Exercise – 3A

1.

Sol:
On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 2x + 3y = 2

2x + 3y = 2

⇒3y = (2 – 2x)

⇒3y = 2(1 – x)

⇒y = $\frac{2(1-x)}{3}$ …(i)

Putting x = 1, we get y = 0
Putting x = -2, we get y = 2
Putting x = 4, we get y = -2

Thus, we have the following table for the equation 2x + 3y = 2

x	1	-2	4
y	0	2	-2

Now, plot the points A(1, 0), B(-2, 2) and C(4, -2) on the graph paper.
Join AB and AC to get the graph line BC. Extend it on both ways.
Thus, the line BC is the graph of 2x + 3y = 2.

Graph of x - 2y = 8

x – 2y = 8

⇒ 2y = (x – 8)

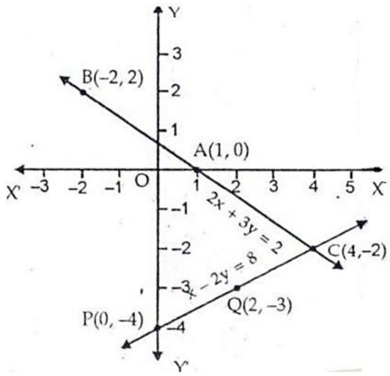
⇒ y = $\frac{x-8}{2}$ …(ii)

Putting x = 2, we get y = -3
Putting x = 4, we get y = -2
Putting x = 0, we get y = -4

Thus, we have the following table for the equation x – 2y = 8.

x	2	4	0
y	-3	-2	-4

Now, plot the points P(0, -4) and Q(2, -3). The point C(4, -2) has already been plotted. Join PQ and QC and extend it on both ways.
Thus, line PC is the graph of x – 2y = 8.



The two graph lines intersect at C(4, -2).

∴ x = 4 and y = -2 are the solutions of the given system of equations.

2.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 3x + 2y = 4

3x + 2y = 4

$\Rightarrow 2y = (4 - 3x)$

$\Rightarrow y = \frac{4 - 3x)}{2}$ …(i)

Putting x = 0, we get y = 2

Putting x = 2, we get y = -1

Putting x = -2, we get y = 5

Thus, we have the following table for the equation 3x + 2y = 4

x	0	2	-2
y	2	-1	5

Now, plot the points A(0, 2), B(2, -1) and C(-2, 5) on the graph paper.

Join AB and AC to get the graph line BC. Extend it on both ways.

Thus, BC is the graph of 3x + 2y = 4.

Graph of 2x - 3y = 7

2x – 3y = 7

$\Rightarrow 3y = (2x - 7)$

$\Rightarrow y = \frac{2x - 7}{3}$ …(ii)

Putting x = 2, we get y = -1

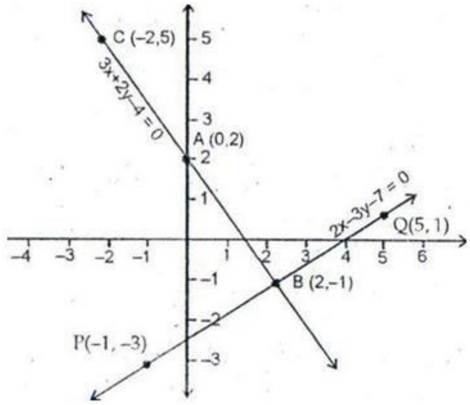
Putting x = -1, we get y = -3

Putting x = 5, we get y = 1

Thus, we have the following table for the equation 2x – 3y = 7.

x	2	-1	5
y	-1	-3	1

Now, plot the points P(-1, -3) and Q(5, 1). The point C(2, -1) has already been plotted. Join PB and QB and extend it on both ways.
Thus, line PQ is the graph of 2x – 3y = 7.



The two graph lines intersect at B(2, -1).

∴x = 2 and y = -1 are the solutions of the given system of equations.

3.

Sol:
On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x + 3y = 8

2x + 3y = 8

⇒3y = (8 – 2x)

⇒y = $\frac{8-2x}{3}$ …(i)

Putting x = 1, we get y = 2.
Putting x = -5, we get y = 6.
Putting x = 7, we get y = -2.
Thus, we have the following table for the equation 2x + 3y = 8.

x	1	-5	7
y	2	6	-2

Now, plot the points A(1, 2), B(5, -6) and C(7, -2) on the graph paper.
Join AB and AC to get the graph line BC. Extend it on both ways.
Thus, BC is the graph of 2x + 3y = 8.

Graph of x - 2y + 3 = 0

x – 2y + 3 = 0

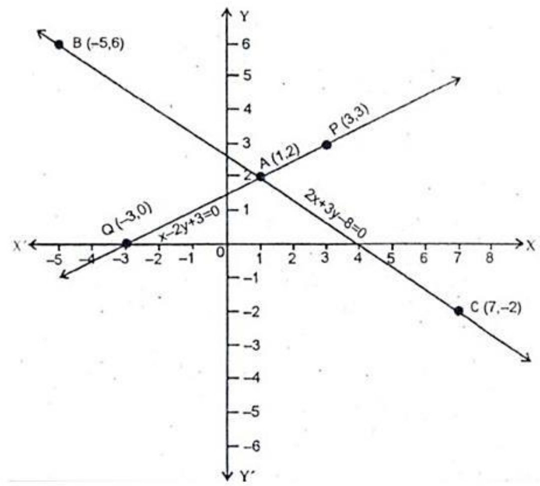
⇒ 2y = (x + 3)

⇒ y = $\frac{x+3}{2}$ …(ii)

Putting x = 1, we get y = 2.
Putting x = 3, we get y = 3.
Putting x = -3, we get y = 0.
Thus, we have the following table for the equation x – 2y + 3 = 0.

x	1	3	-3
y	2	3	0

Now, plot the points P (3, 3) and Q (-3, 0). The point A (1, 2) has already been plotted. Join AP and QA and extend it on both ways.
Thus, PQ is the graph of x – 2y + 3 = 0.



The two graph lines intersect at A (1, 2).

∴ x = 1 and y = 2.

4.

Sol:
On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x - 5y + 4 = 0

2x – 5y + 4 = 0

⇒5y = (2x + 4)

⇒y = $\frac{2x+4}{5}$ …(i)

Putting x = -2, we get y = 0.
Putting x = 3, we get y = 2.
Putting x = 8, we get y = 4.
Thus, we have the following table for the equation 2x - 5y + 4 = 0.

x	-2	3	8
y	0	2	4

Now, plot the points A (-2, 0), B (3, 2) and C(8, 4) on the graph paper.
Join AB and BC to get the graph line AC. Extend it on both ways.
Thus, AC is the graph of 2x - 5y + 4 = 0.

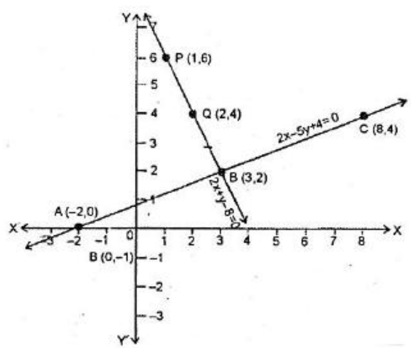
Graph of 2x + y - 8 = 0

2x + y - 8 = 0

$\Rightarrow$ y = (8 – 2x) …(ii)
Putting x = 1, we get y = 6.
Putting x = 3, we get y = 2.
Putting x = 2, we get y = 4.
Thus, we have the following table for the equation 2x + y - 8 = 0.

x	1	3	2
y	6	2	4

Now, plot the points P (1, 6) and Q (2, 4). The point B (3, 2) has already been plotted. Join PQ and QB and extend it on both ways.
Thus, PB is the graph of 2x + y - 8 = 0.



The two graph lines intersect at B (3, 2).

$\therefore$x = 3 and y = 2

5.

Sol:
The given equations are:
3x + 2y = 12 …..(i)

5x – 2y = 4 …..(ii)

From (i), write y in terms of x

y = $\frac{12 - 3x}{2}$ …..(iii)

Now, substitute different values of x in (iii) to get different values of y

For x = 0, y = $\frac{12 - 3x}{2} = \frac{12 - 0}{2}$ = 6

For x = 2, y = $\frac{12 - 3x}{2} = \frac{12 - 6}{2}$ = 3

For x = 4, y = $\frac{12 - 3x}{2} = \frac{12 - 12}{2}$ = 0

Thus, the table for the first equation (3x + 2y = 12) is

x	0	2	4
y	6	3	0

Now, plot the points A (0, 6), B(2, 3) and C(4, 0) on a graph paper and join A, B and C to get the graph of 3x + 2y = 12.

From (ii), write y in terms of x

y = $\frac{5x - 4}{2}$ ….(iv)

Now, substitute different values of x in (iv) to get different values of y

For x = 0, y = $\frac{5x - 4}{2} = \frac{0 - 4}{2}$ = -2

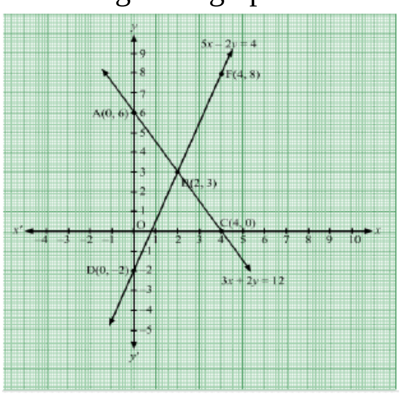
For x = 2, y = $\frac{5x - 4}{2} = \frac{10 - 4}{2}$ = 3

For x = 4, y = $\frac{5x - 4}{2} = \frac{20 - 4}{2}$ = 8

Thus, the table for the first equation (5x – 2y = 4) is

x	0	2	4
y	-2	3	8

Now, plot the points D (0, -2), E (2, 3) and F (4, 8) on the same graph paper and join D, E and F to get the graph of 5x – 2y = 4.



From the graph it is clear that, the given lines intersect at (2, 3).

Hence, the solution of the given system of equations is (2, 3).

6.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 3x + y + 1 = 0

3x + y + 1 = 0

⇒y = (-3x - 1) …(i)

Putting x = 0, we get y = -1.

Putting x = -1, we get y = 2.

Putting x = 1, we get y = -4.

Thus, we have the following table for the equation 3x + y + 1 = 0.

x	0	-1	1
y	-1	2	-4

Now, plot the points A(0, -1), B(-1, 2) and C(1, -4) on the graph paper.

Join AB and AC to get the graph line BC. Extend it on both ways.

Thus, BC is the graph of 3x + y + 1 = 0.

Graph of 2x - 3y + 8 = 0

2x - 3y + 8 = 0

⇒ 3y = (2x + 8)

∴ $y = \frac{2x+8}{3}$

Putting x = -1, we get y = 2.

Putting x = 2, we get y = 4.

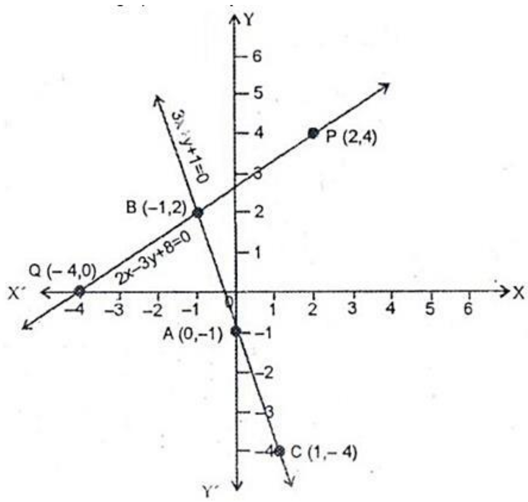
Putting x = -4, we get y = 0.

Thus, we have the following table for the equation 2x - 3y + 8 = 0.

x	-1	2	-4
y	2	4	0

Now, plot the points P(2, 4) and Q(-4, 0). The point B(-1, 2) has already been plotted. Join PB and BQ and extend it on both ways.

Thus, PQ is the graph of 2x + y - 8 = 0.



The two graph lines intersect at B (-1. 2).

∴x = -1 and y = 2

7.

Sol:

From the first equation, write y in terms of x

$y = -\left(\frac{5+2x}{3}\right)$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -1, y = - $\frac{5-2}{3}$ = -1

For x = 2, y = - $\frac{5+4}{3}$ = -3

For x = 5, y = - $\frac{5+10}{3}$ = -5

Thus, the table for the first equation (2x + 3y + 5 = 0) is

x	-1	2	5
y	-1	-3	-5

Now, plot the points A (-1, -1), B (2, -3) and C (5, -5) on a graph paper and join them to get the graph of 2x + 3y + 5 = 0.

From the second equation, write y in terms of x

$y = \frac{3x-12}{2}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = $\frac{0-12}{2}$ = -6

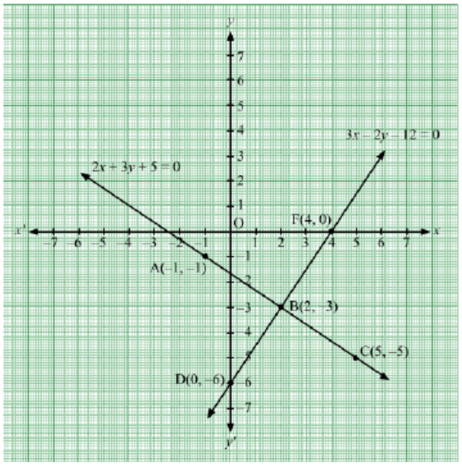
For x = 2, y = $\frac{6-12}{2}$ = -3

For x = 4, y = $\frac{12-12}{2}$ = 0

So, the table for the second equation (3x – 2y – 12 = 0) is

x	0	2	4
y	-6	-3	0

Now, plot the points D (0, -6), E (2, -3) and F (4, 0) on the same graph paper and join D, E and F to get the graph of 3x – 2y – 12 = 0.



From the graph it is clear that, the given lines intersect at (2, -3).
Hence, the solution of the given system of equation is (2, -3).

8.

Sol:
From the first equation, write y in terms of x

y = $\frac{2x+13}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -5, y = $\frac{-10+13}{3}$ = 1
For x = 1, y = $\frac{2+13}{3}$ = 5
For x = 4, y = $\frac{8+13}{3}$ = 7
Thus, the table for the first equation (2x - 3y + 13 = 0) is

x	-5	1	4
y	1	5	7

Now, plot the points A (-5, 1), B (1, 5) and C (4, 7) on a graph paper and join A, B and C to get the graph of 2x - 3y + 13 = 0.
From the second equation, write y in terms of x

y = $\frac{3x+12}{2}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -4, y = $\frac{-12+12}{2}$ = 0

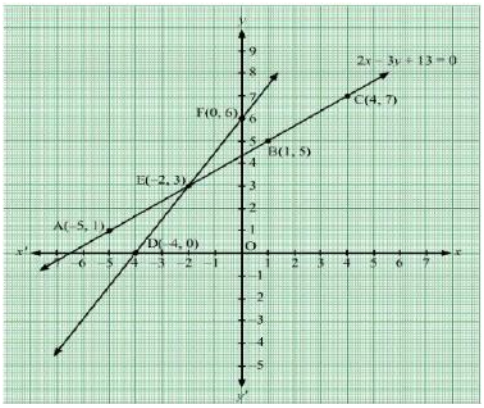
For x = -2, y = $\frac{-6+12}{2}$ = 3

For x = 0, y = $\frac{0+12}{2}$ = 6

So, the table for the second equation (3x – 2y + 12 = 0) is

x	-4	-2	0
y	0	3	6

Now, plot the points D (-4, 0), E (-2, 3) and F (0, 6) on the same graph paper and join D, E and F to get the graph of 3x – 2y + 12 = 0.



From the graph, it is clear that, the given lines intersect at (-2, 3).

Hence, the solution of the given system of equation is (-2, 3).

9.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x + 3y = 4

2x + 3y = 4

⇒3y = (4 – 2x)

∴ y = $\frac{4-2x}{3}$ …(i)

Putting x = -1, we get y = 2.

Putting x = 2, we get y = 0.

Putting x = 5, we get y = -2.

Thus, we have the following table for the equation 2x + 3y = 4.

x	-1	2	5
y	2	0	-2

Now, plot the points A (-1, 2), B (2, 0) and C (5, -2) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of 2x + 3y = 4.

Graph of 3x – y = -5

3x – y = -5

⇒ y = (3x + 5) ……..(ii)

Putting x = -1, we get y = 2.

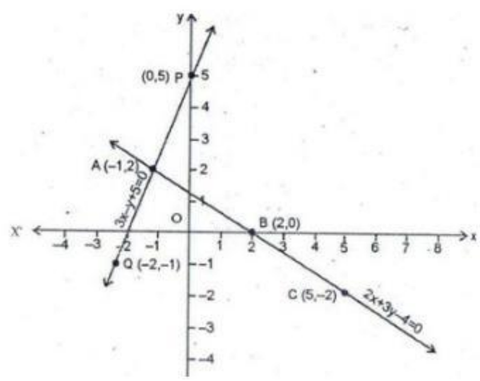
Putting x = 0, we get y = 5.

Putting x = -2, we get y = -1.

Thus, we have the following table for the equation 3x – y = -5.

x	-1	0	-2
y	2	5	-1

Now, plot the points P (0, 5) and Q (-2, -1). The point A (-1, 2) has already been plotted.
Join PA and QA and extend it on both ways.
Thus, PQ is the graph of 3x - y = - 5.



The two graph lines intersect at A (-1. 2).

∴x = -1 and y = 2 are the solutions of the given system of equations.

10.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x + 3y = 4

x + 2y + 2 = 0

⇒2y = (–2 – x)

∴ $y = \frac{-2-x}{2}$ …(i)

Putting x = -2, we get y = 0.

Putting x = 0, we get y = -1.

Putting x = 2, we get y = -2.

Thus, we have the following table for the equation x + 2y + 2 = 0.

x	-2	0	2
y	0	-1	-2

Now, plot the points A (-2, 0), B (0, -1) and C (2, -2) on the graph paper.
Join AB and BC to get the graph line AC. Extend it on both ways.
Thus, AC is the graph of x + 2y + 2 = 0.

Graph of 3x + 2y - 2 = 0

3x + 2y – 2 = 0

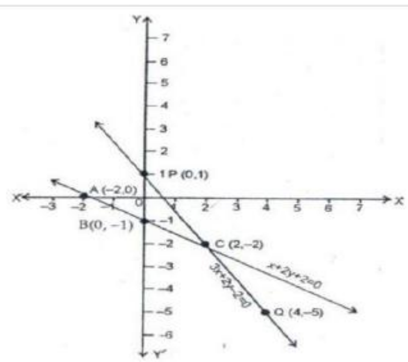
⇒ 2y = (2 – 3x)

∴ $y = \frac{2-3x}{2}$ ……..(ii)

Putting x = 0, we get y = 1.
Putting x = 2, we get y = -2.
Putting x = 4, we get y = -5.
Thus, we have the following table for the equation 3x + 2y – 2 = 0.

x	0	2	4
y	1	-2	-5

Now, plot the points P (0, 1) and Q(4, -5). The point C(2, -2) has already been plotted. Join PC and QC and extend it on both ways.
Thus, PQ is the graph of 3x + 2y – 2 = 0.



The two graph lines intersect at A(2, -2).

∴x = 2 and y = -2.

11.

Sol:
From the first equation, write y in terms of x
y = x + 3 …….(i)
Substitute different values of x in (i) to get different values of y
For x = -3, y = -3 + 3 = 0

For x = -1, y = -1 + 3 = 2
For x = 1, y = 1 + 3 = 4
Thus, the table for the first equation (x - y + 3 = 0) is

x	-3	-1	1
y	0	2	4

Now, plot the points A(-3, 0), B(-1, 2) and C(1, 4) on a graph paper and join A, B and C to get the graph of x - y + 3 = 0.
From the second equation, write y in terms of x

$y = \frac{4-2x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -4, $y = \frac{4+8}{3} = 4$

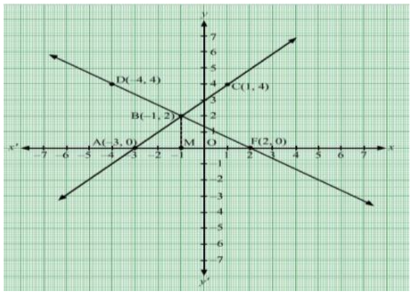
For x = -1, $y = \frac{4+12}{3} = 2$

For x = 2, $y = \frac{4-4}{3} = 0$

So, the table for the second equation (2x + 3y - 4 = 0) is

x	-4	-1	2
y	4	2	0

Now, plot the points D(-4, 4), E(-1, 2) and F(2, 0) on the same graph paper and join D, E and F to get the graph of 2x + 3y - 4 = 0.



From the graph, it is clear that, the given lines intersect at (-1, 2).
So, the solution of the given system of equation is (-1, 2).
The vertices of the triangle formed by the given lines and the x-axis are (-3, 0), (-1, 2) and (2, 0).
Now, draw a perpendicular from the intersection point E on the x-axis. So,

Area (ΔEAF) = $\frac{1}{2} \times AF \times EM$

$= \frac{1}{2} \times 5 \times 2$

= 5 sq. units

Hence, the vertices of the triangle formed by the given lines and the x-axis are (-3, 0), (-1, 2) and (2, 0) and its area is 5 sq. units.

12.

Sol:

From the first equation, write y in terms of x

y = $\frac{2x+4}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -2, y = $\frac{-4+4}{3}$ = 0

For x = 1, y = $\frac{2+4}{3}$ = 2

For x = 4, y = $\frac{8+4}{3}$ = 4

Thus, the table for the first equation (2x - 3y + 4 = 0) is

x	-2	1	4
y	0	2	4

Now, plot the points A(-2, 0), B(1, 2) and C(4, 4) on a graph paper and join A, B and C to get the graph of 2x - 3y + 4 = 0.

From the second equation, write y in terms of x

y = $\frac{5-x}{2}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -3, y = $\frac{5+3}{2}$ = 4

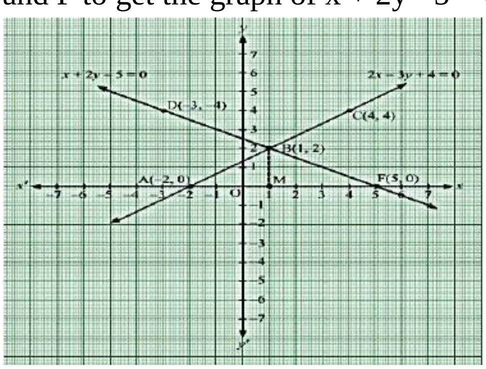
For x = 1, y = $\frac{5-1}{2}$ = 2

For x = 5, y = $\frac{5-5}{2}$ = 0

So, the table for the second equation (x + 2y - 5 = 0) is

x	-3	1	5
y	4	2	0

Now, plot the points D(-3, 4), B(1, 2) and F(5, 0) on the same graph paper and join D, E and F to get the graph of x + 2y - 5 = 0.



From the graph, it is clear that, the given lines intersect at (1, 2).

So, the solution of the given system of equation is (1, 2).

From the graph, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (1, 2) and (5, 0).

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Now, draw a perpendicular from the intersection point B on the x-axis. So,

Area (ΔBAF) = $\frac{1}{2} \times$ AF $\times$ BM

= $\frac{1}{2} \times 7 \times 2$

= 7 sq. units

Hence, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (1, 2) and (5, 0) and the area of the triangle is 7 sq. units.

13.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 4x - 3y + 4 = 0

4x - 3y + 4 = 0

$\Rightarrow$3y = (4x + 4)

$\therefore$ y = $\frac{4x+4}{3}$ …(i)

Putting x = -1, we get y = 0.

Putting x = 2, we get y = 4.

Putting x = 5, we get y = 8.

Thus, we have the following table for the equation 4x - 3y + 4 = 0.

x	-1	2	5
y	0	4	8

Now, plot the points A(-1, 0), B(2, 4) and C(5, 8) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of 4x - 3y + 4 = 0.

Graph of 4x + 3y - 20 = 0

4x + 3y - 20 = 0

$\Rightarrow$ 3y = (–4x + 20)

$\therefore$ y = $\frac{-4x+20}{3}$ ……..(ii)

Putting x = 2, we get y = 4.

Putting x = -1, we get y = 8.

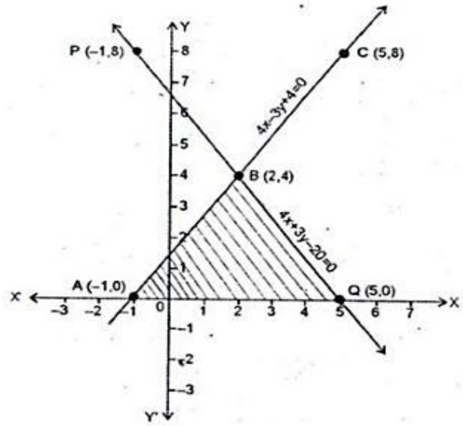
Putting x = 5, we get y = 0.

Thus, we have the following table for the equation 4x + 3y - 20 = 0.

x	2	-1	5
y	4	8	0

Now, plot the points P(1, -8) and Q(5, 0). The point B(2, 4) has already been plotted. Join PB and QB to get the graph line. Extend it on both ways.

Then, line PQ is the graph of the equation 4x + 3y - 20 = 0.



The two graph lines intersect at B(2, 4).

∴ The solution of the given system of equations is x = 2 and y = 4.

Clearly, the vertices of ΔABQ formed by these two lines and the x-axis are Q(5, 0), B(2, 4) and A(-1, 0).

Now, consider ΔABQ.

Here, height = 4 units and base (AQ) = 6 units

∴ Area ΔABQ = $\frac{1}{2}\times$ base $\times$ height sq. units

$= \frac{1}{2}\times 6\times 4$

= 12 sq. units.

14.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of x - y + 1 = 0

x - y + 1 = 0

⇒y = x + 1 ...(i)

Putting x = -1, we get y = 0.

Putting x = 1, we get y = 2.

Putting x = 2, we get y = 3.

Thus, we have the following table for the equation x - y + 1 = 0.

x	-1	1	2
y	0	2	3

Now, plot the points A(-1, 0), B(1, 2) and C(2, 3) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of x - y + 1 = 0.

Graph of 3x + 2y - 12 = 0

3x + 2y - 12 = 0

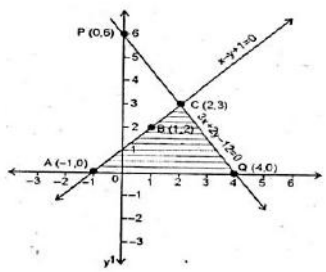
⇒ 2y = (–3x + 12)

∴ $y = \frac{-3x + 12}{2}$ ……..(ii)

Putting x = 0, we get y = 6.
Putting x = 2, we get y = 3.
Putting x = 4, we get y = 0.
Thus, we have the following table for the equation 3x + 2y - 12 = 0.

x	0	2	4
y	6	3	0

Now, plot the points P(0, 6) and Q(4, 0). The point B(2, 3) has already been plotted. Join PC and CQ to get the graph line PQ. Extend it on both ways.
Then, PQ is the graph of the equation 3x + 2y - 12 = 0.



The two graph lines intersect at C(2, 3).

∴ The solution of the given system of equations is x = 2 and y = 3.
Clearly, the vertices of ΔACQ formed by these two lines and the x-axis are Q(4, 0), C(2, 3) and A(-1, 0).
Now, consider ΔACQ.
Here, height = 3 units and base (AQ) = 5 units

∴ Area ΔACQ = $\frac{1}{2} \times$ base $\times$ height sq. units

$= \frac{1}{2} \times 5 \times 3$

= 7.5 sq. units.

15.

Sol:
From the first equation, write y in terms of x

$y = \frac{x + 2}{2}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -2, y = $\frac{-2+2}{2}$ = 0

For x = 2, y = $\frac{2+2}{2}$ = 2

For x = 4, y = $\frac{4+2}{2}$ = 3

Thus, the table for the first equation (x - 2y + 2 = 0) is

x	-2	2	4
y	0	2	3

Now, plot the points A(-2, 0), B(2, 2) and C(4, 3) on a graph paper and join A, B and C to get the graph of x - 2y + 2 = 0.

From the second equation, write y in terms of x

y = 6 – 2x   …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 1, y = 6 – 2 = 4

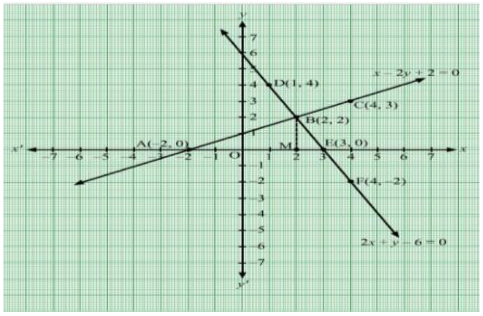
For x = 3, y = 0

For x = 4, y = 6 – 8 = -2

So, the table for the second equation (2x + y - 6 = 0) is

x	1	3	4
y	4	0	-2

Now, plot the points D(1, 4), E(3, 0) and F(4, -2) on the same graph paper and join D, E and F to get the graph of 2x + y - 6 = 0.



