Let, } \log_5 256 = x$$

$$\Rightarrow (.5)^x = 256 \Rightarrow \left(\frac{5}{10}\right)^x = 256$$

$$(vii) \log_{81} x = \frac{3}{2}$$

$$\begin{aligned} \therefore x &= 81^{\frac{3}{2}} = (3^4)^{\frac{3}{2}} = 3^{4 \times \frac{3}{2}} = 3^6 \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729 \end{aligned}$$

$$(viii) \log_9 x = 2.5 = \frac{5}{2}$$

$$\begin{aligned} \therefore x &= (9)^{\frac{5}{2}} = (3^2)^{\frac{5}{2}} = 3^{2 \times \frac{5}{2}} = 3^5 \\ &= 3 \times 3 \times 3 \times 3 \times 3 = 243 \end{aligned}$$

$$(ix) \log_4 x = -1.5 = \frac{-3}{2}$$

$$\begin{aligned} \therefore x &= (4)^{\frac{-3}{2}} = (2^2)^{\frac{-3}{2}} = 2^{2 \times \left(\frac{-3}{2}\right)} \\ &= 2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8} \end{aligned}$$

$$(x) \log_{\sqrt{5}} x = 2 \Rightarrow (\sqrt{5})^2 = x$$

$$\Rightarrow (5)^{2 \times \frac{1}{2}} = x \Rightarrow (5)^1 = x \therefore x = 5$$

$$(xi) \log_x 0.001 = -3 \Rightarrow (x)^{-3} = \frac{1}{1000}$$

$$\Rightarrow (x)^{-3} = \frac{1}{(10)^3} \therefore (x)^{-3} = 10^{-3} \therefore x = 10$$

$$(xii) \log_{\sqrt{3}} (x+1) = 2 \Rightarrow (\sqrt{3})^2 = x+1$$

$$\Rightarrow 3 = x+1 \Rightarrow x+1 = 3$$

$$\Rightarrow x = 3 - 1 \therefore x = 2$$

$$(xiii) \log_4 (2x+3) = \frac{3}{2} \Rightarrow (4)^{\frac{3}{2}} = 2x+3$$

$$\Rightarrow (2 \times 2)^{\frac{3}{2}} = 2x+3 \Rightarrow (2)^{2 \times \frac{3}{2}} = 2x+3$$

$$\Rightarrow (2)^3 = 2x+3 \Rightarrow 2 \times 2 \times 2 = 2x+3$$

$$\Rightarrow 8 = 2x+3 \Rightarrow 2x = 8-3$$

$$\therefore 2x = 5 \therefore x = \frac{5}{2}$$

$$(xiv) \log_{\sqrt{2}} x = 3 \Rightarrow (\sqrt[3]{2})^3 = x \Rightarrow \left[(2)^{\frac{1}{3}} \right]^3 = x$$

$$\Rightarrow (2)^{\frac{1}{3} \times 3} = x \Rightarrow (2)^1 = x \quad \therefore x = 2$$

$$(xv) \log_2(x^2 - 1) = 3 \Rightarrow (2)^3 = x^2 - 1$$

$$\Rightarrow 2 \times 2 \times 2 = x^2 - 1 \Rightarrow 8 = x^2 - 1$$

$$\Rightarrow x^2 - 1 = 8 \Rightarrow x^2 = 8 + 1 \Rightarrow x^2 = 9$$

$$\therefore x = \pm 3$$

$$(xvi) \log x = -1 \Rightarrow (10)^{-1} = x \Rightarrow x = 10^{-1}$$

$$\therefore x = \frac{1}{10}$$

$$(xvii) \log(2x - 3) = 1 \Rightarrow (10)^1 = 2x - 3$$

$$\Rightarrow 10 = 2x - 3 \Rightarrow 2x = 10 + 3 \Rightarrow 2x = 13$$

$$\therefore x = \frac{13}{2} = 6\frac{1}{2}$$

$$(xviii) \log x = -2, 0, \frac{1}{3}$$

$$\log x = -2 \Rightarrow (10)^{-2} = x \Rightarrow \frac{1}{100} = x \Rightarrow x = \frac{1}{100}$$

$$\text{when, } \log x = 0 \Rightarrow (10)^0 = x \Rightarrow x = 1$$

$$\text{when } \log x = \frac{1}{3} \Rightarrow (10)^{\frac{1}{3}} = x \Rightarrow x = \sqrt[3]{10}$$

$$\text{Hence, } x = \frac{1}{100}, 1, \sqrt[3]{10}$$

Question 5.

Given $\log_{10} a = b$, express 10^{2b-3} in terms of a .

Solution:

$$\text{Given } \log_{10} a = b \Rightarrow (10)^b = a$$

$$\text{Now } 10^{2b-3} = \frac{(10)^{2b}}{(10)^3} = \frac{(10^b)^2}{10 \times 10 \times 10} = \frac{(10^b)^2}{1000}$$

$$= \frac{a^2}{1000}$$

Question 6.

Given $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$,

(i) write down 10^{2a-3} in terms of x .

(ii) write down 10^{3b-1} in terms of y .

(iii) if $\log_{10} P = 2a + \frac{b}{2} - 3c$, express P in terms of x , y and z .

Solution:

$$\text{Given that } \log_{10} x = a \Rightarrow (10)^a = x \quad \dots(1)$$

$$\log_{10} y = b \Rightarrow (10)^b = y \quad \dots(2)$$

$$\log_{10} z = c \Rightarrow (10)^c = z \quad \dots(3)$$

$$(i) 10^{2a-3} = \frac{(10)^{2a}}{(10)^3} = \frac{(10^a)^2}{10 \times 10 \times 10} = \frac{(x)^2}{1000} = \frac{x^2}{1000}$$

$$(ii) 10^{3b-1} = \frac{(10)^{3b}}{(10)^1} = \frac{(10^b)^3}{10} = \frac{(y)^3}{10} = \frac{y^3}{10}$$

$$(iii) \log_{10} P = 2a + \frac{b}{2} - 3c$$

Substituting the value of a , b and c from equation (1), (2) and (3) we get

$$\log_{10} P = 2 \log_{10} x + \frac{1}{2} \log_{10} y - 3 \log_{10} z$$

$$\Rightarrow \log_{10} P = \log_{10} (x)^2 + \log_{10} (y)^{\frac{1}{2}} - \log_{10} (z)^3$$

$$\Rightarrow \log_{10} P = \log_{10} (x^2 \times y^{\frac{1}{2}}) - \log_{10} z^3$$

$$\Rightarrow \log_{10} P = \log_{10} \left(\frac{x^2 \sqrt{y}}{z^3} \right) \Rightarrow P = \frac{x^2 \sqrt{y}}{z^3}$$

Question 7.

If $\log_{10} x = a$ and $\log_{10} y = b$, find the value of xy .

Solution:

$$\text{Given that } \log_{10} x = a \text{ and } \log_{10} y = b$$

$$\Rightarrow (10)^a = x \text{ and } (10)^b = y$$

$$\text{Then } xy = (10)^a \times (10)^b = (10)^{a+b}$$

Question 8.

Given $\log_{10} a = m$ and $\log_{10} b = n$, express $\frac{a^3}{b^2}$ in terms of m and n .

Solution:

$$\text{Given } \log_{10} a = m \text{ and } \log_{10} b = n$$

$$\text{Then } (10)^m = a \text{ and } (10)^n = b$$

$$\frac{a^3}{b^2} = \frac{(10^m)^3}{(10^n)^2} = \frac{(10)^{3m}}{(10)^{2n}} = (10)^{3m-2n}$$

Question 9.

Given $\log_{10} a = 2a$ and $\log_{10} y = \frac{b}{2}$

(i) write 10^a in terms of x .

(ii) write 10^{2b+1} in terms of y .

(iii) if $\log_{10} P = 3a - 2b$, express P in terms of x and y .

Solution:

$$\text{Given that } \log_{10} x = 2a \Rightarrow (10)^{2a} = x$$

$$\text{and } \log_{10} y = \frac{b}{2}, \Rightarrow (10)^{\frac{b}{2}} = y$$

$$(i) 10^a = (10^{2a})^{\frac{1}{2}} = (x)^{\frac{1}{2}} = \sqrt{x}$$

$$(ii) 10^{2b+1} = 10^{2b} \times 10^1 = 10^{4\left(\frac{b}{2}\right)} \times 10^1$$

$$= \left(10^{\frac{b}{2}}\right)^4 \times 10 = y^4 \times 10 = 10y^4$$

$$(iii) \log_{10} P = 3a - 2b$$

$$\Rightarrow \log_{10} P = \frac{3}{2} (2a) - 4 \left(\frac{b}{2}\right)$$

$$\Rightarrow \log_{10} P = \frac{3}{2} (\log_{10} x) - 4 = (\log_{10} y)$$

$$\Rightarrow \log_{10} P = \log_{10} (x)^{\frac{3}{2}} - \log_{10} y^4$$

$$\Rightarrow \log_{10} P = \log_{10} \left(\frac{(x)^{\frac{3}{2}}}{y^4} \right) \Rightarrow P = \frac{(x)^{\frac{3}{2}}}{y^4}$$

Question 10.

If $\log_2 y = x$ and $\log_3 z = x$, find 72^x in terms of y and z .

Solution:

$$\log_2 y = x, \log_3 z = x$$

$$y = 2^x \text{ and } z = 3^x \quad \dots(i)$$

$$72^x = (2 \times 2 \times 2 \times 3 \times 3)^x = (2^3 \times 3^2)^x$$

$$= 2^{3x} \times 3^{2x} = (2^x)^3 \times (3^x)^2 = y^3 \cdot z^2 \quad [\text{From}(i)]$$

$$\text{Hence } 72^x = y^3 \cdot z^2$$

Question 11.

