Exercise – 11A

1.

Sol:

(i) The given progression 9, 15, 21, 27,…………

Clearly, 15 – 9 = 21 – 15 = 27 – 21 = 6 (Constant)

Thus, each term differs from its preceding term by 6. So, the given progression is an AP.

First term = 9

Common difference =6

Next term of the AP = 27 + 6 = 33

(ii) The given progression 11, 6, 1, – 4,……..

Clearly,6 – 11 = 1 – 6 = –4 – 1 = –5 (Constant)

Thus, each term differs from its preceding term by 6. So, the given progression is an AP.

First term = 11

Common difference = –5

Next term of the $AP = -4 + (-5) = -9$

(iii) The given progression $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ...$

Clearly, $\frac{-5}{6} - (-1) = \frac{-2}{3} - \left(\frac{-5}{6}\right) = \frac{-1}{2} - \left(\frac{-2}{3}\right) = \frac{1}{6}$ (Constant)

Thus, each term differs from its preceding term by $\frac{1}{6}$. So, the given progression is an AP.

First term = –1

Common difference = $\frac{1}{6}$

Next tern of the $AP = \frac{-1}{2} + \frac{1}{6} = \frac{-2}{6} = \frac{-1}{3}$

(iv) The given progression $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots\ldots$

This sequence can be written as $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots\ldots$

Clearly, $2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$ (Constant)

Thus, each term differs from its preceding term by $\sqrt{2}$, So, the given progression is an AP.

First term $= \sqrt{2}$

Common difference $= \sqrt{2}$

Next tern of the $AP = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

(v) This given progression $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \ldots\ldots\ldots$

This sequence can be re-written as $2\sqrt{5}, 3\sqrt{5}, 4\sqrt{5}, 5\sqrt{5}, \ldots\ldots\ldots$

Clearly, $3\sqrt{5} - 2\sqrt{5} = 4\sqrt{5} - 3\sqrt{5} = 5\sqrt{5} - 4\sqrt{5} = \sqrt{5}$ (Constant)

Thus, each term differs from its preceding term by $\sqrt{5}$. So, the given progression is an AP.

First term $= 2\sqrt{5} = \sqrt{20}$

Common difference $= \sqrt{5}$

Next term of the $AP = 5\sqrt{5} + \sqrt{5} = 6\sqrt{5} = \sqrt{180}$

2.

Sol:

(i) The given AP is $9, 13, 17, 21, \ldots\ldots\ldots$

First term, $a = 9$

Common difference, $d = 13 - 9 = 4$

n^{th} term of the AP, $a_n = a + (n-1)d = 9 + (n-1) \times 4$

$\therefore$ $20th$ term of the AP, $a_{20} = 9 + (20-1) \times 4 = 9 + 76 = 85$

(ii) The given AP is $20, 17, 14, 11, \ldots\ldots\ldots$

First term, $a = 20$

Common difference, $d = 17 - 20 = -3$

n^{th} term of the AP, $a_n = a + (n-1)d = 20 + (n-1) \times (-3)$

$\therefore$ 35*th* term of the AP, $a_{35} = 20 + (35-1)\times(-3) = 20 - 102 = -82$

(iii) The given AP is $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots\ldots\ldots$

This can be re-written as $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, \ldots\ldots\ldots$

First term, $a = \sqrt{2}$

Common difference, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

n^{th} term of the AP, $a_n = a + (n-1)d = \sqrt{2} + (n-1)\times 2\sqrt{2}$

$\therefore$ 18*th* term of the AP, $a_{18} = \sqrt{2} + (18-1)\times 2\sqrt{2} = \sqrt{2} + 34\sqrt{2} = 35\sqrt{2} = \sqrt{2450}$

(iv) The given AP is $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots\ldots\ldots$

First term, $a = \frac{3}{4}$

Common difference, $d = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

n^{th} term of the AP, $a_n = a + (n-1)d = \frac{3}{4} + (n-1)\times\left(\frac{1}{2}\right)$

$\therefore 9^{th}$ *term* of the AP, $a_9 = \frac{3}{4} + (9-1)\times\frac{1}{2} = \frac{3}{4} + 4 = \frac{19}{4}$

(v) The given AP is $-40, -15, 10, 35, \ldots\ldots\ldots$

First term, $a = -40$

Common difference, $d = -15 - (-40) = 25$

n^{th} term of the AP, $a_n = a + (n-1)d = -40 + (n-1)\times 25$

$\therefore$ 15*th* term of the AP, $a_{15} = -40 + (15-1)\times 25 = -40 + 350 = 310$

3.

Sol:

The given AP is $6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \ldots\ldots\ldots$

First term, $a = 6$ and common difference, $d = 7\frac{3}{4} - 6 \Rightarrow \frac{31}{4} - 6 \Rightarrow \frac{31-24}{4} = \frac{7}{4}$

Now, $T_{37} = a + (37-1)d = a + 36d$

$= 6 + 36\times\frac{7}{4} = 6 + 63 = 69$

$\therefore 37^{th}$ term $= 69$

4.

Sol:

The given AP is $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \ldots\ldots$

First term = 5

Common difference $= 4\frac{1}{2} - 5 \Rightarrow \frac{9}{2} - 5 \Rightarrow \frac{9-10}{2} = -\frac{1}{2}$

$\therefore a = 5 \text{ and } d = -\frac{1}{2}$

Now, $T_{25} = a + (25-1)d = a + 24d$

$= 5 + 24 \times \left(-\frac{1}{2}\right) = 5 - 12 = -7$

$\therefore 25^{th}$ term $= -7$

5.

Sol:

(i) (6n – 1)

(ii) (23 – 7n)

6.

Sol:

$T_n = (4_n - 10)$ [Given]

$T_1 = (4 \times 1 - 10) = -6$

$T_2 = (4 \times 2 - 10) = -2$

$T_3 = (4 \times 3 - 10) = 2$

$T_4 = (4 \times 4 - 10) = 6$

Clearly, $[-2-(-6)] = [2-(-2)] = [6-2] = 4$ (Constant)

So, the terms $-6, -2, 2, 6, \ldots\ldots$ forms an AP.

Thus we have

(i) First term $= -6$

(ii) Common difference = 4

(iii) $T_{16} = a + (n-1)d = a + 15d = -6 + 15 \times 4 = 54$

7.

Sol:

In the given AP, $a = 6$ and $d = (10-6) = 4$

Suppose that there are n terms in the given AP.

Then, $T_n = 174$

$\Rightarrow a + (n-1)d = 174$

$\Rightarrow 6 + (n-1) \times 4 = 174$

$\Rightarrow 2 + 4n = 174$

$\Rightarrow 4n = 172$

$\Rightarrow n = 43$

Hence, there are 43 terms in the given AP.

8.

Sol:

In the given AP, $a = 41$ *and* $d = (38-41) = -3$

Suppose that there are n terms in the given AP.

Then $T_n = 8$

$\Rightarrow a + (n-1)d = 8$

$\Rightarrow 41 + (n-1) \times (-3) = 8$

$\Rightarrow 44 - 3n = 8$

$\Rightarrow 3n = 36$

$\Rightarrow n = 12$

Hence, there are 12 terms in the given AP.

9.

Sol:

The given AP is $18, 15\frac{1}{2}, 13, \ldots\ldots, -47$.

First term, $a = 18$

Common difference, $d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{31-36}{2} = -\frac{5}{2}$

Suppose there re n terms in the given AP. Then,

$a_n = -47$

$\Rightarrow 18 + (n-1) \times \left(-\frac{5}{2}\right) = -47$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow -\frac{5}{2}(n-1) = -47 - 18 = -65$

$\Rightarrow n - 1 = -65 \times \left(-\frac{2}{5}\right) = 26$

$\Rightarrow n = 26 + 1 = 27$

Hence, there are 27 terms in the given AP.

10.

Sol:

In the given AP, first term, $a = 3$ and common difference, $a = (8-3) = 5.$

Let's its n^{th} term be 88

Then, $T_n = 88$

$\Rightarrow a + (n-1)d = 88$

$\Rightarrow 3 + (n-1) \times 5 = 88$

$\Rightarrow 5n - 2 = 88$

$\Rightarrow 5n = 90$

$\Rightarrow n = 18$

Hence, the 18^{th} term of the given AP is 88.

11.

Sol:

In the given AP, first term, $a = 72$ and common difference, $d = (68-72) = -4.$

Let its n^{th} term be 0.

Then, $T_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 72 + (n-1) \times (-4) = 0$

$\Rightarrow 76 - 4n = 0$

$\Rightarrow 4n = 76$

$\Rightarrow n = 19$

Hence, the 19^{th} term of the given AP is 0.

12.

Sol:

In the given AP, first term $= \frac{5}{6}$ and common difference, $d = \left(1 - \frac{5}{6} = \frac{1}{6}\right)$

Let its n^{th} term be 3.

Now, $T_n = 3$

$\Rightarrow a + (n-1)d = 3$

$\Rightarrow \frac{5}{6} + (n-1) \times \frac{1}{6} = 3$

$\Rightarrow \frac{2}{3} + \frac{n}{6} = 3$

$\Rightarrow \frac{n}{6} = \frac{7}{3}$

$\Rightarrow n = 14$

Hence, the 14^{th} term of the given AP is 3.

13.

Sol:

The given AP is $21, 18, 15, ...$

First term, $a = 21$

Common difference, $d = 18 - 21 = -3$

Suppose n^{th} term of the given AP is -81. then,

$a_n = -81$

$\Rightarrow 21 + (n-1) \times (-3) = -81 \quad [a_n = a + (n-1)d]$

$\Rightarrow -3(n-1) = -81 - 21 = -102$

$\Rightarrow n - 1 = \frac{102}{3} = 34$

$\Rightarrow n = 34 + 1 = 35$

Hence, the $35th$ term of the given AP is -81.

14.

Sol:

Here, $a = 3$ and $d = (8-3) = 5$

The 20^{th} term is given by

$T_{20} = a + (20-1)d = a + 19d = 3 + 19 \times 5 = 98$

$\therefore$ Required term $= (98 + 55) = 153$

Let this be the n^{th} term.

Then $T_n = 153$

$\Rightarrow 3 + (n-1) \times 5 = 153$

$\Rightarrow 5n = 155$

$\Rightarrow n = 31$

Hence, the 31st term will be 55 more than 20th term.

15.

Sol:

Here, $a = 5$ and $d = (15-5) = 10$

The 31st term is given by

$T_{31} = a + (31-1)d = a + 30d = 5 + 30 \times 10 = 305$

$\therefore$ Required term $= (305 + 130) = 435$

Let this be the n^{th} term.

Then, $T_n = 435$

$\Rightarrow 5 + (n-1) \times 10 = 435$

$\Rightarrow 10n = 440$

$\Rightarrow n = 44$

Hence, the 44th term will be 130 more than its 31st term.

16.

Sol:

In the given AP, let the first term be a and the common difference be d.

Then, $T_n = a + (n-1)d$

Now, we have:

$T_{10} = a + (10-1)d$

$\Rightarrow a + 9d = 52$ ....(1)

$T_{13} = a + (13-1)d = a + 12d$ ....(2)

$T_{17} = a + (17-1)d = a + 16d$ ....(3)

But, it is given that $T_{17} = 20 + T_{13}$

i.e., $a + 16d = 20 + a + 12d$

$\Rightarrow 4d = 20$

$\Rightarrow d = 5$

On substituting $d = 5$ in (1), we get:

$a+9\times 5=52$

$\Rightarrow a=7$

Thus, $a=7$ *and* $d=5$

$\therefore$ The terms of the AP are $7, 12, 17, 22, ...$

17.

Sol:

The given AP is $6, 13, 20, ..., 216.$

First term, $a=6$

Common difference, $d=13-6=7$

Suppose these are *n* terms in the given AP. Then,

$a_n=216$

$\Rightarrow 6+(n-1)\times 7=216 \qquad [a_n=a+(n-1)d]$

$\Rightarrow 7(n-1)=216-6=210$

$\Rightarrow n-1=\frac{210}{7}=30$

$\Rightarrow n=30+1=31$

Thus, the given AP contains 31 terms,

$\therefore$ Middle term of the given AP

$=\left(\frac{31+1}{2}\right)th$ term

$=16th$ term

$=6+(16-1)\times 7$

$=6+105$

$=111$

Hence, the middle term of the given AP is 111.

18.

Sol:

The given AP is $10, 7, 4, ..., -62.$

First term, $a=10$

Common difference, $d=7-10=-3$

Suppose these are *n* terms in the given AP. Then,

$a_n = -62$

$\Rightarrow 10 + (n-1)\times(-3) = -62 \qquad [a_n = a + (n-1)d]$

$\Rightarrow -3(n-1) = -62 - 10 = -72$

$\Rightarrow n - 1 = \frac{72}{3} = 24$

$\Rightarrow n = 24 + 1 = 25$

Thus, the given AP contains 25 terms.

$\therefore$ Middle term of the given AP

$= \left(\frac{25+1}{2}\right)th$ term

$= 13th$ term

$= 10 + (13-1)\times(-3)$

$= 10 - 36$

$= -26$

Hence, the middle term of the given AP is -26.

19.

Sol:

The given AP is $-\frac{4}{3}, -1, \frac{-2}{3}, \ldots\ldots, 4\frac{1}{3}$.

