1.

Sol:

(i) $\frac{\sin 16^0}{\cos 74^0}$

$= \frac{\sin\ (90^0 - 74^0)}{\cos 74^0}$

$= \frac{\cos 74^0}{\cos 74^0}$ [∵ sin (90 – θ) = cos θ]

$= 1$

(ii) $\frac{\sec 11^0}{\text{cosec}\, 79^0}$

$= \frac{\sec\ (90^0 - 79^0)}{\text{cosec}\, 79^0}$

$= \frac{\text{cosec}\, 79^0}{\text{cosec}\, 79^0}$ [∵ sec (90 – θ) = cosec θ]

$= 1$

(iii) $\frac{\tan 27^0}{\cot 63^0}$

$= \frac{\tan\ (90^0 - 63^0)}{\cot 63^0}$

$= \frac{\cos 63^0}{\cos 63^0}$ [∵ tan (90 – θ) = cot θ]

$= 1$

(iv) $\frac{\cos 35^0}{\sin 55^0}$

$= \frac{\cos\ (90^0 - 55^0)}{\sin\ 55^0}$

$= \frac{\sin 55^0}{\sin 55^0}$ [∵ sin (90 – θ) = cos θ]

$= 1$

(v) $\frac{\text{cosec}\ 42^0}{\sec 48^0}$

$= \frac{\text{cosec}\ (90^0 - 48^0)}{\sec 48^0}$

$= \frac{\sec 48^0}{\sec 48^0}$ [∵ sec (90 – θ) = cosec θ]

$= 1$

(vi) $\frac{\cot 38^0}{\tan 52^0}$

$= \frac{\cot\ (90^0 - 52^0)}{\tan 52^0}$

$= \frac{\tan 52^0}{\tan 52^0}$ [∵ tan (90 – θ) = cot θ]

= 1

2.

Sol:

(i) LHS = cos 81^0 – sin 9^0

= cos(90^0 – 9^0) – sin 9^0

= sin 9^0 – sin 9^0

= 0

= RHS

(ii) LHS = tan 71^0 – cot 19^0

= tan (90^0 – 19^0) – cot 19^0

= cot 19^0 – cot 19^0

= 0

= RHS

(iii) LHS = cosec 80^0 – sec 10^0

= cosec (90^0 – 10^0) – sec 10^0

= sec 10^0 – sec 10^0

= 0

= RHS

(iv) LHS = $\text{cosec}^2 72^0$ – $\tan^2 18^0$

= $\text{cosec}^2(90^0 – 18^0)$ – $\tan^2 18^0$

= $\sec^2 18^0$ – $\tan^2 18^0$

= 1

= RHS

(v) LHS = $\cos^2 75^0 + \cos^2 15^0$

= $\cos^2(90^0 – 15^0) + \cos^2 15^0$

= $\sin^2 15^0 + \cos^2 15^0$

= 1

= RHS

(vi) LHS = $\tan^2 66^0$ - $\cot^2 24^0$

= $\tan^2(90^0 – 24^0)$ - $\cot^2 24^0$

= $\cot^2 24^0$ - $\cot^2 24^0$

= 0

= RHS

(vii) LHS = $\sin^2 48^0 + \sin^2 42^0$
$= \sin^2(90^0 – 42^0) + \sin^2 42^0$
$= \cos^2 42^0 + \sin^2 42^0$
$= 1$
= RHS
(viii) LHS = $\cos^2 57^0 - \sin^2 33^0$
$= \cos^2(90^0 – 33^0) - \sin^2 33^0$
$= \sin^2 33^0 - \sin^2 33^0$
$= 0$
= RHS
(ix) LHS = $(\sin 65^0 + \cos 25^0)(\sin 65^0 - \cos 25^0)$
$= \sin^2 65^0 - \cos^2 25^0$
$= \sin^2 (90^0 - 25^0) - \cos^2 25^0$
$= \cos^2 25^0 - \cos^2 25^0$
$= 0$
= RHS

3.