If $\log_2 x = a$ and $\log_5 y = a$, write 100^{2a-1} in terms of x and y .

Solution:

$$\log_2 x = a \text{ and } \log_5 y = a$$

$$\therefore x = 2^a \text{ and } y = 5^a$$

$$100^{2a-1} = (2 \times 2 \times 5 \times 5)^{2a-1} \\ = (2^2 \times 5^2)^{2a-1} = 2^{4a-2} \times 5^{4a-2}$$

$$= \frac{2^{4a}}{2^2} \times \frac{5^{4a}}{5^2} = \frac{(2^a)^4 \times (5^a)^4}{4 \times 25} = \frac{x^4 \times y^4}{100}$$

$$= \frac{x^4 y^4}{100}$$

Exercise 9.2

Question 1.

Simplify the following :

$$(i) \log a^3 - \log a^2 \quad (ii) \log a^3 \div \log a^2$$

$$(iii) \frac{\log 4}{\log 2}$$

$$(iv) \frac{\log 8 \log 9}{\log 27}$$

$$(v) \frac{\log 27}{\log \sqrt{3}}$$

$$(vi) \frac{\log 9 - \log 3}{\log 27}$$

Solution:

$$\begin{aligned} \text{(i)} \quad \log a^3 - \log a^2 &= \log \left(\frac{a^3}{a^2} \right) \text{ (Quotient Law)} \\ &= \log a \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \log a^3 \div \log a^2 &= 3 \log a \div 2 \log a \text{ (Power Law)} \\ &= \frac{3 \log a}{2 \log a} = \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \frac{\log 4}{\log 2} &= \frac{\log(2 \times 2)}{\log 2} = \frac{\log 2}{\log 2} \\ &= \frac{2 \log 2}{\log 2} \text{ (Power Law)} = 2(1) = 2 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \frac{\log 8 \log 9}{\log 27} &= \frac{\log 2^3 \cdot \log 3^2}{\log 3^3} \\ &= \frac{(3 \log 2) \cdot (2 \log 3)}{(3 \log 3)} \text{ (Power Law)} \\ &= \frac{(\log 2) \cdot (2)}{1} = 2 \log 2 = \log 2^2 = \log 4. \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad \frac{\log 27}{\log \sqrt{3}} &= \frac{\log(3 \times 3 \times 3)}{\log(3)^{\frac{1}{2}}} \\ &= \frac{\log(3)^3}{\log(3)^{\frac{1}{2}}} = \frac{3 \log 3}{\frac{1}{2} \log 3} \text{ (Power Law)} \end{aligned}$$

$$= \frac{3 \times 2}{1} \left(\frac{\log 3}{\log 3} \right) = 6(1) = 6$$

$$\begin{aligned} \text{(vi)} \quad \frac{\log 9 - \log 3}{\log 27} &= \frac{\log(3 \times 3) - \log 3}{\log(3 \times 3 \times 3)} \\ &= \frac{\log 3^2 - \log 3}{\log 3^3} = \frac{2 \log 3 - \log 3}{3 \log 3} \text{ (Power Law)} \end{aligned}$$

$$= \frac{\log 3}{3 \log 3} = \frac{1}{3}$$

Question 2.

Evaluate the following:

(i) $\log \left(10 \div \sqrt[3]{10} \right)$ (ii) $2 + \frac{1}{2} \log(10^{-3})$

(iii) $2 \log 5 + \log 8 - \frac{1}{2} \log 4.$

(iv) $2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4$

(v) $2 \log 2 + \log 5 - \frac{1}{2} \log 36 - \log \frac{1}{30}$

(vi) $2 \log 5 + \log 3 + 3 \log 2 - \frac{1}{2} \log 36 - 2 \log 10$

(vii) $\log 2 + 16 \log \frac{6}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}.$

(viii) $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4.$

Solution:

$$\begin{aligned} \text{(i) } \log \left(10 \div \sqrt[3]{10} \right) &= \log \left(10 \div (10)^{\frac{1}{3}} \right) \\ &= \log \left((10)^1 \div (10)^{\frac{1}{3}} \right) = \log \left(10^{1-\frac{1}{3}} \right) = \log \left(10^{\frac{2}{3}} \right) \\ &= \frac{2}{3} \log 10 = \frac{2}{3} (1) = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{(ii) } 2 + \frac{1}{2} \log (10^{-3}) &= 2 + \frac{1}{2} \times (-3) \log 10 \\ &= 2 - \frac{3}{2} \log 10 = 2 - \frac{3}{2} (1) = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{(iii) } 2 \log 5 + \log 8 - \frac{1}{2} \log 4 \\ &= \log (5)^2 + \log 8 - \frac{1}{2} \log (2)^2 \\ &= \log 25 + \log 8 - \frac{1}{2} \times 2 \log 2 \\ &= \log 25 + \log 8 - \log 2 = \log \left(\frac{25 \times 8}{2} \right) \\ &= \log \left(\frac{25 \times 4}{1} \right) = \log (100) = \log (10)^2 \\ &= 2 \log 10 = 2 (1) = 2 \end{aligned}$$

$$\begin{aligned} \text{(iv) } 2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4 \\ &= 2 \times 3 \log 10 + 3 (-2) \log 10 - \frac{1}{3} (-3) \log 5 + \frac{1}{2} \\ &\quad \log (2)^2 \end{aligned}$$

$$\begin{aligned}
&= 6 \log 10 - 6 \log 10 + \frac{3}{3} \log 5 + \frac{1}{2} \times 2 \log 2 \\
&= 0 + 1 \log 5 + \log 2 = \log 5 + \log 2 = \log (5 \times 2) \\
&= \log (10) = 1
\end{aligned}$$

$$\begin{aligned}
\text{(v)} \quad &2 \log 2 + \log 5 - \frac{1}{2} \log 36 - \log \frac{1}{30} \\
&= \log (2)^2 + \log 5 - \frac{1}{2} \log (6)^2 - \log \left(\frac{1}{30} \right) \\
&= \log 4 + \log 5 - \frac{1}{2} \times 2 \log 6 - \log \frac{1}{30} \\
&= \log 4 + \log 5 - \log 6 - (\log 1 - \log 30) \\
&= \log 4 + \log 5 - \log 6 - \log 1 + \log 30 \\
&= (\log 4 + \log 5 + \log 30) - (\log 6 + \log 1) \\
&= \log (4 \times 5 \times 30) - \log (6 \times 1) \\
&= \log \frac{4 \times 5 \times 30}{6 \times 1} = \log \frac{4 \times 5 \times 5}{1 \times 1} = \log 100 \\
&= \log (10)^2 = 2 \log 10 = 2(1) = 2
\end{aligned}$$

$$\begin{aligned}
\text{(vi)} \quad &2 \log 5 + \log 3 + 3 \log 2 - \frac{1}{2} \log 36 - 2 \log 10 \\
&= \log (5)^2 + \log 3 + \log (2)^3 - \frac{1}{2} \log (6)^2 - 2 \log 10 \\
&= \log 25 + \log 3 + \log 8 - \frac{1}{2} \times 2 \log 6 - 2 \log 10 \\
&= \log 25 + \log 3 + \log 8 - \log 6 - \log (10)^2 \\
&= \log (25 \times 3 \times 8) - \log 6 - \log 100 \\
&= \log \left(\frac{25 \times 3 \times 8}{6 \times 100} \right) = \log \left(\frac{1 \times 3 \times 8}{6 \times 4} \right) \\
&= \log \left(\frac{24}{24} \right) = \log 1 = 0.
\end{aligned}$$

$$\begin{aligned}
& \text{(vii) } \log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} \\
&= \log 2 + 16 (\log 16 - \log 15) + 12 (\log 25 - \log 24) \\
&\quad + 7 (\log 81 - \log 80) \\
&= \log 2 + 16 [\log (2)^4 - \log (3 \times 5)] + 12 [\log (5)^2 \\
&\quad - \log (3 \times 2 \times 2 \times 2)] + 7 [\log (3 \times 3 \times 3 \times 3) - \log \\
&\quad (2)^4 \times 5] \\
&= \log 2 + 16 [4 \log 2 - (\log 3 + \log 5)] + 12 [2 \log 5 \\
&\quad - \log (3 \times 2^3)] + 7 [\log (3)^4 - (\log 4 + \log 5)] \\
&= \log 2 + 16 [4 \log 2 - \log 3 - \log 5] + 12 [2 \log 5 - \\
&\quad (\log 3 + \log 2^3)] + 7 [4 \log 3 - \log 4 - \log 5] \\
&= \log 2 + 64 \log 2 - 16 \log 3 - 16 \log 5 + 24 \log 5 - \\
&\quad 12 \log 3 - 12 \log 2^3 + 28 \log 3 - 7 \log 2^4 - 7 \log 5 \\
&= \log 2 + 64 \log 2 - 16 \log 3 - 16 \log 5 + 24 \log 5 - 12 \\
&\quad \log 3 - 36 \log 2 + 28 \log 3 - 28 \log 2 - 7 \log 5 \\
&= (\log 2 + 64 \log 2 - 36 \log 2 - 28 \log 2) + (-16 \log 3 \\
&\quad - 12 \log 3 + 28 \log 3) + (-16 \log 5 + 24 \log 5 - 7 \\
&\quad \log 5) + 28 \log 3 + (-16 \log 5 + 24 \log 5 - 7 \log 5) \\
&= (65 \log 2 - 64 \log 2) + (-28 \log 3 + 28 \log 3) + \\
&\quad (-23 \log 5 + 24 \log 5) \\
&= \log 2 + 0 + \log 5 = \log 2 + \log 5 \\
&= \log (2 \times 5) = \log 10 = 1
\end{aligned}$$

$$\begin{aligned}
& \text{(viii) } 2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 \\
&= \log_{10} (5)^2 + \log_{10} 8 - \log_{10} (4)^{\frac{1}{2}} \\
&= \log_{10} 25 + \log_{10} 8 - \log_{10} (2)^{2 \times \frac{1}{2}} \\
&= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 \\
&= \log_{10} \left(\frac{25 \times 8}{2} \right) = \log_{10} (25 \times 4) \\
&= \log_{10} 100 = \log_{10} (10)^2 = 2 \log_{10} 10 = 2 (1) = 2.
\end{aligned}$$

Question 3.

