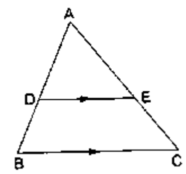
Exercise – 4A

1.



Sol:

(i) In Δ ABC, it is given that DE ∥ BC.
Applying Thales' theorem, we get:
$\frac{AD}{DB} = \frac{AE}{EC}$
∵ AD = 3.6 cm , AB = 10 cm, AE = 4.5cm
∴ DB = 10 − 3.6 = 6.4cm
Or, $\frac{3.6}{6.4} = \frac{4.5}{EC}$
Or, EC = $\frac{6.4\times4.5}{3.6}$
Or, EC= 8 cm
Thus, AC = AE + EC
= 4.5 + 8 = 12.5 cm

(ii) In Δ ABC, it is given that DE || BC.
Applying Thales' Theorem, we get :
$\frac{AD}{DB} = \frac{AE}{EC}$
Adding 1 to both sides, we get :
$\frac{AD}{DB} + 1 = \frac{AE}{EC}+1$
$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$
$\Rightarrow \frac{13.3}{DB} = \frac{11.9}{5.1}$
$\Rightarrow$DB = $\frac{13.3\times5.1}{11.9}$ = 5.7 cm
Therefore, AD=AB-DB=13.5-5.7=7.6 cm

(iii) In Δ ABC, it is given that DE || BC.
Applying Thales' theorem, we get :
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{4}{7} = \frac{AE}{EC}$
Adding 1 to both the sides, we get :
$\frac{11}{7} = \frac{AC}{EC}$
$\Rightarrow$ EC = $\frac{6.6\times7}{11}$ = 4.2 cm
Therefore,

AE = AC – EC = 6.6 – 4.2 = 2.4 cm

(iv) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+EC}$

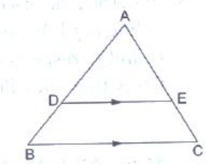
$\Rightarrow \frac{8}{15} = \frac{AE}{AE+3.5}$

⟹ 8AE + 28 = 15AE

⟹ 7AE = 28

⟹ AE = 4cm

2.



Sol:

(i) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

⟹X(x-1) = (x-2) (x+2)

$\Rightarrow x^2 - x = x^2 - 4$

⟹x= 4 cm

(ii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

⟹ 4 (3x-19) = 8 (x-4)

⟹12x – 76 = 8x – 32

⟹4x = 44

⟹ x = 11 cm

(iii) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

⟹3x (7x-4) = (5x-2) (3x+4)

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$

$\Rightarrow 6x^2 - 26x + 8 = 0$

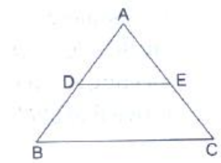
$\Rightarrow (x-4)(6x-2) = 0$

$\Rightarrow x = 4, \frac{1}{3}$

$\because x \neq \frac{1}{3}$ (as if $x = \frac{1}{3}$ $then\ AE\ will\ become\ negative$)

$\therefore x = 4$ cm

3.



Sol:

(i) We have:

$\frac{AD}{DE} = \frac{5.7}{9.5} = 0.6\ cm$

$\frac{AE}{EC} = \frac{4.8}{8} = 0.6\ cm$

Hence, $\frac{AD}{DB} = \frac{AE}{EC}$

Applying the converse of Thales' theorem,

We conclude that DE || BC.

(ii) We have:

AB = 11.7 cm, DB = 6.5 cm

Therefore,

AD = 11.7 -6.5 = 5.2 cm

Similarly,

AC = 11.2 cm, AE = 4.2 cm

Therefore,

EC = 11.2 – 4.2 = 7 cm

Now,

$\frac{AD}{DB} = \frac{5.2}{6.5} = \frac{4}{5}$

$\frac{AE}{EC} = \frac{4.2}{7}$

Thus, $\frac{AD}{DB} \neq \frac{AE}{EC}$

Applying the converse of Thales' theorem,

We conclude that DE is not parallel to BC.

(iii) We have:

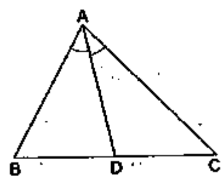
AB = 10.8 cm, AD = 6.3 cm

Therefore,

DB = 10.8 – 6.3 = 4.5 cm
Similarly,
AC = 9.6 cm, EC = 4cm
Therefore,
AE = 9.6 – 4 = 5.6 cm
Now,
$\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$
$\frac{AE}{EC} = \frac{5.6}{4} = \frac{7}{5}$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

(iv) We have :
AD = 7.2 cm, AB = 12 cm
Therefore,
DB = 12 – 7.2 = 4.8 cm
Similarly,
AE = 6.4 cm, AC = 10 cm
Therefore,
EC = 10 – 6.4 = 3.6 cm
Now,
$\frac{AD}{DB} = \frac{7.2}{4.8} = \frac{3}{2}$
$\frac{AE}{EC} = \frac{6.4}{3.6} = \frac{16}{9}$
This, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

4.



Sol:

(i) It is give that AD bisects ∠A.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{5.6}{DC} = \frac{6.4}{8}$
$\Rightarrow DC = \frac{8 \times 5.6}{6.4} = 7\ cm$

(ii) It is given that AD bisects $\angle A$.

Applying angle – bisector theorem in Δ ABC, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

Let BD be x cm.

Therefore, DC = (6- x) cm

$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$

$\Rightarrow$14x = 60-10x

$\Rightarrow$24x = 60

$\Rightarrow$x = 2.5 cm

Thus, BD = 2.5 cm

DC = 6-2.5 = 3.5 cm

(iii) It is given that AD bisector $\angle A$.

Applying angle – bisector theorem in Δ ABC, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

BD = 3.2 cm, BC = 6 cm

Therefore, DC = 6- 3.2 = 2.8 cm

$\Rightarrow \frac{3.2}{2.8} = \frac{5.6}{AC}$

$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2} = 4.9\ cm$

(iv) It is given that AD bisects $\angle A$.

Applying angle – bisector theorem in Δ ABC, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

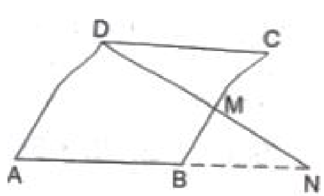
$\Rightarrow \frac{BD}{3} = \frac{5.6}{4}$

$\Rightarrow BD = \frac{5.6 \times 3}{4}$

$\Rightarrow$BD = 4.2 cm

Hence, BC = 3+ 4.2 = 7.2 cm

5.



Sol:

(i) Given: ABCD is a parallelogram

To prove:

(i) $\frac{DM}{MN} = \frac{DC}{BN}$

(ii) $\frac{DN}{DM} = \frac{AN}{DC}$

Proof: In Δ DMC and Δ NMB

∠DMC = ∠NMB (Vertically opposite angle)

∠DCM = ∠NBM (Alternate angles)

By AAA- Similarity

ΔDMC ~ ΔNMB

$\therefore \frac{DM}{MN} = \frac{DC}{BN}$

NOW, $\frac{MN}{DM} = \frac{BN}{DC}$

Adding 1 to both sides, we get

$\frac{MN}{DM} + 1 = \frac{BN}{DC} + 1$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+DC}{DC}$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+AB}{DC}$ [∵ ABCD is a parallelogram]

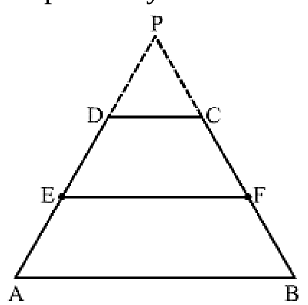
$\Rightarrow \frac{DN}{DM} = \frac{AN}{DC}$

6.

Sol:

(i)

Let the trapezium be ABCD with E and F as the mid Points of AD and BC, Respectively Produce AD and BC to Meet at P.



In Δ PAB, DC || AB.

Applying Thales' theorem, we get

$\frac{PD}{DA} = \frac{PC}{CB}$

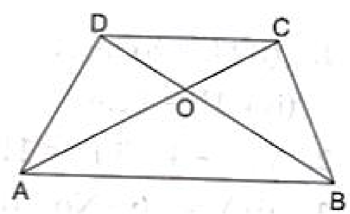
Now, E and F are the midpoints of AD and BC, respectively.

$\Rightarrow \frac{PD}{2DE} = \frac{PC}{2CF}$

$\Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$

Applying the converse of Thales' theorem in Δ PEF, we get that DC
Hence, EF || AB.
Thus. EF is parallel to both AB and DC.
This completes the proof.

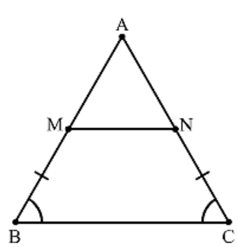
7.



Sol:
In trapezium ABCD, AB ∥ CD and the diagonals AC and BD intersect at O.
Therefore,
$\frac{AO}{OC} = \frac{BO}{OD}$
$\Rightarrow \frac{5x-7}{2x+1} = \frac{7x-5}{7x+1}$
$\Rightarrow$ (5x-7) (7x+1) = (7x-5) (2x+1)
$\Rightarrow 35x^2$ + 5x – 49x – 7 = $14x^2$ – 10x + 7x – 5
$\Rightarrow 21x^2$ – 41x – 2 = 0
$\Rightarrow 21x^2$ – 42x + x -2 =0
$\Rightarrow$21x (x-2) +1 (x-2) =0
$\Rightarrow$ (x-2) (21x + 1) =0
$\Rightarrow$ x = 2, - $\frac{1}{21}$
$\because$ x ≠ - $\frac{1}{21}$
$\therefore$ x = 2

8.

Sol:



In ΔABC, ∠B = ∠C
∴ AB = AC (Sides opposite to equal angle are equal)
Subtracting BM from both sides, we get

AB – BM = AC – BM

$\Rightarrow$AB – BM = AC – CN   ($\because$BM =CN)

$\Rightarrow$AM =AN

$\therefore\angle$AMN =$\angle$ ANM (Angles opposite to equal sides are equal)

Now, in ΔABC,

$\angle A + \angle B + \angle C = 180^0$ ----(1)

(Angle Sum Property of triangle)

Again In In ΔAMN,

$\angle A + \angle AMN + \angle ANM = 180^0$ ----(2)

(Angle Sum Property of triangle)

From (1) and (2), we get

$\angle B + \angle C = \angle AMN + \angle ANM$

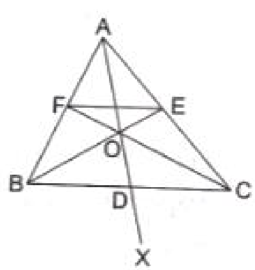
$\Rightarrow 2\angle B = 2\angle AMN$

$\Rightarrow \angle B = \angle AMN$

Since, $\angle$B and $\angle$ AMN are corresponding angles.

$\therefore$ MN || BC.

9.



Sol:

In Δ CAB, PQ || AB.

Applying Thales' theorem, we get:

$\frac{CP}{PB} = \frac{CQ}{QA}$ …(1)

Similarly, applying Thales theorem in ΔBDC , Where PR||DM we get:

$\frac{CP}{PB} = \frac{CR}{RD}$ …(2)

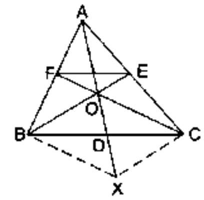
Hence, from (1) and (2), we have :

$\frac{CQ}{QA} = \frac{CR}{RD}$

Applying the converse of Thales' theorem, we conclude that QR ∥ AD in Δ ADC.

This completes the proof.

10.



Sol:

It is give that BC is bisected at D.

∴ BD = DC

It is also given that OD =OX

The diagonals OX and BC of quadrilateral BOCX bisect each other.

Therefore, BOCX is a parallelogram.

∴ BO || CX and BX || CO

BX || CF and CX || BE

BX || OF and CX || OE

Applying Thales' theorem in Δ ABX, we get:

$\frac{AO}{AX} = \frac{AF}{AB}$ …(1)

Also, in Δ ACX, CX || OE.

Therefore by Thales' theorem, we get:

$\frac{AO}{AX} = \frac{AE}{AC}$ …(2)

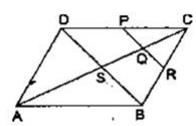
From (1) and (2), we have:

$\frac{AO}{AX} = \frac{AE}{AC}$

Applying the converse of Theorem in Δ ABC, EF || CB.

This completes the proof.

11.



Sol:

We know that the diagonals of a parallelogram bisect each other.

Therefore,

CS = 1/2 AC …(i)

Also, it is given that CQ = 1/4 AC …(ii)

Dividing equation (ii) by (i), we get:

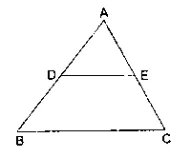
$\frac{CQ}{CS} = \frac{\frac{1}{4}AC}{\frac{1}{2}AC}$

Or, CQ = $\frac{1}{2}$ CS

Hence, Q is the midpoint of CS.

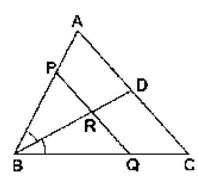
Therefore, according to midpoint theorem in ΔCSD
PQ || DS
If PQ || DS, we can say that QR || SB
In Δ CSB, Q is midpoint of CS and QR ∥ SB.
Applying converse of midpoint theorem , we conclude that R is the midpoint of CB.
This completes the proof.

12.



Sol:
Given:
AD = AE …(i)
AB = AC …(ii)
Subtracting AD from both sides, we get:
⟹ AB – AD = AC – AD
⟹ AB – AD = AC - AE (Since, AD = AE)
⟹ BD = EC …(iii)
Dividing equation (i) by equation (iii), we get:
$\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem, DE∥BC
⟹ ∠DEC + ∠ECB = 180^0 (Sum of interior angles on the same side of a Transversal Line is 0^0 .)
⟹ ∠DEC + ∠CBD = 180^0 (Since, AB = AC ⟹ ∠B = ∠C)
Hence, quadrilateral BCED is cyclic.
Therefore, B,C,E and D are concylic points.

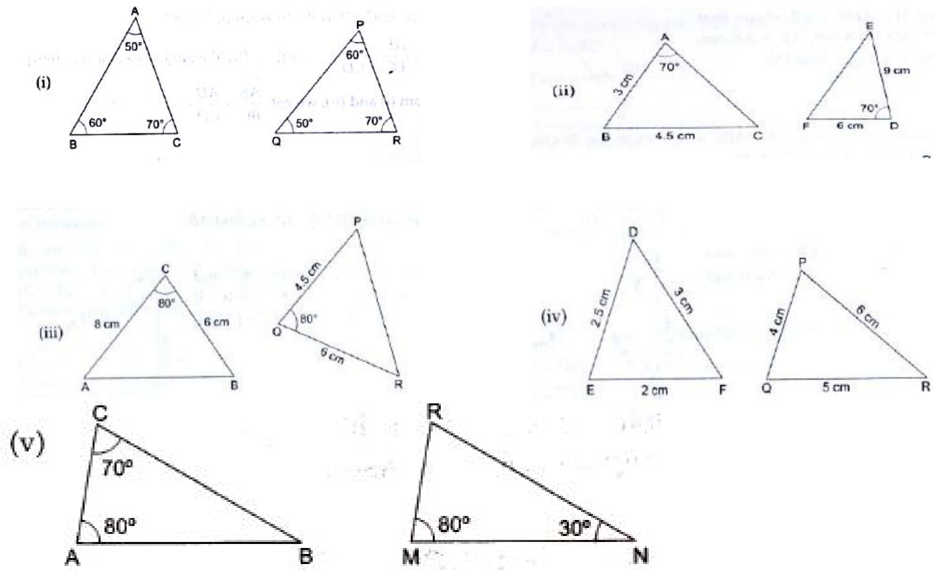
13.



Sol:
In triangle BQO, BR bisects angle B.
Applying angle bisector theorem, we get:
$\frac{QR}{PR} = \frac{BQ}{BP}$
⟹BP × QR = BQ × PR
This completes the proof.

Exercise – 4B

1.