From the graph, it is clear that, the given lines intersect at (2, 2).

So, the solution of the given system of equation is (2, 2).

From the graph, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (2, 2) and (3, 0).

Now, draw a perpendicular from the intersection point B on the x-axis. So,

Area (ΔBAE) = $\frac{1}{2} \times AE \times BM$

= $\frac{1}{2} \times 5 \times 2$

= 5 sq. units

Hence, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (2, 2) and (3, 0) and the area of the triangle is 5 sq. units.

16.

Sol:

From the first equation, write y in terms of x

y = $\frac{2x + 6}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -3, y = $\frac{-6 + 6}{3}$ = 0

For x = 0, y = $\frac{0 + 6}{3}$ = 2

For x = 3, y = $\frac{6 + 6}{3}$ = 4

Thus, the table for the first equation (2x - 3y + 6 = 0) is

x	-3	0	3
y	0	2	4

Now, plot the points A(-3, 0), B(0, 2) and C(3, 4) on a graph paper and join A, B and C to get the graph of 2x - 3y + 6 = 0.

From the second equation, write y in terms of x

y = $\frac{18-2x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = $\frac{18-0}{3}$ = 6

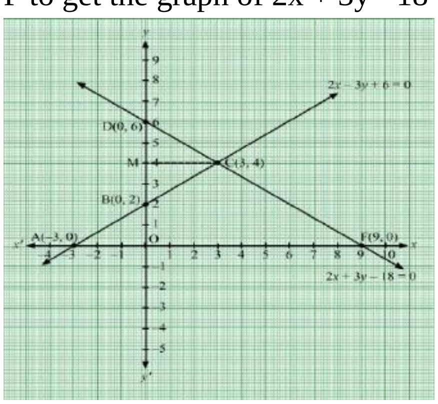
For x = 3, y = $\frac{18 - 6}{3}$ = 4

For x = 9, y = $\frac{18-18}{3}$= 0

So, the table for the second equation (2x + 3y - 18 = 0) is

x	0	3	9
y	6	4	0

Now, plot the points D(0, 6), E(3, 4) and F(9, 0) on the same graph paper and join D, E and F to get the graph of 2x + 3y - 18 = 0.



From the graph, it is clear that, the given lines intersect at (3, 4).

So, the solution of the given system of equation is (3, 4).
From the graph, the vertices of the triangle formed by the given lines and the y-axis are (0, 2), (0, 6) and (3, 4).
Now, draw a perpendicular from the intersection point E (or C) on the y-axis. So,
Area (ΔEDB) = $\frac{1}{2} \times BD \times EM$

$= \frac{1}{2} \times 4 \times 3$

=6 sq. units

Hence, the vertices of the triangle formed by the given lines and the y-axis are (0, 2), (0, 6) and (3, 4) and the area of the triangle is 6 sq. units.

17.

Sol:

From the first equation, write y in terms of x

$y = 4x - 4$ ....(i)

Substitute different values of x in (i) to get different values of y

For $x = 0$, $y = 0 - 4 = -4$

For $x = 1$, $y = 4 - 4 = 0$

For $x = 2$, $y = 8 - 4 = 4$

Thus, the table for the first equation ($4x - y - 4 = 0$) is

x	0	1	2
y	-4	0	4

Now, plot the points A(0, -4), B(1, 0) and C(2, 4) on a graph paper and join A, B and C to get the graph of $4x - y - 4 = 0$.

From the second equation, write y in terms of x

$y = \frac{14-3x}{2}$ ....(ii) $2y = 14 - 3x$ $-3x = 2y - 14$

Now, substitute different values of x in (ii) to get different values of y

For $x = 0$, $y = \frac{14-0}{2} = 7$

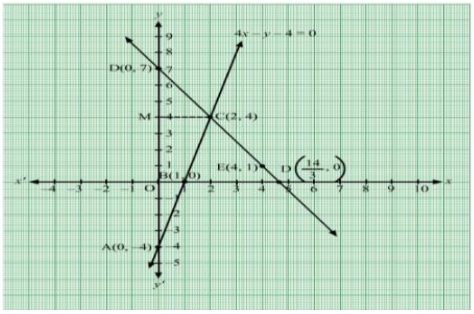
For $x = 4$, $y = \frac{14-12}{2} = 1$

For $x = \frac{14}{3}$, $y = \frac{14-14}{2} = 0$

So, the table for the second equation ($3x + 2y - 14 = 0$) is

x	0	4	$\frac{14}{3}$
y	7	1	0

Now, plot the points D(0, 7), E(4, 1) and F($\frac{14}{3}$, 0) on the same graph paper and join D, E and F to get the graph of $3x + 2y - 14 = 0$.



From the graph, it is clear that, the given lines intersect at (2, 4).
So, the solution of the given system of equation is (2, 4).
From the graph, the vertices of the triangle formed by the given lines and the y-axis are 0, 7), (0, -4) and (2, 4).
Now, draw a perpendicular from the intersection point C on the y-axis. So,

Area (ΔDAB) = $\frac{1}{2} \times DA \times CM$

$= \frac{1}{2} \times 11 \times 2$

= 11 sq. units

Hence, the vertices of the triangle formed by the given lines and the y-axis are (0, 7), (0, -4) and (2, 4) and the area of the triangle is 11 sq. units.

18.

Sol:

From the first equation, write y in terms of x
y = x – 5 …….(i)
Substitute different values of x in (i) to get different values of y
For x = 0, y = 0 – 5 = -5
For x = 2, y = 2 – 5 = -3
For x = 5, y = 5 – 5 = 0
Thus, the table for the first equation (x – y – 5 = 0) is

x	0	2	5
y	-5	-3	0

Now, plot the points A(0, -5), B(2, -3) and C(5, 0) on a graph paper and join A, B and C to get the graph of x – y – 5 = 0.
From the second equation, write y in terms of x

y = $\frac{15-3x}{5}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -5, y = $\frac{15 + 15}{5}$ = 6

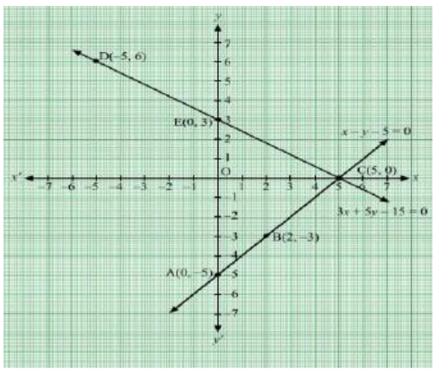
For x = 0, y = $\frac{15 - 0}{5}$ = 3

For x = 5, y = $\frac{15 - 15}{5}$ = 0

So, the table for the second equation (3x + 5y - 15 = 0) is

x	-5	0	5
y	6	3	0

Now, plot the points D(-5, 6), E(0, 3) and F(5, 0) on the same graph paper and join D, E and F to get the graph of 3x + 5y - 15 = 0.



From the graph, it is clear that, the given lines intersect at (5, 0).

So, the solution of the given system of equation is (5, 0).

From the graph, the vertices of the triangle formed by the given lines and the y-axis are 0, 3), (0, -5) and (5, 0).

Now, draw a perpendicular from the intersection point C on the y-axis. So,

Area (ΔCEA) = $\frac{1}{2} \times$ EA $\times$ OC

= $\frac{1}{2} \times 8 \times 5$

= 20 sq. units

Hence, the vertices of the triangle formed by the given lines and the y-axis are (0, 3), (0, -5) and (5, 0) and the area of the triangle is 20 sq. units.

19.

Sol:

From the first equation, write y in terms of x

y = $\frac{2x + 4}{5}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -2, y = $\frac{-4 + 4}{5}$ = 0

For x = 0, y = $\frac{\mathbf{0+4}}{\mathbf{5}} = \frac{4}{5}$

For x = 3, y = $\frac{\mathbf{6+4}}{\mathbf{5}} = 2$

Thus, the table for the first equation (2x – 5y + 4 = 0) is

x	-2	0	3
y	0	$\frac{4}{5}$	2

Now, plot the points A(-2, 0), B(0, $\frac{4}{5}$) and C(3, 2) on a graph paper and join A, B and C to get the graph of 2x – 5y + 4 = 0.

From the second equation, write y in terms of x

y = 8 – 2x ……..(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = 8 – 0 = 8

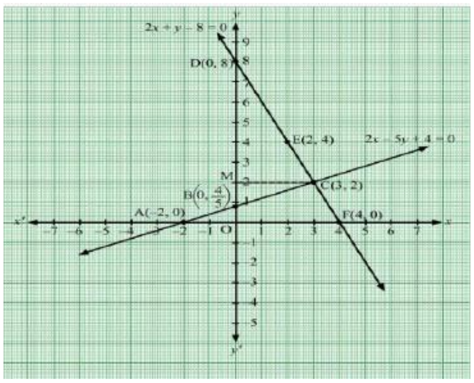
For x = 2, y = 8 – 4 = 3

For x = 4, y = 8 – 8 = 0

So, the table for the second equation (2x – 5y + 4 = 0) is

x	0	2	4
y	8	4	0

Now, plot the points D(0, 8), E(2, 4) and F(4, 0) on the same graph paper and join D, E and F to get the graph of 2x + y - 8 = 0.



From the graph, it is clear that, the given lines intersect at (3, 2).

So, the solution of the given system of equation is (3, 2).

The vertices of the triangle formed by the system of equations and y-axis are (0, 8), (0, $\frac{4}{5}$) and (3, 2).

Draw a perpendicular from point C on the y-axis. So,

Area (ΔDBC) = $\frac{\mathbf{1}}{\mathbf{2}} \times \text{DB} \times \text{CM}$

$= \frac{\mathbf{1}}{\mathbf{2}} \times \left(\mathbf{8} - \frac{\mathbf{4}}{\mathbf{5}}\right) \times 3$

$= \frac{\mathbf{54}}{\mathbf{5}}$ sq. units

Hence, the vertices of the triangle are (0, 8), (0, $\frac{\mathbf{4}}{\mathbf{5}}$) and (3, 2) and its area is $\frac{\mathbf{54}}{\mathbf{5}}$ sq. units.

20.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 5x - y = 7

5x - y = 7

$\Rightarrow y = (5x - 7)$ …(i)

Putting x = 0, we get y = -7.

Putting x = 1, we get y = -2.

Putting x = 2, we get y = 3.

Thus, we have the following table for the equation 5x - y = 7.

x	0	1	2
y	-7	-2	3

Now, plot the points A(0, -7), B(1, -2) and C(2, 3) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of 5x - y = 7.

Graph of x - y + 1 = 0

x - y + 1 = 0

$\Rightarrow y = x + 1$ ……..(ii)

Putting x = 0, we get y = 1.

Putting x = 1, we get y = 2.

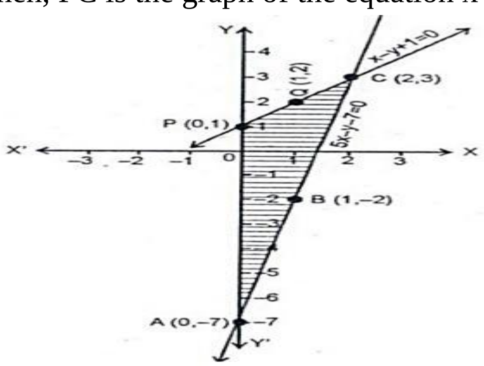
Putting x = 2, we get y = 3.

Thus, we have the following table for the equation x - y + 1 = 0.

x	0	1	2
y	1	2	3

Now, plot the points P(0, 1) and Q(1, 2). The point C(2, 3) has already been plotted. Join PQ and QC to get the graph line PC. Extend it on both ways.

Then, PC is the graph of the equation x - y + 1 = 0.



The two graph lines intersect at C(2, 3).

∴ The solution of the given system of equations is x = 2 and y = 3.

Clearly, the vertices of ΔAPC formed by these two lines and the y-axis are P(0, 1), C(2, 3) and A(0, -7).

Now, consider ΔAPC.

Here, height = 2 units and base (AP) = 8 units

∴ Area ΔAPC = $\frac{1}{2} \times$ base $\times$ height sq. units

$= \frac{1}{2} \times 8 \times 2$

= 8 sq. units.

21.

Sol:

From the first equation, write y in terms of x

$y = \frac{2x - 12}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = 0, $y = \frac{0-12}{3} = -4$

For x = 3, $y = \frac{6 - 12}{3} = -2$

For x = 6, $y = \frac{12-12}{3} = 0$

Thus, the table for the first equation (2x – 3y = 12) is

x	0	3	6
y	-4	-2	0

Now, plot the points A(0, -4), B(3,-2) and C(6, 0) on a graph paper and join A, B and C to get the graph of 2x – 3y = 12.

From the second equation, write y in terms of x

$y = \frac{6 - x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, $y = \frac{6 - 0}{3} = 2$

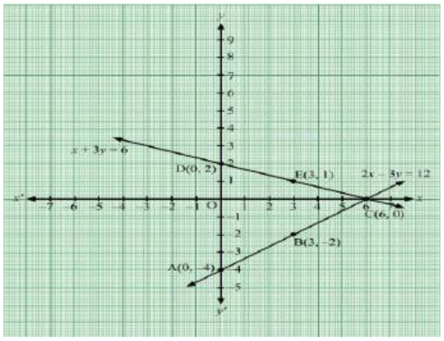
For x = 3, $y = \frac{6 - 3}{3} = 1$

For x = 6, $y = \frac{6 - 6}{3} = 0$

So, the table for the second equation (x + 3y = 6) is

x	0	3	6
y	2	1	0

Now, plot the points D(0, 2), E(3, 1) and F(6, 0) on the same graph paper and join D, E and F to get the graph of x + 3y = 6.



From the graph, it is clear that, the given lines intersect at (6, 0).
So, the solution of the given system of equation is (6, 0).
The vertices of the triangle formed by the system of equations and y-axis are (0, 2), (6, 0) and (0, -4).

Area (ΔDAC) = $\frac{1}{2} \times DA \times OC$

$= \frac{1}{2} \times 6 \times 6$

= 18 sq. units

Hence, the vertices of the triangle are (0, 2), (6, 0) and (0, -4) and its area is 18 sq. units.

22.

Sol:
From the first equation, write y in terms of x

$y = \frac{6 - 2x}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -3, y = $\frac{6+6}{3}$ = 4

For x = 3, y = $\frac{6-6}{3}$= 0

For x = 6, y = $\frac{6-12}{3}$ = -2

Thus, the table for the first equation (2x + 3y = 6) is

x	-3	3	6
y	4	0	-2

Now, plot the points A(-3, 4), B(3, 0) and C(6, -2) on a graph paper and join A, B and C to get the graph of 2x + 3y = 6.
From the second equation, write y in terms of x

$y = \frac{12 - 4x}{6}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -6, y = $\frac{12+24}{6}$ = 6

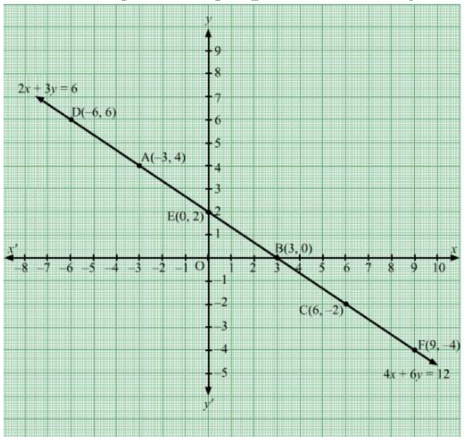
For x = 0, y = $\frac{12-0}{6}$ = 2

For x = 9, y = $\frac{12-36}{6}$ = -4

So, the table for the second equation (4x + 6y = 12) is

x	-6	0	9
y	6	2	-4

Now, plot the points D(-6, 6), E(0, 2) and F(9, -4) on the same graph paper and join D, E and F to get the graph of 4x + 6y = 12.



From the graph, it is clear that, the given lines coincidence with each other.
Hence, the solution of the given system of equations has infinitely many solutions.

23.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 3x - y = 5

3x - y = 5

⇒y = 3x - 5 …(i)

Putting x = 1, we get y = -2

Putting x = 0, we get y = -5

Putting x = 2, we get y = 1

Thus, we have the following table for the equation 3x - y = 5

x	1	0	2
y	-2	-5	1

Now, plot the points A(1, -2), B(0, -5) and C(2, 1) on the graph paper.

Join AB and AC to get the graph line BC. Extend it on both ways.

Thus, the line BC is the graph of 3x - y = 5.

Graph of 6x - 2y = 10

6x - 2y = 10

⇒ 2y = (6x – 10)

⇒ y = $\frac{6x-10}{2}$ …(ii)

Putting x = 0, we get y = -5

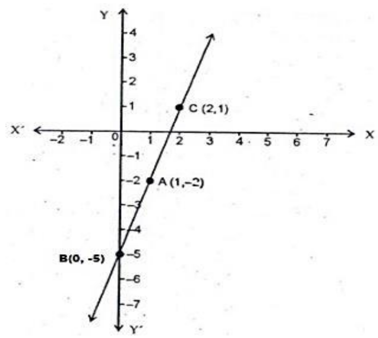
Putting x = 1, we get y = -2

Putting x = 2, we get y = 1

Thus, we have the following table for the equation 6x - 2y = 10.

x	0	1	2
y	-5	-2	1

These are the same points as obtained for the graph line of equation (i).



It is clear from the graph that these two lines coincide.

Hence, the given system of equations has infinitely many solutions.

24.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 2x + y = 6

2x + y = 6

⇒y = (6 – 2x) …(i)

Putting x = 3, we get y = 0

Putting x = 1, we get y = 4

Putting x = 2, we get y = 2

Thus, we have the following table for the equation 2x + y = 6

x	3	1	2
y	0	4	2

Now, plot the points A(3, 0), B(1, 4) and C(2, 2) on the graph paper.

Join AC and CB to get the graph line AB. Extend it on both ways.

Thus, the line AB is the graph of 2x + y = 6.

Graph of 6x + 3y = 18

6x + 3y = 18

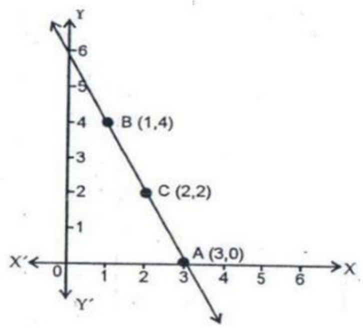
⇒ 3y = (18 - 6x)

⇒ $y = \frac{18 - 6x}{3}$ …(ii)

Putting x = 3, we get y = 0
Putting x = 1, we get y = 4
Putting x = 2, we get y = 2
Thus, we have the following table for the equation 6x + 3y = 18.

x	3	1	2
y	0	4	2

These are the same points as obtained for the graph line of equation (i).



It is clear from the graph that these two lines coincide.
Hence, the given system of equations has an infinite number of solutions.

25.

Sol:
From the first equation, write y in terms of x

$y = \frac{x - 5}{2}$ …….(i)

Substitute different values of x in (i) to get different values of y
For x = -5, $y = \frac{-5-5}{2} = -5$

For x = 1, $y = \frac{1 - 5}{2} = -2$

For x = 3, $y = \frac{3 - 5}{2} = -1$

Thus, the table for the first equation (x - 2y = 5) is

x	-5	1	3
y	-5	-2	-1

Now, plot the points A(-5, -5), B(1, -2) and C(3, -1) on a graph paper and join A, B and C to get the graph of x - 2y = 5.
From the second equation, write y in terms of x

y = $\frac{3x - 15}{6}$ ....(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -3, y = $\frac{-9 - 15}{6}$ = -4

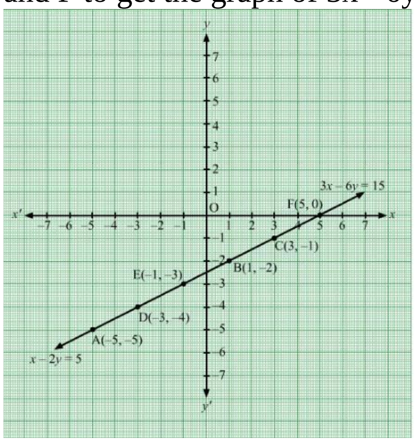
For x = -1, y = $\frac{-3 - 15}{6}$ = -3

For x = 5, y = $\frac{15 - 15}{6}$ = 0

So, the table for the second equation (3x - 6y = 15) is

x	-3	-1	5
y	-4	-3	0

Now, plot the points D(-3, -4), E(-1, -3) and F(5, 0) on the same graph paper and join D, E and F to get the graph of 3x - 6y = 15.



From the graph, it is clear that, the given lines coincide with each other.

Hence, the solution of the given system of equations has infinitely many solutions.

26.

Sol:

From the first equation, write y in terms of x

y = $\frac{x - 6}{2}$ ....(i)

Substitute different values of x in (i) to get different values of y

For x = -2, y = $\frac{-2-6}{2}$ = -4

For x = 0, y = $\frac{0 - 6}{2}$= -3

For x = 2, y = $\frac{2 - 6}{2}$ = -2

Thus, the table for the first equation (x - 2y = 5) is

x	-2	0	2
y	-4	-3	-2

Now, plot the points A(-2, -4), B(0, -3) and C(2, -2) on a graph paper and join A, B and C to get the graph of x - 2y = 6.

From the second equation, write y in terms of x

y = $\frac{1}{2}x$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -4, y = $\frac{-4}{2}$ = -2

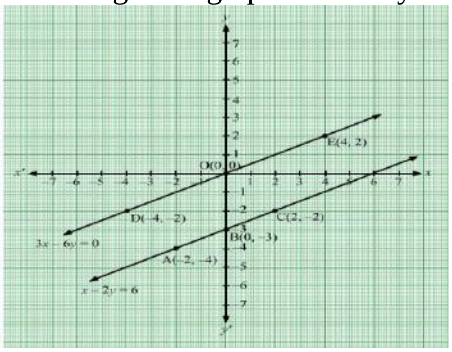
For x = 0, y = $\frac{0}{2}$ = 0

For x = 4, y = $\frac{4}{2}$ = 2

So, the table for the second equation (3x - 6y = 0) is

x	-4	0	4
y	-2	0	2

Now, plot the points D(-4, -2), O(0, 0) and E(4, 2) on the same graph paper and join D, E and F to get the graph of 3x - 6y = 0.



From the graph, it is clear that, the given lines do not intersect at all when produced.
Hence, the system of equations has no solution and therefore is inconsistent.

27.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x +3y = 4

2x + 3y = 4

⇒3y = (–2x + 4) …(i)

Putting x = 2, we get y = 0

Putting x = -1, we get y = 2

Putting x = -4, we get y = 4

Thus, we have the following table for the equation 2x + 3y = 4.

x	2	-1	-4
y	0	2	4

Now, plot the points A(2, 0), B(-1, 2) and C(-4, 4) on the graph paper.
Join AB and BC to get the graph line AC. Extend it on both ways.
Thus, the line AC is the graph of 2x + 3y = 4.

Graph of 4x + 6y = 12

4x + 6y = 12

⇒ 6y = (-4x + 12)

⇒ y = $\frac{-4x+12}{6}$ …(ii)

Putting x = 3, we get y = 0

Putting x = 0, we get y = 2

Putting x = 6, we get y = -2

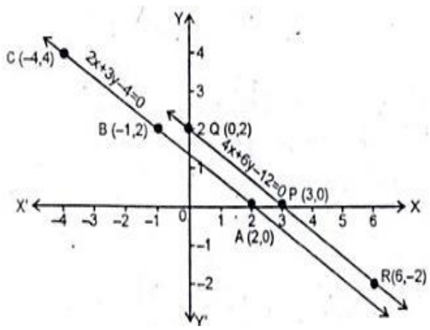
Thus, we have the following table for the equation 4x + 6y = 12.

x	3	0	6
y	0	2	-2

Now, on the same graph, plot the points A(3, 0), B(0, 2) and C(6, -2).

Join PQ and PR to get the graph line QR. Extend it on both ways.

Thus, QR is the graph of the equation 4x + 6y = 12.



It is clear from the graph that these two lines are parallel and do not intersect when produced.

Hence, the given system of equations is inconsistent.

28.

Sol:

From the first equation, write y in terms of x

y = 6 – 2x …….(i)

Substitute different values of x in (i) to get different values of y

For x = 0, y = 6 – 0 = 6

For x = 2, y = 6 – 4 = 2

For x = 4, y = 6 – 8 = -2

Thus, the table for the first equation (2x + y = 6) is

x	0	2	4
y	6	2	-2

Now, plot the points A(0, 6), B(2, 2) and C(4, -2) on a graph paper and join A, B and C to get the graph of 2x + y = 6.

From the second equation, write y in terms of x

y = $\frac{20-6x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = $\frac{20-0}{3} = \frac{20}{3}$

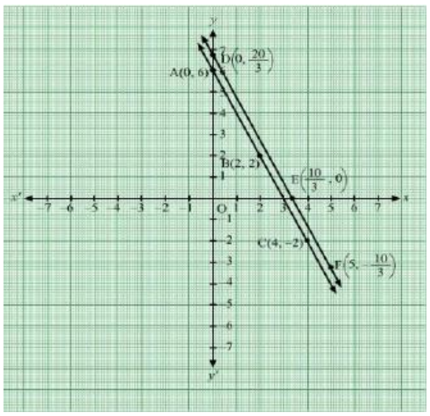
For x = $\frac{10}{3}$, y = $\frac{20-20}{3}$ = 0

For x = 5, y = $\frac{20-30}{3} = -\frac{10}{3}$

So, the table for the second equation (6x + 3y = 20) is

x	0	$\frac{10}{3}$	5
y	$\frac{20}{3}$	0	$-\frac{10}{3}$

Now, plot the points D(0, $\frac{20}{3}$), O($\frac{10}{3}$, 0) and E(5, $-\frac{10}{3}$) on the same graph paper and join D, E and F to get the graph of 6x + 3y = 20.



From the graph, it is clear that, the given lines do not intersect at all when produced. Hence, the system of equations has no solution and therefore is inconsistent.

29.

Sol:

From the first equation, write y in terms of x

y = 2 – 2x ....(i)

Substitute different values of x in (i) to get different values of y

For x = 0, y = 2 – 0 = 2

For x = 1, y = 2 – 2 = 0

For x = 2, y = 2 – 4 = -2

Thus, the table for the first equation (2x + y = 2) is

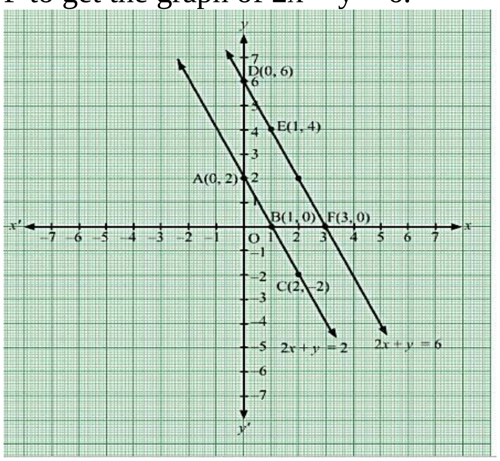
x	0	1	2
y	2	0	-2

Now, plot the points A(0, 2), B(1, 0) and C(2, -2) on a graph paper and join A, B and C to get the graph of 2x + y = 2.

From the second equation, write y in terms of x
y = 6 – 2x ……..(ii)
Now, substitute different values of x in (ii) to get different values of y
For x = 0, y = 6 – 0 = 6
For x = 1, y = 6 – 2 = 4
For x = 3, y = 6 – 6 = 0
So, the table for the second equation (2x + y = 6) is

x	0	1	3
y	6	4	0

Now, plot the points D(0,6), E(1, 4) and F(3,0) on the same graph paper and join D, E and F to get the graph of 2x + y = 6.



From the graph, it is clear that, the given lines do not intersect at all when produced. So, these lines are parallel to each other and therefore, the quadrilateral DABF is a trapezium.
The vertices of the required trapezium are D(0, 6), A (0, 2), B(1, 0) and F(3, 0).
Now,
Area(Trapezium DABF) = Area (ΔDOF) – Area(ΔAOB)

$= \frac{1}{2} \times 3 \times 6 - \frac{1}{2} \times 1 \times 2$

= 9 – 1

= 8 sq. units

Hence, the area of the required trapezium is 8 sq. units.

Exercise – 3B

1.

Sol:
The given system of equation is:

x + y = 3…….(i)
4x – 3y = 26 ……(ii)
On multiplying (i) by 3, we get:

$3x + 3y = 9$ ….(iii)
On adding (ii) and (iii), we get:
$7x = 35$
$\Rightarrow x = 5$
On substituting the value of x = 5 in (i), we get:
$5 + y = 3$
$\Rightarrow y = (3 – 5) = -2$
Hence, the solution is x = 5 and y = -2

2.