Express each of the following as a single logarithm:

(i) $2 \log 3 - \frac{1}{2} \log 16 + \log 12$

(ii) $2 \log_{10} 5 - \log_{10} 2 + 3 \log_{10} 4 + 1.$

(iii) $\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5$

(iv) $\frac{1}{2} \log 25 - 2 \log 3 + 1$

(v) $\frac{1}{2} \log 9 + 2 \log 3 - \log 6 + \log 2 - 2.$

Solution:

(i) $2 \log 3 - \frac{1}{2} \log 16 + \log 12$

$= 2 \log 3 - \frac{1}{2} \log (4)^2 + \log 12$

$$\begin{aligned}
&= 2 \log 3 - \frac{1}{2} \times 2 \log 4 + \log 12 \\
&= 2 \log 3 - \log 4 + \log 12 = \log (3)^2 - \log 4 + \log 12 \\
&= \log 9 - \log 4 + \log 12 = \log \frac{9 \times 12}{4} = \log \left(\frac{9 \times 3}{1} \right) \\
&= \log 27.
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad &2 \log_{10} 5 - \log_{10} 2 + 3 \log_{10} 4 + 1 \\
&= \log_{10} (5)^2 - \log_{10} 2 + \log_{10} (4)^3 + \log_{10} 10 \\
&\qquad\qquad\qquad (\because \log_{10} 10 = 1) \\
&= \log_{10} 25 - \log_{10} 2 + \log_{10} 64 + \log_{10} 10 \\
&= \log_{10} (25 \times 64 \times 10) - \log_{10} 2 = \log_{10} (16000) - \log_{10} 2 \\
&= \log_{10} \left(\frac{16000}{2} \right) = \log_{10} 8000
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad &\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5 \\
&= \log (36)^{\frac{1}{2}} + \log (8)^2 - \log 1.5 \\
&= \log (6)^{2 \times \frac{1}{2}} + \log 64 - \log \left(\frac{15}{10} \right) \\
&= \log 6 + \log 64 - (\log 15 - \log 10) \\
&= \log 6 \times 64 - \log 15 + \log 10 \\
&= \log (6 \times 64 \times 10) - \log 15 \\
&= \log \left(\frac{60 \times 64}{15} \right) = \log (4 \times 64) = \log 256
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad &\frac{1}{2} \log 25 - 2 \log 3 + 1 \\
&= \log (25)^{\frac{1}{2}} - \log (3)^2 + \log 10 \quad (\because \log 10 = 1) \\
&= \log (5)^{\frac{1}{2} \times 2} - \log 9 + \log 10 \\
&= \log 5 - \log 9 + \log 10 = \log (5 \times 10) - \log 9 \\
&= \log \frac{5 \times 10}{9} = \log \frac{50}{9}
\end{aligned}$$

$$(v) \frac{1}{2} \log 9 + 2 \log 3 - \log 6 + \log 2 - 2.$$

$$\frac{1}{2} \log 9 + 2 \log 3 - \log 6 + \log 2 - 2$$

$$= \log 9^{\frac{1}{2}} + \log 3^2 - \log 6 + \log 2 - \log 100$$

$$= \log 3 + \log 9 - \log 6 + \log 2 - \log 100$$

$$= \log \frac{3 \times 9 \times 2}{6 \times 100}$$

$$= \log \frac{9}{100}$$

Question 4.

Prove the following :

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Solution:

$$(i) \text{ L.H.S} = \log_{10} 4 \div \log_{10} 2$$

$$= \log_{10} (2)^2 \div \log_{10} 2 = 2 \log_{10} 2 \div \log_{10} 2$$

$$= \frac{2 \log_{10} 2}{\log_{10} 2} = 2(1) = 2$$

$$\text{R.H.S.} = \log_3 9 = \log_3 (3)^2 = 2 \log_3 3 = 2(1) = 2$$

Hence, Proved. L.H.S. = R.H.S.

$$(ii) \text{ L.H.S.} = \log_{10} 25 + \log_{10} 4 = \log_{10} 25 \times 4$$

$$= \log_{10} 100 = \log_{10} 10^2$$

$$= 2 \log_{10} 10 = 2 \times 1$$

$$= 2 \quad (\because \log_a a = 1)$$

$$\text{R.H.S.} = \log_5 25 = \log_5 (5)^2$$

$$= 2 \log_5 5 = 2 \times 1 = 2$$

$$(\because \log_a a = 1)$$

Hence L.H.S. = R.H.S.

Question 5.

If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, express

$$\log \frac{10\sqrt{y}}{x^2z^3} \text{ in terms of } a, b, c.$$

Solution:

$$\begin{aligned} \text{Given that } x &= (100)^a = [(10)^2]^a = (10)^{2a} \\ y &= (10000)^b = [(10)^4]^b = (10)^{4b} \\ z &= (10)^c = (10)^c \end{aligned}$$

$$\text{Now, } \log \frac{10\sqrt{y}}{x^2z^3}$$

$$\begin{aligned} &= (\log 10 + \log \sqrt{y}) - (\log x^2 + \log z^3) \\ &= \left(1 + \log(y)^{\frac{1}{2}}\right) - (\log(x)^2 + \log(z)^3) \quad [\because \log 10 = 1] \end{aligned}$$

$$= \left(1 + \frac{1}{2} \log y\right) - (2 \log x + 3 \log z)$$

Substituting the value of x , y and z , we get

$$\begin{aligned} &= \left(1 + \frac{1}{2} \log(10)^{4b}\right) - (2 \log(10)^{2a} + 3 \log(10)^c) \\ &= \left(1 + \frac{1}{2} \times 4b \log 10\right) - (2 \times 2a \log 10 + 3 \times c \log 10) \\ &= \left(1 + \frac{1}{2} \times 4b \times 1\right) - (2 \times 2a \times 1 + 3 \times c \times 1) \\ & \qquad \qquad \qquad [\because \log 10 = 1] \\ &= (1 + 2b) - (4a + 3c) = 1 + 2b - 4a - 3c \\ &= 1 - 4a + 2b - 3c \end{aligned}$$

Question 6.

If $a = \log_{10}x$, find the following in terms of a :

(i) x

(ii) $\log_{10}\sqrt[5]{x^2}$

(iii) $\log_{10}5x$

Solution:

(i) Given that,

$$a = \log_{10} x \Rightarrow (10)^a = x \Rightarrow x = (10)^a$$

$$\begin{aligned} \text{(ii) } \log_{10} \sqrt[5]{x^2} &= \log_{10} (x^2)^{\frac{1}{5}} = \log_{10} (x)^{\frac{2}{5}} \\ &= \frac{2}{5} \log_{10} x = \frac{2}{5} (a) = \frac{2}{5} a. \end{aligned}$$

$$\begin{aligned} \text{(iii) } x &= (10)^a = \log_{10} 5x = \log_{10} 5 (10)^a \\ &= \log_{10} 5 + \log_{10} 10 = \log_{10} 5 + a \quad (1) \\ &= a + \log_{10} 5 \end{aligned}$$

Question 7.

$$\text{If } a = \log \frac{2}{3}, b = \log \frac{3}{5} \text{ and } c = 2 \log \sqrt{\frac{5}{2}},$$

find the value of

(i) $a + b + c$

(ii) 5^{a+b+c} .

Solution:

Given that

$$a = \log \frac{2}{3}, b = \log \frac{3}{5}, c = 2 \log \sqrt{\frac{5}{2}}$$

$$\begin{aligned} \text{(i) } a + b + c &= \log \frac{2}{3} + \log \frac{3}{5} + 2 \log \sqrt{\frac{5}{2}} \\ &= (\log 2 - \log 3) + (\log 3 - \log 5) + 2 \log \left(\frac{5}{2}\right)^{\frac{1}{2}} \\ &= \log 2 - \log 3 + \log 3 - \log 5 + 2 \times \frac{1}{2} \log \left(\frac{5}{2}\right) \\ &= \log 2 - \log 3 + \log 3 - \log 5 + \log \frac{5}{2} \\ &= \log 2 + (-\log 3 + \log 3) - \log 5 + \log 5 - \log 2 \\ &= (\log 2 - \log 2) + 0 + (\log 5 - \log 5) \\ &= 0 + 0 + 0 = 0 \\ \text{(ii) } 5^{a+b+c} &= 5^0 = 1 \end{aligned}$$

Question 8.

If $x = \log \frac{3}{5}$, $y = \log \frac{5}{4}$ and $z = 2 \log \frac{\sqrt{3}}{2}$,

find the values of

(i) $x + y - z$, (ii) 3^{x+y-z}

Solution:

$$x = \log \frac{3}{5}, y = \log \frac{5}{4}, z = 2 \log \frac{\sqrt{3}}{2}$$

$$\therefore x = \log 3 - \log 5,$$
$$y = \log 5 - \log 4$$

$$z = \log \left(\frac{\sqrt{3}}{2} \right)^2 = \log \frac{3}{4} = \log 3 - \log 4$$

$$(i) \text{ Now, } x + y - z = \log 3 - \log 5 + \log 5 - \log 4$$
$$- \log 3 + \log 4$$
$$= 0$$

$$(ii) 3^{x+y-z} = 3^0 = 1$$

Question 9.

