Exercise – 10A

1.

Sol:

(i) $\left(x^2 - x + 3\right)$ is a quadratic polynomial

$\therefore x^2 - x + 3 = 0$ is a quadratic equation.

(ii) Clearly, $\left(2x^2 + \frac{5}{2}x - \sqrt{3}\right)$ is a quadratic polynomial.

$\therefore 2x^2 + \frac{5}{2}x - \sqrt{3} = 0$ is a quadratic equation.

(iii) Clearly, $\left(\sqrt{2}x^2 + 7x + 5\sqrt{2}\right)$ is a quadratic polynomial.

$\therefore \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ is a quadratic equation.

(iv) Clearly, $\left(\frac{1}{3}x^2 + \frac{1}{5}x - 2\right)$ is a quadratic polynomial.

$\therefore \frac{1}{3}x^2 + \frac{1}{5}x - 2 = 0$ is a quadratic equation.

(v) $\left(x^2 - 3x - \sqrt{x} + 4\right)$ contains a term with $\sqrt{x}$, i.e., $x^{\frac{1}{2}}$, where $\frac{1}{2}$ is not a ineger.
Therefore, it is not a quadratic polynomial.
$\therefore x^2 - 3x - \sqrt{x} + 4 = 0$ is not a quadratic equation.

(vi) $x - \frac{6}{x} = 3$

$\Rightarrow x^2 - 6 = 3x$

$\Rightarrow x^2 - 3x - 6 = 0$

$\left(x^3 - 3x - 6\right)$ is not quadratic polynomial; therefore, the given equation is quadratic.

(vii) $x^2 + \frac{2}{x} = x^2$

$\Rightarrow x^2 + 2 = x^3$

$\Rightarrow x^3 - x^2 - 2 = 0$

$\left(x^3 - x^2 - 2\right)$ is not a quadratic polynomial.

$\therefore x^3 - x^2 - 2 = 0$ is not a quadratic equation.

(viii) $x^2 - \frac{1}{x^2} = 5$

$\Rightarrow x^4 - 1 = 5x^2$

$\Rightarrow x^4 - 5x^2 - 1 = 0$

$(x^4 - 5x^2 - 1)$ is a polynomial with degree 4.

$\therefore x^4 - 5x^2 - 1 = 0$ is not a quadratic equation.

(ix) $(x+2)^3 = x^3 - 8$

$\Rightarrow x^3 + 6x^2 + 12x + 8 = x^3 - 8$

$\Rightarrow 6x^2 + 12x + 16 = 0$

This is of the form $ax^2 + bx + c = 0$

Hence, the given equation is a quadratic equation.

(x) $(2x+3)(3x+2) = 6(x-1)(x-2)$

$\Rightarrow 6x^2 + 4x + 9x + 6 = 6(x^2 - 3x + 2)$

$\Rightarrow 6x^2 + 13x + 6 = 6x^2 - 18x + 12$

$\Rightarrow 31x - 6 = 0$

This is of the form $ax^2 + bx + c = 0$

Hence, the given equation is not a quadratic equation.

(xi) $\left(x + \frac{1}{x}\right)^2 = 2\left(x + \frac{1}{x}\right) + 3$

$\Rightarrow \left(\frac{x^2+1}{x}\right)^2 = 2\left(\frac{x^2+1}{x}\right) + 3$

$\Rightarrow (x^2+1)^2 = 2x(x^2+1) + 3x^2$

$\Rightarrow x^4 + 2x^2 + 1 = 2x^3 + 2x + 3x^2$

$\Rightarrow x^4 - 2x^3 - x^2 - 2x + 1 = 0$

This is not of the form $ax^2 + bx + c = 0$

Hence, the given equation is not a quadratic equation.

2.

Sol:

The given equation is $\left(3x^2+2x-1=0\right)$.

(i) $x=(-1)$

$$\text{L.H.S.}=x^2+2x-1$$
$$=3\times(-1)^2+2\times(-1)-1$$
$$=3-2-1$$
$$=0$$
$$=R.H.S.$$

Thus, (-1) is a root of $\left(3x^2+2x-1=0\right)$.

(ii) On subtracting $x=\frac{1}{3}$ in the given equation, we get:

$$\text{L.H.S.}=3x^2+2x-1$$
$$=3\times\left(\frac{1}{3}\right)^2+2\times\frac{1}{3}-1$$
$$=3\times\frac{1}{9}+\frac{2}{3}-1$$
$$=\frac{1+2-3}{3}$$
$$=\frac{0}{3}$$
$$=0$$
$$=R.H.S.$$

Thus, $\left(\frac{1}{3}\right)$ is a root of $\left(3x^2+2x-1=0\right)$

(iii) On subtracting $x=\left(-\frac{1}{2}\right)$ in the given equation, we get

$$\text{L.H.S.}=3x^2+2x-1$$
$$=3\times\left(-\frac{1}{2}\right)^2+2\times\left(-\frac{1}{2}\right)-1$$
$$=3\times\frac{1}{4}-1-1$$

$= \frac{3}{4} - 2$

$- \frac{3-8}{4}$

$= \frac{-5}{4} \neq 0$

Thus, $L.H.S = R.H.S$.

Hence, $\left(-\frac{1}{2}\right)$ is a solution of $\left(3x^2 + 2x - 1 = 0\right)$.

3.

Sol:

It is given that $(x=1)$ is a root of $\left(x^2 + kx + 3 = 0\right)$.

Therefore, $(x=1)$ must satisfy the equation.

$\Rightarrow (1)^2 + k \times 1 + 3 = 0$

$\Rightarrow k + 4 = 0$

$\Rightarrow k = -4$

Hence, the required value of k is -4.

4.

Sol:

It is given that $\frac{3}{4}$ is a root of $ax^2 + bx - 6 = 0$; therefore, we have:

$a \times \left(\frac{3}{4}\right)^2 + b \times \frac{3}{4} - 6 = 0$

$\Rightarrow \frac{9a}{16} + \frac{3b}{4} = 6$

$\Rightarrow \frac{9a + 12b}{16} = 6$

$\Rightarrow 9a + 12b - 96 = 0$

$\Rightarrow 3a + 4b = 32 \qquad ....(i)$

Again, (-2) is a root of $ax^2+bx-6=0;$ therefore, we have:

$$a\times(-2)^2+b\times(-2)-6=0$$
$$\Rightarrow 4a-2b=6$$
$$\Rightarrow 2a-b=3 \qquad ....(ii)$$

On multiplying (ii) by 4 and adding the result with (i), we get:

$$\Rightarrow 3a+4b+8a-4b=32+12$$
$$\Rightarrow 11a=44$$
$$\Rightarrow a=4$$

Putting the value of a in (ii), we get:

$$2\times4-b=3$$
$$\Rightarrow 8-b=3$$
$$\Rightarrow b=5$$

Hence, the required values of *a* and *b* are 4 and 5, respectively.

5.

Sol:

$$(2x-3)(3x+1)=0$$
$$\Rightarrow 2x-3=0 \text{ or } 3x+1=0$$
$$\Rightarrow 2x=3 \text{ or } 3x=-1$$
$$\Rightarrow x=\frac{3}{2} \text{ or } x=-\frac{1}{3}$$

Hence the roots of the given equation are $\frac{3}{2}$ *and* $-\frac{1}{3}$.

6.

Sol:

$$4x^2+5x=0$$
$$\Rightarrow x(4x+5)=0$$
$$\Rightarrow x=0 \text{ or } 4x+5=0$$
$$\Rightarrow x=0 \text{ or } x=-\frac{5}{4}$$

Hence, the roots of the given equation are 0 and $-\frac{5}{4}$.

7.

Sol:

Given:

$3x^2 - 243 = 0$

$\Rightarrow 3(x^2 - 81) = 0$

$\Rightarrow (x)^2 - (9)^2 = 0$

$\Rightarrow (x+9)(x-9) = 0$

$\Rightarrow x+9=0 \ or \ x-9=0$

$\Rightarrow x=-9 \ or \ x=9$

Hence, -9 and 9 are the roots of the equation

8.

Sol:

We write, $x = 4x - 3x$ as $2x^2 \times (-6) = -12x^2 = 4x \times (-3x)$

$\therefore 2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (x+2)(2x-3) = 0$

$\Rightarrow x+2=0 \ or \ 2x-3=0$

$\Rightarrow x=-2 \ or \ x=\frac{3}{2}$

Hence, the roots of the given equation are -2 and $\frac{3}{2}$.

9.

Sol:

We write, $6x = x + 5x$ as $x^2 \times 5 = 5x^2 = x \times 5x$

$\therefore x^2 + 6x + 5 = 0$

$\Rightarrow x^2 + x - 5x + 5 = 0$

$\Rightarrow x(x+1)+5(x+1)=0$

$\Rightarrow (x+1)(x+5)=0$

$\Rightarrow x+1=0 \text{ or } x+5=0$

$\Rightarrow x=-1 \text{ or } x=-5$

Hence, the roots of the given equation are -1 and -5.

10.

Sol:

We write, $-3x=3x-6x$ as $9x^2\times(-2)=-18x^2=3x\times(-6x)$

$\therefore 9x^2-3x-2=0$

$\Rightarrow 9x^2+3x-6x-2=0$

$\Rightarrow 3x(3x+1)-2(3x+1)=0$

$\Rightarrow (3x+1)(3x-2)=0$

$\Rightarrow 3x+1=0 \text{ or } 3x-2=0$

$\Rightarrow x=-\frac{1}{3} \text{ or } x=\frac{2}{3}$

Hence, the roots of the given equation are $-\frac{1}{3}$ and $\frac{2}{3}$.

11.

Sol:

Given:

$x^2+12x+35=0$

$\Rightarrow x^2+7x+5x+35=0$

$\Rightarrow x(x+7)+5(x+7)=0$

$\Rightarrow (x+5)(x+7)=0$

$\Rightarrow x+5=0 \text{ or } x+7=0$

$\Rightarrow x=-5 \text{ or } x=-7$

Hence, -5 and -7 are the roots of the equation $x^2+12x+35=0$.

12.

Sol:

Given

$x^2=18x-77$

$\Rightarrow x^2-18x+77=0$

$\Rightarrow x^2-(11x+7x)+77=0$

$\Rightarrow x^2-11x-7x+77=0$

$\Rightarrow x(x-11)-7(x-11)=0$

$\Rightarrow (x-7)(x-11)=0$

$\Rightarrow x-7=0 \text{ or } x-11=0$

$\Rightarrow x=7 \text{ or } x=11$

Hence, 7 and 11 are the roots of the equation $x^2=18x-77$.

13.

Sol:

Given:

$6x^2+11x+3=0$

$\Rightarrow 6x^2+9x+2x+3=0$

$\Rightarrow 3x(2x+3)+1(2x+3)=0$

$\Rightarrow (3x+1)(2x+3)=0$

$\Rightarrow 3x+1=0 \text{ or } 2x+3=0$

$\Rightarrow x=\frac{-1}{3} \text{ or } x=\frac{-3}{2}$

Hence, $\frac{-1}{3}$ and $\frac{-3}{2}$ are the roots of the equation

14.

Sol:

Given:

$6x^2+x-12=0$

$\Rightarrow 6x^2+9x-8x-12=0$

$\Rightarrow 3x(2x+3)-4(2x+3)=0$

$\Rightarrow (3x-4)(2x+3)=0$

$\Rightarrow 3x-4=0 \text{ or } 2x+3=0$

$\Rightarrow x=\frac{4}{3} \text{ or } x=\frac{-3}{2}$

Hence, $\frac{4}{3}$ and $\frac{-3}{2}$ are the roots of the equation

15.

Sol:

We write, $-2x=-3x+x$ as $3x^2\times(-1)=-3x^2=(-3x)\times x$

$\therefore 3x^2-2x-1=0$

$\Rightarrow 3x^2-3x+x-1=0$

$\Rightarrow 3x(x-1)+1(x-1)=0$

$\Rightarrow (x-1)(3x+1)=0$

$\Rightarrow x-1=0$ *or* $3x+1=0$

$\Rightarrow x=1$ or $x=-\frac{1}{3}$

Hence, the roots of the given equation are 1 and $-\frac{1}{3}$.

16.

Sol:

Given:

$4x^2-9x=100$

$\Rightarrow 4x^2-9x-100=0$

$\Rightarrow 4x^2-(25x-16x)-100=0$

$\Rightarrow 4x^2-25x+16x-100=0$

$\Rightarrow x(4x-25)+4(4x-25)=0$

$\Rightarrow (4x-25)(x+4)=0$

$\Rightarrow 4x-25=0$ *or* $x+4=0$

$\Rightarrow x=\frac{25}{4}$ *or* $x=-4$

Hence, the roots of the equation are $\frac{25}{4}$ *and* -4.

17.

Sol:

Given:

$15x^2-28=x$

$\Rightarrow 15x^2-x-28=0$

$\Rightarrow 15x^2-(21x-20x)-28=0$

$\Rightarrow 15x^2-21x+20x-28=0$

$\Rightarrow 3x(5x-7)+4(5x-7)=0$

$\Rightarrow (3x+4)(5x-7)=0$

$\Rightarrow 3x+4=0$ or $5x-7=0$

$\Rightarrow x=\frac{-4}{3}$ *or* $x=\frac{7}{5}$

Hence, the roots of the equation are $\frac{-4}{3}$ and $\frac{7}{5}$.

18.

Sol:

Given:

$4-11x=3x^2$

$\Rightarrow 3x^2+11x-4=0$

$\Rightarrow 3x^2+12x-x-4=0$

$\Rightarrow 3x(x+4)-1(x+4)=0$

$\Rightarrow (x+4)(3x-1)=0$

$\Rightarrow x+4=0$ *or* $3x-1=0$

$\Rightarrow x=-4$ *or* $x=\frac{1}{3}$

Hence, the roots of the equation are -4 and $\frac{1}{3}$.

19.

Sol:

Given:

$48x^2-13x-1=0$

$\Rightarrow 48x^2-(16x-3x)-1=0$

$\Rightarrow 48x^2-16x+3x-1=0$

$\Rightarrow 16x(3x-1)+1(3x-1)=0$

$\Rightarrow (16x+1)(3x-1)=0$

$\Rightarrow 16x+1=0$ *or* $3x-1=0$

$\Rightarrow x=\frac{-1}{16}$ *or* $x=\frac{1}{3}$

Hence, the roots of the equation are $\frac{-1}{16}$ and $\frac{1}{3}$.

20.

Sol:

We write:

$2\sqrt{2}x = 3\sqrt{2}x - \sqrt{2}x$ as $x^2 \times (-6) = -6x^2 = 3\sqrt{2}x \times \left(-\sqrt{2}x\right)$

$\therefore x^2 + 2\sqrt{2}x - 6 = 0$

$\Rightarrow x^2 + 2\sqrt{2}x - \sqrt{2}x - 6 = 0$

$\Rightarrow x\left(x + 3\sqrt{2}\right) - \sqrt{2}\left(x + 3\sqrt{2}\right) = 0$

$\Rightarrow \left(x + 3\sqrt{2}\right)\left(x - \sqrt{2}\right) = 0$

$\Rightarrow x + 3\sqrt{2} = 0$ *or* $x - \sqrt{2} = 0$

$\Rightarrow x = -3\sqrt{2}$ or $x = \sqrt{2}$

Hence, the roots of the given equation are $-3\sqrt{2}$ and $\sqrt{2}$

21.

Sol:

We write: $10x = 3x + 7x$ as $\sqrt{3}x^2 \times 7\sqrt{3} = 21x^2 = 3x \times 7x$

$\therefore \sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x\left(x + \sqrt{3}\right) + 7\left(x + \sqrt{3}\right) = 0$

$\Rightarrow \left(x + \sqrt{3}\right)\left(\sqrt{3}x + 7\right) = 0$

$\Rightarrow x + \sqrt{3} = 0$ *or* $\sqrt{3}x + 7 = 0$

$\Rightarrow x = -\sqrt{3}$ or $x = -\frac{7}{\sqrt{3}} = -\frac{7\sqrt{3}}{3}$

Hence, the roots of the given equation are $-\sqrt{3}$ and $-\frac{7\sqrt{3}}{3}$.

22.

Sol:

Given:

$\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x\left(x + 3\sqrt{3}\right) + 2\left(x + 3\sqrt{3}\right) = 0$

$\Rightarrow \left(x+3\sqrt{3}\right)\left(\sqrt{3}x+2\right)=0$

$\Rightarrow x+3\sqrt{3}=0 \ or \ \sqrt{3}x+2=0$

$\Rightarrow x=-3\sqrt{3} \ or \ x=\frac{-2}{\sqrt{3}}=\frac{-2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{-2\sqrt{3}}{3}$

Hence, the roots of the equation are $-3\sqrt{3}$ and $\frac{-2\sqrt{3}}{3}$.

23.

Sol:

Given:

$3\sqrt{7}x^2+4x-\sqrt{7}=0$

$\Rightarrow 3\sqrt{7}x^2+7x-3x-\sqrt{7}=0$

$\Rightarrow \sqrt{7}x\left(3x+\sqrt{7}\right)-1\left(3x+\sqrt{7}\right)=0$

$\Rightarrow \left(3x+\sqrt{7}\right)\left(\sqrt{7}-1\right)=0$

$\Rightarrow 3x+\sqrt{7}=0 \ or \ \sqrt{7}x-1=0$

$\Rightarrow x=\frac{-\sqrt{7}}{3} or \ x=\frac{1}{\sqrt{7}}=\frac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}}=\frac{\sqrt{7}}{7}$

Hence, the roots of the equation are $\frac{-\sqrt{7}}{3}$ and $\frac{\sqrt{7}}{7}$.

24.

Sol:

We write, $-6x=7x-13x$ as $\sqrt{7}x^2\times\left(-13\sqrt{7}\right)=-91x^2=7x\times\left(-13x\right)$

$\therefore \sqrt{7}x^2-6x-13\sqrt{7}=0$

$\Rightarrow \sqrt{7}x^2+7x-13x-13\sqrt{7}=0$

$\Rightarrow \sqrt{7}x\left(x+\sqrt{7}\right)-13\left(x+\sqrt{7}\right)=0$

$\Rightarrow \left(x+\sqrt{7}\right)\left(\sqrt{7}x-13\right)=0$

$\Rightarrow x+\sqrt{7}=0 \ or \ \sqrt{7}x-13=0$

$\Rightarrow x=-\sqrt{7}$ or $x=\frac{13}{\sqrt{7}}=\frac{13\sqrt{7}}{7}$

Hence, the roots of the given equation are $-\sqrt{7}$ and $\frac{13\sqrt{7}}{7}$.

25.

Sol:
Given:

$4\sqrt{6}x^2 - 13x - 2\sqrt{6} = 0$

$\Rightarrow 4\sqrt{6}x^2 - 16x + 3x - 2\sqrt{6} = 0$

$\Rightarrow 4\sqrt{2}x\left(\sqrt{3}x - 2\sqrt{2}\right) + \sqrt{3}\left(\sqrt{3}x - 2\sqrt{2}\right) = 0$

$\Rightarrow \left(4\sqrt{2}x + \sqrt{3}\right)\left(\sqrt{3}x - 2\sqrt{2}\right) = 0$

$\Rightarrow 4\sqrt{2}x + \sqrt{3} = 0 \text{ or } \sqrt{3}x - 2\sqrt{2} = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{4\sqrt{2}} = \frac{-\sqrt{3}\times\sqrt{2}}{4\sqrt{2}\times\sqrt{2}} = \frac{-\sqrt{8}}{8} \text{ or } x = \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{2\sqrt{6}}{3}$

Hence, the roots of the equation are $\frac{-\sqrt{6}}{8}$ and $\frac{2\sqrt{6}}{3}$.

26.

Sol:
We write, $-2\sqrt{6}x = -\sqrt{6}x$ ad $3x^2 \times 2 = 6x^2 = \left(-\sqrt{6}x\right)\times\left(-\sqrt{6}x\right)$

$\therefore 3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3}x\left(\sqrt{3}x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}x - \sqrt{2}\right) = 0$

$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$

$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)^2 = 0$

$\Rightarrow \sqrt{3}x - \sqrt{2} = 0$

$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

Hence, $\frac{\sqrt{6}}{3}$ is the repeated root of the given equation.

27.

Sol:
We write, $-2\sqrt{2}x = -3\sqrt{2}x + \sqrt{2}x$ as $\sqrt{3}x^2 \times \left(-2\sqrt{3}\right) = -6x^2 = \left(-3\sqrt{2}x\right)\times\left(\sqrt{2}x\right)$

$\therefore \sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x\left(x - \sqrt{6}\right) + \sqrt{2}\left(x - \sqrt{6}\right) = 0$

$\Rightarrow \left(x - \sqrt{6}\right)\left(\sqrt{3}x + \sqrt{2}\right) = 0$

$\Rightarrow x - \sqrt{6} = 0 \text{ or } \sqrt{3}x + \sqrt{2} = 0$

$\Rightarrow x - \sqrt{6} = 0 \text{ or } x = -\frac{\sqrt{2}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}$

Hence, the roots of the given equation are $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$.

28.

Sol:

We write, $-3\sqrt{5}x = -2\sqrt{5}x - \sqrt{5}x$ as $x^2 \times 10 = 10x^2 = \left(-2\sqrt{5}x\right) \times \left(-\sqrt{5}x\right)$

$\therefore x^2 - 3\sqrt{5}x + 10 = 0$

$\Rightarrow x^2 - 2\sqrt{5}x - \sqrt{5}x + 10 = 0$

$\Rightarrow x\left(x - 2\sqrt{5}\right) - \sqrt{5}\left(x - 2\sqrt{5}\right) = 0$

$\Rightarrow \left(x - 2\sqrt{5}\right)\left(x - \sqrt{5}\right) = 0$

$\Rightarrow x\left(x - 2\sqrt{5}\right) - \sqrt{5}\left(x - 2\sqrt{5}\right) = 0$

Hence, the roots of the given equation are $\sqrt{5}$ and $2\sqrt{5}$.

29.

Sol:

$x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$

$\Rightarrow x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$\Rightarrow x\left(x - \sqrt{3}\right) - 1\left(x - \sqrt{3}\right) = 0$

$\Rightarrow \left(x - \sqrt{3}\right)\left(x - 1\right) = 0$

$\Rightarrow x - \sqrt{3} = 0 \text{ or } x - 1 = 0$

$\Rightarrow x = \sqrt{3} \text{ or } x = 1$

Hence, 1 and $\sqrt{3}$ are the roots of the given equation.

30.

Sol:

We write, $3\sqrt{3}x = 5\sqrt{3}x - 2\sqrt{3}x$ as $x^2 \times (-30) = -30x^2 = 5\sqrt{3}x \times \left(-2\sqrt{3}x\right)$

$\therefore x^2 + 3\sqrt{3}x - 30 = 0$

$\Rightarrow x^2 + 5\sqrt{3}x - 2\sqrt{3}x - 30 = 0$

$\Rightarrow x\left(x + 5\sqrt{3}\right) - 2\sqrt{3}\left(x + 5\sqrt{3}\right) = 0$

$\Rightarrow \left(x + 5\sqrt{3}\right)\left(x - 2\sqrt{3}\right) = 0$

$\Rightarrow x + 5\sqrt{3} = 0 \text{ or } x - 2\sqrt{3} = 0$

$\Rightarrow x = -5\sqrt{3} \text{ or } x = 2\sqrt{3}$

Hence, the roots of the given equation are $-5\sqrt{3}$ and $2\sqrt{3}$

31.

Sol:

We write, $7x = 5x + 2x$ as $\sqrt{2}x^2 \times 5\sqrt{2} = 10x^2 = 5x \times 2x$

$\therefore \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x\left(\sqrt{2}x + 5\right) + \sqrt{2}\left(\sqrt{2}x + 5\right) = 0$

$\Rightarrow \left(\sqrt{2}x + 5\right)\left(x + \sqrt{2}\right) = 0$

$\Rightarrow x + \sqrt{2} = 0 \text{ or } \sqrt{2}x + 5 = 0$

$\Rightarrow x = -\sqrt{2} \text{ or } x = -\frac{5}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}$

Hence, the roots of the given equation are $-\sqrt{2}$ and $-\frac{5\sqrt{2}}{2}$.

32.

Sol:

We write, $13x = 5x + 8x$ as $5x^2 \times 8 = 40x^2 = 5x \times 8x$

$\therefore 5x^2 + 13x + 8 = 0$

$\Rightarrow 5x^2 + 5x + 8x + 8 = 0$

$\Rightarrow 5x(x+1) + 8(x+1) = 0$

$\Rightarrow (x+1)(5x+8) = 0$

$\Rightarrow x + 1 = 0 \text{ or } 5x + 8 = 0$

$x=-1 \; or \; x=-\frac{8}{5}$

Hence, -1 and $-\frac{8}{5}$ are the roots of the given equation.

33.

Sol:

Given:

$x^2-\left(1+\sqrt{2}\right)x+\sqrt{2}=0$

$\Rightarrow x^2-x-\sqrt{2}x+\sqrt{2}=0$

$\Rightarrow x\left(x-1\right)-\sqrt{2}\left(x-1\right)=0$

$\Rightarrow \left(x-\sqrt{2}\right)\left(x-1\right)=0$

$\Rightarrow x-\sqrt{2}=0 \; or \; x-1=0$

$\Rightarrow x=\sqrt{2} \; or \; x=1$

Hence, the roots of the equation are $\sqrt{2}$ and 1.

34.

Sol:

Given:

$9x^2+6x+1=0$

$\Rightarrow 9x^2+3x+3x+1=0$

$\Rightarrow 3x\left(3x+1\right)+1\left(3x+1\right)=0$

$\Rightarrow \left(3x+1\right)\left(3x+1\right)=0$

$\Rightarrow 3x+1=0 \; or \; 3x+1=0$

$\Rightarrow x=\frac{-1}{3} \; or \; x=\frac{-1}{3}$

Hence, $\frac{-1}{3}$ is the root of the equation $9x^2+6x+1=0$.

35.

Sol:

We write, $-20x=-10x-10x$ as $100x^2\times1=100x^2=\left(-10x\right)\times\left(-10x\right)$

$\therefore 100x^2-20x+1=0$

$\Rightarrow 100x^2-10x-10x+1=0$

$\Rightarrow 10x(10x-1)-1(10x-1)=0$

$\Rightarrow (10x-1)(10x-1)=0$

$\Rightarrow (10x-1)^2=0$

$\Rightarrow 10x-1=0$

$\Rightarrow x=\frac{1}{10}$

Hence, $\frac{1}{10}$ is the repeated root of the given equation.

36.

