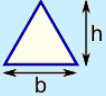
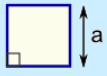
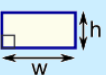
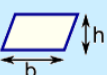
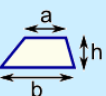

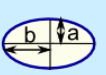
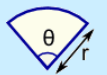


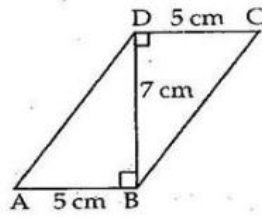
Area

Exercise 10A

	<p><u>Triangle</u> Area = $\frac{1}{2} \times b \times h$ b = base h = vertical height</p>		<p><u>Square</u> Area = a^2 a = length of side</p>
	<p><u>Rectangle</u> Area = $w \times h$ w = width h = height</p>		<p><u>Parallelogram</u> Area = $b \times h$ b = base h = vertical height</p>
	<p><u>Trapezoid (US)</u> <u>Trapezium (UK)</u> Area = $\frac{1}{2}(a+b) \times h$ h = vertical height</p>		<p><u>Circle</u> Area = $\pi \times r^2$ Circumference = $2 \times \pi \times r$ r = radius</p>
	<p><u>Ellipse</u> Area = πab</p>		<p><u>Sector</u> Area = $\frac{1}{2} \times r^2 \times \theta$ r = radius θ = angle in radians</p>

Question 1:

$$\text{Area of } \triangle ABD = \frac{1}{2} \times \text{base} \times \text{height}$$



$$= \left(\frac{1}{2} \times 5 \times 7 \right) \text{ cm}^2 = \frac{35}{2} \text{ cm}^2$$

$$\text{Area of } \triangle CBD = \left(\frac{1}{2} \times 5 \times 7 \right) \text{ cm}^2 = \frac{35}{2} \text{ cm}^2$$

Since the diagonal BD divides ABCD into two triangles of equal area.

\therefore ABCD is a parallelogram.

\therefore Area of parallelogram = Area of $\triangle ABD$ + Area of $\triangle CBD$

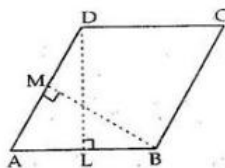
$$= \left(\frac{35}{2} + \frac{35}{2} \right) \text{ cm}^2 = \frac{70}{2} \text{ cm}^2$$

$$= 35 \text{ cm}^2$$

\therefore Area of parallelogram = 35 cm^2

Question 2:

Since ABCD is a parallelogram and DL is perpendicular to AB.



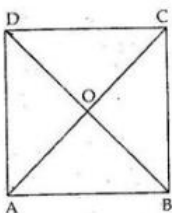
$$\begin{aligned} \text{So, its area} &= AB \times DL \\ &= (10 \times 6) \text{ cm}^2 \\ &= 60 \text{ cm}^2 \end{aligned}$$

Also, in parallelogram ABCD,
 $BM \perp AD$
 \therefore Area of parallelogram ABCD = $AD \times BM$

$$\begin{aligned} 60 &= AD \times 8 \text{ cm} \\ \therefore AD \times 8 &= 60 \\ \Rightarrow AD &= \frac{60}{8} = 7.5 \text{ cm} \\ \therefore AD &= 7.5 \text{ cm} \end{aligned}$$

Question 3:

ABCD is a rhombus in which diagonal $AC = 24$ cm
 and $BD = 16$ cm.
 These diagonals intersect at O.



Since diagonals of a rhombus are perpendicular to each other. So, in ΔACD ,
 OD is its altitude and AC is its base.

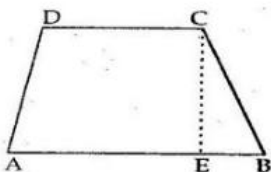
$$\begin{aligned} \text{So, area of } \Delta ACD &= \frac{1}{2} \times AC \times OD \\ &= \frac{1}{2} \times 24 \times \frac{BD}{2} \\ &= \left(\frac{1}{2} \times 24 \times 8 \right) \text{ cm}^2 \quad [\because BD = 16 \text{ cm}] \\ &= 96 \text{ cm}^2 \\ \Rightarrow \text{Area of } \Delta ABC &= \frac{1}{2} \times AC \times OB \\ &= \left(\frac{1}{2} \times 24 \times 8 \right) \text{ cm}^2 = 96 \text{ cm}^2 \\ \text{Now, area of rhombus} &= \text{Area of } \Delta ACD + \text{Area of } \Delta ABC \\ &= (96 + 96) \text{ cm}^2 \\ &= 192 \text{ cm}^2 \end{aligned}$$