Sol:
The given system of equations is
$x – y = 3$ …..(i)
$\frac{x}{3} + \frac{y}{2} = 6$ ……(ii)
From (i), write y in terms of x to get
$y = x – 3$
Substituting y = x – 3 in (ii), we get
$\frac{x}{3} + \frac{x-3}{2} = 6$
$\Rightarrow 2x + 3(x – 3) = 36$
$\Rightarrow 2x + 3x – 9 = 36$
$\Rightarrow x = \frac{45}{5} = 9$
Now, substituting x = 9 in (i), we have
$9 – y = 3$
$\Rightarrow y = 9 – 3 = 6$
Hence, x = 9 and y = 6.

3.

Sol:
The given system of equation is:
$2x + 3y = 0$ ……(i)
$3x + 4y = 5$ ……(ii)
On multiplying (i) by 4 and (ii) by 3, we get:
$8x + 12y = 0$ ……(iii)
$9x + 12y = 15$ …….(iv)
On subtracting (iii) from (iv) we get:

x = 15
On substituting the value of x = 15 in (i), we get:
30 + 3y = 0

$\Rightarrow$3y = -30

$\Rightarrow$y = -10
Hence, the solution is x = 15 and y = -10.

4.

Sol:
The given system of equation is:
2x - 3y = 13 ……(i)
7x - 2y = 20 ……(ii)
On multiplying (i) by 2 and (ii) by 3, we get:
4x - 6y = 26 ……(iii)
21x - 6y = 60 …….(iv)
On subtracting (iii) from (iv) we get:
17x = (60 – 26) = 34

$\Rightarrow$x = 2
On substituting the value of x = 2 in (i), we get:
4 – 3y = 13

$\Rightarrow$3y = (4 – 13) = -9

$\Rightarrow$y = -3
Hence, the solution is x = 2 and y = -3.

5.

Sol:
The given system of equation is:
3x - 5y - 19 = 0 ……(i)
-7x + 3y + 1 = 0 ……(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
9x - 15y = 57 ……(iii)
-35x + 15y = -5 …….(iv)
On subtracting (iii) from (iv) we get:
-26x = (57 – 5) = 52

$\Rightarrow$x = -2
On substituting the value of x = -2 in (i), we get:

$-6 - 5y - 19 = 0$

$\Rightarrow 5y = (-6 - 19) = -25$

$\Rightarrow y = -5$

Hence, the solution is x = -2 and y = -5.

6.

Sol:

The given system of equation is:

$2x - y + 3 = 0$.......(i)

$3x - 7y + 10 = 0$ ......(ii)

From (i), write y in terms of x to get

$y=2x + 3$

Substituting $y = 2x + 3$ in (ii), we get

$3x - 7(2x + 3) + 10 = 0$

$\Rightarrow 3x - 14x - 21 + 10 = 0$

$\Rightarrow -7x = 21 - 10 = 11$

$x = -\frac{11}{7}$

Now substituting $x = -\frac{11}{7}$ in (i), we have

$-\frac{22}{7} - y + 3 = 0$

$y = 3 - \frac{22}{7} = -\frac{1}{7}$

Hence, $x = -\frac{11}{7}$ and $y = -\frac{1}{7}$.

7.

Sol:

The given system of equation can be written as:

$9x - 2y = 108$ ......(i)

$3x + 7y = 105$ ......(ii)

On multiplying (i) by 7 and (ii) by 2, we get:

$63x + 6x = 108 \times 7 + 105 \times 2$

$\Rightarrow 69x = 966$

$\Rightarrow x = \frac{966}{69} = 14$

Now, substituting x = 14 in (i), we get:

$9 \times 14 - 2y = 108$

$\Rightarrow 2y = 126 - 108$

$\Rightarrow y = \frac{18}{2} = 9$

Hence, x = 14 and y = 9.

8.

Sol:

The given equations are:

$\frac{x}{3} + \frac{y}{4} = 11$

$\Rightarrow 4x + 3y = 132$ …….(i)

and $\frac{5x}{6} - \frac{y}{3} + 7 = 0$

$\Rightarrow 5x – 2y = -42$ ……..(ii)

On multiplying (i) by 2 and (ii) by 3, we get:

$8x + 6y = 264$ …...(iii)

$15x – 6y = -126$ ….(iv)

On adding (iii) and (iv), we get:

$23x = 138$

$\Rightarrow x = 6$

On substituting x = 6 in (i), we get:

$24 + 3y = 132$

$\Rightarrow 3y = (132 – 24) = 108$

$\Rightarrow y = 36$

Hence, the solution is x = 6 and y = 36.

9.

Sol:

The given system of equation is:

$4x - 3y = 8$ ……(i)

$6x - y = \frac{29}{3}$ ……(ii)

On multiplying (ii) by 3, we get:

$18x – 3y = 29$ ….(iii)

On subtracting (iii) from (i) we get:

$-14x = -21$

$x = \frac{21}{14} = \frac{3}{2}$

Now, substituting the value of $x = \frac{3}{2}$ in (i), we get:

$4 \times \frac{3}{2} - 3y = 8$

$\Rightarrow 6 - 3y = 8$

$\Rightarrow 3y = 6 - 8 = -2$

$y = \frac{-2}{3}$

Hence, the solution $x = \frac{3}{2}$ and $y = \frac{-2}{3}$.

10.

Sol:

The given equations are:

$2x - \frac{3y}{4} = 3$ …..(i)

$5x = 2y + 7$ ……..(ii)

On multiplying (i) by 2 and (ii) by $\frac{3}{4}$, we get:

$4x - \frac{3}{2}y = 6$ …...(iii)

$\frac{15}{4}x = \frac{3}{2}y + \frac{21}{4}$….(iv)

On subtracting (iii) and (iv), we get:

$-\frac{1}{4}x = -\frac{3}{4}$

$\Rightarrow x = 3$

On substituting x = 3 in (i), we get:

$2 \times 3 - \frac{3y}{4} = 3$

$\Rightarrow \frac{3y}{4} = (6 - 3) = 3$

$\Rightarrow y = \frac{3 \times 4}{3} = 4$

Hence, the solution is x = 3 and y = 4.

11.

Sol:

The given equations are:

$2x - 5y = \frac{8}{3}$ …..(i)

$3x - 2y = \frac{5}{6}$ ……..(ii)

On multiplying (i) by 2 and (ii) by 5, we get:

$4x - 10y = \frac{16}{3}$ …...(iii)

15x – 10y = $\frac{25}{6}$……(iv)
On adding (iii) and (iv), we get:
19x = $\frac{57}{6}$
$\Rightarrow$x = $\frac{57}{6 \times 19} = \frac{3}{6} = \frac{1}{2}$
On substituting x = $\frac{1}{2}$ in (i), we get:
$2 \times \frac{1}{2} + 5y = \frac{8}{3}$
$\Rightarrow 5y = \left(\frac{8}{3} - 1\right) = \frac{5}{3}$
$\Rightarrow$y = $\frac{5}{3 \times 5} = \frac{1}{3}$
Hence, the solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

12.

Sol:
The given equations are:
$\frac{7-4x}{3}$ = y
$\Rightarrow$ 4x + 3y = 7 ……..(i)
and 2x + 3y + 1 = 0
$\Rightarrow$2x + 3y = -1 ……….(ii)
On subtracting (ii) from (i), we get:
2x = 8
$\Rightarrow$x = 4
On substituting x = 4 in (i), we get:
16x + 3y = 7
$\Rightarrow$3y = (7 – 16) = -9
$\Rightarrow$y = -3
Hence, the solution is x = 4 and y = -3.

13.

Sol:
The given system of equations is
0.4x + 0.3y = 1.7 …….(i)
0.7x – 0.2y = 0.8 …….(ii)

Multiplying (i) by 0.2 and (ii) by 0.3 and adding them, we get

0.8x + 2.1x = 3.4 + 2.4

$\Rightarrow 2.9x = 5.8$

$\Rightarrow x = \frac{5.8}{2.9} = 2$

Now, substituting x = 2 in (i), we have

$0.4 \times 2 + 0.3y = 1.7$

$\Rightarrow 0.3y = 1.7 - 0.8$

$\Rightarrow y = \frac{0.9}{0.3} = 3$

Hence, x = 2 and y = 3.

14.

Sol:

The given system of equations is

0.3x + 0.5y = 0.5 …….(i)

0.5x + 0.7y = 0.74 …….(ii)

Multiplying (i) by 5 and (ii) by 3 and subtracting (ii) from (i), we get

2.5y - 2.1y = 2.5 - 2.2

$\Rightarrow 0.4y = 0.28$

$\Rightarrow y = \frac{0.28}{0.4} = 0.7$

Now, substituting y = 0.7 in (i), we have

$0.3x + 0.5 \times 0.7 = 0.5$

$\Rightarrow 0.3x = 0.50 - 0.35 = 0.15$

$\Rightarrow x = \frac{0.15}{0.3} = 0.5$

Hence, x = 0.5 and y = 0.7.

15.

Sol:

The given equations are:

7(y + 3) – 2(x + 2) = 14

$\Rightarrow$ 7y + 21 – 2x – 4 = 14

$\Rightarrow$-2x + 7y = -3 ……..(i)

and 4(y – 2) + 3(x – 3) = 2

$\Rightarrow$4y – 8 + 3x – 9 = 2

$\Rightarrow 3x + 4y = 19$ ……….(ii)
On multiplying (i) by 4 and (ii) by 7, we get:
$-8x + 28y = -12$ ……(iii)
$21x + 28y = 133$ ……(iv)
On subtracting (iii) from (iv), we get:
$29x = 145$

$\Rightarrow x = 5$
On substituting x = 5 in (i), we get:
$-10 + 7y = -3$

$\Rightarrow 7y = (-3 + 10) = 7$

$\Rightarrow y = 1$
Hence, the solution is x = 5 and y = 1.

16.

Sol:
The given equations are:
$6x + 5y = 7x + 3y + 1 = 2(x + 6y – 1)$

$\Rightarrow 6x + 5y = 2(x + 6y – 1)$

$\Rightarrow 6x + 5y = 2x + 12y – 2$

$\Rightarrow 6x – 2x + 5y – 12y = -2$

$\Rightarrow 4x – 7y = -2$ …….(i)
and $7x + 3y + 1 = 2(x + 6y – 1)$

$\Rightarrow 7x + 3y + 1 = 2x + 12y – 2$

$\Rightarrow 7x – 2x + 3y – 12y = -2 – 1$

$\Rightarrow 5x – 9y = -3$ …….(ii)
On multiplying (i) by 9 and (ii) by 7, we get:
$36x - 63y = -18$ ……(iii)
$35x - 63y = -21$ ……(iv)
On subtracting (iv) from (iii), we get:
$x = (-18 + 21) = 3$
On substituting x = 3 in (i), we get:
$12 - 7y = -2$

$\Rightarrow 7y = (2 + 12) = 14$

$\Rightarrow y = 2$
Hence, the solution is x = 3 and y = 2.

17.

Sol:
The given equations are:
$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$
i.e., $\frac{x+y-8}{2} = \frac{3x+y-12}{11}$
By cross multiplication, we get:
11x + 11y – 88 = 6x + 2y – 24
⇒11x – 6x + 11y – 2y = -24 + 88
⇒5x + 9y = 64 …….(i)
and $\frac{x+2y-14}{3} = \frac{3x+y-12}{11}$
⇒11x + 22y – 154 = 9x + 3y – 36
⇒11x – 9x + 22y – 3y = -36 + 154
⇒2x + 19y = 118 …….(ii)
On multiplying (i) by 19 and (ii) by 9, we get:
95x + 171y = 1216 ……(iii)
18x + 171y = 1062 ……(iv)
On subtracting (iv) from (iii), we get:
77x = 154
⇒x = 2
On substituting x = 2 in (i), we get:
10 + 9y = 64
⇒9y = (64 – 10) = 54
⇒y = 6
Hence, the solution is x = 2 and y = 6.

18.

Sol:
The given equations are:
$\frac{5}{x}$ + 6y = 13 ……..(i)
$\frac{3}{x}$ + 4y = 7 ……..(ii)
Putting $\frac{1}{x}$ = u, we get:

5u + 6y = 13 …….(iii)
3u + 4y = 7 ……(iv)
On multiplying (iii) by 4 and (iv) by 6, we get:
20u + 24y = 52 ……..(v)
18u + 24y = 42 ……..(vi)
On subtracting (vi) from (v), we get:

2u = 10 ⇒ u = 5
$\Rightarrow \frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$
On substituting x = $\frac{1}{5}$ in (i), we get:
$\frac{5}{1/3} + 6y = 13$
25 + 6y = 13
6y = (13 – 25) = -12
y = -2
Hence, the required solution is x = $\frac{1}{5}$ and y = -2.

19.

Sol:
The given equations are:
$x + \frac{6}{y} = 6$ ……..(i)
$3x - \frac{8}{y} = 5$ ……..(ii)
Putting $\frac{1}{y}$ = v, we get:
x + 6v = 6 …….(iii)
3x – 8v = 5 ……(iv)
On multiplying (iii) by 4 and (iv) by 3, we get:
4x + 24v = 24 ……..(v)
9x – 24v = 15 ……..(vi)
On adding (v) from (vi), we get:

13x = 39 ⇒ x = 3
On substituting x = 3 in (i), we get:
$3 + \frac{6}{y} = 6$
$\Rightarrow \frac{6}{y} = (6 - 3) = 3 \Rightarrow 3y = 6 \Rightarrow y = 2$
Hence, the required solution is x = 3 and y = 2.

20.

Sol:

The given equations are:

$2x - \frac{3}{y} = 9$ ……..(i)

$3x + \frac{7}{y} = 2$ ……..(ii)

Putting $\frac{1}{y} = v$, we get:

$2x - 3v = 6$ …….(iii)

$3x + 7v = 2$ ……(iv)

On multiplying (iii) by 7 and (iv) by 3, we get:

$14x - 21v = 63$ ……..(v)

$9x + 21v = 6$ ……..(vi)

On adding (v) from (vi), we get:

$23x = 69 \Rightarrow x = 3$

On substituting x = 3 in (i), we get:

$2 \times 3 - \frac{3}{y} = 9$

$\Rightarrow 6 - \frac{3}{y} = 9 \Rightarrow \frac{3}{y} = -3 \Rightarrow y = -1$

Hence, the required solution is x = 3 and y = -1.

21.

Sol:

The given equations are:

$\frac{3}{x} - \frac{1}{y} + 9 = 0,$

$\Rightarrow \frac{3}{x} - \frac{1}{y} = -9$ ……..(i)

$\Rightarrow \frac{2}{x} - \frac{3}{y} = 5$……..(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get:

$3u – v = -9$ …….(iii)

$2u + 3v = 5$ ……(iv)

On multiplying (iii) by 3, we get:

$9u – 3v = -27$ ……..(v)

On adding (iv) and (v), we get:

$11u = -22 \Rightarrow u = -2$

$\Rightarrow \frac{1}{x} = -2 \Rightarrow x = \frac{-1}{2}$

On substituting $x = \frac{-1}{2}$ in (i), we get:

$\frac{3}{-1/2} - \frac{1}{y} = -9$

$\Rightarrow -6 - \frac{1}{y} = -9 \Rightarrow \frac{1}{y} = (-6 + 9) = 3$

$\Rightarrow y = \frac{1}{3}$

Hence, the required solution is $x = \frac{-1}{2}$ and $y = \frac{1}{3}$.

22.

Sol:

The given equations are:

$\frac{9}{x} - \frac{4}{y} = 8$ ……..(i)

$\frac{13}{x} + \frac{7}{y} = 101$ ……..(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get:

9u - 4v = 8 …….(iii)

13u + 7v = 101 ……(iv)

On multiplying (iii) by 7 and (iv) by 4, we get:

63u - 28v = 56 ……..(v)

52u + 28v = 404 ……..(vi)

On adding (v) from (vi), we get:

115u = 460 ⇒ u = 4

$\Rightarrow \frac{1}{x} = 4 \Rightarrow x = \frac{1}{4}$

On substituting $x = \frac{1}{4}$ in (i), we get:

$\frac{9}{1/4} - \frac{4}{y} = 8$

$\Rightarrow 36 - \frac{4}{y} = 8 \Rightarrow \frac{4}{y} = (36 - 8) = 28$

$y = \frac{4}{28} = \frac{1}{7}$

Hence, the required solution is $x = \frac{1}{4}$ and $y = \frac{1}{7}$.

23.

Sol:

The given equations are:

$\frac{5}{x} - \frac{3}{y} = 1$ ……..(i)

$\frac{3}{2x} + \frac{2}{3y} = 5$ ……..(ii)

Putting $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, we get:

5u - 3v = 1 …….(iii)

$\Rightarrow \frac{3}{2}$ u + $\frac{2}{3}$ v = 5

$\Rightarrow \frac{9u+4v}{6}$ = 5

$\Rightarrow$9u + 4v = 30 …….(iv)

On multiplying (iii) by 4 and (iv) by 3, we get:

20u - 12v = 4 ……..(v)

27u + 12v = 90 ……..(vi)

On adding (iv) and (v), we get:

47u = 94 $\Rightarrow$ u = 2

$\Rightarrow \frac{1}{x}$ = 2 $\Rightarrow$ x = $\frac{1}{2}$

On substituting x = $\frac{1}{2}$ in (i), we get:

$\frac{5}{1/2}$ - $\frac{3}{y}$ = 1

$\Rightarrow$10 - $\frac{3}{y}$ = 1 $\Rightarrow$ $\frac{3}{y}$ = (10 – 1) = 9

y = $\frac{3}{9}$ = $\frac{1}{3}$

Hence, the required solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

24.

Sol:

The given equations are:

$\frac{3}{x}$ + $\frac{2}{y}$ = 12 ……..(i)

$\frac{2}{x}$ + $\frac{3}{y}$ = 13 ……..(ii)

Multiplying (i) by 3 and (ii) by 2 and subtracting (ii) from (i), we get:

$\frac{9}{x}$ - $\frac{4}{x}$ = 36 – 26

$\Rightarrow \frac{5}{x}$ = 10

$\Rightarrow$x = $\frac{5}{10}$ = $\frac{1}{2}$

Now, substituting x = $\frac{1}{2}$ in (i), we have

6 + $\frac{2}{y}$ = 12

$\Rightarrow \frac{2}{y}$ = 6

$\Rightarrow$y = $\frac{1}{3}$

Hence, x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

25.

Sol:

The given equations are:

4x + 6y = 3xy .....(i)

8x + 9y = 5xy ......(ii)

From equation (i), we have:

$\frac{4x + 6y}{xy}$ = 3

⇒ $\frac{4}{y} + \frac{6}{x}$ = 3 ......(iii)

For equation (ii), we have:

$\frac{8x + 9y}{xy}$ = 5

⇒ $\frac{8}{y} + \frac{9}{x}$ = 5 ......(iv)

On substituting $\frac{1}{y}$ = v and $\frac{1}{x}$ = u, we get:

4v + 6u = 3 ......(v)

8v + 9u = 5 ....(vi)

On multiplying (v) by 9 and (vi) by 6, we get:

36v + 54u = 27 ....(vii)

48v + 54u = 30 ....(viii)

On subtracting (vii) from (viii), we get:

12v = 3 ⇒ v = $\frac{3}{12} = \frac{1}{4}$

⇒ $\frac{1}{y} = \frac{1}{4}$ ⇒ y = 4

On substituting y =4 in (iii), we get:

$\frac{4}{4} + \frac{6}{x}$ = 3

⇒ 1 + $\frac{6}{x}$ = 3 ⇒ $\frac{6}{x}$ = (3 – 1) = 2

⇒ 2x = 6 ⇒ x = $\frac{6}{2}$ = 3

Hence, the required solution is x = 3 and y = 4.

26.

Sol:

The given equations are:

x + y = 5xy .....(i)

3x + 2y = 13xy ......(ii)

From equation (i), we have:

$\frac{x + y}{xy} = 5$

$\Rightarrow \frac{1}{y} + \frac{1}{x} = 5$ ……(iii)

For equation (ii), we have:

$\frac{3x + 2y}{xy} = 13$

$\Rightarrow \frac{3}{y} + \frac{2}{x} = 13$ ……(iv)

On substituting $\frac{1}{y}$ = v and $\frac{1}{x}$ = u, we get:

v + u = 5 ……(v)

3v + 2u = 13 …….(vi)

On multiplying (v) by 2, we get:

2v + 2u = 10 ….(vii)

On subtracting (vii) from (vi), we get:

v = 3

$\Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

On substituting y = $\frac{1}{3}$ in (iii), we get:

$\frac{1}{1/3} + \frac{1}{x} = 5$

$\Rightarrow 3 + \frac{1}{x} = 5 \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

Hence, the required solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$ or x= 0 and y = 0.

27.

Sol:

The given equations are

$\frac{5}{x+y} - \frac{2}{x-y} = -1$ ……(i)

$\frac{15}{x+y} - \frac{7}{x-y} = 10$ ……(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), we get

5u – 2v = -1 ……..(iii)

15u + 7v = 10 …….(iv)

Multiplying (iii) by 3 and subtracting it from (iv), we get

7v + 6v = 10 + 3

$\Rightarrow$13v = 13

$\Rightarrow$v = 1

$\Rightarrow x - y = 1 \quad \left(\because \frac{1}{x-y} = v\right)$ .....(v)

Now, substituting v = 1 in (iii), we get

5u – 2 = -1

$\Rightarrow 5u = 1$

$\Rightarrow u = \frac{1}{5}$

x + y = 5 ....(vi)

Adding (v) and (vi), we get

$2x = 6 \Rightarrow x = 3$

Substituting x = 3 in (vi), we have

$3 + y = 5 \Rightarrow y = 5 - 3 = 2$

Hence, x = 3 and y = 2.

28.

Sol:

The given equations are

$\frac{3}{x+y} + \frac{2}{x-y} = 2$ ......(i)

$\frac{9}{x+y} - \frac{4}{x-y} = 1$ ......(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, we get:

3u + 2v = 2 ....(iii)

9u - 4v = 1 ....(iv)

On multiplying (iii) by 2, we get:

6u + 4v = 4 .....(v)

On adding (iv) and (v), we get:

15u = 5

$\Rightarrow u = \frac{5}{15} = \frac{1}{3}$

$\Rightarrow \frac{1}{x+y} = \frac{1}{3} \Rightarrow x + y = 3$ ....(vi)

On substituting $u = \frac{1}{3}$ in (iii), we get

1 + 2v = 2

$\Rightarrow 2v = 1$

$\Rightarrow v = \frac{1}{2}$

$\Rightarrow \frac{1}{x-y} = \frac{1}{2} \Rightarrow x - y = 2$ ....(vii)

On adding (vi) and (vii), we get

2x = 5

$\Rightarrow$ x = $\frac{5}{2}$

On substituting x = $\frac{5}{2}$ in (vi), we have

$\frac{5}{2}$ + y = 3

$\Rightarrow$ y = $\left(3 - \frac{5}{2}\right)$ = $\frac{1}{2}$

Hence, the required solution is x = $\frac{5}{2}$ and y = $\frac{1}{2}$.

29.

Sol:

The given equations are

$\frac{5}{x+1} + \frac{2}{y-1} = \frac{1}{2}$ ……(i)

$\frac{10}{x+1} - \frac{2}{y-1} = \frac{5}{2}$ ……(ii)

Substituting $\frac{1}{x+1}$ = u and $\frac{1}{y-1}$ = v, we get:

5u - 2v = $\frac{1}{2}$ ……..(iii)

10u + 2v = $\frac{5}{2}$ …….(iv)

On adding (iii) and (iv), we get:

15u = 3

$\Rightarrow$u = $\frac{3}{15}$ = $\frac{1}{5}$

$\Rightarrow$ $\frac{1}{x+1}$ = $\frac{1}{5}$ $\Rightarrow$ x + 1 = 5 $\Rightarrow$ x = 4

On substituting u = $\frac{1}{5}$ in (iii), we get

5 $\times$ $\frac{1}{5}$ - 2v = $\frac{1}{2}$ $\Rightarrow$ 1 – 2v = $\frac{1}{2}$

$\Rightarrow$2v = $\frac{1}{2}$ $\Rightarrow$v = $\frac{1}{4}$

$\Rightarrow$ $\frac{1}{y-1}$ = $\frac{1}{4}$ $\Rightarrow$ y – 1 = 4 $\Rightarrow$ y = 5

Hence, the required solution is x = 4 and y = 5.

30.

Sol:

The given equations are

$\frac{44}{x+y} + \frac{30}{x-y}$ = 10 ……(i)

$\frac{55}{x+y} - \frac{40}{x-y}$ = 13 ……(ii)

Putting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, we get:

44u + 30v = 10 ……..(iii)

55u + 40v = 13 …….(iv)

On multiplying (iii) by 4 and (iv) by 3, we get:

176u + 120v = 40 …..(v)

165u + 120v = 39 …..(vi)

On subtracting (vi) and (v), we get:

11u = 1

$\Rightarrow$u = $\frac{1}{11}$

$\Rightarrow$ $\frac{1}{x+y} = \frac{1}{11}$ $\Rightarrow$ x + y = 11 …….(vii)

On substituting u = $\frac{1}{11}$ in (iii), we get:

4 + 30v = 10

$\Rightarrow$30v = 6

$\Rightarrow$v = $\frac{6}{30}$ = $\frac{1}{5}$

$\Rightarrow$ $\frac{1}{x-y} = \frac{1}{5}$ $\Rightarrow$ x – y = 5 …….(viii)

On adding (vii) and (viii), we get

2x = 16

$\Rightarrow$x = 8

On substituting x =8 in (vii), we get:

8 + y = 11

$\Rightarrow$ y = 11 – 8 = 3

Hence, the required solution is x = 8 and y =3.

31.

Sol:

The given equations are

$\frac{10}{x+y} + \frac{2}{x-y} = 4$ ……(i)

$\frac{15}{x+y} - \frac{9}{x-y} = -2$ ……(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), we get:

10u + 2v = 4 ……..(iii)

15u - 9v = -2 …….(iv)

Multiplying (iii) by 9 and (iv) by 2 and adding, we get:

90u + 30u = 36 – 4

$\Rightarrow$120u = 32

$\Rightarrow$u = $\frac{32}{120}$ = $\frac{4}{15}$

$\Rightarrow$x + y = $\frac{15}{4}$ $\left(\because \frac{1}{x+y} = u\right)$ .....(v)

On substituting u = $\frac{4}{15}$ in (iii), we get:

10 $\times$ $\frac{4}{15}$ + 2v = 4

$\frac{8}{3}$ + 2v = 4

$\Rightarrow$2v = 4 - $\frac{8}{3}$ = $\frac{4}{3}$

$\Rightarrow$v = $\frac{2}{3}$

$\Rightarrow$ x – y = $\frac{3}{2}$ $\left(\because \frac{1}{x-y} = v\right)$ .....(vi)

Adding (v) and (vi), we get

2x = $\frac{15}{4}$ + $\frac{3}{2}$ $\Rightarrow$2x = $\frac{21}{4}$ $\Rightarrow$x = $\frac{21}{8}$

Substituting x = $\frac{21}{8}$ in (v), we have

$\frac{21}{8}$ + y = $\frac{15}{4}$ $\Rightarrow$y = $\frac{15}{4}$ - $\frac{21}{8}$ = $\frac{9}{8}$

Hence, x = $\frac{21}{8}$ and y =$\frac{9}{8}$.

32.

Sol:

The given equations are:

71x + 37y = 253 .....(i)

37x + 71y = 287 ......(ii)

On adding (i) and (ii), we get:

108x + 108y = 540

$\Rightarrow$108(x + y) = 540

$\Rightarrow$(x + y) = 5 ......(iii)

On subtracting (ii) from (i), we get:

34x – 34y = -34

$\Rightarrow$34(x – y) = -34

$\Rightarrow$(x – y) = -1 ......(iv)

On adding (iii) from (i), we get:

2x = 5 – 1 = 4

$\Rightarrow$x = 2

On subtracting (iv) from (iii), we get:

2y = 5 + 1 = 6

⇒y = 3

Hence, the required solution is x = 2 and y = 3.

33.

Sol:

The given equations are:

217x + 131y = 913 …..(i)

131x + 217y = 827 ……(ii)

On adding (i) and (ii), we get:

348x + 348y = 1740

⇒348(x + y) = 1740

⇒x + y = 5 ……(iii)

On subtracting (ii) from (i), we get:

86x – 86y = 86

⇒86(x – y) = 86

⇒x – y = 1 ……(iv)

On adding (iii) from (i), we get:

2x = 6

⇒x = 3

On substituting x = 3 in (iii), we get:

3 + y = 5

⇒y = 5 – 3 = 2

Hence, the required solution is x = 3 and y = 2.

34.

Sol:

The given equations are:

23x - 29y = 98 …..(i)

29x - 23y = 110 ……(ii)

Adding (i) and (ii), we get: 52x

– 52y = 208

⇒x – y = 4 ……(iii)

Subtracting (i) from (ii), we get:

6x + 6y = 12

$\Rightarrow x + y = 2$ ……(iv)

Now, adding equation (iii) and (iv), we get:

2x = 6

$\Rightarrow x = 3$

On substituting x = 3 in (iv), we have:

3 + y = 2

$\Rightarrow y = 2 - 3 = -1$

Hence, x = 3 and y = -1.

35.