If $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find the values of

(i) $x - y - z$

(ii) 7^{x-y-z}

Solution:

$$\begin{aligned}
x &= \log_{10} 12, y = \log_4 2 \times \log_{10} 9, \\
z &= \log_{10} 0.4 \\
\text{(i) } x - y - z &= \log_{10} 12 - \log_4 2 \times \log_{10} 9 - \log_{10} 0.4 \\
&= \log_{10} (3 \times 4) - \log_4 4^{\frac{1}{2}} \times \log_{10} 3^2 - \log_{10} \frac{4}{10} \\
&= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \log_4 4 \times 2 \log_{10} 3 - (\log_{10} 4 - \log_{10} 10) \\
&= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \times 1 \times 2 \log_{10} 3 - \log_{10} 4 + 1 \\
&= \log_{10} 3 + \log_{10} 4 - \log_{10} 3 - \log_{10} 4 + 1 \\
&= 1 \\
\text{(ii) } 7^{x-y-z} &= 7^1 = 7
\end{aligned}$$

Question 10.

If $\log V + \log 3 = \log \pi + \log 4 + 3 \log r$, find V in terms of other quantities.

Solution:

$$\begin{aligned}
\Rightarrow \log V + \log 3 &= \log \pi + \log 4 + \log (r)^3 \\
\Rightarrow \log (V \times 3) &= \log (\pi \times 4 \times r^3) \\
\Rightarrow \log (3V) = \log 4 \pi r^3 &\Rightarrow 3V = 4 \pi r^3 \\
\Rightarrow V &= \frac{4}{3} \pi r^3
\end{aligned}$$

Question 11.

Given $3 (\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log n$, find n .

Solution:

$$\text{Given that } 3(\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log n,$$

$$\Rightarrow 3 \log 5 - 3 \log 3 - \log 5 + 2 \log 6 = 2 - \log n$$

$$\Rightarrow 2 \log 5 - 3 \log 3 + 2 \log 6 = 2 - \log n$$

$$\Rightarrow \log (5)^2 - \log (3)^3 + \log (6)^2 = 2(1) - \log n$$

$$\Rightarrow \log 25 - \log 27 + \log 36 = 2 \log 10 - \log n$$

[$\because \log 10 = 1$]

$$\Rightarrow \log n = 2 \log 10 - \log 25 + \log 27 - \log 36$$

$$\Rightarrow \log n = \log (10)^2 - \log 25 + \log 27 - \log 36$$

$$\Rightarrow \log n = \log 100 - \log 25 + \log 27 - \log 36$$

$$\Rightarrow \log n = (\log 100 + \log 27) - (\log 25 + \log 36)$$

$$\Rightarrow \log n = \log (100 \times 27) - \log (25 \times 36)$$

$$\Rightarrow \log n = \log \left(\frac{100 \times 27}{25 \times 36} \right)$$

$$\Rightarrow \log n = \log \left(\frac{4 \times 27}{1 \times 36} \right)$$

$$\Rightarrow \log n = \log \left(\frac{1 \times 27}{1 \times 9} \right) \Rightarrow \log n = \log 3$$

$$\Rightarrow n = 3.$$

Question 12.

Given that $\log_{10} y + 2 \log_{10} x = 2$, express y in terms of x .

Solution:

$$\log_{10} y + 2 \log_{10} x = 2$$

$$\Rightarrow \log_{10} y + \log_{10} x^2 = 2 \Rightarrow \log_{10} (yx^2) = 2$$

$$\Rightarrow \log_{10} (yx^2) = 2 \log_{10} 10$$

$$\Rightarrow \log_{10} (yx^2) = \log_{10} (10)^2 \Rightarrow yx^2 = (10)^2$$

$$\Rightarrow yx^2 = 100 \Rightarrow y = \frac{100}{x^2}$$

Question 13.

Express $\log_{10} 2 + 1$ in the form $\log_{10} X$.

Solution:

$$\begin{aligned}\log_{10} 2 + 1 &= \log_{10} 2 + \log_{10} 10 \quad (\because \log_{10} 10 = 1) \\ &= \log_{10} 2 \times 10 = \log_{10} 20\end{aligned}$$

Question 14.

$$\text{If } a^2 = \log_{10} x, b^3 = \log_{10} y \text{ and } \frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$$

express z in terms of x and y .

Solution:

Given that

$$a^2 = \log_{10} x, b^3 = \log_{10} y$$

$$\text{we have, } \frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$$

$$\Rightarrow \frac{1}{2} (\log_{10} x) - \frac{1}{3} (\log_{10} y) = \log_{10} z$$

$$\Rightarrow \log_{10} (x)^{\frac{1}{2}} - \log_{10} (y)^{\frac{1}{3}} = \log_{10} z$$

$$\Rightarrow \log_{10} \sqrt{x} - \log_{10} \sqrt[3]{y} = \log_{10} z$$

$$\Rightarrow \log_{10} \frac{\sqrt{x}}{\sqrt[3]{y}} = \log_{10} z \Rightarrow \frac{\sqrt{x}}{\sqrt[3]{y}} = z$$

$$\text{Hence, } z = \frac{\sqrt{x}}{\sqrt[3]{y}}$$

Question 15.

Given that $\log m = x + y$ and $\log n = x - y$, express the value of $\log m^2 n$ in terms of x and y .

Solution:

Given that $\log m = x + y$ and $\log n = x - y$

$$\log m^2 n = \log m^2 + \log n = 2 \log m + \log n$$

$$= 2(x + y) + x - y = 2x + 2y + x - y = 3x + y$$

Question 16.

Given that $\log x = m+n$ and $\log y = m - n$,
express the value of $\log \left(\frac{10x}{y^2} \right)$ in terms of m
and n .

Solution:

Given that $\log x = m + n$ and $\log y = m - n$

$$\begin{aligned}\text{Then } \log \left(\frac{10x}{y^2} \right) &= \log 10x - \log y^2 \\ &= \log 10 + \log x - 2 \log y = 1 + \log x - 2 \log y \\ &= 1 + (m+n) - 2(m-n) = 1 + m + n - 2m + 2n \\ &= 1 - m + 3n\end{aligned}$$

Question 17.

If $\frac{\log x}{2} = \frac{\log y}{3}$, find the value of $\frac{y^4}{x^6}$.

Solution:

$$\begin{aligned}\frac{\log x}{2} &= \frac{\log y}{3} \Rightarrow 3 \log x = 2 \log y \\ \Rightarrow \log x^3 &= \log y^2 \Rightarrow x^3 = y^2 \\ \text{Squaring both sides, we get} \\ x^6 &= y^4 \Rightarrow y^4 = x^6 \Rightarrow \frac{y^4}{x^6} = 1\end{aligned}$$

Hence, value of $\frac{y^4}{x^6} = 1$

Question 18.

Solve for x :

(i) $\log x + \log 5 = 2 \log 3$ (ii) $\log_3 x - \log_3 2 = 1$

(iii) $x = \frac{\log 125}{\log 25}$ (iv) $\frac{\log 8}{\log 2} \times \frac{\log 3}{\log \sqrt{3}} = 2 \log x$

Solution:

$$(i) \log x + \log 5 = 2 \log 3$$

$$\Rightarrow \log x = 2 \log 3 - \log 5 \Rightarrow \log x = \log (3)^2 - \log 5$$

$$\Rightarrow \log x = \log 9 - \log 5 \Rightarrow \log x = \log \left(\frac{9}{5} \right)$$

$$\therefore x = \frac{9}{5}$$

$$(ii) \log_3 x - \log_3 2 = 1 \Rightarrow \log_3 x = \log_3 2 + 1$$

$$\Rightarrow \log_3 x = \log_3 2 + \log_3 3 \quad (\because \log_3 3 = 1)$$

$$\Rightarrow \log_3 x = \log_3 (2 \times 3) \Rightarrow \log_3 x = \log_3 6$$

$$\therefore x = 6$$

$$(iii) x = \frac{\log 125}{\log 25} \Rightarrow x = \frac{\log(5)^3}{\log(5)^2}$$

$$\Rightarrow x = \frac{3 \log 5}{2 \log 5} = \frac{3}{2} \therefore x = \frac{3}{2}$$

$$(iv) \frac{\log 8}{\log 2} \times \frac{\log 3}{\log \sqrt{3}} = 2 \log x$$

$$\Rightarrow \frac{\log(2)^3}{\log 2} \times \frac{\log 3}{\log(3)^{\frac{1}{2}}} = 2 \log x$$

$$\Rightarrow \frac{3 \log 2}{\log 2} \times \frac{\log 3}{\frac{1}{2} \log 3} = 2 \log x$$

$$\Rightarrow 3(1) \times \frac{1}{\left(\frac{1}{2}\right)} = 2 \log x \Rightarrow 3 \times \frac{2}{1} = 2 \log x$$

$$\Rightarrow 2 \log x = 6 \Rightarrow \log x = \frac{6}{2} \Rightarrow \log x = 3$$

$$\Rightarrow x = (10)^3 \Rightarrow x = 1000$$

Question 19.