Sol:

(i)

We have:

$\angle BAC = \angle PQR = 50^0$

$\angle ABC = \angle QPR = 60^0$

$\angle ACB = \angle PRQ = 70^0$

Therefore, by AAA similarity theorem, Δ ABC – QPR

(ii)

We have:

$\frac{AB}{DF} = \frac{3}{6} = \frac{1}{2}$ and $\frac{BC}{DE} = \frac{4.5}{9} = \frac{1}{2}$

But , $\angle ABC \neq \angle EDF$ (Included angles are not equal)

Thus, this triangles are not similar.

(iii)

We have:

$\frac{CA}{QR} = \frac{8}{6} = \frac{4}{3}$ and $\frac{CB}{PQ} = \frac{6}{4.5} = \frac{4}{3}$

$\Rightarrow \frac{CA}{QR} = \frac{CB}{PQ}$

Also, $\angle ACB = \angle PQR = 80^0$

Therefore, by SAS similarity theorem, Δ ACB - Δ RQP.

(iv)

We have

$\frac{DE}{QR} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{EF}{PQ} = \frac{2}{4} = \frac{1}{2}$

$\frac{DF}{PR} = \frac{3}{6} = \frac{1}{2}$

$\Longrightarrow \frac{DE}{QR} = \frac{EF}{PQ} = \frac{DF}{PR}$

Therefore, by SSS similarity theorem, Δ FED- Δ PQR

(v)

In Δ ABC

$\angle A + \angle B + \angle C = 180^0$ (Angle Sum Property)

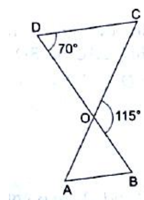
$\Longrightarrow 80^0 + \angle B + 70^0 = 180^0$

$\Longrightarrow \angle B = 30^0$

$\angle A = \angle M \text{ and } \angle B = \angle N$

Therefore, by AA similarity , Δ ABC - Δ MNR

2.



Sol:

(i)

It is given that DB is a straight line.

Therefore,

$\angle DOC + \angle COB = 180^0$

$\angle DOC = 180^0 - 115^0 = 65^0$

(ii)

In Δ DOC, we have:

$\angle ODC + \angle DCO + \angle DOC = 180^0$

Therefore,

$70^0 + \angle DCO + 65^0 = 180^0$

$\Longrightarrow \angle DCO = 180 - 70 - 65 = 45^0$

(iii)

It is given that Δ ODC - Δ OBA

Therefore,

$\angle OAB = \angle OCD = 45^0$

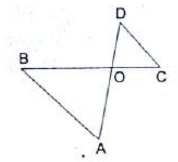
(iv)

Again, Δ ODC- Δ OBA

Therefore,

$\angle OBA = \angle ODC = 70^0$

3.



Sol:

(i) Let OA be X cm.

$\because \Delta$ OAB - Δ OCD

$\therefore \frac{OA}{OC} = \frac{AB}{CD}$

$\Rightarrow \frac{x}{3.5} = \frac{8}{5}$

$\Rightarrow x = \frac{8 \times 3.5}{5} = 5.6$

Hence, OA = 5.6 cm

(ii) Let OD be Y cm

$\because \Delta$ OAB - Δ OCD

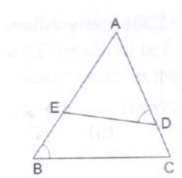
$\therefore \frac{AB}{CD} = \frac{OB}{OD}$

$\Rightarrow \frac{8}{5} = \frac{6.4}{y}$

$\Rightarrow y = \frac{6.4 \times 5}{8} = 4$

Hence, DO = 4 cm

4.



Sol:

Given :

$\angle ADE = \angle ABC \text{ and } \angle A = \angle A$

Let DE be X cm

Therefore, by AA similarity theorem, Δ ADE - Δ ABC

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{3.8}{3.6+2.1} = \frac{x}{4.2}$

$\Rightarrow x = \frac{3.8 \times 4.2}{5.7} = 2.8$

Hence, DE = 2.8 cm

5.

Sol:

It is given that triangles ABC and PQR are similar.

Therefore,

$\frac{Perimeter\ (\Delta ABC)}{Perimeter\ (\Delta PQR)} = \frac{AB}{PQ}$

$\Rightarrow \frac{32}{24} = \frac{AB}{12}$

$\Rightarrow AB = \frac{32 \times 12}{24} = 16\ cm$

6.

Sol:
It is given that Δ ABC - Δ DEF.
Therefore, their corresponding sides will be proportional.
Also, the ratio of the perimeters of similar triangles is same as the ratio of their corresponding sides.

$\Rightarrow \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta DEF} = \frac{BC}{EF}$

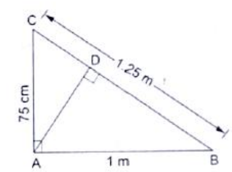
Let the perimeter of ΔABC be X cm
Therefore,

$\frac{x}{25} = \frac{9.1}{6.5}$

$\Rightarrow x = \frac{9.1 \times 25}{6.5} = 35$

Thus, the perimeter of ΔABC is 35 cm.

7.



Sol:
In Δ BDA and Δ BAC, we have :

$\angle BDA = \angle BAC = 90^0$

$\angle DBA = \angle CBA \quad (Common)$

Therefore, by AA similarity theorem, Δ BDA - Δ BAC

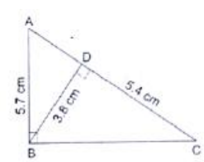
$\Rightarrow \frac{AD}{AC} = \frac{AB}{BC}$

$\Rightarrow \frac{AD}{0.75} = \frac{1}{1.25}$

$\Rightarrow$ AD = $\frac{0.75}{1.25}$

= 0.6 m or 60 cm

8.



Sol:
It is given that ABC is a right angled triangle and BD is the altitude drawn from the right angle to the hypotenuse.
In Δ BDC and Δ ABC, we have :

$\angle ABC = \angle BBC = 90^0 \quad (given)$

$\angle C = \angle C \quad (common)$

By AA similarity theorem, we get :

Δ BDC- Δ ABC

$\frac{AB}{BD} = \frac{BC}{DC}$

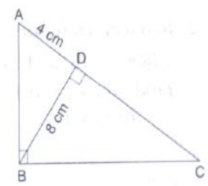
$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$

$\Rightarrow BC = \frac{5.7}{3.8} \times 5.4$

= 8.1

Hence, BC = 8.1 cm

9.



Sol:

It is given that ABC is a right angled triangle

and BD is the altitude drawn from the right angle to the hypotenuse.

In Δ DBA and Δ DCB, we have :

$\angle BDA = \angle CDB$

$\angle DBA = \angle DCB = 90^0$

Therefore, by AA similarity theorem, we get :

ΔDBA - Δ DCB

$\Rightarrow \frac{BD}{CD} = \frac{AD}{BD}$

$\Rightarrow CD = \frac{BD^2}{AD}$

CD = $\frac{8\times8}{4} = 16\ cm$

10.

Sol:

We have :

$\frac{AP}{AB} = \frac{2}{6} = \frac{1}{3}\ and\ \frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

In Δ APQ and Δ ABC, we have:

$\frac{AP}{AB} = \frac{AQ}{AC}$

$\angle A = \angle A$

Therefore, by AA similarity theorem, we get:

Δ APQ - Δ ABC

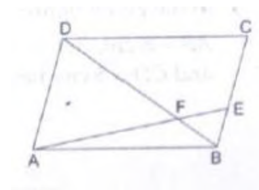
Hence, $\frac{PQ}{BC} = \frac{AQ}{AC} = \frac{1}{3}$

$\Rightarrow \frac{PQ}{BC} = \frac{1}{3}$

$\Rightarrow$ BC = 3PQ

This completes the proof.

11.



Sol:

We have:

$\angle AFD = \angle EFB \quad (Vertically\ Opposite\ angles)$

∵ DA || BC

∴ $\angle DAF = \angle BEF \quad (Alternate\ angles)$

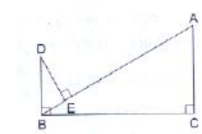
Δ DAF ~ Δ BEF (AA similarity theorem)

$\Longrightarrow \frac{AF}{EF} = \frac{FD}{FB}$

Or, AF × FB = FD × EF

This completes the proof.

12.



Sol:

In ΔBED and ΔACB, we have:

$\angle BED = \angle ACB = 90^0$

∵ $\angle B + \angle C = 180^0$

∴ BD || AC

$\angle EBD = \angle CAB \quad (Alternate\ angles)$

Therefore, by AA similarity theorem, we get :

Δ BED ~ Δ ACB

$\Longrightarrow \frac{BE}{AC} = \frac{DE}{BC}$

$\Longrightarrow \frac{BE}{DE} = \frac{AC}{BC}$

This completes the proof.

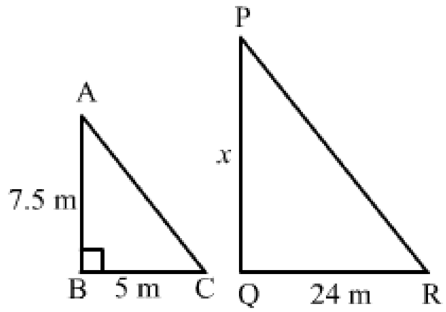
13.

Sol:

Let AB be the vertical stick and BC be its shadow.

Given:

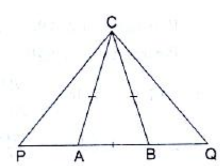
AB = 7.5 m, BC = 5 m



Let PQ be the tower and QR be its shadow.
Given:
QR = 24 m
Let the length of PQ be x m.
In Δ ABC and Δ PQR, we have:
$\angle ABC = \angle PQR = 90^0$
$\angle ACB = \angle PRQ$ (*Angular elevation of the Sun at the same time*)
Therefore, by AA similarity theorem , we get :
Δ ABC ~ Δ PQR
$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$
$\Rightarrow \frac{7.5}{5} = \frac{x}{24}$
$x = \frac{7.5}{5} \times 24 = 36cm$
Therefore, PQ = 36 m
Hence, the height of the tower is 36 m.

14.

Sol:
Disclaimer: It should be ΔAPC ~ ΔBCQ instead of ΔACP ~ ΔBCQ



It is given that ΔABC is an isosceles triangle. Therefore,
CA = CB
$\Rightarrow \angle CAB = \angle CBA$
$\Rightarrow 180^0 - \angle CAB = 180^0 - \angle CBA$
$\Rightarrow \angle CAP = \angle CBQ$
Also,
AP × BQ = AC^2
$\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

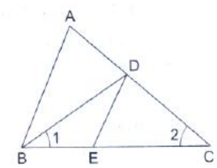
$\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ ($\because AC = BC$)

Thus, by SAS similarity theorem, we get

ΔAPC ~ ΔBCQ

This completes the proof.

15.



Sol:

We have :

$\frac{AC}{BD} = \frac{CB}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{CD}{CE}$ $(Since, BD = DC \; as \; \angle 1 = \angle 2)$

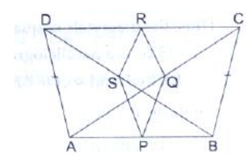
Also, $\angle 1 = \angle 2$

i.e, $\angle DBC = \angle ACB$

Therefore, by SAS similarity theorem, we get :

Δ ACB - Δ DCE

16.



Sol:

In Δ ABC, P and Q are mid points of AB and AC respectively.

So, PQ || BC, and PQ = $\frac{1}{2}$ BC …(1)

Similarly, in ΔADC, …(2)

Now, in ΔBCD, SR $= \frac{1}{2}$ BC …(3)

Similarly, in ΔABD, PS = $\frac{1}{2}$ $AD = \frac{1}{2}$ BC …(4)

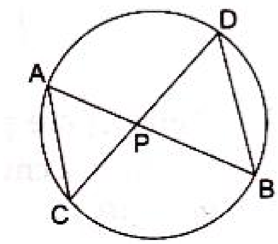
Using (1), (2), (3), and (4).

PQ = QR = SR = PS

Since, all sides are equal

Hence, PQRS is a rhombus.

17.



Sol:

Given : AB and CD are two chords

To Prove:

(a) Δ PAC ~ ΔPDB

(b) PA.PB = PC.PD

Proof: In Δ PAC and Δ PDB

$\angle APC = \angle DPB$ (Vertically Opposite angles)

$\angle CAP = \angle BDP$ (Angles in the same segment are equal)

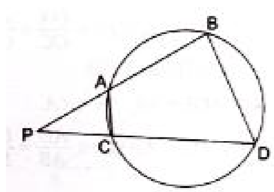
by AA similarity criterion $\Delta PAC \sim PDB$

When two triangles are similar, then the ratios of lengths of their corresponding sides are proportional.

$\therefore \frac{PA}{PD} = \frac{PC}{PB}$

$\Longrightarrow$ PA. PB = PC. PD

18.



Sol:

Given : AB and CD are two chords

To Prove:

(a) Δ PAC - Δ PDB

(b) PA. PB = PC.PD

Proof: $\angle ABD + \angle ACD = 180^0$ …(1) (Opposite angles of a cyclic quadrilateral are supplementary)

$\angle PCA + \angle ACD = 180^0$ …(2) (Linear Pair Angles)

Using (1) and (2), we get

$\angle ABD = \angle PCA$

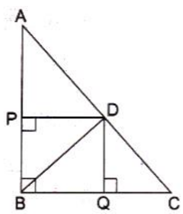
$\angle A = \angle A$ (Common)

By AA similarity-criterion Δ PAC - Δ PDB
When two triangles are similar, then the rations of the lengths of their corresponding sides are proportional.

$\therefore \frac{PA}{PD} = \frac{PC}{PB}$

$\Rightarrow$ PA.PB = PC.PD

19.



Sol:
We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on the both sides of the perpendicular are similar to the whole triangle and also to each other.

(a) Now using the same property in In ΔBDC, we get

ΔCQD ~ ΔDQB

$\frac{CQ}{DQ} = \frac{DQ}{QB}$

$\Rightarrow DQ^2 = QB.CQ$

Now. Since all the angles in quadrilateral BQDP are right angles.
Hence, BQDP is a rectangle.
So, QB = DP and DQ = PB

$\therefore DQ^2 = DP.CQ$

(b)
Similarly, ΔAPD ~ ΔDPB

$\frac{AP}{DP} = \frac{PD}{PB}$

$\Rightarrow DP^2 = AP.PB$

$\Rightarrow DP^2 = AP.DQ$ $\quad [\because DQ = PB]$

Exercise – 4C

1.

Sol:

It is given that Δ ABC ~ Δ DEF.

Therefore, ratio of the areas of these triangles will be equal to the ration of squares of their corresponding sides.

$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{BC^2}{EF^2}$

Let BC be X cm.

$\Rightarrow \frac{64}{121} = \frac{x^2}{(15.4)^2}$

$\Rightarrow x^2 = \frac{64 \times 15.4 \times 15.4}{121}$

$\Rightarrow x = \sqrt{\frac{(64 \times 15.4 \times 15.4)}{121}}$

$= \frac{8 \times 15.4}{11}$

= 11.2

Hence, BC = 11.2 cm

2.

Sol:

It is given that Δ ABC ~ Δ PQR

Therefore, the ration of the areas of triangles will be equal to the ratio of squares of their corresponding sides.

$\frac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{9}{16} = \frac{4^2}{QR^2}$

$\Rightarrow QR^2 = \frac{4.5 \times 4.5 \times 16}{9}$

$\Rightarrow QR = \sqrt{\frac{(4.5 \times 4.5 \times 16)}{9}}$

$= \frac{4.5 \times 4}{3}$

= 6 cm

Hence, QR = 6 cm

3.

Sol:

Given : $ar\ (\Delta ABC) = 4ar\ (\Delta PQR)$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1}$$

∵ ΔABC ~ ΔPQR

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\therefore \frac{BC^2}{QR^2} = \frac{4}{1}$$

$$\Rightarrow QR^2 = \frac{12^2}{4}$$

$$\Rightarrow QR^2 = 36$$

$$\Rightarrow QR = 6\ cm$$

Hence, QR = 6 cm

4.