Question 4:

ABCD is a trapezium in which, $AB \parallel CD$

$AB = 9 \text{ cm}$ and $CD = 6 \text{ cm}$

CE is a perpendicular drawn to AB through C and $CE = 8 \text{ cm}$



Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) \times distance between them

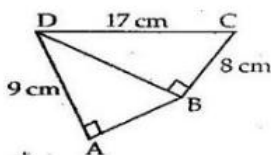
$$= \left[\frac{1}{2} (9 + 6) \times 8 \right] \text{ cm}^2$$

$$= \left[\frac{1}{2} \times 15 \times 8 \right] \text{ cm}^2 = 60 \text{ cm}^2$$

\therefore Area of trapezium = 60 cm^2

Question 5:

(i) ABCD is a quadrilateral.



Now in right angled $\triangle DBC$,

$$DB^2 = DC^2 - CB^2$$

$$= 17^2 - 8^2$$

$$= 289 - 64 = 225 \text{ cm}^2$$

$$\therefore DB = \sqrt{225} = 15 \text{ cm}$$

$$\text{So, area of } \triangle DBC = \left(\frac{1}{2} \times 15 \times 8 \right) \text{ cm}^2 = 60 \text{ cm}^2$$

Again, in right angled $\triangle DAB$,

$$AB^2 = DB^2 - AD^2$$

$$= 15^2 - 9^2$$

$$= 225 - 81 = 144 \text{ cm}^2$$

$$\therefore AB = \sqrt{144} = 12 \text{ cm}$$

$$\therefore \text{area of } \triangle DAB = \left(\frac{1}{2} \times 12 \times 9 \right) \text{ cm}^2 = 54 \text{ cm}^2$$

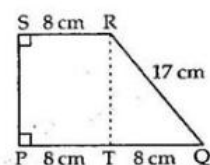
So, area of quadrilateral ABCD

$$= \text{Area of } \triangle DBC + \text{Area of } \triangle DAB$$

$$= (60 + 54) \text{ cm}^2 = 114 \text{ cm}^2$$

$$\therefore \text{area of quadrilateral ABCD} = 114 \text{ cm}^2$$

(ii)



$RT \perp PQ$

In right angled $\triangle RTQ$

$$RT^2 = RQ^2 - TQ^2$$

$$= 17^2 - 8^2$$

$$= 289 - 64 = 225 \text{ cm}^2$$

$$\therefore RT = \sqrt{225} = 15 \text{ cm}$$

\therefore Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) \times distance between them

$$= \frac{1}{2} \times (PQ + SR) \times RT$$

$$= \frac{1}{2} \times (16 + 8) \times 15$$

$$= \left(\frac{1}{2} \times 24 \times 15 \right) \text{ cm}^2 = 180 \text{ cm}^2$$

$$\therefore \text{area of trapezium} = 180 \text{ cm}^2$$

Question 7:

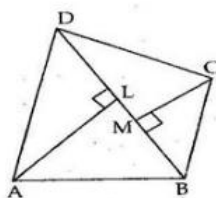
Given: ABCD is a quadrilateral and BD is one of its diagonals.

$AL \perp BD$ and $CM \perp BD$

To Prove: area (quad. ABCD)

$$= \frac{1}{2} \times BD \times (AL + CM)$$

Proof:



$$\text{Area of } \triangle BAD = \frac{1}{2} \times BD \times AL$$

$$\text{Area of } \triangle CBD = \frac{1}{2} \times BD \times CM$$

$$\therefore \text{Area of quad. ABCD} = \text{Area of } \triangle ABD + \text{Area of } \triangle CBD$$

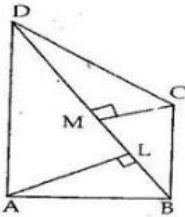
$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$\therefore \text{Area of quad. ABCD} = \frac{1}{2} \times BD [AL + CM]$$

Question 8:

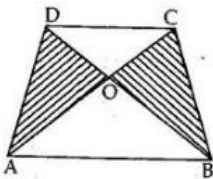
$$\begin{aligned} \text{Area of } \triangle BAD &= \frac{1}{2} \times BD \times AL \\ &= \left(\frac{1}{2} \times 14 \times 8 \right) \text{ cm}^2 = 56 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle CBD &= \frac{1}{2} \times BD \times CM \\ &= \left(\frac{1}{2} \times 14 \times 6 \right) \text{ cm}^2 = 42 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} \therefore \text{ area of quad. } ABCD &= \text{Area of } \triangle ABD + \text{Area of } \triangle CBD \\ &= (56 + 42) \text{ cm}^2 = 98 \text{ cm}^2 \end{aligned}$$

Question 9:



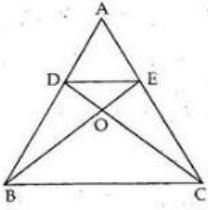
Consider $\triangle ADC$ and $\triangle DCB$. We find they have the same base CD and lie between two parallel lines DC and AB .

Triangles on the same base and between the same parallels are equal in area.

So $\triangle CDA$ and $\triangle CDB$ are equal in area.

$$\begin{aligned} \therefore \quad \text{area}(\triangle CDA) &= \text{area}(\triangle CDB) \\ \text{Now,} \quad \text{area}(\triangle AOD) &= \text{area}(\triangle ADC) - \text{area}(\triangle OCD) \\ \text{and} \quad \text{area}(\triangle BOC) &= \text{area}(\triangle CDB) - \text{area}(\triangle OCD) \\ &= \text{area}(\triangle ADC) - \text{area}(\triangle OCD) \\ \Rightarrow \quad \text{area}(\triangle AOD) &= \text{area}(\triangle BOC) \end{aligned}$$

Question 10:



(i) $\triangle DBE$ and $\triangle DCE$ have the same base DE and lie between parallel lines BC and DE .

So, $\text{area}(\triangle DBE) = \text{area}(\triangle DCE) \dots (1)$

Adding $\text{area}(\triangle ADE)$ on both sides, we get

$$\text{ar}(\triangle DBE) + \text{ar}(\triangle ADE) = \text{ar}(\triangle DCE) + \text{ar}(\triangle ADE)$$

$$\Rightarrow \text{ar}(\triangle ABE) = \text{ar}(\triangle ACD)$$

(ii) Since $\text{ar}(\triangle DBE) = \text{ar}(\triangle DCE)$ [from (1)]

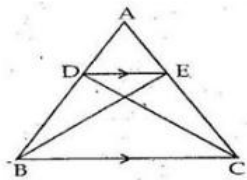
Subtracting $\text{ar}(\triangle ODE)$ from both sides we get

$$\text{ar}(\triangle DBE) - \text{ar}(\triangle ODE) = \text{ar}(\triangle DCE) - \text{ar}(\triangle ODE)$$

$$\Rightarrow \text{ar}(\triangle OBD) = \text{ar}(\triangle OCE)$$

Question 11:

Given: $\triangle ABC$ in which points D and E lie on AB and AC , such that $\text{ar}(\triangle BCE) = \text{ar}(\triangle BCD)$



To Prove: $DE \parallel BC$

Proof : As $\triangle BCE$ and $\triangle BCD$ have same base BC , and are equal in area, they have same altitudes.

This means that they lie between two parallel lines.

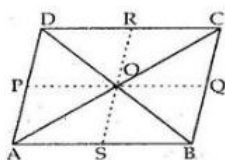
$\therefore DE \parallel BC$

Question 12:

Given : A parallelogram ABCD in which O is a point inside it.

To Prove: (i) $\text{ar}(\triangle OAB) + \text{ar}(\triangle OCD) = \frac{1}{2} \text{ar}(\parallel\text{gm ABCD})$

(ii) $\text{ar}(\triangle OAD) + \text{ar}(\triangle OBC) = \frac{1}{2} \text{ar}(\parallel\text{gm ABCD})$



Construction: Through O draw $PQ \parallel AB$ and $RS \parallel AD$

Proof: (i) $\triangle AOB$ and parallelogram $ABQP$ have same base AB and lie between parallel lines AB and PQ .