Sol:

We write, $-x=-\frac{x}{2}-\frac{x}{2}$ as $2x^2\times\frac{1}{8}=\frac{x^2}{4}=\left(-\frac{x}{2}\right)\times\left(-\frac{x}{2}\right)$

$\therefore 2x^2-x+\frac{1}{8}=0$

$\Rightarrow 2x^2-\frac{x}{2}-\frac{x}{2}+\frac{1}{8}=0$

$\Rightarrow 2x\left(x-\frac{1}{4}\right)-\frac{1}{2}\left(x-\frac{1}{4}\right)=0$

$\Rightarrow \left(x-\frac{1}{4}\right)\left(2x-\frac{1}{2}\right)=0$

$\Rightarrow x-\frac{1}{4}=0 \; or \; 2x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{4} \; or \; x=\frac{1}{4}$

Hence, $\frac{1}{4}$ is the repeated root of the given equation.

37.

Sol:
Given:

$10x-\frac{1}{x}=3$

$\Rightarrow 10x^2-1=3x$ [Multiplying both sides by x]

$\Rightarrow 10x^2 - 3x - 1 = 0$

$\Rightarrow 10x^2 - (5x - 2x) - 1 = 0$

$\Rightarrow 10x^2 - 5x + 2x - 1 = 0$

$\Rightarrow 5x(2x-1) + 1(2x-1) = 0$

$\Rightarrow (2x-1)(5x+1) = 0$

$\Rightarrow 2x - 1 = 0$ or $5x + 1 = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = \frac{-1}{5}$

Hence, the roots of the equation are $\frac{1}{2}$ and $\frac{-1}{5}$.

38.

Sol:

Given:

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$\Rightarrow 2 - 5x + 2x^2 = 0$ [Multiplying both sides by x^2]

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - (4x + x) + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x-2) - 1(x-2) = 0$

$\Rightarrow (2x-1)(x-2) = 0$

$\Rightarrow 2x - 1 = 0$ *or* $x - 2 = 0$

$\Rightarrow x = \frac{1}{2}$ *or* $x = 2$

Hence, the roots of the equation are $\frac{1}{2}$ and 2.

39.

Sol:

We write, $ax = 2ax - ax$ as $2x^2 \times (-a^2) = -2a^2x^2 = 2ax \times (-ax)$

$\therefore 2x^2 + ax - a^2 = 0$

$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x+a) - a(x+a) = 0$

$\Rightarrow (x+a)(2x-a) = 0$

$\Rightarrow x+a=0 \text{ or } 2x-a=0$

$\Rightarrow x=-a \text{ or } x=\dfrac{a}{2}$

Hence, $-a$ and $\dfrac{a}{2}$ are the roots of the given equation.

40.

Sol:

We write, $4bx=2(a+b)x-2(a-b)x$ as

$4x^2\times\left[-\left(a^2-b^2\right)\right]=-4\left(a^2-b^2\right)x^2=2(a+b)x\times\left[-2(a-b)x\right]$

$\therefore 4x^2+4bx-\left(a^2-b^2\right)=0$

$\Rightarrow 4x^2+2(a+b)x-2(a-b)x-(a-b)(a+b)=0$

$\Rightarrow 2x\left[2x+(a+b)\right]-(a-b)\left[2x+(a+b)\right]=0$

$\Rightarrow\left[2x+(a+b)\right]\left[2x-(a-b)\right]=0$

$\Rightarrow 2x+(a+b)=0 \text{ or } 2x-(a-b)=0$

$\Rightarrow x=-\dfrac{a+b}{2} \text{ or } x=\dfrac{a-b}{2}$

Hence, $-\dfrac{a+b}{2}$ and $\dfrac{a-b}{2}$ are the roots of the given equation.

41.

Sol:

We write, $-4a^2x=-2\left(a^2+b^2\right)x-2\left(a^2-b^2\right)x$ as

$4x^2\times\left(a^4-b^4\right)=4\left(a^4-b^4\right)x^2=\left[-2\left(a^2+b^2\right)\right]x\times\left[-2\left(a^2-b^2\right)\right]x$

$\therefore 4x^2-4a^2x+\left(a^4-b^4\right)=0$

$\Rightarrow 4x^2-2\left(a^2+b^2\right)x-2\left(a^2-b^2\right)x+\left(a^2-b^2\right)\left(a^2+b^2\right)=0$

$\Rightarrow 2x\left[2x-\left(a^2+b^2\right)\right]-\left(a^2-b^2\right)\left[2x-\left(a^2+b^2\right)\right]=0$

$\Rightarrow\left[2x-\left(a^2+b^2\right)\right]\left[2x-\left(a^2-b^2\right)\right]=0$

$\Rightarrow 2x-\left(a^2+b^2\right)=0 \text{ or } 2x-\left(a^2-b^2\right)=0$

$\Rightarrow x=\dfrac{a^2+b^2}{2} \text{ or } x=\dfrac{a^2-b^2}{2}$

Hence, $\frac{a^2+b^2}{2}$ and $\frac{a^2-b^2}{2}$ are the roots of the given equation.

42.

Sol:

We write, $5x=(a+3)x-(a-2)x$ as

$x^2\times\left[-\left(a^2+a-6\right)\right]=-\left(a^2+a-6\right)x^2=(a+3)x\times\left[-(a-2)x\right]$

$\therefore x^2+5x-\left(a^2+a-6\right)=0$

$\Rightarrow x^2+(a+3)x-(a-2)x-(a+3)(a-2)=0$

$\Rightarrow x\left[x+(a+3)\right]-(a-2)\left[x+(a+3)\right]=0$

$\Rightarrow\left[x+(a+3)\right]\left[x-(a-2)\right]=0$

$\Rightarrow x+(a+3)=0\ x-(a-2)=0$

$\Rightarrow x=-(a+3)\ or\ x=a-2$

Hence, $-(a+3)$ and $(a-2)$ are the roots of the given equation.

43.

Sol:

We have, $-2ax=(2b-a)x-(2b+a)x$ as

$x^2\times\left[-\left(4b^2-a^2\right)\right]=-\left(4b^2-a^2\right)x^2=(2b-a)x\times\left[-(2b+a)x\right]$

$\therefore x^2-2ax-\left(4b^2-a^2\right)=0$

$\Rightarrow x^2+(2b-a)x-(2b+a)x-(2b-a)(2b+a)=0$

$\Rightarrow x\left[x+(2b-a)\right]-(2b+a)\left[x+(2b-a)\right]=0$

$\Rightarrow\left[x+(2b-a)\right]\left[x-(2b+a)\right]=0$

$\Rightarrow x+(2b-a)=0\ or\ x-(2b+a)=0$

$x=-(2b-a)$ or $x=2b+a$

$\Rightarrow x=a-2b\ or\ x=a+2b$

Hence, $a-2b$ and $a+2b$ are the roots of the given equation.

44.

Sol:

We write, $-(2b-1)x=-(b-5)x-(b+4)x$ as

$x^2 \times (b^2 - b - 20) = (b^2 - b - 20)x^2 = [-(b-5)x] \times [-(b+4)x]$

$\therefore x^2 - (2b-1)x + (b^2 - b - 20) = 0$

$\Rightarrow x^2 - (b-5)x - (b+4)x + (b-5)(b+4) = 0$

$\Rightarrow x[x-(b-5)] - (b+4)[x-(b-5)] = 0$

$\Rightarrow [x-b-5][x-(b+4)] = 0$

$\Rightarrow x-(b-5) = 0$ or $x-(b+4) = 0$

$\Rightarrow x = b-5$ or $x = b+4$

Hence, $b-5$ and $b+4$ are the roots of the given equation.

45.

Sol:

We write, $6x = (a+4)x - (a-2)x$ as

$x^2 \times [-(a^2 + 2a - 8)] = -(a^2 + 2a - 8)x^2 = (a+4)x \times [-(a-2)x]$

$\therefore x^2 + 6x - (a^2 + 2a - 8) = 0$

$\Rightarrow x^2 + (a+4)x - (a-2)x - (a+4)(a-2) = 0$

$\Rightarrow x[x+(a+4)] - (a-2)[x+(a+4)] = 0$

$\Rightarrow [x+(a+4)][x-(a-2)] = 0$

$\Rightarrow x + (a+4) = 0 \ or \ x - (a-2) = 0$

$\Rightarrow x = -(a+4)$ or $x = a-2$

Hence, $-(a+4)$ and $(a-2)$ are the roots of the given equation.

46.

Sol:

$abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow abx^2 + b^2x - acx - bc = 0$

$\Rightarrow bx(ax+b) - c(ax+b) = 0$

$\Rightarrow (bx-c)(ax+b) = 0$

$\Rightarrow bx - c = 0$ or $ax + b = 0$

$\Rightarrow x = \frac{c}{b} \ or \ x = \frac{-b}{a}$

Hence, the roots of the equation are $\frac{c}{b}$ and $\frac{-b}{a}$.

47.

Sol:

We write, $-4ax = -(b+2a)x + (b-2a)x$ as

$x^2 \times (-b^2 + 4a^2) = (-b^2 + 4a^2)x^2 = -(b+2a)x \times (b-2a)x$

$\therefore x^2 - 4ax - b^2 + 4a^2 = 0$

$\Rightarrow x^2 - (b+2a)x(b-2a)x - (b-2a)(b+2a) = 0$

$\Rightarrow x[x-(b+2a)] + (b-2a)[x-(b+2a)] = 0$

$\Rightarrow [x-(b+2a)][x+(b-2a)] = 0$

$\Rightarrow x-(b+2a) = 0 \; or \; x+(b-2a) = 0$

$\Rightarrow x = 2a+b \; or \; x = -(b-2a)$

$\Rightarrow x = 2a+b \; or \; x = 2a-b$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.

48.

Sol:

Given:

$4x^2 - 2(a^2+b^2)x + a^2b^2 = 0$

$\Rightarrow 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$\Rightarrow 2x(2x-a^2) - b^2(2x-a^2) = 0$

$\Rightarrow (2x-b^2)(2x-a^2) = 0$

$\Rightarrow 2x-b^2 = 0 \; or \; 2x-a^2 = 0$

$\Rightarrow x = \frac{b^2}{2} \; or \; x = \frac{a^2}{2}$

Hence, the roots of the equation are $\frac{b^2}{2}$ and $\frac{a^2}{2}$.

49.

Sol:

Given:

$12abx^2-(9a^2-8b^2)x-6ab=0$

$\Rightarrow 12abx^2-9a^2x+8b^2x-6ab=0$

$\Rightarrow 3ax(4bx-3a)+2b(4bx-3a)=0$

$\Rightarrow (3ax+2b)(4bx-3a)=0$

$\Rightarrow 3ax+2b=0 \text{ or } 4bx-3a=0$

$\Rightarrow x=\frac{-2b}{3a} \text{ or } x=\frac{3a}{4b}$

Hence, the roots of the equation are $\frac{-2b}{3a}$ and $\frac{3a}{4b}$.

50.

Sol:

Given:

$a^2b^2x^2+b^2x-a^2x-1=0$

$\Rightarrow b^2x(a^2x+1)-1(a^2x+1)=0$

$\Rightarrow (b^2x-1)(a^2x+1)=0$

$\Rightarrow (b^2x-1)=0 \text{ or } (a^2x+1)=0$

$\Rightarrow x=\frac{1}{b^2} \text{ or } x=\frac{-1}{a^2}$

Hence, $\frac{1}{b^2}$ and $\frac{-1}{a^2}$ are the roots of the given equation.

51.

Sol:

We write, $-9(a+b)x=-3(2a+b)x-3(a+2b)x$ as

$9x^2\times(2a^2+5ab+2b^2)=9(2a^2+5ab+2b^2)x^2=[-3(2a+b)x]\times[-3(a+2b)x]$

$\therefore 9x^2-9(a+b)x+(2a^2+5ab+2b^2)=0$

$\Rightarrow 9x^2-3(2a+b)x-3(a+2b)x+(2a+b)(a+2b)=0$

$\Rightarrow 3x[3x-(2a+b)]-(a+2b)[3x-(2a+b)]=0$

$\Rightarrow [3x-(2a+b)][3x-(a+2b)]=0$

$\Rightarrow 3x-(2a+b)=0 \text{ or } 3x-(a+2b)=0$

$\Rightarrow x = \frac{2a+b}{3}\ or\ x = \frac{a+2b}{3}$

Hence, $\frac{2a+b}{3}$ and $\frac{a+2b}{3}$ are the roots of the given equation.

52.

Sol:

$\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

$\Rightarrow \frac{16}{x} - \frac{15}{x+1} = 1$

$\Rightarrow \frac{16x+16-15x}{x(x+1)} = 1$

$\Rightarrow \frac{x+16}{x^2+x} = 1$

$\Rightarrow x^2 + x = x + 16$ (Cross multiplication)

$\Rightarrow x^2 - 16 = 0$

$\Rightarrow (x+4)(x-4) = 0$

$\Rightarrow x+4 = 0\ or\ x-4 = 0$

$\Rightarrow x = -4\ or\ x = 4$

Hence, -4 and 4 are the roots of the given equation.

53.

Sol:

$\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

$\Rightarrow \frac{4}{x} - \frac{5}{2x+3} = 3$

$\Rightarrow \frac{8x+12-5x}{x(2x+3)} = 3$

$\Rightarrow \frac{3x+12}{2x^2+3x} = 3$

$\Rightarrow \frac{x+4}{2x^2+3x} = 1$

$\Rightarrow 2x^2 + 3x = x + 4$ (Cross multiplication)

$\Rightarrow 2x^2 + 2x - 4 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x+2) - 1(x+2) = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x + 2 = 0$ or $x - 1 = 0$

$\Rightarrow x = -2$ or $x = 1$

Hence, -2 and 1 are the roots of the given equation.

54.

Sol:

$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}, x \neq -1, \frac{1}{3}$

$\Rightarrow \frac{3}{x+1} - \frac{2}{3x-1} = \frac{1}{2}$

$\Rightarrow \frac{9x-3-2x-2}{(x+1)(3x-1)} = \frac{1}{2}$

$\Rightarrow \frac{7x-5}{3x^2+2x-1} = \frac{1}{2}$

$\Rightarrow 3x^2 + 2x - 1 = 14x - 10$ (Cross multiplication)

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x-3) - 1(x-3) = 0$

$\Rightarrow (x-3)(x-1) = 0$

$\Rightarrow x - 3 = 0 \; or \; x - 1 = 0$

$\Rightarrow x = 3 \; or \; x = 1$

Hence, 1 and 3 are the roots of the given equation.

55.

Sol:

$$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x\neq 1,-5$$

$$\Rightarrow \frac{x+5-x+1}{(x-1)(x+5)}=\frac{6}{7}$$

$$\Rightarrow \frac{6}{x^2+4x-5}=\frac{6}{7}$$

$$\Rightarrow x^2+4x-5=7$$

$$\Rightarrow x^2+4x-12=0$$

$$\Rightarrow x^2+6x-2x-12=0$$

$$\Rightarrow x(x+6)-2(x+6)=0$$

$$\Rightarrow (x+6)(x-2)=0$$

$$\Rightarrow x+6=0 \text{ or } x-2=0$$

$$\Rightarrow x=-6 \text{ or } x=2$$

Hence, -6 and 2 are the roots of the given equation.

56.

Sol:

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

$$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$$

$$\Rightarrow \frac{2x-2a-b-2x}{2x(2a+b+2x)}=\frac{2a+b}{2ab}$$

$$\Rightarrow \frac{-(2a+b)}{4x^2+4ax+2bx}=\frac{2a+b}{2ab}$$

$$\Rightarrow 4x^2+4ax+2bx=-2ab$$

$$\Rightarrow 4x^2+4ax+2bx+2ab=0$$

$$\Rightarrow 4x(x+a)+2b(x+a)=0$$

$$\Rightarrow (x+a)(4x+2b)=0$$

$$\Rightarrow x+a=0 \text{ or } 4x+2b=0$$

$$\Rightarrow x=-a \text{ or } x=-\frac{b}{2}$$

Hence, $-a$ and $-\frac{b}{2}$ are the roots of the give equation.

57.

Sol:
Given:

$$\frac{(x+3)}{(x-2)}-\frac{(1-x)}{x}=\frac{17}{4}$$

$$\Rightarrow \frac{x(x+3)-(1-x)(x-2)}{(x-2)x}=\frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x-(x-2-x^2+2x)}{x^2-2x}=\frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x+x^2-3x+2}{x^2-2x}=\frac{17}{4}$$

$$\Rightarrow \frac{2x^2+2}{x^2-2x}=\frac{17}{4}$$

$$\Rightarrow 8x^2+8=17x^2-34x \qquad \text{[On cross multiplying]}$$

$$\Rightarrow -9x^2+34x+8=0$$

$$\Rightarrow 9x^2-34x-8=0$$

$$\Rightarrow 9x^2-36x+2x-8=0$$

$$\Rightarrow 9x(x-4)+2(x-4)=0$$

$$\Rightarrow (x-4)(9x+2)=0$$

$$\Rightarrow x-4=0 \text{ or } 9x+2=0$$

$$\Rightarrow x=4 \text{ or } x=\frac{-2}{9}$$

Hence, the roots of the equation are 4 and $\frac{-2}{9}$.

58.

Sol:

$$\frac{3x-4}{7}+\frac{7}{3x-4}=\frac{5}{2}, x\neq\frac{4}{3}$$

$$\Rightarrow \frac{(3x-4)^2+49}{7(3x-4)}=\frac{5}{2}$$

$$\Rightarrow \frac{9x^2 - 24x + 16 + 49}{21x - 28} = \frac{5}{2}$$

$$\Rightarrow \frac{9x^2 - 24x + 65}{21x - 28} = \frac{5}{2}$$

$$\Rightarrow 18x^2 - 48x + 130 = 105x - 140$$

$$\Rightarrow 18x^2 - 153x + 270 = 0$$

$$\Rightarrow 2x^2 - 17x + 30 = 0$$

$$\Rightarrow 2x^2 - 12x - 5x + 30 = 0$$

$$\Rightarrow 2x(x-6) - 5(x-6) = 0$$

$$\Rightarrow (x-6)(2x-5) = 0$$

$$\Rightarrow x - 6 = 0 \ or \ 2x - 5 = 0$$

$$\Rightarrow x = 6 \ or \ x = \frac{5}{2}$$

Hence, 6 and $\frac{5}{2}$ are the roots of the given equation.

59.

Sol:

$$\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}, x \neq 0,1$$

$$\Rightarrow \frac{x^2 + (x-1)^2}{x(x-1)} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + x^2 - 2x + 1}{x^3 - x} = \frac{17}{4}$$

$$\Rightarrow \frac{2x^2 - 2x + 1}{x^2 - 1} = \frac{17}{4}$$

$$\Rightarrow 8x^2 - 8x + 4 = 17x^2 - 17x$$

$$\Rightarrow 9x^2 - 9x - 4 = 0$$

$$\Rightarrow 9x^2 - 12x + 3x - 4 = 0$$

$$\Rightarrow 3x(3x-4) + 1(3x-4) = 0$$

$$\Rightarrow (3x-4)(3x+1) = 0$$

$$\Rightarrow 3x - 4 = 0 \ or \ 3x + 1 = 0$$

$$\Rightarrow x = \frac{4}{3} \ or \ x = -\frac{1}{3}$$

Hence, $\frac{4}{3}$ and $-\frac{1}{3}$ are the roots of the given equation.

60.

Sol:

$$\frac{x}{x+1}+\frac{x+1}{x}=2\frac{4}{15}, x\neq 0,-1$$
$$\Rightarrow \frac{x^2+(x+1)^2}{x(x+1)}=\frac{34}{15}$$
$$\Rightarrow \frac{x^2+x^2+2x+1}{x^2+x}=\frac{34}{15}$$
$$\Rightarrow \frac{2x^2+2x+1}{x^2+x}=\frac{34}{15}$$
$$\Rightarrow 30x^2+30x+15=34x^2+34x$$
$$\Rightarrow 4x^2+4x-15=0$$
$$\Rightarrow 4x^2+10x-6x-15x=0$$
$$\Rightarrow 2x(2x+5)-3(2x+5)=0$$
$$\Rightarrow (2x+5)(2x-3)=0$$
$$\Rightarrow 2x+5=0 \text{ or } 2x-3=0$$
$$\Rightarrow x=-\frac{5}{2} \text{ or } 2x-3=0$$

Hence, $-\frac{5}{2}$ and $\frac{3}{2}$ are the roots of the given equation.

61.

Sol:

$$\frac{x-4}{x-5}+\frac{x-6}{x-7}=3\frac{1}{3}, x\neq 5,7$$
$$\Rightarrow \frac{(x-4)(x-7)+(x-5)(x-6)}{(x-5)(x-7)}=\frac{10}{3}$$
$$\Rightarrow \frac{x^2-11x+28+x^2-11x+30}{x^2-12x+35}=\frac{10}{3}$$
$$\Rightarrow \frac{2x^2-22x+58}{x^2-12x+35}=\frac{10}{3}$$

$$\Rightarrow \frac{x^2-11x+29}{x^2-12x+35}=\frac{5}{3}$$
$$\Rightarrow 3x^2-33x+87=5x^2-60x+175$$
$$\Rightarrow 2x^2-27x+88=0$$
$$\Rightarrow 2x^2-16x-11x+88=0$$
$$\Rightarrow 2x(x-8)-11(x-8)=0$$
$$\Rightarrow (x-8)(2x-11)=0$$
$$\Rightarrow x-8=0 \text{ or } 2x-11=0$$
$$\Rightarrow x=8 \; or \; x=\frac{11}{2}$$

Hence, 8 and $\frac{11}{2}$ are the roots of the given equation.

62.

Sol:

$$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x\neq 2,4$$
$$\Rightarrow \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)}=\frac{10}{3}$$
$$\Rightarrow \frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8}=\frac{10}{3}$$
$$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$$
$$\Rightarrow \frac{x^2-5x+5}{x^2-6x+8}=\frac{5}{3}$$
$$\Rightarrow 3x^2-15x+15=5x^2-30x+40$$
$$\Rightarrow 2x^2-15x+25=0$$
$$\Rightarrow 2x^2-10x-5x+25=0$$
$$\Rightarrow 2x(x-5)-5(x-5)=0$$
$$\Rightarrow (x-5)(2x-5)=0$$
$$\Rightarrow x-5=0 \; or \; 2x-5=0$$
$$\Rightarrow x=5 \; or \; x=\frac{5}{2}$$

Hence, 5 and $\frac{5}{2}$ are the roots of the given equation.

63.

Sol:

$$\frac{1}{(x-2)}+\frac{2}{(x-1)}=\frac{6}{x}$$

$$\Rightarrow \frac{(x-1)+2(x-2)}{(x-1)(x-2)}=\frac{6}{x}$$

$$\Rightarrow \frac{3x-5}{x^2-3x+2}=\frac{6}{x}$$

$$\Rightarrow 3x^2-5x=6x^2-18x+12$$ [On cross multiplying]

$$\Rightarrow 3x^2-13x+12=0$$

$$\Rightarrow 3x^2-(9+4)x+12=0$$

$$\Rightarrow 3x^2-9x-4x+12=0$$

$$\Rightarrow 3x(x-3)-4(x-3)=0$$

$$\Rightarrow (3x-4)(x-3)=0$$

$$\Rightarrow 3x-4=0 \text{ or } x-3=0$$

$$\Rightarrow x=\frac{4}{3} \text{ or } x=3$$

64.

Sol:

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{5}{x+4}, x\neq -1,-2,-4$$

$$\Rightarrow \frac{x+2+2x+2}{(x+1)(x+2)}=\frac{5}{x+4}$$

$$\Rightarrow \frac{3x+4}{x^2+3x+2}=\frac{5}{x+4}$$

$$\Rightarrow (3x+4)(x+4)=5(x^2+3x+2)$$

$$\Rightarrow 3x^2+16x+16=5x^2+15x+10$$

$$\Rightarrow 2x^2-x-6=0$$

$$\Rightarrow 2x^2-4x+3x-6=0$$

$$\Rightarrow 2x(x-2)+3(x-2)=0$$

$$\Rightarrow (x-2)(2x+3)=0$$

$\Rightarrow 3x^2+16x+16=5x^2+15x+10$

$\Rightarrow 2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0 \text{ or } 2x+3=0$

$\Rightarrow x=2 \text{ or } x=-\frac{3}{2}$

Hence, 2 and $-\frac{3}{2}$ are the roots of the given equation.

65.

Sol:

$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5, x\neq\frac{1}{3},-\frac{3}{2}$

$\Rightarrow \frac{3(3x-1)^2-2(2x+3)^2}{(2x+3)(3x-1)}=5$

$\Rightarrow \frac{3(9x^2-6x+1)-2(4x^2+12x+9)}{6x^2+7x-3}=5$

$\Rightarrow \frac{27x^2-18x+3-8x^2-24x-18}{6x^2+7x-3}=5$

$\Rightarrow \frac{19x^2-42x-15}{6x^2+7x-3}=5$

$\Rightarrow 19x^2-42x-15=30x^2+35x-15$

$\Rightarrow 11x^2+77x=0$

$\Rightarrow 11x(x+7)=0$

$\Rightarrow x=0 \text{ or } x+7=0$

$\Rightarrow x=0 \text{ or } x=-7$

Hence, 0 and -7 are the roots of the given equation.

66.

Sol:

$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11, x\neq\frac{3}{5},-\frac{1}{7}$

$\Rightarrow\frac{3(7x+1)^2-4(5x-3)^2}{(5x-3)(7x+1)}=11$

$\Rightarrow\frac{3(49x^2+14x+1)-4(25x^3-30x+9)}{35x^3-16x-3}=11$

$\Rightarrow\frac{147x^2+42x+3-100x^2+120x-36}{35x^2-16x-3}=11$

$\Rightarrow\frac{47x^2+162x-33}{35x^2-16x-3}=11$

$\Rightarrow 47x^2+162x-33=385x^2-176x-33$

$\Rightarrow 338x^2-338x=0$

$\Rightarrow 338x(x-1)=0$

$\Rightarrow x=0 \text{ or } x-1=0$

$\Rightarrow x=0 \text{ or } x=1$

Hence, 0 and 1 are the roots of the given equation.

67.