Sol:

It is given that the triangles are similar.

Therefore, the ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Let the longest side of smaller triangle be X cm.

$$\frac{ar\ (Larger\ triangle)}{ar\ (Smaller\ triangle)} = \frac{(Longest\ side\ of\ larger\ traingle)^2}{(\ Longest\ side\ of\ smaller\ traingle)^2}$$

$$\Rightarrow \frac{169}{121} = \frac{26^2}{x^2}$$

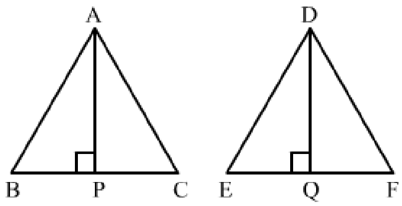
$$\Rightarrow x = \sqrt{\frac{26 \times 26 \times 121}{169}}$$

= 22

Hence, the longest side of the smaller triangle is 22 cm.

5.

Sol:



It is given that ΔABC ~ ΔDEF.

Therefore, the ration of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the altitude of ΔABC be AP, drawn from A to BC to meet BC at P and the altitude of ΔDEF be DQ, drawn from D to meet EF at Q.
Then,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \frac{100}{49} = \frac{5^2}{DQ^2}$$

$$\Rightarrow \frac{100}{49} = \frac{25}{DQ^2}$$

$$\Rightarrow DQ^2 = \frac{49 \times 25}{100}$$

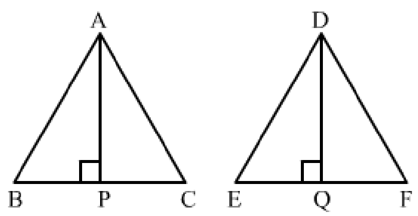
$$\Rightarrow DQ = \sqrt{\frac{49 \times 25}{100}}$$

$$\Rightarrow DQ = 3.5\ cm$$

Hence, the altitude of ΔDEF is 3.5 cm

6.

Sol:
Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



It is given that Δ ABC ~ Δ DEF.
We know that the ration of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AP)^2}{(DQ)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(DEF)} = \frac{6^2}{9^2}$$

$$= \frac{36}{81}$$

$$= \frac{4}{9}$$

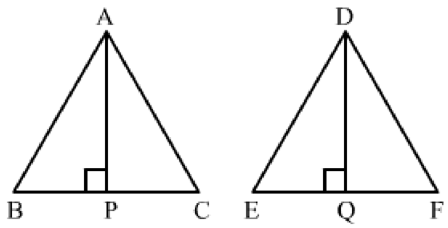
Hence, the ratio of their areas is 4 : 9

7.

Sol:
It is given that the triangles are similar.
Therefore, the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AP^2}{DQ^2}$

$\Longrightarrow \frac{81}{49} = \frac{6.3^2}{DQ}$

$\Longrightarrow DQ^2 = \frac{49}{81} \times 6.3^2$

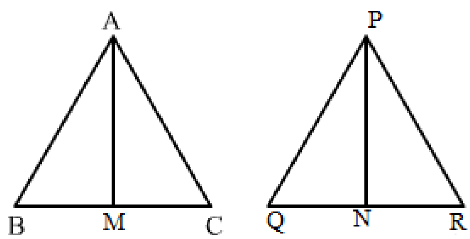
$\Longrightarrow DQ^2 = \sqrt{\frac{49}{81} \times 6.3 \times 6.3}$

Hence, the altitude of the other triangle is 4.9 cm.

8.

Sol:

Let the two triangles be ABC and PQR with medians AM and PN, respectively.



Therefore, the ratio of areas of two similar triangles will be equal to the ratio of squares of their corresponding medians.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$

$\Longrightarrow \frac{64}{100} = \frac{5.6^2}{PN^2}$

$\Longrightarrow PN^2 = \frac{64}{100} \times 5.6^2$

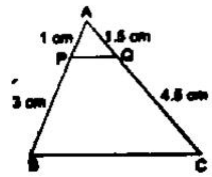
$\Longrightarrow PN^2 = \sqrt{\frac{100}{64} \times 5.6 \times 5.6}$

= 7 cm

Hence, the median of the larger triangle is 7 cm.

9.

Sol:



We have :

$\frac{AP}{AB} = \frac{1}{1+3} = \frac{1}{4}$ and $\frac{AQ}{AC} = \frac{1.5}{1.5+4.5} = \frac{1.5}{6} = \frac{1}{4}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

Also, $\angle A = \angle A$

By SAS similarity, we can conclude that ΔAPQ- ΔABC.

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{1^2}{4^2} = \frac{1}{16}$

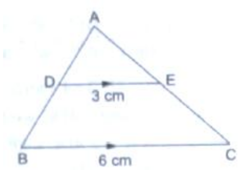
$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16}$

$\Rightarrow ar(\Delta APQ) = \frac{1}{16} \times ar\ (\Delta ABC)$

Hence proved.

10.

Sol:



It is given that DE || BC

$\therefore \angle ADE = \angle ABC\ (Corresponding\ angles)$

$\angle AED = \angle ACB\ (Corresponding\ angles)$

By AA similarity, we can conclude that Δ ADE ~ Δ ABC

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{15}{ar(\Delta ABC)} = \frac{3^2}{6^2}$

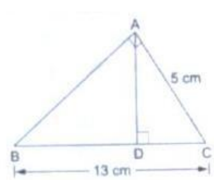
$\Rightarrow ar\ (\Delta ABC) = \frac{15 \times 36}{9}$

= 60 cm^2

Hence, area of triangle ABC is 60 cm^2

11.

Sol:



In ΔABC and ΔADC, we have:

$\angle BAC = \angle ADC = 90^0$

$\angle ACB = \angle ACD\ (common)$

By AA similarity, we can conclude that Δ BAC~ Δ ADC.

Hence, the ratio of the areas of these triangles is equal to the ratio of squares of their corresponding sides.

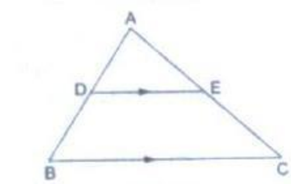
$\therefore \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{BC^2}{AC^2}$

$\Rightarrow \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{13^2}{5^2}$

$= \frac{169}{25}$

Hence, the ratio of areas of both the triangles is 169:25

12.



Sol:

It is given that DE || BC.

$\therefore \angle ADE = \angle ABC \ (Corresponding\ angles)$

$\angle AED = \angle ACB \ (Corresponding\ angles)$

Applying AA similarity theorem, we can conclude that Δ ADE ~ ΔABC.

$\therefore \frac{ar(\Delta ABC)}{ar(ADE)} = \frac{BC^2}{DE^2}$

Subtracting 1 from both sides, we get:

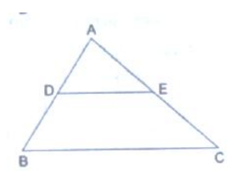
$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} - 1 = \frac{5^2}{3^2} - 1$

$\Rightarrow \frac{ar(\Delta ABC) - ar(\Delta ADE)}{ar(\Delta ADE)} = \frac{25-9}{9}$

$\Rightarrow \frac{ar(BCED)}{ar(\Delta ADE)} = \frac{16}{9}$

Or, $\frac{ar(\Delta ADE)}{ar(BCED)} = \frac{9}{16}$

13.



Sol:

It is given that D and E are midpoints of AB and AC.

Applying midpoint theorem, we can conclude that DE ∥ BC.

Hence, by B.P.T., we get :

$\frac{AD}{AB} = \frac{AE}{AC}$

Also, $\angle A = \angle A$

Applying SAS similarity theorem, we can conclude that Δ ADE~ Δ ABC.

Therefore, the ration of areas of these triangles will be equal to the ratio of squares of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2}$

$= \frac{1}{4}$

Exercise – 4D

1.

Sol:

For the given triangle to be right-angled, the sum of the two sides must be equal to the square of the third side.

Here, let the three sides of the triangle be a, b and c.

(i)

a = 9 cm, b = 16 cm and c = 18 cm

Then,

$a^2 + b^2 = 9^2 + 16^2$

$= 81 + 256$

$= 337$

$c^2 = 19^2$

$= 361$

$a^2 + b^2 \neq c^2$

Thus, the given triangle is not right-angled.

(ii)

A=7 cm, b = 24 cm and c = 25 cm

Then,

$a^2 + b^2 = 7^2 + 24^2$

= 49 + 576

= 625

$c^2 = 25^2$

= 625

$a^2 + b^2 = c^2$

Thus, the given triangle is a right-angled.

(iii)

A= 1.4 cm, b= 4.8 cm and c= 5 cm

Then,

$a^2 + b^2 = (1.4)^2 + (4.8)^2$

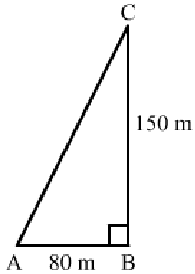
= 1.96 + 23.04

= 25

$c^2 = 5^2$
= 25
$a^2 + b^2 = c^2$
Thus, the given triangle is right-angled.
(iv) A = 1.6 cm, b= 3.8 cm and c= 4 cm
Then
$a^2 + b^2 = (1.6)^2 + (3.8)^2$
= 2.56 + 14.44
= 16
$a^2 + b^2 \neq c^2$
Thus, the given triangle is not right-angled.
(v)
P = (a-1) cm, q = 2 $\sqrt{a}\ cm\ and\ r = (a+1)cm$
Then,
$p^2 + q^2 = (a-1)^2 + \left(2\sqrt{a}\right)^2$
$= a^2 + 1 - 2a + 4a$
$= a^2 + 1 + 2a$
$= (a+1)^2$
$r^2 = (a+1)^2$
$p^2 + q^2 = r^2$
Thus, the given triangle is right-angled.

2.

Sol:
Let the man starts from point A and goes 80 m due east to B.
Then, from B, he goes 150 m due north to c.

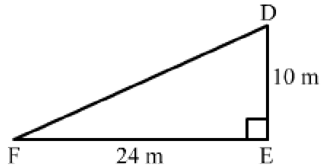


We need to find AC.
In right- angled triangle ABC, we have:
$AC^2 = AB^2 + BC^2$
$AC = \sqrt{80^2 + 150^2}$
$= \sqrt{6400 + 22500}$
$= \sqrt{28900}$
= 170 m

Hence, the man is 170 m away from the starting point.

3.

Sol:
Let the man starts from point D and goes 10 m due south at E. He then goes 24 m due west at F.
In right ΔDEF, we have:
DE = 10 m, EF = 24 m



$DF^2 = EF^2 + DE^2$

$DF = \sqrt{10^2 + 24^2}$

$= \sqrt{100 + 576}$

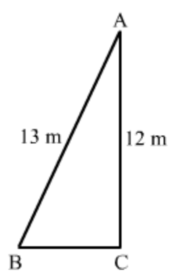
$= \sqrt{676}$

= 26 m

Hence, the man is 26 m away from the starting point.

4.

Sol:
Let AB and AC be the ladder and height of the building.
It is given that :
AB = 13 m and AC = 12 m
We need to find distance of the foot of the ladder from the building, i.e, BC.
In right-angled triangle ABC, we have:



$AB^2 = AC^2 + BC^2$

$\Rightarrow BC = \sqrt{13^2 - 12^2}$

$= \sqrt{169 - 144}$

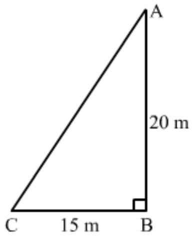
$= \sqrt{25}$

= 5 m

Hence, the distance of the foot ladder from the building is 5 m

5.

Sol:
Let the height of the window from the ground and the distance of the foot of the ladder from the wall be AB and BC, respectively.
We have :
AB = 20 m and BC = 15 m
Applying Pythagoras theorem in right-angled ABC, we get:



$AC^2 = AB^2 + BC^2$

$\Rightarrow AC = \sqrt{20^2 + 15^2}$

$= \sqrt{400 + 225}$

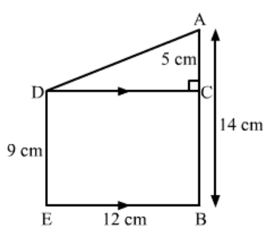
$= \sqrt{625}$

= 25 m

Hence, the length of the ladder is 25 m.

6.

Sol:
Let the two poles be DE and AB and the distance between their bases be BE.
We have:
DE = 9 m, AB = 14 m and BE = 12 m
Draw a line parallel to BE from D, meeting AB at C.
Then, DC = 12 m and AC = 5 m
We need to find AD, the distance between their tops.



Applying Pythagoras theorem in right-angled ACD, we have:

$AD^2 = AC^2 + DC^2$

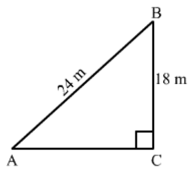
$AD^2 = 5^2 + 12^2 = 25 + 144 = 169$

$AD = \sqrt{169} = 13\ m$

Hence, the distance between the tops to the two poles is 13 m.

7.

Sol:



Let AB be a guy wire attached to a pole BC of height 18 m. Now, to keep the wire taut let it to be fixed at A.

Now, In right triangle ABC

By using Pythagoras theorem, we have

$AB^2 = BC^2 + CA^2$

$\Rightarrow 24^2 = 18^2 + CA^2$

$\Rightarrow CA^2 = 576 - 324$

$\Rightarrow CA^2 = 252$

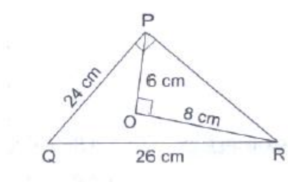
$\Rightarrow CA = 6\sqrt{7}\, m$

Hence, the stake should be driven 6 $\sqrt{7}m$ far from the base of the pole.

8.

Sol:

Applying Pythagoras theorem in right-angled triangle POR, we have:



$PR^2 = PO^2 + OR^2$

$\Rightarrow PR^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow PR = \sqrt{100} = 10\ cm$

IN Δ PQR,

$PQ^2 + PR^2 = 24^2 + 10^2 = 576 + 100 = 676$

And $QR^2 = 26^2 = 676$

$\therefore PQ^2 + PR^2 = QR^2$

Therefore, by applying Pythagoras theorem, we can say that ΔPQR is right-angled at P.

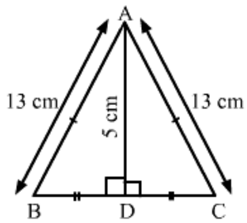
9.

Sol:

It is given that Δ ABC is an isosceles triangle.

Also, AB = AC = 13 cm

Suppose the altitude from A on BC meets BC at D. Therefore, D is the midpoint of BC.
AD =5 cm
$\Delta\, ADB$ and $\Delta\, ADC$ are right-angled triangles.
Applying Pythagoras theorem, we have;



$AB^2 = AD^2 + BD^2$

$BD^2 = AB^2 - AD^2 = 13^2 - 5^2$

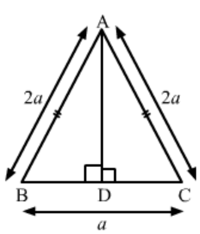
$BD^2 = 169 - 25 = 144$

$BD = \sqrt{144} = 12$

Hence,
BC =2(BD) = 2 × 12 = 24 cm

10.

Sol:
In isosceles Δ ABC, we have:
AB = AC = 2a units and BC = a units
Let AD be the altitude drawn from A that meets BC at D.
Then, D is the midpoint of BC.
BD = BC = $\frac{a}{2}$ *units*
Applying Pythagoras theorem in right-angled ΔABD, we have:



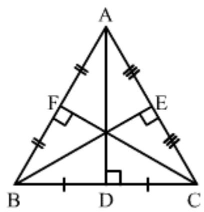
$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2 = (2a)^2 - \left(\frac{a}{2}\right)^2$

$AD^2 = 4a^2 - \frac{a^2}{4} = \frac{15a^2}{4}$

$AD = \sqrt{\frac{15a^2}{4}} = \frac{a\sqrt{15}}{2}\ units.$

11.
Sol:



Let AD, BE and CF be the altitudes of ΔABC meeting BC, AC and AB at D, E and F, respectively.
Then, D, E and F are the midpoint of BC, AC and AB, respectively.
In right-angled ΔABD, we have:
AB = 2a and BD = a
Applying Pythagoras theorem, we get:

$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2$

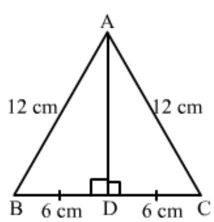
$AD^2 = 4a^2 - a^2 = 3a^2$

$AD = \sqrt{3}a \; units$

Similarly,
BE = $a\sqrt{3} \; units \; and \; CF = a\sqrt{3} \; units$

12.