First term, $a = -\frac{4}{3}$

Common difference, $d = -1 - \left(-\frac{4}{3}\right) = -1 + \frac{4}{3} = \frac{1}{3}$

Suppose there are n terms in the given AP. Then,

$a_n = 4\frac{1}{3}$

$\Rightarrow -\frac{4}{3} + (n-1)\times\left(\frac{1}{3}\right) = \frac{13}{3} \qquad [a_n = a + (n-1)d]$

$\Rightarrow \frac{1}{3}(n-1) = \frac{13}{3} + \frac{4}{3} = \frac{17}{3}$

$\Rightarrow n - 1 = 17$

$\Rightarrow n = 17 + 1 = 18$

Thus, the given AP contains 18 terms. So, there are two middle terms in the given AP.

The middle terms of the given AP are $\left(\frac{18}{2}\right)th$ terms and $\left(\frac{18}{2}+1\right)th$ term i.e. 9th term and 10th term.

$\therefore$ Sum of the middle most terms of the given AP

= 9th term + 10^{th} term

$$= \left[-\frac{4}{3}+(9-1)\times\frac{1}{3}\right]+\left[-\frac{4}{3}+(10-1)\times\frac{1}{3}\right]$$

$$= -\frac{4}{3}+\frac{8}{3}-\frac{4}{3}+3$$

$$= 3$$

Hence, the sum of the middle most terms of the given AP is 3.

20.

Sol:

Here, $a = 7$ and $d = (10-7) = 3, l = 184$ and $n = 8^{th}$ form the end.

Now, n^{th} term from the end $= [l-(n-1)d]$

8^{th} term from the end $= [184-(8-1)\times 3]$

$$= [184-(7\times 3)] = (184-21) = 163$$

Hence, the 8^{th} term from the end is 163.

21.

Sol:

Here, $a = 7$ and $d = (14-17) = -3, l = (-40)$ *and* $n = 6$

Now, nth term from the end $= [l-(n-1)d]$

6^{th} term from the end $= [(-40)-(6-1)\times(-3)]$

$$= [-40+(5\times 3)] = (-40+15) = -25$$

Hence, the 6^{th} term from the end is –25.

22.

Sol:

The given AP is $3, 7, 11, 15, ...$

Here, $a = 3$ and $d = 7-3 = 4$

Let the nth term of the given AP be 184. Then,

$a_n = 184$

$\Rightarrow 3+(n-1)\times 4 = 184$ $\quad [a_n = a+(n-1)d]$

$\Rightarrow 4n-1 = 184$

$\Rightarrow 4n = 185$

$\Rightarrow n = \frac{185}{4} = 46\frac{1}{4}$

But, the number of terms cannot be a fraction.

Hence, 184 is not a term of the given AP.

23.

Sol:

The given AP is 11, 8, 5, 2, ……

Here, $a = 11$ and $d = 8 - 11 = -3$

Let the nth term of the given AP be -150. Then,

$a_n = -150$

$\Rightarrow 11 + (n-1) \times (-3) = -150$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow -3n + 14 = -150$

$\Rightarrow -3n = -164$

$\Rightarrow n = \frac{164}{3} = 54\frac{2}{3}$

But, the number of terms cannot be a fraction.

Hence, -150 is not a term of the given AP.

24.

Sol:

The given AP is 121, 117, 113, …….

Here, $a = 121$ and $d = 117 - 121 = -4$

Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 121 + (n-1) \times (-4) < 0$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow -4n < -125$

$\Rightarrow n > \frac{125}{4} = 31\frac{1}{4}$

$\therefore n = 32$

Hence, the 32^{nd} term is the first negative term of the given AP.

25.

Sol:

The given AP is $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$

Here, $a = 20$ and $d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{77-80}{4} = -\frac{3}{4}$

Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 20 + (n-1) \times \left(-\frac{3}{4}\right) < 0 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 20 + \frac{3}{4} - \frac{3}{4}n < 0$

$\Rightarrow \frac{83}{4} - \frac{3}{4}n < 0$

$\Rightarrow -\frac{3}{4}n < -\frac{83}{4}$

$\Rightarrow n > \frac{83}{3} = 27\frac{2}{3}$

$\therefore n = 28$

Hence, the 28^{th} term is the first negative term of the given AP.

26.

Sol:

We have

$T_7 = a + (n-1)d$

$\Rightarrow a + 6d = -4 \qquad ....(1)$

$T_{13} = a + (n-1)d$

$\Rightarrow a + 12d = -16 \qquad ....(2)$

On solving (1) and (2), we get

$a = 8 \text{ and } d = -2$

Thus, first term = 8 and common difference $= -2$

$\therefore$ The term of the AP are $8, 6, 4, 2, ...$

27.

Sol:

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a + (n-1)d$

Now, $T_4 = a + (4-1)d$

$\Rightarrow a + 3d = 0$ ......(1)

$\Rightarrow a = -3d$

Again, $T_{11} = a + (11-1)d = a + 10d$

$= -3d + 10d = 7d$ [Using (1)]

Also, $T_{25} = a + (25-1)d = a + 24d = -3d + 24d = 21d$ [Using (1)]

i.e., $T_{25} = 3 \times 7d = (3 \times T_{11})$

Hence, 25th term is triple its 11th term.

28.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_8 = 0$ $\left[a_n = a + (n-1)d\right]$

$\Rightarrow a + (8-1)d = 0$

$\Rightarrow a + 7d = 0$

$\Rightarrow a = -7d$ ....(1)

Now,

$$\frac{a_{38}}{a_{18}} = \frac{a + (38-1)d}{a + (18-1)d}$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{-7d + 37d}{-7d + 17d} \qquad [From\ (1)]$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{30d}{10d} = 3$$

$\Rightarrow a_{38} = 3 \times a_{18}$

Hence, the 38th term of the AP id triple its 18th term.

29.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_4 = 11$

$\Rightarrow a + (4-1)d = 11$ $\left[a_n = a + (n-1)d\right]$

$\Rightarrow a + 3d = 11$ ....(1)

Now,

$a_5 + a_7 = 34$ (Given)

$\Rightarrow (a+4d)+(a+6d)=34$

$\Rightarrow 2a+10d=34$

$\Rightarrow a+5d=17 \quad ......(2)$

From (1) and (2), we get

$11-3d+5d=17$

$\Rightarrow 2d=17-11=6$

$\Rightarrow d=3$

Hence, the common difference of the AP is 3.

30.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_9=-32$

$\Rightarrow a+(9-1)d=-32 \quad [a_n=a+(n-1)d]$

$\Rightarrow a+8d=-32 \quad ....(1)$

Now,

$a_{11}+a_{13}=-94 \quad$ (Given)

$\Rightarrow (a+10d)+(a+12d)=-94$

$\Rightarrow 2a+22d=-94$

$\Rightarrow a+11d=-47 \quad ....(2)$

From (1) and (2), we get

$-32-8d+11d=-47$

$\Rightarrow 3d=-47+32=-15$

$\Rightarrow d=-5$

Hence, the common difference of the AP is -5.

31.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_7=-1$

$\Rightarrow a+(7-1)d=-1 \quad [a_n=a+(n-1)d]$

$\Rightarrow a+6d=-1 \quad ....(1)$

Also,

$a_{16}=17$

$\Rightarrow a+15d=17 \quad ....(2)$

From (1) and (2), we get

$-1-6d+15d=17$

$\Rightarrow 9d=17+1=18$

$\Rightarrow d=2$

Putting $d=2$ in (1), we get

$a+6\times 2=-1$

$\Rightarrow a=-1-12=-13$

$\therefore a_n=a+(n-1)d$

$=-13+(n-1)\times 2$

$=2n-15$

Hence, the nth term of the AP is $(2n-15)$.

32.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$4\times a_4=18\times a_{18}$ (Given)

$\Rightarrow 4(a+3d)=18(a+17d)$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2(a+3d)=9(a+17d)$

$\Rightarrow 2a+6d=9a+153d$

$\Rightarrow 7a=-147d$

$\Rightarrow a=-21d$

$\Rightarrow a+21d=0$

$\Rightarrow a+(22-1)d=0$

$\Rightarrow a_{22}=0$

Hence, the 22nd term of the AP is 0.

33.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$10\times a_{10}=15\times a_{15}$ (Given)

$\Rightarrow 10(a+9d)=15(a+14d)$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2(a+9d)=3(a+14d)$

$\Rightarrow 2a+18d=3a+42d$

$\Rightarrow a=-24d$

$\Rightarrow a + 24d = 0$

$\Rightarrow a + (25-1)d = 0$

$\Rightarrow a_{25} = 0$

Hence, the 25th term of the AP is 0.

34.

Sol:

Let the common difference of the AP be d.

First term, $a = 5$

Now,

$a_1 + a_2 + a_3 + a_4 = \frac{1}{2}(a_5 + a_6 + a_7 + a_8)$ (Given)

$\Rightarrow a + (a+d) + (a+2d) + (a+3d) = \frac{1}{2}[(a+4d) + (a+5d) + (a+6d) + (a+7d)]$

$[a_n = a + (n-1)d]$

$\Rightarrow 4a + 6d = \frac{1}{2}(4a + 22d)$

$\Rightarrow 8a + 12d = 4a + 22d$

$\Rightarrow 22d - 12d = 8a - 4a$

$\Rightarrow 10d = 4a$

$\Rightarrow d = \frac{2}{5}a$

$\Rightarrow d = \frac{2}{5} \times 5 = 2$ $(a = 5)$

Hence, the common difference of the AP is 2.

35.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_2 + a_7 = 30$ (Given)

$\therefore (a+d) + (a+6d) = 30$ $[a_n = a + (n-1)d]$

$\Rightarrow 2a + 7d = 30$ ....(1)

Also,

$a_{15} = 2a_8 - 1$ (Given)

$\Rightarrow a+14d=2(a+7d)-1$

$\Rightarrow a+14d=2a+14d-1$

$\Rightarrow -a=-1$

$\Rightarrow a=1$

Putting a = 1in (1), we get

$2\times1+7d=30$

$\Rightarrow 7d=30-2=28$

$\Rightarrow d=4$

So,

$a_2=a+d=1+4=5$

$a_3=a+2d=1+2\times4=9$..........

Hence, the AP is 1, 5, 9, 13, …….

36.

Sol:

Let the term of the given progressions be t_n and T_n, respectively.

The first AP is 63, 65, 67,...

Let its first term be a and common difference be *d*.

Then a = 63 and d = (65 - 63) = 2

So, its nth term is given by

$t_n=a+(n-1)d$

$\Rightarrow 63+(n-1)\times2$

$\Rightarrow 61+2n$

The second AP is 3, 10, 17,...

Let its first term be A and common difference be D.

Then A = 3 and D = (10 - 3) = 7

So, its nth term is given by

$T_n=A+(n-1)D$

$\Rightarrow 3+(n-1)\times7$

$\Rightarrow 7n-4$

Now, $t_n=T_n$

$\Rightarrow 61+2n=7n-4$

$\Rightarrow 65=5n$

$\Rightarrow n=13$

Hence, the l3 terms of the Al’s are the same.

37.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{17} = 2a_8 + 5$ (Given)

$\therefore a + 16d = 2(a + 7d) + 5$ $[a_n = a + (n-1)d]$

$\Rightarrow a + 16d = 2a + 14d + 5$

$\Rightarrow a - 2d = -5$ ....(1)

Also,

$a_{11} = 43$ (Given)

$\Rightarrow a + 10d = 43$ ....(2)

From (1) and (2), we get

$-5 + 2d + 10d = 43$

$\Rightarrow 12d = 43 + 5 = 48$

$\Rightarrow d = 4$

Putting $d = 4$ in (1), we get

$a - 2 \times 4 = -5$

$\Rightarrow a = -5 + 8 = 3$

$\therefore a_n = a + (n-1)d$

$= 3 + (n-1) \times 4$

$= 4n - 1$

Hence, the nth term of the AP is $(4n - 1)$.

38.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{24} = 2a_{10}$ (Given)

$\Rightarrow a + 23d = 2(a + 9d)$ $[a_n = a + (n-1)d]$

$\Rightarrow a + 23d = 2a + 18d$

$\Rightarrow 2a - a = 23d - 18d$

$\Rightarrow a = 5d$ ....(1)

Now,

$\frac{a_{72}}{a_{15}} = \frac{a + 71d}{a + 14d}$

$\Rightarrow \frac{a_{72}}{a_{15}} = \frac{5d + 71d}{5d + 14d}$ $\quad [From(1)]$

$\Rightarrow \frac{a_{72}}{a_{15}} = \frac{76d}{19d} = 4$

$\Rightarrow a_{72} = 4 \times a_{15}$

Hence, the 72nd term of the AP is 4 times its 15th term.

39.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{19} = 3a_6$ (Given)

$\Rightarrow a + 18d = 3(a + 5d)$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow a + 18d = 3a + 15d$

$\Rightarrow 3a - a = 18d - 15d$

$\Rightarrow 2a = 3d$ ....(1)

Also,

$a_9 = 19$ (Given)

$\Rightarrow a + 8d = 19$ ......(2)

From (1) and (2), we get

$\frac{3d}{2} + 8d = 19$

$\Rightarrow \frac{3d + 16d}{2} = 19$

$\Rightarrow 19d = 38$

$\Rightarrow d = 2$

Putting $d = 2$ in (1), we get

$2a = 3 \times 2 = 6$

$\Rightarrow a = 3$

So,

$a_2 = a + d = 3 + 2 = 5$

$a_3 = a + 2d = 3 + 2 \times 2 = 7, ...$

Hence, the AP is $3, 5, 7, 9, ...$

40.