Sol:

The given equations can be written as

$\frac{5}{x} + \frac{2}{y} = 6$ …….(i)

$\frac{-5}{x} + \frac{4}{y} = -3$ ……(ii)

Adding (i) and (ii), we get

$\frac{6}{y} = 3 \Rightarrow y = 2$

Substituting y = 2 in (i), we have

$\frac{5}{x} + \frac{2}{2} = 6 \Rightarrow x = 1$

Hence, x = 1 and y = 2.

36.

Sol:

The given equations are

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$ ……(i)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

$\frac{1}{3x+y} - \frac{1}{3x-y} = -\frac{1}{4}$ (Multiplying by 2) ……(ii)

Substituting $\frac{1}{3x+y}$ = u and $\frac{1}{3x-y}$ = v in (i) and (ii), we get:

$u + v = \frac{3}{4}$ ……..(iii)

$u - v = -\frac{1}{4}$ …….(iv)

Adding (iii) and (iv), we get:

$2u = \frac{1}{2}$

$\Rightarrow$ u = $\frac{1}{4}$

$\Rightarrow$3x + y = 4 $\left(\because \frac{1}{3x+y} = u\right)$ …..(v)

Now, substituting u = $\frac{1}{4}$ in (iii), we get:

$\frac{1}{4}$ + v = $\frac{3}{4}$

v = $\frac{3}{4}$ - $\frac{1}{4}$

$\Rightarrow$v = $\frac{1}{2}$

$\Rightarrow$ 3x – y = 2 $\left(\because \frac{1}{3x-y} = v\right)$ …..(vi)

Adding (v) and (vi), we get

6x = 6 $\Rightarrow$x = 1

Substituting x = 1 in (v), we have

3 + y = 4 $\Rightarrow$y = 1

Hence, x = 1 and y = 1.

37.

Sol:

The given equations are

$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}$ ……(i)

$\frac{1}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$ ……(ii)

Putting $\frac{1}{x+2y}$ = u and $\frac{1}{3x-2y}$ = v, we get:

$\frac{1}{2}$u + $\frac{5}{3}$v = - $\frac{3}{2}$ ……..(iii)

$\frac{5}{4}$u – $\frac{3}{5}$v = $\frac{61}{60}$ …….(iv)

On multiplying (iii) by 6 and (iv) by 20, we get:

3u + 10v = -9 …..(v)

25u – 12v = $\frac{61}{3}$ …..(vi)

On multiplying (v) by 6 and (vi) by 5, we get

18u + 60v = -54 …….(vii)

125u – 60v = $\frac{305}{3}$ ……..(viii)

On adding(vii) and (viii), we get:

143u = $\frac{305}{3}$ – 54 = $\frac{305-162}{3}$ = $\frac{143}{3}$

$\Rightarrow$u = $\frac{1}{3}$ = $\frac{1}{x+2y}$

$\Rightarrow x + 2y = 3$ ....(ix)

On substituting $u = \frac{1}{3}$ in (v), we get:

$1 + 10v = -9$

$\Rightarrow 10v = -10$

$\Rightarrow v = -1$

$\Rightarrow \frac{1}{3x-2y} = -1 \Rightarrow 3x - 2y = -1$ ....(x)

On adding (ix) and (x), we get:

$4x = 2$

$\Rightarrow x = \frac{1}{2}$

On substituting $x = \frac{1}{2}$ in (x), we get:

$\frac{3}{2} - 2y = -1$

$2y = \left(\frac{3}{2} + 1\right) = \frac{5}{2}$

$y = \frac{5}{4}$

Hence, the required solution is $x = \frac{1}{2}$ and $y = \frac{5}{4}$.

38.

Sol:

The given equations are

$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$ ......(i)

$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$ ......(ii)

Substituting $\frac{1}{3x+2y} = u$ and $\frac{1}{3x-2y} = v$, in (i) and (ii), we get:

$2u + 3v = \frac{17}{5}$ ....(iii)

$5u + v = 2$ ....(iv)

Multiplying (iv) by 3 and subtracting from (iii), we get:

$2u - 15u = \frac{17}{5} - 6$

$\Rightarrow -13u = \frac{-13}{5} \Rightarrow u = \frac{1}{5}$

$\Rightarrow 3x + 2y = 5 \quad \left(\because \frac{1}{3x+2y} = u\right)$ .....(v)

Now, substituting $u = \frac{1}{5}$ in (iv), we get

$1 + v = 2 \Rightarrow v = 1$

$\Rightarrow 3x - 2y = 1 \quad \left(\because \frac{1}{3x-2y} = v\right)$ ....(vi)

Adding(v) and (vi), we get:

$\Rightarrow 6x = 6 \Rightarrow x = 1$

Substituting x = 1 in (v), we get:

$3 + 2y = 5 \Rightarrow y = 1$

Hence, x = 1 and y =1.

39.

Sol:

The given equations can be written as

$\frac{3}{x} + \frac{6}{y} = 7$ ....(i)

$\frac{9}{x} + \frac{3}{y} = 11$ ......(ii)

Multiplying (i) by 3 and subtracting (ii) from it, we get

$\frac{18}{y} - \frac{3}{y} = 21 - 11$

$\Rightarrow \frac{15}{y} = 10$

$\Rightarrow y = \frac{15}{10} = \frac{3}{2}$

Substituting $y = \frac{3}{2}$ in (i), we have

$\frac{3}{x} + \frac{6 \times 2}{3} = 7$

$\Rightarrow \frac{3}{x} = 7 - 4 = 3$

Hence, x = 1 and $y = \frac{3}{2}$.

40.

Sol:

The given equations are

$x + y = a + b$ ....(i)

$ax - by = a^2 - b^2$ ....(ii)

Multiplying (i) by b and adding it with (ii), we get

$bx + ax = ab + b^2 + a^2 - b^2$

$\Rightarrow x = \frac{ab + a^2}{a+b} = a$

Substituting x = a in (i), we have

$a + y = a + b$

$\Rightarrow y = b$

Hence, x = a and y = b.

41.

Sol:

The given equations are:

$\frac{x}{a} + \frac{y}{b} = 2$

$\Rightarrow \frac{bx+ay}{ab} = 2$ [Taking LCM]

$\Rightarrow bx + ay = 2ab$ …….(i)

Again, $ax - by = (a^2 - b^2)$ …..(ii)

On multiplying (i) by b and (ii) by a, we get:

$b^2x + bay = 2ab^2$ ……..(iii)

$a^2x - bay = a(a^2 - b^2)$ …….(iv)

On adding (iii) from (iv), we get:

$(b^2 + a^2)x = 2a^2b + a(a^2 - b^2)$

$\Rightarrow (b^2 + a^2)x = 2ab^2 + a^3 - ab^2$

$\Rightarrow (b^2 + a^2)x = ab^2 + a^3$

$\Rightarrow (b^2 + a^2)x = a(b^2 + a^2)$

$\Rightarrow x = \frac{a(b^2 + a^2)}{(b^2 + a^2)} = a$

On substituting x = a in (i), we get:

$ba + ay = 2ab$

$\Rightarrow ay = ab$

$\Rightarrow y = b$

Hence, the solution is x = a and y = b.

42.

Sol:

The given equations are

$px + qy = p - q$ ………(i)

$qx - py = p + q$ ………(ii)

Multiplying (i) by p and (ii) by q and adding them , we get

$p^2x + q^2x = p^2 - pq + pq + q^2$

$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$

Substituting x = 1 in (i), we have

$p + qy = p - q$

$\Rightarrow qy = -p$

$\Rightarrow$y = -1

Hence, x = 1 and y = -1.

43.

Sol:

The given equations can be written as

$\frac{x}{a} - \frac{y}{b} = 0$ ....(i)

$ax + by = a^2 + b^2$ ......(ii)

From (i),

$y = \frac{bx}{a}$

Substituting $y = \frac{bx}{a}$ in (ii), we get

$ax + \frac{b \times bx}{a} = a^2 + b^2$

$\Rightarrow x = \frac{(a^2 + b^2) \times a}{a^2 + b^2} = a$

Now, substitute x = a in (ii) to get

$a^2 + by = a^2 + b^2$

$\Rightarrow by = b^2$

$\Rightarrow$y = b

Hence, x = a and y = b.

44.

Sol:

The given equations are

6(ax + by) = 3a + 2b

$\Rightarrow$6ax + 6by = 3a + 2b ....(i)

and 6(bx – ay) = 3b – 2a

$\Rightarrow$6bx – 6ay = 3b – 2a ....(ii)

On multiplying (i) by a and (ii) by b, we get

$6a^2x + 6aby = 3a^2 + 2ab$ ....(iii)

$6b^2x - 6aby = 3b^2 - 2ab$ ....(iv)

On adding (iii) and (iv), we get

$6(a^2 + b^2)x = 3(a^2 + b^2)$

$x = \frac{3(a^2 + b^2)}{6(a^2 + b^2)} = \frac{1}{2}$

On substituting x = $\frac{1}{2}$ in (i), we get:

$6a \times \frac{1}{2} + 6by = 3a + 2b$

6by = 2b

$y = \frac{2b}{6b} = \frac{1}{3}$

Hence, the required solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

45.

Sol:

The given equations are

$ax - by = a^2 + b^2$ ....(i)

x + y = 2a ....(ii) From

(ii)

y = 2a - x

Substituting y = 2a – x in (i), we get

$ax - b(2a - x) = a^2 + b^2$

$\Rightarrow ax - 2ab + bx = a^2 + b^2$

$\Rightarrow x = \frac{a^2 + b^2 + 2ab}{a+b} = \frac{(a+b)^2}{a+b} = a + b$

Now, substitute x = a + b in (ii) to get

a + b + y = 2a

$\Rightarrow y = a - b$

Hence, x = a + b and y = a – b.

46.

Sol:

The given equations are:

$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$

By taking LCM, we get:

$b^2x - a^2y = -a^2b - b^2a$ .....(i)

and bx – ay + 2ab = 0

bx – ay = -2ab ....(ii)

On multiplying (ii) by a, we get:

$abx - a^2y = -2a^2b$ ......(iii)

On subtracting (i) from (iii), we get:

$abx - b^2x = 2a^2b + a^2b + b^2a = -a^2b + b^2a$

$\Rightarrow x(ab - b^2) = -ab(a - b)$

$\Rightarrow x(a - b)b = -ab(a - b)$

$\therefore x = \frac{-ab(a-b)}{(a-b)b} = -a$

On substituting x = -a in (i), we get:

$b^2(-a) - a^2y = -a^2b - b^2a$

$\Rightarrow -b^2a - a^2y = -a^2b - b^2a$

$\Rightarrow -a^2y = -a^2b$

$\Rightarrow y = b$

Hence, the solution is x = -a and y = b.

47.

Sol:

The given equations are:

$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$

By taking LCM, we get:

$\frac{b^2x + a^2y}{ab} = a^2 + b^2$

$\Rightarrow b^2x + a^2y = (ab)a^2 + b^2$

$\Rightarrow b^2x + a^2y = a^3b + ab^3$ .....(i)

Also, x + y = 2ab ....(ii)

On multiplying (ii) by a^2, we get:

$a^2x + a^2y = 2a^3b$ ......(iii)

On subtracting (iii) from (i), we get:

$(b^2 - a^2)x = a^3b + ab^3 - 2a^3b$

$\Rightarrow (b^2 - a^2)x = -a^3b + ab^3$

$\Rightarrow (b^2 - a^2)x = ab(b^2 - a^2)$

$\Rightarrow (b^2 - a^2)x = ab(b^2 - a^2)$

$\therefore x = \frac{ab(b^2 - a^2)}{(b^2 - a^2)} = ab$

On substituting x = ab in (i), we get:

$b^2(ab) + a^2y = a^3b + ab^3$

$\Rightarrow a^2y = a^3b$

$\Rightarrow \frac{a^3b}{a^2} = ab$

Hence, the solution is x = ab and y = ab.

48.

Sol:
The given equations are
$x + y = a + b$ ....(i)
$ax - by = a^2 - b^2$ ....(ii)
From (i)
$y = a + b - x$
Substituting $y = a + b - x$ in (ii), we get
$ax - b(a + b - x) = a^2 - b^2$
$\Rightarrow ax - ab - b^2 + bx = a^2 - b^2$
$\Rightarrow x = \frac{a^2 + ab}{a+b} = a$
Now, substitute $x = a$ in (i) to get
$a + y = a + b$
$\Rightarrow y = b$
Hence, $x = a$ and $y = b$.

49.

Sol:
The given equations are
$a^2x + b^2y = c^2$ ....(i)
$b^2x + a^2y = d^2$ ....(ii)
Multiplying (i) by a^2 and (ii) by b^2 and subtracting, we get
$a^4x - b^4x = a^2c^2 - b^2 d^2$
$\Rightarrow x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$
Now, multiplying (i) by b^2 and (ii) by a^2 and subtracting, we get
$b^4y - a^4y = b^2c^2 - a^2 d^2$
$\Rightarrow y = \frac{b^2c^2 - a^2d^2}{b^4 - a^4}$
Hence, $x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$ and $y = \frac{b^2c^2 - a^2d^2}{b^4 - a^4}$.

50.

Sol:
The given equations are
$\frac{x}{a} + \frac{y}{b} = a + b$ ....(i)
$\frac{x}{a^2} + \frac{y}{b^2} = 2$ ......(ii)

Multiplying (i) by b and (ii) by b^2 and subtracting, we get

$\frac{bx}{a} - \frac{b^2x}{a^2} = ab + b^2 - 2b^2$

$\Rightarrow \frac{ab - b^2}{a^2} x = ab - b^2$

$\Rightarrow x = \frac{(ab - b^2)a^2}{ab - b^2} = a^2$

Now, substituting $x = a^2$ in (i) we get

$\frac{a^2}{a} + \frac{y}{b} = a + b$

$\Rightarrow \frac{y}{b} = a + b – a = b$

$\Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$.

Exercise – 3C

1.

Sol:

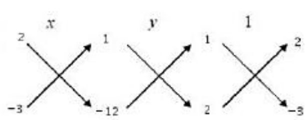
The given equations are:

$x + 2y + 1 = 0$ …….(i)

$2x – 3y – 12 = 0$ …….(ii)

Here $a_1 = 1$, $b_1 = 2$, $c_1 = 1$, $a_2 = 2$, $b_2 = -3$ and $c_2 = -12$

By cross multiplication, we have:



$\therefore \frac{x}{[2 \times (-12) - 1 \times (-3)]} = \frac{y}{[1 \times 2 - 1 \times (-12)]} = \frac{1}{[1 \times (-3) - 2 \times 2]}$

$\Rightarrow \frac{x}{(-24+3)} = \frac{y}{(2+12)} = \frac{1}{(-3-4)}$

$\Rightarrow \frac{x}{(-21)} = \frac{y}{(14)} = \frac{1}{(-7)}$

$\Rightarrow x = \frac{-21}{-7} = 3$, $y = \frac{14}{-7} = -2$

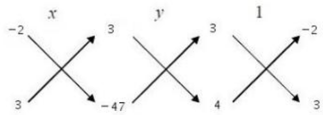
Hence, $x = 3$ and $y = -2$ is the required solution.

2.

Sol:

The given equations are:

$3x - 2y + 3 = 0$ ....(i)
$4x + 3y - 47 = 0$ ....(ii)
Here $a_1 = 3$, $b_1 = -2$, $c_1 = 3$, $a_2 = 4$, $b_2 = 3$ and $c_2 = -47$
By cross multiplication, we have:



$$\therefore \frac{x}{[(-2)\times(-47)-3\times 3]} = \frac{y}{[3\times 4-(-47)\times 3]} = \frac{1}{[3\times 3-(-2)\times 4]}$$

$$\Rightarrow \frac{x}{(94-9)} = \frac{y}{(12+141)} = \frac{1}{(9+8)}$$

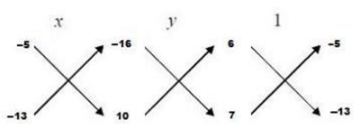
$$\Rightarrow \frac{x}{(85)} = \frac{y}{(153)} = \frac{1}{(17)}$$

$$\Rightarrow x = \frac{85}{17} = 5,\ y = \frac{153}{17} = 9$$

Hence, x = 5 and y = 9 is the required solution.

3.

Sol:
The given equations are:
$6x - 5y - 16 = 0$ ....(i)
$7x - 13y + 10 = 0$ ....(ii)
Here $a_1 = 6$, $b_1 = -5$, $c_1 = -16$, $a_2 = 7$, $b_2 = -13$ and $c_2 = 10$
By cross multiplication, we have:



$$\therefore \frac{x}{[(-5)\times 10-(-16)\times(-13)]} = \frac{y}{[(-16)\times 7-10\times 6]} = \frac{1}{[6\times(-13)-(-5)\times 7]}$$

$$\Rightarrow \frac{x}{(-50-208)} = \frac{y}{(-112-60)} = \frac{1}{(-78+35)}$$

$$\Rightarrow \frac{x}{(-258)} = \frac{y}{(-172)} = \frac{1}{(43)}$$

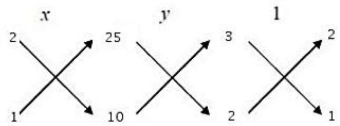
$$\Rightarrow x = \frac{-258}{-43} = 6,\ y = \frac{-172}{-43} = 4$$

Hence, x = 6 and y = 4 is the required solution.

4.

Sol:
The given equations are:
$3x + 2y + 25 = 0$ ....(i)

$2x + y + 10 = 0$ …….(ii)
Here $a_1 = 3$, $b_1 = 2$, $c_1 = 25$, $a_2 = 2$, $b_2 = 1$ and $c_2 = 10$
By cross multiplication, we have:



$\therefore \frac{x}{[2\times10-25\times1]} = \frac{y}{[25\times2-10\times3]} = \frac{1}{[3\times1-2\times2]}$

$\Rightarrow \frac{x}{(20-25)} = \frac{y}{(50-30)} = \frac{1}{(3-4)}$

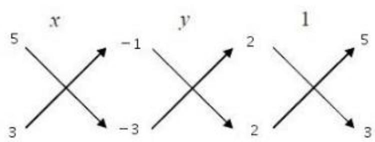
$\Rightarrow \frac{x}{(-5)} = \frac{y}{20} = \frac{1}{(-1)}$

$\Rightarrow x = \frac{-5}{-1} = 5,\ y = \frac{20}{(-1)} = -20$

Hence, x = 5 and y = -20 is the required solution.

5.

Sol:
The given equations may be written as:
$2x + 5y - 1 = 0$ …….(i)
$2x + 3y - 3 = 0$ …….(ii)
Here $a_1 = 2$, $b_1 = 5$, $c_1 = -1$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -3$
By cross multiplication, we have:



$\therefore \frac{x}{[5\times(-3)-3\times(-1)]} = \frac{y}{[(-1)\times2-(-3)\times2]} = \frac{1}{[2\times3-2\times5]}$

$\Rightarrow \frac{x}{(-15+3)} = \frac{y}{(-2+6)} = \frac{1}{(6-10)}$

$\Rightarrow \frac{x}{-12} = \frac{y}{4} = \frac{1}{-4}$

$\Rightarrow x = \frac{-12}{-4} = 3,\ y = \frac{4}{-4} = -1$

Hence, x = 3 and y = -1 is the required solution.

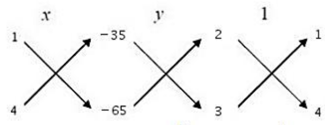
6.

Sol:
The given equations may be written as:
$2x + y - 35 = 0$ …….(i)

3x + 4y – 65 = 0 …….(ii)

Here $a_1 = 2$, $b_1 = 1$, $c_1 = -35$, $a_2 = 3$, $b_2 = 4$ and $c_2 = -65$

By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-65)-4\times(-35)]} = \frac{y}{[(-35)\times 3-(-65)\times 2]} = \frac{1}{[2\times 4-3\times 1]}$

$\Rightarrow \frac{x}{(-65+140)} = \frac{y}{(-105+130)} = \frac{1}{(8-3)}$

$\Rightarrow \frac{x}{75} = \frac{y}{25} = \frac{1}{5}$

$\Rightarrow x = \frac{75}{5} = 15,\ y = \frac{25}{5} = 5$

Hence, x = 15 and y = 5 is the required solution.

7.

Sol:

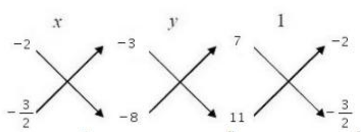
The given equations may be written as:

7x - 2y – 3 = 0 …….(i)

$11x - \frac{3}{2}y - 8 = 0$ …….(ii)

Here $a_1 = 7$, $b_1 = -2$, $c_1 = -3$, $a_2 = 11$, $b_2 = -\frac{3}{2}$ and $c_2 = -8$

By cross multiplication, we have:



$\therefore \frac{x}{\left[(-2)\times(-8)-\left(-\frac{3}{2}\right)\times(-3)\right]} = \frac{y}{[(-3)\times 11-(-8)\times 7]} = \frac{1}{\left[7\times\left(-\frac{3}{2}\right)-11\times(-2)\right]}$

$\Rightarrow \frac{x}{(16-\frac{9}{2})} = \frac{y}{(-33+56)} = \frac{1}{(-\frac{21}{2}+22)}$

$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)} = \frac{y}{23} = \frac{1}{\left(\frac{23}{2}\right)}$

$\Rightarrow x = \frac{\frac{23}{2}}{\frac{23}{2}} = 1,\ y = \frac{23}{\frac{23}{2}} = 2$

Hence, x = 1 and y = 2 is the required solution.

8.

Sol:

The given equations may be written as:

$\frac{x}{6} + \frac{y}{15} - 4 = 0$ ....(i)

$\frac{x}{3}$ - $\frac{y}{12} - \frac{19}{4} = 0$ ....(ii)

Here $a_1 = \frac{1}{6}$, $b_1 = \frac{1}{15}$, $c_1 = -4$, $a_2 = \frac{1}{3}$, $b_2 = -\frac{1}{12}$ and $c_2 = -\frac{19}{4}$

By cross multiplication, we have:

$\therefore \frac{x}{\left[\frac{1}{15} \times \left(-\frac{19}{4}\right) - \left(-\frac{1}{12}\right) \times (-4)\right]} = \frac{y}{\left[(-4) \times \frac{1}{3} - \left(\frac{1}{6}\right) \times \left(-\frac{19}{4}\right)\right]} = \frac{1}{\left[\frac{1}{6} \times \left(-\frac{1}{12}\right) \times \frac{1}{3} \times \frac{1}{15}\right]}$

$\Rightarrow \frac{x}{\left(-\frac{19}{60} - \frac{1}{3}\right)} = \frac{y}{\left(-\frac{4}{3} + \frac{19}{34}\right)} = \frac{1}{\left(-\frac{1}{72} - \frac{1}{45}\right)}$

$\Rightarrow \frac{x}{\left(-\frac{39}{60}\right)} = \frac{y}{\left(-\frac{13}{24}\right)} = \frac{1}{\left(-\frac{13}{360}\right)}$

$\Rightarrow x = \left[\left(-\frac{39}{60}\right) \times \left(-\frac{360}{13}\right)\right] = 18$, $y = \left[\left(-\frac{13}{24}\right) \times \left(-\frac{360}{13}\right)\right] = 15$

Hence, x = 18 and y = 15 is the required solution.

9.

Sol:

Taking $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, the given equations become:

u + v = 7

2u + 3v = 17

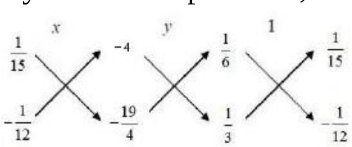
The given equations may be written as:

u + v – 7 = 0 .....(i)

2u + 3v – 17 = 0 ......(ii)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -7$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -17$

By cross multiplication, we have:



$\therefore \frac{u}{[1 \times (-17) - 3 \times (-7)]} = \frac{v}{[(-7) \times 2 - 1 \times (-17)]} = \frac{1}{[3-2]}$

$\Rightarrow \frac{u}{(-17+21)} = \frac{v}{(-14+17)} = \frac{1}{(1)}$

$\Rightarrow \frac{u}{4} = \frac{v}{3} = \frac{1}{1}$

$\Rightarrow u = \frac{4}{1} = 4$, $v = \frac{3}{1} = 3$

$\Rightarrow \frac{1}{x} = 4, \frac{1}{y} = 3$

$\Rightarrow x = \frac{1}{4}, y = \frac{1}{3}$

Hence, x = $\frac{1}{4}$ and y = $\frac{1}{3}$ is the required solution.

10.

Sol:

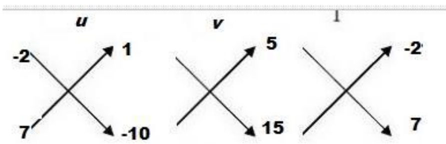
Taking $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, the given equations become:

5u - 2v + 1 = 0 .....(i)

15u + 7v – 10 = 0 ......(ii)

Here, a_1 = 5, b_1 = -2, c_1 = 1, a_2 = 15, b_2 = -7 and c_2 = -10

By cross multiplication, we have:



$\therefore \frac{u}{[-2\times(-10)-1\times7]} = \frac{v}{[1\times15-(-10)\times5]} = \frac{1}{[35+30]}$

$\Rightarrow \frac{u}{(20-7)} = \frac{v}{(15+50)} = \frac{1}{65}$

$\Rightarrow \frac{u}{13} = \frac{v}{65} = \frac{1}{65}$

$\Rightarrow$u = $\frac{13}{65} = \frac{1}{5}$, v = $\frac{65}{65}$ = 1

$\Rightarrow \frac{1}{x+y} = \frac{1}{5}, \frac{1}{x-y} = 1$

So, (x + y) = 5 ....(iii)

and (x – y) = 1 ......(iv)

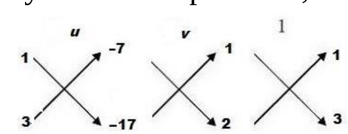
Again, the above equations (ii) and (iv) may be written as:

x + y – 5 = 0 .....(i)

x – y – 1 = 0 ......(ii)

Here, a_1 = 1, b_1 = 1, c_1 = -5, a_2 = 1, b_2 = -1 and c_2 = -1

By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-1)-(-5)\times(-1)]} = \frac{y}{[(-5)\times1-(-1)\times1]} = \frac{1}{[1\times(-1)-1\times1]}$

$\Rightarrow \frac{x}{(-1-5)} = \frac{y}{(-5+1)} = \frac{1}{(-1-1)}$

$\Rightarrow \frac{x}{-6} = \frac{v}{-4} = \frac{1}{-2}$

$\Rightarrow$x = $\frac{-6}{-2}$ = 3, y = $\frac{-4}{-2}$ = 2

Hence, x = 3 and y = 2 is the required solution.

11.

Sol:

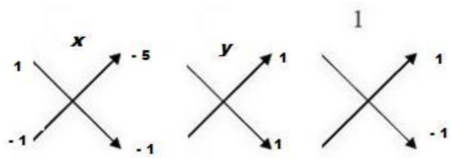
The given equations may be written as:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$ ……(i)

$ax - by - 2ab = 0$ ……(ii)

Here, $a_1 = \frac{a}{b}$, $b_1 = \frac{-b}{a}$, $c_1 = -(a + b)$, $a_2 = a$, $b_2 = -b$ and $c_2 = -2ab$

By cross multiplication, we have:



$\therefore \frac{x}{\left[\left(-\frac{b}{a}\right)\times(-2ab) - (-b)\times(-(a+b))\right]} = \frac{y}{\left[-(a+b)\times a - (-2ab)\times\frac{a}{b}\right]} = \frac{1}{\left[\frac{a}{b}\times(-b) - a\times\left(-\frac{b}{a}\right)\right]}$

$\Rightarrow \frac{x}{(2b^2 - b(a+b))} = \frac{y}{-a(a+b)+2a^2} = \frac{1}{-a+b}$

$\Rightarrow \frac{x}{2b^2 - ab - b^2} = \frac{y}{-a^2 - ab + 2a^2} = \frac{1}{-a+b}$

$\Rightarrow \frac{x}{b^2 - ab} = \frac{y}{a^2 - ab} = \frac{1}{-(a-b)}$

$\Rightarrow \frac{x}{-b(a-b)} = \frac{y}{a(a-b)} = \frac{1}{-(a-b)}$

$\Rightarrow x = \frac{-b(a-b)}{-(a-b)} = b$, $y = \frac{a(a-b)}{-(a-b)} = -a$

Hence, $x = b$ and $y = -a$ is the required solution.

12.