Sol:

Given:

$\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3$

Putting $\frac{4x-3}{2x+1}=y,$ we get:

$y-\frac{10}{y}=3$

$\Rightarrow\frac{y^2-10}{y}=3$

$\Rightarrow y^2-10=3y$ [On cross multiplying]

$\Rightarrow y^2-3y-10=0$

$\Rightarrow y^2-(5-2)y-10=0$

$\Rightarrow y^2-5y+2y-10=0$

$\Rightarrow y(y-5)+2(y-5)=0$

$\Rightarrow(y-5)(y+2)=0$

$\Rightarrow y-5=0 \text{ or } y+2=0$

$\Rightarrow y=5 \text{ or } y=-2$

Case I:

If $y=5$, we get:

$\frac{4x-3}{2x+1}=5$

$\Rightarrow 4x-3=5(2x+1)$ [On cross multiplying]

$\Rightarrow 4x-3=10x+5$

$\Rightarrow -6x=8$

$\Rightarrow -6x=8$

$\Rightarrow x=\frac{8}{6}$

$\Rightarrow x=-\frac{4}{3}$

Case II:

If $y=-2$, we get:

$\frac{4x-3}{2x+1}=-2$

$\Rightarrow 4x-3=-2(2x+1)$

$\Rightarrow 4x-3=-4x-2$

$\Rightarrow 8x=1$

$\Rightarrow x=\frac{1}{8}$

Hence, the roots of the equation are $-\frac{4}{3}$ and $\frac{1}{8}$.

68.

Sol:

$\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+6=0$

Putting $\frac{x}{x+1}=y$, we get:

$y^2-5y+6=0$

$\Rightarrow y^2-5y+6=0$

$\Rightarrow y^2-(3+2)y+6=0$

$\Rightarrow y^2-3y-2y+6=0$

$\Rightarrow y(y-3)-2(y-3)=0$

$\Rightarrow (y-3)(y-2)=0$

$\Rightarrow y-3=0 \; or \; y-2=0$

$\Rightarrow y=3 \; or \; y=2$

Case I:

If $y=3$, we get

$\frac{x}{x+1}=3$

$\Rightarrow x=3(x+1)$ [On cross multiplying]

$\Rightarrow x=3x+3$

$\Rightarrow x=\frac{-3}{2}$

Case II:

If $y=2$, we get:

$\frac{x}{x+1}=2$

$\Rightarrow x=2(x+1)$

$\Rightarrow x=2x+2$

$\Rightarrow -x=2$

$\Rightarrow x=-2$

Hence, the roots of the equation are $\frac{-3}{2}$ and -2.

69.

Sol:

$\frac{a}{(x-b)}+\frac{b}{(x-a)}=2$

$\Rightarrow \left[\frac{a}{(x-b)}-1\right]+\left[\frac{b}{(x-b)}-1\right]=0$

$\Rightarrow \frac{a-(x-b)}{x-b}+\frac{b-(x-b)}{x-b}=0$

$$\Rightarrow (a-x+b)\left[\frac{1}{(x-b)}+\frac{1}{(x-a)}\right]=0$$

$$\Rightarrow (a-x+b)\left[\frac{(x-a)+(x-b)}{(x-b)(x-a)}\right]=0$$

$$\Rightarrow (a-x+b)\left[\frac{2x-(a+b)}{(x-b)(x-a)}\right]=0$$

$$\Rightarrow (a-x+b)\left[2x-(a+b)\right]=0$$

$$\Rightarrow a-x+b=0 \ or \ 2x-(a+b)=0$$

$$\Rightarrow x=a+b \ or \ x=\frac{a+b}{2}$$

Hence, the roots of the equation are $(a+b)$ and $\left(\frac{a+b}{2}\right)$.

70.

Sol:

$$\frac{a}{(ax-1)}+\frac{b}{(bx-1)}=(a+b)$$

$$\Rightarrow \left[\frac{a}{(ax-1)}-b\right]+\left[\frac{b}{(bx-1)}-a\right]=0$$

$$\Rightarrow \frac{a-b(ax-1)}{ax-1}+\frac{b-a(bx-1)}{bx-1}=0$$

$$\Rightarrow \frac{a-abx+b}{ax-1}+\frac{a-abx+b}{bx-1}=0$$

$$\Rightarrow (a-abx+b)\left[\frac{1}{ax-1}+\frac{1}{(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[\frac{(bx-1)+(ax-1)}{(ax-1)(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[\frac{(a+b)x-2}{(ax-1)(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[(a+b)x-2\right]=0$$

$$\Rightarrow a-abx+b=0 \ or \ (a+b)x-2=0$$

$\Rightarrow x = \frac{(a+b)}{ab}$ or $x = \frac{2}{(a+b)}$

Hence, the roots of the equation are $\frac{(a+b)}{ab}$ and $\frac{2}{(a+b)}$.

71.

Sol:

$3^{(x+2)} + 3^{-x} = 10$

$3^x.9 + \frac{1}{3^x} = 10$

Let 3^x be equal to y.

$\therefore 9y + \frac{1}{y} = 10$

$\Rightarrow 9y^2 + 1 = 10y$

$\Rightarrow 9y^2 - 10y + 1 = 0$

$\Rightarrow (y-1)(9y-1) = 0$

$\Rightarrow y - 1 = 0$ or $9y - 1 = 0$

$\Rightarrow y = 1$ or $y = \frac{1}{9}$

$\Rightarrow 3x^x = 1$ or $3^x = \frac{1}{9}$

$\Rightarrow 3^x = 3^0$ or $3^x = 3^{-2}$

$\Rightarrow x = 0$ or $x = -2$

Hence, 0 and -2 are the roots of the given equation.

72.

Sol:

Given:

$4^{(x+1)} + 4^{(1-x)} = 10$

$\Rightarrow 4^x.4 + 4^1.\frac{1}{4^4} = 10$

Let 4^x be y.

$\therefore 4y + \frac{4}{y} = 10$

$\Rightarrow 4y^2 - 10y + 4 = 0$

$\Rightarrow 4y^2 - 8y - 2y + 4 = 0$

$\Rightarrow 4y(y-2)-2(y-2)=0$

$\Rightarrow y=2 \text{ or } y=\frac{2}{4}=\frac{1}{2}$

$\Rightarrow 4^x=2 \text{ or } \frac{1}{2}$

$\Rightarrow 4^x=2^{2x}=2^1 \text{ or } 2^{2x}=2^{-1}$

$\Rightarrow x=\frac{1}{2} \text{ or } x=\frac{-1}{2}$

Hence, $\frac{1}{2}$ and $\frac{-1}{2}$ are roots of the given equation.

73.

Sol:

$2^{2x}-3.2^{(x+2)}+32=0$

$\Rightarrow (2^x)^2-3.2^x.2^2+32=0$

Let 2^x be y.

$\therefore y^2-12y+32=0$

$\Rightarrow y^2-8y-4y+32=0$

$\Rightarrow y(y-8)-4(y-8)=0$

$\Rightarrow (y-8)=0 \text{ or } (y-4)=0$

$\Rightarrow y=8 \text{ or } y=4$

$\therefore 2^x=8 \text{ or } 2^x=4$

$\Rightarrow 2^x=2^3 \text{ or } 2^x=2^2$

$\Rightarrow x-2 \text{ or } 3$

Hence, 2 and 3 are the roots of the given equation.

Exercise - 10B

1.

Sol:

$x^2-6x+3=0$

$\Rightarrow x^2-6x=-3$

$\Rightarrow x^2-2\times x\times 3+3^2=-3+3^2$ (Adding 3^2 on both sides)

$\Rightarrow (x-3)^2=-3+9=6$

$\Rightarrow x-3=\pm\sqrt{6}$ (Taking square root on the both sides)

$\Rightarrow x-3=\sqrt{6}\ or\ x-3=-\sqrt{6}$

$\Rightarrow x=3+\sqrt{6}\ or\ x=3-\sqrt{6}$

Hence, $3+\sqrt{6}$ and $3-\sqrt{6}$ are the roots of the given equation.

2.

Sol:

$x^2-4x+1=0$

$\Rightarrow x^2-4x=-1$

$\Rightarrow x^2-2\times x\times 2+2^2=-1+2^2$ (Adding 2^2 on both sides)

$\Rightarrow (x-2)^2=-1+4=3$

$\Rightarrow x-2=\pm\sqrt{3}$ (Taking square root on the both sides)

$\Rightarrow x-2=\sqrt{3}\ or\ x-2=-\sqrt{3}$

$\Rightarrow x=2+\sqrt{3}\ or\ x=2-\sqrt{3}$

Hence, $2+\sqrt{3}$ and $2-\sqrt{3}$ are the roots of the given equation.

3.

Sol:

$x^2+8x-2=0$

$\Rightarrow x^2+8x=2$

$\Rightarrow x^2+2\times x\times 4+4^2=2+4^2$ (Adding 4^2 on both sides)

$\Rightarrow (x+4)^2=2+16=18$

$\Rightarrow x+4=\pm\sqrt{18}=\pm 3\sqrt{2}$ (Taking square root on the both sides)

$\Rightarrow x+4=3\sqrt{2}\ or\ x+4=-3\sqrt{2}$

$\Rightarrow x=-4+3\sqrt{2}\ or\ x=-4-3\sqrt{2}$

Hence, $\left(-4+3\sqrt{2}\right)$ and $\left(-4-3\sqrt{2}\right)$ are the roots of the given equation.

4.

Sol:

$4x^2+4\sqrt{3}x+3=0$

$\Rightarrow 4x^2+4\sqrt{3}x=-3$

$\Rightarrow (2x)^2+2\times 2x\times\sqrt{3}+\left(\sqrt{3}\right)^2=-3+\left(\sqrt{3}\right)^2$ [Adding $\left(\sqrt{3}\right)^2$ on both sides]

$\Rightarrow \left(2x+\sqrt{3}\right)^2 = -3+3=0$

$\Rightarrow 2x+\sqrt{3}=0$

$\Rightarrow x=-\frac{\sqrt{3}}{2}$

Hence, $-\frac{\sqrt{3}}{2}$ is the repeated root of the given equation.

5.

Sol:

$2x^2+5x-3=0$

$\Rightarrow 4x^2+10x-6=0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2+10x=6$

$\Rightarrow (2x)^2+2\times 2x\times\frac{5}{2}+\left(\frac{5}{2}\right)^2=6+\left(\frac{5}{2}\right)^2$ [Adding $\left(\frac{5}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x+\frac{5}{2}\right)^2=6+\frac{25}{4}=\frac{24+25}{4}=\frac{49}{4}=\left(\frac{7}{2}\right)^2$

$\Rightarrow 2x+\frac{5}{2}=\pm\frac{7}{2}$ (Taking square root on both sides)

$\Rightarrow 2x+\frac{5}{2}=\frac{7}{2}$ *or* $2x+\frac{5}{2}=-\frac{7}{2}$

$\Rightarrow 2x=\frac{7}{2}-\frac{5}{2}=\frac{2}{3}=1$ or $2x=-\frac{7}{2}-\frac{5}{2}=-\frac{12}{2}=-6$

$x=\frac{1}{2}$ or $x=-3$

Hence, $\frac{1}{2}$ and -3 are the roots of the given equation.

6.

Sol:

$3x^2-x-2=0$

$\Rightarrow 9x^2-3x-6=0$ (Multiplying both sides by 3)

$\Rightarrow 9x^2-3x=6$

$\Rightarrow (3x)^2-2\times 3x\times\frac{1}{2}+\left(\frac{1}{2}\right)^2=6+\left(\frac{1}{2}\right)^2$ [Adding $\left(\frac{1}{2}\right)^2$ on both sides]

$$\Rightarrow \left(3x-\frac{1}{2}\right)^2 = 6+\frac{1}{4}=\frac{25}{4}=\left(\frac{5}{2}\right)^2$$

$\Rightarrow 3x-\frac{1}{2}=\pm\frac{5}{2}$ (Taking square root on both sides)

$$\Rightarrow 3x-\frac{1}{2}=\frac{5}{2} \; or \; 3x-\frac{1}{2}=-\frac{5}{2}$$

$$\Rightarrow 3x=\frac{5}{2}+\frac{1}{2}=\frac{6}{2}=3 \text{ or } 3x=-\frac{5}{2}+\frac{1}{2}=-\frac{4}{2}=-2$$

$$\Rightarrow x=1 \; or \; x=-\frac{2}{3}$$

Hence, 1 and $-\frac{2}{3}$ are the roots of the given equation.

7.

Sol:

$8x^2-14x-15=0$

$\Rightarrow 16x^2-28x-30=0$ (Multiplying both sides by 2)

$\Rightarrow 16x^2-28x=30$

$\Rightarrow (4x)^2-2\times 4x\times\frac{7}{2}+\left(\frac{7}{2}\right)^2=30+\left(\frac{7}{2}\right)^2$ [Adding $\left(\frac{7}{2}\right)^2$ on both sides]

$$\Rightarrow \left(4x-\frac{7}{2}\right)^2=30+\frac{49}{4}=\frac{169}{4}=\left(\frac{13}{2}\right)^2$$

$\Rightarrow 4x-\frac{7}{2}=\pm\frac{13}{2}$ (Taking square root on both sides)

$$\Rightarrow 4x-\frac{7}{2}=\frac{13}{2} \; or \; 4x-\frac{7}{2}=\frac{13}{2}$$

$$\Rightarrow 4x=\frac{13}{2}+\frac{7}{2}=\frac{20}{2}=10 \text{ or } 4x=-\frac{13}{2}+\frac{7}{2}=-\frac{6}{2}=-3$$

$$\Rightarrow x=\frac{5}{2} \; or \; x=-\frac{3}{4}$$

Hence, $\frac{5}{2}$ and $-\frac{3}{4}$ are the roots of the given equation.

8.

Sol:

$7x^2+3x-4=0$

$\Rightarrow 49x^2+21x-28=0$ (Multiplying both sides by 7)

$\Rightarrow 49x^2 + 21x = 28$

$\Rightarrow (7x)^2 + 2\times 7x\times \frac{3}{2} + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\Rightarrow \left(7x+\frac{3}{2}\right)^2 = 28 + \frac{9}{4} = \frac{121}{4} = \left(\frac{11}{2}\right)^2$

$\Rightarrow 7x + \frac{3}{2} = \pm\frac{11}{2}$ (Taking square root on both sides)

$\Rightarrow 7x + \frac{3}{2} = \frac{11}{2}$ *or* $7x + \frac{3}{2} = -\frac{11}{2}$

$\Rightarrow 7x = \frac{11}{2} - \frac{3}{2} = \frac{8}{2} = 4$ or $7x = -\frac{11}{2} - \frac{3}{2} = -\frac{14}{2} = -7$

$\Rightarrow x = \frac{4}{7}$ *or* $x = -1$

Hence, $\frac{4}{7}$ and -1 are the roots of the given equation.

9.

Sol:

$3x^2 - 2x - 1 = 0$

$\Rightarrow 9x^2 - 6x - 3 = 0$ (Multiplying both sides by 3)

$\Rightarrow 9x^2 - 6x = 3$

$\Rightarrow (3x)^2 - 2\times 3x\times 1 + 1^2 = 3 + 1^2$ [Adding 1^2 on both sides]

$\Rightarrow (3x-1)^2 = 3 + 1 = 4 = (2)^2$

$\Rightarrow 3x - 1 = \pm 2$ (Taking square root on both sides)

$\Rightarrow 3x - 1 = 2$ or $3x - 1 = -2$

$\Rightarrow 3x = 3$ or $3x = -1$

$\Rightarrow x = 1$ or $x = -\frac{1}{3}$

Hence, 1 and $-\frac{1}{3}$ are the roots of the given equation.

10.

Sol:

$5x^2 - 6x - 2 = 0$

$\Rightarrow 25x^2 - 30x - 10 = 0$ (Multiplying both sides by 5)

$\Rightarrow 25x^2 - 30x = 10$

$\Rightarrow (5x)^2 - 2\times 5x\times 3 + 3^2 = 10 + 3^2$ (Adding 3^2 on both sides)

$\Rightarrow (5x-3)^2 = 10 + 9 - 19$

$\Rightarrow 5x - 3 = \pm\sqrt{19}$ (Taking square root on both)

$\Rightarrow 5x - 3 = \sqrt{19}$ *or* $5x - 3 = -\sqrt{19}$

$\Rightarrow 5x = 3 + \sqrt{19}$ *or* $5x = 3 - \sqrt{19}$

$\Rightarrow x = \frac{3+\sqrt{19}}{5}$ *or* $x = \frac{3-\sqrt{19}}{5}$

Hence, $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$ are $\frac{3-\sqrt{19}}{5}$ are the roots of the given equation.

11.

Sol:

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$\Rightarrow \frac{2 - 5x + 2x^2}{x^2} = 0$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 4x^2 - 10x + 4 = 0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2 - 10x = -4$

$\Rightarrow (2x)^2 - 2\times 2x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$ [Adding $\left(\frac{5}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x - \frac{5}{2}\right)^2 = -4 + \frac{25}{4} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

$\Rightarrow 2x - \frac{5}{2} = \pm\frac{3}{2}$ (Taking square root on both sides)

$\Rightarrow 2x - \frac{5}{2} = \frac{3}{2}$ or $2x - \frac{5}{2} = -\frac{3}{2}$

$\Rightarrow 2x = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$ or $2x = -\frac{3}{2} + \frac{5}{2} = \frac{2}{2} = 1$

$\Rightarrow x = 2$ or $x = \frac{1}{2}$

Hence, 2 and $\frac{1}{2}$ are the roots of the given equation.

12.

Sol:

$4x^2+4bx-\left(a^2-b^2\right)=0$

$\Rightarrow 4x^2+4bx=a^2-b^2$

$\Rightarrow (2x)^2+2\times 2x\times b+b^2=a^2-b^2+b^2$ (Adding b^2 on both sides)

$\Rightarrow (2x+b)^2=a^2$

$\Rightarrow 2x+b=\pm a$ (Taking square root on both sides)

$\Rightarrow 2x+b=a \text{ or } 2x+b=-a$

$\Rightarrow 2x=a-b \text{ or } 2x=-a-b$

$\Rightarrow x=\frac{a-b}{2} \text{ or } x=-\frac{a+b}{2}$

Hence, $\frac{a-b}{2}$ and $-\frac{a+b}{2}$ are the roots of the given equation.

13.

Sol:

$x^2-\left(\sqrt{2}+1\right)x+\sqrt{2}=0$

$\Rightarrow x^2-\left(\sqrt{2}+1\right)x=-\sqrt{2}$

$\Rightarrow x^2-2\times x\times\left(\frac{\sqrt{2}+1}{2}\right)+\left(\frac{\sqrt{2}+1}{2}\right)^2=-\sqrt{2}+\left(\frac{\sqrt{2}+1}{2}\right)^2$

[Adding $\left(\frac{\sqrt{2}+1}{2}\right)^2$ on both sides]

$\Rightarrow \left[x-\left(\frac{\sqrt{2}+1}{2}\right)\right]^2=\frac{-4\sqrt{2}+2+1+2\sqrt{2}}{4}=\frac{2-2\sqrt{2}+1}{4}=\left(\frac{\sqrt{2}-1}{2}\right)^2$

$\Rightarrow x-\left(\frac{\sqrt{2}+1}{2}\right)=\pm\left(\frac{\sqrt{2}-1}{2}\right)$ (Taking square root on both sides)

$\Rightarrow x-\left(\frac{\sqrt{2}+1}{2}\right)=\left(\frac{\sqrt{2}-1}{2}\right) \text{ or } x-\left(\frac{\sqrt{2}+1}{2}\right)=-\left(\frac{\sqrt{2}-1}{2}\right)$

$\Rightarrow x=\frac{\sqrt{2}+1}{2}+\frac{\sqrt{2}-1}{2}$ or $x=\frac{\sqrt{2}+1}{2}-\frac{\sqrt{2}-1}{2}$

$\Rightarrow x = \frac{2\sqrt{2}}{2} = \sqrt{2}$ or $x = \frac{2}{2} = 1$

Hence, $\sqrt{2}$ and 1 are the roots of the given equation.

14.

Sol:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$\Rightarrow 2x^2 - 3\sqrt{2}x - 4 = 0$ (Multiplying both sides by $\sqrt{2}$)

$\Rightarrow 2x^2 - 3\sqrt{2}x = 4$

$\Rightarrow \left(\sqrt{2}x\right)^2 - 2\times\sqrt{2}x\times\frac{3}{2} + \left(\frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\Rightarrow \left(\sqrt{2}x - \frac{3}{2}\right)^2 = 4 + \frac{9}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$

$\Rightarrow \sqrt{2}x - \frac{3}{2} = \pm\frac{5}{2}$ (Taking square root on both sides)

$\Rightarrow \sqrt{2}x - \frac{3}{2} = \frac{5}{2}$ or $\sqrt{2}x - \frac{3}{2} = -\frac{5}{2}$

$\Rightarrow \sqrt{2}x = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$ or $\sqrt{2}x = -\frac{5}{2} + \frac{3}{2} = -\frac{2}{2} = -1$

$\Rightarrow x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ or $x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Hence, $2\sqrt{2}$ and $-\frac{\sqrt{2}}{2}$ are the roots of the given equation.

15.

Sol:

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow 3x^2 + 10\sqrt{3}x + 21 = 0$ (Multiplying both sides by $\sqrt{3}$)

$\Rightarrow 3x^2 + 10\sqrt{3}x = -21$

$\Rightarrow \left(\sqrt{3}x\right)^2 + 2\times\sqrt{3}x\times5 + 5^2 = -21 + 5^2$ (Adding 5^2 on both sides)

$\Rightarrow \left(\sqrt{3}x + 5\right)^2 = 21 + 25 = 4 = 2^2$

$\Rightarrow \sqrt{3}x + 5 = \pm2$ (Taking square root on both sides)

$\Rightarrow \sqrt{3}x + 5 = 2$ or $\sqrt{3}x + 5 = -2$

$\Rightarrow \sqrt{3}x = -3$ or $\sqrt{3}x = -7$

$\Rightarrow x = -\frac{3}{\sqrt{3}} = -\sqrt{3}$ or $x = -\frac{7}{\sqrt{3}} = -\frac{7\sqrt{3}}{3}$

Hence, $-\sqrt{3}$ and $-\frac{7\sqrt{3}}{3}$ are the roots of the given equation.

16.

Sol:

$2x^2 + x + 4 = 0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2 + 2x + 8 = 0$

$\Rightarrow 4x^2 + 2x = -8$

$\Rightarrow (2x)^2 + 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 = -8 + \left(\frac{1}{2}\right)^2$ [Adding $\left(\frac{1}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x + \frac{1}{2}\right)^2 = -8 + \frac{1}{4} = -\frac{31}{4} < 0$

But, $\left(2x + \frac{1}{2}\right)^2$ cannot be negative for any real value of x.

So, there is no real value of x satisfying the given equation.
Hence, the given equation has no real roots.

Exercise -10C

1.

Sol:

(i) $2x^2 - 7x + 6 = 0$

Here,

$a = 2,$

$b = -7,$

$c = 6$

Discriminant D is diven by:

$D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 6$

$= 49 - 48$

$= 1$

(ii) $3x^2 - 2x + 8 = 0$

Here,

$a = 3,$

$b = -2,$

$c = 8$

Discriminant D is given by:

$D = b^2 - 4ac$

$= (-2)^2 - 4 \times 3 \times 8$

$= 4 - 96$

$= -92$

(iii) $2x^2 - 5\sqrt{2x} + 4 = 0$

Here,

$a = 2,$

$b = -5\sqrt{2},$

$c = 4$

Discriminant D is given by:

$D = b^2 - 4ac$

$= \left(-5\sqrt{2}\right)^2 - 4 \times 2 \times 4$

$= (25 \times 2) - 32$

$= 50 - 32$

$= 18$

(iv) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Here,

$a = \sqrt{3}$

$b = 2\sqrt{2},$

$c = -2\sqrt{3}$

Discriminant D is given by:

$D = b^2 - 4ac$

$= \left(2\sqrt{2}\right)^2 - 4 \times \sqrt{3} \times \left(-2\sqrt{3}\right)$

$= (4 \times 2) + (8 \times 3)$

$= 8 + 24$

$= 32$

(v) $(x-1)(2x-1) = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = -3 \ and \ c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 1 = 9 - 8 = 1$

(vi) $1 - x = 2x^2$

$\Rightarrow 2x^2 + x - 1 = 0$

Here,

$a = 2,$

$b = 1,$

$c = -1$

Discriminant D is given by:

$D = b^2 - 4ac$

$= 1^2 - 4 \times 2(-1)$

$= 1 + 8$

$= 9$

Find the roots of the each of the following equations, if they exist, by applying the quadratic formula:

2.

Sol:

Given:

$x^2 - 4x - 1 = 0$

On comparing it with $ax^2 + bx + c = 0,$ we get:

$a = 1, b = -4 \ and \ c = -1$

Discriminant D is given by:

$D = (b^2 - 4ac)$

$= (-4)^2 - 4 \times 1 \times (-1)$

$= 16 + 4$

$= 20$

$= 20 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-4) + \sqrt{20}}{2 \times 1} = \frac{4 + 2\sqrt{5}}{2} = \frac{2(2 + \sqrt{5})}{2} = (2 + \sqrt{5})$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-4) - \sqrt{20}}{2} = \frac{4 - 2\sqrt{5}}{2} = \frac{2(2 - \sqrt{5})}{2} = (2 - \sqrt{5})$$

Thus, the roots of the equation are $\left(2+\sqrt{5}\right)$ and $\left(2-\sqrt{5}\right)$.

3.

Given:

$x^2-6x+4=0$

On comparing it with $ax^2+bx+c=0,$ we get:

$a=1, b=-6$ *and* $c=4$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=\left(-6\right)^2-4\times1\times4$

$=36-16$

$=20>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-\left(-6\right)+\sqrt{20}}{2\times1}=\frac{6+2\sqrt{5}}{2}=\frac{2\left(3+\sqrt{3}\right)}{2}=\left(3+\sqrt{5}\right)$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-\left(-6\right)-\sqrt{20}}{2}=\frac{6-2\sqrt{5}}{2}=\frac{2\left(3-\sqrt{5}\right)}{2}=\left(3-\sqrt{5}\right)$$

Thus, the roots of the equation are $\left(3+2\sqrt{5}\right)$ and $\left(3-2\sqrt{5}\right)$.

4.

Sol:

The given equation is $2x^2+x-4=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=2, b=1$ and $c=-4$

$\therefore$ Discriminant, $D=b^2-4ac=\left(1\right)^2-4\times2\times\left(-4\right)=1+32=33>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{33}$

$$\therefore\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-1+\sqrt{33}}{2\times2}=\frac{-1+\sqrt{33}}{4}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-1-\sqrt{33}}{2\times2}=\frac{-1-\sqrt{33}}{4}$$

Hence, $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$ are the roots of the given equation.

5.

Sol:

Given:

$25x^2+30x+7=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=25, b=30 \text{ and } c=7$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=30^2-4\times25\times7$

$=900-700$

$=200$

$=200>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-30+\sqrt{200}}{2\times25}=\frac{-30+10\sqrt{2}}{50}=\frac{10\left(-3+\sqrt{2}\right)}{50}=\frac{\left(-3+\sqrt{2}\right)}{5}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-30-\sqrt{200}}{2\times25}=\frac{-30-10\sqrt{2}}{50}=\frac{10\left(-3-\sqrt{2}\right)}{50}=\frac{\left(-3-\sqrt{2}\right)}{5}$$

Thus, the roots of the equation are $\frac{\left(-3+\sqrt{2}\right)}{5}$ *and* $\frac{\left(-3-\sqrt{2}\right)}{5}$.