Sol:

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a + (n-1)d$

$\Rightarrow Tp = a + (p-1)d = q \qquad ....(i)$

$\Rightarrow T_q = a + (q-1)d = p \qquad ....(ii)$

On subtracting (i) from (ii), we get:

$(q-p)d = (p-q)$

$\Rightarrow d = -1$

Putting $d = -1$ in (i), we get:

$a = (p+q-1)$

Thus, $a = (p+q-1)$ *and* $d = -1$

Now, $T_{p+q} = a + (p+q-1)d$

$= (p+q-1) + (p+q-1)(-1)$

$= (p+q-1) - (p+q-1) = 0$

Hence, the $(p+q)^{th}$ term is 0 (zero).

41.

Sol:

In the given AP, first term = *a* and last term = *l*.

Let the common difference be *d*.

Then, nth term from the beginning is given by

$T_n = a + (n-1)d \qquad .....(1)$

Similarly, nth term from the end is given by

$T_n = \{l - (n-1)d\} \qquad ......(2)$

Adding (1) and (2), we get

$a + (n-1)d + \{l - (n-1)d\}$

$= a + (n-1)d + l - (n-1)d$

$= a + 1$

Hence, the sum of the nth term from the beginning and the nth term from the end $(a+1)$.

42.

Sol:

The two digit numbers divisible by 6 are 12, 18, 24,……, 96

Clearly, these number are in AP.

Here, $a = 12$ and $d = 18 - 12 = 6$

Let this AP contains n terms. Then,

$a_n = 96$

$\Rightarrow 12 + (n-1) \times 6 = 96 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 6n + 6 = 96$

$\Rightarrow 6n = 96 - 6 = 90$

$\Rightarrow n = 15$

Hence, these are 15 two-digit numbers divisible by 6.

43.

Sol:

The two-digit numbers divisible by 3 are 12, 15, 18, ..., 99.

Clearly, these number are in AP.

Here, $a = 12$ and $d = 15 - 12 = 3$

Let this AP contains n terms. Then,

$a_n = 99$

$\Rightarrow 12 + (n-1) \times 3 = 99 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 3n + 9 = 99$

$\Rightarrow 3n = 99 - 9 = 90$

$\Rightarrow n = 30$

Hence, there are 30 two-digit numbers divisible by 3.

44.

Sol:

The three-digit numbers divisible by 9 are 108, 117, 126,...., 999.

Clearly, these number are in AP.

Here. a = 108 and d = 117 – 108 = 9

Let this AP contains n terms. Then.

$a_n = 999$

$\Rightarrow 108 + (n-1) \times 9 = 999 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 99 = 999$

$\Rightarrow 9n = 999 - 99 = 900$

$\Rightarrow n = 100$

Hence: there are 100 three-digit numbers divisible by 9.

45.

Sol:

The numbers which are divisible by both 2 and 5 are divisible by 10 also.

Now, the numbers between 101 and 999 which are divisible 10 are 110, 120, 130, .., 990.

Clearly, these number are in AP

Here, $a = 110$ and $d = 120 - 110 = 10$
Let this AP contains n terms. Then,
$a_n = 990$
$\Rightarrow 110 + (n-1) \times 10 = 990$ $[a_n = a + (n-1)d]$
$\Rightarrow 10n + 100 = 990$
$\Rightarrow 10n = 990 - 100 = 890$
$\Rightarrow n = 89$
Hence, there are 89 numbers between 101 and 999 which are divisible by both 2 and 5.

46.

Sol:
The numbers of rose plants in consecutive rows are 43, 41, 39,..., 11.
Difference of rose plants between two consecutive rows = (41 – 43) = (39 – 41) = –2 [Constant]
So, the given progression is an AP
Here, first term = 43
Common difference = –2
Last term 11
Let n be the last term, then we have:
$T_n = a + (n-1)d$
$\Rightarrow 11 = 43 + (n-1)(-2)$
$\Rightarrow 11 = 45 - 2n$
$\Rightarrow 34 = 2n$
$\Rightarrow n = 17$
Hence, the 17^{th} term is 11 or there are 17 rows in the flower bed.

47.

Sol:
Let the amount of the first prize be ₹a
Since each prize after the first is ₹200 less than the preceding prize, so the amounts of the four prizes are in AP.
Amount of the second prize = ₹ $(a - 200)$
Amount of the third prize = ₹ $(a - 2 \times 200) = (a - 400)$
Amount of the fourth prize = ₹$(a - 3 \times 200) = (a - 600)$
Now,

Total sum of the four prizes = 2,800

$\therefore$ ₹a+₹$(a-200)+$ ₹$(a-400)+$ ₹$(a-600)=$ ₹2,800

$\Rightarrow 4a-1200=2800$

$\Rightarrow 4a=2800+1200=4000$

$\Rightarrow a=1000$

$\therefore$ Amount of the first prize = ₹1,000

Amount of the second prize $=$ ₹$(1000-200)$ = ₹800

Amount of the third prize $=$ ₹$(1000=400)=$ ₹600

Amount of the fourth prize $=$ ₹$(1000-600)$ = ₹400

Hence, the value of each of the prizes is ₹1,000, ₹800, ₹600 and ₹400.

Exercise - 11B

1.

Sol:

It is given that $(3k-2),(4k-6)$ and $(k+2)$ are three consecutive terms of an AP.

$\therefore (4k-6)-(3k-2)=(k+2)-(4k-6)$

$\Rightarrow 4k-6-3k+2=k+2-4k+6$

$\Rightarrow k-4=-3k+8$

$\Rightarrow k+3k=8+4$

$\Rightarrow 4k=12$

$\Rightarrow k=3$

Hence, the value of *k* is 3.

2.

Sol:

It is given that $(5x+2),(4x-1)$ and $(x+2)$ are in AP.

$\therefore (4x-1)-(5x+2)=(x+2)-(4x-1)$

$\Rightarrow 4x-1-5x-2=x+2-4x+1$

$\Rightarrow -x-3=-3x+3$

$\Rightarrow 3x-x=3+3$

$\Rightarrow 2x=6$

$\Rightarrow x=3$

Hence, the value of *x* is 3.

3.

Sol:

It is given that $(3y-1),(3y+5)$ and $(5y+1)$ are three consecutive terms of an AP.

$\therefore (3y+5)-(3y-1)=(5y+1)-(3y+5)$

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow 6=2y-4$

$\Rightarrow 2y=6+4=10$

$\Rightarrow y=5$

Hence, the value of y is 5.

4.

Sol:

Since $(x+2),2x$ and $(2x+3)$ are in AP, we have

$2x-(x+2)=(2x+3)-2x$

$\Rightarrow x-2=3$

$\Rightarrow x=5$

$\therefore x=5$

5.

Sol:

The given numbers are $(a-b)^2,(a^2+b^2)$ and $(a+b)^2$.

Now,

$(a^2+b^2)-(a-b)^2=a^2+b^2-(a^2-2ab+b^2)=a^2+b^2-a^2+2ab-b^2=2ab$

$(a+b)^2-(a^2+b^2)=a^2+2ab+b^2-a^2-b^2=2ab$

So, $(a^2+b^2)-(a-b)^2=(a+b)^2-(a^2+b^2)=2ab$ (Constant)

Since each term differs from its preceding term by a constant, therefore, the given numbers are in AP.

6.

Sol:

Let the required numbers be $(a-d),a$ and $(a+d)$.

Then $(a-d)+a+(a+d)=15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Also, $(a-d).a.(a+d) = 80$

$\Rightarrow a(a^2 - d^2) = 80$

$\Rightarrow 5(25 - d^2) = 80$

$\Rightarrow d^2 = 25 - 16 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 5$ and $d = \pm 3$

Hence, the required numbers are $(2, 5 \text{ and } 8)$ *or* $(8, 5 \text{ and } 2)$.

7.

Sol:

Let the required numbers be $(a-d), a$ and $(a+d)$.

Then $(a-d) + a + (a+d) = 3$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

Also, $(a-d).a.(a+d) = -35$

$\Rightarrow a(a^2 - d^2) = -35$

$\Rightarrow 1.(1 - d^2) = -35$

$\Rightarrow d^2 = 36$

$\Rightarrow d = \pm 6$

Thus, $a = 1$ and $d = \pm 6$

Hence, the required numbers are $(-5, 1 \text{ and } 7)$ *or* $(7, 1 \text{ and } -5)$.

8.

Sol:

Let the required parts of 24 be $(a-d), a$ and $(a+d)$ such that they are in AP.

Then $(a-d) + a + (a+d) = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Also, $(a-d).a.(a+d) = 440$

$\Rightarrow a(a^2 - d^2) = 440$

$\Rightarrow 8(64 - d^2) = 440$

$\Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 8$ and $d = \pm 3$

Hence, the required parts of 24 are $(5,8,11)$ *or* $(11,8,5)$.

9.

Sol:

Let the required terms be $(a-d), a$ and $(a+d)$.

Then $(a-d)+a+(a+d)=21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

Also, $(a-d)^2 + a^2 + (a+d)^2 = 165$

$\Rightarrow 3a^2 + 2d^2 = 165$

$\Rightarrow (3 \times 49 + 2d^2) = 165$

$\Rightarrow 2d^2 = 165 - 147 = 18$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 7$ and $d = \pm 3$

Hence, the required terms are $(4,7,10)$ *or* $(10,7,4)$.

10.

Sol:

Let the required angles be $(a-15)°,(a-5)°,(a+5)°$ *and* $(a+15)°$, as the common difference is 10 (given).

Then $(a-15)° + (a-5)° + (a+5)° + (a+15)° = 360°$

$\Rightarrow 4a = 360$

$\Rightarrow a = 90$

Hence, the required angles of a quadrilateral are

$(90-15)°,(90-5)°,(90+5)°$ *and* $(90+15)°$; *or* $75°, 85°, 95°$ *and* $105°$.

11.

Sol:

(4, 6, 8, 10) or (10, 8, 6, 4)

12.

Sol:
Let the four parts in AP be $(a-3d),(a-d),(a+d)$ *and* $(a+3d)$. Then,

$(a-3d)+(a-d)+(a+d)+(a+3d)=32$

$\Rightarrow 4a=32$

$\Rightarrow a=8 \quad ....(1)$

Also,

$(a-3d)(a+3d):(a-d)(a+d)=7:15$

$\Rightarrow \frac{(8-3d)(8+3d)}{(8-d)(8+d)}=\frac{7}{15} \quad [From\ (1)]$

$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$

$\Rightarrow 15(64-9d^2)=7(64-d^2)$

$\Rightarrow 960-135d^2=448-7d^2$

$\Rightarrow 135d^2-7d^2=960-448$

$\Rightarrow 128d^2=512$

$\Rightarrow d^2=4$

$\Rightarrow d=\pm 2$

When $a=8$ and $d=2$,

$a-3d=8-3\times 2=8-6=2$

$a-d=8-2=6$

$a+d=8+2=10$

$a+3d=8+3\times 2=8+6=14$

When $a=8$ and $d=-2$,

$a-3d=8-3\times(-2)=8+6=14$

$a-d=8-(-2)=8+2=10$

$a+d=8-2=6$

$a+3d=8+3\times(-2)=8-6=2$

Hence, the four parts are 2, 6, 10 and 14.

13.

Sol:
Let the first three terms of the AP be $(a-d), a$ *and* $(a+d)$. Then,

$(a-d)+a+(a+d)=48$

$\Rightarrow 3a=48$

$\Rightarrow a=16$

Now,

$(a-d)\times a=4(a+d)+12$ (Given)

$\Rightarrow (16-d)\times 16=4(16+d)+12$

$\Rightarrow 256-16d=64+4d+12$

$\Rightarrow 16d+4d=256-76$

$\Rightarrow 20d=180$

$\Rightarrow d=9$

When $a=16$ and $d=9$,

$a-d=16-9=7$

$a+d=16+9=25$

Hence, the first three terms of the AP are 7, 16, and 25.

Exercise - 11C

1.

Sol:

The terms $(3y-1),(3y+5)$ and $(5y+1)$ are in AP.

$\therefore (3y+5)-(3y-1)=(5y+1)-(3y+5)$

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow 6=2y-4$

$\Rightarrow 2y=10$

$\Rightarrow y=5$

Hence, the value of y is 5.

2.

Sol:

It is given that $k,(2k-1)$ and $(2k+1)$ are the three successive terms of an AP.

$\therefore (2k-1)-k=(2k+1)-(2k-1)$

$\Rightarrow k-1=2$

$\Rightarrow k=3$

Hence, the value of k is 3.

3.

Sol:

It is given that 18, $a, (b-3)$ are in AP.

$\therefore a-18=(b-3)-a$

$\Rightarrow a+a-b=18-3$

$\Rightarrow 2a-b=15$

Hence, the required value is 15.

4.

Sol:

It is given that the numbers $a, 9, b, 25$ from an AP.