Given $2 \log_{10} x + 1 = \log_{10} 250$, find

(i) x

(ii) $\log_{10} 2x$

Solution:

$$(i) 2 \log_{10} x + 1 = \log_{10} 250$$

$$\Rightarrow \log_{10} x^2 + 1 = \log_{10} 250 \quad [\log_a m^n = n \log_a m]$$

$$\Rightarrow \log_{10} x^2 \times 10 = \log_{10} 250 \quad [\log_{10} 10 = 1]$$

$$\Rightarrow \log_{10} x^2 \times \log_{10} 10 = \log_{10} 250$$

$$\Rightarrow x^2 \times 10 = 250 \Rightarrow x^2 = \frac{250}{10} \Rightarrow x^2 = 25$$

$$\Rightarrow (x)^2 = (5)^2 \therefore x = 5$$

(ii) $x = 5$ (proved in (i) above)

$$\log_{10} 2x = \log_{10} 2 \times 5 \quad [\text{Putting } x = 5]$$

$$= \log_{10} 10 = 1 \quad [\log_{10} 10 = 1]$$

Question 20.

$$\text{If } \frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{\log 9}{\log \frac{1}{3}}, \text{ find } x \text{ and } y.$$

Solution:

$$\frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{\log 9}{\log \frac{1}{3}}$$

Taking first and third terms,

$$\frac{\log x}{\log 5} = \frac{\log 9}{\log \frac{1}{3}} \Rightarrow \log x = \frac{\log 9}{\log \frac{1}{3}} \times \log 5$$

$$\Rightarrow \log x = \frac{\log(3 \times 3)}{\log 1 - \log 3} \times \log 5$$

$$\Rightarrow \log x = \frac{\log(3)^2}{0 - \log 3} \times \log 5 \quad [\log 1 = 0]$$

$$\Rightarrow \log x = \frac{2 \log 3}{-\log 3} \times \log 5$$

$$\Rightarrow \log x = \frac{2 \log 3}{\log 3} \times \log 5$$

$$\Rightarrow \log x = -2 (1) \times \log 5 \Rightarrow \log x = -2 \log 5$$

$$\Rightarrow \log x = \log (5)^{-2} \Rightarrow x = (5)^{-2}$$

$$\Rightarrow x = \frac{1}{(5)^2} \Rightarrow x = \frac{1}{25}$$

taking second and third terms,

$$\frac{\log y^2}{\log 2} = \frac{\log 9}{\log\left(\frac{1}{3}\right)} \Rightarrow \log y^2 = \frac{\log 9}{\log\left(\frac{1}{3}\right)} \times \log 2$$

$$\Rightarrow \log y^2 = \frac{\log(3)^2}{\log 1 - \log 3} \times \log 2$$

$$\Rightarrow \log y^2 = \frac{2 \log 3}{0 - \log 3} \times \log 2 \quad [\log 1 = 0]$$

$$\Rightarrow \log y^2 = \frac{2 \log 3}{-\log 3} \times \log 2$$

$$\Rightarrow \log y^2 = \frac{-2 \log 3}{\log 3} \times \log 2$$

$$\Rightarrow \log y^2 = -2 \times \log 2 \Rightarrow \log y^2 = \log(2)^{-2}$$

$$\Rightarrow y^2 = (2)^{-2} \Rightarrow y = (2)^{-2 \times \frac{1}{2}} \Rightarrow y = (2)^{-1}$$

$$\Rightarrow y = \frac{1}{2}$$

Question 21.

Prove the following :

(i) $3_{\log 4} = 4_{\log 3}$

(ii) $27_{\log 2} = 8_{\log 3}$

Solution:

(i) $3^{\log 4} = 4^{\log 3}$ is true
if $\log 3^{\log 4} = \log 4^{\log 3}$

(Taking log both sides)

if $\log 4 \cdot \log 3 = \log 3 \cdot \log 4$

if $\log_2 2 \cdot \log 3 = \log 3 \cdot \log 2^2$

if $2 \log 2 \times \log 3 = \log 3 \times 2 \log 2$

if $2 \log 2 \log 3 = 2 \log 2 \log 3$

which is true

Hence proved

(ii) $27^{\log 2} = 8^{\log 3}$ is true

if $\log 27^{\log 2} = \log 8^{\log 3}$

(Taking log both sides)

if $\log 2 \log 27 = \log 3 \log 8$

if $\log 2 \log 3^3 = \log 3 \log 2^3$

if $\log 2 \cdot 3 \log 3 = \log 3 \cdot 3 \log 2$

if $3 \log 2 \cdot \log 3 = 3 \cdot \log 2 \log 3$

which is true

Hence proved

Question 22.

Solve the following equations :

(i) $\log (2x + 3) = \log 7$

(ii) $\log (x + 1) + \log (x - 1) = \log 24$

(iii) $\log (10x + 5) - \log (x - 4) = 2$

(iv) $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

(v) $\log (4y - 3) = \log (2y + 1) - \log 3$

(vi) $\log_{10} (x + 2) + \log_{10} (x - 2) = \log_{10} 3 + 31 \log_{10} 4$.

(vii) $\log (3x + 2) + \log (3x - 2) = 5 \log 2$.

Solution:

$$(i) \log(2x + 3) = \log 7$$

$$\Rightarrow 2x + 3 = 7 \Rightarrow 2x = 7 - 3 \Rightarrow 2x = 4$$

$$\Rightarrow x = \frac{4}{2} \quad \therefore x = 2$$

$$(ii) \log(x + 1) + \log(x - 1) = \log 24$$

$$\Rightarrow \log(x + 1)(x - 1) = \log 24$$

$$\Rightarrow \log(x^2 - 1) = \log 24 \Rightarrow x^2 - 1 = 24$$

$$\Rightarrow x^2 = 24 + 1 \Rightarrow x^2 = 25 \Rightarrow x^2 = (5)^2$$

$$\therefore x^2 = 5$$

$$(iii) \log(10x + 5) - \log(x - 4) = 2$$

$$\Rightarrow \log \frac{(10x + 5)}{(x - 4)} = 2 (\log 10) \quad [\because \log 10 = 1]$$

$$\Rightarrow \log \frac{10x + 5}{x - 4} = \log (10)^2$$

$$\Rightarrow \log \left(\frac{10x + 5}{x - 4} \right) = \log 100 \Rightarrow \frac{10x + 5}{(x - 4)} = 100$$

$$\Rightarrow 10x + 5 = 100(x - 4)$$

$$\Rightarrow 10x + 5 = 100x - 400 \Rightarrow 10x = 100x - 400 - 5$$

$$\Rightarrow -90x = -405 \Rightarrow x = \frac{-405}{-90}$$

$$\Rightarrow x = \frac{405}{90} = \frac{81}{18} = \frac{9}{2} \quad \therefore x = 4.5$$

$$\begin{aligned}
 \text{(iv)} \quad & \log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1 \\
 \Rightarrow & \log_{10} 5 \times (5x + 1) = \log_{10}(x + 5) + \log_{10} 10 \\
 & \qquad \qquad \qquad [\log_{10} 10 = 1] \\
 \Rightarrow & \log_{10} 5(5x + 1) = \log_{10} [10 \times (x + 5)] \\
 \Rightarrow & 5(5x + 1) = 10(x + 5) \Rightarrow 25x + 5 = 10x + 50 \\
 \Rightarrow & 25x - 10x = 50 - 5 \Rightarrow 15x = 45 \\
 \Rightarrow & x = \frac{45}{15} \quad \therefore x = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & \log(4y - 3) = \log(2y + 1) - \log 3 \\
 \Rightarrow & \log 4y - 3 = \log \frac{(2y + 1)}{3} \\
 \Rightarrow & 4y - 3 = \frac{2y + 1}{3} \Rightarrow 3(4y - 3) = 2y + 1 \\
 \Rightarrow & 12y - 9 = 2y + 1 \Rightarrow 12y - 2y = 1 + 9 \\
 \Rightarrow & 10y = 10 \Rightarrow y = \frac{10}{10} = 1
 \end{aligned}$$

$$\begin{aligned}
 \therefore y &= 1 \\
 \text{(vi)} \quad & \log_{10}(x + 2) + \log_{10}(x - 2) = \log_{10} 3 + 3 \log_{10} 4 \\
 \Rightarrow & \log_{10}(x + 2)(x - 2) = \log_{10} 3 + \log_{10}(4)^3 \\
 \Rightarrow & \log_{10}(x^2 - 2^2) = \log_{10} 3 + \log_{10}(4 \times 4 \times 4) \\
 \Rightarrow & \log_{10}(x^2 - 4) = \log_{10} 3 + \log_{10} 64 \\
 \Rightarrow & \log_{10}(x^2 - 4) = \log_{10} 3 \times 64 \\
 \Rightarrow & x^2 - 4 = 3 \times 64 \Rightarrow x^2 - 4 = 192 \\
 \Rightarrow & x^2 = 192 + 4 \Rightarrow x^2 = 196 \\
 \Rightarrow & x^2 = (14)^2 \\
 \therefore x &= 14
 \end{aligned}$$

Hence proved

$$\begin{aligned}
 \text{(vii)} \quad & \log(3x + 2) + \log(3x - 2) = 5 \log 2 \\
 \Rightarrow & \log(3x + 2)(3x - 2) = \log 2^5 \\
 \Rightarrow & \log(9x^2 - 4) = \log 32 \\
 & \text{Comparing both sides} \\
 & 9x^2 - 4 = 32 \Rightarrow 9x^2 = 32 + 4 = 36
 \end{aligned}$$

$$x^2 = \frac{36}{9} = 4 = (\pm 2)^2$$

$$\begin{aligned}
 \therefore x &= \pm 2 \\
 x &= 2
 \end{aligned}$$

Question 23.