Sol:

(i) LHS = $\sin 53^0 \cos 37^0 + \cos 53^0 \sin 37^0$
$= \sin (90^0 – 37^0) \cos 37^0 + \cos(90^0 – 37^0) \sin 37^0$
$= \cos 37^0 \cos 37^0 + \sin 37^0 \sin 37^0$
$= \cos^2 37^0 + \sin^2 37^0$
$= 1$
= RHS
(ii) LHS = $\cos 54^0 \cos 36^0 - \sin 54^0 \sin 36^0$
$= \cos (90^0 – 36^0) \cos 36^0 - \sin(90^0 – 36^0) \sin 36^0$
$= \sin 36^0 \cos 36^0 - \cos 36^0 \sin 36^0$
$= 0$
= RHS
(iii) LHS = $\sec 70^0 \sin 20^0 + \cos 20^0 \operatorname{cosec} 70^0$
$= \sec (90^0 – 20^0) \sin 20^0 + \cos 20^0 \operatorname{cosec} (90^0 – 20^0)$
$= \operatorname{cosec} 20^0 . \frac{1}{\operatorname{cosec} 20^0} + \frac{1}{\sec 20^0} . \sec 20^0$
$= 1 + 1$

= 2

= RHS

(iv) LHS = $\sin 35^0 \sin 55^0 - \cos 35^0 \cos 55^0$

= $\sin 35^0 \cos(90^0 - 55^0)$ - $\cos 35^0 \sin(90^0 - 55^0)$

= $\sin 35^0 \cos 35^0 - \cos 35^0 \sin 35^0$

= 0

= RHS

(v) LHS = $(\sin 72^0 + \cos 18^0)(\sin 72^0 - \cos 18^0)$

= $(\sin 72^0 + \cos 18^0)[\cos(90^0 - 72^0) - \cos 18^0]$

= $(\sin 72^0 + \cos 18^0)(\cos 18^0 - \cos 18^0)$

= $(\sin 72^0 + \cos 18^0)(0)$

= RHS

(vi) LHS = $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0$

= $\cot(90^0 - 48^0) \cot(90^0 - 23^0) \tan 42^0 \tan 67^0$

= $\cot 42^0 \cot 67^0 \tan 42^0 \tan 67^0$

= $\frac{1}{\tan 42^0} \times \frac{1}{\tan 67^0} \times \tan 42^0 \times \tan 67^0$

= 1

= RHS

4.

Sol:

(i) LHS = $\frac{\sin 70^0}{\cos 20^0} + \frac{\operatorname{cosec} 20^0}{\sec 70^0} - 2\cos 70^0 \operatorname{cosec} 20^0$

= $\frac{\sin 70^0}{\sin(90^0 - 20^0)} + \frac{\sec(90^0 - 20^0)}{\sec 70^0} - 2\cos 70^0 \sec(90^0 - 20^0)$

= $\frac{\sin 70^0}{\sin 70^0} + \frac{\sec 70^0}{\sec 70^0} - 2\cos 70^0 \sec 70^0$

= $1 + 1 - 2 \times \cos 70^0 \times \frac{1}{\cos 70^0}$

= 2 – 2

= 0

= RHS

(ii) LHS = $\frac{\cos 80^0}{\sin 10^0}$ + cos59^0 cosec31^0

= $\frac{\cos 80^0}{\sin(90^0 - 10^0)}$ + sin(90^0 - 59^0) cosec31^0

= $\frac{\cos 80^0}{\cos 80^0}$ + sin31^0 cosec31^0

= 1 + sin31^0 $\times$ $\frac{1}{\sin 31^0}$

= 1 + 1

= 2

= RHS

(iii) LHS = $\frac{2\sin 68^0}{\cos 22^0}$ - $\frac{2\cot 15^0}{5\tan 75^0}$ - $\frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5}$