So, $\therefore 9-a=b-9=25-b$

$b-9=25-b$

$\Rightarrow 2b=34$

$\Rightarrow b=17$

Also,

$9-a=b-9$

$\Rightarrow a=18-b$

$\Rightarrow a=18-17 \qquad (b=17)$

$\Rightarrow a=1$

Hence, the required values of a and b are 1 and 17, respectively.

5.

Sol:

It is given that the numbers $(2n-1), (3n+2)$ and $(6n-1)$ are in AP.

$\therefore (3n+2)-(2n-1)=(6n-1)-(3n+2)$

$\Rightarrow 3n+2-2n+1=6n-1-3n-2$

$\Rightarrow n+3=3n-3$

$\Rightarrow 2n=6$

$\Rightarrow n=3$

When, $n=3$

$2n-1=2\times3-1=6-1=5$

$3n+2=3\times3+2=9+2=11$

$6n-1=6\times3-1=18-1=17$

Hence, the required value of n is 3 and the numbers are 5, 11 and 17.

6.

Sol:

The three digit natural numbers divisible by 7 are $105, 112, 119......, 994$

Clearly, these number are in AP.

Here, $a = 105$ *and* $d = 112 - 105 = 7$

Let this AP contains *n* terms. Then,

$a_n = 994$

$\Rightarrow 105 + (n-1) \times 7 = 994 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 7n + 98 = 994$

$\Rightarrow 7n = 994 - 98 = 986$

$\Rightarrow n = 128$

Hence, there are 128 three digit numbers divisible by 7.

7.

Sol:

The three-digit natural numbers divisible by 9 are 108, 117, 126 ,….. 999.

Clearly, these number are in AP.

Here. a = 108 and d = 117 – 108 = 9

Let this AP contains *n* terms. Then,

$a_n = 999$

$\Rightarrow 108 + (n-1) \times 9 = 999 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 99 = 999$

$\Rightarrow 9n = 999 - 99 = 900$

$\Rightarrow n = 100$

Hence, there are 100 three-digit numbers divisible by 9.

8.

Sol:

Let S_m denotes the sum of first m terms of the AP.

$\therefore S_m = 2m^2 + 3m$

$\Rightarrow S_{m-1} = 2(m-1)^2 + 3(m-1) = 2(m^2 - 2m + 1) + 3(m-1) = 2m^2 - m - 1$

Now,

m^{th} term of AP, $a_m = S_m - S_{m-1}$

$\therefore a_m = (2m^2 + 3m) - (2m^2 - m - 1) = 4m + 1$

Putting $m = 2$, we get

$a_2 = 4 \times 2 + 1 = 9$

Hence, the second term of the AP is 9.

9.

Sol:

The given AP is $3a, 5a, \ldots\ldots..$

Here,

First term, A = a

Common difference, $D = 3a - a = 2a$

$\therefore$ Sum of the n terms, S_n

$= \frac{n}{2}\left[2\times a + (n-1)\times 2a\right]$ $\left\{S_n = \frac{n}{2}\left[2A + (n-1)D\right]\right\}$

$= \frac{n}{2}(2a + 2an - 2a)$

$= \frac{n}{2}\times 2an$

$= an^2$

Hence, the required sum is an^2.

10.

Sol:

The given AP is 2, 7, 12, ..., 47.

Let us re-write the given AP in reverse order i.e. 47, 42, .., 12, 7, 2.

Now, the 5th term from the end of the given AP is equal to the 5th term from beginning of the AP 47, 42,.... ,12, 7, 2.

Consider the AP 47, 42,..., 12, 7, 2.

Here, a = 47 and d = 42 – 47 = –5

5th term of this AP

= 47 + (5 – 1) × (–5)

= 47 – 20

= 27

Hence, the 5th term from the end of the given AP is 27.

11.

Sol:

The given AP is 2, 7, 12, 17,………

Here, $a = 2$ and $d = 7 - 2 = 5$

$\therefore a_{30} - a_{20}$

$= [2+(30-1)\times 5] - [2+(20-1)\times 5]$ $\quad [a_n = a+(n-1)d]$

$= 147 - 97$

$= 50$

Hence, the required value is 50.

12.

Sol:

We have

$T_n = (3n+5)$

Common difference $= T_2 - T_1$

$T_1 = 3\times 1 + 5 = 8$

$T_2 = 3\times 2 + 5 = 11$

$d = 11 - 8 - 3$

Hence, the common difference is 3.

13.

Sol:

We have

$T_n = (7-4n)$

Common difference $= T_2 - T_1$

$T_1 = 7 - 4\times 1 = 3$

$T_2 = 7 - 4\times 2 = -1$

$d = -1 - 3 = -4$

Hence, the common difference is -4.

14.

Sol:

The given AP is $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

On simplifying the terms, we get:

$2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, ...$

Here, $a = 2\sqrt{2}$ and $d = (3\sqrt{2} - 2\sqrt{2}) = \sqrt{2}$

$\therefore$ Next term, $T_4 = a + 3d = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} = \sqrt{50}$

15.

Sol:

The given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}, ...$

On simplifying the terms, we get:

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, ...$

Here, $a = \sqrt{2}$ and $d = \left(2\sqrt{2} - \sqrt{2}\right) = \sqrt{2}$

$\therefore$ Next term, $T_4 = a + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2} = \sqrt{32}$

16.

Sol:

In the given AP, first term, $a = 21$ and common difference, $d = \left(18 - 21\right) = -3$

Let's its n^{th} term be 0.

Then, $T_n = 0$

$\Rightarrow a + \left(n-1\right)d = 0$

$\Rightarrow 21 + \left(n-1\right) \times \left(-3\right) = 0$

$\Rightarrow 24 - 3n = 0$

$\Rightarrow 3n = 24$

$\Rightarrow n = 8$

Hence, the 8^{th} term of the given AP is 0.

17.

Sol:

The first n natural numbers are 1, 2, 3, 4, 5,……..,n

Here, a = 1 and d = (2 – 1) = 1

Sum of n terms of an AP is given by

$S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$= \left(\frac{n}{2}\right) \times \left[2 \times 1 + \left(n-1\right) \times 1\right]$

$= \left(\frac{n}{2}\right) \times \left[2 + n - 1\right] = \left(\frac{n}{2}\right) \times \left(n+1\right) = \frac{n\left(n+1\right)}{2}$

18.

Sol:

The first *n* even natural numbers are 2, 4, 6, 8, 10, …., *n*.

Here, $a = 2$ and $d = (4-2) = 2$

Sum of n terms of an AP is given by

$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \left(\frac{n}{2}\right)\times\left[2\times 2+(n-1)\times 2\right]$

$= \left(\frac{n}{2}\right)\times\left[4+2n-2\right] = \left(\frac{n}{2}\right)\times(2n+2) = n(n+1)$

Hence, the required sum is $n(n+1)$.

19.

Sol:

Here, $a = p$ and $d = q$

Now, $T_n = a+(n-1)d$

$\Rightarrow T_n = p+(n-1)q$

$\therefore T_{10} = p+9q$

20.

Sol:

If 45, a and 2 are three consecutive terms of an AP, then we have:

$a-45 = 2-a$

$\Rightarrow 2a = 2+45$

$\Rightarrow 2a = 145$

$\Rightarrow a = 75$

21.

Sol:

Let $(2p+1), 13, (5p-3)$ be three consecutive terms of an AP.

Then $13-(2p+1) = (5p-3)-13$

$\Rightarrow 7p = 28$

$\Rightarrow p = 4$

$\therefore$ When $p = 4, (2p+1), 13$ and $(5p-3)$ from three consecutive terms of an AP.

22.

Sol:

Let $(2p-1), 7$ and $3p$ be three consecutive terms of an AP.

Then $7-(2p-1)=3p-7$

$\Rightarrow 5p=15$

$\Rightarrow p=3$

$\therefore$ When $p=3, (2p-1), 7$ and $3p$ form three consecutive terms of an AP.

23.

Sol:

Let S_p denotes the sum of first p terms of the AP.

$\therefore S_p = ap^2 + bp$

$\Rightarrow S_{p-1} = a(p-1)^2 + b(p-1)$

$= a(p^2 - 2p + 1) + b(p-1)$

$= ap^2 - (2a-b)p + (a-b)$

Now,

p^{th} term of $AP, a_p = S_p - S_{p-1}$

$= (ap^2 + bp) - [ap^2 - (2a-b)p + (a-b)]$

$= ap^2 + bp - ap^2 + (2a-b)p - (a-b)$

$= 2ap - (a-b)$

Let d be the common difference of the AP.

$\therefore d = a_p - a_{p-1}$

$= [2ap - (a-b)] = [2a(p-1) - (a-b)]$

$= 2ap - (a-b) - 2a(p-1) + (a-b)$

$= 2a$

Hence, the common difference of the AP is 2a.

24.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2 + 5n$

$\Rightarrow S_{n-1} = 3(n-1)^2 + 5(n-1)$

$=3\left(n^2-2n+1\right)+5(n-1)$

$=3n^2-n-2$

Now,

n^{th} term of $AP, a_n=S_n-S_{n-1}$

$=\left(3n^2+5n\right)-\left(3n^2-n-2\right)$

$=6n+2$

Let d be the common difference of the AP.

$\therefore d=a_n-a_{n-1}$

$=(6n+2)-\left[6(n-1)+2\right]$

$=6n+2-6(n-1)-2$

$=6$

Hence, the common difference of the AP is 6.

25.

Sol:

Let *a* be the first term and d be the common difference of the AP. Then,

$a_4=9$

$\Rightarrow a+(4-1)d=9 \qquad \left[a_n=a+(n-1)d\right]$

$\Rightarrow a+3d=9 \qquad ....(1)$

Now,

$a_6+a_{13}=40$ (Given)

$\Rightarrow (a+5d)+(a+12d)=40$

$\Rightarrow 2a+17d=40 \qquad ....(2)$

From (1) and (2), we get

$2(9-3d)+17d=40$

$\Rightarrow 18-6d+17d=40$

$\Rightarrow 11d=40-18=22$

$\Rightarrow d=2$

Putting $d=2$ in (1), we get

$a+3\times 2=9$

$\Rightarrow a=9-6=3$

Hence, the AP is 3, 5, 7, 9, 11,…….

Exercise – 11D

1.

Sol:

(i) The given AP is 2, 7, 12, 17,………

Here, $a = 2$ and $d = 7 - 2 = 5$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{19} = \frac{19}{2}\left[2\times 2 + (19-1)\times 5\right]$

$= \frac{19}{2}\times(4+90)$

$= \frac{19}{2}\times 94$

$= 893$

(ii) The given AP is 9, 7, 5, 3,……….

Here, $a = 9$ and $d = 7 - 9 = -2$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{14} = \frac{14}{2}\left[2\times 9 + (14-1)\times(-2)\right]$

$= 7\times(18-26)$

$= 7\times(-8)$

$= -56$

(iii) The given AP is -37, -33, -29,……..

Here, $a = -37$ and $d = -33 - (-37) = -33 + 37 = 4$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{12} = \frac{12}{2}\left[2\times(-37) + (12-1)\times 4\right]$

$= 6\times(-74+44)$

$= 6\times(-30)$

$= -180$

(iv) The given AP is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$

Here, $a = \frac{1}{15}$ and $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{11} = \frac{11}{2}\left[2 \times \left(\frac{1}{15}\right) + (11-1) \times \frac{1}{60}\right]$

$= \frac{11}{2} \times \left(\frac{2}{15} + \frac{10}{60}\right)$

$= \frac{11}{2} \times \left(\frac{18}{60}\right)$

$= \frac{33}{20}$

(v) The given AP is $0.6, 1.7, 2.8, \ldots\ldots\ldots$

Here, $a = 0.6$ and $d = 1.7 - 0.6 = 1.1$

Using formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{100} = \frac{100}{2}\left[2 \times 0.6 + (100-1) \times 1.1\right]$

$= 50 \times (1.2 + 108.9)$

$= 50 \times 110.1$

$= 5505$

2.

Sol:

(i) The given arithmetic series is $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$.

Here, $a = 7, d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-4}{2} = \frac{7}{2}$ and $l = 84$.

Let the given series contains n terms. Then,

$a_n = 84$

$\Rightarrow 7 + (n-1) \times \frac{7}{2} = 84 \qquad [a_n = a + (n-1)d]$

$\Rightarrow \frac{7}{2}n + \frac{7}{2} = 84$

$\Rightarrow \frac{7}{2}n = 84 - \frac{7}{2} = \frac{161}{2}$

$\Rightarrow n = \frac{161}{7} = 23$

$\therefore$ Required sum $= \frac{23}{2} \times (7 + 84) \qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$= \frac{23}{2} \times 91$

$= \frac{2030}{2}$

$1046\frac{1}{2}$

(ii) The given arithmetic series is $34 + 32 + 30 + ... + 10$.

Here, $a = 34, d = 32 - 34 = -2$ and $l = 10$.

Let the given series contain n terms. Then,

$a_n = 10$

$\Rightarrow 34 + (n-1) \times (-2) = 10 \qquad [a_n = a + (n-1)d]$

$\Rightarrow -2n + 36 = 10$

$\Rightarrow -2n = 10 - 36 = -26$

$\Rightarrow n = 13$

$\therefore$ Required sum $= \frac{13}{2} \times (34 + 10) \qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$= \frac{13}{2} \times 44$

$= 286$

(iii) The given arithmetic series is $(-5) + (-8) + (-11) + ... + (-230)$.