Sol:

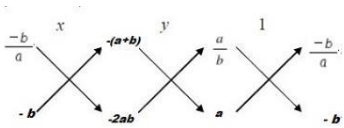
The given equations may be written as:

$2ax + 3by - (a + 2b) = 0$ ……(i)

$3ax + 2by - (2a + b) = 0$ ……(ii)

Here, $a_1 = 2a$, $b_1 = 3b$, $c_1 = -(a + 2b)$, $a_2 = 3a$, $b_2 = 2b$ and $c_2 = -(2a + b)$

By cross multiplication, we have:



$\therefore \frac{x}{[3b\times(-(2a+b)) - 2b\times(-(a+2b))]} = \frac{y}{[-(a+2b)\times 3a - 2a\times(-(2a+b))]} = \frac{1}{[2a\times 2b - 3a\times 3b]}$

$\Rightarrow \frac{x}{(-6ab - 3b^2 + 2ab + 4b^2)} = \frac{y}{(-3a^2 - 6ab + 4a^2 + 2ab)} = \frac{1}{4ab - 9ab}$

$\Rightarrow \frac{x}{b^2-4ab} = \frac{y}{a^2-4ab} = \frac{1}{-5ab}$

$\Rightarrow \frac{x}{-b(4a-b)} = \frac{y}{-a(4b-a)} = \frac{1}{-5ab}$

$\Rightarrow x = \frac{-b(4a-b)}{-5ab} = \frac{(4a-b)}{5a}$, $y = \frac{-a(4b-a)}{-5ab} = \frac{(4b-a)}{5b}$

Hence, $x = \frac{(4a-b)}{5a}$ and $y = \frac{(4b-a)}{5b}$ is the required solution.

13.

Sol:

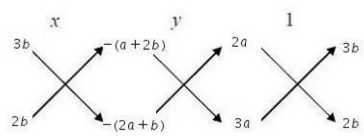
Substituting $\frac{1}{x}$ = u and $\frac{1}{y}$ = v in the given equations, we get

au – bv + 0 = 0 …….(i)

$ab^2u + a^2bv - (a^2 + b^2) = 0$ ……(ii)

Here, $a_1 = a$, $b_1 = -b$, $c_1 = 0$, $a_2 = ab^2$, $b_2 = a^2b$ and $c_2 = -(a^2 + b^2)$.

So, by cross-multiplication, we have



$\frac{u}{b_1c_2 - b_2c_1} = \frac{v}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{u}{(-b)[-(a^2+b^2)]-(a^2b)(0)} = \frac{v}{(0)(ab^2)-(-a^2-b^2)(a)} = \frac{1}{(a)(a^2b)-(ab^2)(-b)}$

$\Rightarrow \frac{u}{b(a^2+b^2)} = \frac{v}{a(a^2+b^2)} = \frac{1}{ab(a^2+b^2)}$

$\Rightarrow u = \frac{b(a^2+b^2)}{ab(a^2+b^2)}$, $v = \frac{a(a^2+b^2)}{ab(a^2+b^2)}$

$\Rightarrow u = \frac{1}{a}$, $v = \frac{1}{b}$

$\Rightarrow \frac{1}{x} = \frac{1}{a}, \frac{1}{y} = \frac{1}{b}$

$\Rightarrow x = a, y = b$

Hence, x = a and y = b.

Exercise – 3D

1.

Sol:

The given system of equations is:

3x + 5y = 12
5x + 3y = 4
These equations are of the forms:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= 5$, $c_1 = -12$ and $a_2 = 5$, $b_2 = 3$, $c_2 = -4$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{3}{5} \neq \frac{5}{3}$
Hence, the given system of equations has a unique solution.
Again, the given equations are:
3x + 5y = 12 .....(i)
5x + 3y = 4 .....(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
9x + 15y = 36 ....(iii)
25x + 15y = 20 ......(iv)
On subtracting (iii) from (iv), we get:
16x = -16
$\Rightarrow$x = -1
On substituting x = -1 in (i), we get:
3(-1) + 5y = 12
$\Rightarrow$5y = (12 + 3) = 15
$\Rightarrow$y = 3
Hence, x = -1 and y = 3 is the required solution.

2.

Sol:
The given system of equations is:
2x - 3y - 17 = 0 ....(i)
4x + y - 13 = 0 .....(ii)
The given equations are of the form
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= -3$, $c_1 = -17$ and $a_2 = 4$, $b_2 = 1$, $c_2 = -13$
Now,

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{1} = -3$
Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the system of equations has unique solution.
Using cross multiplication method, we have

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{x}{-3(-13)-1\times(-17)} = \frac{y}{-17 \times 4-(-13)\times 2} = \frac{1}{2 \times 1-4\times(-3)}$

$\Rightarrow \frac{x}{39+17} = \frac{y}{-68+26} = \frac{1}{2+12}$

$\Rightarrow \frac{x}{56} = \frac{y}{-42} = \frac{1}{14}$

$\Rightarrow$ x = $\frac{56}{14}$, y = $\frac{-42}{14}$

$\Rightarrow$ x = 4, y = -3

Hence, x = 4 and y = -3.

3.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

$\frac{x}{3} + \frac{y}{2} = 3$

$\Rightarrow \frac{2x+3y}{6} = 3$

2x + 3y = 18

$\Rightarrow$2x + 3y – 18 = 0 ….(i)

and

x – 2y = 2

x – 2y – 2 = 0 …..(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -18$ and $a_2 = 1$, $b_2 = -2$, $c_2 = -2$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{2}{1} \neq \frac{3}{-2}$

Hence, the given system of equations has a unique solution.

Again, the given equations are:

2x + 3y – 18 = 0 …..(iii)

x – 2y – 2 = 0 …..(iv)

On multiplying (i) by 2 and (ii) by 3, we get:

4x + 6y – 36 = 0 …….(v)

3x - 6y – 6 = 0 ……(vi)

On adding (v) from (vi), we get:

7x = 42

$\Rightarrow$x = 6

On substituting x = 6 in (iii), we get:

$2(6) + 3y = 18$

$\Rightarrow 3y = (18 - 12) = 6$

$\Rightarrow y = 2$

Hence, x = 6 and y = 2 is the required solution.

4.

Sol:

The given system of equations are

$2x + 3y - 5 = 0$

$kx - 6y - 8 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -5$ and $a_2 = k$, $b_2 = -6$, $c_2 = -8$

Now, for the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{k} \neq \frac{3}{-6}$

$\Rightarrow k \neq -4$

Hence, $k \neq -4$

5.

Sol:

The given system of equations are

$x - ky - 2 = 0$

$3x + 2y + 5 = 0$

This system of equations is of the form:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= -k$, $c_1 = -2$ and $a_2 = 3$, $b_2 = 2$, $c_2 = 5$

Now, for the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{3} \neq \frac{-k}{2}$

$\Rightarrow k \neq -\frac{2}{3}$

Hence, $k \neq -\frac{2}{3}$.

6.

Sol:
The given system of equations are
5x - 7y – 5 = 0 ....(i)
2x + ky - 1 = 0 ...(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 5$, $b_1= -7$, $c_1 = -5$ and $a_2 = 2$, $b_2 = k$, $c_2 = -1$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{5}{2} \neq \frac{-7}{k}$
$\Rightarrow k \neq -\frac{14}{5}$
Hence, $k \neq -\frac{14}{5}$.

7.

Sol:
The given system of equations are
4x + ky + 8 = 0
x + y + 1 = 0
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 4$, $b_1= k$, $c_1 = 8$ and $a_2 = 1$, $b_2 = 1$, $c_2 = 1$
For the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{4}{1} \neq \frac{k}{1}$
$\Rightarrow k \neq 4$
Hence, $k \neq 4$.

8.
Sol:

The given system of equations are

$4x - 5y = k$

$\Rightarrow 4x - 5y - k = 0$ ….(i)

And, $2x - 3y = 12$

$\Rightarrow 2x - 3y - 12 = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 4$, $b_1= -5$, $c_1 = -k$ and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{4}{2} \neq \frac{-5}{-3}$

$\Rightarrow 2 \neq \frac{5}{3} \Rightarrow 6 \neq 5$

Thus, for all real values of k, the given system of equations will have a unique solution.

9.

Sol:

The given system of equations:

$kx + 3y = (k – 3)$

$\Rightarrow kx + 3y – (k - 3) = 0$ ….(i)

And, $12x + ky = k$

$\Rightarrow 12x + ky - k = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = k$, $b_1= 3$, $c_1 = -(k – 3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{k}{12} \neq \frac{3}{k}$

$\Rightarrow k^2 \neq 36 \Rightarrow k \neq \pm 6$

Thus, for all real values of k, other than ±6, the given system of equations will have a unique solution.

10.

Sol:

The given system of equations: 2x - 3y = 5

$\Rightarrow$ 2x - 3y – 5 = 0 ….(i)

6x - 9y = 15

$\Rightarrow$6x - 9y - 15 = 0 …(ii)

These equations are of the following forms:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 2$, $b_1= -3$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -9$, $c_2 = -15$

$\therefore \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has an infinite number of solutions.

11.

Sol:

The given system of equations can be written as

6x + 5y – 11 = 0 ….(i)

$\Rightarrow 9x + \frac{15}{2}y - 21 = 0$ …(ii)

This system is of the form

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Here, $a_1 = 6$, $b_1= 5$, $c_1 = -11$ and $a_2 = 9$, $b_2 = \frac{15}{2}$, $c_2 = -21$

Now,

$\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{5}{\frac{15}{2}} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{-11}{-21} = \frac{11}{21}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, therefore the given system has no solution.

12.

Sol:

The given system of equations:

kx + 2y = 5

$\Rightarrow$ kx + 2y - 5 = 0 ....(i)

3x - 4y = 10

$\Rightarrow$3x - 4y - 10 = 0 ...(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = -4$, $c_2 = -10$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{k}{3} \neq \frac{2}{-4} \Rightarrow k \neq \frac{-3}{2}$

Thus for all real values of k other than $\frac{-3}{2}$, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4} \neq \frac{-5}{-10}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4}$ and $\frac{k}{3} \neq \frac{1}{2}$

$\Rightarrow k = \frac{-3}{2}, k \neq \frac{3}{2}$

Hence, the required value of k is $\frac{-3}{2}$.

13.

Sol:

The given system of equations:

x + 2y = 5

$\Rightarrow$ x + 2y - 5 = 0 ....(i)

3x + ky + 15 = 0 ...(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = k$, $c_2 = 15$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

Thus for all real values of k other than 6, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow k = 6, k \neq -6$

Hence, the required value of k is 6.

14.

Sol:

The given system of equations:

x + 2y = 3

$\Rightarrow$ x + 2y - 3 = 0 ….(i)

And, 5x + ky + 7 = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -3$ and $a_2 = 5$, $b_2 = k$, $c_2 = 7$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10$

Thus for all real values of k other than 10, the given system of equations will have a unique solution.

(ii) In order that the given system of equations has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow k = 10, k \neq \frac{14}{-3}$

Hence, the required value of k is 10.

There is no value of k for which the given system of equations has an infinite number of solutions.

15.

Sol:

The given system of equations:

2x + 3y = 7,

$\Rightarrow$ 2x + 3y - 7 = 0 ….(i)

And, (k – 1)x + (k + 2)y = 3k

$\Rightarrow$(k – 1)x + (k + 2)y - 3k = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (k – 1)$, $b_2 = (k + 2)$, $c_2 = -3k$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{-7}{-3k}$

$\Rightarrow\frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{7}{3k}$

Now, we have the following three cases:

Case I:

$\frac{2}{(k-1)} = \frac{3}{k+2}$

$\Rightarrow$ 2(k + 2) = 3(k – 1) $\Rightarrow$ 2k + 4 = 3k – 3 $\Rightarrow$ k = 7

Case II:

$\frac{3}{(k+2)} = \frac{7}{3k}$

$\Rightarrow$ 7(k + 2) = 9k $\Rightarrow$ 7k + 14 = 9k $\Rightarrow$ 2k = 14 $\Rightarrow$ k = 7

Case III:

$\frac{2}{(k-1)} = \frac{7}{3k}$

$\Rightarrow$ 7k – 7 = 6k $\Rightarrow$ k = 7

Hence, the given system of equations has an infinite number of solutions when k is equal to 7.

16.

Sol:

The given system of equations:

2x + (k – 2)y = k

⇒ 2x + (k – 2)y - k = 0 ….(i)

And, 6x + (2k - 1) y = (2k + 5)

⇒ 6x + (2k - 1) y - (2k + 5) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= (k – 2)$, $c_1 = -k$ and $a_2 = 6$, $b_2 = (2k - 1)$, $c_2 = -(2k + 5)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{6} = \frac{(k-2)}{(2k-1)} = \frac{-k}{-(2k+5)}$

$\Rightarrow \frac{1}{3} = \frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

Now, we have the following three cases:

Case I:

$\frac{1}{3} = \frac{(k-2)}{(2k-1)}$

⇒ (2k – 1) = 3(k – 2)

⇒ 2k - 1 = 3k – 6 ⇒ k = 5

Case II:

$\frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

⇒ (k - 2) (2k + 5) = k(2k - 1)

⇒ $2k^2 + 5k – 4k – 10 = 2k^2 – k$

⇒ k + k = 10 ⇒ 2k = 10 ⇒ k = 5

Case III:

$\frac{1}{3} = \frac{k}{(2k+5)}$

⇒ 2k + 5 = 3k ⇒ k = 5

Hence, the given system of equations has an infinite number of solutions when k is equal to 5.

17.

Sol:

The given system of equations:

kx + 3y = (2k + 1)

⇒ kx + 3y - (2k + 1) = 0 ….(i)

And, $2(k + 1)x + 9y = (7k + 1)$

$\Rightarrow 2(k + 1)x + 9y - (7k + 1) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1 = 3$, $c_1 = -(2k + 1)$ and $a_2 = 2(k + 1)$, $b_2 = 9$, $c_2 = -(7k + 1)$

For an infinite number of solutions, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e., $\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

Now, we have the following three cases:

Case I:

$\frac{k}{2(k+1)} = \frac{1}{3}$

$\Rightarrow 2(k + 1) = 3k$

$\Rightarrow 2k + 2 = 3k$

$\Rightarrow k = 2$

Case II:

$\frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow (7k + 1) = 6k + 3$

$\Rightarrow k = 2$

Case III:

$\frac{k}{2(k+1)} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow k(7k + 1) = (2k + 1) \times 2(k + 1)$

$\Rightarrow 7k^2 + k = (2k + 1)(2k + 2)$

$\Rightarrow 7k^2 + k = 4k^2 + 4k + 2k + 2$

$\Rightarrow 3k^2 – 5k – 2 = 0$

$\Rightarrow 3k^2 – 6k + k – 2 = 0$

$\Rightarrow 3k(k – 2) + 1(k – 2) = 0$

$\Rightarrow (3k + 1)(k – 2) = 0$

$\Rightarrow k = 2$ or $k = \frac{-1}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 2.

18.

Sol:

The given system of equations:

5x + 2y = 2k

⇒ 5x + 2y - 2k = 0 ….(i)

And, 2(k + 1)x + ky = (3k + 4)

⇒ 2(k + 1)x + ky - (3k + 4) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1= 2$, $c_1 = -2k$ and $a_2 = 2(k + 1)$, $b_2 = k$, $c_2 = -(3k + 4)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{5}{2(k+1)} = \frac{2}{k} = \frac{-2k}{-(3k+4)}$

$\Rightarrow \frac{5}{2(k+1)} = \frac{2}{k} = \frac{2k}{(3k+4)}$

Now, we have the following three cases:

Case I:

$\frac{5}{2(k+1)} = \frac{2}{k}$

⇒ 2 × 2(k + 1) = 5k

⇒ 4(k + 1) = 5k

⇒ 4k + 4 = 5k

⇒ k = 4

Case II:

$\frac{2}{k} = \frac{2k}{(3k+4)}$

⇒ $2k^2 = 2 \times (3k + 4)$

⇒ $2k^2 = 6k + 8 \Rightarrow 2k^2 – 6k – 8 = 0$

⇒ $2(k^2 – 3k – 4) = 0$

⇒ $k^2 – 4k + k – 4 = 0$

⇒ k(k – 4) + 1(k – 4) = 0

⇒ (k + 1) (k – 4) = 0

⇒ (k + 1) = 0 or (k – 4) = 0

$\Rightarrow$ k = -1 or k = 4

Case III:

$\frac{5}{2(k+1)} = \frac{2k}{(3k+4)}$

$\Rightarrow$ 15k + 20 = $4k^2$ + 4k

$\Rightarrow$ $4k^2$ – 11k – 20 = 0

$\Rightarrow$ $4k^2$– 16k + 5k – 20 = 0

$\Rightarrow$ 4k(k – 4) + 5(k – 4) = 0

$\Rightarrow$ (k – 4) (4k + 5) = 0

$\Rightarrow$ k = 4 or k = $\frac{-5}{4}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 4.

19.

Sol:

The given system of equations:

(k – 1)x – y = 5

$\Rightarrow$ (k – 1)x – y – 5 = 0 ….(i)

And, (k + 1)x + (1 – k)y = (3k + 1)

$\Rightarrow$ (k + 1)x + (1 – k)y – (3k + 1) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, a_1 = (k – 1), b_1= -1, c_1 = -5 and a_2 = (k + 1), b_2 = (1 – k), c_2 = -(3k + 1)

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{(k-1)}{(k+1)} = \frac{-1}{-(k-1)} = \frac{-5}{-(3k+1)}$

$\Rightarrow \frac{(k-1)}{(k+1)} = \frac{1}{(k-1)} = \frac{5}{(3k+1)}$

Now, we have the following three cases:

Case I:

$\frac{(k-1)}{(k+1)} = \frac{1}{(k-1)}$

$\Rightarrow$ $(k - 1)^2$ = (k + 1)

$\Rightarrow$ k^2 + 1 – 2k = k + 1

$\Rightarrow k^2 - 3k = 0 \Rightarrow k(k - 3) = 0$

$\Rightarrow k = 0$ or $k = 3$

Case II:

$\frac{1}{(k-1)} = \frac{5}{(3k+1)}$

$\Rightarrow 3k + 1 = 5k - 5$

$\Rightarrow 2k = 6 \Rightarrow k = 3$

Case III:

$\frac{(k-1)}{(k+1)} = \frac{5}{(3k+1)}$

$\Rightarrow (3k + 1)(k - 1) = 5(k + 1)$

$\Rightarrow 3k^2 + k - 3k - 1 = 5k + 5$

$\Rightarrow 3k^2 - 2k - 5k - 1 - 5 = 0$

$\Rightarrow 3k^2 - 7k - 6 = 0$

$\Rightarrow 3k^2 - 9k + 2k - 6 = 0$

$\Rightarrow 3k(k - 3) + 2(k - 3) = 0$

$\Rightarrow (k - 3)(3k + 2) = 0$

$\Rightarrow (k - 3) = 0$ or $(3k + 2) = 0$

$\Rightarrow k = 3$ or $k = \frac{-2}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 3.

20.

Sol:

The given system of equations can be written as

$(k - 3)x + 3y - k = 0$

$kx + ky - 12 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1 = 3$, $c_1 = -k$ and $a_2 = k$, $b_2 = k$, $c_2 = -12$

For the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$

$\Rightarrow k - 3 = 3$ and $k^2 = 36$

⇒ k = 6 and k = ±6
⇒ k = 6
Hence, k = 6.

21.

Sol:
The given system of equations can be written as
(a – 1) x + 3y = 2
⇒(a – 1) x + 3y – 2 = 0 ….(i)
and 6x + (1 – 2b)y = 6
⇒6x + (1 – 2b)y – 6 = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = (a – 1)$, $b_1= 3$, $c_1 = -2$ and $a_2 = 6$, $b_2 = (1 – 2b)$, $c_2 = -6$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{-2}{-6}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow \frac{a-1}{6} = \frac{1}{3}$ and $\frac{3}{(1-2b)} = \frac{1}{3}$
⇒ 3a – 3 = 6 and 9 = 1 – 2b
⇒ 3a = 9 and 2b = -8
⇒ a = 3 and b = -4
∴ a = 3 and b = -4

22.

Sol:
The given system of equations can be written as
(2a – 1) x + 3y = 5
⇒(2a – 1) x + 3y – 5 = 0 ….(i)
and 3x + (b – 1)y = 2
⇒3x + (b – 1)y – 2 = 0 ….(ii)
These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = (2a – 1)$, $b_1= 3$, $c_1 = -5$ and $a_2 = 3$, $b_2 = (b – 1)$, $c_2 = -2$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{(2a-1)}{3} = \frac{3}{(b-1)} = \frac{-5}{-2}$

$\Rightarrow \frac{(2a-1)}{6} = \frac{3}{(b-1)} = \frac{5}{2}$

$\Rightarrow \frac{(2a-1)}{6} = \frac{5}{2}$ and $\frac{3}{(b-1)} = \frac{5}{2}$

$\Rightarrow 2(2a – 1) = 15$ and $6 = 5(b – 1)$

$\Rightarrow 4a – 2 = 15$ and $6 = 5b – 5$

$\Rightarrow 4a = 17$ and $5b = 11$

$\therefore a = \frac{17}{4}$ and $b = \frac{11}{5}$

23.

Sol:

The given system of equations can be written as

$2x - 3y = 7$

$\Rightarrow 2x - 3y - 7 = 0$ ….(i)

and $(a + b)x - (a + b – 3)y = 4a + b$

$\Rightarrow (a + b)x - (a + b – 3)y - 4a + b = 0$ ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 2$, $b_1= -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$, $c_2 = -(4a + b)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$

$\Rightarrow \frac{2}{a+b} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

$\Rightarrow \frac{2}{a+b} = \frac{7}{(4a+b)}$ and $\frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

$\Rightarrow 2(4a + b) = 7(a + b)$ and $3(4a + b) = 7(a + b - 3)$

$\Rightarrow 8a + 2b = 7a + 7b$ and $12a + 3b = 7a + 7b - 21$

$\Rightarrow 4a = 17$ and $5b = 11$

$\therefore a = 5b$ …….(iii)

and $5a = 4b – 21$ ……(iv)

On substituting $a = 5b$ in (iv), we get:

25b = 4b – 21
⇒21b = -21
⇒b = -1
On substituting b = -1 in (iii), we get:
a = 5(-1) = -5
∴a = -5 and b = -1.

24.

Sol:
The given system of equations can be written as
2x + 3y = 7
⇒2x + 3y - 7 = 0 ….(i)
and (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1
(a + b + 1)x - (a + 2b + 2)y – [4(a + b) + 1] = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (a + b + 1)$, $b_2 = (a + 2b + 2)$, $c_2 = – [4(a + b) + 1]$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{-7}{-[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)}$ and $\frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
⇒ 2(a + 2b + 2) = 3(a + b + 1) and 3[4(a + b) + 1] = 7(a + 2b + 2)
⇒ 2a + 4b + 4 = 3a + 3b + 3 and 3(4a + 4b + 1) = 7a + 14b + 14
⇒ a – b - 1=0 and 12a + 12b + 3 = 7a + 14b + 14
⇒ a - b = 1 and 5a – 2b = 11
a = (b + 1) …….(iii)
5a - 2b = 11 ……(iv)
On substituting a = (b + 1) in (iv), we get:
5(b + 1) – 2b = 11
⇒5b + 5 – 2b = 11
⇒ 3b = 6
⇒ b = 2
On substituting b = 2 in (iii), we get:
a = 3

∴ a = 3 and b = 2.

25.

Sol:
The given system of equations can be written as
2x + 3y - 7 = 0 ....(i)
(a + b)x + (2a - b)y – 21 = 0 ....(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = a + b$, $b_2 = 2a - b$, $c_2 = -21$
For the given system of linear equations to have an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{2}{a+b} = \frac{3}{2a-b} = \frac{-7}{-21}$
$\Rightarrow \frac{2}{a+b} = \frac{-7}{-21} = \frac{1}{3}$ and $\frac{3}{2a-b} = \frac{-7}{-21} = \frac{1}{3}$
⇒ a + b = 6 and 2a –b = 9
Adding a + b = 6 and 2a – b = 9 ,we get
$3a = 15 \Rightarrow a = \frac{15}{3} = 3$
Now substituting a = 5 in a + b = 6, we have
5 + b = 6 ⇒ b = 6 – 5 = 1
Hence, a = 5 and b = 1.

26.

Sol:
The given system of equations can be written as
2x + 3y - 7 = 0 ....(i)
2ax + (a + b)y – 28 = 0 ....(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = 2a$, $b_2 = a + b$, $c_2 = -28$
For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

$\Rightarrow \frac{2}{2a} = \frac{-7}{-28} = \frac{1}{4}$ and $\frac{3}{a+b} = \frac{-7}{-28} = \frac{1}{4}$

$\Rightarrow$ a = 4 and a + b = 12

Substituting a = 4 in a + b = 12, we get

4 + b = 12 $\Rightarrow$ b = 12 – 4 = 8

Hence, a = 4 and b = 8.

27.

Sol:

The given system of equations:

8x + 5y = 9

8x + 5y - 9 = 0 ….(i)

kx + 10y = 15

kx + 10y - 15 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 8$, $b_1= 5$, $c_1 = -9$ and $a_2 = k$, $b_2 = 10$, $c_2 = -15$

In order that the given system has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{8}{k} = \frac{5}{10} \neq \frac{-9}{-15}$

i.e., $\frac{8}{k} = \frac{1}{2} \neq \frac{3}{5}$

$\frac{8}{k} = \frac{1}{2}$ and $\frac{8}{k} \neq \frac{3}{5}$

$\Rightarrow$ k = 16 and k $\neq \frac{40}{3}$

Hence, the given system of equations has no solutions when k is equal to 16.

28.

Sol:

The given system of equations:

kx + 3y = 3

kx + 3y - 3 = 0 ….(i)

12x + ky = 6

12x + ky - 6 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -3$ and $a_2 = 12$, $b_2 = k$, $c_2 = –6$

In order that the given system has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{k}{12} = \frac{3}{k} \neq \frac{-3}{-6}$

$\frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{1}{2}$

$\Rightarrow k^2 = 36$ and $k \neq 6$

$\Rightarrow k = \pm 6$ and $k \neq 6$

Hence, the given system of equations has no solution when k is equal to -6.

29.

Sol:

The given system of equations:

$3x - y - 5 = 0$ ….(i)

And, $6x - 2y + k = 0$ ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 3$, $b_1= -1$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -2$, $c_2 = k$

In order that the given system has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k}$

$\Rightarrow \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow k \neq -10$

Hence, equations (i) and (ii) will have no solution if $k \neq -10$.

30.

Sol:

The given system of equations can be written as

$kx + 3y + 3 - k = 0$ ….(i)

$12x + ky - k = 0$ ….(ii)

This system of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = 3 - k$ and $a_2 = 12$, $b_2 = k$, $c_2 = –k$

For the given system of linear equations to have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow k^2 = 36$ and $-3 \neq 3 - k$

$\Rightarrow k = \pm 6$ and $k \neq 6$

$\Rightarrow k = -6$

Hence, k = -6.

31.

Sol:

The given system of equations:

$5x - 3y = 0$ ….(i)

$2x + ky = 0$ ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0,\ a_2x+b_2y+c_2 = 0$

where, $a_1 = 5,\ b_1= -3,\ c_1 = 0$ and $a_2 = 2,\ b_2 = k,\ c_2 = 0$

For a non-zero solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{5}{2} = \frac{-3}{k}$

$\Rightarrow 5k = -6 \Rightarrow k = \frac{-6}{5}$

Hence, the required value of k is $\frac{-6}{5}$.

Linear equations in two variables – 3E

32.

Sol:

Let the cost of a chair be ₹ x and that of a table be ₹ y, then

$5x + 4y = 5600$ ……..(i)

$4x + 3y = 4340$ ……..(ii)

Multiplying (i) by 3 and (ii) by 4, we get

$15x – 16x = 16800 – 17360$

$\Rightarrow -x = -560$

$\Rightarrow$ x = 560

Substituting x = 560 in (i), we have

$5 \times 560 + 4y = 5600$

$\Rightarrow 4y = 5600 - 2800$

$\Rightarrow y = \frac{2800}{4} = 700$

Hence, the cost of a chair and that a table are respectively ₹ 560 and ₹ 700.

33.

Sol:

Let the cost of a spoon be Rs.x and that of a fork be Rs.y. Then

$23x + 17y = 1770$ ……….(i)

$17x + 23y = 1830$ ………(ii)

Adding (i) and (ii), we get

$40x + 40y = 3600$

$\Rightarrow x + y = 90$ ………..(iii)

Now, subtracting (ii) from (i), we get

$6x - 6y = -60$

$\Rightarrow x - y = -10$ ………..(iv)

Adding (iii) and (iv), we get

$2x = 80 \Rightarrow x = 40$

Substituting x = 40 in (iii), we get

$40 + y = 90 \Rightarrow y = 50$

Hence, the cost of a spoon that of a fork is Rs.40 and Rs.50 respectively.

34.

Sol:

Let x and y be the number of 50-paisa and 25-paisa coins respectively. Then

$x + y = 50$ ……….(i)

$0.5x + 0.25y = 19.50$ ………(ii)

Multiplying (ii) by 2 and subtracting it from (i), we get

$0.5y = 50 - 39$

$\Rightarrow y = \frac{11}{0.5} = 22$

Subtracting y = 22 in (i), we get

$x + 22 = 50$

$\Rightarrow x = 50 - 22 = 28$

Hence, the number of 25-paisa and 50-paisa coins is 22 and 28 respectively.

35.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 137$ ……….(i)

$x - y = 43$ ………(ii)

On adding (i) and (ii), we get

$2x = 180 \Rightarrow x = 90$

On substituting $x = 90$ in (i), we get

$90 + y = 137$

$\Rightarrow y = (137 - 90) = 47$

Hence, the required numbers are 90 and 47.

