1.

Sol:
On substituting the values of various T-ratios, we get:
$\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = \frac{4}{4} = 1$

2.

Sol:
On substituting the values of various T-ratios, we get:
$\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0$

3.

Sol:
On substituting the values of various T-ratios, we get:
$\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

4.

Sol:
$\frac{\sin 30^0}{\cos 45^0} + \frac{\cot 45^0}{\sec 60^0} - \frac{\sin 60^0}{\tan 45^0} + \frac{\cos 30^0}{\sin 90^0}$

$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)} + \frac{1}{2} - \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} + \frac{\left(\frac{\sqrt{3}}{2}\right)}{1}$

$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+1}{2}$

5.

Sol:
$\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{\left(\frac{5}{4} + \frac{4 \times 4}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)}$

$= \frac{\left(\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right)}{\left(\frac{4}{4}\right)}$

$= \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{4}{4}\right)}$

$= \frac{\left(\frac{67}{12}\right)}{(1)}$

$= \frac{67}{12}$

6.

Sol:

On substituting the values of various T-ratios, we get:

$2\cos^2 60^0 + 3\sin^2 45^0 - 3\sin^2 30^0 + 2\cos^2 90^0$

$= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2 \times (0)^2$

$= 2 \times \frac{1}{4} + 3 \times \frac{1}{2} - 3 \times \frac{1}{4} + 0$

$= \left(\frac{1}{2} + \frac{3}{2} - \frac{3}{4}\right) = \left(\frac{2+6-3}{4}\right) = \frac{5}{4}$

7.

Sol:

On substituting the values of various T-ratios, we get:

$\cot^2 30^0 - 2\cos^2 30^0 - \frac{3}{4}\sec^2 45^0 + \frac{1}{4}\text{cosec}^2 30^0$

$= \left(\sqrt{3}\right)^2 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times \left(\sqrt{2}\right)^2 + \frac{1}{4} \times (2)^2$

$= 3 - 2 \times \frac{3}{4} - \frac{3}{4} \times 2 + \frac{1}{4} \times 4$

$= 3 - \frac{3}{2} - \frac{3}{2} + 1$

$= 4 - \left(\frac{3}{2} + \frac{3}{2}\right)$

$= 4 - 3 = 1$

8.

Sol:
On substituting the values of various T-ratios, we get:
($\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0$) ($\text{cosec}^2 45^0 \sec^2 30^0$)

$= \left[\left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2\right]\left[\left(\sqrt{2}\right)^2 \left(\frac{2}{\sqrt{3}}\right)^2\right]$

$= \left(\frac{1}{4} + 4 - 4\right)\left(2 \times \frac{4}{3}\right)$

$= \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

9.

Sol:
On substituting the values of various T-ratios, we get:

$\frac{4}{cot^2 30^0} + \frac{1}{sin^2 30^0} - 2\cos^2 45^0 - \sin^2 0^0$

$= \frac{4}{\left(\sqrt{3}\right)^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - (0)^2$

$= \frac{4}{3} + \frac{1}{\frac{1}{4}} - 2 \times \frac{1}{2} - 0$

$= \frac{4}{3} + 4 - 1$

$= \frac{4}{3} + 3 = \frac{4+9}{3} = \frac{13}{3}$

10.

Sol:
(i)

LHS $= \frac{1-\sin 60^0}{\cos 60^0} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\left(\frac{2-\sqrt{3}}{2}\right)}{\frac{1}{2}} = \left(\frac{2-\sqrt{3}}{2}\right) \times 2 = 2 - \sqrt{3}$

RHS $= \frac{\tan 60^0 - 1}{\tan 60^0 + 1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$

Hence, LHS = RHS

$\therefore \frac{1-\sin 60^0}{\cos 60^0} = \frac{\tan 60^0 - 1}{\tan 60^0 + 1}$

(ii)

LHS $= \frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \frac{\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\frac{\sqrt{3}+\sqrt{3}}{2}}{\frac{2+1+1}{2}} = \frac{\sqrt{3}}{2}$

Also, RHS = $\cos 30^0 = \frac{\sqrt{3}}{2}$

Hence, LHS = RHS

$\therefore \frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \cos 30^0$

11.

Sol:

(i) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Also, $\sin 30^0 = \frac{1}{2}$

$\therefore \sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0 = \sin 30^0$

(ii) $\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Also, $\cos 30^0 = \frac{\sqrt{3}}{2}$

$\therefore \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0 = \cos 30^0$

(iii) $2 \sin 30^0 \cos 30^0$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Also, $\sin 60^0 = \frac{\sqrt{3}}{2}$

$\therefore 2 \sin 30^0 \cos 30^0 = \sin 60^0$

(iv) $2 \sin 45^0 \cos 45^0$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

Also, $\sin 90^0 = 1$

$\therefore 2 \sin 45^0 \cos 45^0 = \sin 90^0$

12.