Here, $a = -5, d = -8 - (-5) = -8 + 5 = -3$ and $l = 230$.

Let the given series contain n terms. Then,

$a_n = -230$

$\Rightarrow -5+(n-1)\times(-3) = -230 \qquad [a_n = a+(n-1)d]$

$\Rightarrow -3n-2 = -230$

$\Rightarrow -3n = -230+2 = -228$

$\Rightarrow n = 76$

$\therefore \text{Required sum} = \frac{76}{2}\times[(-5)+(-230)] \qquad \left[S_n = \frac{n}{2}(a+l)\right]$

$= \frac{76}{2}\times(-235)$

$= -8930$

3.

Sol:

Let a_n be the nth term of the AP.

$\therefore a_n = 5-6n$

Putting $n=1$, we get

First term, $a = a_1 = 5-6\times1 = -1$

Putting $n=2$, we get

$a_2 = 5-6\times2 = -7$

Let d be the common difference of the AP.

$\therefore d = a_2 - a_1 = -7-(-1) = -7+1 = -6$

Sum of first n tern of the AP, S_n

$= \frac{n}{2}[2\times(-1)+(n-1)\times(-6)] \qquad \left\{S_n = \frac{n}{2}[2a+(n-1)d]\right\}$

$= \frac{n}{2}(-2-6n+6)$

$= n(2-3n)$

$= 2n-3n^2$

Putting $n=20$, we get

$S_{20} = 2\times20-3\times20^2 = 40-1200 = -1160$

4.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2 + 6n$

$\Rightarrow S_{n-1} = 3(n-1)^2 + 6(n-1)$

$= 3(n^2 - 2n + 1) + 6(n-1)$

$= 3n^2 - 3$

$\therefore n^{th}$ term of the AP, a_n

$= S_n - S_{n-1}$

$= (3n^2 + 6n) - (3n^2 - 3)$

$= 6n + 3$

Putting $n = 15$, we get

$a_{15} = 6 \times 15 + 3 = 90 + 3 = 93$

Hence, the n^{th} term is $(6n+3)$ and 15[th] term is 93.

5.

Sol:

Given: $S_n = (3n^2 - n)$ ......(i)

Replacing n by $(n-1)$ in (i), we get:

$S_{n-1} = 3(n-1)^2 - (n-1)$

$= 3(n^2 - 2n + 1) - n + 1$

$= 3n^2 - 7n + 4$

(i) Now, $T_n = (S_n - S_{n-1})$

$= (3n^2 - n) - (3n^2 - 7n + 4) = 6n - 4$

$\therefore n^{th}$ term, $T_n = (6n - 4)$ ....(ii)

(ii) Putting $n = 1$ in (ii), we get:

$T_1 = (6 \times 1) - 4 = 2$

(iii) Putting $n = 2$ in (ii), we get:

$T_2 = (6 \times 2) - 4 = 8$

$\therefore$ Common difference, $d = T_2 - T_1 = 8 - 2 = 6$

6.

Sol:

$$S_n = \left(\frac{5n^2}{2} + \frac{3n}{2}\right) = \frac{1}{2}\left(5n^2 + 3n\right) \qquad ....(i)$$

Replacing n by $(n-1)$ in (i), we get:

$$S_{n-1} = \frac{1}{2} \times \left[5(n-1)^2 + 3(n-1)\right]$$

$$= \frac{1}{2} \times \left[5n^2 - 10n + 5 + 3n - 3\right] = \frac{1}{2} \times \left[5n^2 - 7n + 2\right]$$

$$\therefore T_n = S_n - S_{n-1}$$

$$= \frac{1}{2}\left(5n^2 + 3n\right) - \frac{1}{2} \times \left[5n^2 - 7n + 2\right]$$

$$= \frac{1}{2}(10n - 2) = 5n - 1 \qquad ....(ii)$$

Putting $n = 20$ in (ii), we get

$$T_{20} = (5 \times 20) - 1 = 99$$

Hence, the 20th term is 99.

7.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$$\therefore S_n = \frac{3n^2}{2} + \frac{5n}{2}$$

$$\Rightarrow S_{n-1} = \frac{3(n-1)^2}{2} + \frac{5(n-1)}{2}$$

$$= \frac{3\left(n^2 - 2n + 1\right)}{2} + \frac{5(n-1)}{2}$$

$$= \frac{3n^2 - n - 2}{2}$$

$\therefore n^{th}$ term of the AP, a_n

$$= S_n - S_{n-1}$$

$= \left(\frac{3n^2+5n}{2}\right) - \left(\frac{3n^2-n-2}{2}\right)$

$= \frac{6n+2}{2}$

$= 3n+2$

Putting $n = 25$, we get

$a_{25} = 3\times 25+1 = 75+1 = 76$

Hence, the nth term is $(3n+1)$ and 25^{th} term is 76.

8.

Sol:

Thee given AP is 21, 18, 15,……….

Here, $a = 21$ and $d = 18-21 = -3$

Let the required number of terms be n. Then,

$S_n = 0$

$\Rightarrow \frac{n}{2}\left[2\times 21+(n-1)\times(-3)\right] = 0 \qquad \left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow \frac{n}{2}(42-3n+3) = 0$

$\Rightarrow n(45-3n) = 0$

$\Rightarrow n = 0 \text{ or } 45-3n = 0$

$\Rightarrow n = 0 \text{ or } n = 15$

$\therefore n = 15$ (Number of terms cannot be zero)

Hence, the required number of terms is 15.

9.

Sol:

The given AP is 9, 17, 25,……

Here, $a = 9$ and $d = 17-9 = 8$

Let the required number of terms be n. Then,

$S_n = 636$

$\Rightarrow \frac{n}{2}\left[2\times 9+(n-1)\times 8\right] = 636 \qquad \left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow \frac{n}{2}(18+8n-8) = 636$

$\Rightarrow \frac{n}{2}(10+8n) = 636$

$\Rightarrow n(5+4n)=636$

$\Rightarrow 4n^2+5n-636=0$

$\Rightarrow 4n^2-48n+53n-636=0$

$\Rightarrow 4n(n-12)+53(n-12)=0$

$\Rightarrow (n-12)(4n+53)=0$

$\Rightarrow n-12=0 \text{ or } 4n+53=0$

$\Rightarrow n=12 \text{ or } n=-\frac{53}{4}$

$\therefore n=12$ (Number of terms cannot negative)

Hence, the required number of terms is 12.

10.

Sol:

The given AP is 63, 60, 57, 54,………..

Here, $a=63$ and $d=60-63=-3$

Let the required number of terms be *n*. Then,

$S_n=693$

$\Rightarrow \frac{n}{2}\left[2\times 63+(n-1)\times(-3)\right]=693$ $\left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow \frac{n}{2}(126-3n+3)=693$

$\Rightarrow n(129-3n)=1386$

$\Rightarrow 3n^2-129n+1386=0$

$\Rightarrow 3n^2-66n-63n+1386=0$

$\Rightarrow 3n(n-22)-63(n-22)=0$

$\Rightarrow (n-22)(3n-63)=0$

$\Rightarrow n-22=0 \text{ or } 3n-63=0$

$\Rightarrow n=22 \text{ or } n=21$

So, the sum of 21 terms as well as that of 22 terms is 693. This is because the 22^{nd} term of the AP is 0.

$a_{22}=63+(22-1)\times(-3)=63-63=0$

Hence, the required number of terms is 21 or 22.

11.

Sol:

The given AP is 20, $19\frac{1}{3}, 18\frac{2}{3}, ...$

Here, $a = 20$ and $d = 19\frac{1}{3} - 20 = \frac{58}{3} - 20 = \frac{58-60}{3} = -\frac{2}{3}$

Let the required number of terms be *n*. Then,

$S_n = 300$

$\Rightarrow \frac{n}{2}\left[2\times 20 + (n-1)\times\left(-\frac{2}{3}\right)\right] = 300 \qquad \left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow \frac{n}{2}\left(40 - \frac{2}{3}n + \frac{2}{3}\right) = 300$

$\Rightarrow \frac{n}{2}\times\frac{(122-2n)}{3} = 300$

$\Rightarrow 122n - 2n^2 = 1800$

$\Rightarrow 2n^2 - 122n + 1800 = 0$

$\Rightarrow 2n^2 - 50n - 72n + 1800 = 0$

$\Rightarrow 2n(n-25) - 72(n-25) = 0$

$\Rightarrow (n-25)(2n-72) = 0$

$\Rightarrow n - 25 = 0 \; or \; 2n - 72 = 0$

$\Rightarrow n = 25 \; or \; n = 36$

So, the sum of first 25 terms as well as that of first 36 terms is 300. This is because the sum of all terms from 26^{th} to 36^{th} is 0.

12.

Sol:

All odd numbers between 0 and 50 are 1, 3, 5, 7,……… 49.

This is an AP in which $a = 1, d = (3-1) = 2$ and $l = 49$.

Let the number of terms be n.

Then, $T_n = 49$

$\Rightarrow a + (n-1)d = 49$

$\Rightarrow 1 + (n-1)\times 2 = 49$

$\Rightarrow 2n = 50$

$\Rightarrow n = 25$

$\therefore$ Required sum $= \frac{n}{2}(a+l)$

$= \frac{25}{2}[1+49] = 25 \times 25 = 625$

Hence, the required sum is 625.

13.

Sol:

Natural numbers between 200 and 400 which are divisible by 7 are 203, 210 ,.... 399.

This is an AP with a = 203, d = 7 and l = 399.

Suppose there are n terms in the AP. Then,

$a_n = 399$

$\Rightarrow 203 + (n-1) \times 7 = 399 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 7n + 196 = 399$

$\Rightarrow 7n = 399 - 196 = 203$

$\Rightarrow n = 29$

$\therefore$ Required sum $= \frac{29}{2}(203+399) \qquad \left[S_n = \frac{n}{2}(a+l)\right]$

$= \frac{29}{2} \times 602$

$= 8729$

Hence, the required sum is 8729.

14.

Sol:

The positive integers divisible by 6 are 6, 12, 18,…..

This is an AP with $a = 6$ and $d = 6$.

Also, $n = 40$ (Given)

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{40} = \frac{40}{2}[2 \times 6 + (40-1) \times 6]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Hence, the required sum is 4920.

15.

Sol:

The first 15 multiples of 8 are 8, 16, 24, 32,…….

This is an AP in which $a=8, d=(16-8)=8$ and $n=15$.

Thus, we have:

$l=a+(n-1)d$

$=8+(15-1)8$

$=120$

$\therefore$ Required sum $=\frac{n}{2}(a+l)$

$=\frac{15}{2}[8+120]=15\times 64=960$

Hence, the required sum is 960.

16.

Sol:

The multiples of 9 lying between 300 and 700 are 306, 315,………, 693.

This is an AP with $a=306, d=9$ and $l=693$.

Suppose these are *n* terms in the AP. Then,

$a_n=693$

$\Rightarrow 306+(n-1)\times 9=693$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 9n+297=693$

$\Rightarrow 9n=693-297=396$

$\Rightarrow n=44$

$\therefore$ Required sum $=\frac{44}{2}(306=693)$ $\quad \left[S_n=\frac{n}{2}(a+l)\right]$

$=22\times 999$

$=21978$

Hence, the required sum is 21978.

17.

Sol:

All three-digit numbers which are divisible by 13 are 104, 117, 130, 143,……. 938.

This is an AP in which a = 104, d = (117 – 104) = 13 and l = 938

Let the number of terms be n

Then $T_n=938$

$\Rightarrow a+(n-1)d=988$

$\Rightarrow 104+(n-1)\times 13=988$

$\Rightarrow 13n=897$

$\Rightarrow n=69$

$\therefore$ Required sum $=\frac{n}{2}(a+l)$

$=\frac{69}{2}[104+988]=69\times 546=37674$

Hence, the required sum is 37674.

18.

Sol:

The first few even natural numbers which are divisible by 5 are 10, 20, 30, 40, ...

This is an AP in which a = 10, d = (20 – l0) = 10 and n = 100

The sum of n terms of an AP is given by

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$=\left(\frac{100}{2}\right)\times\left[2\times 10+(100-1)\times 10\right] \qquad [\because a=10, d=10 \text{ and } n=100]$

$=50\times[20+990]=50\times 1010=50500$

Hence, the sum of the first hundred even natural numbers which are divisible by 5 is 50500.

19.

Sol:

On simplifying the given series, we get:

$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots n \text{ terms}$

$=(1+1+1+\ldots\ldots.n \text{ terms})-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots\ldots..+\frac{n}{n}\right)$

$=n-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots\ldots..+\frac{n}{n}\right)$

Here, $\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots\ldots+\frac{n}{n}\right)$ is an AP whose first term is $\frac{1}{n}$ and the common difference is $\left(\frac{2}{n}-\frac{1}{n}\right)=\frac{1}{n}$.

The sum of terms of an AP is given by

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$=n-\left[\frac{n}{2}\left\{2\times\left(\frac{1}{n}\right)+(n-1)\times\left(\frac{1}{n}\right)\right\}\right]$$

$$=n-\left[\frac{n}{2}\left[\left(\frac{2}{n}\right)+\left(\frac{n-1}{n}\right)\right]\right]=n-\left\{\frac{n}{2}\left(\frac{n+1}{n}\right)\right\}$$

$$=n-\left(\frac{n+1}{2}\right)=\frac{n-1}{2}$$

20.