36.

Sol:

Let the first number be x and the second number be y.

Then, we have:

$2x + 3y = 92$ ……….(i)

$4x - 7y = 2$ ………(ii)

On multiplying (i) by 7 and (ii) by 3, we get

$14x + 21y = 644$ ………..(iii)

$12x - 21y = 6$ ………..(iv)

On adding (iii) and (iv), we get

$26x = 650$

$\Rightarrow x = 25$

On substituting $x = 25$ in (i), we get

$2 \times 25 + 3y = 92$

$\Rightarrow 50 + 3y = 92$

$\Rightarrow 3y = (92 - 50) = 42$

$\Rightarrow y = 14$

Hence, the first number is 25 and the second number is 14.

37.

Sol:

Let the first number be x and the second number be y.

Then, we have:

$3x + y = 142$ ……….(i)
$4x - y = 138$ ………(ii)
On adding (i) and (ii), we get
$7x = 280$

$\Rightarrow x = 40$
On substituting x = 40 in (i), we get:

$3 \times 40 + y = 142$

$\Rightarrow y = (142 - 120) = 22$

$\Rightarrow y = 22$
Hence, the first number is 40 and the second number is 22.

38.

Sol:
Let the greater number be x and the smaller number be y.
Then, we have:
$25x - 45 = y$ or $2x - y = 45$ ……….(i)
$2y - 21 = x$ or $-x + 2y = 21$ ………(ii)
On multiplying (i) by 2, we get:
$4x - 2y = 90$ ………..(iii)
On adding (ii) and (iii), we get
$3x = (90 + 21) = 111$

$\Rightarrow x = 37$

On substituting x = 37 in (i), we get

$2 \times 37 - y = 45$

$\Rightarrow 74 - y = 45$

$\Rightarrow y = (74 - 45) = 29$
Hence, the greater number is 37 and the smaller number is 29.

39.

Sol:
We know:

Dividend = Divisor × Quotient + Remainder
Let the larger number be x and the smaller be y.

Then, we have:

$3x = y \times 4 + 8$ or $3x - 4y = 8$ ……….(i)

$5y = x \times 3 + 5$ or $-3x + 5y = 5$ ………(ii)

On adding (i) and (ii), we get:

$y = (8 + 5) = 13$

On substituting y = 13 in (i) we get

$3x - 4 \times 13 = 8$

$\Rightarrow 3x = (8 + 52) = 60$

$\Rightarrow x = 20$

Hence, the larger number is 20 and the smaller number is 13.

40.

Sol:

Let the required numbers be x and y.

Now, we have:

$\frac{x+2}{y+2} = \frac{1}{2}$

By cross multiplication, we get:

$2x + 4 = y + 2$

$\Rightarrow 2x - y = -2$ ……(i)

Again, we have:

$\frac{x-4}{y-4} = \frac{5}{11}$

By cross multiplication, we get:

$11x - 44 = 5y - 20$

$\Rightarrow 11x - 5y = 24$ ……(ii)

On multiplying (i) by 5, we get:

$10x - 5y = -10$

On subtracting (iii) from (ii), we get:

$x = (24 + 10) = 34$

On substituting x = 34 in (i), we get:

$2 \times 34 - y = -2$

$\Rightarrow 68 - y = -2$

$\Rightarrow y = (68 + 2) = 70$

Hence, the required numbers are 34 and 70.

41.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x - y = 14$ or $x = 14 + y$ ……….(i)

$x^2 - y^2 = 448$ ………(ii)

On substituting $x = 14 + y$ in (ii) we get

$(14 + y)^2 - y^2 = 448$

$\Rightarrow 196 + y^2 + 28y - y^2 = 448$

$\Rightarrow 196 + 28y = 448$

$\Rightarrow 28y = (448 - 196) = 252$

$\Rightarrow y = \frac{252}{28} = 9$

On substituting $y = 9$ in (i), we get:

$x = 14 + 9 = 23$

Hence, the required numbers are 23 and 9.

42.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 12$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 18$

$\Rightarrow 10y + x - 10x - y = 18$

$\Rightarrow 9y - 9x = 18$

$\Rightarrow y - x = 2$ ……..(ii)

On adding (i) and (ii), we get:

$2y = 14$

$\Rightarrow y = 7$

On substituting $y = 7$ in (i) we get

$x + 7 = 12$

$\Rightarrow x = (12 - 7) = 5$

Number = $(10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$

Hence, the required number is 57.

43.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$10x + y = 7(x + y)$

$10x + 7y = 7x + 7y$ or $3x - 6y = 0$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$(10x + y) - 27 = (10y + x)$

$\Rightarrow 10x - x + y - 10y = 27$

$\Rightarrow 9x - 9y = 27$

$\Rightarrow 9(x - y) = 27$

$\Rightarrow x - y = 3$ ……..(ii)

On multiplying (ii) by 6, we get:

$6x - 6y = 18$ ………(iii)

On subtracting (i) from (ii), we get:

$3x = 18$

$\Rightarrow x = 6$

On substituting $x = 6$ in (i) we get

$3 \times 6 - 6y = 0$

$\Rightarrow 18 - 6y = 0$

$\Rightarrow 6y = 18$

$\Rightarrow y = 3$

Number = $(10x + y) = 10 \times 6 + 3 = 60 + 3 = 63$

Hence, the required number is 63.

44.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 15$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 9$

$\Rightarrow 10y + x - 10x - y = 9$

$\Rightarrow 9y - 9x = 9$

$\Rightarrow y - x = 1$ ……..(ii)

On adding (i) and (ii), we get:

$2y = 16$

$\Rightarrow y = 8$

On substituting $y = 8$ in (i) we get

$x + 8 = 15$

$\Rightarrow x = (15 - 8) = 7$

Number $= (10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$

Hence, the required number is 78.

45.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number $= (10x + y)$

$10x + y = 4(x + y) + 3$

$\Rightarrow 10x + y = 4x + 4y + 3$

$\Rightarrow 6x - 3y = 3$

$\Rightarrow 2x - y = 1$ ……….(i)

Again, we have:

$10x + y + 18 = 10y + x$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow x - y = -2$ ……..(ii)

On subtracting (ii) from (i), we get:

$x = 3$

On substituting $x = 3$ in (i) we get

$2 \times 3 - y = 1$

$\Rightarrow y = 6 - 1 = 5$

Required number $= (10x + y) = 10 \times 3 + 5 = 30 + 5 = 35$

Hence, the required number is 35.

46.

Sol:

We know:

Dividend = Divisor $\times$ Quotient + Remainder

Let the tens and the units digits of the required number be x and y, respectively.
Required number = (10x + y)

$10x + y = (x + y) \times 6 + 0$

$\Rightarrow 10x - 6x + y - 6y = 0$

$\Rightarrow 4x - 5y = 0$ …….(i)

Number obtained on reversing its digits = (10y + x)

$\therefore 10x + y - 9 = 10y + x$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow x - y = 1$ ……..(ii)

On multiplying (ii) by 5, we get:

$5x - 5y = 5$ ……..(iii)

On subtracting (i) from (iii), we get:

x = 5

On substituting x = 5 in (i) we get

$4 \times 5 - 5y = 0$

$\Rightarrow 20 - 5y = 0$

$\Rightarrow y = 4$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 4 = 50 + 4 = 54$

Hence, the required number is 54.

47.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Then, we have:

$xy = 35$ …….(i)

Required number = (10x + y)

Number obtained on reversing its digits = (10y + x)

$\therefore (10x + y) + 18 = 10y + x$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow 9(y - x) = 18$

$\Rightarrow y - x = 2$ ……..(ii)

We know:

$(y + x)^2 - (y - x)^2 = 4xy$

$\Rightarrow (y + x) = \pm \sqrt{(y-x)^2 + 4xy}$

$\Rightarrow (y + x) = \pm \sqrt{4 + 4 \times 35} = \pm \sqrt{144} = \pm 12$

$\Rightarrow y + x = 12$ ……..(iii) ($\because$ x and y cannot be negative)

On adding (ii) and (iii), we get:

$2y = 2 + 12 = 14$

$\Rightarrow y = 7$

On substituting y = 7in (ii) we get

$7 - x = 2$

$\Rightarrow x = (7 - 2) = 5$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$

Hence, the required number is 57.

48.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Then, we have:

$xy = 18$ …….(i)

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10x + y) - 63 = 10y + x$

$\Rightarrow 9x - 9y = 63$

$\Rightarrow 9(x - y) = 63$

$\Rightarrow x - y = 7$ ……..(ii)

We know:

$(x + y)^2 - (x - y)^2 = 4xy$

$\Rightarrow (x + y) = \pm \sqrt{(x-y)^2 + 4xy}$

$\Rightarrow (x + y) = \pm \sqrt{49 + 4 \times 18}$

$= \pm \sqrt{49 + 72}$

$= \pm \sqrt{121} = \pm 11$

$\Rightarrow x + y = 11$ ……..(iii) ($\because$ x and y cannot be negative)

On adding (ii) and (iii), we get:

$2x = 7 + 11 = 18$

$\Rightarrow x = 9$

On substituting x = 9in (ii) we get

$9 - y = 7$

$\Rightarrow y = (9 - 7) = 2$

$\therefore$ Number = $(10x + y) = 10 \times 9 + 2 = 90 + 2 = 92$

Hence, the required number is 92.

49.

Sol:

Let x be the ones digit and y be the tens digit. Then

Two digit number before reversing = 10y + x

Two digit number after reversing = 10x + y

As per the question

$(10y + x) + (10x + y) = 121$

$\Rightarrow 11x + 11y = 121$

$\Rightarrow x + y = 11$ …….(i)

Since the digits differ by 3, so

$x - y = 3$ ……….(ii)

Adding (i) and (ii), we get

$2x = 14 \Rightarrow x = 7$

Putting x = 7 in (i), we get

$7 + y = 11 \Rightarrow y = 4$

Changing the role of x and y, x = 4 and y = 7

Hence, the two-digit number is 74 or 47.

50.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$x + y = 8$ ……(i)

And, $\frac{x+3}{y+3} = \frac{3}{4}$

$\Rightarrow 4(x + 3) = 3(y + 3)$

$\Rightarrow 4x + 12 = 3y + 9$

$\Rightarrow$ 4x – 3y = -3 ……(ii)
On multiplying (i) by 3, we get:
3x + 3y = 24
On adding (ii) and (iii), we get:
7x = 21
$\Rightarrow$ x = 3
On substituting x = 3 in (i), we get:
3 + y = 8
$\Rightarrow$ y = (8 – 3) = 5
$\therefore$ x = 3 and y = 5
Hence, the required fraction is $\frac{3}{5}$.

51.

Sol:
Let the required fraction be $\frac{x}{y}$.
Then, we have:
$\frac{x+2}{y} = \frac{1}{2}$
$\Rightarrow$ 2(x + 2) = y
$\Rightarrow$ 2x + 4 = y
$\Rightarrow$2x - y = -4 ……(i)
Again, $\frac{x}{y-1} = \frac{1}{3}$
$\Rightarrow$3x = 1(y – 1)
$\Rightarrow$3x – y = -1 ……(ii)
On subtracting (i) from (ii), we get:
x = (-1 + 4) = 3
On substituting x = 3 in (i), we get:
2 $\times$ 3 – y = -4
$\Rightarrow$ 6 – y = -4
$\Rightarrow$ y = (6 + 4) = 10
$\therefore$ x = 3 and y = 10
Hence, the required fraction is $\frac{3}{10}$.

52.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

y = x + 11

⇒ y – x = 11 ……(i)

Again, $\frac{x+8}{y+8} = \frac{3}{4}$

⇒4(x + 8) = 3(y + 8)

⇒4x + 32 = 3y + 24

⇒ 4x – 3y = -8 ……(ii)

On multiplying (i) by 4, we get:

4y – 4x = 44

On adding (ii) and (iii), we get:

y = (-8 + 44) = 36

On substituting y = 36 in (i), we get:

36 – x = 11

⇒ x = (36 – 11) = 25

∴ x = 25 and y = 36

Hence, the required fraction is $\frac{25}{36}$.

53.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$\frac{x-1}{y+2} = \frac{1}{2}$

⇒ 2(x – 1) = 1(y + 2)

⇒2x – 2 = y + 2

⇒2x – y = 4 ……(i)

Again, $\frac{x-7}{y-2} = \frac{1}{3}$

⇒3(x – 7) = 1(y – 2)

$\Rightarrow 3x - 21 = y - 2$

$\Rightarrow 3x - y = 19$ ……(ii)

On subtracting (i) from (ii), we get:

$x = (19 - 4) = 15$

On substituting x = 15 in (i), we get:

$2 \times 15 - y = 4$

$\Rightarrow 30 - y = 4$

$\Rightarrow y = 26$

$\therefore$ x = 15 and y = 26

Hence, the required fraction is $\frac{15}{26}$.

54.

Sol:

Let the required fraction be x/y

As per the question

$x + y = 4 + 2x$

$\Rightarrow y - x = 4$ ……(i)

After changing the numerator and denominator

New numerator = x + 3

New denominator = y + 3

Therefore

$\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow 3(x + 3) = 2(y + 3)$

$\Rightarrow 3x + 9 = 2y + 6$

$\Rightarrow 2y - 3x = 3$ ……(ii)

Multiplying (i) by 3 and subtracting (ii), we get:

$3y - 2y = 12 - 3$

$\Rightarrow y = 9$

Now, putting y = 9 in (i), we get:

$9 - x = 4 \Rightarrow x = 9 - 4 = 5$

Hence, the required fraction is $\frac{5}{9}$.

55.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 16$ ……(i)

And, $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$ ……(ii)

$\Rightarrow 3(x + y) = xy$

$\Rightarrow 3 \times 16 = xy$ [Since from (i), we have: $x + y = 16$]

$\therefore xy = 48$ …….(iii)

We know:

$(x - y)^2 = (x + y)^2 - 4xy$

$(x - y)^2 = (16)^2 - 4 \times 48 = 256 - 192 = 64$

$\therefore (x - y) = \pm\sqrt{64} = \pm 8$

Since x is larger and y is smaller, we have:

$x - y = 8$ ………(iv)

On adding (i) and (iv), we get:

$2x = 24$

$\Rightarrow x = 12$

On substituting $x = 12$ in (i), we get:

$12 + y = 16 \Rightarrow y = (16 - 12) = 4$

Hence, the required numbers are 12 and 4.

56.

Sol:

Let the number of students in classroom A be x

Let the number of students in classroom B be y.

If 10 students are transferred from A to B, then we have:

$x - 10 = y + 10$

$\Rightarrow x - y = 20$ …..(i)

If 20 students are transferred from B to A, then we have:

$2(y - 20) = x + 20$

$\Rightarrow 2y - 40 = x + 20$

$\Rightarrow -x + 2y = 60$ …..(ii)

On adding (i) and (ii), we get:
y = (20 + 60) = 80
On substituting y = 80 in (i), we get:
x – 80 = 20

$\Rightarrow$x = (20 + 80) = 100
Hence, the number of students in classroom A is 100 and the number of students in classroom B is 80.

57.

Sol:
Let fixed charges be Rs.x and rate per km be Rs.y.
Then as per the question
x + 80y = 1330 ………(i)
x + 90y = 1490 ……..(ii)
Subtracting (i) from (ii), we get
$10y = 160 \Rightarrow y = \frac{160}{10} = 16$
Now, putting y = 16, we have
x + 80 × 16 = 1330

$\Rightarrow$x = 1330 – 1280 = 50

Hence, the fixed charges be Rs.50 and the rate per km is Rs.16.

58.

Sol:
Let the fixed charges be Rs.x and the cost of food per day be Rs.y.
Then as per the question
x + 25y = 4500 ………(i)
x + 30y = 5200 ……..(ii)
Subtracting (i) from (ii), we get
$5y = 700 \Rightarrow y = \frac{700}{5} = 140$
Now, putting y = 140, we have
x + 25 × 140 = 4500

$\Rightarrow$x = 4500 – 3500 = 1000

Hence, the fixed charges be Rs.1000 and the cost of the food per day is Rs.140.

59.

Sol:

Let the amounts invested at 10% and 8% be Rs.x and Rs.y respectively.

Then as per the question

$\frac{x \times 10 \ \times 1}{100} = \frac{y \times 8 \times 1}{100} = 1350$

10x + 8y = 135000 …………..(i)

After the amounts interchanged but the rate being the same, we have

$\frac{x \times 8 \ \times 1}{100} = \frac{y \times 10 \times 1}{100} = 1350 - 45$

8x + 10y = 130500 …………….(ii)

Adding (i) and (ii) and dividing by 9, we get

2x + 2y = 29500 ……………(iii)

Subtracting (ii) from (i), we get

2x – 2y = 4500

Now, adding (iii) and (iv), we have

4x = 34000

x = $\frac{34000}{4}$ = 8500

Putting x = 8500 in (iii), we get

2 × 8500 + 2y = 29500

2y = 29500 – 17000 = 12500

y = $\frac{12500}{2}$ = 6250

Hence, the amounts invested are Rs. 8,500 at 10% and Rs. 6,250 at 8%.

60.

Sol:

Let the monthly income of A and B are Rs.x and Rs.y respectively.

Then as per the question

$\frac{x}{y} = \frac{5}{4}$

⇒ y = $\frac{4x}{5}$

Since each save Rs.9,000, so

Expenditure of A = Rs.(x – 9000)

Expenditure of B = Rs.(y – 9000)

The ratio of expenditures of A and B are in the ratio 7:5.

$\therefore \frac{x-9000}{y-9000} = \frac{7}{5}$

$\Rightarrow 7y - 63000 = 5x - 45000$

$\Rightarrow 7y - 5x = 18000$

From (i), substitute $y = \frac{4x}{5}$ in (ii) to get

$7 \times \frac{4x}{5} - 5x = 18000$

$\Rightarrow 28x - 25x = 90000$

$\Rightarrow 3x = 90000$

$\Rightarrow x = 30000$

Now, putting x = 30000, we get

$y = \frac{4 \times 30000}{5} = 4 \times 6000 = 24000$

Hence, the monthly incomes of A and B are Rs. 30,000 and Rs.24,000.

61.

Sol:

Let the cost price of the chair and table be Rs.x and Rs.y respectively.

Then as per the question

Selling price of chair + Selling price of table = 1520

$\frac{100+25}{100} \times x + \frac{100+10}{100} \times y = 1520$

$\Rightarrow \frac{125}{100} x + \frac{110}{100} y = 1520$

$\Rightarrow 25x + 22y - 30400 = 0$ ……….(i)

When the profit on chair and table are 10% and 25% respectively, then

$\frac{100+10}{100} \times x + \frac{100+25}{100} \times y = 1535$

$\Rightarrow \frac{110}{100} x + \frac{125}{100} y = 1535$

$\Rightarrow 22x + 25y - 30700 = 0$ ……….(ii)

Solving (i) and (ii) by cross multiplication, we get

$\frac{x}{(22)(-30700)-(25)(-30400)} = \frac{y}{(-30400)(22)-(-30700)(25)} = \frac{1}{(25)(25)-(22)(22)}$

$\Rightarrow \frac{x}{7600-6754} = \frac{y}{7675-6688} = \frac{100}{3 \times 47}$

$\Rightarrow \frac{x}{846} = \frac{y}{987} = \frac{100}{3 \times 47}$

$\Rightarrow x = \frac{100 \times 846}{3 \times 47}, y = \frac{100 \times 987}{3 \times 47}$

⇒ x = 600, y = 700

Hence, the cost of chair and table are Rs.600 and Rs.700 respectively.

62.

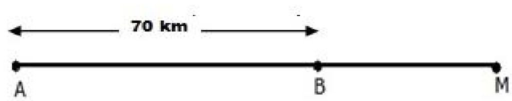
Sol:

Let X and Y be the cars starting from points A and B, respectively and let their speeds be x km/h and y km/h, respectively.

Then, we have the following cases:

Case I: When the two cars move in the same direction

In this case, let the two cars meet at point M.



Distance covered by car X in 7 hours = 7x km

Distance covered by car Y in 7 hours = 7y km

∴ AM = (7x) km and BM = (7y) km

⇒(AM – BM) = AB

⇒(7x – 7y) = 70

⇒7(x – y) = 70

⇒(x – y) = 10 ……..(i)

Case II: When the two cars move in opposite directions

In this case, let the two cars meet at point N.

Distance covered by car X in 1 hour = x km

Distance covered by car Y in 1 hour = y km

∴ AN = x km and BN = y km

⇒ AN + BN = AB

⇒ x + y = 70 ………(ii)

On adding (i) and (ii), we get:

2x = 80

⇒x = 40

On substituting x = 40 in (i), we get:

40 – y = 10

⇒y = (40 – 10) = 30

Hence, the speed of car X is 40km/h and the speed of car Y is 30km/h.

63.

Sol:
Let the original speed be x kmph and let the time taken to complete the journey be y hours.
∴ Length of the whole journey = (xy) km
Case I:
When the speed is (x + 5) kmph and the time taken is (y – 3) hrs:
Total journey = (x + 5) (y – 3) km

$\Rightarrow (x + 5)(y - 3) = xy$

$\Rightarrow xy + 5y - 3x - 15 = xy$

$\Rightarrow 5y - 3x = 15$ ………(i)

Case II:
When the speed is (x – 4) kmph and the time taken is (y + 3) hrs:
Total journey = (x – 4) (y + 3) km

$\Rightarrow (x - 4)(y + 3) = xy$

$\Rightarrow xy - 4y + 3x - 12 = xy$

$\Rightarrow 3x - 4y = 12$ ………(ii)

On adding (i) and (ii), we get:
y = 27
On substituting y = 27 in (i), we get:

$5 \times 27 - 3x = 15$

$\Rightarrow 135 - 3x = 15$

$\Rightarrow 3x = 120$

$\Rightarrow x = 40$

∴ Length of the journey = (xy) km = (40×27) km = 1080 km

64.

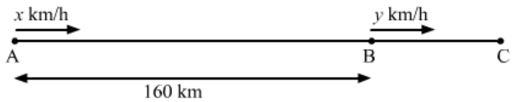
Sol:
Let the speed of the train and taxi be x km/h and y km/h respectively. Then as per the question

$\frac{3}{x} + \frac{2}{y} = \frac{11}{200}$ …………(i)

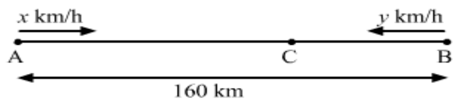
When the speeds of the train and taxi are 260 km and 240 km respectively, then
$\frac{260}{x} + \frac{240}{y} = \frac{11}{2} + \frac{6}{60}$
$\Rightarrow \frac{13}{x} + \frac{12}{y} = \frac{28}{100}$ …………..(ii)
Multiplying (i) by 6 and subtracting (ii) from it, we get
$\frac{18}{x} - \frac{13}{x} = \frac{66}{200} - \frac{28}{100}$
$\Rightarrow \frac{5}{x} = \frac{10}{200} \Rightarrow$ x = 100
Putting x = 100 in (i), we have
$\frac{3}{100} + \frac{2}{y} = \frac{11}{200}$
$\Rightarrow \frac{2}{y} = \frac{11}{200} - \frac{3}{100} = \frac{1}{40}$
$\Rightarrow$ y = 80
Hence, the speed of the train and that of the taxi are 100 km/h and 80 km/h respectively.

65.

Sol:
Let the speed of the car A and B be x km/h and y km/h respectively. Let x > y.
Case-1: When they travel in the same direction



From the figure
AC – BC = 160
$\Rightarrow x \times 8 - y \times 8 = 160$
$\Rightarrow x - y = 20$
Case-2: When they travel in opposite direction



From the figure
AC + BC = 160
$\Rightarrow x \times 2 + y \times 2 = 160$
$\Rightarrow x + y = 80$
Adding (i) and (ii), we get
$2x = 100 \Rightarrow x = 50$ km/h

Putting x = 50 in (ii), we have

50 + y = 80 ⇒ y = 80 – 50 = 30 km/h

Hence, the speeds of the cars are 50 km/h and 30 km/h.

66.

Sol:

Let the speed of the sailor in still water be x km/h and that of the current y km/h.

Speed downstream = (x + y) km/h

Speed upstream = (x – y) km/h

As per the question

$(x + y) \times \frac{40}{60} = 8$

$\Rightarrow x + y = 12$ ………(i)

When the sailor goes upstream, then

$(x – y) \times 1 = 8$

$x – y = 8$ ………(ii)

Adding (i) and (ii), we get

2x = 20 ⇒ x = 10

Putting x = 10 in (i), we have

10 + y = 12 ⇒ y = 2

Hence, the speeds of the sailor in still water and the current are 10 km/h and 2 km/h respectively.

67.

Sol:

Let the speed of the boat in still water be x km/h and the speed of the stream be y km/h.

Then we have

Speed upstream = (x – y) km/hr

Speed downstream = (x + y) km/hr

Time taken to cover 12 km upstream = $\frac{12}{(x-y)}$ hrs

Time taken to cover 40 km downstream = $\frac{40}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{12}{(x-y)} + \frac{40}{(x+y)} = 8$ ……….(i)

Again, we have:

Time taken to cover 16 km upstream = $\frac{16}{(x-y)}$ hrs

Time taken to cover 32 km downstream = $\frac{32}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{16}{(x-y)} + \frac{32}{(x+y)} = 8$ ……….(ii)

Putting $\frac{1}{(x-y)}$ = u and $\frac{1}{(x+y)}$ = v in (i) and (ii), we get:

12u + 40v = 8

3u + 10v = 2 ………(a)

And, 16u + 32v = 8

⇒2u + 4v = 1 ……….(b)

On multiplying (a) by 4 and (b) by 10, we get:

12u + 40v = 8 …….(iii)

And, 20u + 40v = 10 …….(iv)

On subtracting (iii) from (iv), we get:

8u = 2

⇒ u = $\frac{2}{8} = \frac{1}{4}$

On substituting u = $\frac{1}{4}$ in (iii), we get:

40v = 5

⇒ v = $\frac{5}{40} = \frac{1}{8}$

Now, we have:

u = $\frac{1}{4}$

⇒ $\frac{1}{(x-y)} = \frac{1}{4}$ ⇒ x – y = 4 ……..(v)

v = $\frac{1}{8}$

⇒ $\frac{1}{(x+y)} = \frac{1}{8}$ ⇒ x + y = 8 ……..(vi)

On adding (v) and (vi), we get:

2x = 12

⇒ x = 6

On substituting x = 6 in (v), we get:

6 – y = 4

y = (6 – 4) = 2

∴ Speed of the boat in still water = 6km/h

And, speed of the stream = 2 km/h

68.

Sol:

Let us suppose that one man alone can finish the work in x days and one boy alone can finish it in y days.

$\therefore$ One man's one day's work $= \frac{1}{x}$

And, one boy's one day's work $= \frac{1}{y}$

2 men and 5 boys can finish the work in 4 days.

$\therefore$ (2 men's one day's work) + (5 boys' one day's work) $= \frac{1}{4}$

$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

$\Rightarrow 2u + 5v = \frac{1}{4}$ ....(i) Here, $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Again, 3 men and 6 boys can finish the work in 3days.

$\therefore$ (3 men's one day's work) + (6 boys' one day's work) $= \frac{1}{3}$

$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

$\Rightarrow 3u + 6v = \frac{1}{3}$ ....(ii) Here, $\frac{1}{x} = u$ and $\frac{1}{y} = v$

On multiplying (iii) from (iv), we get:

$3u = \left(\frac{5}{3} - \frac{6}{4}\right) = \frac{2}{12} = \frac{1}{6}$

$\Rightarrow u = \frac{1}{6 \times 3} = \frac{1}{18} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$

On substituting $u = \frac{1}{18}$ in (i), we get:

$2 \times \frac{1}{18} + 5v = \frac{1}{4} \Rightarrow 5v = \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}$

$\Rightarrow v = \left(\frac{5}{36} \times \frac{1}{5}\right) = \frac{1}{36} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$

Hence, one man alone can finish the work is 18days and one boy alone can finish the work in 36 days.

69.

Sol:

Let the length of the room be x meters and he breadth of the room be y meters.

Then, we have:

Area of the room = xy

According to the question, we have:

x = y + 3

$\Rightarrow x - y = 3$ ....(i)

And, $(x + 3)(y - 2) = xy$

$\Rightarrow xy - 2x + 3y - 6 = xy$

$\Rightarrow 3y - 2x = 6$ ....(ii)

On multiplying (i) by 2, we get:

$2x - 2y = 6$ ....(iii)

On adding (ii) and (iii), we get:

$y = (6 + 6) = 12$

On substituting y = 12 in (i), we get:

$x - 12 = 3$

$\Rightarrow x = (3 + 12) = 15$

Hence, the length of the room is 15 meters and its breadth is 12 meters.

70.

Sol:

Let the length and the breadth of the rectangle be x m and y m, respectively.