If a triangle and a parallelogram are on the same base, and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

$$\therefore \text{ar}(\triangle AOB) = \frac{1}{2} \text{ar}(\parallel\text{gm ABQP})$$

$$\text{Similarly, } \text{ar}(\triangle COD) = \frac{1}{2} \text{ar}(\parallel\text{gm PQCD})$$

$$\begin{aligned} \text{So, } \text{ar}(\triangle AOB) + \text{ar}(\triangle COD) &= \frac{1}{2} \text{ar}(\parallel\text{gm ABQP}) + \frac{1}{2} \text{ar}(\parallel\text{gm PQCD}) \\ &= \frac{1}{2} [\text{ar}(\parallel\text{gm ABQP}) + \text{ar}(\parallel\text{gm PQCD})] \\ &= \frac{1}{2} [\text{ar}(\parallel\text{gm ABCD})] \end{aligned}$$

(ii) $\triangle AOD$ and $\parallel\text{gm ASRD}$ have the same base AD and lie between same parallel lines AD and RS .

$$\text{So, } \text{ar}(\triangle AOD) = \frac{1}{2} \text{ar}(\parallel\text{gm ASRD})$$

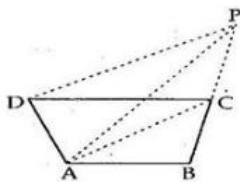
$$\text{Similarly, } \text{ar}(\triangle BOC) = \frac{1}{2} \text{ar}(\parallel\text{gm RSBC})$$

$$\begin{aligned} \therefore \text{ar}(\triangle AOD) + \text{ar}(\triangle BOC) &= \frac{1}{2} [\text{ar}(\parallel\text{gm ASRD}) + \text{ar}(\parallel\text{gm RSBC})] \\ &= \frac{1}{2} [\text{ar}(\parallel\text{gm ABCD})] \end{aligned}$$

Question 13:

Given: ABCD is a quadrilateral in which through D, a line is drawn parallel to AC which meets BC produced in P.

To Prove : $\text{ar}(\triangle ABP) = \text{ar}(\text{quad. ABCD})$



Proof : $\triangle ACP$ and $\triangle ACD$ have same base AC and lie between parallel lines AC and DP .

$$\therefore \text{ar}(\triangle ACP) = \text{ar}(\triangle ACD)$$

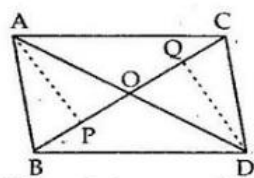
Adding $\text{ar}(\triangle ABC)$ on both sides, we get;

$$\text{ar}(\triangle ACP) + \text{ar}(\triangle ABC) = \text{ar}(\triangle ACD) + \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\triangle ABP) = \text{ar}(\text{quad. ABCD})$$

Question 14:

Given: Two triangles, i.e. $\triangle ABC$ and $\triangle DBC$ which have same base BC and points A and D lie on opposite sides of BC and
 $ar(\triangle ABC) = ar(\triangle DBC)$



To Prove: $OA = OD$

Construction: Draw $AP \perp BC$ and $DQ \perp BC$

Proof: We have

$$ar(\triangle ABC) = \frac{1}{2} \times BC \times AP \text{ and}$$

$$ar(\triangle DBC) = \frac{1}{2} \times BC \times DQ$$

So, $\frac{1}{2} \times BC \times AP = \frac{1}{2} \times BC \times DQ$ [from (1)]

$\Rightarrow AP = DQ$ (2)

Now, in $\triangle AOP$ and $\triangle DQO$, we have

$$\angle APO = \angle DQO = 90^\circ$$

and $\angle AOP = \angle DOQ$ [vertically opp. angles]

$$AP = DQ \text{ [from (2)]}$$

Thus, by Angle-Angle-Side criterion of congruence, we have

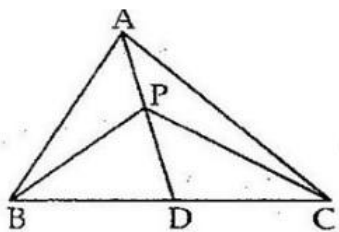
$$\therefore \triangle AOP \cong \triangle DQO \text{ [AAS]}$$

The corresponding parts of the congruent triangles are equal.

$$\therefore OA = OD \text{ [C.P.C.T.]}$$

Question 15:

Given: A $\triangle ABC$ in which AD is the median and P is a point on AD .