= $\frac{2\sin 68^0}{\sin(90^0 - 22^0)}$ - $\frac{2\cot 15^0}{5\tan(90^0 - 75^0)}$ - $\frac{3 \times 1 \times \cot(90^0 - 20^0) \times \cot(90^0 - 40^0) \times \tan 50^0 \times \tan 70^0}{5}$

= $\frac{2\sin 68^0}{\sin 68^0)}$ - $\frac{2\cot 15^0}{5\cot 15^0}$ - $\frac{3 \times \cot 70^0 \ \cot 50^0 \tan 50^0 \ \tan 70^0}{5}$

= 2 - $\frac{2}{5}$ - $\frac{3 \times \frac{1}{\tan 70^0} \times \frac{1}{\tan 50^0} \times \tan 50^0 \times \tan 70^0}{5}$

= 2 - $\frac{2}{5}$ - $\frac{3}{5}$

= $\frac{10-2-3}{5}$

= $\frac{5}{5}$

= 1

= RHS

(iv) LHS = $\frac{\sin 18^0}{\cos 72^0}$ + $\sqrt{3}$ (tan 10^0 tan 30^0 tan 40^0 tan 50^0 tan 80^0)

= $\frac{\sin 18^0}{\sin(90^0 - 72^0)}$ + $\sqrt{3}$ [cot (90^0 – 10^0) $\times$ $\frac{1}{\sqrt{3}}$ $\times$ cot(90^0 – 40^0) $\times$ tan 50^0 $\times$ tan 80^0]

= $\frac{\sin 18^0}{\sin 18^0}$ + $\sqrt{3}$ $\left(\frac{\cot 80^0 \times \cot 50^0 \times \tan 50^0 \times \tan 80^0}{\sqrt{3}}\right)$

= 1 + $\left(\frac{1}{\tan 80^0} \times \frac{1}{\tan 50^0} \times \tan 50^0 \times \tan 80^0\right)$

= 1 + 1

= 2

= RHS

(v) LHS = $\frac{7\cos 55^0}{3\sin 35^0}$ - $\frac{4\,(\cos 70^0 cosec\ 20^0)}{3(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)}$

= $\frac{7\cos 55^0}{3\cos(90^0 - 35^0)}$ - $\frac{4\,[\sin(90^0 - 70^0) cosec\ 20^0]}{3\,[\cot(90^0 - 5^0) \times \cot(90^0 - 25^0) \times 1 \times \tan 65^0 \times \tan 85^0}$

= $\frac{7\cos 55^0}{3\cos 55^0}$ - $\frac{4\,(\sin 20^0\ cosec\ 20^0)}{3\,(\cot 85^0 \cot 65^0 \tan 65^0\ \tan 85^0)}$

= $\frac{7}{3}$ - $\frac{4\left(\sin 20^0 \times \frac{1}{\sin 20^0}\right)}{3\left(\frac{1}{\tan 85^0} \times \frac{1}{\tan 65^0} \times \tan 65^0 \times \tan 85^0\right)}$

= $\frac{7}{3}$ - $\frac{4}{3}$

$= \frac{3}{3}$

= 1

= RHS

5.

Sol:

(i) LHS = sin θ cos(90^0 – θ) + sin(90^0 – θ) cos θ

= sin θ sin θ + cos θ cos θ

= $\sin^2\theta + \cos^2\theta$

= 1

= RHS

Hence proved.

(ii) LHS = $\frac{\sin\theta}{\cos(90^0-\theta)} + \frac{\cos\theta}{\sin(90^0-\theta)}$

$= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\cos\theta}$

= 1 + 1

= 2

= RHS

Hence proved.

(iii) LHS = $\frac{\sin\theta\cos(90^0-\theta)cos\theta}{\sin(90^0-\theta)} + \frac{cos\theta\sin(90^0-\theta)\,sin\theta}{\cos(90^0-\theta)}$

$= \frac{\sin\theta\sin\theta\,cos\theta}{\cos\theta} + \frac{cos\theta\cos\theta\,sin\theta}{\sin\theta}$

= $\sin^2\theta + \cos^2\theta$

= 1

= RHS

Hence proved.