6.

Sol:

Given:

$16x^2+24x+1$

$\Rightarrow 16x^2-24x-1=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=16, b=-24 \text{ and } c=-1$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=\left(-24\right)^2-4\times16\times\left(-1\right)$

$= 576 + (64)$

$= 640 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-24)+\sqrt{640}}{2\times 16} = \frac{24+8\sqrt{10}}{32} = \frac{8(3+\sqrt{10})}{32} = \frac{(3+\sqrt{10})}{4}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-24)-\sqrt{640}}{2\times 16} = \frac{24-8\sqrt{10}}{32} = \frac{8(3-\sqrt{10})}{32} = \frac{(3-\sqrt{10})}{4}$$

Thus, the roots of the equation are $\frac{(3+\sqrt{10})}{4}$ *and* $\frac{(3-\sqrt{10})}{4}$.

7.

Sol:

Given:

$15x^2 - 28 = x$

$\Rightarrow 15x^2 - x - 28 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get;

$a = 25, b = -1$ *and* $c = -28$

Discriminant D is given by:

$D = (b^2 - 4ac)$

$= (-1)^2 - 4\times 15\times(-28)$

$= 1 - (-1680)$

$= 1 + 1680$

$= 1681$

$= 1681 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-1)+\sqrt{1681}}{2\times 25} = \frac{1+41}{30} = \frac{42}{30} = \frac{7}{5}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-1)-\sqrt{1681}}{2\times 25} = \frac{1-41}{30} = \frac{-40}{30} = \frac{-4}{3}$$

Thus, the roots of the equation are $\frac{7}{5}$ and $\frac{-4}{3}$.

8.

Sol:

The given equation is $2x^2-2\sqrt{2}x+1=0$

Comparing it with $ax^2+bx+c=0,$ we get

$a=2, b=-2\sqrt{2}$ *and* $c=1$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-2\sqrt{2}\right)^2-4\times2\times1=8-8=0$

So, the given equation has real roots.

Now, $\sqrt{D}=0$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-\left(-2\sqrt{2}\right)}{2\times2}=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-\left(-2\sqrt{2}\right)-\sqrt{0}}{2\times2}=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}$$

Hence, $\frac{\sqrt{2}}{2}$ is the repeated root of the given equation.

9.

Sol:

The given equation is $\sqrt{2}x^2+7+5\sqrt{2}=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=\sqrt{2}, b=7$ *and* $c=5\sqrt{2}$

$\therefore$ Discriminant, $D=b^2-4ac=(7)^2-4\times\sqrt{2}\times5\sqrt{2}=49-40=9>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{9}=3$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-7+3}{2\times\sqrt{2}}=\frac{-4}{2\sqrt{2}}=-\sqrt{2}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-7-3}{2\times\sqrt{2}}=\frac{-10}{2\sqrt{2}}=-\frac{5\sqrt{2}}{2}$$

Hence, $-\sqrt{2}$ and $-\frac{5\sqrt{2}}{2}$ are the root of the given equation.

10.

Sol:

Given:

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

On comparing it with $ax^2 + bx + x = 0,$ we get;

$a = \sqrt{3}, b = 10 \ and \ c = -8\sqrt{3}$

Discriminant D is given by:

$D = \left(b^2 - 4ac\right)$

$= (10)^2 - 4 \times \sqrt{3} \times \left(-8\sqrt{3}\right)$

$= 100 + 96$

$= 196 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-10 + \sqrt{196}}{2\sqrt{3}} = \frac{-10 + 14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(10) - \sqrt{196}}{2\sqrt{3}} = \frac{-10 - 14}{2\sqrt{3}} = \frac{-24}{2\sqrt{3}} = \frac{-12}{\sqrt{3}} = \frac{-12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-12\sqrt{3}}{3} = -4\sqrt{3}$$

Thus, the roots of the equation are $\frac{2\sqrt{3}}{3}$ and $-4\sqrt{3}$.

11.

Sol:

The given equation is $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$.

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = \sqrt{3}, b = -2\sqrt{2} \ and \ c = -2\sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4 \times \sqrt{3} \times \left(-2\sqrt{3}\right) = 8 + 24 = 32 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{32} = 4\sqrt{2}$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right) + 4\sqrt{2}}{2 \times \sqrt{3}} = \frac{6\sqrt{2}}{2\sqrt{3}} = \sqrt{6}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right) - 4\sqrt{2}}{2 \times \sqrt{3}} = \frac{-2\sqrt{2}}{2\sqrt{3}} = -\frac{\sqrt{6}}{3}$$

Hence, $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$ are the root of the given equation.

12.

Sol:

The given equation is $2x^2 + 6\sqrt{3}x - 60 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = 6\sqrt{3}$ *and* $c = -60$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(6\sqrt{3}\right)^2 - 4 \times 2 \times (-60) = 180 + 480 = 588 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{588} = 14\sqrt{3}$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-6\sqrt{3} + 14\sqrt{3}}{2 \times 2} = \frac{8\sqrt{3}}{4} = 2\sqrt{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-6\sqrt{3} - 14\sqrt{3}}{2 \times 2} = \frac{-20\sqrt{3}}{4} = -5\sqrt{3}$$

Hence, $2\sqrt{3}$ and $-5\sqrt{3}$ are the root of the given equation.

13.

Sol:

The given equation is $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 4\sqrt{3}, b = 5$ *and* $c = -2\sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = 5^2 - 4 \times 4\sqrt{3} \times \left(-2\sqrt{3}\right) = 25 + 96 = 121 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{121} = 11$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-5 - 11}{2 \times 4\sqrt{3}} = \frac{6}{8\sqrt{3}} = \frac{\sqrt{3}}{4}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-5 - 11}{2 \times 4\sqrt{3}} = \frac{-16}{8\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Hence, $\frac{\sqrt{3}}{4}$ and $-\frac{2\sqrt{3}}{3}$ are the root of the given equation.

14.

Sol:

The given equation is $3x^2 - 2\sqrt{6}x + 2 = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 3, b = -2\sqrt{6}$ *and* $c = 2$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{6}\right)^2 - 4\times3\times2 = 24 - 24 = 0$

So, the given equation has real roots.

Now, $\sqrt{D} = 0$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{6}\right)+0}{2\times3} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{6}\right)}{2\times3} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$$

Hence, $\frac{\sqrt{6}}{3}$ are the repeated of the given equation.

15.

Sol:

The given equation is $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2\sqrt{3}, b = -5$ *and* $c = \sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-5)^2 - 4\times2\sqrt{3}\times\sqrt{3} = 25 - 25 = 1 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{1} = 1$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-5+1)+1}{2\times2\sqrt{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-5)-1}{2\times2\sqrt{3}} = \frac{4}{4\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Hence, $\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{3}$ are the roots of the given equation.

16.

Sol:

The given equation is $x^2 + x + 2 = 0.$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1, b = 1$ and $c = 2$

$\therefore$ Discriminant $D = b^2 - 4ac = 1^2 - 4\times1\times2 = 1 - 8 = -7 < 0$

Hence, the given equation has no real roots (or real roots does not exist).

17.

Sol:

The given equation is $2x^2 + ax - a^2 = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 2, B = a \text{ and } C = -a^2$

$\therefore$ Discriminant, $D = B^2 - 4AC = a^2 - 4\times 2\times -a^2 = a^2 + 8a^2 = 9a^2 \geq 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{9a^2} = 3a$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-a+3a}{2\times 2} = \frac{2a}{4} = \frac{a}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-a-3a}{2\times 2} = \frac{-4a}{4} = -a$$

Hence, $\frac{a}{2}$ and $-a$ are the roots of the given equation.

18.

Sol:

The given equation is $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 1, b = -\left(\sqrt{3}+1\right)$ and $c = \sqrt{3}$

$\therefore$ Discriminant,

$$D = b^2 - 4ac = \left[-\left(\sqrt{3}+1\right)\right]^2 - 4\times 1\times \sqrt{3} = 3+1+2\sqrt{3}-4\sqrt{3} = 3-2\sqrt{3}+1 = \left(\sqrt{3}-1\right)^2 > 0$$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{\left(\sqrt{3}-1\right)^2} = \sqrt{3}-1$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-\left[-\left(\sqrt{3}+1\right)\right]+\left(\sqrt{3}-1\right)}{2\times 1} = \frac{\sqrt{3}+1+\sqrt{3}-1}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-\left[-\left(\sqrt{3}+1\right)\right]-\left(\sqrt{3}-1\right)}{2\times 1} = \frac{\sqrt{3}+1-\sqrt{3}+1}{2} = \frac{2}{2} = 1$$

Hence, $\sqrt{3}$ and 1 are the roots of the given equation.

19.

Sol:

The given equation is $2x^2 + 5\sqrt{3}x + 6 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = 5\sqrt{3}$ *and* $c = 6$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(5\sqrt{3}\right)^2 - 4\times 2\times 6 = 75 - 48 = 27 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{27} = 3\sqrt{3}$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-5\sqrt{3}+3\sqrt{3}}{2\times 2} = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-5\sqrt{3}-3\sqrt{3}}{2\times 2} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}$$

Hence, $-\frac{\sqrt{3}}{2}$ and $-2\sqrt{3}$ are the roots of the given equation.

20.

Sol:

The given equation is $3x^2 - 2x + 2 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 3, b = -2$ and $c = 2$

$\therefore$ Discriminant $D = b^2 - 4ac = (-2)^2 - 4\times 3\times 2 = 4 - 24 = -20 < 0$

Hence, the given equation has no real roots (or real roots does not exist).

21.

Sol:

The given equation is

$$x + \frac{1}{x} = 3, x \neq 0$$

$$\Rightarrow \frac{x^2+1}{x} = 3$$

$$\Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

This equation is of the form $ax^2 + bx + c = 0,$ where, $a = 1, b = -3$ and $c = 1.$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4\times 1\times 1 = 9 - 4 = 5 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{5}$

$\therefore a = \frac{-b+\sqrt{D}}{2a} = \frac{-(-3)+\sqrt{5}}{2\times 1} = \frac{3+\sqrt{5}}{2}$

$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-3)-\sqrt{5}}{2\times 1} = \frac{3-\sqrt{5}}{2}$

Hence, $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$ are the roots of the given equation.

22. –

Sol:

The given equation is

$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$

$\Rightarrow \frac{x-2-x}{x(x-2)} = 3$

$\Rightarrow \frac{-2}{x^2-2x} = 3$

$\Rightarrow -2 = 3x^2 - 6x$

$\Rightarrow 3x^2 - 6x + 2 = 0$

This equation is of the form $ax^2 + bx + c = 0$, where $a = 3, b = -6$ and $c = 2$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-6)^2 - 4\times 3\times 2 = 36 - 24 = 12 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{12} = 2\sqrt{3}$

$\therefore a = \frac{-b+\sqrt{D}}{2a} = \frac{-(-6)+2\sqrt{3}}{2\times 3} = \frac{6+2\sqrt{3}}{6} = \frac{3+\sqrt{3}}{3}$

$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-6)-2\sqrt{3}}{2\times 3} = \frac{6-2\sqrt{3}}{6} = \frac{3-\sqrt{3}}{3}$

Hence, $\frac{3+\sqrt{3}}{3}$ and $\frac{3-\sqrt{3}}{3}$ are the roots of the given equation.

23.

Sol:

The given equation is

$x - \frac{1}{x} = 3, x \neq 0$

$\Rightarrow \frac{x^2 - 1}{x} = 3$

$\Rightarrow x^2 - 1 = 3x$

$\Rightarrow x^2 - 3x - 1 = 0$

This equation is of the form $ax^2 + bx + c = 0$, where $a = 1, b = -3$ and $c = -1$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1) = 9 + 4 = 13 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{13}$

$\therefore a = \frac{-b + \sqrt{D}}{2a} = \frac{-(-3) + \sqrt{13}}{2 \times 1} = \frac{3 + \sqrt{13}}{2}$

$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-3) - \sqrt{13}}{2 \times 1} = \frac{3 - \sqrt{13}}{2 \times 1} = \frac{3 - \sqrt{13}}{2}$

Hence, $\frac{3 + \sqrt{13}}{3}$ and $\frac{3 - \sqrt{13}}{3}$ are the roots of the given equation.

24.

Sol:

The given equation is

$\frac{m}{n} x^2 \frac{n}{m} = 1 - 2x$

$\Rightarrow \frac{m^2 x^2 + n^2}{mn} = 1 - 2x$

$\Rightarrow m^2 x^2 + n^2 = mn - 2mnx$

$\Rightarrow m^2 x^2 + 2mnx + n^2 - mn = 0$

This equation is of the form $ax^2 + bx + c = 0$, where $a = m^2, b = 2mn$ and $c = n^2 - mn$

$\therefore$ Discriminant,

$D = b^2 - 4ac = (2mn)^2 - 4 \times m^2 \times (n^2 - mn) = 4m^2n^2 - 4m^2n^2 + 4m^3n^2 = 4m^3n > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{4m^3n} = 2m\sqrt{mn}$

$\therefore a = \frac{-b + \sqrt{D}}{2a} = \frac{-2mn + 2m\sqrt{mn}}{2 \times m^2} = \frac{2mn(-n + \sqrt{mn})}{2m^2} = \frac{-n + \sqrt{mn}}{m}$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-2mn-2m\sqrt{mn}}{2\times m^2} = \frac{-2m\left(n+\sqrt{mn}\right)}{2m^2} = \frac{-n+\sqrt{mn}}{m}$$

Hence, $\frac{-n+\sqrt{mn}}{m}$ and $\frac{-n-\sqrt{mn}}{m}$ are the roots of the given equation.

25.

Sol:

The given equation is $36x^2 - 12ax + \left(a^2 - b^2\right) = 0$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 36, B = -12a \ and \ C = a^2 - b^2$

$\therefore$ Discriminant,

$$D = B^2 - 4AC = (-12a)^2 - 4\times 36\times\left(a^2 - b^2\right) = 144a^2 - 144a^2 + 144b^2 = 144b^2 > 0$$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{144b^2} = 12b$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-12a)+12b}{2\times 36} = \frac{12(a+b)}{72} = \frac{a+b}{0}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-12a)-12b}{2\times 36} = \frac{12(a-b)}{72} = \frac{a-b}{6}$$

Hence, $\frac{a+b}{6}$ and $\frac{a-b}{6}$ are the roots of the given equation.

26.

Sol:

Given:

$x^2 - 2ax + \left(a^2 - b^2\right) = 0$

On comparing it with $Ax^2 + Bx + C = 0,$ we get:

$A = 1, B = -2a$ and $C = \left(a^2 - b^2\right)$

Discriminant D is given by:

$D = B^2 - 4AC$

$= (-2a)^2 - 4\times 1\times\left(a^2 - b^2\right)$

$= 4a^2 - 4a^2 + 4b^2$

$= 4b^2 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(-2a)+\sqrt{4b^2}}{2\times 1}=\frac{2a+2b}{2}=\frac{2(a+b}{2}=(a+b)$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-2a)-\sqrt{4b^2}}{2\times 1}=\frac{2a-2b}{2}=\frac{2(a-b)}{2}=(a-b)$$

Hence, the roots of the equation are $(a+b)$ and $(a-b)$.

27.

Sol:

The given equation is $x^2-2ax-(4b^2-a^2)=0$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-2a \ and \ C=-(4b^2-a^2)$

$\therefore$ Discriminant,

$B^2-4AC=(-2a)^2-4\times 1\times\left[-(4b^2-a^2)\right]=4a^2+16b^2-4a^2=16b^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{16b^2}=4b$

$$\therefore \alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-(-2a)+4b}{2\times 1}=\frac{2(a+2b)}{2}=a+2b$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-(-2a)-4b}{2\times 1}=\frac{2(a-2b)}{2}=a-2b$$

Hence, $a+2b$ and $a-2b$ are the roots of the given equation.

28.

Sol:

The given equation is $x^2+6x-(a^2+2a-8)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=6 \ and \ C=-(a^2+2a-8)$

$\therefore$ Discriminant, $D=$

$B^2-4AC=6^2-4\times 1\times\left[-(a^2+2a-8)\right]=36+4a^2+8a-32=4a^2+8a-32=4a^2+8a+4$

$=4(a^2+2a+1)=4(a+1)^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{4(a+1)^2}=2(a+1)$

$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-6+2(a+1)}{2\times1} = \frac{2a-4}{2} = a-2$

$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-6-2(a+1)}{2\times1} = \frac{-2a-8}{2} = -a-4 = -(a+4)$

Hence, $(a-2)$ and $-(a+4)$ are the roots of the given equation.

29.

Sol:

The given equation is $x^2+5x-(a^2+a-6)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=5$ *and* $C=-(a^2+a-8)$

$\therefore$ Discriminant, $D =$

$B^2-4AC=5^2-4\times1\times\left[-(a^2+a-6)\right]=25+4a^2+4a-24=4a^2+4a^2+4a+1$

$=(2a+1)^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{(2a+1)^2}=2a+1$

$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-5+2a+1}{2\times1} = \frac{2a-4}{2} = a-2$

$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-5-(2a+1)}{2\times1} = \frac{-2a-6}{2} = -a-3 = -(a+3)$

Hence, $(a-2)$ and $-(a+3)$ are the roots of the given equation.

30.

Sol:

The given equation is $x^2-4ax-b^2+4a^2=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-4a$ *and* $C=-b^2+4a^2$

$\therefore$ Discriminant, $D=B^2-4AC=(-4a)^2-4\times1\times(-b^2+4a^2)=16a^2+4b^2-16a^2=4b^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{4b^2}=2b$

$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-4a)+2b}{2\times1} = \frac{4a+2b}{2} = 2a+b$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-4a)-2b}{2\times1} = \frac{4a-2b}{2} = 2a-b$$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.

31.

Sol:

The given equation is $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 4, B = -4a^2 \ and \ C = a^4 - b^4$

$\therefore$ Discriminant, $B^2 - 4AC = (-4a^2)^2 - 4\times4\times(a^2 - b^4) = 16a^4 - 16a^4 + 16b^4 = 16b^4 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16b^4} = 4b^2$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-4a^2)+4b^2}{2\times4} = \frac{4(a^2+b^2)}{8} = \frac{a^2+b^2}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-4a^2)-4b^2}{2\times4} = \frac{4(a^2-b^2)}{8} = \frac{a^2-b^2}{2}$$

Hence, $\frac{1}{2}(a^2+b^2)$ and $\frac{1}{2}(a^2-b^2)$ are the roots of the given equation.

32.

Sol:

The given equation is $4x^2 - 4bx - (a^2 - b^2) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 4, B = 4b \ and \ C = -(a^2 - b^2)$

$\therefore$ Discriminant,

$D = B^2 - 4AC = (4b)^2 - 4\times4\times[-(a^2 - b^2)] = 16b^2 + 16a^2 - 16b^2 = 16a^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16a^2} = 4a$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-4b+4a}{2\times4} = \frac{4(a-b)}{8} = \frac{a-b}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-4b-4a}{2\times4} = \frac{-4(a+b)}{8} = -\frac{a+b}{2}$$

Hence, $\frac{1}{2}(a-b)$ and $-\frac{1}{2}(a+b)$ are the roots of the given equation.

33.

Sol:

The given equation is $x^2-(2b-1)x+(b^2-b-20)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-(2b-1)\ and\ C=b^2-b-20$

$\therefore$ Discriminant,

$D=B^2-4AC=\left[-(2b-1)\right]^2-4\times1\times(b^2-b-20)=4b^2-4b+1-4b^2+4b+80=81>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{18}=9$

$$\therefore \alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-\left[-(2b-1)\right]+9}{2\times1}=\frac{2b+8}{2}=b+4$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-\left[-(2b-1)\right]-9}{2\times1}=\frac{2b-10}{2}=b-5$$

Hence, $(b+4)$ and $(b-5)$ are the roots of the given equation.

34.

Sol:

Given:

$3a^2x^2+8abx+4b^2=0$

On comparing it with $Ax^2+Bx+C=0,$ we get:

$A=3a^2, B=8ab\ and\ C=4b^2$

Discriminant D is given by:

$D=(B^2-4AC)$

$=(8ab)^2-4\times3a^2\times4b^2$

$=16a^2b^2>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-8ab+\sqrt{16a^2b^2}}{2\times3a^2}=\frac{-8ab+4ab}{6a^2}=\frac{-4ab}{6a^2}=\frac{-2b}{3a}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-8ab-\sqrt{16a^2b^2}}{2\times3a^2}=\frac{-8ab-4ab}{6a^2}=\frac{-12ab}{6a^2}=\frac{-2b}{a}$$

Thus, the roots of the equation are $\frac{-2b}{3a}\ and\ \frac{-2b}{a}.$

35.

Sol:

The given equation is $a^2b^2x^2 - \left(4b^4 - 3a^4\right)x - 12a^2b^2 = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = a^2b^2, B = -\left(4b^4 - 3a^4\right)$ *and* $c = -12a^2b^2$

$\therefore$ Discriminant,

$B^2 - 4AC = \left[-\left(4b^4 - 3a^4\right)\right]^2 - 4 \times a^2b^2 \times \left(-12a^2b^2\right) = 16b^8 - 24a^4b^4 + 9a^8 + 48a^4b^4$

$= 16b^8 + 24a^4b^4 + 9a^8 = \left(4b^4 + 3a^4\right)^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{\left(4b^4 + 3a^4\right)^2} = 4b^4 + 3a^4$

$$\therefore \alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] + \left(4b^4 + 3a^4\right)}{2 \times a^2b^2} = \frac{8b^4}{2a^2b^2} = \frac{4b^2}{a^2}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] - \left(4b^4 + 3a^4\right)}{2 \times a^2b^2} = \frac{-6a^4}{2a^2b^2} = -\frac{3a^2}{b^2}$$

Hence, $\frac{4b^2}{a^2}$ and $-\frac{3a^2}{b^2}$ are the roots of the given equation.

36.

Sol:

Given:

$12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0$

On comparing it with $Ax^2 + Bx + C = 0,$ we get:

$A = 12ab, B = -\left(9a^2 - 8b^2\right)$ and $C = -6ab$

Discriminant D is given by:

$D = B^2 - 4AC$

$= \left[-\left(9a^2 - 8b^2\right)\right]^2 - 4 \times 12ab \times \left(-6ab\right)$

$= 81a^4 - 144a^2b^2 + 64b^4 + 288a^2b^2$

$= 81a^4 + 144a^2b^2 + 64b^4$

$= \left(9a^2 + 8b^2\right)^2 > 0$

Hence, the roots of the equation are equal.
Roots α and β are given by:

$$\alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-\left[-\left(9a^2-8b^2\right)\right]+\sqrt{\left(9a^2+8b^2\right)^2}}{2\times 12ab} = \frac{9a^2-8b^2+9a^2+8b^2}{24ab} = \frac{18a^2}{24ab} = \frac{3a}{4b}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-\left[-\left(9a^2-8b^2\right)\right]-\sqrt{\left(9a^2+8b^2\right)^2}}{2\times 12ab} = \frac{9a^2-8b^2-9a^2-8b^2}{24ab} = \frac{-16a^2}{24ab} = \frac{-2b}{3a}$$

Thus, the roots of the equation are $\frac{3a}{4b}$ and $\frac{-2b}{3a}$.

Exercise - 10D

1.

Sol:

(i) The given equation is $2x^2-8x+5=0$.
This is of the form $ax^2+bx+c=0$, where $a=2, b=-8$ *and* $c=5$.
$\therefore$ Discriminant, $D=b^2-4ac=(-8)^2-4\times 2\times 5=64-40=24>0$
Hence, the given equation has real and unequal roots.

(ii) The given equation is $3x^2-2\sqrt{6}x+2=0$.
This is of the form $ax^2+bx+c=0$, where $a=3, b=-2\sqrt{6}$ *and* $c=2$.
$\therefore$ Discriminant, $D=b^2-4ac=\left(-2\sqrt{6}\right)^2-4\times 3\times 2=24-24=0$
Hence, the given equation has real and equal roots.

(iii) The given equation is $5x^2-4x+1=0$.
This is of the form $ax^2+bx+c=0$, where $a=5, b=-4$ *and* $c=1$.
$\therefore$ Discriminant, $D=b^2-4ac=(-4)^2-4\times 5\times 1=16-20=-4<0$
Hence, the given equation has no real roots.

(iv) The given equation is

$5x(x-2)+6=0$

$\Rightarrow 5x^2-10x+6=0$

This is of the form $ax^2+bx+c=0$, where $a=5, b=-10$ *and* $c=6$.

Discriminant, $D=b^2-4ac=(-10)^2-4\times5\times6=100-120=-20<0$

Hence, the given equation has no real roots.

(v) The given equation is $12x^2-4\sqrt{15}x+5=0$

This is of the form $ax^2+bx+c=0$, where $a=12, b=-4\sqrt{15}$ *and* $c=5$.

Discriminant, $D=b^2-4ac=\left(-4\sqrt{15}\right)^2-4\times12\times5=240-240=0$

Hence, the given equation has real and equal roots.

(vi) The given equation is $x^2-x+2=0$.

This is of the form $ax^2+bx+c=0$, where $a=1, b=-1$ *and* $c=2$.

Discriminant, $D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7<0$

Hence, the given equation has no real roots.

2.

Sol:

The given equation is $2\left(a^2+b^2\right)x^2+2(a+b)x+1=0$.

$\therefore D=\left[2(a+b)\right]^2-4\times2\left(a^2+b^2\right)\times1$

$=4\left(a^2+2ab+b^2\right)-8\left(a^2+b^2\right)$

$=4a^2+8ab+4b^2-8a^2-8b^2$

$=-4a^2+8ab-4b^2$

$=-4\left(a^2-2ab+b^2\right)$

$=-4(a-b)^2<0$

Hence, the given equation has no real roots.

3.

Sol:

Given:

$x^2+px-q^2=0$

Here,

$a=1, b=p$ *and* $c=-q^2$

Discriminant D is given by:

$D = \left(b^2 - 4ac\right)$

$= p^2 - 4 \times 1 \times \left(-q^2\right)$

$= \left(p^2 + 4q^2\right) > 0$

$D > 0$ for all real values of p and q.

Thus, the roots of the equation are real.

4.

Sol:

Given:

$3x^2 + 2kx + 27 = 0$

Here,

$a = 3, b = 2k \ and \ c = 27$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow (2k)^2 - 4 \times 3 \times 27 = 0$

$\Rightarrow 4k^2 - 324 = 0$

$\Rightarrow 4k^2 = 324$

$\Rightarrow k^2 = 81$

$\Rightarrow k = \pm 9$

$\therefore k = 9 \ or \ k = -9$

5.