Sol:

Let a be the first term and d be the common difference of thee AP. Then,

$S_5+S_7=167$

$\Rightarrow \frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d)=167$ $\quad\left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 5a+10d+7a+21d=167$

$\Rightarrow 12a+31d=167$ ....(1)

Also,

$S_{10}=235$

$\Rightarrow \frac{10}{2}(2a+9d)=235$

$\Rightarrow 5(2a+9d)=235$

$\Rightarrow 2a+9d=47$

Multiplying both sides by 6, we get

$12a+54d=282$ ....(2)

Subtracting (1) from (2), we get

$12a+54d-12a-31d=282-167$

$\Rightarrow 23d=115$

$\Rightarrow d=5$

Putting $d=5$ in (1), we get

$12a + 31 \times 5 = 167$

$\Rightarrow 12a + 155 = 167$

$\Rightarrow 12a = 167 - 155 = 12$

$\Rightarrow a = 1$

Hence, the AP is 1, 6, 11, 16,…….

21.

Sol:

Here, $a = 2, l = 29 \text{ and } S_n = 155$

Let d be the common difference of the given AP and n be the total number of terms.

Then, $T_n = 29$

$\Rightarrow a + (n-1)d = 29$

$\Rightarrow 2 + (n-1)d = 29 \qquad ....(i)$

The sum of *n* terms of an AP is given by

$S_n = \frac{n}{2}[a + l] = 155$

$\Rightarrow \frac{n}{2}[2 + 29] = \left(\frac{n}{2}\right) \times 31 = 155$

$\Rightarrow n = 10$

Putting the value of *n* in (i), we get:

$\Rightarrow 2 + 9d = 29$

$\Rightarrow 9d = 27$

$\Rightarrow d = 3$

Thus, the common difference of the given AP is 3.

22.

Sol:

Suppose there are *n* terms in the AP.

Here, $a = -4, l = 29$ and $S_n = 150$

$S_n = 150$

$\Rightarrow \frac{n}{2}(-4 + 29) = 150 \qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow n = \frac{150 \times 2}{25} = 12$

Thus, the AP contains 12 terms.

Let *d* be the common difference of the AP.

$\therefore a_{12} = 29$

$\Rightarrow -4 + (12-1) \times d = 29$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow 11d = 29 + 4 = 33$

$\Rightarrow d = 3$

Hence, the common difference of the AP is 3.

23.

Sol:

Suppose there are n terms in the AP.

Here, a = 17, d = 9 and $l = 350$

$\therefore a_n = 350$

$\Rightarrow 17 + (n-1) \times 9 = 350$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 8 = 350$

$\Rightarrow 9n = 350 - 8 = 342$

$\Rightarrow n = 38$

Thus, there are 38 terms in the AP.

$\therefore S_{38} = \frac{28}{2}(17 + 350)$ $\quad \left[S_n = \frac{n}{2}(a + l)\right]$

$= 19 \times 367$

$= 6973$

Hence, the required sum is 6973.

24.

Sol:

Suppose there are n term in the AP.

Here, $a = 5, l = 45$ and $S_n = 400$

$S_n = 400$

$\Rightarrow \frac{n}{2}(5 + 45) = 400$ $\quad \left[S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow \frac{n}{2} \times 50 = 400$

$\Rightarrow n = \frac{400 \times 2}{50} = 16$

Thus, there are 16 terms in the AP.

Let d be the common difference of the AP.

$\therefore a_{16} = 45$

$\Rightarrow 5 + (16-1) \times d = 45 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 15d = 45 - 5 = 40$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

Hence, the common difference of the AP is $\frac{8}{3}$.

25.

Sol:

Here, $a = 22, T_n = -11$ and $S_n = 66$

Let d be the common difference of the given AP.

Then, $T_n = -11$

$\Rightarrow a + (n-1)dd = 22 + (n-1)d = -11$

$\Rightarrow (n-1)d = -33 \qquad ....(i)$

The sum of *n* terms of an AP is given by

$S_n = \frac{n}{2}[2a + (n-1)d] = 66$ [Substituting the value off $(n-1)d$ from (i)]

$\Rightarrow \frac{n}{2}[2 \times 22 + (-33)] = \left(\frac{n}{2}\right) \times 11 = 66$

$\Rightarrow n = 12$

Putting the value of *n* in (i), we get:

$11d = -33$

$\Rightarrow d = -3$

Thus, $n = 12$ and $d = -3$

26.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{12} = -13$

$\Rightarrow a + 11d = -13 \qquad ....(1) \qquad [a_n = a + (n-1)d]$

Also,

$S_4 = 24$

$\Rightarrow \frac{4}{2}(2a+3d)=24$ $\quad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 2a+3d=12$ $\quad$ ....(2)

Solving (1) and (2), we get

$2(-13-11d)+3d=12$

$\Rightarrow -26-22d+3d=12$

$\Rightarrow -19d=12+26=38$

$\Rightarrow d=-2$

Putting $d=-2$ in (1), we get

$a+11\times(-2)=-13$

$\Rightarrow a=-13+22=9$

$\therefore$ Sum of its first 10 terms, S_{10}

$=\frac{10}{2}\left[2\times 9+(10-1)\times(-2)\right]$

$=5\times(18-18)$

$=5\times 0$

$=0$

Hence, the required sum is 0.

27.

Sol:

Let a be the first term and d be the common difference of the AP.

$\therefore S_7=182$

$\Rightarrow \frac{7}{2}(2a+6d)=182$ $\quad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow a+3d=26$ $\quad$ ....(1)

Also,

$a_4:a_{17}=1:5$ $\quad$ (Given)

$\Rightarrow \frac{a+3d}{a+16d}=\frac{1}{5}$ $\quad \left[a_n=a+(n-1)d\right]$

$\Rightarrow 5a+15d=a+16d$

$\Rightarrow d=4a$ $\quad$ ....(2)

Solving (1) and (2), we get

$a+3\times 4a=26$

$\Rightarrow 13a=26$

$\Rightarrow a=2$

Putting $a=2$ in (2), we get

$d=4\times 2=8$

Hence, the required AP is 2, 10, 18, 26,………

28.

Sol:

Here, $a=4, d=7$ *and* $l=81$

Let the nth term be 81.

Then $T_n=81$

$\Rightarrow a+(n-1)d=4+(n-1)7=81$

$\Rightarrow (n-1)7=77$

$\Rightarrow (n-1)=11$

$\Rightarrow n=12$

Thus, there are 12 terms in the AP.

The sum of *n* terms of an AP is given by

$$S_n=\frac{n}{2}[a+l]$$

$$\therefore S_{12}=\frac{12}{2}[4+81]=6\times 85=510$$

Thus, the required sum is 510.

29.

Sol:

Let a be the first term and d be the common difference of the given AP.

Then, we have:

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_7=\frac{7}{2}[2a+6d]=7[a+3d]$$

$$S_{17}=\frac{17}{2}[2a+16d]=17[a+8d]$$

However, $S_7=49$ and $S_{17}=289$

Now, $7(a+3d)=49$

$\Rightarrow a+3d=7 \quad ....(i)$

Also, $17[a+8d]=289$

$\Rightarrow a+8d=17 \quad ....(ii)$

Subtracting (i) from (ii), we get:

$5d=10$

$\Rightarrow d=2$

Putting $d=2$ in (i), we get

$a+6=7$

$\Rightarrow a=1$

Thus, $a=1$ and $d=2$

$\therefore$ Sum of n terms of AP = $\frac{n}{2}[2\times1+(n-1)\times2]=n[1+(n-1)]=n^2$

30.

Sol:

Let a_1 and a_2 be the first terms of the two APs.

Here, $a_1=8$ *and* $a_2=3$

Suppose d be the common difference of the two Aps

Let S_{50} and $S'_{50}=\frac{50}{2}[2a_1+(50-1)d]-\frac{50}{2}[2a_2+(50-1)d]$

$=25(2\times8\times49d)-25(2\times3+49d)$

$=25\times(16-6)$

$=250$

Hence, the required difference between the two sums is 250.

31.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$S_{10}=-150$ (Given)

$\Rightarrow \frac{10}{2}(2a+9d)=-150 \quad \left\{S_n=\frac{n}{2}[2a+(n-1)d]\right\}$

$\Rightarrow 5(2a+9d)=-150$

$\Rightarrow 2a+9d=-30 \quad ....(1)$

It is given that the sum of its next 10 terms is -550.

Now,

$S_{20}=$ Sum of first 20 terms = Sum of first 10 terms + Sum of the next 10 terms =

$-150+(-550)=-700$

$\therefore S_{20}=-700$

$\Rightarrow \frac{20}{2}(2a+19d)=-700$

$\Rightarrow 10(2a+19d)=-700$

$\Rightarrow 2a+19d=-70 \qquad ....(2)$

Subtracting (1) from (2), we get

$(2a+19d)-(2a+9d)=-70-(-30)$

$\Rightarrow 10d=-40$

$\Rightarrow d=-4$

Putting $d=-4$ in (1), we get

$2a+9\times(-4)=-30$

$\Rightarrow 2a=-30+36=6$

$\Rightarrow a=3$

Hence, the required AP is $3,-1,-5,-9,.............$

32.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{13}=4\times a_3$ (Given)

$\Rightarrow a+12d=4(a+2d) \qquad [a_n=a+(n-1)d]$

$\Rightarrow a+12d=4a+8d$

$\Rightarrow 3a=4d \qquad ....(1)$

Also,

$a_5=16$ (Given)

$\Rightarrow a+4d=16 \qquad ....(2)$

Solving (1) and (2), we get

$a+3a=16$

$\Rightarrow 4a=16$

$\Rightarrow a=4$

Putting $a=4$ in (1), we get

$4d=3\times4=12$

$\Rightarrow d=3$

Using the formula, $S_4 = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$S_{10} = \frac{10}{2}\left[2\times 4+(10-1)\times 3\right]$

$= 5\times(8+27)$

$= 5\times 35$

$= 175$

Hence, the required sum is 175.

33.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{16} = 5\times a_3$ (Given)

$\Rightarrow a+15d = 5(a+2d)$ $\left[a_n = a+(n-1)d\right]$

$\Rightarrow a+15d = 5a+10d$

$\Rightarrow 4a = 5d$

Also,

$a_{10} = 41$ (Given)

$\Rightarrow a+9d-41$ ....(2)

Solving (1) and (2), we get

$a+9\times\frac{4a}{5} = 41$

$\Rightarrow \frac{5a+36a}{5} = 41$

$\Rightarrow \frac{41a}{5} = 41$

$\Rightarrow a = 5$

Putting $a = 5$ in (1), we get

$5d = 4\times 5 = 20$

$\Rightarrow d = 4$

Using the formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$S_{15} = \frac{15}{2}\left[2\times5+(15-1)\times4\right]$

$= \frac{15}{2}\times(10+56)$

$= \frac{15}{2}\times66$

$= 495$

Hence, the required sum is 495.

34.

Sol:

The given AP is $5,12,19,.......$

Here, $a = 5, d = 12-5 = 7$ and $n = 50$.

Since there are 50 terms in the AP, so the last term of the AP is a_{50}.

$l = a_{50} = 5+(50-1)\times7$ $\qquad\left[a_n = a+(n-1)d\right]$

$= 5+343$

$= 348$

Thus, the last term of the AP is 348.

Now,

Sum of the last 15 terms of the AP

$= S_{50} - S_{35}$

$= \frac{50}{2}\left[2\times5+(50-1)\times7\right] - \frac{35}{2}\left[2\times5+(35-1)\times7\right]$

$\left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$= \frac{50}{2}\times(10+343) - \frac{35}{2}\times(10+238)$

$= \frac{50}{2}\times353 - \frac{35}{2}\times248$

$= \frac{17650-8680}{2}$

$= \frac{8970}{2}$

$= 4485$

Hence, the require sum is 4485.

35.

Sol:

The given AP is 8, 10, 12,……

Here, $a = 8, d = 10 - 8 = 2$ and $n = 60$

Since there are 60 terms in the AP, so the last term of the AP is a_{60}.

$l = a_{60} = 8 + (60 - 1) \times 2$ $\quad\quad [a_n = a + (n-1)d]$

$= 8 + 118$

$= 126$

Thus, the last term of the AP is 126.

Now,

Sum of the last 10 terms of the AP

$= S_{60} - S_{50}$

$= \frac{60}{2}[2 \times 8 + (60 - 1) \times 2] - \frac{50}{2}[2 \times 8 + (50 - 1) \times 2]$

$\left\{S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$= 30 \times (16 + 118) - 25 \times (16 + 98)$

$= 30 \times 134 - 25 \times 114$

$= 4020 - 2850$

$= 1170$

Hence, the required sum is 1170.

36.

Sol:

Let a be the first and d be the common difference of the AP.