(iv) LHS = $\frac{\cos(90^0-\theta)\sec(90^0-\theta)\tan\theta}{cosec\,(90^0-\theta)\sin\,(90^0-\theta)\cot\,(90^0-\theta)} + \frac{\tan(90^0-\theta)}{\cot\theta}$

$= \frac{\sin\theta\,cosec\,\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$

= 1 + 1

= 2

= RHS

Hence proved.

(v) LHS = $\frac{\cos(90^0 - \theta)}{1+\sin(90^0 - \theta)} + \frac{1+\sin(90^0 - \theta)}{\cos(90^0 - \theta)}$

$= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \frac{sin^2\theta+1+cos^2\theta+2\cos\theta}{(1+cos\theta)\sin\theta}$

$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= 2\,\frac{1}{\sin\theta}$

= 2 cosec θ

= RHS

Hence proved.

(vi) LHS = $\frac{\sec(90^0 - \theta)cosec\,\theta - \tan(90^0 - \theta)\cot\theta + cos^2 25^0 + cos^2 65^0}{3\tan 27^0 \tan 63^0}$

$= \frac{cosec\,\theta\ cosec\,\theta - \cot\theta\cot\theta + sin^2(90^0 - 25^0) + cos^2 65^0}{3\tan 27^0 \cot(90^0 - 63^0)}$

$= \frac{cosec^2\theta - cot^2\theta + sin^2 65^0 + cos^2 65^0}{3\tan 27^0 \cot 27^0}$

$= \frac{1+1}{3\times\tan 27^0\times\frac{1}{\tan 27^0}}$

$= \frac{2}{3}$

= RHS

(vii) LHS = cot θ tan (90^0 – θ) – sec (90^0 – θ) cosec θ + $\sqrt{3}$ tan 12^0 tan 60^0 tan 78^0

= cot θ cot θ - cosec θ cosec θ + $\sqrt{3}$ tan 12^0 × $\sqrt{3}$ × cot (90^0 – 78^0)

= $\cot^2\theta - cosec^2\theta$ + 3 tan 12^0 cot 12^0

= -1 + 3 × tan 12^0 × $\frac{1}{\tan 12^0}$

= -1 + 3

= 2

= RHS

6.

Sol:

(i) LHS = $\tan 5^0 \tan 25^0 \tan 30^0 \tan 65^0 \tan 85^0$

$= \tan (90^0 - 85^0) \tan (90^0 - 65^0) \times \frac{1}{\sqrt{3}} \times \frac{1}{\cot 60^0} \frac{1}{\cot 85^0}$

$= \cot 85^0 \cot 65^0 \frac{1}{\sqrt{3}} \frac{1}{\cot 60^0} \frac{1}{\cot 85^0}$

$= \frac{1}{\sqrt{3}}$ = RHS

(ii) LHS = $\cot 12^0 \cot 38^0 \cot 52^0 \cot 60^0 \cot 78^0$

$= \tan(90^0 - 12^0) \times \tan(90^0 - 38^0) \times \cot 52^0 \times \frac{1}{\sqrt{3}} \times \cot 78^0$

$= \frac{1}{\sqrt{3}} \times \tan 78^0 \times \tan 52^0 \times \cot 52^0 \times \cot 78^0$

$= \frac{1}{\sqrt{3}} \times \tan 78^0 \times \tan 52^0 \times \frac{1}{\tan 52^0} \times \frac{1}{\tan 78^0}$

$= \frac{1}{\sqrt{3}}$

= RHS

(iii) LHS = $\cos 15^0 \cos 35^0 \operatorname{cosec} 55^0 \cos 60^0 \operatorname{cosec} 75^0$

$= \cos (90^0 - 75^0) \cos (90^0 - 55^0) \frac{1}{\sin 55^0} \times \frac{1}{2} \times \frac{1}{\sin 75^0}$