Sol:

$A = 45^0$

$\Rightarrow 2A = 2 \times 45^0 = 90^0$

(i) $\sin 2A = \sin 90^0 = 1$

$2 \sin A \cos A = 2 \sin 45^0 \cos 45^0 = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

$\therefore \sin 2A = 2 \sin A \cos A$

(ii) $\cos 2A = \cos 90^0 = 0$

$2 \cos^2 A - 1 = 2 \cos^2 45^0 - 1 = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$

Now, $1 - 2 \sin^2 A = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$

$\therefore \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

13.

Sol:

$A = 30^0$

$\Rightarrow 2A = 2 \times 30^0 = 60^0$

(i) $\sin 2A = \sin 60^0 = \frac{\sqrt{3}}{2}$

$$\frac{2 \tan A}{1+ tan^2 A} = \frac{2 \tan 30^0}{1+ tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1+\frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{4}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

$\therefore \sin 2A = \frac{2 \tan A}{1+ tan^2 A}$

(ii) $\cos 2A = \cos 60^0 = \frac{1}{2}$

$$\frac{1- tan^2 A}{1+ tan^2 A} = \frac{1- tan^2 30^0}{1+ tan^2 30^0} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(1-\frac{1}{3}\right)}{1+\frac{1}{3}} = \frac{\left(\frac{2}{3}\right)}{\frac{4}{3}} = \left(\frac{2}{3}\right) \times \frac{3}{4} = \frac{1}{2}$$

$\therefore \cos 2A = \frac{1- tan^2 A}{1+ tan^2 A}$

(iii) $\tan 2A = \tan 60^0 = \sqrt{3}$

$$\frac{2 \tan A}{1 - tan^2 A} = \frac{2 \tan 30^0}{1 - tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$$

$\therefore \tan 2A = \frac{2 \tan A}{1 - tan^2 A}$

14.

Sol:

A = 60^0 and B = 30^0

Now, A + B = $60^0 + 30^0 = 90^0$

Also, A – B = $60^0 – 30^0 = 30^0$

(i) sin (A + B) = sin 90^0 = 1

sin A cos B + cos A sin B = sin 60^0 cos 30^0 + cos 60^0 sin 30^0

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = 1$

∴ sin (A + B) = sin A cos B + cos A sin B

(ii) cos (A + B) = cos 90^0 = 0

cos A cos B - sin A sin B = cos 60^0 cos 30^0 - sin 60^0 sin 30^0

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0$

∴ cos (A + B) = cos A cos B - sin A sin B

15.

Sol:

(i) sin (A – B) = sin $30^0 = \frac{1}{2}$

sin A cos B – cos A sin B = sin 60^0 cos 30^0 – cos 60^0 sin 30^0

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{2}{4} = \frac{1}{2}$

∴ sin (A – B) = sin A cos B – cos A sin B

(ii) cos (A – B) = cos $30^0 = \frac{\sqrt{3}}{2}$

cos A cos B + sin A sin B = cos 60^0 cos 30^0 + sin 60^0 sin 30^0

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

∴ cos (A – B) = cos A cos B + sin A sin B

(iii) tan (A – B) = tan $60^0 = \frac{1}{\sqrt{3}}$

$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan 60^0 - \tan 30^0}{1 + \tan 60^0 \tan 30^0} = \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\sqrt{3} \times \frac{1}{\sqrt{3}}\right)} = \frac{1}{2} \times \frac{3-1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$

∴ tan (A – B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

16.

Sol:
Given:

tan A = $\frac{1}{3}$ and tan B = $\frac{1}{2}$

tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A \tan B}$

On substituting these values in RHS of the expression, we get:

$\frac{\tan A+\tan B}{1-\tan A \tan B} = \frac{\left(\frac{1}{3}+\frac{1}{2}\right)}{\left(1-\frac{1}{3}\times\frac{1}{3}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(1-\frac{1}{6}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{5}{6}\right)} = 1$

⇒ tan (A + B) = 1 = tan 45^0 [∵ tan 450 = 1]

∴ A + B = 45^0

17.

Sol:
A = 30^0

⇒ 2A = 2 × 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

tan 2A = $\frac{2\tan A}{1-tan^2 A}$

⇒ tan 60^0 = $\frac{2\tan 30^0}{1-tan^2 30^0} = \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right)\times\frac{3}{2} = \sqrt{3}$

∴ tan 60^0 = $\sqrt{3}$

18.

Sol:
A = 30^0

⇒ 2A = 2 × 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

Cos A = $\sqrt{\frac{1+\cos 2A}{2}}$

Cos 30^0 = $\sqrt{\frac{1+\cos 60^0}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

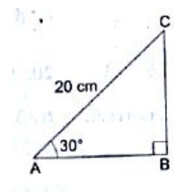
∴ cos A = $\frac{\sqrt{3}}{2}$

19.