Sol:
Let ABC be the equilateral triangle with AD as an altitude from A meeting BC at D. Then, D will be the midpoint of BC.
Applying Pythagoras theorem in right-angled triangle ABD, we get:



$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = 12^2 - 6^2 \; (\because BD = \frac{1}{2} BC = 6)$

$\Rightarrow AD^2 = 144 - 36 = 108$

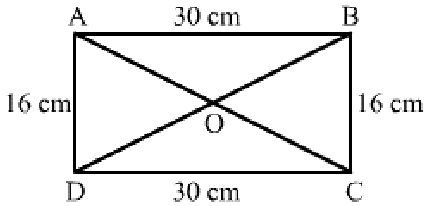
$\Rightarrow AD = \sqrt{108} = 6\sqrt{3} \; cm.$

Hence, the height of the given triangle is $6\sqrt{3} \; cm$.

13.

Sol:

Let ABCD be the rectangle with diagonals AC and BD meeting at O.
According to the question:
AB = CD = 30 cm and BC = AD = 16 cm



Applying Pythagoras theorem in right-angled triangle ABC, we get:

$AC^2 = AB^2 + BC^2 = 30^2 + 16^2 = 900 + 256 = 1156$

$AC = \sqrt{1156} = 34\ cm$

Diagonals of a rectangle are equal.
Therefore, AC = BD = 34 cm

14.

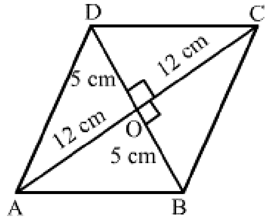
Sol:

Let ABCD be the rhombus with diagonals (AC = 24 cm and BD = 10 cm) meeting at O.
We know that the diagonals of a rhombus bisect each other at angles.
Applying Pythagoras theorem in right-angled AOB, we get:

$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$

$AB^2 = 144 + 25 = 169$

$AB = \sqrt{169} = 13\ cm$



Hence, the length of each side of the rhombus is 13 cm.

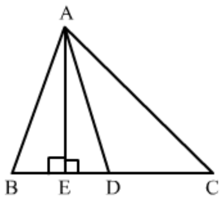
15.

Sol:

In right-angled triangle AED, applying Pythagoras theorem, we have:

$AB^2 = AE^2 + ED^2 \quad ...(i)$

In right-angled triangle AED, applying Pythagoras theorem, we have:



$AD^2 = AE^2 + ED^2$

$\Longrightarrow AE^2 = AD^2 - ED^2 \quad ... (ii)$

Therefore,

$AB^2 = AD^2 - ED^2 + EB^2$ ($from\ (i) and\ (ii)$)

$AB^2 = AD^2 - ED^2 + (BD - DE)^2$

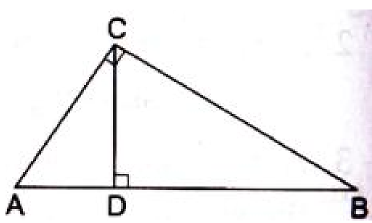
$= AD^2 - ED^2 + \left(\frac{1}{2} BC - DE\right)^2$

$= AD^2 - DE^2 + \frac{1}{4} BC^2 + DE^2 - BC.DE$

$= AD^2 + \frac{1}{4} BC^2 - BC.DE$

This completes the proof.

16.



Sol:

Given: $\angle ACB = 90^0\ and\ CD \perp AB$

To Prove; $\frac{BC^2}{AC^2} = \frac{BD}{AD}$

Proof: In Δ ACB and Δ CDB

$\angle ACB = \angle CDB = 90^0\ (Given)$

$\angle ABC = \angle CBD\ (Common)$

By AA similarity-criterion Δ ACB ~ ΔCDB

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

$\therefore \frac{BC}{BD} = \frac{AB}{BC}$

$\Longrightarrow BC^2 = BD.AB \quad ... (1)$

In Δ ACB and Δ ADC

$\angle ACB = \angle ADC = 90^0\ (Given)$

$\angle CAB = \angle DAC\ (Common)$

By AA similarity-criterion Δ ACB ~ ΔADC

When two triangles are similar, then the ratios of their corresponding sides are proportional.

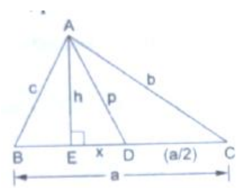
$\therefore \frac{AC}{AD} = \frac{AB}{AC}$

$\Rightarrow AC^2$ = AD. AB ....(2)

Dividing (2) by (1), we get

$\frac{BC^2}{AC^2} = \frac{BD}{AD}$

17.



Sol:

(i)

In right-angled triangle AEC, applying Pythagoras theorem, we have:

$AC^2 = AE^2 + EC^2$

$\Rightarrow b^2 = h^2 + \left(x + \frac{a}{2}\right)^2 = h^2 + x^2 + \frac{a^2}{4} + ax \ldots (i)$

In right – angled triangle AED, we have:

$AD^2 = AE^2 + ED^2$

$\Rightarrow p^2 = h^2 + x^2 \ldots (ii)$

Therefore,

from (i) and (ii),

$b^2 = p^2 + ax + \frac{a^2}{x}$

(ii)

In right-angled triangle AEB, applying Pythagoras, we have:

$AB^2 = AE^2 + EB^2$

$\Rightarrow c^2 = h^2 + \left(\frac{a}{2} - x\right)^2 \; (\because BD = \frac{a}{2} \; and \; BE = BD - x)$

$\Rightarrow c^2 = h^2 + x^2 - \frac{a^2}{4} \; (\because h^2 + x^2 = p^2)$

$\Rightarrow c^2 = p^2 - ax + \frac{a^2}{x}$

(iii)

Adding (i) and (ii), we get:

$\Rightarrow b^2 + c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$

$= 2p^2 + ax - ax + \frac{a^2+a^2}{4}$

$= 2p^2 + \frac{a^2}{2}$

(iv)

Subtracting (ii) from (i), we get:

$b^2 - c^2 = p^2 + ax + \frac{a^2}{4} - (p^2 - ax + \frac{a^2}{4})$

$= p^2 - p^2 + ax + ax + \frac{a^2}{4} - \frac{a^2}{4}$

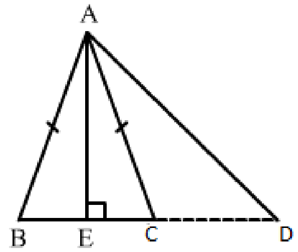
= 2ax

18.

Sol:

Draw AE⊥BC, meeting BC at D.

Applying Pythagoras theorem in right-angled triangle AED, we get:



Since, ABC is an isosceles triangle and AE is the altitude and we know that the altitude is also the median of the isosceles triangle.

So, BE = CE

And DE+CE=DE+BE=BD

$AD^2 = AE^2 + DE^2$

$\Rightarrow AE^2 = AD^2 - DE^2$ …(i)

In ΔACE,

$AC^2 = AE^2 + EC^2$

$\Rightarrow AE^2 = AC^2 - EC^2$ ... (ii)

Using (i) and (ii),

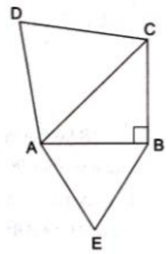
$\Rightarrow AD^2 - DE^2 = AC^2 - EC^2$

$\Rightarrow AD^2 - AC^2 = DE^2 - EC^2$

= (DE + CE) (DE – CE)

= (DE + BE) CD

= BD.CD

19. 

Sol:

We have, ABC as an isosceles triangle, right angled at B.

Now, AB = BC

Applying Pythagoras theorem in right-angled triangle ABC, we get:

$AC^2 = AB^2 + BC^2 = 2AB^2 \ (\because AB = AC) \ldots (i)$

$\because$ ΔACD ~ Δ ABE

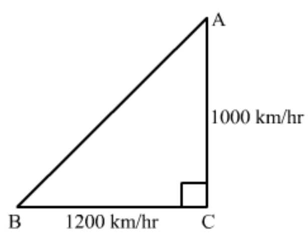
We know that ratio of areas of 2 similar triangles is equal to squares of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABE)}{ar(\Delta ACD)} = \frac{AB^2}{AC^2} = \frac{AB^2}{2AB^2} \ [from\ (i)]$

$= \frac{1}{2} = 1 : 2$

20.

Sol:



Let A be the first aeroplane flied due north at a speed of 1000 km/hr and B be the second aeroplane flied due west at a speed of 1200 km/hr

Distance covered by plane A in $1\frac{1}{2}\ hours = 1000 \times \frac{3}{2} = 1500\ km$

Distance covered by plane B in $1\frac{1}{2}\ hours = 1200 \times \frac{3}{2} = 1800\ km$

Now, In right triangle ABC

By using Pythagoras theorem, we have

$AB^2 = BC^2 + CA^2$

$= (1800)^2 + (1500)^2$

= 3240000 + 2250000

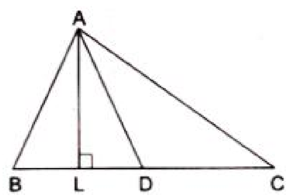
= 5490000

$\therefore AB^2 = 5490000$

$\Rightarrow$ AB = 300 $\sqrt{61}\,m$

Hence, the distance between two planes after $1\frac{1}{2}$ $hours\ is$ $300\sqrt{61}m$

21.



Sol:

(a) In right triangle ALD

Using Pythagoras theorem, we have

$AC^2 = AL^2 + LC^2$

$= AD^2 - DL^2 + (DL + DC)^2$ [Using (1)]

$= AD^2 - DL^2 + \left(DL + \frac{BC}{2}\right)^2$ [$\because$ AD is a median]

$= AD^2 - DL^2 + DL^2 + \left(\frac{BC}{2}\right)^2 + BC.DL$

$\therefore AC^2 = AD^2 + BC.DL + \left(\frac{BC}{2}\right)^2$ .... (2)

(b) In right triangle ALD

Using Pythagoras theorem, we have

$AL^2 = AD^2 - DL^2$ ... (3)

Again, In right triangle ABL

Using Pythagoras theorem, we have

$AB^2 = AL^2 + LB^2$

$= AD^2 - DL^2 + LB^2$ [$Using$ (3)]

$= AD^2 - DL^2 + (BD - DL)^2$

$= AD^2\ DL^2 + \left(\frac{1}{2}\ BC - DL\right)^2$

$= AD^2 - DL^2 + \left(\frac{BC}{2}\right)^2 - BC.DL + DL^2$

$\therefore AB^2 = AD^2 - BC.DL + \left(\frac{BC}{2}\right)^2$ ..... (4)

(c) Adding (2) and (4), we get,

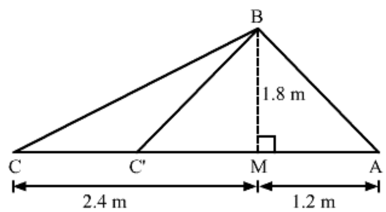
$= AC^2 + AB^2 = AD^2 + BC.DL + \left(\frac{BC}{2}\right)^2 + AD^2 - BC.DL + \left(\frac{BC}{2}\right)^2$

$= 2\ AD^2 + \frac{BC^2}{4} + \frac{BC^2}{4}$

$= 2\ AD^2 + \frac{1}{2}\ BC^2$

22.

Sol:



Naman pulls in the string at the rate of 5 cm per second.
Hence, after 12 seconds the length of the string he will pulled is given by
12 × 5 = 60 cm or 0.6 m
Now, in ΔBMC
By using Pythagoras theorem, we have
$BC^2 = CM^2 + MB^2$
$= (2.4)^2 + (1.8)^2$
= 9
∴ BC = 3 m
Now, BC' = BC – 0.6
= 3 – 0.6
= 2.4 m
Now, In ΔBC'M
By using Pythagoras theorem, we have
$C'M^2 = BC'^2 - MB^2$

$= (2.4)^2 - (1.8)^2$
= 2.52
∴ C'M = 1.6 m
The horizontal distance of the fly from him after 12 seconds is given by
C'A = C'M + MA
= 1.6 + 1.2
= 2.8 m

Exercise – 4E

1.
Sol:
The two triangles are similar if and only if
1. The corresponding sides are in proportion.
2. The corresponding angles are equal.

2.
Sol:
If a line is draw parallel to one side of a triangle intersect the other two sides, then it divides the other two sides in the same ratio.

3.
Sol:
If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

4.
Sol:
The line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is equal to one half of the third side.

5.
Sol:
If the corresponding angles of two triangles are equal, then their corresponding sides are proportional and hence the triangles are similar.

6.
Sol:
If two angles are correspondingly equal to the two angles of another triangle, then the two triangles are similar.

7.

Sol:

If the corresponding sides of two triangles are proportional then their corresponding angles are equal, and hence the two triangles are similar.

8.

Sol:

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional then the two triangles are similar.

9.

Sol:

The square of the hypotenuse is equal to the sum of the squares of the other two sides. Here, the hypotenuse is the longest side and it's always opposite the right angle.

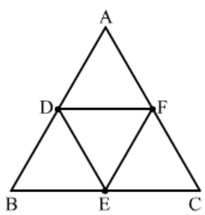
10.

Sol:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

11.

Sol:



By using mid theorem i.e., the segment joining two sides of a triangle at the midpoints of those sides is parallel to the third side and is half the length of the third side.

∴ DF || BC

And $DF = \frac{1}{2} BC$

⟹ DF = BE

Since, the opposite sides of the quadrilateral are parallel and equal.

Hence, BDFE is a parallelogram

Similarly, DFCE is a parallelogram.

Now, in ΔABC and ΔEFD

$\angle ABC = \angle EFD$ (*Opposite angles of a parallelogram*)

$\angle BCA = \angle EDF$ (*Opposite angles of a parallelogram*)

By AA similarity criterion, ΔABC ~ ΔEFD

If two triangles are similar, then the ratio of their areas is equal to the squares of their corresponding sides.

$\therefore \frac{area(\Delta DEF)}{area(\Delta ABC)} = \left(\frac{DF}{BC}\right)^2 = \left(\frac{DF}{2\,DF}\right)^2 = \frac{1}{4}$

Hence, the ratio of the areas of ΔDEF and ΔABC is 1 : 4.

12.

Sol:

Now, In ΔABC and ΔPQR

$\angle A = \angle P = 70^0 \quad (Given)$

$\frac{AB}{PQ} = \frac{AC}{PR} \left[\because \frac{3}{4.5} = \frac{6}{9} \Rightarrow \frac{1}{1.5} = \frac{1}{1.5}\right]$

By SAS similarity criterion, ΔABC~ ΔPQR

13.

Sol:

When two triangles are similar, then the ratios of the lengths of their corresponding sides are equal.

Here, ΔABC ~ΔDEF

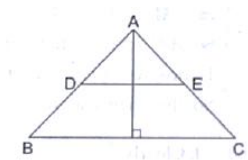
$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{AB}{2\,AB} = \frac{6}{EF}$

$\Rightarrow$ EF = 12 cm

14.