Solve for x :

$$\log_3 (x + 1) - 1 = 3 + \log_3 (x - 1)$$

Solution:

$$\log_3 (x + 1) - 1 = 3 + \log_3 (x - 1)$$

$$\Rightarrow \log_3 (x + 1) - 3 \log_3 (x - 1) = 3 + 1$$

$$\Rightarrow \log_3 \frac{x+1}{x-1} = 4 = 4 \times 1 = 4 \log_3 3$$

$$(\because \log_a a = 1)$$

$$\Rightarrow \log_3 \frac{x+1}{x-1} = \log_3 3^4 = \log_3 81$$

$$\therefore \frac{x+1}{x-1} = \frac{81}{1}$$

$$\Rightarrow 81x - 81 = x + 1$$

$$\Rightarrow 81x - x = 1 + 81 \Rightarrow 80x = 82$$

$$\therefore x = \frac{82}{80} = \frac{41}{40} = 1\frac{1}{40}$$

Question 24.

Solve for x : $5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$.

Solution:

$$5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$$

$$5^{\log x} + 3^{\log x} = 3^{\log x} \cdot 3^1 - 5^{\log x} \cdot 5^{-1}$$

$$5^{\log x} + 3^{\log x} = 3 \cdot 3^{\log x} - \frac{1}{5} \cdot 5^{\log x}$$

$$5^{\log x} + \frac{1}{5} \cdot 5^{\log x} = 3 \cdot 3^{\log x} - 3^{\log x}$$

$$\Rightarrow \left(1 + \frac{1}{5}\right) (5^{\log x}) = (3 - 1) (3^{\log x})$$

$$\Rightarrow \frac{6}{5} (5^{\log x}) = 2 \times 3^{\log x}$$

$$\Rightarrow \frac{5^{\log x}}{3^{\log x}} = \frac{2 \times 5}{6} = \left(\frac{5}{3}\right)^1 \Rightarrow \left(\frac{5}{3}\right)^{\log x} = \left(\frac{5}{3}\right)^1$$

Comparing, we get

$$\log x = 1 = \log 10$$

$$\therefore x = 10$$

Question 25.

If $\log \left(\frac{x-y}{2}\right) = \frac{1}{2} (\log x + \log y)$, prove that

$$x^2 + y^2 = 6xy.$$

Solution:

$$\log \left(\frac{x-y}{2} \right) = \frac{1}{2} (\log x + \log y)$$

$$\Rightarrow \log \left(\frac{x-y}{2} \right) = \frac{1}{2} \log xy$$

$$[\because \log m + \log n = \log mn]$$

$$\Rightarrow \log \left(\frac{x-y}{2} \right) = \log (xy)^{\frac{1}{2}} \Rightarrow \frac{x-y}{2} = (xy)^{\frac{1}{2}}$$

Squaring both sides, we get

$$\Rightarrow \left(\frac{x-y}{2} \right)^2 = \left[(xy)^{\frac{1}{2}} \right]^2 \Rightarrow \frac{(x-y)^2}{4} = (xy)^{\frac{1}{2} \times 2}$$

$$\Rightarrow (x-y)^2 = 4 \times xy \Rightarrow x^2 + y^2 - 2xy = 4xy$$

$$[\because (A-B)^2 = A^2 + B^2 - 2AB]$$

$$\Rightarrow x^2 + y^2 = 4xy + 2xy$$

$$\Rightarrow x^2 + y^2 = 6xy$$

Proved.

Question 26.

If $x^2 + y^2 = 23xy$, Prove that

$$\log \frac{x+y}{5} = \frac{1}{2} (\log x + \log y)$$

Solution:

$$\text{Given } x^2 + y^2 = 23xy \Rightarrow x^2 + y^2 = 25xy - 2xy$$

$$\Rightarrow x^2 + y^2 + 2xy = 25xy$$

$$\Rightarrow (x)^2 + (y)^2 + 2 \times x \times y = 25xy$$

$$\Rightarrow (x + y)^2 = 25xy \Rightarrow \frac{(x + y)^2}{25} = xy$$

taking log on both sides, we get

$$\Rightarrow \log \frac{(x + y)^2}{25} = \log xy$$

$$\Rightarrow \log \left(\frac{x + y}{5} \right)^2 = \log x + \log y$$

$$\Rightarrow 2 \log \frac{x + y}{5} = \log x + \log y \Rightarrow \log \frac{x + y}{5}$$

$$= \frac{1}{2} (\log x + \log y) \text{ Proved.}$$

Question 27.

If $p = \log_{10} 20$ and $q = \log_{10} 25$, find the value of x if $2 \log_{10} (x + 1) = 2p - q$.

Solution:

Given that $p = \log_{10} 20$ and $q = \log_{10} 25$

Then, $2 \log_{10} (x + 1) = 2p - q$

Substituting the value of p and q , we get

$$\Rightarrow 2 \log_{10} (x + 1) = 2 \log_{10} 20 - \log_{10} 25$$

$$\Rightarrow 2 \log_{10} (x + 1) = 2 \log_{10} 20 - \log_{10} (5)^2$$

$$\Rightarrow 2 \log_{10} (x + 1) = 2 \log_{10} 20 - 2 \log_{10} 5$$

$$\Rightarrow 2 \log_{10} (x + 1) = 2 (\log_{10} 20 - \log_{10} 5)$$

$$\Rightarrow \log_{10} (x + 1) = 2 \frac{(\log_{10} 20 - \log_{10} 5)}{2}$$

$$\Rightarrow \log_{10} (x + 1) = \log_{10} 20 - \log_{10} 5$$

$$\Rightarrow \log_{10} (x + 1) = \log_{10} \left(\frac{20}{5} \right)$$

$$\Rightarrow \log_{10} (x + 1) = \log_{10} 4 \Rightarrow x + 1 = 4$$

$$\Rightarrow x = 4 - 1 \quad \therefore x = 3$$

Question 28.

Show that:

$$(i) \frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$$

$$(ii) \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$$

Solution:

$$(i) \frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$$

$$\text{L.H.S.} = \frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42}$$

$$\left\{ \because \log_n m = \frac{\log m}{\log n} \right\}$$

$$= \frac{1}{\frac{\log 42}{\log_2}} + \frac{1}{\frac{\log 42}{\log_3}} + \frac{1}{\frac{\log 42}{\log_7}}$$

$$= \frac{\log_2}{\log 42} + \frac{\log_3}{\log 42} + \frac{\log_7}{\log 42}$$

$$= \frac{\log_2 + \log_3 + \log_7}{\log 42} = \frac{\log_{(2 \times 3 \times 7)}}{\log 42}$$

$$\left\{ \begin{array}{l} \because \log_m + \log_n + \log_p \\ = \log_{mnp} \end{array} \right\}$$

$$= \frac{\log 42}{\log 42} = 1 = \text{R.H.S.}$$

$$(ii) \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$$

$$\text{L.H.S.} = \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36}$$

$$= \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36}$$

$$= \frac{\log_8}{\log 36} + \frac{\log_9}{\log 36} + \frac{\log_{18}}{\log 36}$$

$$= \frac{\log_8 + \log_9 + \log_{18}}{\log 36}$$

$$= \frac{\log(8 \times 9 \times 18)}{\log 36} = \frac{\log(36)^2}{\log 36}$$

$$= \frac{2 \log 36}{\log 36} = 2 = \text{R.H.S.}$$

Question 29.

Prove the following identities:

$$(i) \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$$

$$(ii) \log_b a \cdot \log_c b \cdot \log_a c = \log_a a$$

Solution:

$$(i) \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$$

$$\text{L.H.S.} = \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$\left\{ \because \log_n m = \frac{\log m}{\log n} \right\}$$

$$= \frac{\log_a}{\log abc} + \frac{\log_b}{\log abc} + \frac{\log_c}{\log abc}$$

$$= \frac{\log_a + \log_b + \log_c}{\log abc}$$

$$= \frac{\log abc}{\log abc} = 1 = \text{R.H.S.}$$

$$\left\{ \begin{array}{l} \because \log mnp \\ = \log_m + \log_n + \log_p \end{array} \right\}$$

$$(ii) \log_b a \cdot \log_c b \cdot \log_d c = \log_d a$$

$$\text{L.H.S.} = \log_b a \times \log_c b \times \log_d c$$

$$= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log d} = \frac{\log a}{\log d}$$

$$= \log_d a = \text{R.H.S.}$$

Question 30.

. Given that $\log_a x = \frac{1}{\alpha}$, $\log_b x = \frac{1}{\beta}$, $\log_c x = \frac{1}{\gamma}$, find $\log_{abc} x$.

Solution:

$$\log_a x = \frac{1}{\alpha}, \log_b x = \frac{1}{\beta}, \log_c x = \frac{1}{\gamma}$$

$$\log_a x = \frac{1}{\alpha} \Rightarrow \frac{\log x}{\log a} = \frac{1}{\alpha} \Rightarrow \log_a = \alpha \log x$$

$$\log_b x = \frac{1}{\beta} \Rightarrow \frac{\log x}{\log b} = \frac{1}{\beta} \Rightarrow \log_b = \beta \log x$$

$$\log_c x = \frac{1}{\gamma} \Rightarrow \frac{\log x}{\log c} = \frac{1}{\gamma} \Rightarrow \log_c = \gamma \log x$$

$$\text{Now } \log_{abc} x = \frac{\log x}{\log abc}$$

$$= \frac{\log x}{\log a + \log b + \log c}$$

$$= \frac{\log x}{\alpha \log x + \beta \log x + \gamma \log x}$$

$$= \frac{\log x}{\log x (\alpha + \beta + \gamma)} = \frac{1}{\alpha + \beta + \gamma}$$

Question 31.