To Prove: (i) $ar(\triangle BPD) = ar(\triangle CDP)$

(ii) $ar(\triangle ABP) = ar(\triangle APC)$

Proof : (i) In $\triangle BPC$, PD is the median. Since median of a triangle divides the triangle into two triangles of equal areas

So, $ar(\triangle BPD) = ar(\triangle CDP)$(1)

(ii) In $\triangle ABC$, AD is the median

So, $ar(\triangle ABD) = ar(\triangle ADC)$

But, $ar(\triangle BPD) = ar(\triangle CDP)$ [from (1)]

Subtracting $ar(\triangle BPD)$ from both the sides of the equation, we have

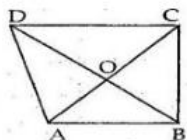
$$\therefore ar(\triangle ABD) - ar(\triangle BPD) = ar(\triangle ADC) - ar(\triangle BPD)$$

$$= ar(\triangle ADC) - ar(\triangle CDP) \text{ from (1)}$$

$$\Rightarrow ar(\triangle ABP) = ar(\triangle ACP).$$

Question 16:

Given : A quadrilateral ABCD in which diagonals AC and BD intersect at O and $BO = OD$



To Prove : $\text{ar}(\triangle ABC) = \text{ar}(\triangle ADC)$

Proof: Since $OB = OD$ [Given]

So, AO is the median of $\triangle ABD$

$$\therefore \text{ar}(\triangle AOD) = \text{ar}(\triangle AOB) \dots (i)$$

As OC is the median of $\triangle CBD$

$$\text{ar}(\triangle DOC) = \text{ar}(\triangle BOC) \dots (ii)$$

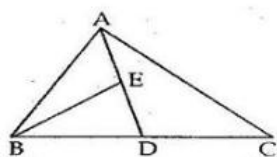
Adding both sides of (i) and (ii), we get

$$\text{ar}(\triangle AOD) + \text{ar}(\triangle DOC) = \text{ar}(\triangle AOB) + \text{ar}(\triangle BOC)$$

$$\therefore \text{ar}(\triangle ADC) = \text{ar}(\triangle ABC)$$

Question 17:

Given : A $\triangle ABC$ in which AD is a median and E is the mid-point of AD



To Prove : $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$

Proof: Since, $\text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$ [\because AD is the median]

$$\text{i.e.} \quad \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \dots (1)$$

$$[\because \text{ar}(\triangle ABC) = \text{ar}(\triangle ABD) + \text{ar}(\triangle ACD)]$$

Now, as BE is the median of $\triangle ABD$

$$\text{ar}(\triangle ABE) = \text{ar}(\triangle BED) \dots (2)$$

Since $\text{ar}(\triangle ABD) = \text{ar}(\triangle ABE) + \text{ar}(\triangle BED) \dots (3)$

$$\therefore \text{ar}(\triangle BED) = \text{ar}(\triangle ABE) \quad [\text{from (2)}]$$

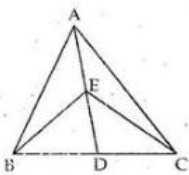
$$= \frac{1}{2} \text{ar}(\triangle ABD) \quad [\text{from (2) and (3)}]$$

$$= \frac{1}{2} \left[\frac{1}{2} \text{ar}(\triangle ABC) \right] \quad [\text{from (1)}]$$

$$= \frac{1}{4} \text{ar}(\triangle ABC)$$

Question 18:

Given: A $\triangle ABC$ in which E is the mid-point of line segment AD where D is a point on BC.



To Prove: $\text{ar}(\triangle BEC) = \frac{1}{2} \text{ar}(\triangle ABC)$

Proof: Since BE is the median of $\triangle ABD$

So, $\text{ar}(\triangle BDE) = \text{ar}(\triangle ABE)$

$\therefore \text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle ABD) \dots (i)$

As, CE is median of $\triangle ADC$

So, $\text{ar}(\triangle CDE) = \frac{1}{2} \text{ar}(\triangle ACD) \dots (ii)$

Adding (i) and (ii), we get

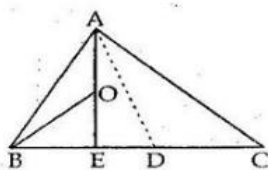
$$\text{ar}(\triangle BDE) + \text{ar}(\triangle CDE) = \frac{1}{2} \text{ar}(\triangle ABD) + \frac{1}{2} \text{ar}(\triangle ACD)$$

$$\text{ar}(\triangle BEC) = \frac{1}{2} [\text{ar}(\triangle ABD) + \text{ar}(\triangle ACD)]$$

$$= \frac{1}{2} \text{ar}(\triangle ABC).$$

Question 19:

Given: A $\triangle ABC$ in which AD is the median and E is the mid-point of BD. O is the mid-point of AE.