Sol:



In ΔADE and ΔABC

$\angle ADE = \angle ABC \quad (Corresponding\ angles\ in\ DE \parallel BC)$

$\angle AED = \angle ACB \quad (Corresponding\ angles\ in\ DE \parallel BC$

By AA similarity criterion, ΔADE ~ ΔABC

If two triangles are similar, then the ratio of their corresponding sides are proportional

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{AD}{AD+DB} = \frac{AE}{AE+EC}$

$\Rightarrow \frac{x}{x+3x+4} = \frac{x+3}{x+3+3x+19}$

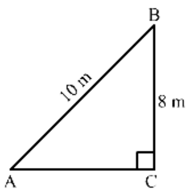
$\Rightarrow \frac{x}{4x+4} = \frac{x+3}{x+3+3x+19}$

$\Rightarrow \frac{x}{2x+2} = \frac{x+3}{2x+11}$

$\Longrightarrow 2x^2 + 11x = 2x^2 + 2x + 6x + 6$
$\Longrightarrow$ 3x = 6
$\Longrightarrow$ x = 2
Hence, the value of x is 2.

15.

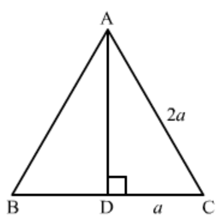
Sol:



Let AB be A ladder and B is the window at 8 m above the ground C.
Now, In right triangle ABC
By using Pythagoras theorem, we have
$AB^2 = BC^2 + CA^2$
$\Longrightarrow 10^2 = 8^2 + CA^2$
$\Longrightarrow CA^2 = 100 - 64$
$\Longrightarrow CA^2 = 36$
$\Longrightarrow$ CA = 6m
Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

16.

Sol:



We know that the altitude of an equilateral triangle bisects the side on which it stands and forms right angled triangles with the remaining sides.
Suppose ABC is an equilateral triangle having AB = BC = CA = 2a.
Suppose AD is the altitude drawn from the vertex A to the side BC.
So, it will bisects the side BC
$\therefore$ DC = a
Now, In right triangle ADC
By using Pythagoras theorem, we have
$AC^2 = CD^2 + DA^2$
$\Longrightarrow (2a)^2 = a^2 + DA^2$

$\Rightarrow DA^2 = 4a^2 - a^2$
$\Rightarrow DA^2 = 3a^2$
$\Rightarrow$ DA = $\sqrt{3}a$
Hence, the length of the altitude of an equilateral triangle of side 2a cm is $\sqrt{3}a\ cm$

17.

Sol:
We have Δ ABC ~ Δ DEF
If two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta ABC)}{area(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$

$\Rightarrow \frac{64}{169} = \left(\frac{BC}{EF}\right)^2$

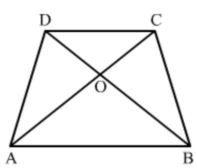
$\Rightarrow \left(\frac{8}{13}\right)^2 = \left(\frac{4}{EF}\right)^2$

$\Rightarrow \frac{8}{13} = \frac{4}{EF}$

$\Rightarrow$ EF = 6.5 cm

18.

Sol:



In Δ AOB and COD
$\angle ABO = \angle CDO \qquad (Alternte\ angles\ in\ AB \parallel CD)$
$\angle AOB = \angle COD \qquad (Vertically\ opposite\ angles)$
By AA similarity criterion, ΔAOB ~ ΔCOD
If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta AOB)}{area(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$

$\Rightarrow \frac{84}{area(\Delta COD)} = \left(\frac{2\ CD}{CD}\right)^2$

$\Rightarrow area\ (\Delta\ COD\ ) = 12\ cm^2$

19.

Sol:

If two triangles are similar, then the ratio of their areas is equal to the squares of their corresponding sides.

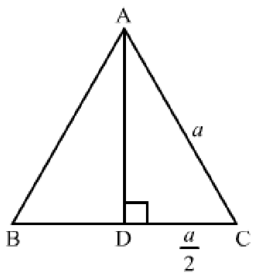
$\therefore \frac{area\ of\ smaller\ triangle}{area\ of\ lager\ triangle} = \left(\frac{Side\ of\ smaller\ triangle}{Side\ of\ larger\ triangle}\right)^2$

$\Rightarrow \frac{48}{area\ of\ larger\ triangle} = \left(\frac{2}{3}\right)^2$

$\Rightarrow area\ of\ larger\ triangle = 108\ cm^2$

20

Sol:



We know that the altitude of an equilateral triangle bisects the side on which it stands and forms right angled triangles with the remaining sides.

Suppose ABC is an equilateral triangle having AB =BC = CA = a.

Suppose AD is the altitude drawn from the vertex A to the side BC.

So, It will bisects the side BC

$\therefore DC = \frac{1}{2}\ a$

Now, In right triangle ADC

By using Pythagoras theorem, we have

$AC^2 = CD^2 + DA^2$

$\Rightarrow a^2 - \left(\frac{1}{2}a\right)^2 + DA^2$

$\Rightarrow DA^2 = a^2 - \frac{1}{4}a^2$

$\Rightarrow DA^2 = \frac{3}{4}a^2$

$\Rightarrow DA = \frac{\sqrt{3}}{2}\ a$

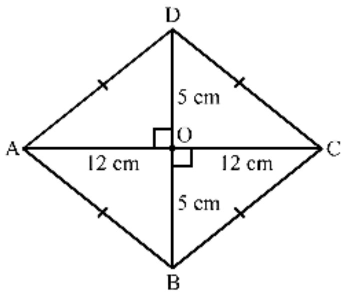
$Now, area\ (\Delta ABC) = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}\ a$

$= \frac{\sqrt{3}}{4}\ a^2$

21.

Sol:



Suppose ABCD is a rhombus.

We know that the diagonals of a rhombus perpendicularly bisect each other.

$\therefore \angle AOB = 90^0$, $AO = 12\ cm\ and\ BO = 5\ cm$

Now, In right triangle AOB

By using Pythagoras theorem we have

$AB^2 = AO^2 + BO^2$

$= 12^2 + 5^2$

$= 144 + 25$

$= 169$

$\therefore AB^2 = 169$

$\Longrightarrow$ AB = 13 cm

Since, all the sides of a rhombus are equal.

Hence, AB = BC = CD = DA = 13 cm

22.

Sol:

If two triangles are similar then the corresponding angles of the two triangles are equal.

Here, ΔDEF ~ ΔGHK

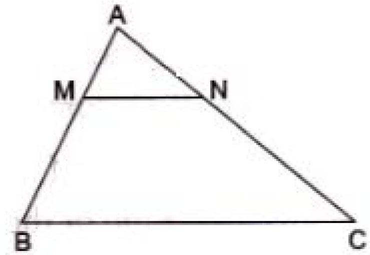
$\therefore \angle E = \angle H = 57^0$

Now, In Δ DEF

$\angle D + \angle E + \angle F = 180^0$ $\quad (Angle\ sum\ property\ of\ triangle)$

$\Longrightarrow \angle F = 180^0 - 48^0 - 57^0 = 75^0$

23.



Sol:

We have

AM : MB = 1 : 2

$\Rightarrow \frac{MB}{AM} = \frac{2}{1}$

Adding 1 to both sides, we get

$\Rightarrow \frac{MB}{AM} + 1 = \frac{2}{1} + 1$

$\Rightarrow \frac{MB+AM}{AM} = \frac{2+1}{1}$

$\Rightarrow \frac{AB}{AM} = \frac{3}{1}$

Now, In ΔAMN and ΔABC

$\angle AMN = \angle ABC \quad (Corresponding\ angles\ in\ MN \parallel BC)$

$\angle ANM = \angle ACB \quad (Corresponding\ angles\ in\ MN \parallel BC)$

By AA similarity criterion, ΔAMN ~ Δ ABC

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta AMN)}{area(\Delta ABC)} = \left(\frac{AM}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

24.

Sol:

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

Here, ΔBMP ~ ΔCNR

$\therefore \frac{BM}{CN} = \frac{BP}{CR} = \frac{MP}{NR} \quad ..(1)$

$Now, \frac{BM}{CN} = \frac{MP}{NR} \quad [Using\ (1)]$

$\Rightarrow CN = \frac{BM \times NR}{MP} = \frac{9 \times 9}{6} = 13.5\ cm$

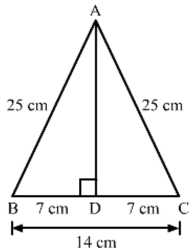
$Again, \frac{BM}{CN} = \frac{BP}{CR} = [Using\ (1)]$

$\Rightarrow CR = \frac{BP \times CN}{BM} = \frac{5 \times 13.5}{9} = 7.5\ cm$

Perimeter of ΔCNR = CN + NR + CR = 13.5 + 9 + 7.5 = 30 cm

25.

Sol:



We know that the altitude drawn from the vertex opposite to the non-equal side bisects the non-equal side.

Suppose ABC is an isosceles triangle having equal sides AB and BC.

So, the altitude drawn from the vertex will bisect the opposite side.

Now, In right triangle ABD

By using Pythagoras theorem, we have

$AB^2 = BD^2 + DA^2$

$\Longrightarrow 25^2 = 7^2 + DA^2$

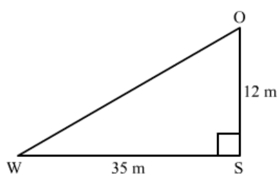
$\Longrightarrow DA^2 = 625 - 49$

$\Longrightarrow DA^2 = 576$

$\Longrightarrow DA = 24\ cm$

26.

Sol:



In right triangle SOW

By using Pythagoras theorem, we have

$OW^2 = WS^2 + SO^2$

$= 35^2 + 12^2$

$= 1225 + 144$

$= 1369$

$\therefore OW^2 = 1369$

$\Longrightarrow OW = 37\ m$

Hence, the man is 37 m away from the starting point.

27.

Sol:

Let DC = X

∴ BD = a-X

By using angle bisector there in ΔABC, we have

$\frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \frac{c}{b} = \frac{a-x}{x}$

$\Rightarrow cx = ab - bx$

$\Rightarrow x\,(b + c) = ab$

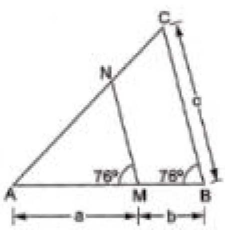
$\Rightarrow x = \frac{ab}{(b+c)}$

Now, $a - x = a - \frac{ab}{b+c}$

$= \frac{ab+ac-ab}{b+c}$

$= \frac{ac}{a+b}$

28.



Sol:

In ΔAMN and ΔABC

$\angle AMN = \angle ABC = 76^0 \quad (Given)$

$\angle A = \angle A \quad (Common)$

By AA similarity criterion, ΔAMN ~ ΔABC

If two triangles are similar, then the ratio of their corresponding sides are proportional

$\therefore \frac{AM}{AB} = \frac{MN}{BC}$

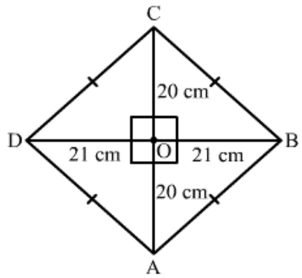
$\Rightarrow \frac{AM}{AM+MB} = \frac{MN}{BC}$

$\Rightarrow \frac{a}{a+b} = \frac{MN}{c}$

$\Rightarrow MN = \frac{ac}{a+b}$

29.

Sol:

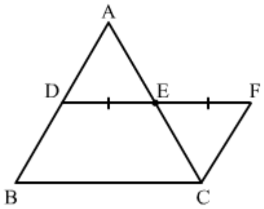


Suppose ABCD is a rhombus.
We know that the diagonals of a rhombus perpendicularly bisect each other.
$\therefore \angle AOB = 90^0, AO = 20\ cm\ and\ BO = 21\ cm$
Now, In right triangle AOB
By using Pythagoras theorem we have
$AB^2 = AO^2 + OB^2$
$= 20^2 + 21^2$
$= 400 + 441$
$= 841$
$\therefore AB^2 = 841$
$\Longrightarrow$ AB = 29 cm
Since, all the sides of a rhombus are equal.
Hence, AB = BC = CD = DA = 29 cm

30.

Sol:
(i)
Two rectangles are similar if their corresponding sides are proportional.
(ii) True

Two circles of any radii are similar to each other.
(iii)false
If two triangles are similar, their corresponding angles are equal and their corresponding sides are proportional.
(iv) True
Suppose ABC is a triangle and M, N are



Construction: DE is expanded to F such that EF = DE

To proof = DE = $\frac{1}{2}BC$

Proof: In ΔADE and ΔCEF

AE = EC (E is the mid point of AC)

DE = EF (By construction)

AED = CEF (Vertically Opposite angle)

By SAS criterion, ΔADE ~= ΔCEF

CF = AD (CPCT)

⟹ BD = CF

$\angle ADE = \angle EFC \quad (CPCT)$

Since, $\angle ADE$ and $\angle EFC$ are $alternate$ $angle$

Hence, AD ∥ CF and BD ∥ CF

When two sides of a quadrilateral are parallel, then it is a parallelogram

∴ DF = BC and BD ∥ CF

∴BDFC is a parallelogram

Hence, DF = BC

⟹ DE + EF = BC

$\Longrightarrow DE = \frac{1}{2}\ BC$

(v) False

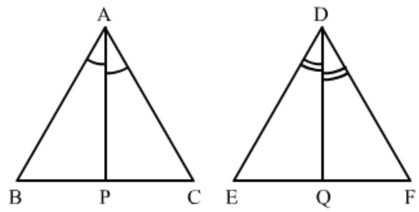
In ΔABC, AB = 6 cm, $\angle A = 45^0$ and $AC = 8$ cm and in $\Delta DEF, DF = 9$ $cm, \angle D = 45^0$ and $DE = 12$ $cm,$ $then$ $\Delta ABC \sim \Delta DEF.$

In ΔABC and ΔDEF

(vi) False

The polygon formed by joining the mid points of the sides of a quadrilateral is a parallelogram.

(vii) True



Given: ΔABC ~ ΔDEF

To prove = $\frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AP}{DQ}\right)^2$

Proof: in ΔABP and ΔDEQ

$\angle BAP = \angle EDQ \quad (As\ \angle A = \angle D, so\ their\ Half\ is\ also\ equal)$

$\angle B = \angle E \qquad (\angle ABC \sim \Delta DEF)$

By AA criterion, ΔABP and ΔDEQ

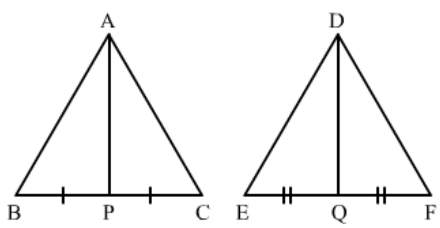
$\frac{AB}{DE} = \frac{AP}{DQ} \quad ....(1)$

Since, ΔABC ~ ΔDEF

$\therefore \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$

$\Longrightarrow \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AP}{DQ}\right)^2 \quad [Using\ (1)]$

(viii)



Given: ΔABC ~ ΔDEF

To Prove = $\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AP}{DQ}$

Proof: In ΔABP and ΔDEQ

$\angle B = \angle E \qquad (\therefore \Delta ABC \sim \Delta DEF\ )$

∴ ΔABC ~ ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Longrightarrow \frac{AB}{DE} = \frac{2\ BP}{2\ EQ}$

$\Longrightarrow \frac{AB}{DE} = \frac{BP}{EQ}$

By SAS criterion, ΔABP ~ ΔDEQ

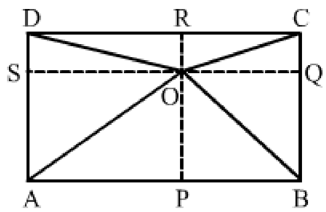
$\frac{AB}{DE} = \frac{AP}{DQ} \quad ....(1)$

Since, ΔABC ~ ΔDEF

$\therefore \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$

$\Rightarrow \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AP}{DQ}$ $[Using\ (1)]$

(ix) True



Suppose ABCD is a rectangle with O is any point inside it.

Construction: $OA^2 + OC^2 = OB^2 + OD^2$

Proof:

$OA^2 + OC^2 = (AS^2 + OS^2) + (OQ^2 + QC^2)$ [Using Pythagoras theorem in right triangle AOP and COQ]

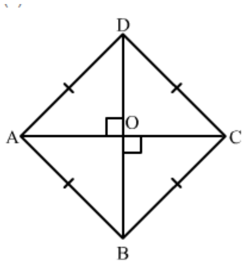
= $(BQ^2 + OS^2) + (OQ^2 + DS^2)$

= $(BQ^2 + OQ^2) + (OS^2 + DS^2)$ [Using Pythagoras theorem in right triangle BOQ and DOS]

= $OB^2 + OD^2$

Hence, LHS = RHS

(x) True



Suppose ABCD is a rhombus having AC and BD its diagonals.