$= \sin 75^0 \sin 55^0 \quad \frac{1}{\sin 55^0} \times \frac{1}{2} \times \frac{1}{\sin 75^0}$

$= \frac{1}{2}$ = RHS

(iv) LHS = $\cos 1^0 \cos 2^0 \cos 3^0 \ldots . \cos 180^0$

$= \cos 1^0 \times \cos 2^0 \times \cos 3^0 \times \ldots . \times \cos 90^0 \times \ldots \times \cos 180^0$

$= \cos 1^0 \times \cos 2^0 \times \cos 3^0 \times \ldots . \times 0 \times \ldots \times \cos 180^0$

= 0

= RHS

(v) LHS = $\left(\frac{\sin 49^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\sin 49^0}\right)$

$= \left(\frac{\cos(90^0 - 49^0)}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\cos(90^0 - 49^0)^0}\right)^2$

$= \left(\frac{\cos 41^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\cos 41^0}\right)^2$

$= 1^2 + 1^2$

= 1 + 1
= 2
= RHS
Disclaimer: The RHS of (v) given in textbook is incorrect. There should be 2 instead 1. The same has been corrected in the solution here.

7.

Sol:

(i) LHS = $\sin(70^0 + \theta) - \cos(20^0 - \theta)$
$= \sin\{90^0 - (20^0 - \theta)\} - \cos(20^0 - \theta)$
$= \cos(20^0 - \theta) - \cos(20^0 - \theta)$
= 0
= RHS

(ii) LHS = $\tan(55^0 - \theta) - \cot(35^0 + \theta)$
$= \tan\{90^0 - (35^0 + \theta)\} - \cot(35^0 + \theta)$
$= \cot(35^0 + \theta) - \cot(35^0 + \theta)$
= 0
= RHS

(iii) LHS = $\text{cosec}(67^0 + \theta) - \sec(23^0 - \theta)$
$= \text{cosec}\{90^0 - (23^0 - \theta)\} - \sec(23^0 - \theta)$
$= \sec(23^0 - \theta) - \sec(23^0 - \theta)$
= 0
= RHS

(iv) LHS = $\text{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta)$
$= \text{cosec}\{90^0 - (25^0 - \theta)\} - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot\{90^0 - (55^0 - \theta)\}$
$= \sec(25^0 - \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \tan(55^0 - \theta)$
= 0
= RHS

(v) LHS = $\sin(50^0 + \theta) - \cos(40^0 - \theta) + \tan 1^0 \tan 10^0 \tan 80^0 \tan 89^0$
$= \sin\{90^0 - (40^0 - \theta)\} - \cos(40^0 - \theta) + \{\tan 1^0 \tan(90^0 - 1^0)\}\{\tan 10^0 \tan(90^0 - 10)\}$
$= \cos(40^0 - \theta) - \cos(40^0 - \theta) + (\tan 1^0 \cot 1^0)(\tan 10^0 \cot 10^0)$

$= \left(\frac{1}{\cot 1^0} \times \cot 1^0\right)\left(\tan 10^0 \times \frac{1}{\tan 10^0}\right)$
$= 1 \times 1$
$= 1$
= RHS

8.

Sol:

(i) $\sin 67^0 + \cos 75^0$
$= \cos (90^0 - 67^0) + \sin (90^0 - 75^0)$
$= \cos 23^0 + \sin 15^0$
(ii) $\cot 65^0 + \tan 49^0$
$= \cos (90^0 - 65^0) + \cot (90^0 - 49^0)$
$= \cos 25^0 + \cot 41^0$
(iii) $\sec 78^0 + \operatorname{cosec} 56^0$
$= \sec (90^0 - 12^0) + \operatorname{cosec} (90^0 - 34^0)$
$= \operatorname{cosec} 12^0 + \sec 34^0$
(iv) $\operatorname{cosec} 54^0 + \sin 72^0$
$= \sec (90^0 - 54^0) + \cos (90^0 - 72^0)$
$= \sec 36^0 + \cos 18^0$

9.