Solve for x :

$$(i) \log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$$

$$(ii) \log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$$

Solution:

$$(i) \log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3} + \frac{1}{\log_x 9} + \frac{1}{\log_x 81} = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3^1} + \frac{1}{\log_x 3^2} + \frac{1}{\log_x 3^4} = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3} + \frac{1}{2\log_x 3} + \frac{1}{4\log_x 3} = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3} \left[1 + \frac{1}{2} + \frac{1}{4} \right] = \frac{7}{4}$$

$$\Rightarrow \log_x 3 \times \frac{7}{4} = \frac{7}{4}$$

$$\Rightarrow \log_x 3 = \frac{7}{4} \times \frac{4}{7} = 1 = \log_3 3$$

$$\{\because \log_a a = 1\}$$

Comparing, we get

$$\therefore x = 3$$

$$(ii) \log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2} + \frac{1}{\log_x 8} + \frac{1}{\log_x 32} = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2^1} + \frac{1}{\log_x 2^3} + \frac{1}{\log_x 2^5} = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2} + \frac{1}{3\log_x 2} + \frac{1}{5\log_x 2} = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2} \left[-1 + \frac{1}{3} + \frac{1}{5} \right] = \frac{23}{15}$$

$$\Rightarrow \frac{23}{15} [\log_x 2] = \frac{23}{15}$$

$$\log_x 2 = \frac{23}{15} \times \frac{15}{23} = 1 = \log_2 2$$

$$\{\because \log_a a = 1\}$$

Comparing,

$$x = 2$$

Multiple Choice Questions

correct Solution from the given four options (1 to 7):

Question 1.

If $\log_{\sqrt{3}} 27 = x$, then the value of x is

- (a) 3
- (b) 4
- (c) 6
- (d) 9

Solution:

$$\log_{\sqrt{3}} 27 = x$$

$$(\sqrt{3})^x = 27$$

$$\Rightarrow (3)^{\frac{1}{2}x} = 3^3$$

$$\Rightarrow 3^{\frac{x}{2}} = 3^3 \Rightarrow \frac{x}{2} = 3$$

$$\Rightarrow x = 6$$

(c)

Question 2.

If $\log_5 (0.04) = x$, then the value of x is

- (a) 2
- (b) 4
- (c) -4
- (d) -2

Solution:

$$\log_5 (0.04) = x$$

$$5^x = 0.04 = \frac{4}{100} = \frac{1}{25} = 5^{-2}$$

$$\therefore x = -2 \quad \text{(d)}$$

Question 3.

If $\log_{0.5} 64 = x$, then the value of x is

- (a) -4
- (b) -6
- (c) 4
- (d) 6

Solution:

$$\log_{0.5} 64 = x \Rightarrow 0.5^x = 64$$

$$= \left(\frac{1}{2}\right)^x = 2^6 \Rightarrow 2^{-x} = 2^6$$

$$\therefore -x = 6 \Rightarrow x = -6 \quad \text{(b)}$$

Question 4.

If $\log_{10} \sqrt[3]{5} x = -3$, then the value of x is

- (a) $\frac{1}{5}$
- (b) $-\frac{1}{5}$
- (c) -1
- (d) 5

Solution:

$$\log_{\sqrt[3]{5}} x = -3, \quad (\sqrt[3]{5})^{-3} = x$$

$$x = (5^{\frac{1}{3}})^{-3} = 5^{\frac{1}{3}(-3)} = 5^{-1}$$

$$x = \frac{1}{5} \quad \text{(b)}$$

Question 5.

If $\log(3x + 1) = 2$, then the value of x is

(a) $\frac{1}{3}$ (b) 99

(c) 33 (d) $\frac{19}{3}$

Solution:

$$\log(3x + 1) = 2 = \log 100 \quad (\because \log 100 = 2)$$

$$\therefore 3x + 1 = 100 \Rightarrow 3x = 100 - 1 = 99$$

$$\Rightarrow x = \frac{99}{3} = 33 \quad \text{(c)}$$

Question 6.

The value of $2 + \log_{10}(0.01)$ is

- (a) 4
- (b) 3
- (c) 1
- (d) 0

Solution:

$$\begin{aligned} & 2 + \log_{10}(0.01) \\ & = 2 + (-2) = 2 - 2 = 0 \end{aligned} \quad \text{(d)}$$

Question 7.

The value of $\frac{\log 8 - \log 2}{\log 32}$ is

(a) $\frac{2}{5}$ (b) $\frac{1}{4}$

(c) $-\frac{2}{5}$ (d) $\frac{1}{3}$

Solution:

$$\begin{aligned}\frac{\log 8 - \log 2}{\log 32} &= \frac{\log \frac{8}{2}}{\log 2^5} \\ &= \frac{\log 4}{\log 2^5} = \frac{\log 2^2}{\log 2^5} \\ &= \frac{2 \log 2}{5 \log 2} = \frac{2}{5}\end{aligned}\quad (\text{a})$$

Chapter Test

Question 1.

Expand $\log_a \sqrt[3]{x^7 y^8 \div \sqrt[4]{z}}$

Solution:

$$\begin{aligned}\log_a \sqrt[3]{x^7 y^8 \div \sqrt[4]{z}} \\ &= \log_a (x^7 y^8 \div \sqrt[4]{z})^{1/3} = \frac{1}{3} \log_a (x^7 y^8 \div \sqrt[4]{z}) \\ &= \frac{1}{3} [\log_a x^7 y^8 - \log_a \sqrt[4]{z}] \\ &= \frac{1}{3} [7 \log_a x + 8 \log_a y - \log_a (z)^{1/4}] \\ &= \frac{1}{3} \left[7 \log_a x + 8 \log_a y - \frac{1}{4} \log_a z \right] \\ &= \frac{7}{3} \log_a x + \frac{8}{3} \log_a y - \frac{1}{12} \log_a z\end{aligned}$$

Question 2.

Find the value of $\log \sqrt[3]{3} \sqrt[3]{3} - \log_5 (0.04)$

Solution:

$$\begin{aligned}
& \log_{\sqrt{3}} 3\sqrt{3} - \log_5 (0.04) \\
&= \log_{\sqrt{3}} 3 + \log_{\sqrt{3}} \sqrt{3} - \log_5 \frac{4}{100} \\
&= \log_{\sqrt{3}} 3 + 1 - \log_5 \frac{1}{25} \\
&= \log_{\sqrt{3}} 3 + 1 - \log_5 5^{-2} \\
&= \log_{\sqrt{3}} 3 + 1 - (-2) \log_5 5 \\
&= \log_{\sqrt{3}} (\sqrt{3})^2 + 1 + 2 \times 1 = 2 \log_{\sqrt{3}} \sqrt{3} + 1 + 2 \\
&= 2 \times 1 + 1 + 2 = 2 \times 1 + 2 = 5
\end{aligned}$$

Question 3.

Prove the following:

(i) $(\log x)^2 - (\log y)^2 = \log \frac{x}{y} \cdot \log xy$

(ii) $2 \log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91} = \log 2.$

Solution:

$$(i) (\log x)^2 - (\log y)^2 = \log \frac{x}{y} \cdot \log xy$$

$$\text{L.H.S} = (\log x)^2 - (\log y)^2 = (\log x - \log y) (\log x + \log y)$$

$$[\because A^2 - B^2 = (A - B)(A + B)]$$

$$= \left(\log \frac{x}{y} \right) (\log xy) = \log \frac{x}{y} \cdot \log xy = \text{R.H.S.}$$

Result is proved.

$$(ii) 2 \log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91} = \log 2$$

$$\text{L.H.S.} = 2 \log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91}$$

$$= 2[\log 11 - \log 13] + [\log 130 - \log 77] - [\log 55 - \log 91]$$

$$= 2 [\log 11 - \log 13] + [\log 13 \times 10 - \log 7 \times 11] - [\log 11 \times 5 - \log 13 \times 7]$$

$$= 2 [\log 11 - \log 13] + [(\log 13 + \log 10) - (\log 11 + \log 7)] - [(\log 11 + \log 5) - (\log 13 + \log 7)]$$

$$= 2 \log 11 - 2 \log 13 + \log 10 - \log 11 - \log 7 - \log 11 - \log 5 + \log 13 + \log 7$$

$$= (2 \log 11 - \log 11 - \log 11) + (-2 \log 13 + \log 13 + \log 13) + \log 10 - \log 5 + (\log 7 - \log 7)$$

$$= 0 + 0 + \log 10 - \log 5 + 0 = \log 10 - \log 5$$

$$= \log \left(\frac{10}{5} \right) = \log 2 = \text{R.H.S}$$

Hence, Result is proved.

Question 4.

If $\log(m+n) = \log m + \log n$, show that $n = \frac{m}{m-1}$

Solution:

Question 5.

If $\log \frac{x+y}{2} = \frac{1}{2}(\log x + \log y)$, prove that $x = y$.

Solution:

$$\log \frac{x+y}{2} = \frac{1}{2}(\log x + \log y)$$

$$\Rightarrow \log \frac{x+y}{2} = \frac{1}{2} \log (x \times y)$$

$$\Rightarrow \log \frac{x+y}{2} = \log (x \times y)^{\frac{1}{2}}$$

Comparing, we get,

$$\therefore \frac{x+y}{2} = (x \times y)^{\frac{1}{2}} = xy^{\frac{1}{2}} \Rightarrow x+y = 2(xy)^{\frac{1}{2}}$$

squaring

$$\Rightarrow (x+y)^2 = 4xy \quad \Rightarrow x^2 + y^2 + 2xy = 4xy$$

$$\Rightarrow x^2 + y^2 + 2xy - 4xy = 0 \Rightarrow x^2 + y^2 - 2xy = 0$$

$$\Rightarrow (x-y)^2 = 0 \Rightarrow x-y = 0$$

$$\therefore x = y$$

Hence proved.

Question 6.