To Prove: $\text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$

Proof: Since O is the midpoint of AE.

So, BO is the median of $\triangle BAE$

$\therefore \text{ar}(\triangle BOE) = \frac{1}{2} \text{ar}(\triangle ABE) \dots (1)$

Now, E is the mid-point of BD

So AE divides $\triangle ABD$ into two triangles of equal area.

$\therefore \text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(\triangle ABD) \dots (2)$

As D is the mid point of BC

So $\text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \dots (3)$

$\Rightarrow \text{ar}(\triangle BOE) = \frac{1}{2} \text{ar}(\triangle ABE)$ [from (1)]

$$= \frac{1}{2} \left[\frac{1}{2} \text{ar}(\triangle ABD) \right]$$
 [from (2)]

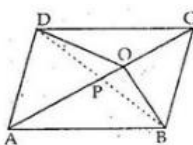
$$= \frac{1}{4} \text{ar}(\triangle ABD)$$

$$= \frac{1}{4} \times \frac{1}{2} \text{ar}(\triangle ABC)$$
 [from (3)]

$$= \frac{1}{8} \text{ar}(\triangle ABC)$$

Question 20:

Given: A parallelogram ABCD in which O is any point on the diagonal AC.



To Prove: $\text{ar}(\triangle AOB) = \text{ar}(\triangle AOD)$.

Construction: Join BD which intersects AC at P.

Proof: As diagonals of a parallelogram bisect each other,

so, OP is the median of $\triangle ODB$

$\therefore \text{ar}(\triangle ODP) = \text{ar}(\triangle OBP)$.

Also, AP is the median of $\triangle ABD$

$\therefore \text{ar}(\triangle ADP) = \text{ar}(\triangle ABP)$

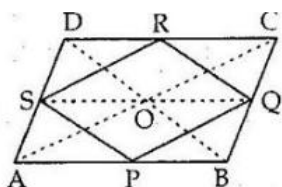
Adding both sides, we get

$\text{ar}(\triangle ODP) + \text{ar}(\triangle ADP) = \text{ar}(\triangle OBP) + \text{ar}(\triangle ABP)$

$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle AOB)$.

Question 21:

Given: ABCD is a parallelogram and P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.



To Prove: PQRS is a parallelogram and $\text{ar}(\text{gmPQRS})$

$$= \frac{1}{2} \text{ar}(\text{gmABCD})$$

Construction: Join AC, BD and SQ.

Proof: As S and R are the midpoints of AD and CD. So, in $\triangle ADC$,

$SR \parallel AC$ [By mid point theorem]

Also, as P and Q are the midpoints of AB and BC. So, in $\triangle ABC$,

$PQ \parallel AC$

$\therefore PQ \parallel AC \parallel SR$

$\therefore PQ \parallel SR$

Similarly, we can prove $SP \parallel RQ$.

Thus PQRS is a parallelogram as its opposite sides are parallel since diagonals of a parallelogram bisect each other.

So in $\triangle ABD$,

O is the midpoint of AC and S is the midpoint of AD.

$\therefore OS \parallel AB$ [By midpoint theorem]

Similarly in $\triangle ABC$, we can prove that,

$OQ \parallel AB$

i.e. $SQ \parallel AB$

Thus, ABQS is a parallelogram.

Now, $\text{ar}(\triangle SPQ) = \frac{1}{2} \text{ar}(\text{gmABQS}) \dots (i)$

[$\because \triangle SPQ$ and gm ABQS have the same base and lie between same parallel lines]

Similarly, we can prove that;

$\text{ar}(\triangle SRQ) = \frac{1}{2} \text{ar}(\text{gmSQCD}) \dots (ii)$

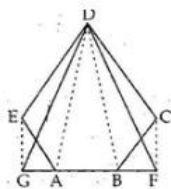
Adding (i) and (ii) we get

$$\text{ar}(\triangle SPQ) + \text{ar}(\triangle SRQ) = \frac{1}{2} [\text{ar}(\text{gmABQS}) + \text{ar}(\text{gmSQCD})]$$

$$\therefore \text{ar}(\text{gmPQRS}) = \frac{1}{2} \text{ar}(\text{gmABCD})$$

Question 22:

Given: ABCDE is a pentagon. EG, drawn parallel to DA, meets BA produced at G, and CF, drawn parallel to DB, meets AB produced at F.