Sol:

The given equation is

$kx\left(x - 2\sqrt{5}\right) + 10 = 0$

$\Rightarrow kx^2 - 2\sqrt{5}kx + 10 = 0$

This is of the form $ax^2 + bx + c = 0$, where $a = k, b = -2\sqrt{5}k \ and \ c = 10$.

$\therefore D = b^2 - 4ac = \left(-2\sqrt{5}k\right)^2 - 4 \times k \times 10 = 20k^2 - 40k$

The given equation will have real and equal roots if $D = 0$.

$\therefore 20k^2 - 40k = 0$

$\Rightarrow 20k(k-2) = 0$

$\Rightarrow k = 0 \ or \ k - 2 = 0$

$\Rightarrow k = 0 \ or \ k = 2$

But, for $k = 0$, we get $10 = 0$, which is not true

Hence, 2 is the required value of *k*.

6.

Sol:

The given equation is $4x^2 + px + 3 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 4, b = p \ and \ c = 3$.

$\therefore D = b^2 - 4ac = p^2 - 4 \times 4 \times 3 = p^2 - 48$

The given equation will have real and equal roots if $D = 0$.

$\therefore p^2 - 48 = 0$

$\Rightarrow p^2 = 48$

$\Rightarrow p = \pm\sqrt{48} = \pm 4\sqrt{3}$

Hence, $4\sqrt{3}$ and $-4\sqrt{3}$ are the required values of *p*.

7.

Sol:

The given equation is $9x^2 - 3kx + k = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 9, b = -3k \ and \ c = k$.

$\therefore D = b^2 - 4ac = (-3k)^2 - 4 \times 9 \times k = 9k^2 - 36k$

The given equation will have real and equal roots if $D = 0$.

$\therefore 9k^2 - 36k = 0$

$\Rightarrow 9k(k-4) = 0$

$\Rightarrow k = 0 \ or \ k - 4 = 0$

$\Rightarrow k = 0 \ or \ k = 4$

But, $k \neq 0$ (Given)

Hence, the required values of *k* is 4.

8.

Sol:

The given equation is $(3k+1)x^2+2(k+1)x+1=0.$

This is of the form $ax^2+bx+c=0,$ where $a=3k+1, b=2(k+1)$ *and* $c=1.$

$\therefore D=b^2-4ac$

$=\left[2(k+1)\right]^2-4\times(3k+1)\times 1$

$=4(k^2+2k+1)-4(3k+1)$

$=4k^2+8k+4-12k-4$

$=4k^2-4k$

The given equation will have real and equal roots if $D=0.$

$\therefore 4k^2-4k=0$

$\Rightarrow 4k(k-1)=0$

$\Rightarrow k=0 \text{ or } k-1=0$

$\Rightarrow k=0 \text{ or } k=1$

Hence, 0 and $1$are the required values of $k.$

9.

Sol:

The given equation is $(2p+1)x^2-(7p+2)x+(7p-3)=0.$

This is of the form $ax^2+bx+c=0,$ where $a=2p+1, b=-(7p+2)$ *and* $c=7p-3.$

$\therefore D=b^2-4ac$

$=-\left[-(7p+2)\right]^2-4\times(2p+1)\times(7p-3)$

$=(49p^2+28p+4)-4(14p^2+p-3)$

$=49p^2+28p+4-56p^2-4p+12$

$=-7p^2+24p+16$

The given equation will have real and equal roots if $D=0.$

$\therefore -7p^2+24p+16=0$

$\Rightarrow 7p^2-24p-16=0$

$\Rightarrow 7p^2-28p+4p-16=0$

$\Rightarrow 7p(p-4)+4(p-4)=0$

$\Rightarrow (p-4)(7p+4)=0$

$\Rightarrow p-4=0 \text{ or } 7p+4=0$

$\Rightarrow p=4 \ or \ p=-\frac{4}{7}$

Hence, 4 and $-\frac{4}{7}$ are the required values of p.

10.

Sol:

The given equation is $(p+1)x^2-6(p+1)x+3(p+9)=0.$

This is of the form $ax^2+bx+c=0,$ where $a=p+1, b=-6(p+1) \ and \ c=3(p+9).$

$\therefore D=b^2-4ac$

$=[-6(p+1)]^2-4\times(p+1)\times3(p+9)$

$=12(p+1)[3(p+1)-(p+9)]$

$=12(p+1)(2p-6)$

The given equation will have real and equal roots if $D=0.$

$\therefore 12(p+1)(2p-6)=0$

$\Rightarrow p+1=0 \ or \ 2p-6=0$

$\Rightarrow p=-1 \ or \ p=3$

But, $p\neq-1$ (Given)

Thus, the value of p is 3

Putting $p=3,$ the given equation becomes $4x^2-24x+36=0$

$4x^2-24x+36=0$

$\Rightarrow 4(x^2-6x+9)=0$

$\Rightarrow (x-3)^2=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

Hence, 3 is the repeated root of this equation.

11.

Sol:

It is given that -5 is a root of the quadratic equation $2x^2+px-15=0.$

$\therefore 2(-5)^2+p\times(-5)-15=0$

$\Rightarrow -5p + 35 = 0$

$\Rightarrow p = 7$

The roots of the equation $px^2 + px + k = 0 = 0$ are equal.

$\therefore D = 0$

$\Rightarrow p^2 - 4pk = 0$

$\Rightarrow (7)^2 - 4 \times 7 \times k = 0$

$\Rightarrow 49 - 28k = 0$

$\Rightarrow k = \frac{49}{28} = \frac{7}{4}$

Thus, the value of k is $\frac{7}{4}$.

12.

Sol:

It is given that 3 is a root of the quadratic equation $x^2 - x + k = 0$.

$\therefore (3)^2 - 3 + k = 0$

$\Rightarrow k + 6 = 0$

$\Rightarrow k = -6$

The roots of the equation $x^2 + 2kx + \left(k^2 + 2k + p\right) = 0$ are equal.

$\therefore D = 0$

$\Rightarrow (2k)^2 - 4 \times 1 \times \left(k^2 + 2k + p\right) = 0$

$\Rightarrow 4k^2 - 4k^2 - 8k - 4p = 0$

$\Rightarrow -8k - 4p = 0$

$\Rightarrow p = \frac{8k}{-4} = -2k$

$\Rightarrow p = -2 \times (-6) = 12$

Hence, the value of p is 12.

13.

Sol:

It is given that -4 is a root of the quadratic equation $x^2 + 2x + 4p = 0$.

$\therefore (-4)^2 + 2\times(-4) + 4p = 0$

$\Rightarrow 16 - 8 + 4p = 0$

$\Rightarrow 4p + 8 = 0$

$\Rightarrow p = -2$

The equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has real roots.

$\therefore D = 0$

$\Rightarrow \left[p(1+3k)\right]^2 - 4\times1\times7(3+2k) = 0$

$\Rightarrow \left[-2(1+3k)\right]^2 - 28(3+2k) = 0$

$\Rightarrow 4(1+6k+9k^2) - 28(3+2k) = 0$

$\Rightarrow 4(1+6k+9k^2 - 21 - 14k) = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow 9k(k-2) + 10(k-2) = 0$

$\Rightarrow (k-2)(9k+10) = 0$

$\Rightarrow k - 2 = 0 \ or \ 9k + 10 = 0$

$\Rightarrow k = 2 \ or \ k = -\frac{10}{9}$

Hence, the required value of k is *2* or $-\frac{10}{9}$.

14.

Sol:
Given:

$(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

Here,

$a = (1+m^2), b = 2mc \ and \ c = (c^2 - a^2)$

It is given that the roots of the equation are equal; therefore, we have:

$D = 0$

$\Rightarrow (b^2 - 4ac) = 0$

$\Rightarrow (2mc)^2 - 4\times(1+m^2)\times(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4\left(c^2 - a^2 + m^2c^2 - m^2a^2\right) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$\Rightarrow -4c^2 + 4a^2 + 4m^2a^2 = 0$

$\Rightarrow a^2 + m^2a^2 = c^2$

$\Rightarrow a^2\left(1 + m^2\right) = c^2$

$\Rightarrow c^2 = a^2\left(1 + m^2\right)$

Hence proved.

15.

Sol:

Given:

$\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$

Here,

$a = \left(c^2 - ab\right), b = -2\left(a^2 - bc\right), c = \left(b^2 - ac\right)$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow \left(b^2 - 4ac\right) = 0$

$\Rightarrow \left\{-2\left(a^2 - bc\right)\right\}^2 - 4 \times \left(c^2 - ab\right) \times \left(b^2 - ac\right) = 0$

$\Rightarrow 4\left(a^4 - 2a^2bc + b^2c^2\right) - 4\left(b^2c^2 - ac^3 - ab^3 + a^2bc\right) = 0$

$\Rightarrow a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$

$\Rightarrow a^4 - 3a^2bc + ac^3 + ab^3 = 0$

$\Rightarrow a\left(a^3 - 3abc + c^3 + b^3\right) = 0$

Now,

$a = 0 \ or \ a^3 - 3abc + c^3 + b^3 = 0$

$a = 0 \ or \ a^3 + b^3 + c^3 = 3abc$

16. **Sol:**

Given:

$2x^2 + px + 8 = 0$

Here,

$a = 2, b = p \ and \ c = 8$

Discriminant D is given by:

$D = \left(b^2 - 4ac\right)$

$= p^2 - 4 \times 2 \times 8$

$= \left(p^2 - 64\right)$

If $D \geq 0$, the roots of the equation will be real

$\Rightarrow \left(p^2 - 64\right) \geq 0$

$\Rightarrow (p+8)(p-8) \geq 0$

$\Rightarrow p \geq 8 \ and \ p \leq -8$

Thus, the roots of the equation are real for $p \geq 8$ and $p \leq -8$.

17.

Sol:

Given:

$(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$

Here,

$a = (\alpha = 12), b = 2(\alpha - 12) \ and \ c = 2$

It is given that the roots of the equation are equal; therefore, we have

$D = 0$

$\Rightarrow \left(b^2 - 4ac\right) = 0$

$\Rightarrow \left\{2(\alpha - 12)\right\}^2 - 4 \times (\alpha - 12) \times 2 = 0$

$\Rightarrow 4\left(\alpha^2 - 24\alpha + 144\right) - 8\alpha + 96 = 0$

$\Rightarrow 4\alpha^2 - 96\alpha + 576 - 8\alpha + 96 = 0$

$\Rightarrow 4\alpha^2 - 104\alpha + 672 = 0$

$\Rightarrow \alpha^2 - 26\alpha + 168 = 0$

$\Rightarrow \alpha^2 - 14\alpha - 12\alpha + 168 = 0$

$\Rightarrow \alpha(\alpha - 14) - 12(\alpha - 14) = 0$

$\Rightarrow (\alpha - 14)(\alpha - 12) = 0$

$\therefore \alpha = 14 \ or \ \alpha = 12$

If the value of α is 12, the given equation becomes non-quadratic.

Therefore, the value of α will be 14 for the equation to have equal roots.

18.

Sol:

Given:

$9x^2+8kx+16=0$

Here,

$a=9, b=8k \text{ and } c=16$

It is given that the roots of the equation are real and equal; therefore, we have:

$D=0$

$\Rightarrow (b^2-4ac)=0$

$\Rightarrow (8k)^2-4\times 9\times 16=0$

$\Rightarrow 64k^2-576=0$

$\Rightarrow 64k^2=576$

$\Rightarrow k^2=9$

$\Rightarrow k=\pm 3$

$\therefore k=3 \text{ or } k=-3$

19.

Sol:

(i) The given equation is $kx^2+6x+1=0.$

$\therefore D=6^2-4\times k\times 1=36-4k$

The given equation has real and distinct roots if $D>0.$

$\therefore 36-4k>0$

$\Rightarrow 4k<36$

$\Rightarrow k<9$

(ii) The given equation is $x^2-kx+9=0.$

$\therefore D=(-k)^2-4\times 1\times 9=k^2-36$

The given equation has real and distinct roots if $D>0.$

$\therefore k^2-36>0$

$\Rightarrow (k-6)(k+6)>0$

$\Rightarrow k<-6 \text{ or } k>6$

(iii) The given equation is $9x^2+3kx+4=0.$

$\therefore D=(3k)^2-4\times 9\times 4=9k^2-144$

The given equation has real and distinct roots if $D>0.$

$\therefore 9k^2 - 144 > 0$

$\Rightarrow 9(k^2 - 16) > 0$

$\Rightarrow (k-4)(k+4) > 0$

$\Rightarrow k < -4 \text{ or } k > 4$

(iv) The given equation is $5x^2 - kx + 1 = 0.$

$\therefore D = (-k)^2 - 4 \times 5 \times 1 = k^2 - 20$

The given equation has real and distinct roots if $D > 0$.

$\therefore k^2 - 20 > 0$

$\Rightarrow k^2 - (2\sqrt{5})^2 > 0$

$\Rightarrow (k - 2\sqrt{5})(k + 2\sqrt{5}) > 0$

$\Rightarrow k < -2\sqrt{5} \text{ or } k > 2\sqrt{5}$

20.

Sol:

The given equation is $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0.$

$\therefore D = [5(a+b)]^2 - 4 \times (a-b) \times [-2(a-b)]$

$= 25(a+b)^2 + 8(a-b)^2$

Since a and b are real and $a \neq b,$ so $(a-b)^2 > 0 \text{ and } (a+b)^2 > 0.$

$\therefore 8(a-b)^2 > 0$ ....(1) (Product of two positive numbers is always positive)

Also, $25(a+b)^2 > 0$ ....(2) (Product of two positive numbers is always positive)

Adding (1) and (2), we get

$25(a+b)^2 + 8(a-b)^2 > 0$ (Sum of two positive numbers is always positive)

$\Rightarrow D > 0$

Hence, the roots of the given equation are real and unequal.

21.

Sol:

It is given that the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal.

$\therefore D = 0$

$\Rightarrow \left[-2(ac+bd)\right]^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\Rightarrow 4(a^2c^2+b^2d^2+2abcd) - 4(a^2c^2+a^2d^2+b^2c^2+b^2d^2) = 0$

$\Rightarrow 4(a^2c^2+b^2d^2+2abcd-a^2c^2-a^2d^2-b^2c^2-b^2d^2) = 0$

$\Rightarrow (-a^2d^2+2abcd-b^2c^2) = 0$

$\Rightarrow -(a^2d^2-2abcd+b^2c^2) = 0$

$\Rightarrow (ad-bc)^2 = 0$

$\Rightarrow ad-bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

Hence proved.

22.

Sol:

It is given that the roots of the equation $ax^2+2bx+c=0$ are real.

$\therefore D_1 = (2b)^2 - 4\times a\times c \ge 0$

$\Rightarrow 4(b^2-ac) \ge 0$

$\Rightarrow b^2-ac \ge 0 \qquad ....(1)$

Also, the roots of the equation $bx^2-2\sqrt{ac}x+b=0$ are real.

$\therefore D_2 = (-2\sqrt{ac})^2 - 4\times b\times b \ge 0$

$\Rightarrow 4(ac-b^2) \ge 0$

$\Rightarrow -4(b^2-ac) \ge 0$

$\Rightarrow b^2-ac \ge 0 \qquad ....(2)$

The roots of the given equations are simultaneously real if (1) and (2) holds true together.

This is possible if

$b^2-ac=0$

$\Rightarrow b^2 = ac$

Exercise 10E

1.

Sol:

Let the required natural number be *x*.

According to the given condition,

$x + x^2 = 156$

$\Rightarrow x^2 + x - 156 = 0$

$\Rightarrow x^2 + 13x - 12x - 156 = 0$

$\Rightarrow x(x+13) - 12(x+13) = 0$

$\Rightarrow (x+13)(x-12) = 0$

$\Rightarrow x + 13 = 0 \; or \; x - 12 = 0$

$\Rightarrow x = -13 \; or \; x = 12$

$\therefore x = 12$ (*x* cannot be negative)

Hence, the required natural number is 12.

2.

Sol:

Let the required natural number be *x*.

According to the given condition,

$x + \sqrt{x} = 132$

Putting $\sqrt{x} = y \; or \; x = y^2$, we get

$y^2 + y = 132$

$\Rightarrow y^2 + y - 132 = 0$

$\Rightarrow y^2 + 12y - 11y - 132 = 0$

$\Rightarrow y(y+12) - 11(y+12) = 0$

$\Rightarrow (y+12)(y-11) = 0$

$\Rightarrow y + 12 = 0 \; or \; y - 11 = 0$

$\Rightarrow y = -12 \; or \; y = 11$

$\therefore y = 11$ (y cannot be negative)

Now,

$\sqrt{x} = 11$

$\Rightarrow x = (11)^2 = 121$

Hence, the required natural number is 121.

3.

Sol:

Let the required number be x and $(28-x)$.

According to the given condition,

$x(28-x)=192$

$\Rightarrow 28x-x^2=192$

$\Rightarrow x^2-28x+192=0$

$\Rightarrow x^2-16x-12x+192=0$

$\Rightarrow x(x-16)-12(x-16)=0$

$\Rightarrow (x-12)(x-16)=0$

$\Rightarrow x-12=0 \text{ or } x-16=0$

$\Rightarrow x=12 \text{ or } x=16$

When $x=12$,

$28-x=28-12=16$

When $x=16$,

$28-x=28-16=12$

Hence, the required numbers are 12 and 16.

4.

Sol:

Let the required two consecutive positive integers be x and $(x+1)$.

According to the given condition,

$x^2+(x+1)^2=365$

$\Rightarrow x^2+x^2+2x+1=365$

$\Rightarrow 2x^2+2x-364=0$

$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

$\Rightarrow x+14=0 \text{ or } x-13=0$

$\Rightarrow x=-14 \text{ or } x=13$

$\therefore x=13$ (x is a positive integers)

When $x=13$,

$x+1=13+1=14$

Hence, the required positive integers are 13 and 14.

5.

Sol:

Let the two consecutive positive odd numbers be x and $(x+2)$.

According to the given condition,

$x^2+(x+2)^2=514$

$\Rightarrow x^2+x^2+4x+4=514$

$\Rightarrow 2x^2+4x-510=0$

$\Rightarrow x^2+2x-255=0$

$\Rightarrow x^2+17x-15x-255=0$

$\Rightarrow x(x+17)-15(x+17)=0$

$\Rightarrow (x+17)(x-15)=0$

$\Rightarrow x+17=0$ or $x-15=0$

$\Rightarrow x=-17$ or $x=15$

$\therefore x=15$ (x is a positive odd number)

When $x=15$,

$x+2=15+2=17$

Hence, the required positive integers are 15 and 17.

6.

Sol:

Let the two consecutive positive even numbers be x and $(x+2)$.

According to the given condition,

$x^2+(x+2)^2=452$

$\Rightarrow x^2+x^2+4x+4=452$

$\Rightarrow 2x^2+4x-448=0$

$\Rightarrow x^2+2x-224=0$

$\Rightarrow x^2+16x-14x-224=0$

$\Rightarrow x(x+16)-14(x+16)=0$

$\Rightarrow (x+16)(x-14)=0$

$\Rightarrow x+16=0$ or $x-14=0$

$\Rightarrow x=-16$ or $x=14$

$\therefore x=14$ (x is a positive even number)

When $x=14$,

$x+2=14+2=16$

Hence, the required numbers are 14 and 16.

7.

Sol:

Let the two consecutive positive integers be x and $(x+1)$.

According to the given condition,

$x(x+1)=306$

$\Rightarrow x^2+x-306=0$

$\Rightarrow x^2+18x-17x-306=0$

$\Rightarrow x(x+18)-17(x+18)=0$

$\Rightarrow (x+18)(x-17)=0$

$\Rightarrow x+18=0 \; or \; x-17=0$

$\Rightarrow x=-18 \; or \; x=17$

$\therefore x=17$ (x is a positive integers)

When $x=17$,

$x+1=17+1=18$

Hence, the required integers are 17 and 18.

8.

Sol:

Let the required numbers be x and $(x+3)$.

According to the question:

$x(x+3)=504$

$\Rightarrow x^2+3x=504$

$\Rightarrow x^2+3x-504=0$

$\Rightarrow x^2+(24-21)x-504=0$

$\Rightarrow x^2+24x-21x-504=0$

$\Rightarrow x(x+24)-21(x+24)=0$

$\Rightarrow (x+24)(x-21)=0$

$\Rightarrow x+24=0 \; or \; x-21=0$

$\Rightarrow x=-24 \; or \; x=21$

If $x=-24$, the numbers are -24 and $\{(-24+3)=-21\}$.

If $x=21$, the numbers are 21 and $\{(21+3)=24\}$.

Hence, the numbers are $(-24,-21)$ and $(21,24)$.

9.

Sol:

Le the required consecutive multiples of 3 be $3x$ and $3(x+1)$.

According to the given condition,

$3x \times 3(x+1) = 648$

$\Rightarrow 9(x^2 + x) = 648$

$\Rightarrow x^2 + x = 72$

$\Rightarrow x^2 + x - 72 = 0$

$\Rightarrow x^2 + 9x - 8x - 72 = 0$

$\Rightarrow x(x+9) - 8(x+9) = 0$

$\Rightarrow (x+9)(x-8) = 0$

$\Rightarrow x+9 = 0 \text{ or } x-8 = 0$

$\Rightarrow x = -9 \text{ or } x = 8$

$\therefore x = 8$ (Neglecting the negative value)

When $x = 8$,

$3x = 3 \times 8 = 24$

$3(x+1) = 3 \times (8+1) = 3 \times 9 = 27$

Hence, the required multiples are 24 and 27.

10.

Sol:

Let the two consecutive positive odd integers be x and $(x+2)$.

According to the given condition,

$x(x+2) = 483$

$\Rightarrow x^2 + 2x - 483 = 0$

$\Rightarrow x^2 + 23x - 21x - 483 = 0$

$\Rightarrow x(x+23) - 21(x+23) = 0$

$\Rightarrow (x+23)(x+21) = 0$

$\Rightarrow x+23 = 0 \text{ or } x-21 = 0$

$\Rightarrow x = -23 \text{ or } x = 21$

$\therefore x = 21$ (x is a positive odd integer)

When $x = 21$,

$x + 2 = 21 + 2 = 23$

Hence, the required integers are 21 and 23.

11.

Sol:

Let the two consecutive positive even integers be x and $(x+2)$.

According to the given condition,

$x(x+2)=288$

$\Rightarrow x^2+2x-288=0$

$\Rightarrow x^2+18x-16x-288=0$

$\Rightarrow x(x+18)-16(x+18)=0$

$\Rightarrow (x+18)(x-16)=0$

$\Rightarrow x+18=0 \text{ or } x-16=0$

$\Rightarrow x=-18 \text{ or } x=16$

$\therefore x=16$ (x is a positive even integer)

When $x=16$,

$x+2=16+2=18$

Hence, the required integers are 16 and 18.

12.

Sol:

Let the required natural numbers be x and $(9,-x)$.

According to the given condition,

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$

$$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$$

$$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$$

$\Rightarrow 9x-x^2=18$

$\Rightarrow x^2-9x+18=0$

$\Rightarrow x^2-6x-3x+18=0$

$\Rightarrow x(x-6)-3(x-6)=0$

$\Rightarrow x-3=0 \text{ or } x-6=0$

$\Rightarrow x=3 \text{ or } x=6$

When $x=3$,

$9-x=9-3=6$

When $x=6,$

$9-x=9-6=3$

Hence, the required natural numbers are 3 and 6.

13.

Sol:

Let the required natural numbers be x and $(15-x)$.

According to the given condition,

$\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$

$\Rightarrow \frac{15-x+x}{x(15-x)}=\frac{3}{10}$

$\Rightarrow \frac{15}{15x-x^2}=\frac{3}{10}$

$\Rightarrow 15x-x^2=50$

$\Rightarrow x^2-15x+50=0$

$\Rightarrow x^2-10x-5x+50=0$

$\Rightarrow x(x-10)-5(x-10)=0$

$\Rightarrow (x-5)(x-10)=0$

$\Rightarrow x-5=0 \ or \ x-10=0$

$\Rightarrow x=5 \ or \ x=10$

When $x=5,$

$15-x=15-5=10$

When $x=10,$

$15-x=15-10=5$

Hence, the required natural numbers are 5 and 10.

14.

Sol:

Let the required natural numbers be x and $(x+3)$.

Now, $x<x+3$

$\therefore \frac{1}{x} > \frac{1}{x+3}$

According to the given condition,

$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28}$

$\Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$

$\Rightarrow \frac{3}{x^2+3x} = \frac{3}{28}$

$\Rightarrow x^2 + 3x = 28$

$\Rightarrow x^2 + 3x - 28 = 0$

$\Rightarrow x^2 + 7x - 4x - 28 = 0$

$\Rightarrow x(x+7) - 4(x+7) = 0$

$\Rightarrow (x+7)(x-4) = 0$

$\Rightarrow x+7 = 0 \; or \; x-4 = 0$

$\Rightarrow x = -7 \; or \; x = 4$

$\therefore x = 4$ (-7 is not a natural number)

When $x = 4$,

$x + 3 = 4 + 3 = 7$

Hence, the required natural numbers are 4 and 7.

15.

Sol:

Let the required natural numbers be x and $(x+5)$.

Now, $x < x+5$

$\therefore \frac{1}{x} > \frac{1}{x+5}$

According to the given condition,

$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$

$\Rightarrow \frac{x+5-x}{x(x+5)} = \frac{5}{14}$

$\Rightarrow \frac{5}{x^2+5x} = \frac{5}{14}$

$\Rightarrow x^2 + 5x = 14$

$\Rightarrow x^2 + 5x - 14 = 0$

$\Rightarrow x^2 + 7x - 2x - 14 = 0$

$\Rightarrow x(x+7) - 2(x+7) = 0$

$\Rightarrow (x+7)(x-2) = 0$

$\Rightarrow x + 7 = 0$ *or* $x - 2 = 0$

$\Rightarrow x = -7$ *or* $x = 2$

$\therefore x = 2$ $(-7$ is not a natural number)

When $x = 2$,

$x + 5 = 2 + 5 = 7$

Hence, the required natural numbers are 2 and 7.

16.

Sol:

Let the required consecutive multiplies of 7 be $7x$ and $7(x+1)$.