If a, b are positive real numbers, $a > b$ and $a^2 + b^2 = 27 ab$, prove that

$$\log \left(\frac{a-b}{5} \right) = \frac{1}{2} (\log a + \log b)$$

Solution:

$$a^2 + b^2 = 27ab$$

$$\Rightarrow a^2 + b^2 - 2ab = 25ab$$

$$\Rightarrow \frac{a^2 + b^2 - 2ab}{25} = ab \Rightarrow \left(\frac{a-b}{5}\right)^2 = ab$$

Taking log both sides, $\log\left(\frac{a-b}{5}\right)^2 = \log ab$

$$\Rightarrow 2 \log\left(\frac{a-b}{5}\right) = \log a + \log b$$

$$\Rightarrow \log\left(\frac{a-b}{5}\right) = \frac{1}{2} (\log a + \log b)$$

Hence proved.

Solve the following equations for x

Question 7.

Solve the following equations for x :

(i) $\log_x \frac{1}{49} = -2$ (ii) $\log_x \frac{1}{4\sqrt{2}} = -5$

(iii) $\log_x \frac{1}{243} = 10$ (iv) $\log_4 32 = x - 4$

(v) $\log_7 (2x^2 - 1) = 2$ (vi) $\log (x^2 - 21) = 2$

(vii) $\log_6 (x - 2)(x + 3) = 1$

(viii) $\log_6 (x - 2) + \log_6 (x + 3) = 1$

(ix) $\log (x + 1) + \log (x - 1) = \log 11 + 2 \log 3.$

Solution:

$$(i) \log_x \frac{1}{49} = -2 \Rightarrow (x)^{-2} = \frac{1}{49}$$

$$\Rightarrow (x)^{-2} = \left(\frac{1}{7}\right)^2 \Rightarrow (x)^{-2} = (7)^{-2} \Rightarrow x = 7$$

$$(ii) \log_x \frac{1}{4\sqrt{2}} = -5$$

$$\Rightarrow \frac{-1}{5} \log_x \frac{1}{4\sqrt{2}} = 1 \Rightarrow -\frac{1}{5} \log_x \frac{1}{\sqrt{32}} = 1$$

$$\Rightarrow -\frac{1}{5} \log_x \frac{1}{2^{\frac{5}{2}}} = 1 \Rightarrow -\frac{1}{5} \log_x 2^{-\frac{5}{2}} = 1$$

$$\Rightarrow -\frac{1}{5} \times \left(\frac{-5}{2}\right) \log_x 2 = 1 \Rightarrow \frac{1}{2} \log_x 2 = 1$$

$$\Rightarrow \log_x (2^{\frac{1}{2}}) = 1 \Rightarrow \log_x \sqrt{2} = \log_x x$$

$$\therefore x = \sqrt{2}$$

$$(iii) \log_x \frac{1}{243} = 10$$

$$\begin{aligned}
\Rightarrow \frac{1}{10} \log_x \frac{1}{243} = 1 &\Rightarrow \frac{1}{10} \log_x \frac{1}{3^5} = 1 \\
\Rightarrow \frac{1}{10} \log_x (3)^{-5} = 1 &\Rightarrow \frac{1}{10} \times (-5) \times \log_x 3 = 1 \\
\Rightarrow -\frac{1}{2} \log_x 3 = \log_x x &\Rightarrow \log_x 3^{-\frac{1}{2}} = \log_x x \\
\Rightarrow \log_x \frac{1}{\sqrt{3}} = \log_x x & \\
\therefore x = \frac{1}{\sqrt{3}} &
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad \log_4 32 = x - 4 &\Rightarrow (4)^{x-4} = 32 \\
\Rightarrow [(2)^2]^{x-4} = 2 \times 2 \times 2 \times 2 \times 2 &\Rightarrow (2)^{2(x-4)} = (2)^5 \\
\Rightarrow (2)^{2x-8} = (2)^5 &\Rightarrow 2x - 8 = 5 \Rightarrow 2x = 5 + 8 \\
\Rightarrow 2x = 13 &\Rightarrow x = \frac{13}{2} = 6 \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{(v)} \quad \log_7 (2x^2 - 1) = 2 &\Rightarrow (7)^2 = 2x^2 - 1 \\
\Rightarrow 49 = 2x^2 - 1 &\Rightarrow 50 = 2x^2 \Rightarrow 2x^2 = 50 \\
\Rightarrow x^2 = \frac{50}{2} &\Rightarrow x^2 = 25 \Rightarrow x^2 = \pm \sqrt{25} \\
\Rightarrow x = +5, -5 &
\end{aligned}$$

$$(vi) \log(x^2 - 21) = 2$$

$$\Rightarrow (10)^2 = x^2 - 21 \Rightarrow 100 = x^2 - 21$$

$$\Rightarrow x^2 - 21 = 100 \Rightarrow x^2 = 100 + 21$$

$$\Rightarrow x^2 = 121 \Rightarrow x = \pm \sqrt{121} \Rightarrow x = \pm 11$$

$$\therefore x = 11, -11$$

$$(vii) \log_6(x-2)(x+3) = 1 = \log_6 6 \quad \{\because \log_a a = 1\}$$

Comparing,

$$(x-2)(x+3) = 6$$

$$\Rightarrow x^2 + 3x - 2x - 6 = 6$$

$$\Rightarrow x^2 + x - 6 - 6 = 0$$

$$\Rightarrow x^2 + x - 12 = 0$$

$$\Rightarrow x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow x(x+4) - 3(x+4) = 0$$

$$\Rightarrow (x+4)(x-3) = 0$$

Either $x+4=0$, then $x=-4$

or $x-3=0$, then $x=3$

Hence $x = 3, -4$

$$(viii) \log_6 (x-2) + \log_6 (x+3) = 1$$

$$\Rightarrow \log_6 (x-2)(x+3) = 1 = \log_6 6 \quad \{\because \log_a a = 1\}$$

Comparing,

$$(x-2)(x+3) = 6 \Rightarrow x^2 + 3x - 2x - 6 = 6$$

$$\Rightarrow x^2 + x - 6 - 6 = 0 \Rightarrow x^2 + x - 12 = 0$$

$$\Rightarrow x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow x(x+4) - 3(x+4) = 0$$

$$\Rightarrow (x+4)(x-3) = 0$$

Either $x+4=0$, then $x=-4$

or $x-3=0$, then $x=3$

$$\therefore x = 3, -4$$

$$(ix) \quad \log (x+1) + \log (x-1) = \log 11 + 2 \log 3$$

$$\Rightarrow \log [(x+1)(x-1)] = \log 11 + \log (3)^2$$

$$\Rightarrow \log (x^2 - 1) = \log 11 + \log 9$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\Rightarrow \log (x^2 - 1) = \log (11 \times 9) \Rightarrow x^2 - 1 = 11 \times 9$$

$$\Rightarrow x^2 - 1 = 99 \Rightarrow x^2 = 99 + 1 \Rightarrow x^2 = 100$$

$$\Rightarrow x^2 = (10)^2 \Rightarrow x = 10$$

Question 8.

Solve for x and y :

$$\frac{\log x}{3} = \frac{\log y}{2} \quad \text{and} \quad \log (xy) = 5$$

Solution:

$$\frac{\log x}{3} = \frac{\log y}{2}$$

$$\Rightarrow 2 \log x = 3 \log y$$

$$\Rightarrow 2 \log x - 3 \log y = 0 \quad \dots(i)$$

and $\log xy = 5$

$$\Rightarrow \log x + \log y = 5 \quad \dots(ii)$$

Multiply (ii) by 3 and (i) by 1,

$$2 \log x - 3 \log y = 0$$

$$3 \log x + 3 \log y = 15$$

Adding, $5 \log x = 15$

$$\Rightarrow \log x = \frac{15}{5} = 3 \Rightarrow \frac{1}{3} \log x = 1 = \log 10$$

$$\Rightarrow \log x^{\frac{1}{3}} = \log 10$$

$$\therefore x^{\frac{1}{3}} = 10$$

$$\Rightarrow x = 10^3 = 1000 \quad (\because \log 10 = 1)$$

Hence $x = 1000$

Substituting the value of $\log x = 3$ in (ii)

$$3 + \log y = 5$$

$$\Rightarrow \log y = 5 - 3 = 2 \Rightarrow \frac{1}{2} \log y = 1$$

$$\Rightarrow \log y^{\frac{1}{2}} = \log 10 \quad (\because \log 10 = 1)$$

$$\therefore y^{\frac{1}{2}} = 10$$

$$\Rightarrow y = (10)^2 = 100$$

Hence $x = 1000$ and $y = 100$

Question 9.

If $a = 1 + \log_x yz$, $6 = 1 + \log_y zx$ and $c = 1 + \log_z xy$, then show that $ab + bc + ca = abc$.

Solution:

$$a = 1 + \log_x yz$$

$$b = 1 + \log_y zx$$

$$c = 1 + \log_z xy$$

$$a = 1 + \log_x yz = \log_x x + \log_x yz$$

$$\Rightarrow a = \log_x xyz \Rightarrow \frac{1}{a} = \log_{xyz} x$$

Similarly,

$$\frac{1}{b} = \log_{xyz} y \text{ and } \frac{1}{c} = \log_{xyz} z$$

$$\text{Now } \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \log_{xyz} x + \log_{xyz} y + \log_{xyz} z$$

$$= \frac{\log x}{\log_{xyz}} + \frac{\log y}{\log_{xyz}} + \frac{\log z}{\log_{xyz}}$$

$$= \frac{\log x + \log y + \log z}{\log_{xyz}}$$

$$= \frac{\log xyz}{\log_{xz}} = 1$$

$$\Rightarrow \frac{bc + ca + ab}{abc} = 1 \Rightarrow ab + bc + ca = abc$$

Hence proved.