$\therefore a_4 + a_8 = 24$ $\quad\quad$ (Given)

$\Rightarrow (a + 3d) + (a + 7d) = 24$ $\quad\quad [a_n = a + (n-1)d]$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12$ $\quad\quad$ ......(1)

Also,

$\therefore a_6 + a_{10} = 44$ $\quad\quad$ (Given)

$\Rightarrow (a + 5d) + (a + 9d) = 44$ $\quad\quad [a_n = a + (n-1)d]$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22$ $\quad\quad$ ......(2)

Subtracting (1) from (2), we get

$(a + 7d) - (a + 5d) = 22 - 12$

$\Rightarrow 2d = 10$

$\Rightarrow d = 5$

Putting $d = 5$ in (1), we get

$a + 5 \times 5 = 12$

$\Rightarrow a = 12 - 25 = -13$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$S_{10} = \frac{10}{2}\left[2 \times (-13) + (10-1) \times 5\right]$

$= 5 \times (-26 + 45)$

$= 5 \times 19$

$= 95$

Hence, the required sum is 95.

37.

Sol:

Let S_m denotes the sum of the first m terms of the AP. Then,

$S_m = 4m^2 - m$

$\Rightarrow S_{m-1} = 4(m-1)^2 - (m-1)$

$= 4(m^2 - 2m + 1) - (m-1)$

$= 4m^2 - 9m + 5$

Suppose a_m denote the m^{th} term of the AP.

$\therefore a_m = S_m - S_{m-1}$

$= (4m^2 - m) - (4m^2 - 9m + 5)$

$= 8m - 5$ ....(1)

Now,

$a_n = 107$ (Given)

$\Rightarrow 8n - 5 = 107$ [From (1)]

$\Rightarrow 8n = 107 + 5 = 112$

$\Rightarrow n = 14$

Thus, the value of n is 14.

Putting $m = 21$ in (1), we get

$a_{21} = 8 \times 21 - 5 = 168 - 5 = 163$

Hence, the 21^{st} term of the AP is 163.

38.

Sol:

Let S_q denote the sum of the first q terms of the AP. Then,

$S_q = 63q - 3q^2$

$\Rightarrow S_{q-1} = 63(q-1) - 3(q-1)^2$

$= 63q - 63 - 3(q^2 - 2q + 1)$

$= -3q^2 + 69q - 66$

Suppose a_q denote the q^{th} term of the AP.

$\therefore a_q = S_q - S_{q-1}$

$= (63q - 3q^2) - (-3q^2 + 69q - 66)$

$= -6q + 66 \qquad ....(1)$

Now,

$a_p = -60 \qquad$ (Given)

$\Rightarrow -6p + 66 = -60 \qquad$ [From (1)]

$\Rightarrow -6p = -60 - 66 = -126$

$\Rightarrow p = 21$

Thus, the value of p is 21.

Putting $q = 11$ in (1), we get

$a_{11} = -6 \times 11 + 66 = -66 + 66 = 0$

Hence, the 11th term of the AP is 0.

39.

Sol:

The given AP is $-12, -9, -6, ..., 21.$

Here, $a = -12, d = -9 - (-12) = -9 + 12 = 3 \text{ and } l = 21$

Suppose there are n terms in the AP.

$\therefore l = a_n = 21$

$\Rightarrow -12 + (n-1) \times 3 = 21 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 3n - 15 = 21$

$\Rightarrow 3n = 21 + 15 = 36$

$\Rightarrow n = 12$

Thus, there are 12 terms in the AP.

If 1 is added to each term of the AP, then the new AP so obtained is $-11, -8, -5, ..., 22$.

Here, first term, $A = -11$; last term, $L = 22$ *and* $n = 12$

$\therefore$ Sum of the terms of this AP

$= \frac{12}{2}(-11+22)$ $\left[S_n = \frac{n}{2}(a+l)\right]$

$= 6 \times 11$

$= 66$

Hence, the required sum is 66.

40.

Sol:

Let d be the common difference of the AP.

Here, $a = 10$ and $n = 14$

Now,

$S_{14} = 1505$ (Given)

$\Rightarrow \frac{14}{2}\left[2 \times 10 + (14-1) \times d\right] = 1505$ $\left\{S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right\}$

$\Rightarrow 7(20 + 13d) = 1505$

$\Rightarrow 20 + 13d = 215$

$\Rightarrow 13d = 215 - 20 = 195$

$\Rightarrow d = 15$

$\therefore 25^{\text{th}}$ term of the AP, a_{25}

$= 10 + (25-1) \times 15$ $\left[a_n = a + (n-1)d\right]$

$= 10 + 360$

$= 370$

Hence, the required term is 370.

41.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$d = a_3 - a_2 = 18 - 14 = 4$

Now,

$a_2 = 14$ (Given)

$\Rightarrow a + d = 14$ $\left[a_n = a + (n-1)d\right]$

$\Rightarrow a + 4 = 14$

$\Rightarrow a = 14 - 4 = 10$

Using the formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_{51}=\frac{51}{2}\left[2\times10+(51-1)\times4\right]$$

$$=\frac{51}{2}(20+200)$$

$$=\frac{51}{2}\times220$$

$$=5610$$

Hence, the required sum is 5610.

42.

Sol:

Number of trees planted by the students of each section of class 1 = 2

There are two sections of class 1.

$\therefore$ Number of trees planted by the students of class $1=2\times2=4$

Number of trees planted by the students of each section of class 2 = 4

There are two sections of class 2.

$\therefore$ Number of trees planted by the students of class $2=2\times4=8$

Similarly,

Number of trees planted by the students of class $3=2\times6=12$

So, the number of trees planted by the students of different classes are 4, 8, 12, ...

$\therefore$ Total number of trees planted by the students $=4+8+12+......$ up to 12 terms

This series is an arithmetic series.

Here, $a=4, d=8-4=4$ *and* $n=12$

Using the formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_{12}=\frac{12}{2}\left[2\times4+(12-1)\times4\right]$$

$$=6\times(8+44)$$

$$=6\times52$$

$$=312$$

Hence, the total number of trees planted by the students is 312.

The values shown in the question are social responsibility and awareness for conserving nature.

43.

Sol:

Distance covered by the competitor to pick and drop the first potato $= 2 \times 5m = 10m$

Distance covered by the competitor to pick and drop the second potato

$= 2 \times (5+3)m = 2 \times 8m = 16\ m$

Distance covered by the competitor to pick and drop the third potato

$= 2 \times (5+3+3)m = 2 \times 11m = 22$ m and so on.

$\therefore$ Total distance covered by the competitor $= 10m + 16m + 22m + ...$ up to 10 terms

This is an arithmetic series.

Here, $a = 10, d = 16 - 10 = 6$ and $n = 10$

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{10} = \frac{10}{2}[2 \times 10 + (10-1) \times 6]$

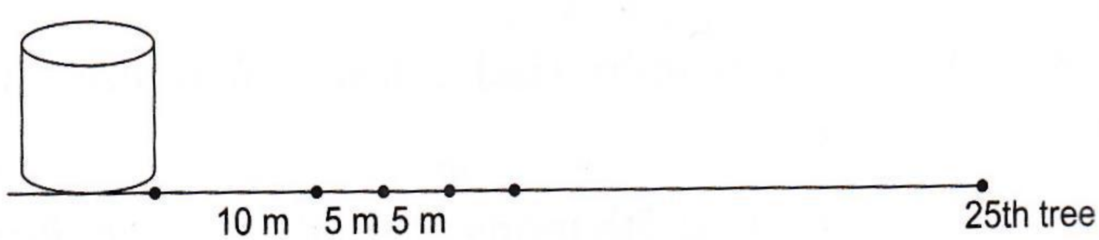
$= 5 \times (20 + 54)$

$= 5 \times 74$

$= 370$

Hence, the total distance the competitor has to run is 370 m.

44.



Sol:

Distance covered by the gardener to water the first tree and return to the water tank $= 10m + 10m = 20m$

Distance covered by the gardener to water the second tree and return to the water tank $= 15m + 15m = 30m$

Distance covered by the gardener to water the third tree and return to the water tank $= 20m + 20m = 40m$ and so on.

$\therefore$ Total distance covered by the gardener to water all the trees $= 20m + 30m + 40m + ...$ up to 25 terms

This series is an arithmetic series.

Here, $a = 20, d = 30 - 20 = 10$ *and* $n = 25$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d,\right]$ we get

$S_{25} = \frac{25}{2}\left[2 \times 20 + (25-1) \times 10\right]$

$= \frac{25}{2}(40 + 240)$

$= \frac{25}{2} = 280$

$= 3500$

Hence, the total distance covered by the gardener to water all the trees 3500 m.

45.

Sol:

Let the value of the first prize be *a*.

Since the value of each prize is 20 less than its preceding prize, so the values of the prizes are in AP with common difference – ₹20.

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$$\Rightarrow \frac{40}{2}\left[2a+(40-1)d\right]=36000$$

$\therefore d = -$₹ $\Rightarrow 20(2a+39d)=36000$

$$\Rightarrow 2a+39d=1800 \qquad ....(2)$$

Number of cash prizes to be given to the students, n = 7

Total sum of the prizes, $S_7 =$ ₹700

Using the formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_7=\frac{7}{2}\left[2a+(7-1)\times(-20)\right]=700$$

$$\Rightarrow \frac{7}{2}(2a-120)=700$$

$$\Rightarrow 7a-420=700$$

$$\Rightarrow 7a=700+420=1120$$

$$\Rightarrow a=160$$

Thus, the value of the first prize is ₹160.

Hence, the value of each prize is ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 and ₹40.

46.

Sol:

Let the money saved by the man in the first month be ₹a

It is given that in each month after the first, he saved ₹100 more than he did in the preceding month. So, the money saved by the man every month is in AP with common difference ₹100.

$\therefore d =$ ₹100

Number of months, n = 10

Sum of money saved in 10 months, $S_{10} =$ ₹ 33,000

Using the formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_{10}=\frac{10}{2}\left[2a+(10-1)\times 100\right]=33000$$

$$\Rightarrow 5(2a+900)=33000$$

$$\Rightarrow 2a+900=6600$$

$$\Rightarrow 2a=6600-900=5700$$

$$\Rightarrow a=2850$$

Hence, the money saved by the man in the first month is ₹2,850.

47.

Sol:

Let the value of the first installment be ₹a.

Since the monthly installments form an arithmetic series, so let us suppose the man increases the value of each installment by ₹d every month.

∴ Common difference of the arithmetic series = ₹d

Amount paid in 30 installments = ₹$36,000-\frac{1}{3}\times$ ₹ $36,000=$ ₹ $36,000-$ ₹ $12,000=$ ₹ $24,000$

Let S_n denote the total amount of money paid in the n installments. Then,

$S_{30}=24,000$

$\Rightarrow \frac{30}{2}\left[2a+(30-1)d\right]=24000$ $\quad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 15(2a+29d)=24000$

$\Rightarrow 2a+29d=1600$ ....(1)

Also,

$S_{40}=$ ₹$36,000$

$\Rightarrow \frac{40}{2}\left[2a+(40-1)d\right]=36000$

$\Rightarrow 20(2a+39d)=36000$

$\Rightarrow 2a+39d=1800$ ....(2)

Subtracting (1) from (2), we get

$(2a+39d)-(2a+29d)=1800-1600$

$\Rightarrow 10d=200$

$\Rightarrow d=20$

Putting $d=20$ in (1), we get

$2a+29\times 20=1600$

$\Rightarrow 2a+580=1600$

$\Rightarrow 2a=1600-580=1020$

$\Rightarrow a=510$

Thus, the value of the first installment is ₹510.

48. Sol:

It is given that the penalty for each succeeding day is 50 more than for the preceding day, so the amount of penalties are in AP with common difference ₹50

Number of days in the delay of the work = 30

The amount of penalties are ₹200, ₹250, ₹300,... up to 30 terms.

$\therefore$ Total amount of money paid by the contractor as penalty,

$S_{30} = ₹\ 200 + ₹\ 250 + ₹\ 300 + ...$ up to 30 terms

Here, $a = ₹\ 200, d = ₹\ 50\ and\ n = 30$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$S_{30} = \frac{30}{2}\left[2 \times 200 + (30-1) \times 50\right]$

$= 15(400 - 1450)$

$= 15 \times 1850$

$= 27750$

Hence, the contractor has to pay ₹27,750 as penalty

Exercise - Multiple Choice Questions

1.

Answer: (c) -1

Sol:

The given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{2}, ...$

$\therefore$ Common difference, $d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = -1$

2.

Answer: (d) -b

Sol:

The given AP is $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, ...$

$\therefore$ Common difference, $d = \frac{1-3b}{3} - \frac{1}{3} = \frac{1-3b-1}{3} = \frac{-3b}{3} = -b$

3.

Answer: (d) $\sqrt{112}$

Sol:

The given terms of the AP can be written as $\sqrt{7}, \sqrt{4\times7}, \sqrt{9\times7}, ...i.e. \sqrt{7}, 2\sqrt{7}, 3\sqrt{7}, ...$

$\therefore$ Next term $= 4\sqrt{7} = \sqrt{16\times7} = \sqrt{112}$

4.

Answer: (c) 22

Sol:

Here, $a = 4, l = 28$ *and* $n = 5$

Then, $T_5 = 28$

$\Rightarrow a + (n-1)d = 28$

$\Rightarrow 4 + (5-1)d = 28$

$\Rightarrow 4d = 24$

$\Rightarrow d = 6$

Hence, $x_3 = 28 - 6 = 22$

5.