Since, the diagonals of a rhombus perpendicular bisect each other.

Hence, AOC is a right angle triangle

In right triangle AOC

By using Pythagoras theorem, we have

$AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

[∴ Diagonals of a rhombus perpendicularly bisect each other]

$\Rightarrow AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$

$\Rightarrow 4\ AB^2 = AC^2 + BD^2$

$\Rightarrow AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$

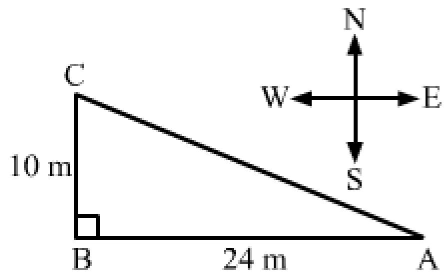
$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$ $[\because All\ sides\ of\ a\ rhombus\ are\ equal]$

Exercise – MCQ

1.

Sol:

(c) 26 m



Suppose, the man starts from point A and goes 24 m due west to point B. From here, he goes 10 m due north and stops at C.
In right triangle ABC, we have:
AB = 24 m, BC = 10 m
Applying Pythagoras theorem, we get:

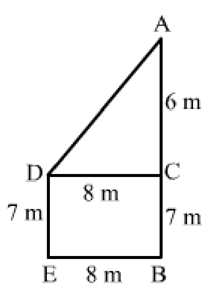
$AC^2 = AB^2 + BC^2 = 24^2 + 10^2$

$AC^2 = 576 + 100 = 676$

$AC = \sqrt{676} = 26$

2.

Sol:
(b) 10 m



Let AB and DE be the two poles.
According to the question:
AB = 13 m
DE = 7 m
Distance between their bottoms = BE = 8 m
Draw a perpendicular DC to AB from D, meeting AB at C. We get:
DC = 8m, AC = 6 m

Applying, Pythagoras theorem in right-angled triangle ACD, we have

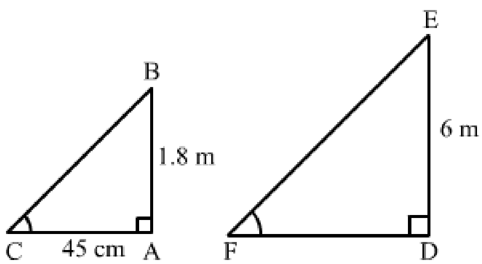
$AD^2 = DC^2 + AC^2$

$= 8^2 + 6^2 = 64 + 36 = 100$

$AD = \sqrt{100} = 10\ M$

3.

Sol:



Let AB and AC be the vertical stick and its shadow, respectively.
According to the question:
AB = 1.8 m
AC = 45 cm = 0.45 m
Again, let DE and DF be the pole and its shadow, respectively.
According to the question:
DE = 6 m
DF =?
Now, in right-angled triangles ABC and DEF, we have:

$\angle BAC = \angle EDF = 90^0$

$\angle ACB = \angle DFE \quad (Angular\ elevation\ of\ the\ Sun\ at\ the\ same\ time)$

Therefore, by AA similarity theorem, we get:

Δ ABC ~ ΔDFE

$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$

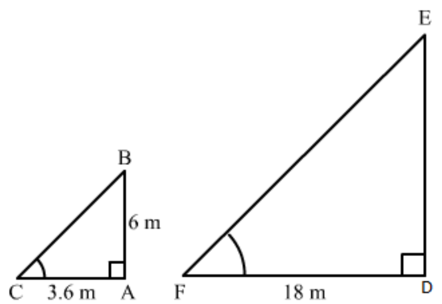
$\Rightarrow \frac{1.8}{0.45} = \frac{6}{DF}$

$\Rightarrow DF = \frac{6 \times 0.45}{1.8} = 1.5\ m$

4.

Sol:

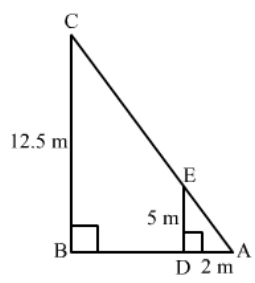
(d)



Let AB and AC be the vertical pole and its shadow, respectively.
According to the question:
AB = 6 m
AC = 3.6 m
Again, let DE and DF be the tower and its shadow.
According to the question:
DF = 18 m
DE =?
Now, in right -angled triangles ABC and DEF, we have:
$\angle BAC = \angle EDF = 90^0$
$\angle ABC = \angle DFE =$ (*Angular elevation of the sun at the same time*)
Therefore, by AA similarity theorem, we get:
Δ ABC - Δ DEF
$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$
$\Rightarrow \frac{6}{3.6} = \frac{DE}{18}$
$\Rightarrow DE = \frac{6\times18}{3.6} = 30\ m$

5.

Sol:



Suppose DE is a 5 m long stick and BC is a 12.5 m high tree.
Suppose DA and BA are the shadows of DE and BC respectively.
Now, In ΔABC and ΔADE
$\angle ABC = \angle ADE = 90^0$

$\angle A = \angle A \quad (Common)$

By AA- similarity criterion

$\Delta ABC \sim \Delta ADE$

If two triangles are similar, then the ratio of their corresponding sides are equal.

$\therefore \frac{AB}{AD} = \frac{BC}{DE}$

$\Rightarrow \frac{AB}{2} = \frac{12.5}{5}$

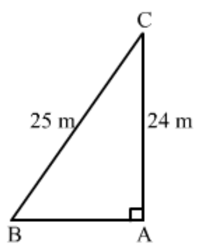
$\Rightarrow$ AB = 5 cm

Hence, the correct answer is option (d).

6.

Sol:

(a) 7 m



Let the ladder BC reaches the building at C.

Let the height of building where the ladder reaches be AC.

According to the question:

BC = 25 m

AC = 24 m

In right-angled triangle CAB, we apply Pythagoras theorem to find the value of AB.

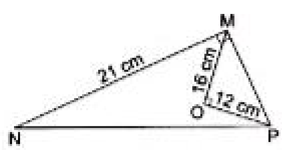
$BC^2 = AC^2 + AB^2$

$\Rightarrow AB^2 = BC^2 - AC^2 = 25^2 - 24^2$

$\Rightarrow AB^2 = 625 - 576 = 49$

$\Rightarrow AB = \sqrt{49} = 7\ m$

7.



Sol:

Now, In right triangle MOP

By using Pythagoras theorem, we have

$MP^2 = PO^2 + OM^2$
$= 12^2 + 16^2$
= 144 + 256
= 400
$\therefore MP^2 = 400$
$\Longrightarrow$ MO = 20 cm
Now, In right triangle MPN
By using Pythagoras theorem, we have
$PN^2 = NM^2 + MP^2$
$= 21^2 + 20^2$
$= 441 + 400$
= 841
$\therefore MP^2 = 841$
$\Longrightarrow$ MP = 29 cm
Hence, the correct answer is option (b).

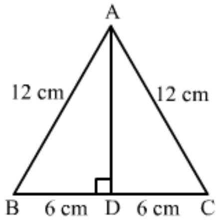
8.

Sol:

(b) 15 cm, 20 cm
It is given that length of hypotenuse is 25 cm.
Let the other two sides be x cm and (x−5) cm.
Applying Pythagoras theorem, we get:
$25^2 = x^2 + (x - 5)^2$
$\Longrightarrow 625 = x^2 + x^2 + 25 - 10x$
$\Longrightarrow 2x^2 - 10x - 600 = 0$
$\Longrightarrow x^2 - 5x - 300 = 0$
$\Longrightarrow x^2 - 20x + 15x - 300 = 0$
$\Longrightarrow x(x - 20) + 15(x - 20) = 0$
$\Longrightarrow$ (x – 20) (x + 15) =0
$\Longrightarrow x - 20 = 0 \text{ or } x + 15 = 0$
$\Longrightarrow x = 20 \text{ or } x = -15$
Side of a triangle cannot be negative.
Therefore, x = 20 cm
Now,
x – 5 = 20 – 5 = 15 cm

9.

Sol:
(b) $6\sqrt{3}cm$



Let ABC be the equilateral triangle with AD as its altitude from A.
In right-angled triangle ABD, we have

$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2$

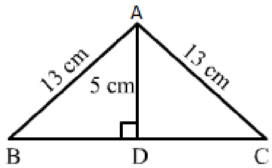
$= 12^2 - 6^2$

= 144 – 36 = 108

AD = $\sqrt{108} = 6\sqrt{3}cm$

10.

Sol:
(d) 24 cm



In triangle ABC, let the altitude from A on BC meets BC at D.
We have:
AD = 5 cm, AB = 13 cm and D is the midpoint of BC.
Applying Pythagoras theorem in right-angled triangle ABD, we get:

$AB^2 = AD^2 + BD^2$

$\Rightarrow BD^2 = AB^2 - AD^2$

$\Rightarrow BD^2 = 13^2 - 5^2$

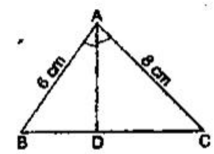
$\Rightarrow BD^2 = 169 - 25$

$\Rightarrow BD^2 = 144$

$\Rightarrow BD = \sqrt{144} = 12\ cm$

Therefore, BC = 2BD = 24 cm

11.



Sol:

(a) 3 : 4

In Δ ABD and ΔACD, we have:

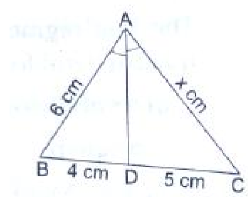
$\angle BAD = \angle CAD$

Now,

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$

BD : DC = 3 : 4

12.



Sol:

(d) 7.5 cm

It is given that AD bisects angle A

Therefore, applying angle bisector theorem, we get:

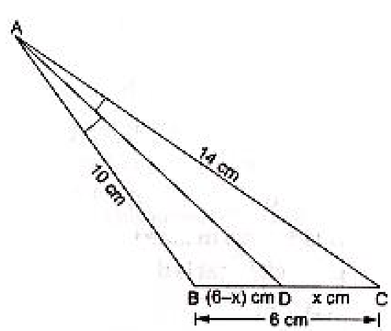
$\frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{4}{5} = \frac{6}{x}$

$\Rightarrow x = \frac{5 \times 6}{4} = 7.5$

Hence, AC = 7.5 cm

13.



Sol:

By using angle bisector in ΔABC, we have

$\frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \frac{10}{14} = \frac{6-x}{x}$

$\Rightarrow$ 10x = 84 – 14x
$\Rightarrow$ 24x = 84
$\Rightarrow$ x = 3.5
Hence, the correct answer is option (b).

14.

Sol:

(b) Isosceles
In an isosceles triangle, the perpendicular from the vertex to the base bisects the base.

15.

Sol:

(c) $3AB^2 = 4AD^2$
Applying Pythagoras theorem in right-angled triangles ABD and ADC, we get:

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = \left(\frac{1}{2}\ AB\right)^2 + AD^2 \quad \left(\because\ \Delta ABC \text{ is equilateral and } AD = \frac{1}{2}\ AB\right)$

$\Rightarrow AB^2 = \frac{1}{4}\ AB^2 + AD^2$

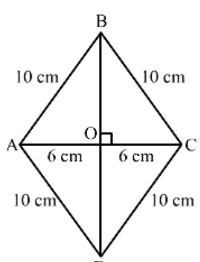
$\Rightarrow AB^2 - \frac{1}{4}\ AB^2 = AD^2$

$\Rightarrow \frac{3}{4}\ AB^2 = AD^2$

$\Rightarrow 3AB^2 = 4AD^2$

16.

Sol:
(c) 16 cm

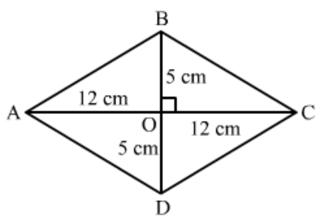


Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.
Also, diagonals of a rhombus bisect each other at right angles.

If AC = 12 cm, AO = 6 cm
Applying Pythagoras theorem in right-angled triangle AOB. We get:
$AB^2 = AO^2 + BO^2$
$\Rightarrow BO^2 = AB^2 - AO^2$
$\Rightarrow BO^2 = 10^2 - 6^2 = 100 - 36 = 64$
$\Rightarrow BO = \sqrt{64} = 8$
$\Rightarrow BD = 2 \times BO = 2 \times 8 = 16\ cm$
Hence, the length of the second diagonal BD is 16 cm.

17.

Sol:
(b) 13 cm



Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.
We have:
AC = 24 cm and BD = 10 cm
We know that diagonals of a rhombus bisect each other at right angles.
Therefore applying Pythagoras theorem in right-angled triangle AOB, we get:
$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$
= 144 + 25 = 169
$AB = \sqrt{169} = 13$
Hence, the length of each side of the rhombus is 13 cm.

18.

Sol:

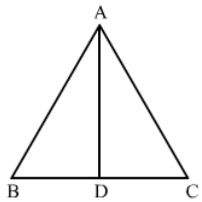
(b) trapezium
Diagonals of a trapezium divide each other proportionally.

19.
Sol:

(a) A parallelogram
The line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

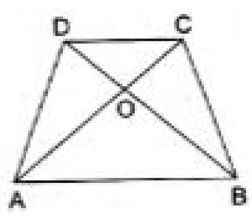
20.

Sol:
(c) isosceles



Let AD be the angle bisector of angle A in triangle ABC.
Applying angle bisector theorem, we get:
$\frac{AB}{AC} = \frac{BD}{DC}$
It is given that AD bisects BC.
Therefore, BD = DC
$\Rightarrow \frac{AB}{AC} = 1$
$\Rightarrow$ AB = AC
Therefore, the triangle is isosceles.

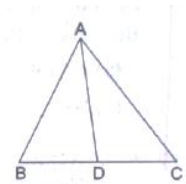
21.



Sol:
(a) 2
We know that the diagonals of a trapezium are proportional.
Therefore $\frac{OA}{OC} = \frac{OB}{OD}$
$\Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$
$\Rightarrow$ (3X – 1) (6X – 5) = (2X + 1) (5X -3)

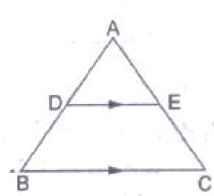
$\Longrightarrow 18X^2 - 15X - 6X + 5 = 10X^2 - 6X + 5X - 3$
$\Longrightarrow 18X^2 - 21X + 5 = 10X^2 - X - 3$
$\Longrightarrow 18X^2 - 21X + 5 - 10X^2 + X + 3 = 0$
$\Longrightarrow 8X^2 - 20X + 8 = 0$
$\Longrightarrow 4\,(2X^2 - 5X + 2) = 0$
$\Longrightarrow 2X^2 - 5X + 2 = 0$
$\Longrightarrow 2X^2 - 4X - X + 2 = 0$
$\Longrightarrow 2X(X - 2) - 1\,(X - 2) = 0$
$\Longrightarrow (X - 2)(2X - 1) = 0$
$\Longrightarrow Either\ x - 2 = 0\ or\ 2x - 1 = 0$
$\Longrightarrow Either\ x = 2\ or\ x = \frac{1}{2}$
$When\ x = \frac{1}{2}, 6x - 5 = -2 < 0, which\ is\ not\ possible.$
Therefore, x = 2

22.



Sol:
(a) 30^0
We have:
$\frac{AB}{AC} = \frac{BD}{DC}$
Applying angle bisector theorem, we can conclude that AD bisects $\angle A$.
In ΔABC,
$\angle A + \angle B + \angle C = 180^0$
$\Longrightarrow \angle A = 180 - \angle B - \angle C$
$\Longrightarrow \angle A = 180 - 70 - 50 = 60^0$
$\because \angle BAD = \angle CAD = \frac{1}{2} \angle BAC$
$\therefore \angle BAD = \frac{1}{2} \times 60 = 30^0$

23.