Sol:
In ΔABC,
$A + B + C = 180^0$
$\Rightarrow A + C = 180^0 - B$ ....(i)
Now,
LHS $= \tan\left(\frac{C+A}{2}\right)$
$= \tan\left(\frac{180^0-B}{2}\right)$ [Using (i)]
$= \tan\left(90^0 - \frac{B}{2}\right)$
$= \cot\frac{B}{2}$
= RHS

10.

Sol:

We have,

$\cos 2\theta = \sin 4\theta$

$\Rightarrow \sin (90^0 - 2\theta) = \sin 4\theta$

Comparing both sides, we get

$90^0 - 2\theta = 4\theta$

$\Rightarrow 2\theta + 4\theta = 90^0$

$\Rightarrow 6\theta = 90^0$

$\Rightarrow \theta = \frac{90^0}{6}$

$\therefore \theta = 15^0$

Hence, the value of θ is 15^0.

11.

Sol:

We have,

$\sec 2A = \operatorname{cosec} (A - 42^0)$

$\Rightarrow \operatorname{cosec} (90^0 - 2A) = \operatorname{cosec} (A - 42^0)$

Comparing both sides, we get

$90^0 - 2A = A - 42^0$

$\Rightarrow 2A + A = 90^0 + 42^0$

$\Rightarrow 3A = 132^0$

$\Rightarrow A = \frac{132^0}{3}$

$\therefore A = 44^0$

Hence, the value of A is 44^0.

12.

Sol:

$\sin 3A = \cos (A - 26^0)$

$\Rightarrow \cos (90^0 - 3A) = \cos (A - 26^0)$ $[\because \sin \theta = \cos (90^0 - \theta)]$

$\Rightarrow 90^0 - 3A = A - 26^0$

$\Rightarrow 116^0 = 4A$

$\Rightarrow A = \frac{116^0}{4} = 29^0$

13.

Sol:

tan2A = cot (A – 12^0)

$\Rightarrow$ cot (90^0 – 2A) = cot (A – 12^0) [$\because$ tan θ = cot (90^0 – θ)]

$\Rightarrow$ (90^0 – 2A) = (A – 12^0)

$\Rightarrow$ 102^0 = 3A

$\Rightarrow$ A = $\frac{102^0}{3}$ = 34^0

14.

Sol:

sec4A = cosec (A – 15^0)

$\Rightarrow$ cosec (90^0 – 4A) = cosec (A – 15^0) [$\because$ sec θ = cosec (90^0 – θ)]

$\Rightarrow$ (90^0 – 4A) = (A – 15^0)

$\Rightarrow$ 105^0 = 5A

$\Rightarrow$ A = $\frac{105^0}{5}$ = 21^0

15.

Sol:

$\frac{2}{3}$ cosec$^2 58^0$ $-\frac{2}{3}$ cot 58^0 tan 32^0 $-\frac{5}{3}$ tan 13^0 tan 37^0 tan 45^0 tan 53^0 tan 77^0

$= \frac{2}{3}(cosec^2\ 58^0\ -\ \cot 58^0 \tan 32^0)\ -\frac{5}{3}$ tan 13^0 tan (90^0 – 13^0) tan37^0 tan(90^0 – 37^0)

(tan 45^0)

$= \frac{2}{3}\{cosec^2\ 58^0\ -\ cot\ 58^0\ tan(90^0 -\ 58^0)\} - \frac{5}{3}$ tan 13^0 cot 13^0 tan 37^0 cot 37^0 (1)

$= \frac{2}{3}(cosec^2\ 58^0\ -\ cot\ 58^0\ tan\ 58^0) - \frac{5}{3}$ tan 13^0 $\frac{1}{\tan 13^0}$ tan 37^0 $\frac{1}{\tan 37^0}$

$= \frac{2}{3}(cosec^2\ 58^0\ -\ \ cot^2 58^0) - \frac{5}{3}$

$= \frac{2}{3} - \frac{5}{3}$

$= -1$

Hence proved.