Sol:
A = 30^0

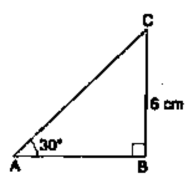
$\Rightarrow$ 2A = 2 × 30^0 = 60^0
By substituting the value of the given T-ratio, we get:
$\sin A = \sqrt{\frac{1-\cos 2A}{2}}$
$\sin 30^0 = \sqrt{\frac{1-\cos 60^0}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
$\therefore \sin 30^0 = \frac{1}{2}$

20.



Sol:
From the given right-angled triangle, we have:
$\frac{BC}{AC} = \sin 30^0$
$\Rightarrow \frac{BC}{20} = \frac{1}{2}$
$\Rightarrow BC = \frac{20}{2} = 10\text{cm}$
Also, $\frac{AB}{AC} = \cos 30^0$
$\Rightarrow \frac{AB}{20} = \frac{\sqrt{3}}{2}$
$\Rightarrow AB = \left(20 \times \frac{\sqrt{3}}{2}\right) = 10\sqrt{3}$ cm
$\therefore$ BC = 10cm and AB = $10\sqrt{3}$ cm

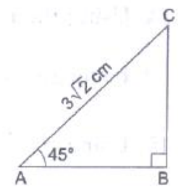
21.
Sol:
From the given right-angled triangle, we have:



$\frac{BC}{AB} = \tan 30^0$
$\Rightarrow \frac{6}{AB} = \frac{1}{\sqrt{3}}$
$\Rightarrow AB = 6\sqrt{3}\text{cm}$
Also, $\frac{BC}{AC} = \sin 30^0$
$\Rightarrow \frac{6}{AC} = \frac{1}{2}$
$\Rightarrow$ AC = (2 × 6) = 12 cm
$\therefore$ AB = $6\sqrt{3}$ cm and AC = 12 cm

22.

Sol:



From the right-angled ΔABC, we have:

$\frac{BC}{AC} = \sin 45^0$

$\Rightarrow \frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow BC = 3cm$

Also, $\frac{AB}{AC} = \cos 45^0$

$\Rightarrow \frac{AB}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow AB = 3$ cm

∴ BC = 3 cm and AB = 3 cm

23.

Sol:

Here, sin (A + B) = 1

⇒ sin (A + B) = 90^0 [∵ sin 90^0 = 1]

⇒ (A + B) = 90^0 …….(i)

Also, cos (A – B) = 1

⇒ cos (A – B) = 0^0 [∵ cos 0^0 = 1]

⇒ A – B = 0^0 ….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 45^0

24.

Sol:

Here, sin (A – B) = $\frac{1}{2}$

⇒ sin (A – B) = 30^0 [∵ sin $30^0 = \frac{1}{2}$]

⇒ (A – B) = 30^0 …….(i)

Also, cos (A + B) = $\frac{1}{2}$

⇒ cos (A + B) = cos 60^0 [∵ cos $60^0 = \frac{1}{2}$]

⇒ A + B = 60^0 ….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 15^0

25.

Sol:

Here, tan (A – B) = $\frac{1}{\sqrt{3}}$

⇒ tan (A – B) = tan 30^0 [∵ tan $30^0 = \frac{1}{\sqrt{3}}$]

⇒ (A – B) = 30^0 …….(i)

Also, tan (A + B) = $\sqrt{3}$

$\Rightarrow$ tan (A + B) = tan 60^0 [$\because$ tan $60^0 = \sqrt{3}$]

$\Rightarrow$ A + B = 60^0 …….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 15^0

26.

Sol:

$3\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{9}{3}\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right)$

$= \frac{1}{3}\left[(3x^2) - \left(\frac{3}{x}\right)^2\right]$

$= \frac{1}{3}[(cosec\ \theta)^2 - (cot\ \theta)^2]$

$= \frac{1}{3}(\text{cosec}^2\theta - \cot^2\theta)$

$= \frac{1}{3}(1) = \frac{1}{3}$

27.

Sol:

Let A = 45^0 and B = 30^0

(i) As, sin(A + B) = sin A cos B + cos A sin B

$\Rightarrow$ sin (45^0 + 30^0) = sin 45^0 cos 30^0 + cos 45^0 sin 30^0

$\Rightarrow$ sin (75^0) = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\Rightarrow$ sin (75^0) = $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ sin (75^0) = $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) As, cos (A – B) = cos A cos B + sin A sin B

$\Rightarrow$ cos (45^0 – 30^0) = cos 45^0 cos 30^0 + sin 45^0 sin 30^0

$\Rightarrow$ cos (15^0) = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\Rightarrow$ cos (15^0) = $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ cos (15^0) = $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Disclaimer: cos 15^0 can also be written by taking A = 60^0 and B = 45^0.