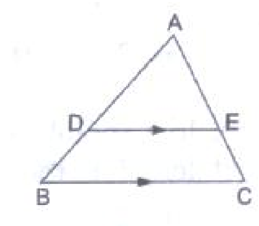


Sol:
(b) 6 cm
It is given that DE || BC.
Applying basic proportionality theorem, we have:
$\frac{AD}{BD} = \frac{AE}{EC}$
$\Longrightarrow \frac{2.4}{BD} = \frac{3.2}{4.8}$

$\Rightarrow BD = \frac{2.4 \times 4.8}{3.2} = 3.6\ cm$

Therefore, AB = AD + BD = 2.4 + 3.6 = 6 cm

24.



Sol:

(b) 4cm

It is given that DE || BC.

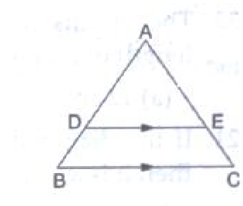
Applying basic proportionality theorem, we get:

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{4.5}{7.2} = \frac{AE}{6.4}$

$\Rightarrow AE = \frac{4.5 \times 6.4}{7.2} = 4\ cm$

25.



Sol:

(c) x = 4

It is given that DE || BC.

Applying Thales' theorem. We get:

$\frac{AD}{BD} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

$\Rightarrow 3x(7x-4) = (5x-2)(3x+4)$

$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$

$\Rightarrow 6x^2 - 26x + 8 = 0$

$\Rightarrow 2\ (3x^2 - 13x + 4) = 0$

$\Rightarrow 3x^2 - 13x + 4 = 0$

$\Rightarrow 3x^2 - 12x - x + 4 = 0$

$\Rightarrow 3x\ (x-4) - 1\ (x-4) = 0$

$\Rightarrow (x-4)(3x-1) = 0$

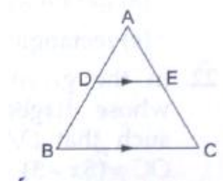
$\Rightarrow x - 4 = 0\ or\ 3x - 1 = 0$

$\Rightarrow x - 4\ or\ x = \frac{1}{3}$

If $x = \frac{1}{3}, 7x - 4 = -\frac{5}{3} < 0\,; it\ is\ not\ possible.$

Therefore, x = 4

26.



Sol:

(d) 2.1 cm

It is given that DE || BC.

Applying Thales' theorem, we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

Let AE be x cm.

Therefore, EC = (5.6 – x) cm

$\Rightarrow \frac{3}{5} = \frac{x}{5.6-x}$

$\Rightarrow 3(5.6 - x) = 5x$

$\Rightarrow 16.8 - 3x = 5x$

$\Rightarrow 8x = 16.8$

⟹ x = 2.1 cm

27.

Sol:

(b) 5.4 cm

ΔABC ~ ΔDEF

Therefore,

$\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{BC}{EF}$

$\Rightarrow \frac{30}{18} = \frac{9}{EF}$

$\Rightarrow EF = \frac{9\times18}{30} = 5.4\ cm$

28.

Sol:

(a) 35 cm

∵ΔABC ~ ΔDEF

$\therefore \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$

$\Rightarrow \frac{Perimeter(\Delta ABC)}{25} = \frac{9.1}{6.5}$

$\Rightarrow Perimeter\ (\Delta ABC) = \frac{9.1\times25}{6.5} = 35\ cm$

29.

Sol:
(d) 30 cm
Perimeter of ΔABC = AB + BC + CA = 9 + 6 + 7.5 = 22.5 cm
$\because$ ΔDEF ~ ΔABC
$\therefore \frac{Perimeter\Delta(ABC)}{Perimeter(\Delta DEF)} = \frac{BC}{EF}$
$\Longrightarrow \frac{22.5}{Perimeter(\Delta DEF)} = \frac{6}{8}$
Perimeter(ΔDEF) = $\frac{22.5\times 8}{6} = 30\ cm$

30.

Sol:
Give: ABC and BDE are two equilateral triangles
Since, D is the midpoint of BC and BDE is also an equilateral triangle.
Hence, E is also the midpoint of AB.
Now, D and E are the midpoint of BC and AB.
In a triangle, the line segment that joins midpoint of the two sides of a triangle is parallel to the third side and is half of it.

$DE \parallel CA \text{ and } DE = \frac{1}{2}\ CA$
Now, in ΔABC and ΔEBD
$\angle BED = \angle BAC \quad (Corresponding\ angles)$
$\angle B = \angle B \quad (Common)$
By AA-similarity criterion
ΔABC ~ ΔEBD
If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

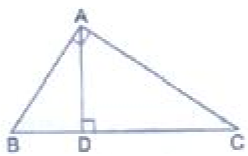
$\therefore \frac{area(\Delta ABC)}{area(\Delta DBE)} = \left(\frac{AC}{ED}\right)^2 = \left(\frac{2ED}{ED}\right)^2 = \frac{4}{1}$
Hence, the correct answer is option (d).

31.

Sol:
(b) DE = 12 cm, $\angle F = 100^0$

Disclaimer: In the question, it should be ΔABC ~ ΔDFE instead of ΔABC ~ ΔDEF.
In triangle ABC,
$\angle A + \angle B + \angle C = 180^0$
$\therefore \angle B = 180 - 30 - 50 = 100^0$
∵ ΔABC ~ ΔDFE
$\therefore \angle D = \angle A = 30^0$
$\angle F = \angle B = 100^0$
And $\angle E = \angle C = 50^0$
Also,
$\frac{AB}{DF} = \frac{AC}{DE} \Rightarrow \frac{5}{7.5} = \frac{8}{DE}$
$\Rightarrow DE = \frac{8 \times 7.5}{5} = 12\ cm$

32.



Sol:

(c) BD . CD = AD^2
In Δ BDA and ΔADC, we have:
$\angle BDA = \angle ADC = 90^0$
$\angle ABD = 90^0 - \angle DAB$
$= 90^0 - (90^0 - \angle DAC)$
$= 90^0 - 90^0 + \angle DAC$
$= \angle DAC$
Applying AA similarity, we conclude that $\Delta BDA - \Delta ADC$.
$\Rightarrow \frac{BD}{AD} = \frac{AD}{CD}$
$\Rightarrow AD^2 = BD.CD$

33.

Sol:
$AB = 6\sqrt{3}cm$
$\Rightarrow AB^2 = 108\ cm^2$
AC = 12 cm
$\Rightarrow AC^2 = 144\ cm^2$

BC = 6 cm
$\Rightarrow BC^2 = 36\ cm$
$\therefore AC^2 = AB^2 + BC^2$
Since, the square of the longest side is equal to the sum of two sides, so ΔABC is a right angled triangle.
∴ The angle opposite to ∠90°
Hence, the correct answer is option (c)

34.

Sol:
(c)$\angle B = \angle D$

Disclaimer: In the question, the ratio should be $\frac{AB}{DE} = \frac{BC}{FD} = \frac{AC}{EF}$.
We can write it as:
$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{FE}$
Therefore, Δ ABC − EDF
Hence, the corresponding angles, i.e.,$\angle B\ and\ \angle D, will\ be\ equal.$
$i.e., \angle B = \angle D$

35.

Sol:
(b) $\frac{DE}{PQ} = \frac{EF}{RP}$
In ΔDEF and ΔPQR, we have:
$\angle D = \angle Q\ and\ \angle R = \angle E$
Applying AA similarity theorem, we conclude that ΔDEF ~ ΔQRP.
$Hence, \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{PR}$

36.

Sol:

(c) BC. DE = AB. EF
ΔABC ~ ΔEDF

Therefore,
$\frac{AB}{DE} = \frac{AC}{EF} = \frac{BC}{DF}$
$\Rightarrow$ BC. DE $\neq$ AB. EF

37.

Sol:

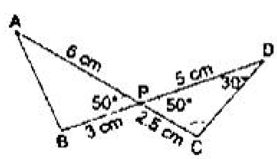
(b) similar but not congruent
In ΔABC and ΔDEF, we have:

$\angle B = \angle E \text{ and } \angle F = \angle C$
Applying AA similarity theorem, we conclude that ΔABC - ΔDEF.
Also,
AB = 3DE
$\Rightarrow$ AB $\neq$ DE
Therefore, ΔABC and ΔDEF are not congruent.

38.

Sol:
(a) ΔPQR ~ ΔCAB
In ΔABC and ΔPQR, we have:
$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$
$\Rightarrow$ ΔABC ~ ΔQRP
We can also write it as ΔPQR ~ ΔCAB.

39.



Sol:
(d)100^0
In Δ APB and Δ DPC, we have:
$\angle APB = \angle DPC = 50^0$
$\frac{AP}{BP} = \frac{6}{3} = 2$

$\frac{DP}{CP} = \frac{5}{2.5} = 2$

Hence, $\frac{AP}{BP} = \frac{DP}{CP}$

Applying SAS theorem, we conclude that Δ APB- Δ DPC.

$\therefore \angle PBA = \angle PCD$

In Δ DPC, we have:

$\angle CDP + \angle CPD + \angle PCD = 180^0$

$\Rightarrow \angle PCD = 180^0 - \angle CDP - \angle CPD$

$\Rightarrow \angle PCD = 180^0 - 30^0 - 50^0$

$\Rightarrow \angle PCD = 100^0$

Therefore, $\angle PBA = 100^0$

40.

Sol:

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area\ of\ first\ triangle}{area\ of\ second\ triangle} = \left(\frac{Side\ of\ first\ triangle}{Side\ of\ second\ triangle}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is option (d).

41.

Sol:

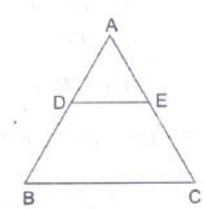
(d) 9:4

It is given that ΔABC ~ ΔPQR and $\frac{BC}{QR} = \frac{2}{3}$

Therefore,

$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{QR^2}{BC^2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

42.



Sol:

(b) 4:1

In ΔABC, D is the midpoint of AB and E is the midpoint of AC.

Therefore, by midpoint theorem,

Also, by Basic Proportionality Theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

Also,

AB = AC = BC (∵ΔABC is an equilateral triangle)

So, $\frac{AD}{DB} = \frac{AE}{EC} = 1$

In ΔABC and ΔADE, we have:

$\angle A = \angle A$

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{2}$

∴ Δ ABC - ΔADE (SAS criterion)

$\therefore ar\ (\Delta ABC) : ar\ (\Delta ADE) = (AB)^2 : (AD)^2$

$\Longrightarrow ar\ (\Delta ABC) : ar\ (\Delta ADE) = 2^2 : 1^2$

$\Longrightarrow ar(\Delta ABC) : ar\ (\Delta ADE) = 4 : 1$

43.

Sol:

(b) 25 :49

In ΔABC and ΔDEF, we have :

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DE} = \frac{5}{7}$

Therefore, by SSS criterion, we conclude that ΔABC ~ ΔDEF.

$\Longrightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \left(\frac{5}{7}\right)^2 = \frac{25}{49} = 25 : 49$

44.

Sol:

(b) 6:7

∵ ΔABC ~ ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ... (i)

Also,

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2}$

$\Longrightarrow \frac{36}{49} = \frac{AB^2}{DE^2}$

$\Longrightarrow \frac{6}{7} = \frac{AB}{DE}$

$\Longrightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{6}{7}$ $\left(from\ (i)\right)$

Thus, the ratio of corresponding sides is 6 : 7.

45.

Sol:

(c) 5:6
Let x and y be the corresponding heights of the two triangles.
It is given that the corresponding angles of the triangles are equal.
Therefore, the triangles are similar. (By AA criterion)
Hence,

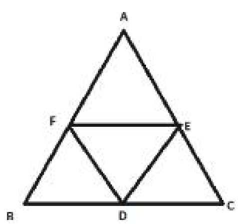
$$\frac{ar(\Delta_1)}{ar(\Delta_2)} = \frac{25}{36} = \frac{x^2}{y^2}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{25}{36}$$

$$\Rightarrow \frac{x^2}{y^2} = \sqrt{\frac{25}{36}} = \frac{5}{6} = 5 : 6$$

46.

Sol:
(b) similar to the original triangle



The line segments joining the midpoint of the sides of a triangle form four triangles, each of which is similar to the original triangle.

47.

Sol:
(b) 10 cm
∵ ΔABC ~ ΔQRP

$\therefore \frac{AB}{QR} = \frac{BC}{PR}$

Now,

$\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$

$\Longrightarrow \left(\frac{AB}{QR}\right)^2 = \frac{9}{4}$

$\Longrightarrow \frac{AB}{QR} = \frac{BC}{PR} = \frac{3}{2}$

Hence, 3PR = 2BC = 2 × 15 = 30

PR = 10 cm

48.

Sol:

(c) isosceles and similar

In ΔAOC and ΔODB, we have:

$\angle AOC = \angle DOB$ (Vertically opposite angles)

and $\angle OAC = \angle ODB$ (Angles in the same segment)

Therefore, by AA similarity theorem, we conclude that $\Delta AOC - \Delta DOB$.

$\Longrightarrow \frac{OC}{OB} = \frac{OA}{OD} = \frac{AC}{BD}$

Now, OB = OD

$\Longrightarrow \frac{OC}{OA} = \frac{OB}{OD} = 1$

$\Longrightarrow$ OC = OA

Hence, ΔOAC and ΔODB are isosceles and similar.

49.

Sol:

(d) 90^0

Given:

AC = BC

$AB^2 = 2AC^2 = AC^2 + AC^2 = AC^2 + BC^2$

Applying Pythagoras theorem, we conclude that ΔABC is right angled at C.

Or, $\angle C = 90^0$

50.

Sol:

(b) right-angled

We have:

$AB^2 + BC^2 = 16^2 + 12^2 = 256 + 144 = 400$

$and, AC^2 = 20^2 = 400$

$\therefore AB^2 + BC^2 = AC^2$

Hence, ΔABC is a right-angled triangle.

51.

Sol:

(c)Two triangles are similar if their corresponding sides are proportional.

According to the statement:

ΔABC~ ΔDEF

$if\ \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

52.

Sol:

(b) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

Because the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

53.

Sol:

(a) −(s)

Let AE be X.

Therefore, EC = 5.6 – X

It is given that DE || BC.

Therefore, by B.P.T.,we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{3}{5} = \frac{x}{5.6-x}$

$\Rightarrow 3(5.6 - x) = 5x$

$\Rightarrow 16.8 - 3x = 5x$

$\Rightarrow 8x = 16.8$

$\Rightarrow x = 2.1\ cm$

(b) −(q)

∵ ΔABC-ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{3}{2} = \frac{6}{EF}$

$EF = \frac{6\times 2}{3} = 4\ cm$

(c) −(p)

∵ ΔABC ~ ΔPQR

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{9}{16} = \frac{4.5^2}{QR^2} \Rightarrow QR = \sqrt{\frac{4.5\times 4.5\times 16}{9}} = \frac{4.5\times 4}{3} = 6\ cm$

(d) −(r)

∵ AB || CD

$\therefore \frac{OA}{OB} = \frac{OC}{OD}\ (Thales' theorem)$

$\Rightarrow \frac{2x+4}{9x-21} = \frac{2x-1}{3}$

$3(2x+4) = (2x-1(9x-21))$

$\Rightarrow 6x + 12 = 18x^2 - 42x - 9x + 21$

$\Rightarrow 18x^2 - 57x + 9 = 0$

$\Rightarrow 6x^2 - 19x + 3 = 0$

$\Rightarrow 6x^2 - 18x - x + 3 = 0$

⟹ (6x – 1) (x -3) = 0

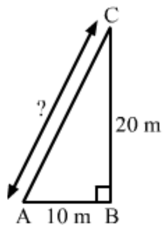
$\Rightarrow x = 3\ or\ x = -\frac{1}{6}$

$But\ x = -\frac{1}{6} makes\ (2x-1) < 0, which\ is\ not\ possible.$

Therefore, x= 3

54.