$\therefore$ Area of the rectangle = (xy) sq.m

Case 1:

When the length is reduced by 5m and the breadth is increased by 3 m:

New length = $(x - 5)$ m

New breadth = $(y + 3)$ m

$\therefore$ New area = $(x - 5)(y + 3)$ sq.m

$\therefore xy - (x - 5)(y + 3) = 8$

$\Rightarrow xy - [xy - 5y + 3x - 15] = 8$

$\Rightarrow xy - xy + 5y - 3x + 15 = 8$

$\Rightarrow 3x - 5y = 7$ ....(i)

Case 2:

When the length is increased by 3 m and the breadth is increased by 2 m:

New length = $(x + 3)$ m

New breadth = $(y + 2)$ m

$\therefore$ New area = $(x + 3)(y + 2)$ sq.m

$\Rightarrow (x + 3)(y + 2) - xy = 74$

$\Rightarrow [xy + 3y + 2x + 6] - xy = 74$

$\Rightarrow 2x + 3y = 68$ ....(ii)

On multiplying (i) by 3 and (ii) by 5, we get:
$9x - 15y = 21$ ……….(iii)
$10x + 15y = 340$ ………(iv)
On adding (iii) and (iv), we get:
$19x = 361$

$\Rightarrow x = 19$
On substituting x = 19 in (iii), we get:

$9 \times 19 - 15y = 21$

$\Rightarrow 171 - 15y = 21$

$\Rightarrow 15y = (171 - 21) = 150$

$\Rightarrow y = 10$
Hence, the length is 19m and the breadth is 10m.

71.

Sol:
Let the length and the breadth of the rectangle be x m and y m, respectively.
Case 1: When length is increased by 3m and the breadth is decreased by 4m:
$xy - (x + 3)(y - 4) = 67$

$\Rightarrow xy - xy + 4x - 3y + 12 = 67$

$\Rightarrow 4x - 3y = 55$ ………(i)
Case 2: When length is reduced by 1m and breadth is increased by 4m:
$(x - 1)(y + 4) - xy = 89$

$\Rightarrow xy + 4x - y - 4 - xy = 89$

$\Rightarrow 4x - y = 93$ ………(ii)
Subtracting (i) and (ii), we get:

$2y = 38 \Rightarrow y = 19$
On substituting y = 19 in (ii), we have
$4x - 19 = 93$

$\Rightarrow 4x = 93 + 19 = 112$

$\Rightarrow x = 28$
Hence, the length = 28m and breadth = 19m.

72.

Sol:

Let the basic first class full fare be Rs.x and the reservation charge be Rs.y.

Case 1: One reservation first class full ticket cost Rs.4, 150

x + y = 4150 ………(i)

Case 2: One full and one and half reserved first class tickets cost Rs.6,255

$(x + y) + \left(\frac{1}{2}x + y\right) = 6255$

⇒ 3x + 4y = 12510 ………(ii)

Substituting y = 4150 – x from (i) in (ii), we get

3x + 4(4150 – x) = 12510

⇒3x – 4x + 16600 = 12510

⇒x = 16600 – 12510 = 4090

Now, putting x = 4090 in (i), we have

4090 + y = 4150

⇒ y = 4150 – 4090 = 60

Hence, cost of basic first class full fare = Rs.4,090 and reservation charge = Rs.60.

73.

Sol:

Let the present age of the man be x years and that of his son be y years.

After 5 years man's age = x + 5

After 5 years ago son's age = y + 5

As per the question

x + 5 = 3(y + 5)

⇒ x – 3y = 10 ……………(i)

5 years ago man's age = x – 5

5 years ago son's age = y – 5

As per the question

x – 5 = 7(y – 5)

⇒ x – 7y = -30 …….(ii)

Subtracting (ii) from (i), we have

4y = 40 ⇒ y = 10

Putting y = 10 in (i), we get

$x – 3 \times 10 = 10$

⇒ x = 10 + 30 = 40

Hence, man's present age = 40 years and son's present age = 10 years.

74.

Sol:
Let the man's present age be x years.
Let his son's present age be y years.
According to the question, we have:
Two years ago:
Age of the man = Five times the age of the son

$\Rightarrow (x - 2) = 5(y - 2)$

$\Rightarrow x - 2 = 5y - 10$

$\Rightarrow x - 5y = -8$ …….(i)

Two years later:
Age of the man = Three times the age of the son + 8

$\Rightarrow (x + 2) = 3(y + 2) + 8$

$\Rightarrow x + 2 = 3y + 6 + 8$

$\Rightarrow x - 3y = 12$ …………(ii)

Subtracting (i) from (ii), we get:
$2y = 20$

$\Rightarrow y = 10$

On substituting y = 10 in (i), we get:

$x - 5 \times 10 = -8$

$\Rightarrow x - 50 = -8$

$\Rightarrow x = (-8 + 50) = 42$

Hence, the present age of the man is 42 years and the present age of the son is 10 years.

75.

Sol:
Let the mother's present age be x years.
Let her son's present age be y years.
Then, we have:
$x + 2y = 70$ …….(i)
And, $2x + y = 95$ ……(ii)

On multiplying (ii) by 2, we get:
$4x + 2y = 190$ ……..(iii)
On subtracting (i) from (iii), we get:
$3x = 120$
$\Rightarrow x = 40$
On substituting x = 40 in (i), we get:
$40 + 2y = 70$
$\Rightarrow 2y = (70 - 40) = 30$
$\Rightarrow y = 15$
Hence, the mother's present age is 40 years and her son's present age is 15 years.

76.

Sol:
Let the woman's present age be x years.
Let her daughter's present age be y years.
Then, we have:
$x = 3y + 3$
$\Rightarrow x - 3y = 3$ …….(i)
After three years, we have:
$(x + 3) = 2(y + 3) + 10$
$\Rightarrow x + 3 = 2y + 6 + 10$
$\Rightarrow x - 2y = 13$ ……(ii)
Subtracting (ii) from (i), we get:
$-y = (3 - 13) = -10$
$\Rightarrow y = 10$
On substituting y = 10 in (i), we get:
$x - 3 \times 10 = 3$
$\Rightarrow x - 30 = 3$
$\Rightarrow x = (3 + 30) = 33$
Hence, the woman's present age is 33 years and her daughter's present age is 10 years.

77.

Sol:

Let the actual price of the tea and lemon set be Rs.x and Rs.y respectively.
When gain is Rs.7, then

$\frac{y}{100} \times 15 - \frac{x}{100} \times 5 = 7$

$\Rightarrow 3y - x = 140$ ……..(i)

When gain is Rs.14, then

$\frac{y}{100} \times 5 + \frac{x}{100} \times 10 = 14$

$\Rightarrow y + 2x = 280$ ……..(ii)

Multiplying (i) by 2 and adding with (ii), we have

$7y = 280 + 280$

$\Rightarrow y = \frac{560}{7} = 80$

Putting y = 80 in (ii), we get

$80 + 2x = 280$

$\Rightarrow x = \frac{200}{2} = 100$

Hence, actual price of the tea set and lemon set are Rs.100 and Rs.80 respectively.

78.

Sol:

Let the fixed charge be Rs.x and the charge for each extra day be Rs.y.
In case of Mona, as per the question

$x + 4y = 27$ ………(i)

In case of Tanvy, as per the question

$x + 2y = 21$ ………(ii)

Subtracting (ii) from (i), we get

$2y = 6 \Rightarrow y = 3$

Now, putting y = 3 in (ii), we have

$x + 2 \times 3 = 21$

$\Rightarrow x = 21 - 6 = 15$

Hence, the fixed charge be Rs.15 and the charge for each extra day is Rs.3.

79.

Sol:

Let x litres and y litres be the amount of acids from 50% and 25% acid solutions respectively.
As per the question

50% of x + 25% of y = 40% of 10

$\Rightarrow$ 0.50x + 0.25y = 4

$\Rightarrow$ 2x + y = 16 ………(i)

Since, the total volume is 10 liters, so

x + y = 10

Subtracting (ii) from (i), we get

x = 6

Now, putting x = 6 in (ii), we have

6 + y = 10 $\Rightarrow$ y = 4

Hence, volume of 50% acid solution = 6litres and volume of 25% acid solution = 4litres.

80.

Sol:

Let x g and y g be the weight of 18-carat and 12- carat gold respectively.

As per the given condition

$\frac{18x}{24} + \frac{12y}{24} = \frac{120 \times 16}{24}$

$\Rightarrow$3x + 2y = 320 …………(i)

And

x + y = 120 …………(ii)

Multiplying (ii) by 2 and subtracting from (i), we get

x = 320 – 240 = 80

Now, putting x = 80 in (ii), we have

80 + y = 120 $\Rightarrow$ y = 40

Hence, the required weight of 18-carat and 12-carat gold bars are 80 g and 40 g respectively.

81.

Sol:

Let x litres and y litres be respectively the amount of 90% and 97% pure acid solutions.

As per the given condition

0.90x + 0.97y = 21 $\times$ 0.95

$\Rightarrow$ 0.90x + 0.97y = 21 $\times$ 0.95 ……….(i)

And

x + y = 21

From (ii), substitute y = 21 – x in (i) to get

0.90x + 0.97(21 – x) = 21 $\times$ 0.95

$\Rightarrow$ 0.90x + 0.97$\times$ 21 – 0.97x = 21 $\times$ 0.95

$\Rightarrow 0.07x = 0.97\times 21 - 21 \times 0.95$

$\Rightarrow x = \frac{21 \times 0.02}{0.07} = 6$

Now, putting x = 6 in (ii), we have

$6 + y = 21 \Rightarrow y = 15$

Hence, the request quantities are 6 litres and 15 litres.

82.

Sol:

Let x and y be the supplementary angles, where x > y.

As per the given condition

$x + y = 180^0$ …….(i)

And

$x - y = 18^0$ ……..(ii)

Adding (i) and (ii), we get

$2x = 198^0 \Rightarrow x = 99^0$

Now, substituting $x = 99^0$ in (ii), we have

$99^0 - y = 18^0 \Rightarrow x = 99^0 - 18^0 = 81^0$

Hence, the required angles are $99^0$and 81^0.

83.

Sol:

$\because \angle C - \angle B = 9^0$

$\therefore y^0 - (3x - 2)^0 = 9^0$

$\Rightarrow y^0 - 3x^0 + 2^0 = 9^0$

$\Rightarrow y^0 - 3x^0 = 7^0$

The sum of all the angles of a triangle is 180^0, therefore

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x^0 + (3x - 2)^0 + y^0 = 180^0$

$\Rightarrow 4x^0 + y^0 = 182^0$

Subtracting (i) from (ii), we have

$7x^0 = 182^0 - 7^0 = 175^0$

$\Rightarrow x^0 = 25^0$

Now, substituting $x^0 = 25^0$ in (i), we have

$y^0 = 3x^0 + 7^0 = 3 \times 25^0 + 7^0 = 82^0$

Thus

$\angle A = x^0 = 25^0$

$\angle B = (3x - 2)^0 = 75^0 - 2^0 = 73^0$
$\angle C = y^0 = 82^0$
Hence, the angles are 25^0, 73^0 and 82^0.

84.

Sol:
The opposite angles of cyclic quadrilateral are supplementary, so
$\angle A + \angle C = 180^0$
$\Rightarrow (2x + 4)^0 + (2y + 10)^0 = 180^0$
$\Rightarrow x + y = 83^0$
And
$\angle B + \angle D = 180^0$
$\Rightarrow (y + 3)^0 + (4x - 5)^0 = 180^0$
$\Rightarrow 4x + y = 182^0$
Subtracting (i) from (ii), we have
$3x = 99 \Rightarrow x = 33^0$
Now, substituting $x = 33^0$ in (i), we have
$33^0 + y = 83^0 \Rightarrow y = 83^0 - 33^0 = 50^0$
Therefore
$\angle A = (2x + 4)^0 = (2 \times 33 + 4)^0 = 70^0$
$\angle B = (y + 3)^0 = (50 + 3)^0 = 53^0$
$\angle C = (2y + 10)^0 = (2 \times 50 + 10)^0 = 110^0$
$\angle D = (4x - 5)^0 = (4 \times 33 - 5)^0 = 132^0 - 5^0 = 127^0$
Hence, $\angle A = 70^0$, $\angle B = 53^0$, $\angle C = 110^0$ and $\angle D = 127^0$.

Exercise – 3F

1.

Sol:
The given equations are
$x + 2y - 8 = 0$ ……(i)
$2x + 4y - 16 = 0$ ……(ii)
Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where
$a_1 = 1, b_1 = 2, c_1 = -8, a_2 = 2, b_2 = 4$ and $c_2 = -18$

Now

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Thus, the pair of linear equations are coincident and therefore has infinitely many solutions.

2.

Sol:

The given equations are

$2x + 3y - 7 = 0$ ……(i)

$(k - 1)x + (k + 2)y - 3k = 0$ ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 2$, $b_1 = 3$, $c_1 = -7$, $a_2 = k - 1$, $b_2 = k + 2$ and $c_2 = -3k$

For the given pair of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$

$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2}, \frac{3}{k+2} = \frac{-7}{-3k}$ and $\frac{2}{k-1} = \frac{-7}{-3k}$

$\Rightarrow 2(k + 2) = 3(k - 1)$, $9k = 7k + 14$ and $6k = 7k - 7$

$\Rightarrow k = 7$, $k = 7$ and $k = 7$

Hence, $k = 7$.

3.

Sol:

The given pair of linear equations are

$10x + 5y - (k - 5) = 0$ ……(i)

$20x + 10y - k = 0$ ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 10$, $b_1 = 5$, $c_1 = -(k - 5)$, $a_2 = 20$, $b_2 = 10$ and $c_2 = -k$

For the given pair of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{10}{20} = \frac{5}{10} = \frac{-(k-5)}{-k}$

$\Rightarrow \frac{1}{2} = \frac{k-5}{k}$

$\Rightarrow 2k - 10 = k \Rightarrow k = 10$

Hence, k = 10.

4.

Sol:

The given pair of linear equations are

$2x + 3y - 9 = 0$ ……(i)

$6x + (k - 2)y - (3k - 2) = 0$ ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 2$, $b_1 = 3$, $c_1 = -9$, $a_2 = 6$, $b_2 = k - 2$ and $c_2 = -(3k - 2)$

For the given pair of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{6} = \frac{3}{k-2} \neq \frac{-9}{-(3k-2)}$

$\Rightarrow \frac{2}{6} = \frac{3}{k-2}, \frac{3}{k-2} \neq \frac{-9}{-(3k-2)}$

$\Rightarrow k = 11, \frac{3}{k-2} \neq \frac{9}{(3k-2)}$

$\Rightarrow k = 11, 3(3k - 2) \neq 9(k - 2)$

$\Rightarrow k = 11, 1 \neq 3$ (true)

Hence, k = 11.

5.

Sol:

The given pair of linear equations are

$x + 3y - 4 = 0$ ……(i)

$2x + 6y - 7 = 0$ ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 1$, $b_1 = 3$, $c_1 = -4$, $a_2 = 2$, $b_2 = 6$ and $c_2 = -7$

Now

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-4}{-7} = \frac{4}{7}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the pair of the given linear equations has no solution.

6.

Sol:

The given pair of linear equations are

$3x + ky = 0$ ……(i)

$2x – y = 0$ ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 3$, $b_1 = k$, $c_1 = 0$, $a_2 = 2$, $b_2 = -1$ and $c_2 = 0$

For the system to have a unique solution, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{3}{2} \neq \frac{k}{-1}$

$\Rightarrow k \neq -\frac{3}{2}$

Hence, $k \neq -\frac{3}{2}$.

7.

Sol:

Let the numbers be x and y, where x > y.

Then as per the question

$x – y = 5$ …….(i)

$x^2 – y^2 = 65$ …….(ii)

Dividing (ii) by (i), we get

$\frac{x^2 - y^2}{x-y} = \frac{65}{5}$

$\Rightarrow \frac{(x-y)(x+y)}{x-y} = 13$

$\Rightarrow x + y = 13$ ……….(iii)

Now, adding (i) and (ii), we have

$2x = 18 \Rightarrow x = 9$

Substituting x = 9 in (iii), we have

$9 + y = 13 \Rightarrow y = 4$

Hence, the numbers are 9 and 4.

8.

Sol:

Let the cost of 1 pen and 1 pencil are ₹x and ₹y respectively.

Then as per the question

$5x + 8y = 120$ ....(i)

$8x + 5y = 153$ ....(ii)

Adding (i) and (ii), we get

$13x + 13y = 273$

$\Rightarrow x + y = 21$ ....(iii)

Subtracting (i) from (ii), we get

$3x - 3y = 33$

$\Rightarrow x - y = 11$ ....(iv)

Now, adding (iii) and (iv), we get

$2x = 32 \Rightarrow x = 16$

Substituting $x = 16$ in (iii), we have

$16 + y = 21 \Rightarrow y = 5$

Hence, the cost of 1 pen and 1 pencil are respectively ₹16 and ₹5.

9.

Sol:

Let the larger number be x and the smaller number be y.

Then as per the question

$x + y = 80$ ....(i)

$x = 4y + 5$

$x - 4y = 5$ ....(ii)

Subtracting (ii) from (i), we get

$5y = 75 \Rightarrow y = 15$

Now, putting $y = 15$ in (i), we have

$x + 15 = 80 \Rightarrow x = 65$

Hence, the numbers are 65 and 15.

10.

Sol:

Let the ones digit and tens digit be x and y respectively.

Then as per the question

$x + y = 10$ …….(i)

$(10y + x) - 18 = 10x + y$

$x - y = -2$ …….(ii)

Adding (i) and (ii), we get

$2x = 8 \Rightarrow x = 4$

Now, putting $x = 4$ in (i), we have

$4 + y = 10 \Rightarrow y = 6$

Hence, the number is 64.

11.

Sol:

Let the number of stamps of 20p and 25p be x and y respectively.

Then as per the question

$x + y = 47$ …….(i)

$0.20x + 0.25y = 10$

$4x + 5y = 200$ …….(ii)

From (i), we get

$y = 47 - x$

Now, substituting $y = 47 - x$ in (ii), we have

$4x + 5(47 - x) = 200$

$\Rightarrow 4x - 5x + 235 = 200$

$\Rightarrow x = 235 - 200 = 35$

Putting $x = 35$ in (i), we get

$35 + y = 47$

$\Rightarrow y = 47 - 35 = 12$

Hence, the number of 20p stamps and 25p stamps are 35 and 12 respectively.

12.

Sol:

Let the number of hens and cow be x and y respectively.

As per the question

$x + y = 48$ …….(i)

$2x + 4y = 140$

$x + 2y = 70$ …….(ii)

Subtracting (i) from (ii), we have

$y = 22$

Hence, the number of cows is 22.

13.

Sol:
The given pair of equation is
$\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$ ………(i)
$\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$ ………(ii)
Multiplying (i) and (ii) by xy, we have
3x + 2y = 9 ……….(iii)
9x + 4y = 21 ………(iv)
Now, multiplying (iii) by 2 and subtracting from (iv), we get
$9x - 6x = 21 - 18 \Rightarrow x = \frac{3}{3} = 1$
Putting x = 1 in (iii), we have
$3 \times 1 + 2y = 9 \Rightarrow y = \frac{9-3}{2} = 3$
Hence, x = 1 and y = 3.

14.

Sol:
The given pair of equations is
$\frac{x}{4} + \frac{y}{3} = \frac{5}{12}$ ………(i)
$\frac{x}{2} + y = 1$ ………(ii)
Multiplying (i) by 12 and (ii) by 4, we have
3x + 4y = 5 ……….(iii)
2x + 4y = 4 ………(iv)
Now, subtracting (iv) from (iii), we get
x = 1
Putting x = 1 in (iv), we have
2 + 4y =4
$\Rightarrow 4y = 2$
$\Rightarrow y = \frac{1}{2}$
$\therefore x + y = 1 + \frac{1}{2} = \frac{3}{2}$
Hence, the value of x + y is $\frac{3}{2}$.

15.

Sol:

The given pair of equations is

$12x + 17y = 53$ ……..(i)

$17x + 12y = 63$ …….(ii)

Adding (i) and (ii), we get

$29x + 29y = 116$

$\Rightarrow x + y = 4$ (Dividing by 4)

Hence, the value of x + y is 4.

16.

Sol:

The given system is

$3x + 5y = 0$ ……(i)

$kx + 10y = 0$ ……(ii)

This is a homogeneous system of linear differential equation, so it always has a zero solution i.e., x = y = 0.

But to have a non-zero solution, it must have infinitely many solutions.

For this, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{3}{k} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow k = 6$

Hence, k = 6.

17.

Sol:

The given system is

$kx - y - 2 = 0$ ……(i)

$6x - 2y - 3 = 0$ ……(ii)

Here, $a_1 = k$, $b_1 = -1$, $c_1 = -2$, $a_2 = 6$, $b_2 = -2$ and $c_2 = -3$

For the system, to have a unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2} = \frac{1}{2}$

$\Rightarrow k \neq 3$

Hence, $k \neq 3$.

18.

Sol:
The given system is
$2x + 3y - 5 = 0$ ……(i)
$4x + ky - 10 = 0$ ……(ii)
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$, $a_2 = 4$, $b_2 = k$ and $c_2 = -10$
For the system, to have an infinite number of solutions, we must have
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{2}{4} = \frac{3}{k} = \frac{-5}{-10}$
$\Rightarrow \frac{1}{2} = \frac{3}{k} = \frac{1}{2}$
$\Rightarrow k = 6$
Hence, $k = 6$.

19.
Sol:
The given system is
$2x + 3y - 1 = 0$ ……(i)
$4x + 6y - 4 = 0$ ……(ii)
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -1$, $a_2 = 4$, $b_2 = 6$ and $c_2 = -4$
Now,
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$
$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$
$\frac{c_1}{c_2} = \frac{-1}{-4} = \frac{1}{4}$
Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and therefore the given system has no solution.

20.

Sol:
The given system is
$x + 2y - 3 = 0$ ……(i)
$5x + ky + 7 = 0$ ……(ii)
Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$, $a_2 = 5$, $b_2 = k$ and $c_2 = 7$.
For the system, to be consistent, we must have
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\Rightarrow \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow \frac{1}{5} = \frac{2}{k}$

$\Rightarrow$ k = 10

Hence, k = 10.

21.

Sol:

The given system of equations is

$\frac{3}{x+y} + \frac{2}{x-y} = 2$ ………..(i)

$\frac{9}{x+y} - \frac{4}{x-y} = 1$ ………..(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), the given equations are changed to

3u + 2v = 2 ………(iii)

9u – 4v = 1 ………(iv)

Multiplying (i) by 2 and adding it with (ii), we get

15u = 4 + 1 $\Rightarrow$ u = $\frac{1}{3}$

Multiplying (i) by 3 and subtracting (ii) from it, we get

6u + 4v = 6 – 1 $\Rightarrow$ u = $\frac{5}{10} = \frac{1}{2}$

Therefore

x + y = 3 ………….(v)

x – y = 2 ………..(vi)

Now, adding (v) and (vi) we have

2x = 5 $\Rightarrow$ x = $\frac{5}{2}$

Substituting x = $\frac{5}{2}$ in (v), we have

$\frac{5}{2}$ + y = 3 $\Rightarrow$ y = 3 – $\frac{5}{2} = \frac{1}{2}$

Hence, x = $\frac{5}{2}$ and y = $\frac{1}{2}$.

Exercise – MCQ

1.

Answer: (c) x = 3, y = 2

Sol:

The given system of equations is

2x + 3y = 12 ……..(i)

3x – 2y = 5 ……..(ii)

Multiplying (i) by 2 and (ii) by 3 and then adding, we get

4x + 9x = 24 + 15

⇒ x = $\frac{39}{13}$ = 3

Now, putting x = 3 in (i), we have

2 × 3 + 3y = 12 ⇒ y = $\frac{12-6}{3}$ = 2

Thus, x = 3 and y = 2.

2.

Answer: (c) x = 6, y = 4

Sol:

The given system of equations is

x – y = 2 ……..(i)

x + y = 10 ……..(ii)

Adding (i) and (ii), we get

2x = 12 ⇒ x = 6

Now, putting x = 6 in (ii), we have

6 + y = 10 ⇒ y = 10 – 6 = 4

Thus, x = 6 and y = 4.

3.

Answer: (a) x = 2, y = 3

Sol:

The given system of equations is

$\frac{2x}{3} - \frac{y}{2} = -\frac{1}{6}$ ……..(i)

$\frac{x}{2} + \frac{2y}{3} = 3$ ……..(ii)

Multiplying (i) and (ii) by 6, we get

4x – 3y = –1 ……..(iii)

3x + 4y = 18 ………(iv)

Multiplying (iii) by 4 and (iv) by 3 and adding, we get

16x + 9x = –4 + 54

⇒ x = $\frac{50}{25}$ = 2

Now, putting x = 2 in (iv), we have

3 × 2 + 4y = 18 ⇒ y = $\frac{18-6}{4}$ = 3

Thus, x = 2 and y = 3.

4.

Answer: (d) x = , y =

Sol:
The given system of equations is

$\frac{1}{x} + \frac{2}{y} = 4$ ……..(i)

$\frac{3}{y} - \frac{1}{x} = 11$ ……..(ii)

Adding (i) and (ii), we get

$\frac{2}{y} + \frac{3}{y} = 15$

$\Rightarrow \frac{5}{y} = 15 \Rightarrow y = \frac{5}{15} = \frac{1}{3}$

Now, putting $y = \frac{1}{3}$ in (i), we have

$\frac{1}{x} + 2 \times 3 = 4 \Rightarrow \frac{1}{x} = 4 - 6 \Rightarrow x = -\frac{1}{2}$

Thus, $x = -\frac{1}{2}$ and $y = \frac{1}{3}$.

5.

Answer: (a) x = 1, y = 1

Sol:
Consider $\frac{2x+y+2}{5} = \frac{3x-y+1}{3}$ and $\frac{3x-y+1}{3} = \frac{3x+2y+1}{3}$. Now, simplifying these equations, we get

$3(2x + y + 2) = 5(3x - y + 1)$

$\Rightarrow 6x + 3y + 6 = 15x - 5y + 5$

$\Rightarrow 9x - 8y = 1$ ……….(i)

And

$6(3x - y + 1) = 3(3x + 2y + 1)$

$\Rightarrow 18x - 6y + 6 = 9x + 6y + 3$

$\Rightarrow 3x - 4y = -1$ ……………(ii)

Multiplying (ii) by 2 and subtracting it from (i)

$9x - 6x = 1 + 2 \Rightarrow x = 1$

Now, putting x = 1 in (ii), we have

$3 \times 1 - 4y = -1 \Rightarrow y = \frac{3+1}{4} = 1$

Thus, x = 1, y = 1.

6.

Answer: (b)

Sol:

The given equations are

$\frac{3}{x+y} + \frac{2}{x-y} = 2$ ……..(i)

$\frac{9}{x+y} - \frac{4}{x-y} = 1$ ……..(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), the new system becomes

3u + 2v = 2 ……..(iii)

9u – 4v = 1 …….(iv)

Now, multiplying (iii) by 2 and adding it with (iv), we get

6u + 9u = 4 + 1 ⇒ u = $\frac{5}{15} = \frac{1}{3}$

Again, multiplying (iii) by 2 and subtracting (iv) from , we get

6v + 4v = 6 – 1 ⇒ v = $\frac{5}{10} = \frac{1}{2}$

Therefore

x + y = 3 ……….(v)

x – y = 2 ………(vi)

Adding (v) and (vi), we get

2x = 3 + 2 ⇒ x = $\frac{5}{2}$

Substituting x = $\frac{5}{2}$, in (v), we have

$\frac{5}{2}$ + y = 3 ⇒ y = 3 – $\frac{5}{2} = \frac{1}{2}$.

Thus, x = $\frac{5}{2}$ and y = $\frac{1}{2}$.

7.

Answer: (c) x = 3, y = 4

Sol:

The given equations are

4x + 6y = 3xy ………(i)

8x + 9y = 5xy ………(ii)

Dividing (i) and (ii) by xy, we get

$\frac{6}{x} + \frac{4}{y} = 3$ ………(iii)

$\frac{9}{x} + \frac{8}{y} = 5$ ....(iv)

Multiplying (iii) by 2 and subtracting (iv) from it, we get

$\frac{12}{x} - \frac{9}{x} = 6 - 5 \Rightarrow \frac{3}{x} = 1 \Rightarrow x = 3$

Substituting x = 3 in (iii), we get

$\frac{6}{3} + \frac{4}{y} = 3 \Rightarrow \frac{4}{y} = 1 \Rightarrow y = 4$

Thus, x = 3 and y = 4.

8.

Answer: (a) x = 1, y = 2

Sol:

The given system of equations is

$29x + 37y = 103$ ....(i)

$37x + 29y = 95$ ....(ii)

Adding (i) and (ii), we get

$66x + 66y = 198$

$\Rightarrow x + y = 3$ ....(iii)

$\Rightarrow$ Subtracting (i) from (ii), we get

$8x - 8y = -8$

$\Rightarrow x - y = -1$

Adding (iii) and (iv), we get

$2x = 2 \Rightarrow x = 1$

Substituting x = 1 in (iii), we have

$1 + y = 3 \Rightarrow y = 2$

Thus, x = 1 and y = 2.