According to the given condition,

$(7x)^2 + [7(x+1)]^2 = 1225$

$\Rightarrow 49x^2 + 49(x^2 + 2x + 1) = 1225$

$\Rightarrow 49x^2 + 49x^2 + 98x + 49 = 1225$

$\Rightarrow 98x^2 + 98x - 1176 = 0$

$\Rightarrow x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x+4) - 3(x+4) = 0$

$\Rightarrow (x+4)(x-3) = 0$

$\Rightarrow x + 4 = 0$ *or* $x - 3 = 0$

$\Rightarrow x = -4$ *or* $x = 3$

$\therefore x = 3$ (Neglecting the negative value)

When $x = 3$,

$7x = 7 \times 3 = 21$

$7(x+1) = 7(3+1) = 7 \times 4 = 28$

Hence, the required multiples are 21 and 28.

17.

Sol:

Let the natural number be x.
According to the given condition,

$$x+\frac{1}{x}=\frac{65}{8}$$
$$\Rightarrow \frac{x^2+1}{x}=\frac{65}{8}$$
$$\Rightarrow 8x^2+8=65x$$
$$\Rightarrow 8x^2-65x+8=0$$
$$\Rightarrow 8x^2-64x-x+8=0$$
$$\Rightarrow 8x(x-8)-1(x-8)=0$$
$$\Rightarrow (x-8)(8x-1)=0$$
$$\Rightarrow x-8=0 \text{ or } 8x-1=0$$
$$\Rightarrow x=8 \text{ or } x=\frac{1}{8}$$

$\therefore x=8$ (x is a natural number)

Hence, the required number is 8.

18.

Sol:

Let the two parts be x and $(57-x)$.
According to the given condition,

$$x(57-x)=680$$
$$\Rightarrow 57x-x^2=680$$
$$\Rightarrow x^2-57x+680=0$$
$$\Rightarrow x^2-40x-17x+680=0$$
$$\Rightarrow x(x-40)-17(x-40)=0$$
$$\Rightarrow (x-40)(x-17)=0$$
$$\Rightarrow x-40=0 \text{ or } x-17=0$$
$$\Rightarrow x=40 \text{ or } x=17$$

When $x=40$,
$57-x=57-40=17$

When $x=17$,
$57-x=57-17=40$

Hence, the required parts are 17 and 40.

19.

Sol:

Let the two parts be x and $(27-x)$.

According to the given condition,

$$\frac{1}{x}+\frac{1}{27-x}=\frac{3}{20}$$

$$\Rightarrow \frac{27-x+x}{x(27-x)}=\frac{3}{20}$$

$$\Rightarrow \frac{27}{27x-x^2}=\frac{3}{20}$$

$$\Rightarrow 27x-x^2=180$$

$$\Rightarrow x^2-27x+180=0$$

$$\Rightarrow x^2-15x-12x+180=0$$

$$\Rightarrow x(x-15)-12(x-15)=0$$

$$\Rightarrow (x-12)(x-15)=0$$

$$\Rightarrow x-12=0 \ or \ x-15=0$$

$$\Rightarrow x=12 \ or \ x=15$$

When $x=12$,

$27-x=27-12=15$

When $x=15$,

$27-x=27-15=12$

Hence, the required parts are 12 and 15.

20.

Sol:

Let the larger and smaller parts be x and y, respectively.

According to the question:

$x+y=16 \quad \quad ....(i)$

$2x^2=y^2+164 \quad \quad ....(ii)$

From (i), we get:

$x=16-y \quad \quad ......(iii)$

From (ii) and (iii), we get:

$2(16-y)^2 = y^2 + 164$

$\Rightarrow 2(256-32y+y^2) = y^2 + 164$

$\Rightarrow 512-64y+2y^2 = y^2 + 164$

$\Rightarrow y^2 - 64y + 348 = 0$

$\Rightarrow y^2 - (58+6)y + 348 = 0$

$\Rightarrow y^2 - 58y - 6y + 348 = 0$

$\Rightarrow y(y-58) - 6(y-58) = 0$

$\Rightarrow (y-58)(y-6) = 0$

$\Rightarrow y-58 = 0 \text{ or } y-6 = 0$

$\Rightarrow y = 6 (\because y < 16)$

Putting the value of y in equation (iii), we get

$x = 16 - 6 = 10$

Hence, the two natural numbers are 6 and 10.

21.

Sol:

Let the two natural numbers be *x* and *y*.

According to the question:

$x^2 + y^2 = 25(x+y) \quad ......(i)$

$x^2 + y^2 = 50(x-y) \quad .....(ii)$

From (i) and (ii), we get:

$25(x+y) = 50(x-y)$

$\Rightarrow x + y = 2(x-y)$

$\Rightarrow x + y = 2x - 2y$

$\Rightarrow y + 2y = 2x - x$

$\Rightarrow 3y = x \qquad ....(iii)$

From (ii) and (iii), we get:

$(3y)^2 + y^2 = 50(3y - y)$

$\Rightarrow 9y^2 + y^2 = 100y$

$\Rightarrow 10y^2 = 100y$

$\Rightarrow y = 10$

From (iii), we have:

$3\times10=x$

$\Rightarrow 30=x$

Hence, the two natural numbers are 30 and 10.

22.

Sol:

Let the greater number be x and the smaller number be y.

According to the question:

$x^2-y^2=45$ ....(i)

$y^2=4x$ ....(ii)

From (i) and (ii), we get:

$x^2-4x=45$

$\Rightarrow x^2-4x-45=0$

$\Rightarrow x^2-(9-5)x-45=0$

$\Rightarrow x^2-9x+5x-45=0$

$\Rightarrow x(x-9)+5(x-9)=0$

$\Rightarrow (x-9)(x+5)=0$

$\Rightarrow x-9=0$ or $x+5=0$

$\Rightarrow x=9$ or $x=-5$

$\Rightarrow x=9$ ($\because x$ is a natural number)

Putting the value of x in equation (ii), we get:

$y^2=4\times9$

$\Rightarrow y^2=36$

$\Rightarrow y=6$

Hence, the two numbers are 9 and 6.

23.

Sol:

Let the three consecutive positive integers be $x, x+1$ and $x+2$.

According to the given condition,

$x^2+(x+1)(x+2)=46$

$\Rightarrow x^2+x^2+3x+2=46$

$\Rightarrow 2x^2+3x-44=0$

$\Rightarrow 2x^2+11x-8x-44=0$

$\Rightarrow x(2x+11)-4(2x+11)=0$

$\Rightarrow (2x+11)(x-4)=0$

$\Rightarrow 2x+11=0 \ or \ x-4=0$

$\Rightarrow x=-\frac{11}{2} \ or \ x=4$

$\therefore x=4$ (x is a positive integer)

When $x=4$,

$x+1=4+1=5$

$x+2=4+2=6$

Hence, the required integers are 4, 5 and 6.

24.

Sol:

Let the digits at units and tens places be *x* and *y*, respectively.

Original number $=10y+x$

According to the question:

$10y+x=4(x+y)$

$\Rightarrow 10y+x=4x+4y$

$\Rightarrow 3x-6y=0$

$\Rightarrow 3x=6y$

$\Rightarrow x=2y$ ......(i)

Also,

$10y+x=2xy$

$\Rightarrow 10y+2y=2.2y.y$ $[From(i)]$

$\Rightarrow 12y=4y^2$

$\Rightarrow y=3$

From (i), we get:

$x=2\times3=6$

$\therefore$ Original number $=10\times3+6=36$

25.

Sol:

Let the digits at units and tens places be *x* and *y*, respectively.

$\therefore xy=14$

$\Rightarrow y=\frac{14}{x}$ ....(i)

According to the question:

$(10y+x)+45=10x+y$

$\Rightarrow 9y-9x=-45$

$\Rightarrow y-x=-5$ ....(ii)

From (i) and (ii), we get:

$\frac{14}{x}-x=-5$

$\Rightarrow \frac{14-x^2}{x}=-5$

$\Rightarrow 14-x^2=-5x$

$\Rightarrow x^2-5x-14=0$

$\Rightarrow x^2-(7-2)x-14=0$

$\Rightarrow x^2-7x+2x-14=0$

$\Rightarrow x(x-7)+2(x-7)=0$

$\Rightarrow (x-7)(x+2)=0$

$\Rightarrow x-7=0 \text{ or } x+2=0$

$\Rightarrow x=7 \text{ or } x=-2$

$\Rightarrow x=7$ ($\because$ the digit cannot be negative)

Putting $x=7$ in equation (i), we get:

$y=2$

$\therefore$ Required number $=10\times 2+7=27$

26.

Sol:

Let the numerator be x.

$\therefore$ Denominator $=x+3$

$\therefore$ Original number $=\frac{x}{x+3}$

According to the question:

$\frac{x}{x+3}+\frac{1}{\left(\frac{x}{x+3}\right)}=2\frac{9}{10}$

$\Rightarrow \frac{x}{x+3}+\frac{x+3}{x}=\frac{29}{10}$

$\Rightarrow \frac{x^2+(x+3)^2}{x(x+3)}=\frac{29}{10}$

$\Rightarrow \frac{x^2+x^2+6x+9}{x^2+3x}=\frac{29}{10}$

$\Rightarrow \frac{2x^2+6x+9}{x^2+3x}=\frac{29}{10}$

$\Rightarrow 29x^2+87x=20x^2+60x+90$

$\Rightarrow 9x^2+27x-90=0$

$\Rightarrow 9(x^2+3x-10)=0$

$\Rightarrow x^2+3x-10=0$

$\Rightarrow x^2+5x-2x-10=0$

$\Rightarrow x(x+5)-2(x+5)=0$

$\Rightarrow (x-2)(x+5)=0$

$\Rightarrow x-2=0 \text{ or } x+5=0$

$\Rightarrow x=2 \text{ or } x=-5 \text{(rejected)}$

So, number $= x = 2$

denominator $= x+3=2+3=5$

So, required fraction $=\frac{2}{5}$

27.

Sol:

Let the denominator of the required fraction be x.

Numerator of the required fraction $= x-3$

$\therefore$ Original fraction $=\frac{x-3}{x}$

If 1 is added to the denominator, then the new fraction obtained is $\frac{x-3}{x+1}$

According to the given condition,

$\frac{x-3}{x+1}=\frac{x-3}{x}-\frac{1}{15}$

$$\Rightarrow \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$$

$$\Rightarrow \frac{(x-3)(x+1) - x(x-3)}{x(x+1)} = \frac{1}{15}$$

$$\Rightarrow \frac{x^2 - 2x - 3 - x^2 + 3x}{x^2 + x} = \frac{1}{15}$$

$$\Rightarrow \frac{x-3}{x^2 + x} = \frac{1}{15}$$

$$\Rightarrow x^2 + x = 15x - 45$$

$$\Rightarrow x^2 - 14x + 45 = 0$$

$$\Rightarrow x^2 - 9x - 5x + 45 = 0$$

$$\Rightarrow x(x-9) - 5(x-9) = 0$$

$$\Rightarrow (x-5)(x-9) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } x - 9 = 0$$

$$\Rightarrow x = 5 \text{ or } x = 9$$

When $x = 5$,

$$\frac{x-3}{x} = \frac{5-3}{5} = \frac{2}{5}$$

When $x = 9$,

$\frac{x-3}{x} = \frac{9-3}{9} = \frac{6}{9} = \frac{2}{3}$ (This fraction is neglected because this does not satisfies the given condition.)

Hence, the required fraction is $\frac{2}{5}$.

28.

Sol:

Let the required number be *x*.

According to the given condition,

$$x + \frac{1}{x} = 2\frac{1}{30}$$

$$\Rightarrow \frac{x^2 + 1}{x} = \frac{61}{30}$$

$$\Rightarrow 30x^2 + 30 = 61x$$

$$\Rightarrow 30x^2 - 61x + 30 = 0$$

$\Rightarrow 30x^2 - 36x - 25x + 30 = 0$

$\Rightarrow 6x(5x-6) - 5(5x-6) = 0$

$\Rightarrow (5x-6)(6x-5) = 0$

$\Rightarrow 5x - 6 = 0 \text{ or } 6x - 5 = 0$

$\Rightarrow x = \frac{6}{5} \text{ or } x = \frac{5}{6}$

Hence, the required number is $\frac{5}{6}$ or $\frac{6}{5}$.

29.

Sol:
Let there be x rows.
Then, the number of students in each row will also be x.

$\therefore$ Total number of students $= (x^2 + 24)$

According to the question:

$(x+1)^2 - 25 = x^2 + 24$

$\Rightarrow x^2 + 2x + 1 - 25 - x^2 - 24 = 0$

$\Rightarrow 2x - 48 = 0$

$\Rightarrow 2x = 48$

$\Rightarrow x = 24$

$\therefore$ Total number of students $= 24^2 + 24 = 576 + 24 = 600$

30.

Sol:
Let the total number of students be x.
According to the question:

$\frac{300}{x} - \frac{300}{x+10} = 1$

$\Rightarrow \frac{300(x+10) - 300x}{x(x+10)} = 1$

$\Rightarrow \frac{300x + 3000 - 300x}{x^2 + 10x} = 1$

$\Rightarrow 3000 = x^2 + 10x$

$\Rightarrow x^2 + 10x - 3000 = 0$

$\Rightarrow x^2 + (60 - 50)x = 3000 = 0$

$\Rightarrow x^2 + 60x - 50x - 3000 = 0$

$\Rightarrow x(x+60) - 50(x+60) = 0$

$\Rightarrow (x+60)(x-50) = 0$

$\Rightarrow x = 50 \ or \ x = -60$

x cannot be negative; therefore, the total number of students is 50.

31.

Sol:

Let the marks of Kamal in mathematics and English be x and y, respectively. According to the question: $x + y = 40$ ......(i)

Also, $(x+3)(y-4) = 360$

$\Rightarrow (x+3)(40 - x - 4) = 360$ [From (i)]

$\Rightarrow (x+3)(36 - x) = 360$

$\Rightarrow 36x - x^2 + 108 - 3x = 360$

$\Rightarrow 33x - x^2 - 252 = 0$

$\Rightarrow -x^2 + 33x - 252 = 0$

$\Rightarrow x^2 - 33x - 252 = 0$

$\Rightarrow x^2 - (21 + 12)x + 252 = 0$

$\Rightarrow x^2 - 21x - 12x + 252 = 0$

$\Rightarrow x(x-21) - 12(x-21) = 0$

$\Rightarrow (x-21)(x-12) = 0$

$\Rightarrow x = 21 \ or \ x = 12$

If $x = 21$,

$y = 40 - 21 = 119$

Thus, Kamal scored 21 and 19 marks in mathematics and English, respectively.

If $x = 12$,

$y = 40 - 12 = 28$

Thus, Kamal scored 12 and 28 marks in mathematics and English, respectively.

32.

Sol:

Let x be the number of students who planned a picnic.

$\therefore$ Original cost of food for each member = ₹ $\frac{2000}{x}$

Five students failed to attend the picnic. So, $(x-5)$ students attended the picnic.

$\therefore$ New cost of food for each member = ₹ $\frac{2000}{(x-5)}$

According of the given condition,

₹ $\frac{2000}{x-5}$ − ₹ $\frac{2000}{x}$ = ₹ 20

$\Rightarrow \frac{2000x-2000x+10000}{x(x-5)}=20$

$\Rightarrow \frac{10000}{x^2-5x}=20$

$\Rightarrow x^2-5x=500$

$\Rightarrow x^2-5x-500=0$

$\Rightarrow x^2-25x+20x-500=0$

$\Rightarrow x(x-25)+20(x-25)=0$

$\Rightarrow (x-25)(x+20)=0$

$\Rightarrow x-25=0 \ or \ x+20=0$

$\Rightarrow x=25 \ or \ x=-20$

$\therefore x=25$ (Number of students cannot be negative)

Number of students who attended the picnic $=x-5=23-5=20$

Amount paid by each student for the food = ₹ $\frac{2000}{(25-5)}$ = ₹ $\frac{2000}{20}$ =₹ 100

33.

Sol:

Let the original price of the book be ₹ x.

$\therefore$ Number of books bought at original price for ₹ 600 $= \frac{600}{x}$

If the price of a book is reduced by ₹ 5, then the new price of the book is ₹$(x-5)$.

$\therefore$ Number of books bought at reduced price for ₹ 600 $= \frac{600}{x-5}$

According to the given condition,

$$\frac{600}{x-5} - \frac{600}{x} = 4$$

$$\Rightarrow \frac{600x - 600x + 3000}{x(x-5)} = 4$$

$$\Rightarrow \frac{3000}{x^2 - 5x} = 4$$

$$\Rightarrow x^2 - 5x = 750$$

$$\Rightarrow x^2 - 5x - 750 = 0$$

$$\Rightarrow x^2 - 30x + 25x - 750 = 0$$

$$\Rightarrow x(x-30) + 25(x-30) = 0$$

$$\Rightarrow x - 30 = 0 \ or \ x + 25 = 0$$

$$\Rightarrow x = 30 \ or \ x = -25$$

$\therefore x = 30$ (Price cannot be negative)

Hence, the original price of the book is ₹30.

34.

Sol:

Let the original duration of the tour be x days.

$\therefore$ Original daily expenses = ₹ $\frac{10,800}{x}$

If he extends his tour by 4 days, then his new daily expenses =₹ $\frac{10,800}{x+4}$

According to the given condition,

$$₹\frac{10,800}{x} - ₹\frac{10,800}{x+4} = ₹\ 90$$

$$\Rightarrow \frac{10800x + 43200 - 10800x}{x(x+4)} = 90$$

$$\Rightarrow \frac{43200}{x^2 + 4x} = 90$$

$\Rightarrow x^2 + 4x = 480$
$\Rightarrow x^2 + 4x - 480 = 0$
$\Rightarrow x^2 + 24x - 20x - 480 = 0$
$\Rightarrow x(x+24) - 20x - 480 = 0$
$\Rightarrow x(x+24) - 20(x+24) = 0$
$\Rightarrow (x+24)(x-20) = 0$
$\Rightarrow x + 24 = 0 \text{ or } x - 20 = 0$
$\Rightarrow x = -24 \text{ or } x = 20$

$\therefore x = 20$ (Number of days cannot be negative)

Hence, the original duration of the tour is 20 days.

35.

Sol:

Let the marks obtained by P in mathematics and science be x and $(28-x)$, respectively.

According to the given condition,

$(x+3)(28-x-4) = 180$
$\Rightarrow (x+3)(24-x) = 180$
$\Rightarrow -x^2 + 21x + 72 = 180$
$\Rightarrow x^2 - 21x + 108 = 0$
$\Rightarrow x^2 - 12x - 9x + 108 = 0$
$\Rightarrow x(x-12) - 9(x-12) = 0$
$\Rightarrow (x-12)(x-9) = 0$
$\Rightarrow x - 12 = 0 \text{ or } x - 9 = 0$
$\Rightarrow x = 12 \text{ or } x = 9$

When $x = 12$,

$28 - x = 28 - 12 = 16$

When $x = 9$,

$28 - x = 28 - 9 = 19$

Hence, he obtained 12 marks in mathematics and 16 marks in science or 9 marks in mathematics and 19 marks in science.

36.

Sol:
Let the total number of pens be x.
According to the question:

$$\frac{80}{x}-\frac{80}{x+4}=1$$

$$\Rightarrow \frac{80(x+4)-80x}{x(x+4)}=1$$

$$\Rightarrow \frac{80+320-80x}{x^2+4x}=1$$

$$\Rightarrow 320=x^2+4x$$

$$\Rightarrow x^2+4x-320=0$$

$$\Rightarrow x^2+(20-16)x-320=0$$

$$\Rightarrow x^2+20x-16x-320=0$$

$$\Rightarrow x(x+20)-16(x+20)=0$$

$$\Rightarrow (x+20)(x-16)=0$$

$$\Rightarrow x=-20 \ or \ x=16$$

The total number of pens cannot be negative; therefore, the total number of pens is 16.

37.

Sol:
Let the cost price of the article be x

$\therefore$ Gain percent $= x\%$

According to the given condition,

$$₹\, x+₹\left(\frac{x}{100}\times x\right)= ₹75 \qquad \text{(Cost price + Gain = Selling price)}$$

$$\Rightarrow \frac{100x+x^2}{100}=75$$

$$\Rightarrow x^2+100x=7500$$

$$\Rightarrow x^2+100x-7500=0$$

$$\Rightarrow x^2+150x-50x-7500=0$$

$$\Rightarrow x(x+150)-50(x+150)=0$$

$$\Rightarrow (x-50)(x+150)=0$$

$$\Rightarrow x-50=0 \ or \ x+150=0$$

$\Rightarrow x = 50 \text{ or } x = -150$

$\therefore x = 50$ (Cost price cannot be negative)

Hence, the cost price of the article is ₹50.

38.

Sol:

Let the present age of the son be x years.

$\therefore$ Present age of the man $= x^2$ years

One year ago,

Age of the son $= (x-1)$ years

Age of the man $= (x^2 - 1)$ years

According to the given condition,

Age of the man $= 8 \times$ Age of the son

$\therefore x^2 - 1 = 8(x-1)$

$\Rightarrow x^2 - 1 = 8x - 8$

$\Rightarrow x^2 - 8x + 7 = 0$

$\Rightarrow x^2 - 7x - x + 7 = 0$

$\Rightarrow x(x-7) - 1(x-7) = 0$

$\Rightarrow (x-1)(x-7) = 0$

$\Rightarrow x - 1 = 0 \text{ or } x - 7 = 0$

$\Rightarrow x = 1 \text{ or } x = 7$

$\therefore x = 7$ (Man's age cannot be 1 year)

Present age of the son $= 7$ years

Present age of the man $= 7^2$ years = 49 years.

39.

Sol:

Let the present age of Meena be x years

Meena's age 3 years ago $= (x-3)$ years

Meena's age 5 years hence $= (x+5)$ years

According to the given condition,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2+2x-15}=\frac{1}{3}$

$\Rightarrow x^2+2x-15=6x+6$

$\Rightarrow x^2-4x-21=0$

$\Rightarrow x^2-7x+3x-21=0$

$\Rightarrow x(x-7)+3(x-7)=0$

$\Rightarrow (x-7)(x+3)=0$

$\Rightarrow x-7=0 \text{ or } x+3=0$

$\Rightarrow x=7 \text{ or } x=-3$

$\therefore x=7$ (Age cannot be negative)

Hence, the present age of Meena is 7 years.

40.

Sol:

Let the present ages of the boy and his brother be x years and $(25-x)$ years.

According to the question:

$x(25-x)=126$

$\Rightarrow 25x-x^2=126$

$\Rightarrow x^2-(18-7)x+126=0$

$\Rightarrow x^2-18x-7x+126=0$

$\Rightarrow x(x-18)-7(x-18)=0$

$\Rightarrow (x-18)(x-7)=0$

$\Rightarrow x-18=0 \text{ or } x-7=0$

$\Rightarrow x=18 \text{ or } x=7$

$\Rightarrow x=18$ ($\because$ Present age of the boy cannot be less than his brother)

If $x=18$, we have

Present ages of the boy = 18 years

Present age of his brother $=(25-18)$ years = 7 years

Thus, the present ages of the boy and his brother are 18 years and 7 years, respectively.

41.

Sol:
Let the present age of Meena be x years.
According to the question:
$(x-5)(x+8)=30$
$\Rightarrow x^2+3x-40=30$
$\Rightarrow x^2+3x-70=0$
$\Rightarrow x^2+(10-7)x-70=0$
$\Rightarrow x^2+10x-7x-70=0$
$\Rightarrow x(x+10)-7(x+10)=0$
$\Rightarrow (x+10)(x-7)=0$
$\Rightarrow x+10=0$ or $x-7=0$
$\Rightarrow x=-10$ or $x=7$
$\Rightarrow x=7$ ($\because$ Age cannot be negative)
Thus, the present age of Meena is 7 years.

42.

Sol:
Let son's age 2 years ago be x years. Then,
Man's age 2 years ago $=3x^2$ years
$\therefore$ Sons present age $=(x+2)$ years
Man's present age $=(3x^2+2)$ years
In three years time,
Son's age $=(x+2+3)$ years $=(x+5)$ years
Man's age $=(3x^2+2+3)$ years $=(3x^2+5)$ years
According to the given condition,
Man's age $= 4\times$ Son's age
$\therefore 3x^2+5=4(x+5)$
$\Rightarrow 3x^2+5=4x+20$
$\Rightarrow 3x^2-4x-15=0$
$\Rightarrow 3x^2-9x+5x-15=0$
$\Rightarrow 3x(x-3)+5(x-3)=0$

$\Rightarrow (x-3)(3x+5)=0$

$\Rightarrow x-3=0 \; or \; 3x+5=0$

$\Rightarrow x=3 \; or \; x=-\frac{5}{3}$

$\therefore x=3$ (Age cannot be negative)

Son's present age $=(x+2)$ years $=(3+2)$ years $=5$ years

Man's present age $=(3x^2+2)$ years $=(3\times 9+2)$ years $=29$ years

43.

Sol:

Let the first speed of the truck be $x\ km/h.$

$\therefore$ Time taken to cover 150 $km=\frac{150}{x}h$ $\left(Time=\frac{Distance}{Speed}\right)$

New speed of the truck $=(x+20)\,km/h$

$\therefore$ Time taken to cover 200 km $=\frac{200}{x+20}h$

According to the given condition,

Time taken to cover 150 km + Time taken to cover 200 km = 5 h

$\therefore \frac{150}{x}+\frac{200}{x+20}=5$

$\Rightarrow \frac{150x+3000+200x}{x(x+20)}=5$

$\Rightarrow 350x+3000=5(x^2+20x)$

$\Rightarrow 350x+3000=5x^2+100x$

$\Rightarrow 5x^2-250x-3000=0$

$\Rightarrow x^2-50x-600=0$

$\Rightarrow x^2-60x+10x-600=0$

$\Rightarrow x(x-60)+10(x-60)=0$

$\Rightarrow (x-60)(x+10)=0$

$\Rightarrow x-60=0 \; or \; x+10=0$

$\Rightarrow x=60 \; or \; x=-10$

$\therefore x=60$ (Speed cannot be negative)

Hence, the first speed of the truck is 60 km/h.

44.

Sol:

Let the original speed of the plane be *x* km/h.

$\therefore$ Actual speed of the plane $=(x+100)\, km/h$

Distance of the journey $=1500\ km$

Time taken to reach the destination at original speed $=\dfrac{1500}{x}h$ $\left(Time=\dfrac{Dis\tan ce}{Speed}\right)$

Time taken to reach the destination at actual speed $=\dfrac{1500}{x+100}h$

According to the given condition,

Time taken to reach the destination at original speed = Time taken to reach the destination at actual speed + 30 min

$\therefore \dfrac{1500}{x}=\dfrac{1500}{x+100}+\dfrac{1}{2}$ $\left(30 \text{ min}=\dfrac{30}{60}h=\dfrac{1}{2}h\right)$

$\Rightarrow \dfrac{1500}{x}-\dfrac{1500}{x+100}=\dfrac{1}{2}$

$\Rightarrow \dfrac{1500x+150000-1500x}{x(x+100)}=\dfrac{1}{2}$

$\Rightarrow \dfrac{150000}{x^2+100x}=\dfrac{1}{2}$

$\Rightarrow x^2+100x=300000$

$\Rightarrow x^2+100x=300000=0$

$\Rightarrow x^2+600x-500x-300000=0$

$\Rightarrow x(x+600)-500(x+600)=0$

$\Rightarrow (x+600)(x-500)=0$

$\Rightarrow x+600=0\ or\ x-500=0$

$\Rightarrow x=-600\ or\ x=500$

$\therefore x=500$ (Speed cannot be negative)

Hence, the original speed of the plane is 500 km/h.