Sol:



(a) −(r)

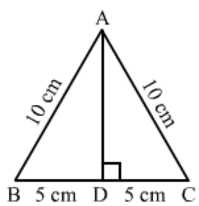
Let the man starts from A and goes 10 m due east at B and then 20 m due north at C.

Then, in right-angled triangle ABC, we have:

$AB^2 + BC^2 = AC^2$

$\Rightarrow AC = \sqrt{10^2 + 20^2} = \sqrt{100 + 200} = 10\sqrt{3}$

Hence, the man is 10 $\sqrt{3}m\ away\ from\ the\ staring\ point.$



(b) −(q)

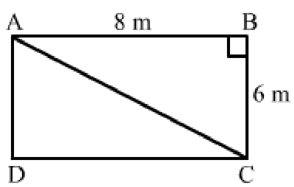
Let the triangle be ABC with altitude AD.

In right-angled triangle ABC, we have:

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = 10^2 - 5^2 \left(\because BD = \frac{1}{2} BC\right)$

$\Rightarrow AD = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm$



(c) – (p)

Area of an equilateral triangle with side a = $\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 10^2 = \sqrt{3} \times 5 \times 5$

$= 25\sqrt{3}\ cm^2$

(d) – (s)

Let the rectangle be ABCD with diagonals AC and BD.

In right-angled triangle ABC, we have:

$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36$

$\Rightarrow AC = \sqrt{100} = 10\ m$

Exercise – Formative Assessment

1.

Sol:

(b) 7.5 cm

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$

$\Rightarrow \frac{32}{24} = \frac{10}{DE}$

$\Rightarrow DE = \frac{10 \times 24}{32} = 7.5\ cm$

2.

Sol:

(a) 5.6 cm

$\because DE \parallel BC$

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \quad (Thales' theorem)$

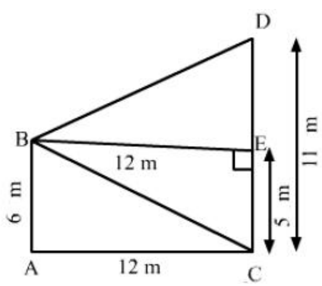
$\Rightarrow \frac{3.5}{AB} = \frac{5}{8}$

$\Rightarrow AB = \frac{3.5 \times 8}{5} = 5.6\ cm$

3.

Sol:

(b) 13 m



Let the poles be and CD

It is given that:

AB = 6 m and CD = 11 m

Let AC be 12 m

Draw a perpendicular farm Bon CD, meeting CD at E

Then,
BE = 12 m
We have to find BD.
Applying Pythagoras theorem in right-angled triangle BED, we have:
$BD^2 = BE^2 + ED^2$
$= 12^2 + 5^2 \ (\because ED = CD - CE = 11 - 6)$
= 144 + 25 = 169
BD = 13 m

4.

Sol:
(c)
We know that the ratio of areas of similar triangles is equal to the ratio of squares of their corresponding altitudes.
Let ɦ be the altitude of the other triangle.
Therefore,

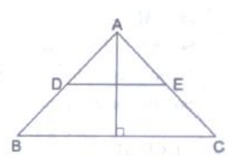
$\frac{25}{36} = \frac{(3.5)^2}{h^2}$
$\Rightarrow h^2 = \frac{(3.5)^2 \times 36}{25}$
$\Rightarrow h^2 = 17.64$
$\Rightarrow h = 4.2\ cm$

5.

Sol:
∵ ΔABC ~ Δ DEF
$\therefore \frac{AB}{DE} = \frac{BC}{EF}$
$\Rightarrow \frac{1}{2} = \frac{6}{EF}$
$\Rightarrow EF = 12\ cm$

6.

Sol:
∵ DE || BC
$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (Basic\ proportionality\ theorem)$

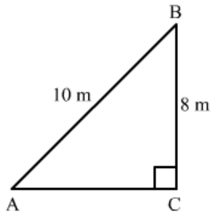


$\frac{x}{3x+4} = \frac{x+3}{3x+19}$
$\Rightarrow x\ (3x + 19) = (x + 3)(3x + 4)$
$\Rightarrow 3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$\Longrightarrow 19x - 13x = 12$
$\Longrightarrow 6x = 12$
$\Longrightarrow x = 2$

7.

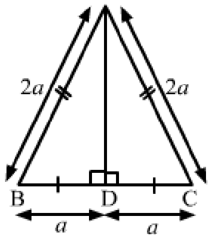
Sol:
Let the ladder be AB and BC be the height of the window from the ground.



We have:
AB 10 m and BC = 8 m
Applying theorem in right-angled triangle ACB, we have:
$AB^2 = AC^2 + BC^2$
$\Longrightarrow AC^2 = AB^2 - BC^2 = 10^2 - 8^2 = 100 - 64 = 36$
$\Longrightarrow AC = 6\,m$
Hence, the ladder is 6 m away from the base of the wall.

8.

Sol:
Let the triangle be ABC with AD as its altitude. Then, D is the midpoint of BC.
In right-angled triangle ABD, we have:



$AB^2 = AD^2 + DB^2$
$\Longrightarrow AD^2 = AB^2 - DB^2 = 4a^2 - a^2 \qquad \left(\because BD = \frac{1}{2} BC\right)$
$= 3a^2$
$AD = \sqrt{3}a$
Hence, the length of the altitude of an equilateral triangle of side 2a cm is $\sqrt{3}a\ cm$.

9.

Sol:

$\because$ ΔABC ~ ΔDEF

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$

$\Rightarrow \frac{64}{169} = \frac{4^2}{EF^2}$

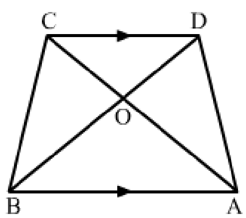
$\Rightarrow EF^2 = \frac{16\times169}{64}$

$\Rightarrow EF = \frac{4\times13}{8} = 6.5\ cm$

10.

Sol:

In ΔAOB and ΔCOD, we have:



$\angle AOB = \angle COD$ $(Vertically\ opposite\ angles)$

$\angle OAB = \angle OCD$ $(Alternate\ angles\ as\ AB \parallel CD\ )$

Applying AA similarity criterion, we get :

Δ AOB - ΔCOD

$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$

$\Rightarrow \frac{84}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$

$\Rightarrow \frac{84}{ar(\Delta COD)} = \left(\frac{2CD}{CD}\right)^2$

$\Rightarrow ar(\Delta COD) = \frac{84}{4} = 21\ cm^2$

11.

Sol:

It is given that the triangles are similar.

Therefore, the ratio of areas of similar triangles will be equal to the ratio of squares of their corresponding sides.

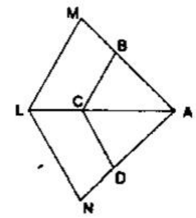
$\therefore \frac{48}{Area\ of\ larger\ triangle} = \frac{2^2}{3^2}$

$\Rightarrow \frac{48}{Area\ of\ larger\ triangle} = \frac{4}{9}$

$\Rightarrow Area\ of\ larger\ triangle = \frac{48\times9}{4} = 108\ cm^2$

12.

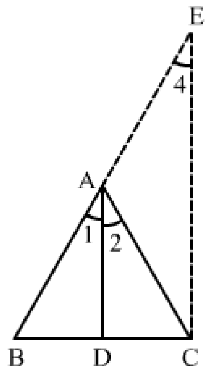
Sol:



LM || CB and LN || CD
Therefore, applying Thales' theorem, we have:
$\frac{AB}{AM} = \frac{AC}{AL}$ and $\frac{AD}{AN} = \frac{AC}{AL}$
$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN}$
$\therefore \frac{AM}{AB} = \frac{AN}{AD}$
This completes the proof.

13.

Sol:
Let the triangle be ABC with AD as the bisector of ∠A which meets BC at D.
We have to prove:
$\frac{BD}{DC} = \frac{AB}{AC}$



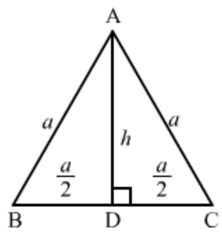
Draw CE || DA, meeting BA produced at E.
CE || DA
Therefore,
$\angle 2 = \angle 3 \quad (Alternate\ angles)$
$and\ \angle 1 = \angle 4 \qquad (Corresponding\ angles)$
But,
$\angle 1 = \angle 2$
Therefore,
$\angle 3 = \angle 4$
$\Rightarrow$ AE = AC
In ΔBCE, DA || CE.
Applying Thales' theorem, we gave:
$\frac{BD}{DC} = \frac{AB}{AE}$

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$

This completes the proof.

14.

Sol:



Let ABC be the equilateral triangle with each side equal to a.

Let AD be the altitude from A, meeting BC at D.

Therefore, D is the midpoint of BC.

Let AD be h.

Applying Pythagoras theorem in right-angled ABD, we have:

$AB^2 = AD^2 + BD^2$

$\Rightarrow a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow h = \frac{\sqrt{3}}{2}\ a$

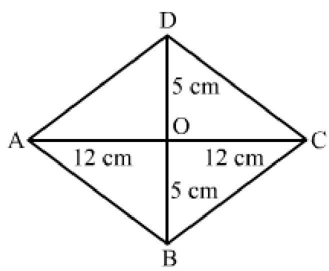
Therefore,

$Area\ of\ triangle\ ABC = \frac{1}{2} \times base \times height = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}\ a$

This completes the proof.

15.

Sol:



Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.

We know that the diagonals of a rhombus bisect each other at right angles.

∴ If AC – 24 cm and BD = 10 cm, AO = 12 cm and BO = 5 cm

Applying Pythagoras theorem in right-angled triangle AOB, we get:

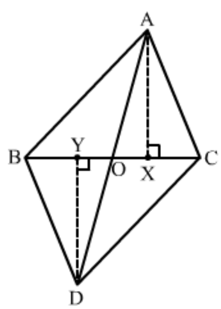
$AB^2 = AO^2 + BO^2 = 12^2 + 5^2 = 144 + 25 = 169$
AB = 13 cm
Hence, the length of each side of the given rhombus is 13 cm.

16.

Sol:
Let the two triangles be ABC and PQR.
We have:
ΔABC ~ ΔPQR,
Here,
BC = a, AC = b and AB = c
PQ = r, PR = q and QR = p
We have to prove:
$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r}$
ΔABC ~ ΔPQR; therefore, their corresponding sides will be proportional.
$\Rightarrow \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k \quad (say) \quad ...(i)$
$\Rightarrow a = kp, b = kq \text{ and } c = kr$
$\therefore \frac{Premieter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r} = k \quad ...(ii)$
$From\ (i) and\ (ii), we\ get:$
$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r} = \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR}$
This completes the proof.

17.

Sol:



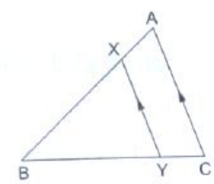
$Construction: Draw\ AX \perp CO\ and\ DY \perp BO.$
As,
$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times AX \times BC}{\frac{1}{2} \times DY \times BC}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AX}{DY} \ldots (i)$

$In\ \Delta\ ABC\ and\ \Delta DBC, \angle AXY = \angle DYO = 90^0\ (BY\ constructin) \angle AOX = \angle DOY\ (Vertically\ opposite\ anges) \therefore \Delta AXO \sim \Delta DYO (BY\ AA\ criterion) \therefore \frac{AX}{DY} = \frac{AO}{DO}\ (Thales' stheorem) \ldots\ (ii) From\ (i) and\ (ii), we\ have : \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AX}{DY} = \frac{AO}{DO}\ or, \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

This completes the proof.

18.



Sol:

In Δ ABC and ΔBXY, we have:

$\angle B = \angle B$

$\angle BXY = \angle BAC \quad (Corresponding\ angles)$

$Thus, \Delta ABC - \Delta BXY \quad (AA\ criterion\ )$

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta BXY)} = \frac{AB^2}{BX^2} = \frac{AB^2}{(AB-AX)^2} \quad \ldots (i)$

$Also, \frac{ar(\Delta ABC)}{ar(\Delta BXY)} = \frac{2}{1}\ \{\therefore ar\ (\Delta BXY) = ar(trapezium\ AXYV)\} \ldots (ii)$

$From\ (i) and\ (ii), we\ have:$

$\frac{AB^2}{(AB-AX)^2} = \frac{2}{1}$

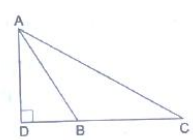
$\Rightarrow \frac{AB}{(AB-AX)} = \sqrt{2}$

$\Rightarrow \frac{(AB-AX)}{AB} = \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{AX}{AB} = \frac{1}{\sqrt{2}}$

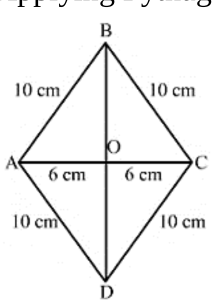
$\Rightarrow \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{(2-\sqrt{2})}{2}$

19.



Sol:

Applying Pythagoras theorem in right-angled triangle ADC, we get:



$AC^2 = AD^2 + DC^2$

$\Longrightarrow AC^2 - DC^2 = AD^2$

$\Longrightarrow AD^2 = AC^2 - DB^2$ ...(1)

Applying Pythagoras theorem in right-angled triangle ADB, we get:

$AB^2 = AD^2 + DB^2$

$\Longrightarrow AB^2 - DB^2 = AD^2$

$\Longrightarrow AD^2 = AB^2 - DB^2$ ...(2)

$From\ equation\ (1) and\ (2), we\ have:$

$AC^2 - DC^2 = AB^2 - DB^2$

$\Longrightarrow AC^2 = AB^2 + DC^2 - DB^2$

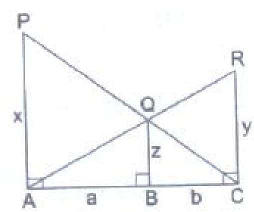
$\Longrightarrow AC^2 = AB^2 + (DB + BC)^2 - DB^2 \quad (\because DB + BC = DC)$

$\Longrightarrow AC^2 = AB^2 + DB^2 + BC^2 + 2DB.BC - DB^2$

$\Longrightarrow AC^2 = AB^2 + BC^2 + 2BC.BD$

This completes the proof.

20.



Sol:

In Δ PAC and ΔQBC, we have:

$\angle A = \angle B \quad (Both\ angles\ are\ 90^0)$

$\angle P = \angle Q \quad (Corresponding\ angles)$

And

$\angle C = \angle C \quad (common\ angles)$

$Therefore, \Delta PAC \sim \Delta QBC$

$\frac{AP}{BQ} = \frac{AC}{BC}$

$\Longrightarrow \frac{x}{2} = \frac{a+b}{b}$

$\Longrightarrow a + b = \frac{ay}{z}$ ... (1)

In Δ RCA and ΔQBA, we have:

$\angle C = \angle B \quad (Both\ angles\ are\ 90^0)$

$\angle R = \angle Q \quad (Corresponding\ angles)$

And

$\angle A = \angle A \quad (common\ angles)$

$Therefore, \Delta RCA \sim \Delta QBA$

$\frac{RC}{BQ} = \frac{AC}{AB}$

$\Longrightarrow \frac{y}{z} = \frac{a+b}{a}$

$\Longrightarrow a + b = \frac{ay}{z}$ ... (2)

From equation (1) and (2), we have:

$\frac{bx}{z} = \frac{ay}{z}$

$\Longrightarrow bx = ay$

$\Longrightarrow \frac{a}{b} = \frac{x}{y} \quad \ldots (3)$

Also,

$\frac{x}{z} = \frac{a+b}{b}$

$\Longrightarrow \frac{x}{z} = \frac{a}{b} + 1$

$Using\ the\ value\ of\ \frac{a}{b}\ from\ equation\ (3), we\ have:$

$\Longrightarrow \frac{x}{z} = \frac{x}{y} + 1$

$Dividing\ both\ sides\ by\ x, we\ get:$

$\frac{1}{z} = \frac{1}{y} + \frac{1}{x}$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

$This\ completes\ the\ proof.$