Answer: (b) 15

Sol:

nth term of the AP, $a_n = 2n+1$ (Given)

$\therefore$ First term, $a_1 = 2\times1+1 = 2+1 = 3$

Second term, $a_2 = 2\times2+1 = 4+1 = 5$

Third term, $a_3 = 2\times3+1 = 6+1 = 7$

$\therefore$ Sum of the first three terms $a_1 + a_2 + a_3 = 3+5+7 = 15$

6.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2 + 6n$

$\Rightarrow S_{n-1} = 3(n-1)^2 + 6(n-1)$

$= 3(n^2 - 2n + 1) + 6(n-1)$

$= 3n^2 - 3$

So,

n^{th} term of the AP, $a_n = S_n - S_{n-1}$

$= (3n^2 + 6n) - (3n^2 - 3)$

$= 6n + 3$

Let d be the common difference of the AP.

$\therefore d = a_n - a_{n-1}$

$= (6n+3) - [6(n-1)+3]$

$= 6n + 3 - 6(n-1) - 3$

$= 6$

Thus, the common difference of the AP is 6.

7.

Answer: (b) (6 – 2n)

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 5n - n^2$

$\Rightarrow S_{n-1} = 5(n-1) - (n-1)^2$

$= 5n - 5 - n^2 + 2n - 1$

$= 7n - n^2 - 6$

$\therefore n^{th}$ term of the AP, $a_n = S_n - S_{n-1}$

$= (5n - n^2) - (7n - n^2 - 6)$

$= 6 - 2n$

Thus, the nth term of the AP is $(6 - 2n)$.

8.

Answer: (c) (8n – 2)

Sol:

Let S_n denotes the sum of first *n* terms of the AP.

$\therefore S_n = 4n^2 + 2n$

$\Rightarrow S_{n-1} = 4(n-1)^2 + 2(n-1)$

$= 4(n^2 - 2n + 1) + 2(n-1)$

$= 4n^2 - 6n + 2$

$\therefore n^{th}$ term of the AP, $a_n = S_n - S_{n-1}$

$= (4n^2 + 2n) - (4n^2 - 6n + 2)$

$= 8n - 2$

Thus, the n^{th} term of thee AP is $(8n-2)$

9.

Answer: (d) (2n – 15)

Sol:

Let *a* be the first term and *d* be the common difference of the AP. Then,

nth term of the AP, $a_n = a + (n-1)d$

Now,

$a_7 = -1$ (Given)

$\Rightarrow a + 6d = -1$ ....(1)

Also,

$a_{16} = 17$ (Given)

$\Rightarrow a + 15d = 17$ ....(2)

Subtracting (1) from (2), we get

$(a + 15d) - (a + 6d) = 17 - (-1)$

$\Rightarrow 9d = 18$

$\Rightarrow d = 2$

Putting *d* = 2 in (1), we get

$a + 6 \times 2 = -1$

$\Rightarrow a = -1 - 12 = -13$

$\therefore$ nth term of the AP, $a_n = -13 + (n-1) \times 2 = 2n - 15$

10.

Answer: (b) -50

Sol:

Let a be the first term of the AP.

Here. $d = -4$

$a_5 = -3$ (Given)

$\Rightarrow a + (5-1)\times(-4) = -3$ $\left[a_n = a + (n-1)d\right]$

$\Rightarrow a - 16 = -3$

$\Rightarrow a = 16 - 3 = 13$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$S_{10} = \frac{10}{2}\left[2\times13 + (10-1)\times(-4)\right]$

$= 5\times(26-36)$

$= 5\times(-10)$

$= -50$

Thus, the sum of its first 10 terms is -50.

11.

Answer: (c) 3

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_5 = 20$

$\Rightarrow a + (5-1)d = 20$ $\left[a_n = a + (n-1)d\right]$

$\Rightarrow a + 4d = 20$ ....(1)

Now,

$a_7 + a_{11} = 64$ (Given)

$\Rightarrow (a + 6d) + (a + 10d) = 64$

$\Rightarrow 2a + 16d = 64$

$\Rightarrow a + 8d = 32$ ....(2)

From (1) and (2), we get

$20 - 4d + 8d = 32$

$\Rightarrow 4d = 32 - 20 = 12$

$\Rightarrow d = 3$

Thus, the common difference of the AP is 3.

12.

Answer: (b) 175

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{13} = 4 \times a_3$ (Given)

$\Rightarrow a + 12d = 4(a + 2d)$ $[a_n = a + (n-1)d]$

$\Rightarrow a + 12d = 4a + 8d$

$\Rightarrow 3a = 4d$ ....(1)

Also

$a_5 = 16$ (Given)

$\Rightarrow a + 4d = 16$ ....(2)

Solving (1) and (2), we get

$a + 3a = 16$

$\Rightarrow 4a = 16$

$\Rightarrow a = 4$

Putting a = 4 in (1), we get

$4d = 3 \times 4 = 12$

$\Rightarrow d = 3$

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{10} = \frac{10}{2}[2 \times 4 + (10-1) \times 3]$

$= 5 \times (8 + 27)$

$= 5 \times 35$

$= 175$

Thus, the sum of its first 10 terms is 175.

13.

Answer: (c) 348

Sol:

The given AP is 5, 12, 19,……

Here, $a = 5, d = 12 - 5 = 7$ and $n = 50$

Since there are 50 terms in the AP, so the last term of the AP is a_{50}.

$a_{50} = 5 + (50-1) \times 7$ $\quad [a_n = a + (n-1)d]$

$= 5 + 343$

$= 348$

Thus, the last term of the AP is 348.

14.

Answer: (c) 400

Sol:

The first 20 odd natural numbers are 1, 3, 5,.., 39.

These numbers are in AP.

Here. a = 1, l = 39 and n = 20

$\therefore$ Sum of first 20 odd natural numbers

$= \frac{20}{2}(1+39)$ $\quad \left[S_n = \frac{n}{2}(a+l)\right]$

$= 10 \times 40$

$= 400$

15.

Answer: (c) 4920

Sol:

The positive integers divisible by 6 are 6, 12, 18,....

This is an AP with a = 6 and d = 6.

Also, n = 40 (Given)

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{40} = \frac{40}{2}[2 \times 6 + (40-1) \times 6]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Thus, the required sum is 4920

16.

Answer: (b) 30

Sol:

The two-digit numbers divisible by 3 are 12, 15, 18,…… 99.
Clearly, these number are in AP.
Here, a = 12 and d = $15-12=3$
Let this AP contains n terms. Then.

$a_n = 99$

$\Rightarrow 12+(n-1)\times 3 = 99 \qquad [a_n = a+(n-1)d]$

$\Rightarrow 3n+9=99$

$\Rightarrow 3n = 99-9 = 90$

$\Rightarrow n = 30$

Thus, there are 30 two-digit numbers divisible by 3.

17.

Answer: (d) 100
Sol:

The three-digit numbers divisible by 9 are 108, 117, 126 ,…. 999.
Clearly, these numbers are in AP.
Here, $a = 108$ *and* $d = 117-108 = 9$
Let this AP contains n terms. Then,

$a_n = 999$

$\Rightarrow 108+(n-1)\times 9 = 999 \qquad [a_n = a+(n-1)d]$

$\Rightarrow 9n+99 = 999$

$\Rightarrow 9n = 999-99 = 900$

$\Rightarrow n = 100$

Thus, there are 100 three-digit numbers divisible by 9.

18.

Answer: (a) 8
Sol:
Let a be the first term and d be the common difference of the AP. Then.

$a_{18} - a_{14} = 32$

$\Rightarrow (a+17d)-(a+13d) = 32 \qquad [a_n = a+(n-1)d]$

$\Rightarrow 4d = 32$

$\Rightarrow d = 8$

Thus, the common difference of the AP is 8.

19.

Answer: (c) 50

Sol:

The given AP is 3, 8,13,18....

Here, a = 3 and d = 8 – 3 = 5

$\therefore a_{30} - a_{20}$

$= \left[3 + (30-1)\times 5\right] - \left[3 + (20-1)\times 5\right]$ $\qquad \left[a_n = a + (n-1)d\right]$

$= 148 - 98$

$= 50$

Thus, the required value is 50.

20.

Answer: (b) 9^{th}

Sol:

The given AP is 72, 63, 54,.....

Here, a = 72 and d = 63 – 72 = –9

Suppose nth term of the given AP is 0. Then.

$a_n = 0$

$\Rightarrow 72 + (n-1)\times(-9) = 0$ $\qquad \left[a_n = a + (n-1)d\right]$

$\Rightarrow -9n + 81 = 0$

$\Rightarrow n = \frac{81}{9} = 9$

Thus, the 9th term of the given AP is 0.

21.

Answer: (d) 7^{th}

Sol:

The given AP is 25, 20, 15,……

Here, a = 25 and $d = 20 - 25 = -5$

Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 25 + (n-1)\times(-5) < 0$ $\qquad \left[a_n = a + (n-1)d\right]$

$\Rightarrow 30 - 5n < 0$

$\Rightarrow -5n < -30$

$\Rightarrow n > \frac{30}{5} = 6$

$\therefore n = 7$

Thus, the 7th term is the first negative term of the given AP.

22.

Answer: (b) 10^{th}

Sol:

Here, $a = 21$ and $d = (42 - 21) = 21$

Let 210 be the nth term of the given AP.

Then, $T_n = 210$

$\Rightarrow a + (n-1)d = 210$

$\Rightarrow 21 + (n-1) \times 21 = 210$

$\Rightarrow 21n = 210$

$\Rightarrow n = 10$

Hence, 210 is the 10^{th} term of the AP.

23.

Answer: (b) 158

Sol:

The given AP is 3, 8, 13,..., 253.

Let us re-write the given AP in reverse order i.e. 253, 248, .., 13, 8, 3.

Now, the 20th term from the end of the given AP is equal to the 20th term from beginning of the AP 253, 248,., 13, 8, 3.

Consider the AP 253, 248,….. 13, 8, 3.

Here, a = 253 and $d = 248 - 253 = -5$

$\therefore$ 20th term of this AP

$= 253 + (20-1) \times (-5)$

$= 253 - 95$

$= 158$

Thus, the 20th term from the end of the given AP is 158.

24.

Answer: (d) 2139

Sol:

Here, $a = 5, d = (13 - 5) = 8$ and $l = 181$

Let the number of terms be *n*.

Then, $T_n = 181$

$\Rightarrow a + (n-1)d = 181$

$\Rightarrow 5 + (n-1) \times 8 = 181$

$\Rightarrow 8n = 184$

$\Rightarrow n = 23$

$\therefore$ Required sum $= \frac{n}{2}(a + l)$

$= \frac{23}{2}(5 + 181) = 23 \times 93 = 2139$

Hence, the required sum is 2139.

25.

Answer: (b) -320

Sol:

Here, $a = 10, d = (6 - 10) = -4$ *and* $n = 16$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$S_{16} = \frac{16}{2}\left[2 \times 10 + (16 - 1) \times (-4)\right]$

$\left[\because a = 10, d = -4 \text{ and } n = 16\right]$

$= 8 \times [20 - 60] = 8 \times (-40) = -320$

Hence, the sum of the first 16 terms of the given AP is -320.

26.

Answer: (c) 14

Sol:

Here, $a = 3$ and $d = (7 - 3) = 4$

Let the sum of *n* terms be 406.

Then, we have:

$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = 406$

$\Rightarrow \frac{n}{2}\left[2 \times 3 + (n-1) \times 4\right] = 406$

$\Rightarrow n[3+2n-2]=406$

$\Rightarrow 2n^2-28n+29n-406$

$\Rightarrow 2n^2+n-406=0$

$\Rightarrow 2n^2-28n+29n-406=0$

$\Rightarrow 2n(n-14)+29(n-14)=0$

$\Rightarrow (2n+29)(n-14)=0$

$\Rightarrow n=14$ $\quad$ ($\because n$ can't be a fraction)

Hence, 14 terms will make the sum 406.

27.

Answer: (b) 73

Sol:

$T_2=a+d=13 \quad ....(i)$

$T_5=a+4d=25 \quad ....(ii)$

On subtracting (i) from (ii), we get:

$\Rightarrow 3d=12$

$\Rightarrow d=4$

On putting the value of *d* in (i), we get:

$\Rightarrow a+4=13$

$\Rightarrow a=9$

Now, $T_{17}=a+16d=9+16\times 4=73$

Hence, the 17th term is 73.

28.

Answer: (a) 3

Sol:

$T_{10}=a+9d$

$T_{17}=a+16d$

Also, $a+16d=21+T_{10}$

$\Rightarrow a+16d=21+9d$

$\Rightarrow 7d=21$

$\Rightarrow d=3$

Hence, the common difference of the AP is 3.

29.

Answer: (b) 2
Sol:

$T_8 = a + 7d = 17 \quad ......(i)$

$T_{14} = a + 13d = 29 \quad ......(ii)$

On subtracting (i) from (ii), we get:

$\Rightarrow 6d = 12$

$\Rightarrow d = 12$

Hence, the common difference is 2.

30.

Answer: (d) 28
Sol:

$T_7 = a + 6d$

$\Rightarrow a + 6 \times (-4) = 4$

$\Rightarrow a = 4 + 24 = 28$

Hence, the first term us 28.