Yes, we appreciate the values shown by the pilot, namely promptness in providing help to the injured and his efforts to reach in time. This reflects the caring nature of the pilot and his dedication to the work.

45.

Sol:
Let the usual speed of the train be x km/h.

$\therefore$ Reduced speed of the train $= (x-8) km/h$

Total distance to be covered = 480 km

Time taken by the train to cover the distance at usual speed $= \frac{480}{x} h$ $\left(Time = \frac{Distance}{Speed} \right)$

Time taken by the train to cover the distance at reduced speed $= \frac{480}{x-8} h$

According to the given condition,
Time taken by the train to cover the distance at reduced speed = Time taken by the train to cover the distance at usual speed + 3 h

$\therefore \frac{480}{x-8} = \frac{480}{x} + 3$

$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 3$

$\Rightarrow \frac{480x - 480x + 3840}{x(x-8)} = 3$

$\Rightarrow \frac{3840}{x^2 - 8x} = 3$

$\Rightarrow x^2 - 8x = 1280$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x(x-40) + 32(x-40) = 0$

$\Rightarrow (x-40)(x+32) = 0$

$\Rightarrow x - 40 = 0 \ or \ x + 32 = 0$

$\Rightarrow x = 40 \ or \ x = -32$

$\therefore x = 40$ (Speed cannot be negative)

Hence, the usual speed of the train is 40 km/h.

46.

Sol:
Let the first speed of the train be x km/h.

Time taken to cover $54\ km = \frac{54}{x} h$ $\left(Time = \frac{Distance}{Speed} \right)$

New speed of the train $= (x+6)\ km/h$

$\therefore$ Time taken to cover $63 km = \frac{63}{x+6} h$

According to the given condition,

Time taken to cover 54 km + Time taken to cover 63 km = 3 h

$\therefore \frac{54}{x} + \frac{63}{x+6} = 3$

$\Rightarrow \frac{54x + 324 + 63x}{x(x+6)} = 3$

$\Rightarrow 117x + 324 = 3(x^2 + 6x)$

$\Rightarrow 117x + 324 = 3x^2 + 18x$

$\Rightarrow 3x^2 - 99x - 324 = 0$

$\Rightarrow x^2 - 33x - 324 = 0$

$\Rightarrow x^2 - 33x - 108 = 0$

$\Rightarrow x^2 - 36x + 3x - 108 = 0$

$\Rightarrow x(x-36) + 3(x-36) = 0$

$\Rightarrow (x-36)(x+3) = 0$

$\Rightarrow x - 36 = 0\ or\ x + 3 = 0$

$\Rightarrow x = 36\ or\ x = -3$

$\therefore x = 36$ (Speed cannot be negative)

Hence, the first speed of the train is 36 km/h.

47.

Sol: 36km/hr

48.

Sol:

Let the original speed of the train be *x* km/hr.

According to the question:

$\frac{90}{x} - \frac{90}{(x+15)} = \frac{1}{2}$

$$\Rightarrow \frac{90(x+15)-90x}{x(x+15)}=\frac{1}{2}$$

$$\Rightarrow \frac{90x+1350-90x}{x^2+15x}=\frac{1}{2}$$

$$\Rightarrow \frac{1350}{x^2+15x}=\frac{1}{2}$$

$$\Rightarrow 2700=x^2+15x$$

$$\Rightarrow x^2+(60-45)x-2700=0$$

$$\Rightarrow x^2+60x-45x-2700=0$$

$$\Rightarrow x(x+60)-45x(x+60)=0$$

$$\Rightarrow (x+60)(x-45)=0$$

$$\Rightarrow x=-60 \text{ or } x=45$$

x cannot be negative; therefore, the original speed of train is 45 km/hr.

49.

Sol:

Let the usual speed x km/hr.

According to the question:

$$\frac{300}{x}-\frac{300}{(x+5)}=2$$

$$\Rightarrow \frac{300(x+5)-300x}{x(x+5)}=2$$

$$\Rightarrow \frac{300x+1500-300x}{x^2+5x}=2$$

$$\Rightarrow 1500=2(x^2+5x)$$

$$\Rightarrow 1500=2x^2+10x$$

$$\Rightarrow x^2+5x-750=0$$

$$\Rightarrow x^2+(30-25)x-750=0$$

$$\Rightarrow x^2+30x-25x-750=0$$

$$\Rightarrow x(x+30)-25(x+30)=0$$

$$\Rightarrow (x+30)(x-25)=0$$

$$\Rightarrow x=-30 \text{ or } x=25$$

The usual speed cannot be negative; therefore, the speed is 25 km/hr.

50.

Sol:

Let the speed of the Deccan Queen be x km/hr.

According to the question:

Speed of another train $=(x-20)km/hr$

$\therefore \frac{192}{x-20}-\frac{192}{x}=\frac{48}{60}$

$\Rightarrow \frac{4}{x-20}-\frac{4}{x}=\frac{1}{60}$

$\Rightarrow \frac{4x-4(x-20)}{(x-20)x}=\frac{1}{60}$

$\Rightarrow \frac{4x-4x+80}{x^2-20x}=\frac{1}{60}$

$\Rightarrow \frac{80}{x^2-20x}=\frac{1}{60}$

$\Rightarrow x^2-20x=4800$

$\Rightarrow x^2-20x-4800=0$

$\Rightarrow x^2-(80-60)x-4800=0$

$\Rightarrow x^2-80x+60x-4800=0$

$\Rightarrow x(x-80)+60(x-80)=0$

$\Rightarrow (x-80)(x+60)=0$

$\Rightarrow x=80 \text{ or } x=-60$

The value of

x cannot be negative; therefore, the original speed of Deccan Queen 180 km/hr.

51.

Sol:

Let the speed of the stream be $x\ km/hr$.

Given:

Speed of the boat $=18\ km/hr$

$\therefore$ Speed downstream $=(18+x)\ km/hr$

Speed upstream $=(18-x)\ km/hr$

$\therefore \frac{24}{(18-x)} - \frac{24}{(18-x)} = 1$

$\Rightarrow \frac{1}{(18-x)} - \frac{1}{(18+x)} = \frac{1}{24}$

$\Rightarrow \frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$

$\Rightarrow \frac{2x}{18^2 - x^2} = \frac{1}{24}$

$\Rightarrow 324 - x^2 = 48x$

$\Rightarrow 324 - x^2 - 48x = 0$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow x^2 + (54-6)x - 324 = 0$

$\Rightarrow x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x+54) - 6(x+54) = 0$

$\Rightarrow (x+54)(x-6) = 0$

$\Rightarrow x = -54 \text{ or } x = 6$

The value of x cannot be negative; therefore, the speed of the stream is 6 km/hr.

52.

Sol:

Speed of the boat in still water $= 8\ km/hr.$

Let the speed of the stream be x km/hr.

$\therefore$ Speed upstream $= (8-x)km/hr.$

Speed downstream $= (8+x)km/hr.$

Time taken to go 22 km downstream $= \frac{22}{(8+x)}hr$

Time taken to go 15 km upstream $= \frac{15}{(8-x)}hr$

According to the question:

$\Rightarrow \frac{22}{(8+x)} + \frac{15}{(8-x)} = 5$

$\Rightarrow \frac{22}{(8+x)} + \frac{15}{(8-x)} - 5 = 0$

$$\Rightarrow \frac{22(8-x)+15(8+x)-5(8-x)(8+x)}{(8-x)(8+x)}=0$$

$\Rightarrow 176-22x+120+15x-320+5x^2=0$

$\Rightarrow 5x^2-7x-24=0$

$\Rightarrow 5x^2-(15-8)x-24=0$

$\Rightarrow 5x^2-15x+8x-24=0$

$\Rightarrow 5x(x-3)-8(x-3)=0$

$\Rightarrow (x-3)(5x-8)=0$

$\Rightarrow x-3=0$ *or* $5x-8=0$

$\Rightarrow x=3$ *or* $x=\frac{8}{5}$

$\Rightarrow x=3$ ($\because$ Speed cannot be a fraction)

$\therefore$ Speed of the stream $=3\ km/hr$

53.

Sol:

Let the speed of the stream be *x* km/hr.

$\therefore$ Downstream speed $=(9+x)km/hr.$

Upstream speed $=(9-x)km/hr$

Distance covered downstream = Distance covered upstream = 15 km

Total time taken = 3hours 45 minutes $=\left(3+\frac{45}{60}\right)$ minutes $=\frac{225}{60}$ minutes $=\frac{15}{4}$ minutes

$$\therefore \frac{15}{(9+x)}+\frac{15}{(9-x)}=\frac{15}{4}$$

$$\Rightarrow \frac{1}{(9+x)}+\frac{1}{(9-x)}=\frac{1}{4}$$

$$\Rightarrow \frac{9-x+9+x}{(9+x)(9-x)}=\frac{1}{4}$$

$$\Rightarrow \frac{18}{9^2-x^2}=\frac{1}{4}$$

$$\Rightarrow \frac{18}{81-x^2}=\frac{1}{4}$$

$\Rightarrow 81-x^2=72$

$\Rightarrow 81-x^2-72=0$

$\Rightarrow -x^2 + 9 = 0$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3 \text{ or } x = -3$

The value of x cannot be negative; therefore, the speed of the stream is 3 km/hr.

54.

Sol:

Let B takes x days to complete the work.

Therefore, A will take $(x-10)$ days.

$\therefore \frac{1}{x} + \frac{1}{(x-10)} = \frac{1}{12}$

$\Rightarrow \frac{(x-10)+x}{x(x-10)} = \frac{1}{12}$

$\Rightarrow \frac{2x-10}{x^2-10x} = \frac{1}{12}$

$\Rightarrow x^2 - 10x = 12(2x-10)$

$\Rightarrow x^2 - 10x = 24x - 120$

$\Rightarrow x^2 - 34x + 120 = 0$

$\Rightarrow x^2 - (30+4)x + 120 = 0$

$\Rightarrow x^2 - 30x - 4x + 120 = 0$

$\Rightarrow x(x-30) - 4(x-30) = 0$

$\Rightarrow (x-30)(x-4) = 0$

$\Rightarrow x = 30 \text{ or } x = 4$

Number of days to complete the work by

B cannot be less than that by A; therefore, we get: $x = 30$

Thus, B completes the work in 30 days.

55.

Sol:

Let one pipe fills the cistern in x mins.

Therefore, the other pipe will fill the cistern in $(x+3)$ mins.

Time taken by both, running together, to fill the cistern $= 3\frac{1}{13} \text{ mins} = \frac{40}{13} \text{ mins}$

Part filled by one pipe in 1 min $= \frac{1}{x}$

Part filled by the other pipe in 1 min $= \frac{1}{x+3}$

Part filled by both pipes, running together, in 1 min $= \frac{1}{x} + \frac{1}{x+3}$

$\therefore \frac{1}{x} + \frac{1}{x+3} = \frac{1}{\frac{40}{13}}$

$\Rightarrow \frac{(x+3)+x}{x(x+3)} = \frac{13}{40}$

$\Rightarrow \frac{2x+3}{x^2+3a} = \frac{13}{40}$

$\Rightarrow 13x^2 + 39x = 80x + 120$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - (65-24)x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0$

$\Rightarrow (x-5)(13x+24) = 0$

$\Rightarrow x - 5 = 0 \text{ or } 13x + 24 = 0$

$\Rightarrow x = 5 \text{ or } x = \frac{-24}{13}$

$\Rightarrow x = 5$ ($\because$ Speed cannot be a negative fraction)

Thus, one pipe will take 5 mins and other will take $\{(5+3) = 8\}$ mins to fill the cistern.

56.

Sol:

Let the time taken by one pipe to fill the tank be *x* minutes.

$\therefore$ Time taken by the other pipe to fill the tank $= (x+5)$ min

Suppose the volume of the tank be *V*.

Volume of the tank filled by one pipe in *x* minutes = *V*

$\therefore$ Volume of the tank filled by one pipe in 1 minute $= \frac{V}{x}$

$\Rightarrow$ Volume of the tank filled by one pipe in $11\frac{1}{9}$ minutes $= \frac{V}{x} \times 11\frac{1}{9} = \frac{V}{x} \times \frac{100}{9}$

Similarly,

Volume of the tank filled by the other pipe in $11\frac{1}{9}$ minutes $= \frac{V}{(x+5)} \times 11\frac{1}{9} = \frac{V}{(x+5)} \times \frac{100}{9}$

Now,

Volume of the tank filled by one pipe in $11\frac{1}{9}$ minutes + Volume of the tank filled by the other pipe in $11\frac{1}{9}$ minutes $= V$

$\therefore V\left(\frac{1}{x} + \frac{1}{x+5}\right) \times 100 = V$

$\Rightarrow \frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$

$\Rightarrow \frac{x+5+x}{x(x+5)} = \frac{9}{100}$

$\Rightarrow \frac{2x+5}{x^2+5x} = \frac{9}{100}$

$\Rightarrow 200x + 500 = 9x^2 + 45x$

$\Rightarrow 9x^2 - 155x - 500 = 0$

$\Rightarrow 9x^2 - 180x + 25x - 500 = 0$

$\Rightarrow 9x(x-20) + 25(x-20) = 0$

$\Rightarrow (x-20)(9x+25) = 0$

$\Rightarrow x - 20 = 0 \text{ or } 9x + 25 = 0$

$\Rightarrow x = 20 \text{ or } x = -\frac{25}{9}$

$\therefore x = 20$ (Time cannot be negative)

Time taken by one pipe to fill the tank = 20 min

Time taken by other pipe to fill the tank = (20 + 5) 25 min

57.

Sol:

Let the tap of smaller diameter fill the tank in x hours.

$\therefore$ Time taken by the tap of larger diameter to fill the tank $= (x-9)h$

Suppose the volume of the tank be V.

Volume of the tank filled by the tap of smaller diameter in x hours = V

$\therefore$ Volume of the tank filled by the tap of smaller diameter in 1 hour $= \frac{V}{x}$

$\Rightarrow$ Volume of the tank filled by the tap of smaller diameter in 6 hours $= \frac{V}{x} \times 6$

Similarly,

Volume of the tank filled by the tap of larger diameter in 6 hours $= \frac{V}{(x-9)} \times 6$

Now,

Volume of the tank filled by the tap of smaller diameter in 6 hours + Volume of the tank filled by the tap of larger diameter in 6 hours = V

$\therefore V\left(\frac{1}{x} + \frac{1}{x-9}\right) \times 6 = V$

$\Rightarrow \frac{1}{x} + \frac{1}{x-9} = \frac{1}{6}$

$\Rightarrow \frac{x-9+x}{x(x-9)} = \frac{1}{6}$

$\Rightarrow \frac{2x-9}{x^2-9x} = \frac{1}{6}$

$\Rightarrow 12x - 54 = x^2 - 9x$

$\Rightarrow x^2 - 21x + 54 = 0$

$\Rightarrow x^2 - 81x - 3x + 54 = 0$

$\Rightarrow x(x-18) - 3(x-18) = 0$

$\Rightarrow (x-18)(x-3) = 0$

$\Rightarrow x - 18 = 0 \ or \ x - 3 = 0$

$\Rightarrow x = 18 \ or \ x = 3$

For $x = 3$, time taken by the tap of larger diameter to fill the tank is negative which is not possible.

$\therefore x = 18$

Time taken by the tap of smaller diameter to fill the tank = 18 h

Time taken by the tap of larger diameter to fill the tank $= (18-9) = 9h$

Hence, the time taken by the taps of smaller and larger diameter to fill the tank is 18 hours and 9 hours, respectively.

58.

Sol:

Let the length and breadth of the rectangle be $2x$ m and x m, respectively.

According to the question:

$2x \times x = 288$

$\Rightarrow 2x^2 = 288$

$\Rightarrow x^2 = 144$

$\Rightarrow x = 12 \; or \; x = -12$

$\Rightarrow x = 12$ $\quad$ ($\because x$ cannot be negative)

$\therefore \text{Length} = 2 \times 12 = 24m$

Breath $= 12m$

59.

Sol:

Let the length and breadth of the rectangle be $3x$ m and x m, respectively.

According to the question:

$3x \times x = 147$

$\Rightarrow 3x^2 = 147$

$\Rightarrow x^2 = 49$

$\Rightarrow x = 7 \; or \; x = -7$

$\Rightarrow x = 7$ ($\because x$ cannot be negative)

$\therefore \text{Length} = 3 \times 7 = 21m$

Breath $= 7m$

60.

Sol:

Let the breath of the rectangular hall be x meter.

Therefore, the length of the rectangular hall will be $(x+3)$ meter.

According to the question:

$x(x+3) = 238$

$\Rightarrow x^2 + 3x = 238$

$\Rightarrow x^2 + 3x - 238 = 0$

$\Rightarrow x^2 + (17 - 14)x - 238 = 0$

$\Rightarrow x^2 + 17x - 14x - 238 = 0$

$\Rightarrow x(x+17) - 14(x+17) = 0$

$\Rightarrow (x+17)(x-14) = 0$

$\Rightarrow x = -17 \; or \; x = 14$

But the value x cannot be negative.
Therefore, the breath of the hall is 14 meter and the length is 17 meter.

61.

Sol:
Let the length and breadth of the rectangular plot be x and y meter, respectively. Therefore, we have:
Perimeter $=2(x+y)=62$ .....(i) and
Area $= xy = 228$

$$\Rightarrow y=\frac{228}{x}$$

Putting the value of y in (i), we get

$$\Rightarrow 2\left(x+\frac{228}{x}\right)=62$$

$$\Rightarrow x+\frac{228}{x}=31$$

$$\Rightarrow \frac{x^2+228}{x}=31$$

$$\Rightarrow x^2+228=31x$$

$$\Rightarrow x^2-31x+228=0$$

$$\Rightarrow x^2-(19+12)x+228=0$$

$$\Rightarrow x^2-19x-12x+228=0$$

$$\Rightarrow x(x-19)-12(x-19)=0$$

$$\Rightarrow (x-19)(x-12)=0$$

$$\Rightarrow x=19 \ or \ x=12$$

If $x=19\,m, y=\frac{228}{19}=12\,m$

Therefore, the length and breadth of the plot are 19 m and 12 m, respectively.

62.

Sol:
Let the width of the path be x m.
$\therefore$ Length of the field including the path $=16+x+x=16+2x$
Breadth of the field including the path $=10+x+x=10+2x$
Now,

(Area of the field including path) - (Area of the field excluding path) =
Area of the path

$\Rightarrow (16+2x)(10+2x)-(16\times 10)=120$

$\Rightarrow 160+32x+20x+4x^2-160=120$

$\Rightarrow 4x^2+52x-120=0$

$\Rightarrow x^2+13x-30=0$

$\Rightarrow x^2+(15-2)x+30=0$

$\Rightarrow x^2+15x-2x+30=0$

$\Rightarrow x(x+15)-2(x+15)=0$

$\Rightarrow (x-2)(x+15)=0$

$\Rightarrow x-2=0 \; or \; x+15=0$

$\Rightarrow x=2 \; or \; x=-15$

$\Rightarrow x=2$ ($\because$ Width cannot be negative)

Thus, the width of the path is 2 m.

63.

Sol:
Let the length of the side of the first and the second square be *x* and *y*. respectively.
According to the question:

$x^2+y^2=640 \qquad ....(i)$

Also,

$4x-4y=64$

$\Rightarrow x-y=16$

$\Rightarrow x=16+y$

Putting the value of *x* in (i), we get:

$x^2+y^2=640$

$\Rightarrow (16+y)^2+y^2=640$

$\Rightarrow 256+32y+y^2+y^2=640$

$\Rightarrow 2y^2+32y-384=0$

$\Rightarrow y^2+16y-192=0$

$\Rightarrow y^2+(24-8)y-192=0$

$\Rightarrow y^2+24y-8y-192=0$

$\Rightarrow y(y+24)-8(y+24)=0$

$\Rightarrow (y+24)(y-8)=0$

$\Rightarrow y=-24 \text{ or } y=8$

$\therefore y=8$ ($\because$ Side cannot be negative)

$\therefore x=16+y=16+8=24\ m$

Thus, the sides of the squares are 8 m and 24 m.

64.

Sol:

Let the breadth of rectangle be *x* cm.

According to the question:

Side of the square $=(x+4)cm$

Length of the rectangle = {3(*x* + 4)} cm

It is given that the areas of the rectangle and square are same.

$\therefore 3(x+4)\times x=(x+4)^2$

$\Rightarrow 3x^2+12x=(x+4)^2$

$\Rightarrow 3x^2+12x=x^2+8x+16$

$\Rightarrow 2x^2+4x-16=0$

$\Rightarrow x^2+2x-8=0$

$\Rightarrow x^2+(4-2)x-8=0$

$\Rightarrow x^2+4x-2x-8=0$

$\Rightarrow x(x+4)-2(x+4)=0$

$\Rightarrow (x+4)(x-2)=0$

$\Rightarrow x=-4 \text{ or } x=2$

$\therefore x=2$ ($\because$ The value of *x* cannot be negative)

Thus, the breadth of the rectangle is 2 cm and length is {3(2 + 4) =18} cm.

Also, the side of the square is 6 cm.

65.

Sol:

Let the length and breadth of the rectangular garden be *x* and *y* meter, respectively. Given:

$xy=180\ sq\ m$ ...(i)

and

$2y + x = 39$

$\Rightarrow x = 39 - 2y$

Putting the value of x in (i), we get:

$(39 - 2y)\, y = 180$

$\Rightarrow 39 - 2y^2 = 180$

$\Rightarrow 39y - 2y^2 - 180 = 0$

$\Rightarrow 2y^2 - 39y + 180 = 0$

$\Rightarrow 2y^2 - (24 + 15)\, y + 180 = 0$

$\Rightarrow 2y^2 - 24y - 15y + 180 = 0$

$\Rightarrow 2y(y - 12) - 15(y - 12) = 0$

$\Rightarrow (y - 12)(2y - 15) = 0$

$\Rightarrow y = 12 \;\; or \;\; y = \frac{15}{2} = 7.5$

If $y = 12, x = 39 - 24 = 15$

If $y = 7.5, x = 39 - 15 = 24$

Thus, the

length and breadth of the garden are (15 m and 12 m) or (24 m and 7.5 m), respectively.

66.

Sol:

Let the altitude of the triangle be x cm

Therefore, the base of the triangle will be $(x + 10)cm$

Area of triangle $= \frac{1}{2} x(x + 10) = 600$

$\Rightarrow (x + 10) = 1200$

$\Rightarrow x^2 + 10x - 1200 = 0$

$\Rightarrow x^2 + (40 - 30)x - 1200 = 0$

$\Rightarrow x^2 + 40x - 30x - 1200 = 0$

$\Rightarrow x(x + 40) - 30(x + 40) = 0$

$\Rightarrow (x + 40)(x - 30) = 0$

$\Rightarrow x = -40 \;\; or \;\; x = 30$

$\Rightarrow x = 30$ [$\because$ Altitude cannot be negative]

Thus, the altitude and base of the triangle are 30 cm and (30 + 10 = 40) cm, respectively.

$(Hypotenuse)^2 = (Altitude)^2 + (Base)^2$

$\Rightarrow (Hypotenuse)^2 = (30)^2 (40)^2$

$\Rightarrow (Hypotenuse)^2 = 900 + 1600 = 2500$

$\Rightarrow (Hypotenuse)^2 = (50)^2$

$\Rightarrow (Hypotenuse) = 50$

Thus, the dimensions of the triangle are:
Hypotenuse = 50 cm
Altitude = 30 cm
Base = 40 cm

67.

Sol:
Let the altitude of the triangle be x m.
Therefore, the base will be $3x$ m.

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$\therefore \frac{1}{2} \times 3x \times x = 96 (\because Area = 96\, sq\, m)$

$\Rightarrow \frac{x^2}{2} = 32$

$\Rightarrow x^2 = 64$

$\Rightarrow x = \pm 8$

The value of x cannot be negative
Therefore, the altitude and base of the triangle are 8 m and $(3 \times 8 = 24 m)$, respectively.

68.

Sol:
Let the base be x m.
Therefore, the altitude will be $(x+7)m$.

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$\therefore \frac{1}{2} \times x \times (x+7) = 165$

$\Rightarrow x^2 + 7x = 330$

$\Rightarrow x^2 + 7x - 330 = 0$

$\Rightarrow x^2+(22-15)x-330=0$

$\Rightarrow x^2+22x-15x-330=0$

$\Rightarrow x(x+22)-15(x+22)=0$

$\Rightarrow (x+22)(x-15)=0$

$\Rightarrow x=-22 \text{ or } x=15$

The value of x cannot be negative
Therefore, the base is 15 m and the altitude is {(15 + 7) = 22 m}.

69.

Sol:
Let one side of the right-angled triangle be *x* m and the other side be $(x+4)m$.
On applying Pythagoras theorem, we have:

$20^2=(x+4)^2+x^2$

$\Rightarrow 400=x^2+8x+16+x^2$

$\Rightarrow 2x^2+8x-384=0$

$\Rightarrow x^2+4x-192=0$

$\Rightarrow x^2+(16-12)x-192=0$

$\Rightarrow x^2+16x-12x-192=0$

$\Rightarrow x^2(x+16)-12(x+16)=0$

$\Rightarrow (x+16)(x-12)=0$

$\Rightarrow x=-16 \text{ or } x=12$

The value of *x* cannot be negative.
Therefore, the base is 12 m and the other side is {(12 + 4) = 16 m).

70.

Sol:
Let the base and altitude of the right-angled triangle be *x* and *y* cm, respectively
Therefore, the hypotenuse will be (*x* + 2) cm.

$\therefore (x+2)^2=y^2+x^2 \qquad ....(i)$

Again, the hypotenuse exceeds twice the length of the altitude by 1 cm.