9.

Answer: (c) 0

Sol:

$\because 2^{x+y} = 2^{x-y} = \sqrt{8}$

$\therefore x + y = x - y$

$\Rightarrow y = 0$

10.

Answer: (b) x = , y = 1

Sol:

The given equations are

$\frac{2}{x} + \frac{3}{y} = 6$ ……..(i)

$\frac{1}{x} + \frac{1}{2y} = 2$ ……..(ii)

Multiplying (ii) by 2 and subtracting it from (ii), we get

$\frac{3}{y} - \frac{1}{y} = 6 - 4$

$\Rightarrow \frac{2}{y} = 2 \Rightarrow y = 1$

Substituting y = 1 in (ii), we get

$\frac{1}{x} + \frac{1}{2} = 2$

$\Rightarrow \frac{1}{x} = 2 - \frac{1}{2} \Rightarrow \frac{3}{2}$

$\Rightarrow x = \frac{2}{3}$.

11.

Answer: (d) $\Rightarrow k \neq 3$

Sol:

The given equations are

kx – y – 2 = 0 ………(i)

6x – 2y – 3 = 0 ……..(ii)

Here, $a_1 = k$, $b_1 = -1$, $c_1 = -2$, $a_2 = 6$, $b_2 = -2$ and $c_2 = -3$.

For the given system to have a unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2}$

$\Rightarrow k \neq 3$

12.

Answer: (b) k ≠ -6

Sol:

The correct option is (b).

The given system of equations can be written as follows:

x – 2y – 3 = 0 and 3x + ky – 1 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -3$, $a_2 = 3$, $b_2 = k$ and $c_2 = -1$.
$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{k}$ and $\frac{c_1}{c_2} = \frac{-3}{-1} = 3$
These graph lines will intersect at a unique point when we have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{3} \neq \frac{-2}{k} \Rightarrow k \neq -6$
Hence, k has all real values other than –6.

13.

Answer: (a) k = 10
Sol:
The correct option is (a).
The given system of equations can be written as follows:
x + 2y – 3 = 0 and 5x + ky + 7 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$, $a_2 = 5$, $b_2 = k$ and $c_2 = 7$.
$\therefore \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{2}{k}$ and $\frac{c_1}{c_2} = \frac{-3}{7}$
For the system of equations to have no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\therefore \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7} \Rightarrow k = 10$

14.

Answer: (d) $\frac{15}{4}$
Sol:
The given system of equations can be written as follows:
3x + 2ky – 2 = 0 and 2x + 5y + 1 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 3$, $b_1 = 2k$, $c_1 = -2$, $a_2 = 2$, $b_2 = 5$ and $c_2 = 1$
$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2k}{5}$ and $\frac{c_1}{c_2} = \frac{-2}{1}$
For parallel lines, we have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$

$\Rightarrow$ k = $\frac{15}{4}$

15.

Answer: (d) all real values except -6

Sol:

The given system of equations can be written as follows:

kx – 2y – 3 = 0 and 3x + y – 5 = 0

The given equations are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = k$, $b_1 = -2$, $c_1 = -3$ and a2 = 3, $b_2 = 1$ and $c_2 =$ -5

$\therefore \frac{a_1}{a_2} = \frac{k}{3}, \frac{b_1}{b_2} = \frac{-2}{1}$ and $\frac{c_1}{c_2} = \frac{-3}{-5} = \frac{3}{5}$

Thus, for these graph lines to intersect at a unique point, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{3} \neq \frac{-2}{1} \Rightarrow k \neq -6$

Hence, the graph lines will intersect at all real values of k except –6.

16.

Answer: (d) no solution

Sol:

The given system of equations can be written as:

x + 2y + 5 = 0 and -3x - 6y + 1 = 0

The given equations are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$, $a_2 = -3$, $b_2 = -6$ and $c_2 = 1$

$\therefore \frac{a_1}{a_2} = \frac{1}{-3}, \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{-3}$ and $\frac{c_1}{c_2} = \frac{5}{1}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given system has no solution.

17.

Answer: (d) no solution

Sol:
The given system of equations can be written as:
2x + 3y - 5 = 0 and 4x + 6y - 15 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$, $a_2 = 4$, $b_2 = 6$ and $c_2 = -15$
$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$
$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Hence, the given system has no solution.

18.

Answer: (d) intersecting or coincident
Sol:
If a pair of linear equations is consistent, then the two graph lines either intersect at a point or coincidence.

19.

Answer: (a) parallel
Sol:
If a pair of linear equations in two variables is inconsistent, then no solution exists as they have no common point. And, since there is no common solution, their graph lines do not intersect. Hence, they are parallel.

20.

Answer: (b) 40^0
Sol:
Let $\angle A = x^0$ and $\angle B = y^0$
$\therefore \angle A = 3\angle B = (3y)^0$
Now, $\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x + y + 3y = 180$
$\Rightarrow x + 4y = 180$ ……(i)
Also, $\angle C = 2(\angle A + \angle B)$
$\Rightarrow 3y = 2(x + y)$
$\Rightarrow 2x – y = 0$ ……..(ii)

On multiplying (ii) by 4, we get:
$8x - 4y = 0$ ....(iii)
On adding (i) and (iii) we get:
$9x = 180 \Rightarrow x = 20$
On substituting x = 20 in (i), we get:
$20 + 4y = 180 \Rightarrow 4y = (180 - 20) = 160 \Rightarrow y = 40$
$\therefore x = 20$ and $y = 40$
$\therefore \angle B = y^0 = 40^0$

21.

Answer: (b) 80^0
Sol:
The correct option is (b).
In a cyclic quadrilateral ABCD:
$\angle A = (x + y + 10)^0$
$\angle B = (y + 20)^0$
$\angle C = (x + y - 30)^0$
$\angle D = (x + y)^0$
We have:

$\angle A + \angle C = 180^0$ and $\angle B + \angle D = 180^0$ [Since ABCD is a cyclic quadrilateral]
Now, $\angle A + \angle C = (x + y + 10)^0 + (x + y - 30)^0 = 180^0$
$\Rightarrow 2x + 2y - 20 = 180$
$\Rightarrow x + y - 10 = 90$
$\Rightarrow x + y = 160$ ......(i)
Also, $\angle B + \angle D = (y + 20)^0 + (x + y)^0 = 180^0$
$\Rightarrow x + 2y + 20 = 180$
$\Rightarrow x + 2y = 160$
On subtracting (i) from (ii), we get:
$y = (160 - 100) = 60$
On substituting y = 60 in (i), we get:
$x + 60 = 100 \Rightarrow x = (100 - 60) = 40$
$\therefore \angle B = (y + 20)^0 = (60 + 20)^0 = 80^0$

22.

Answer: (d) 40 years
Sol:

Let the man's present age be x years.
Let his son's present age be y years.
Five years later:
$(x + 5) = 3(y + 5)$
$\Rightarrow x + 5 = 3y + 15$
$\Rightarrow x - 3y = 10$ ....(i)
Five years ago:
$(x - 5) = 7(y - 5)$
$\Rightarrow x - 5 = 7y - 35$
$\Rightarrow x - 7y = -30$ ....(ii)
On subtracting (i) from (ii), we get:
$-4y = -40 \Rightarrow y = 10$
On substituting y = 10 in (i), we get:
$x - 3 \times 10 = 10 \Rightarrow x - 30 = 10 \Rightarrow x = (10 + 30) = 40$ years
Hence, the man's present age is 40 years.

23.

Answer: (c)
Sol:
Option (c) is the correct answer.
Clearly, Reason (R) is false.
On solving $x + y = 8$ and $x - y = 2$, we get:
$x = 5$ and $y = 3$
Thus, the given system has a unique solution. So, assertion (A) is true. ∴ Assertion (A) is true and Reason (R) is false.

24.
Answer: (b) parallel
Sol:

The given equations are as follows:

6x - 2y + 9 = 0 and 3x - y + 12 = 0

They are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 6$, $b_1 = -2$, $c_1 = 9$ and $a_2 = 3$, $b_2 = -1$ and $c_2 = 12$

$\therefore \frac{a_1}{a_2} = \frac{6}{3} = \frac{2}{1}, \frac{b_1}{b_2} = \frac{-2}{-1} = \frac{2}{1}$ and $\frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

The given system has no solution.

Hence, the lines are parallel.

25.

Answer:

Sol:

The given equations are as follows:

2x + 3y – 2 = 0 and x – 2y – 8 = 0

They are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -2$ and $a_2 = 1$, $b_2 = -2$ and $c_2 = -8$

$\therefore \frac{a_1}{a_2} = \frac{2}{1}, \frac{b_1}{b_2} = \frac{3}{-2}$ and $\frac{c_1}{c_2} = \frac{-2}{-8} = \frac{1}{4}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given system has a unique solution.

Hence, the lines intersect exactly at one point.

26.

Answer: (a) coincident

Sol:

The correct option is (a).

The given system of equations can be written as follows:

$5x - 15y - 8 = 0$ and $3x - 9y - \frac{24}{5} = 0$

The given equations are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 5$, $b_1 = -15$, $c_1 = -8$ and $a_2 = 3$, $b_2 = -9$ and $c_2 = -\frac{24}{5}$

$\therefore \frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}$ and $\frac{c_1}{c_2} = -8 \times \frac{5}{-24} = \frac{5}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The given system of equations will have an infinite number of solutions.

Hence, the lines are coincident.

27.

Answer: (a) 96

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = (10x + y)

According to the question, we have:

$x + y = 15$ ……..(i)

Number obtained on reversing its digits = (10y + x)

$\therefore (10y + x) = (10x + y) + 9$

$\Rightarrow 10y + x - 10x - y = 9$

$\Rightarrow 9y - 9x = 9$

$\Rightarrow y - x = 1$ …….(ii)

On adding (i) and (ii), we get:

$2y = 16 \Rightarrow y = 8$

On substituting y = 8 in (i), we get:

$x + 8 = 15 \Rightarrow x = (15 - 8) = 7$

Number = $(10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$

Hence, the required number is 78.

Exercise – Formative Assessment

1.

Answer: (a) parallel lines

Sol:

The given system of equations can be written as follows:

$x + 2y - 3 = 0$ and $2x + 4y + 7 = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$ and $a_2 = 2$, $b_2 = 4$ and $c_2 = 7$

$\therefore \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-3}{7}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system has no solution.

Hence, the lines are parallel.

2.

Answer: (d) a = -5, b = -1

Sol:

The given system of equations can be written as follows:

$2x - 3y - 7 = 0$ and $(a + b)x - (a + b - 3)y - (4a + b) = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$ and $c_2 = -(4a + b)$

$\therefore \frac{a_1}{a_2} = \frac{2}{(a+b)}, \frac{b_1}{b_2} = \frac{-3}{-(a+b-3)} = \frac{3}{(a+b-3)}$ and $\frac{c_1}{c_2} = \frac{-7}{-(4a+b)} = \frac{7}{(4a+b)}$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{2}{(a+b)} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

Now, we have:

$\frac{2}{(a+b)} = \frac{3}{(a+b-3)} \Rightarrow 2a + 2b - 6 = 3a + 3b$

$\Rightarrow a + b + 6 = 0$ ……(i)

Again, we have:

$\frac{3}{(a+b-3)} = \frac{7}{(4a+b)} \Rightarrow 12a + 3b = 7a + 7b - 21$

$\Rightarrow 5a - 4b + 21 = 0$ …….(ii)

On multiplying (i) by 4, we get:

$4a + 4b + 24 = 0$ ……(iii)

On adding (ii) and (iii), we get:

$9a = -45 \Rightarrow a = -5$

On substituting a = -5 in (i), we get:

$-5 + b + 6 = 0 \Rightarrow b = -1$

$\therefore a = -5$ and $b = -1$.

3.

Answer: (a) a unique solution

Sol:

The given system of equations can be written as follows:

2x + y – 5 = 0 and 3x + 2y – 8 = 0

The given equations are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 2$, $b_1 = 1$, $c_1 = -5$ and $a_2 = 3$, $b_2 = 2$ and $c_2 = -8$

$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given system has a unique solution.

Hence, the lines intersect at one point.

4.

Answer: (d) $\frac{1}{x} - \frac{1}{y} = 0$

Sol:

Given:

x = -y and y > 0

Now, we have:

(i) x^2y

On substituting x = -y, we get:

$(-y)^2y = y^3 > 0$ ($\because$ y > 0)

This is true.

(ii) x + y

On substituting x = -y, we get:

(-y) + y = 0

This is also true.

(iii) xy

On substituting x = -y, we get:

$(-y)y = -y^2$ ($\because$ y > 0)

This is again true.

(iv) $\frac{1}{x} - \frac{1}{y} = 0$

$\Rightarrow \frac{y-x}{xy} = 0$

On substituting x = -y, we get:

$\frac{y-(-y)}{(-y)y} = 0 \Rightarrow \frac{2y}{-y^2} = 0 \Rightarrow 2y = 0 \Rightarrow y = 0.$

5.

Sol:

The given system of equations:

$-x + 2y + 2 = 0$ and $\frac{1}{2}x - \frac{1}{4}y - 1 = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = -1$, $b_1 = 2$, $c_1 = 2$ and $a_2 = \frac{1}{2}$, $b_2 = -\frac{1}{4}$ and $c_2 = -1$

$\therefore \frac{a_1}{a_2} = \frac{-1}{(1/2)} = -2$, $\frac{b_1}{b_2} = \frac{2}{(-1/4)} = -8$ and $\frac{c_1}{c_2} = \frac{2}{-1} = -2$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given system has a unique solution.

Hence, the lines intersect at one point.

6.

Sol:

The given system of equations can be written as follows:

$kx + 3y - (k - 2) = 0$ and $12x + ky - k = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = -(k - 2)$ and $a_2 = 12$, $b_2 = k$ and $c_2 = -k$

$\therefore \frac{a_1}{a_2} = \frac{k}{12}, \frac{b_1}{b_2} = \frac{3}{k}$ and $\frac{c_1}{c_2} = \frac{-(k-2)}{-k} = \frac{(k-2)}{k}$

For inconsistency, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{(k-2)}{k} \Rightarrow k^2 = (3 \times 12) = 36$

$\Rightarrow k = \sqrt{36} = \pm 6$

Hence, the pair of equations is inconsistent if $k = \pm 6$.

7.

Sol:

The given system of equations can be written as follows:

9x – 10y – 21 = 0 and $\frac{3x}{2}$x – $\frac{5y}{3}$ – $\frac{7}{2}$ = 0

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 9$, $b_1 = –10$, $c_1 = –21$ and $a_2 = \frac{3}{2}$, $b_2 = \frac{-5}{3}$ and $c_2 = \frac{-7}{2}$

$\therefore \frac{a_1}{a_2} = \frac{9}{{}^3/_2} = 6, \frac{b_1}{b_2} = \frac{-10}{(-{}^5/_3)} = 6$ and $\frac{c_1}{c_2} = –21 \times \frac{2}{-7} = 6$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

This shows that the given system of equations has an infinite number of solutions.

8.

Sol:

The given equations are as follows:

x – 2y = 0 ....(i)

3x + 4y = 20 ....(ii)

On multiplying (i) by 2, we get:

2x – 4y = 0 ....(iii)

On adding (ii) and (iii), we get:

5x = 20 ⇒ x = 4

On substituting x = 4 in (i), we get:

4 – 2y = 0 ⇒ 4 = 2y ⇒ y = 2

Hence, the required solution is x = 4 and y = 2.

9. **Sol:**

The given system of equations can be written as follows:

x – 3y – 2 = 0 and –2x + 6y – 5 = 0

The given equations are of the following form:

$a_1x + b_{1y} + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 1$, $b_1 = –3$, $c_1 = –2$ and $a_2 = –2$, $b_2 = 6$ and $c_2 = –5$

$\therefore \frac{a_1}{a_2} = \frac{1}{-2} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the given system of equations has no solution.

Hence, the paths represented by the equations are parallel.

10.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x - y = 26$ ....(i)

$x = 3y$ ....(ii)

On substituting x = 3y in (i), we get:

$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$

On substituting y = 13 in (i), we get:

$x - 13 = 26 \Rightarrow x = 26 + 13 = 39$

Hence, the required numbers are 39 and 13.

11.

Sol:

The given equations are as follows:

$23x + 29y = 98$ ....(i)

$29x + 23y = 110$ ....(ii)

On adding (i) and (ii), we get:

$52x + 52y = 208$

$\Rightarrow x + y = 4$ ....(iii)

On subtracting (i) from (ii), we get:

$6x - 6y = 12$

$\Rightarrow x - y = 2$ ....(iv)

On adding (iii) and (iv), we get:

$2x = 6 \Rightarrow x = 3$

On substituting x = 3 in (iii), we get:

$3 + y = 4$

$\Rightarrow y = 4 - 3 = 1$

Hence, the required solution is x = 3 and y = 1.

12.

Sol:

The given equations are as follows:

$6x + 3y = 7xy$ ....(i)

$3x + 9y = 11xy$ ....(ii)

For equation (i), we have:

$\frac{6x+3y}{xy} = 7$

$\Rightarrow \frac{6x}{xy} + \frac{3y}{xy} = 7 \Rightarrow \frac{6}{y} + \frac{3}{x} = 7$ …….(iii)

For equation (ii), we have:

$\frac{3x+9y}{xy} = 11$

$\Rightarrow \frac{3x}{xy} + \frac{9y}{xy} = 11 \Rightarrow \frac{3}{y} + \frac{9}{x} = 11$ …….(iii)

On substituting $\frac{3}{y}$ = v and $\frac{1}{x}$ = u in (iii) and (iv), we get:

6v + 3u = 7 ………..(v)

3v + 9u = 11 ……….(vi)

On multiplying (v) by 3, we get:

18v + 9u = 21 ………(vii)

On substituting y = $\frac{3}{2}$ in (iii), we get:

$\frac{6}{(3/2)} + \frac{3}{x} = 7$

$\Rightarrow 4 + \frac{3}{x} = 7 \Rightarrow \frac{3}{x} = 3 \Rightarrow 3x = 3$

$\Rightarrow x = 1$

Hence, the required solution is x = 1 and y = $\frac{3}{2}$.

13.

Sol:

The given system of equations can be written as follows:

3x + y = 1

$\Rightarrow$ 3x + y – 1 = 0 ……….(i)

kx + 2y = 5

$\Rightarrow$ kx + 2y – 5 = 0 ………(ii)

These equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 3$, $b_1 = 1$, $c_1 = -1$ and $a_2 = k$, $b_2 = 2$ and $c_2 = -5$

(i) For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{3}{k} \neq \frac{1}{2}$ $\Rightarrow k \neq 6$

Thus, for all real values of k other than 6, the given system of equations will have a unique solution.

(ii) In order that the given equations have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{k} = \frac{1}{2} \neq \frac{-1}{-5}$

$\Rightarrow \frac{3}{k} = \frac{1}{2}$ and $\frac{3}{k} = \frac{-1}{-5}$

$\Rightarrow k = 6, k \neq 15$

Thus, for k = 6, the given system of equations will have no solution.

14.

Sol:

Let $\angle A = x^0$ and $\angle B = y^0$

Then, $\angle C = 3\angle B = 3y^0$

Now, we have:

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x + y + 3y = 180$

$\Rightarrow x + 4y = 180$ ……..(i)

Also, $\angle C = 2(\angle A + \angle B)$

$\Rightarrow 3y = 2(x + y)$

$\Rightarrow 2x - y = 0$ ……..(ii)

On multiplying (ii) by 4, we get:

$8x - 4y = 0$ ………(iii)

On adding (i) and (iii), we get:

$9x = 180 \Rightarrow x = 20$

On substituting x = 20 in (i), we get:

$20 + 4y = 180 \Rightarrow 4y = (180 - 20) = 160 \Rightarrow y = 40$

$\therefore$ x = 20 and y = 40

$\therefore \angle A = 20^0, \angle B = 40^0, \angle C = (3 \times 40^0) = 120^0$.

15.

Sol:

Let the cost of each pencil be Rs. x and that of each pen be Rs. y.

Then, we have:

$5x + 7y = 195$ ………(i)

$7x + 5y = 153$ ………(ii)

Adding (i) and (ii), we get:

$12x + 12y = 348$

$\Rightarrow 12(x + y) = 348$

$\Rightarrow x + y = 29$ ………(iii)

Subtracting (i) from (ii), we get:

$2x - 2y = -42$

⇒ 2(x – y) = –42

⇒ x – y = –21 ………(iv)

On adding (iii) and (iv), we get:

4 + y = 29 ⇒ y = (29 – 4) = 25

Hence, the cost of each pencil is Rs. 4 and the cost of each pen is Rs. 25.

16.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and the y-axis, respectively.

Graph of 2x – 3y = 1

2x – 3y = 1

⇒ 3y = (2x – 1)

∴ y = $\frac{2x-1}{3}$ ……..(i)

Putting x = -1, we get:

y = -1

Putting x = 2, we get:

y = 1

Putting x = 5, we get:

y = 3

Thus, we have the following table for the equation 2x – 3y = 1.

x	-1	2	5
y	-1	1	3

Now, plots the points A(-1, -1), B(2, 1) and C(5, 3) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both the sides.

Thus, the line AC is the graph of 2x – 3y = 1.

Graph of 4x – 3y + 1 = 0

4x – 3y + 1 = 0

⇒ 3y = (4x + 1)

∴ y = $\frac{4x+1}{3}$ ……..(ii)

Putting x = -1, we get:

y = -1

Putting x = 2, we get:

y = 3

Putting x = 5, we get:

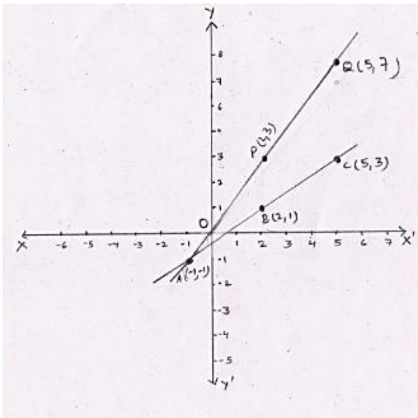
y = 7

Thus, we have the following table for the equation 4x – 3y + 1 = 0.

x	-1	2	5
y	-1	3	7

Now, Plot the points P(2, 3) and Q(5, 7). The point A(-1, -1) has already been plotted. Join PA and QP to get the graph line AQ. Extend it on both sides.

Thus, the line AQ is the graph of the equation $4x - 3y + 1 = 0$.



The two lines intersect at A(-1. -1).

Thus, x = -1 and y = -1 is the solution of the given system of equations.

17.

Sol:

Given:

In a cyclic quadrilateral ABCD, we have:

$\angle A = (4x + 20)^0$

$\angle B = (3x - 5)^0$

$\angle C = 4y^0$

$\angle D = (7y + 5)^0$

$\angle A + \angle C = 180^0$ and $\angle B + \angle D = 180^0$ [Since ABCD is a cyclic quadrilateral] Now,

$\angle A + \angle C = (4x + 20)^0 + (4y^0) = 180^0$

$\Rightarrow 4x + 4y + 20 = 180$

$\Rightarrow 4x + 4y = 180 - 20 = 160$

$\Rightarrow x + y = 40$ …….(i)

Also, $\angle B + \angle D = (3x - 5)^0 + (7y + 5)^0 = 180^0$

$\Rightarrow 3x + 7y = 180$ …….(ii)

On multiplying (i) by 3, we get:

$3x + 3y = 120$ ……(iii)

On subtracting (iii) from (ii), we get:

$4y = 60 \Rightarrow y = 15$

On substituting y = 15 in (i), we get:

$x + 15 = 40 \Rightarrow x = (40 - 15) = 25$

Therefore, we have:

$\angle A = (4x + 20)^0 = (4 \times 25 + 20)^0 = 120°$

$\angle B = (3x - 5)^0 = (3 \times 25 - 5)^0 = 70°$

$\angle C = 4y^0 = (4 \times 15)^0 = 60^0$

$\angle D = (7y + 5)^0 = (7 \times 15 + 5)^0 = (105 + 5)^0 = 110^0$.

18.

Sol:

We have:

$\frac{35}{x+y} + \frac{14}{x-y} = 19$ and $\frac{14}{x+y} + \frac{35}{x-y} = 37$

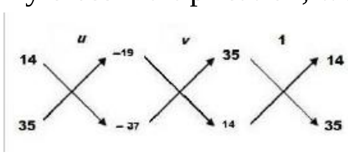
Taking $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$.

$35u + 14v - 19 = 0$ ……….(i)

$14u + 35v - 37 = 0$ ……….(ii)

Here, $a_1 = 35$, $b_1 = 14$, $c_1 = -19$ and $a_2 = 14$, $b_2 = 35$ and $c_2 = -37$

By cross multiplication, we have:



$\therefore \frac{u}{[14 \times (-37) - 35 \times (-19)]} = \frac{v}{[(-19) \times 14 - (-37) \times (35)]} = \frac{1}{[35 \times 35 - 14 \times 14]}$

$\Rightarrow \frac{u}{-518+665} = \frac{v}{-266+1295} = \frac{1}{1225-196}$

$\Rightarrow \frac{u}{147} = \frac{v}{1029} = \frac{1}{1029}$

$\Rightarrow u = \frac{147}{1029} = \frac{1}{7}$, $v = \frac{1029}{1029} = 1$

$\Rightarrow \frac{1}{x+y} = \frac{1}{7}, \frac{1}{x-y} = 1$

$\therefore (x + y) = 7$ ………(iii)

And, $(x - y) = 1$ ……....(iv)

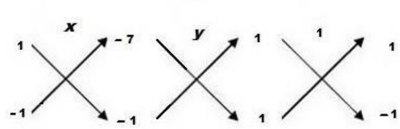
Again, the equations (iii) and (iv) can be written as follows:

$x + y - 7 = 0$ ……….(v)

$x - y - 1 = 0$ ………(vi)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -7$ and $a_2 = 1$, $b_2 = -1$ and $c_2 = -1$

By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-1)-(-1)\times(-7)]} = \frac{y}{[(-7)\times 1 \ -(-1)\times 1]} = \frac{1}{[1\times(-1)\ -1\times 1]}$

$\Rightarrow \frac{x}{-1-7} = \frac{y}{-7+1} = \frac{1}{-1-1}$

$\Rightarrow \frac{x}{-8} = \frac{y}{-6} = \frac{1}{-2}$

$\Rightarrow x = \frac{-8}{-2} = 4$, $y = \frac{-6}{-2} = 3$

Hence, x = 4 and y = 3 is the required solution.

19.

Sol:

Let the required fraction be x/y

Then, we have:

$\frac{x+1}{y+1} = \frac{4}{5}$

$\Rightarrow 5(x + 1) = 4(y + 1)$

$\Rightarrow 5x + 5 = 4y + 4$

$\Rightarrow 5x - 4y = -1$ ………(i)

Again, we have:

$\frac{x-5}{y-5} = \frac{1}{2}$

$\Rightarrow 2(x - 5) = 1(y - 5)$

$\Rightarrow 2x - 10 = y - 5$

$\Rightarrow 2x - y = 5$ ………(ii)

On multiplying (ii) by 4, we get:

$8x - 4y = 20$ ……..(iii)

On subtracting (i) from (iii), we get:

$3x = (20 - (-1)) = 20 + 1 = 21$

$\Rightarrow 3x = 21$

$\Rightarrow x = 7$

On substituting x = 7 in (i), we get

$5 \times 7 - 4y = -1$

$\Rightarrow 35 - 4y = -1$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

$\therefore x = 7$ and $y = 9$

Hence, the required fraction is $\frac{7}{9}$.

20.

Sol:

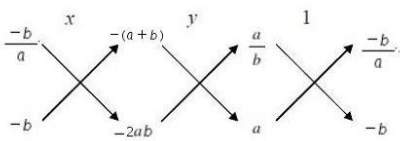
The given equations may be written as follows:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$ ....(i)

$ax - by - 2ab = 0$ ....(ii)

Here, $a_1 = \frac{a}{b}$, $b_1 = \frac{-b}{a}$, $c_1 = -(a + b)$ and $a_2 = a$, $b_2 = -b$ and $c_2 = -2ab$

By cross multiplication, we have:



$$\therefore \frac{x}{\left(-\frac{b}{a}\right)\times(-2ab)-(-b)\times(-(a+b))} = \frac{y}{-(a+b)\times a-(-2ab)\times\frac{a}{b}} = \frac{1}{\frac{a}{b}\times(-b)-a\times\left(-\frac{b}{a}\right)}$$

$$\Rightarrow \frac{x}{2b^2-b(a+b)} = \frac{y}{-a(a+b)+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{2b^2-ab-b^2} = \frac{y}{-a^2-ab+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{b^2-ab} = \frac{y}{a^2-ab} = \frac{1}{-(a-b)}$$

$$\Rightarrow \frac{x}{-b(a-b)} = \frac{y}{a(a-b)} = \frac{1}{-(a-b)}$$

$$\Rightarrow x = \frac{-b(a-b)}{-(a-b)} = b,\ y = \frac{a(a-b)}{-(a-b)} = -a$$

Hence, $x = b$ and $y = -a$ is the required solution.