$\therefore h=(2y+1)$

$\Rightarrow x+2=2y+1$

$\Rightarrow x=2y-1$

Putting the value of *x* in (i), we get:

$(2y-1+2)^2 = y^2 + (2y-1)^2$

$\Rightarrow (2y+1)^2 = y^2 + 4y^2 - 4y + 1$

$\Rightarrow 4y^2 + 4y + 1 = 5y^2 - 4y + 1$

$\Rightarrow -y^2 + 8y = 0$

$\Rightarrow y^2 - 8y = 0$

$\Rightarrow y(y-8) = 0$

$\Rightarrow y = 8\ cm$

$\therefore x = 16 - 1 = 15\ cm$

$\therefore h = 16 + 1 = 17\ cm$

Thus, the base, altitude and hypotenuse of the triangle are 15 cm, 8 cm and 17 cm, respectively.

71.

Sol:

Let the shortest side be x m.

Therefore, according to the question:

Hypotenuse $= (2x-1)m$

Third side $= (x+1)m$

On applying Pythagoras theorem, we get:

$(2x-1)^2 = (x+1)^2 + x^2$

$\Rightarrow 4x^2 - 4x + 1 = x^2 + 2x + 1 + x^2$

$\Rightarrow 2x^2 - 6x = 0$

$\Rightarrow 2x(x-3) = 0$

$\Rightarrow x = 0\ or\ x = 3$

The length of the side cannot be 0; therefore, the shortest side is 3 m.

Therefore,

Hypotenuse $= (2\times 3-1) = 5m$

Third side $= (3+1) = 4m$

Exercise - 10F

1.

Answer: (d) $2x^2 - 5x = (x-1)^2$

Sol:

A quadratic equation is the equation with degree 2.

$\because 2x^2 - 5x = (x-1)^2$

$\Rightarrow 2x^2 - 5x = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 5x - x^2 + 2x - 1 = 0$

$\Rightarrow x^2 - 3x - 1 = 0$, which is a quadratic equation

2.

Answer: (b) $x^3 - x^2 = (x-1)^3$

Sol:

$\because x^3 - x^2 = (x-1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$\Rightarrow 2x^2 - 3x + 1 = 0$, which is a quadratic equation

3.

Answer: (c) $\left(\sqrt{2}x + 3\right)^2 = 2x^2 + 6$

Sol:

$\because \left(\sqrt{2}x + 3\right)^2 = 2x^2 + 6$

$\Rightarrow 2x^2 + 9 + 6\sqrt{2}x = 2x^2 + 6$

$\Rightarrow 6\sqrt{2}x + 3 = 0$, which is not a quadratic equation

4.

Answer: (b) -11

Sol:

It is given that $x=3$ is a solution of $3x^2+(k-1)x+9=0$; therefore, we have:

$3(3)^2+(k-1)\times3+9=0$

$\Rightarrow 27+3(k-1)+9=0$

$\Rightarrow 3(k-1)=-36$

$\Rightarrow (k-1)=-12$

$\Rightarrow k=-11$

5.

Answer: (b) -7

Sol:

It is given that one root of the equation $2x^2+ax+6=0$ is 2.

$\therefore 2\times2^2+a\times2+6=0$

$\Rightarrow 2a+14=0$

$\Rightarrow a=-7$

6.

Answer: (b) -2

Sol:

Sum of the roots of the equation $x^2-6x+2=0$ is

$\alpha+\beta=\frac{-b}{a}=\frac{-(-6)}{1}=6$, where α and β are the roots of the equation.

7.

Answer: (c) 8

Sol:

It is given that the product of the roots of the equation $x^2-3x+k=10$ is -2.

The equation can be rewritten as:

$x^2-3x+(k-10)=0$

Product of the roots of a quadratic equation $= \frac{c}{a}$

$\Rightarrow \frac{c}{a} = -2$

$\Rightarrow \frac{(k-10)}{1} = -2$

$\Rightarrow k = 8$

8.

Answer : (d)2 : 3

Sol:

Given:

$7x^2 - 12x + 18 = 0$

$\therefore \alpha + \beta = \frac{12}{7}$ *and* $\beta = \frac{18}{7}$, where α and β are the roots of the equation

$\therefore$ Ratio of the sum and product of the roots $= \frac{12}{7} : \frac{18}{7}$

$= 12 : 18$

$= 2 : 3$

9.

Answer: (d) 3

Sol:

Given:

$3x^2 - 10x + 3 = 0$

One root of the equation is $\frac{1}{3}$.

Let the other root be α.

Product of the roots $= \frac{c}{a}$

$\Rightarrow \frac{1}{3} \times \alpha = \frac{3}{3}$

$\Rightarrow \alpha = 3$

10. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root then the value of k is

(a) 0 (b) 1 (c)2(d)5

Answer: (d)5

Sol:

Let the roots of the equation $\frac{-2}{3}$ be α and $\frac{1}{\alpha}$.

$\therefore$ Product of the roots $= \frac{c}{a}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{5}$

$\Rightarrow a = \frac{k}{5}$

$\Rightarrow k = 5$

11.

Answer: (d) -2/3

Sol:

Given:

$kx^2 + 2x + 3k = 0$

Sum of the roots = Product of the roots

$\Rightarrow \frac{-2}{k} = \frac{3k}{k}$

$\Rightarrow 3k = -2$

$\Rightarrow k = \frac{-2}{3}$

12.

Answer: (b) $x^2 - 3x - 10 = 0$

Sol:

It is given that the roots of the quadratic equation are 5 and -2.

Then, the equation is:

$x^2 - (5-2)x + 5 \times (-2) = 0$

$\Rightarrow x^2 - 3x - 10 = 0$

13.

Answer: (a) $x^2 - 6x + 6 = 0$

Sol:

Given:

Sum of roots = 6

Product of roots = 6

Thus, the equation is:

$x^2 - 6x + 6 = 0$

14.

Answer: (c) -4

Sol:

It is given that α and β are the roots of the equation $3x^2 + 8x + 2 = 0$

$$\therefore \alpha + \beta = -\frac{8}{3} \text{ and } \alpha\beta = \frac{2}{3}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{8}{3}}{\frac{2}{3}} = -4$$

15.

Answer: (c) c = a

Sol:

Let the roots of the equation $\left(ax^2 + bx + c = 0\right)$ be α and $\frac{1}{\alpha}$.

$\therefore$ Product of the roots $= \alpha \times \frac{1}{\alpha} = 1$

$$\Rightarrow \frac{c}{a} = 1$$

$$\Rightarrow c = a$$

16. If the roots of the equation $ax^2 + bx + c = 0$ are equal then c=?

(a) $\frac{-b}{2a}$ (b) $\frac{b}{2a}$ (c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

Answer: (d) $\frac{b^2}{4a}$

Sol:

It is given that the roots of the equation $\left(ax^2+bx+c=0\right)$ are equal.

$\therefore \left(b^2-4ac\right)=0$

$\Rightarrow b^2=4ac$

$\Rightarrow c=\frac{b^2}{4a}$

17.

Answer: (c) 2 or -2

Sol:

It is given that the roots of the equation $\left(9x^2+6kx+4=0\right)$ are equal.

$\therefore \left(b^2-4ac\right)=0$

$\Rightarrow \left(6k\right)^2-4\times9\times4=0$

$\Rightarrow 36k^2=144$

$\Rightarrow k^2=4$

$\Rightarrow k=\pm2$

18.

Answer: (a) 1 or 4

Sol:

It is given that the roots of the equation $\left(x^2+2\left(k+2\right)x+9k=0\right)$ are equal.

$\therefore \left(b^2-4ac\right)=0$

$\Rightarrow \left\{2\left(k+2\right)\right\}^2-4\times1\times9k=0$

$\Rightarrow 4\left(k^2+4k+4\right)-36k=0$

$\Rightarrow 4k^2+16k+16-36k=0$

$\Rightarrow 4k^2-20+16=0$

$\Rightarrow k^2 - 5k + 4 = 0$
$\Rightarrow k^2 - 4k - k + 4 = 0$
$\Rightarrow k(k-4) - (k-4) = 0$
$\Rightarrow (k-4)(k-1) = 0$
$\Rightarrow k = 4 \text{ or } k = 1$

19.

Answer: (d) $\pm\frac{4}{3}$

Sol:

It is given that the roots of the equation $(4x^2 - 3kx + 1 = 0)$ are equal.

$\therefore (b^2 - 4ac) = 0$
$\Rightarrow (3k)^2 - 4 \times 4 \times 1 = 0$
$\Rightarrow 9k^2 = 16$
$\Rightarrow k^2 = \frac{16}{9}$
$\Rightarrow k = \pm\frac{4}{3}$

20.

Answer: (a) > 0

Sol:

The roots of the equation are real and unequal when $(b^2 - 4ac) > 0$.

21.

Answer: (b) real and unequal

Sol:

We know that when discriminant, $D > 0$, the roots of the given quadratic equation are real and unequal.

22.

Answer: (d) imaginary

Sol:

$\because D = \left(b^2 - 4ac\right)$

$= \left(-6\right)^2 - 4 \times 2 \times 7$

$= 36 - 56$

$= -20 < 0$

Thus, the roots of the equation are imaginary

23.

Answer: (b) real, unequal and irrational

Sol:

$\because D = \left(b^2 - 4ac\right)$

$= \left(-6\right)^2 - 4 \times 2 \times 3$

$= 36 - 24$

$= 12$

12 is greater than

0 and it is not a perfect square; therefore, the roots of the equation are real, unequal and irrational.

24.

Answer: (d) either $k > 2\sqrt{5}$ *or* $k < -2\sqrt{5}$

Sol:

It is given that the roots of the equation $\left(5x^2 - k + 1 = 0\right)$ are real and distinct.

$\therefore \left(b^2 - 4ac\right) > 0$

$\Rightarrow \left(-k\right)^2 - 4 \times 5 \times 1 > 0$

$\Rightarrow k^2 - 20 > 0$

$\Rightarrow k^2 > 20$

$\Rightarrow k > \sqrt{20} \text{ or } k < -\sqrt{20}$

$\Rightarrow k > 2\sqrt{5} \text{ or } k < -2\sqrt{5}$

25.

Answer: (c) $\frac{-8}{5} < k < \frac{8}{5}$

Sol:

It is given that the equation $\left(x^2 + 5kx + 16 = 0\right)$ has no real roots.

$\therefore \left(b^2 - 4ac\right) < 0$

$\Rightarrow (5k)^2 - 4 \times 1 \times 16 < 0$

$\Rightarrow 25k^2 - 64 < 0$

$\Rightarrow k^2 < \frac{64}{25}$

$\Rightarrow \frac{-8}{5} < k < \frac{8}{5}$

26.

Answer: c) -2 < k < 2

Sol:

It is given that the equation $x^2 - kx + 1 = 0$ has no real roots.

$\therefore \left(b^2 - 4ac\right) < 0$

$\Rightarrow (-k)^2 - 4 \times 1 \times 1 < 0$

$\Rightarrow k^2 < 4$

$\Rightarrow -2 < k < 2$

27.

Answer: (b) $k \geq \frac{-9}{2}$

Sol:

It is given that the roots of the equation $\left(kx^2 - 6x - 2 = 0\right)$ are real.

$\therefore D \geq 0$

$\Rightarrow \left(b^2 - 4ac\right) \geq 0$

$\Rightarrow (-6)^2 - 4 \times k \times (-2) \geq 0$

$\Rightarrow 36+8k \geq 0$

$\Rightarrow k \geq \frac{-36}{8}$

$\Rightarrow k \geq \frac{-9}{2}$

28.

Answer: (a) $\frac{5}{4}$ *or* $\frac{4}{5}$

Sol:

Let the required number be *x*.

According to the question:

$x+\frac{1}{x}=\frac{41}{20}$

$\Rightarrow \frac{x^2+1}{x}=\frac{41}{20}$

$\Rightarrow 20x^2-41x+20=0$

$\Rightarrow 20x^2-25x-16x+20=0$

$\Rightarrow 5x(4x-5)-4(4x-5)=0$

$\Rightarrow (4x-5)(5x-4)=0$

$\Rightarrow x=\frac{5}{4} \text{ or } x=\frac{4}{5}$

29.

Answer: (c) 16 m

Sol:

Let the length and breadth of the rectangle be *l* and *b*.

Perimeter of the rectangle $=82\,m$

$\Rightarrow 2\times(l+b)=82$

$\Rightarrow l+b=41$

$\Rightarrow l=(41-b)$ ....(i)

Area of the rectangle $=400\ m^2$

$\Rightarrow l\times b=400\,m^2$

$\Rightarrow (41-b)b = 400$ (using (i))

$\Rightarrow 41b - b^2 = 400$

$\Rightarrow b^2 - 41b + 400 = 0$

$\Rightarrow b^2 - 25b - 16b + 400 = 0$

$\Rightarrow b(b-25) - 16(b-25) = 0$

$\Rightarrow (b-25)(b-16) = 0$

$\Rightarrow b = 25 \text{ or } b = 16$

If $b = 25$, we have:

$l = 41 - 25 = 16$

Since, l cannot be less than b,

$\therefore b = 16 m$

30.

Sol:

Let the breadth of the rectangular field be x m.

$\therefore$ Length of the rectangular field$=(x+8)m$

Area of the rectangular field $= 240 m^2$ (Given)

$\therefore (x+8) \times x = 240$ (Area=Length$\times$Breadth)

$\Rightarrow x^2 + 8x - 240 = 0$

$\Rightarrow x^2 + 20x - 12x - 240 = 0$

$\Rightarrow x(x+20) - 12(x+20) = 0$

$\Rightarrow (x+20)(x-12) = 0$

$\Rightarrow x + 20 = 0 \text{ or } x - 12 = 0$

$\Rightarrow x = -20 \text{ or } x = 12$

$\therefore x = 12$ (Breadth cannot be negative)

Thus, the breadth of the field is 12 m

Hence, the correct answer is option C.

31.

Answer: (b) 2, -3/2

Sol:

The given quadratic equation is $2x^2-x-6=0.$

$2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0 \ or \ 2x+3=0$

$\Rightarrow x=2 \ or \ x=\frac{-3}{2}$

Thus, the roots of the given equation are 2 and $\frac{-3}{2}$

Hence, the correct answer is option B.

32.

Sol:

Let the required natural numbers be x and $(8-x)$.

It is given that the product of the two numbers is 15.

$\therefore x(8-x)=15$

$\Rightarrow 8x-x^2=15$

$\Rightarrow x^2-8x+15=0$

$\Rightarrow x^2-5x-3x+15=0$

$\Rightarrow x(x-5)-3(x-5)=0$

$\Rightarrow (x-5)(x-3)=0$

$\Rightarrow x-5=0 \ or \ x-3=0$

$\Rightarrow x=5 \ or \ x=3$

Hence, the required numbers are 3 and 5.

33.

Sol:

The given equation is $x^2+6x+9=0$

Putting $x=-3$ in the given equation, we get

$LHS=(-3)^2+6\times(-3)+9=9-18+9=0=RHS$

$\therefore x=-3$ is a solution of the given equation.

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34.

Sol:

The given equation is $3x^2+13x+14=0.$

Putting $x=-2$ in the given equation, we get

$LHS-3\times(-2)^2+13\times(-2)+14=12-26+14=0=RHS$

$\therefore x=-2$ is a solution of the given equation.

35.

Sol:

It is given that $x=\frac{-1}{2}$ is a solution of the quadratic equation $3x^2+2kx-3=0.$

$\therefore 3\times\left(\frac{-1}{2}\right)^2+2k\times\left(\frac{-1}{2}\right)-3=0$

$\Rightarrow \frac{3}{4}-k-3=0$

$\Rightarrow k=\frac{3-12}{4}=-\frac{9}{4}$

Hence, the value of k is $-\frac{9}{4}$.

36.

Sol:

The given quadratic equation is $2x^2-x-6=0.$

$2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0 \ or \ 2x+3=0$

$\Rightarrow x=2 \ or \ x=-\frac{3}{2}$

Hence, the roots of the given equation are 2 and $-\frac{3}{2}$.

37.

Sol:

The given quadratic equation is $3\sqrt{3}x^2+10x+\sqrt{3}=0$

$3\sqrt{3}x^2+10x+\sqrt{3}=0$

$\Rightarrow 3\sqrt{3}x^2+9x+x+\sqrt{3}=0$

$\Rightarrow 3\sqrt{3}x\left(x+\sqrt{3}\right)+1\left(x+\sqrt{3}\right)=0$

$\Rightarrow \left(x+\sqrt{3}\right)\left(3\sqrt{3}x+1\right)=0$

$\Rightarrow x+\sqrt{3}=0 \ or \ 3\sqrt{3}x+1=0$

$\Rightarrow x=-\sqrt{3} \ or \ x=-\frac{1}{3\sqrt{3}}=-\frac{\sqrt{3}}{9}$

Hence, $-\sqrt{3}$ and $-\frac{\sqrt{3}}{9}$ are the solutions of the given equation.

38.

Sol:

It is given that the roots of the quadratic equation $2x^2+8x+k=0$ are equal.

$\therefore D=0$

$\Rightarrow 8^2-4\times 2\times k=0$

$\Rightarrow 64-8k=0$

$\Rightarrow k=8$

Hence, the value of k is 8.

39.

Sol:

It is given that the quadratic equation $px^2-2\sqrt{5}px+15=0$ has two equal roots.

$\therefore D=0$

$\Rightarrow \left(-2\sqrt{5}p\right)^2-4\times p\times 15=0$

$\Rightarrow 20p^2-60p=0$

$\Rightarrow 20p\left(p-3\right)=0$

$\Rightarrow p=0 \ or \ p-3=0$

$\Rightarrow p=0 \ or \ p=3$

For $p=0$, we get $15=0$, which is not true.

$\therefore p\neq 0$

Hence, the value of p is 3.

40.

Sol:

It is given that $y=1$ is a root of the equation $ay^2+ay+3=0$.

$\therefore a\times(1)^2+a\times 1+3=0$

$\Rightarrow a+a+3=0$

$\Rightarrow 2a+3=0$

$\Rightarrow a=-\frac{3}{2}$

Also, $y = 1$ is a root of the equation $y^2+y+b=0$.

$\therefore (1)^2+1+b=0$

$\Rightarrow 1+1+b=0$

$\Rightarrow b+2=0$

$\Rightarrow b=-2$

$\therefore ab=\left(-\frac{3}{2}\right)\times(-2)=3$

Hence, the value of ab is 3.

41.

Sol:

Let the other zero of the given polynomial be α.

Now,

Sum of the zeroes of the given polynomial $=\frac{-(-4)}{1}=4$

$\therefore \alpha+\left(2+\sqrt{3}\right)=4$

$\Rightarrow \alpha=4-2-\sqrt{3}=2-\sqrt{3}$

Hence, the other zero of the given polynomial is $\left(2-\sqrt{3}\right)$.

42.

Sol:

Let α and β be the roots of the equation $3x^2-10x+k=0$.

$\therefore \alpha = \frac{1}{\beta}$ (Given)

$\Rightarrow \alpha\beta = 1$

$\Rightarrow \frac{k}{3} = 1$ (Product of the roots $= \frac{c}{a}$)

$\Rightarrow k = 3$

Hence, the value of k is 3.

43.

Sol:

It is given that the roots of the quadratic equation $px^2 - 2px + 6 = 0$ are equal.

$\therefore D = 0$

$\Rightarrow (-2p)^2 - 4 \times p \times 6 = 0$

$\Rightarrow 4p^2 - 24p = 0$

$\Rightarrow 4p(p-6) = 0$

$\Rightarrow p = 0 \text{ or } p - 6 = 0$

$\Rightarrow p = 0 \text{ or } p = 6$

For $p = 0$, we get $6 = 0$, which is not true.

$\therefore p \neq 0$

Hence, the value of p is 6.

44.

Sol:

It is given that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

$\therefore D = 0$

$\Rightarrow (-4k)^2 - 4 \times 1 \times k = 0$

$\Rightarrow 16k^2 - 4k = 0$

$\Rightarrow 4k(4k-1) = 0$

$\Rightarrow k = 0 \text{ or } 4k - 1 = 0$

$\Rightarrow k = 0 \text{ or } k = \frac{1}{4}$

Hence, 0 and $\frac{1}{4}$ are the required values of k.

45.

Sol:

It is given that the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

$\therefore D = 0$

$\Rightarrow (-3k)^2 - 4 \times 9 \times k = 0$

$\Rightarrow 9k^2 - 36k = 0$

$\Rightarrow 9k(k-4) = 0$

$\Rightarrow k = 0 \; or \; k - 4 = 0$

$\Rightarrow k = 0 \; or \; k = 4$

Hence, 0 and 4 are the required values of *k*.

46.

Sol:

$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$\Rightarrow x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$\Rightarrow x(x-\sqrt{3}) - 1(x-\sqrt{3}) = 0$

$\Rightarrow (x-\sqrt{3})(x-1) = 0$

$\Rightarrow x - \sqrt{3} = 0 \; or \; x - 1 = 0$

$\Rightarrow x = \sqrt{3} \; or \; x = 1$

Hence, 1 and $\sqrt{3}$ are the roots of the given equation.

47.

Sol:

$2x^2 + ax - a^2 = 0$

$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x+a) - a(x+a) = 0$

$\Rightarrow (x+a)(2x-a) = 0$

$\Rightarrow x + a = 0 \; or \; 2x - a = 0$

$\Rightarrow x = -a \; or \; x = \frac{a}{2}$

Hence, $-a$ and $\frac{a}{2}$ are the roots of the given equation.

48.

Sol:

$3x^2 + 5\sqrt{5}x - 10 = 0$

$\Rightarrow 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$

$\Rightarrow 3x\left(x + 2\sqrt{5}\right) - \sqrt{5}\left(x + 2\sqrt{5}\right) = 0$

$\Rightarrow \left(x + 2\sqrt{5}\right)\left(3x - \sqrt{5}\right) = 0$

$\Rightarrow x + 2\sqrt{5} = 0 \ or \ 3x - \sqrt{5} = 0$

$\Rightarrow x = -2\sqrt{5} \ or \ x = \frac{\sqrt{5}}{3}$

Hence, $-2\sqrt{5}$ and $\frac{\sqrt{5}}{3}$ are the roots of the given equation.

49.

Sol:

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 + 12x - 2x - 8\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x\left(x + 4\sqrt{3}\right) - 2\left(x + 4\sqrt{3}\right) = 0$

$\Rightarrow \left(x + 4\sqrt{3}\right)\left(\sqrt{3}x - 2\right) = 0$

$\Rightarrow x + 4\sqrt{3} = 0 \ or \ \sqrt{3}x - 2 = 0$

$\Rightarrow x = -4\sqrt{3} \ or \ x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Hence, $-4\sqrt{3}$ and $\frac{2\sqrt{3}}{3}$ are the roots of the given equation.

50.

Sol:

$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x\left(x - \sqrt{6}\right) + \sqrt{2}\left(x - \sqrt{6}\right) = 0$

$\Rightarrow \left(x - \sqrt{6}\right)\left(\sqrt{3} + \sqrt{2}\right) = 0$

$\Rightarrow x - \sqrt{6} = 0 \ or \ \sqrt{3}x + \sqrt{2} = 0$

$\Rightarrow x = \sqrt{6} \ or \ x = -\frac{\sqrt{2}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}$

Hence, $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$ are the roots of the given equation.

51.

Sol:

$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$

$\Rightarrow \left(\sqrt{3}x + 2\right)\left(4x - \sqrt{3}\right) = 0$

$\Rightarrow \sqrt{3}x + 2 = 0 \ or \ 4x - \sqrt{3} = 0$

$\Rightarrow x = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \ or \ x = \frac{\sqrt{3}}{4}$

Hence, $-\frac{2\sqrt{3}}{3}$ and $\frac{\sqrt{3}}{4}$ are the roots of the given equation.

52.

Sol:

$4x^2 + 4bx - \left(a^2 - b^2\right) = 0$

$\Rightarrow 4x^2 + 4bx - (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + 2\left[(a+b) - (a-b)\right]x - (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + 2(a+b)x - 2(a-b)x - (a-b)(a+b) = 0$

$\Rightarrow 2x\left[2x + (a+b)\right] - (a-b)\left[2x + (a+b)\right] = 0$

$\Rightarrow \left[2x + (a+b)\right]\left[2x - (a-b)\right] = 0$

$\Rightarrow 2x + (a+b) = 0 \ or \ 2x - (a-b) = 0$

$\Rightarrow x = -\frac{a+b}{2} \ or \ x = \frac{a-b}{2}$

Hence, $-\frac{a+b}{2}$ and $\frac{a-b}{2}$ are the roots of the given equation.

53.

Sol:

$x^2+5x-\left(a^2+a-6\right)=0$

$\Rightarrow x^2+5x-(a+3)(a-2)=0$

$\Rightarrow x^2+\left[(a+3)-(a-2)\right]x-(a+3)(a-2)=0$

$\Rightarrow x^2+(a+3)x-(a-2)x-(a+3)(a-2)=0$

$\Rightarrow x\left[x+(a+3)\right]-(a-2)\left[x+(a+3)\right]=0$

$\Rightarrow \left[x+(a+3)\right]\left[x-(a-2)\right]=0$

$\Rightarrow x+(a+3)=0 \text{ or } x-(a-2)=0$

$\Rightarrow x=-(a+3) \text{ or } x=(a-2)$

Hence, $-(a+3)$ and $(a-2)$ are the roots of the given equation.

54.

Sol:

$x^2+6x-\left(a^2+2a-8\right)=0$

$\Rightarrow x^2+6x-(a+4)(a-2)=0$

$\Rightarrow x^2+\left[(a+4)-(a-2)\right]x-(a+4)(a-2)=0$

$\Rightarrow x^2+(a+4)x-(a-2)x-(a+4)(a-2)=0$

$\Rightarrow x\left[x+(a+4)\right]-(a-2)\left[x+(a+4)\right]=0$

$\Rightarrow \left[x+(a+4)\right]\left[x-(a-2)\right]=0$

$\Rightarrow x+(a+4)=0 \text{ or } x-(a-2)=0$

$\Rightarrow x=-(a+4) \text{ or } x=(a-2)$

Hence, $-(a+4)$ and $(a-2)$ are the roots of the given equation.

55.

Sol:

$x^2-4ax+4a^2-b^2=0$

$\Rightarrow x^2-4ax+(2a+b)(2a-b)=0$

$\Rightarrow x^2-\left[(2a+b)+(2a-b)\right]x+(2a+b)(2a-b)=0$

$\Rightarrow x^2-(2a+b)x-(2a-b)x+(2a+b)(2a-b)=0$

$\Rightarrow x\left[x-(2a+b)\right]-(2a-b)\left[x-(2a+b)\right]=0$

$\Rightarrow \left[x-(2a+b)\right]\left[x-(2a-b)\right]=0$

$\Rightarrow x-(2a+b)=0 \ or \ x-(2a-b)=0$

$\Rightarrow x=(2a+b) \ or \ x=(2a-b)$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.