To Prove: $\text{ar}(\text{Pentagon } ABCDE) = \text{ar}(\triangle DGF)$

Proof:

Triangles on the same base and between the same parallels are equal in area.

Since $\triangle DGA$ and $\triangle AED$ have same base AD and lie between parallel lines AD and EG

$$\therefore \text{ar}(\triangle DGA) = \text{ar}(\triangle AED) \dots (1)$$

Similarly, $\triangle DBC$ and $\triangle BFD$ have same base DB and lie between parallel lines BD and CF.

$$\therefore \text{ar}(\triangle DBF) = \text{ar}(\triangle DBC) \dots (2)$$

Adding both the sides of the equations (1) and (2), we have

$$\therefore \text{ar}(\triangle DGA) + \text{ar}(\triangle DBF) = \text{ar}(\triangle AED) + \text{ar}(\triangle BCD)$$

Adding $\text{ar}(\triangle ABD)$ to both sides, we get,

$$\begin{aligned} \text{ar}(\triangle DGA) + \text{ar}(\triangle DBF) + \text{ar}(\triangle ABD) \\ = \text{ar}(\triangle AED) + \text{ar}(\triangle BCD) + \text{ar}(\triangle ABD) \end{aligned}$$

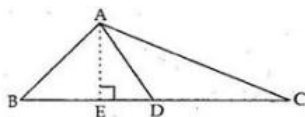
$$\therefore \text{ar}(\triangle DGA) = \text{ar}(\text{pentagon } ABCDE)$$

Question 23:

Given: ABC is a triangle in which AD is the median.

To Prove: $\text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$

Construction: Draw $AE \perp BC$



Proof: $\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AE$

and, $\text{ar}(\triangle ADC) = \frac{1}{2} \times DC \times AE$

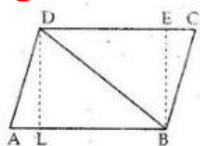
Since, $BD = DC$ [Since D is the median]

So, $\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AE$

$$= \frac{1}{2} \times DC \times AE = \text{ar}(\triangle ADC)$$

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$$

Question 24:



Given : ABCD is a parallelogram in which BD is its diagonal.

To Prove: $\text{ar}(\triangle ABD) = \text{ar}(\triangle CBD)$

Construction : Draw $DL \perp AB$ and $BE \perp CD$

Proof: $\text{ar}(\triangle ABD) = \frac{1}{2} \times AB \times DL$ (i)

and, $\text{ar}(\triangle CBD) = \frac{1}{2} \times CD \times BE$ (ii)

Now, since ABCD is a parallelogram.

$\therefore AB \parallel CD$

and $AB = CD$ (iii)

Since distance between two parallel lines is constant,

$\Rightarrow DL = BE$ (iv)

From (i), (ii), (iii), and (iv) we have

$$\begin{aligned} \text{ar}(\triangle ABD) &= \frac{1}{2} \times AB \times DL \\ &= \frac{1}{2} \times CD \times BE = \text{ar}(\triangle CBD) \end{aligned}$$

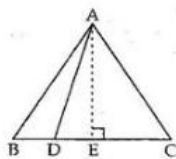
$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle CBD)$

Question 25:

Given : A $\triangle ABC$ in which D is a point on BC such that;

$BD = \frac{1}{2}DC$

To Prove: $\text{ar}(\triangle ABD) = \frac{1}{3} \text{ar}(\triangle ABC)$



Construction: Draw $AE \perp BC$

Proof: $\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AE$ (1)

and, $\text{ar}(\triangle ABC) = \frac{1}{2} \times BC \times AE$ (2)

Given that $BD = \frac{1}{2}BC$

So, $BC = BD + DC = BD + 2BD = 3BD$

$\therefore BD = \frac{1}{3}BC$ (3)

From (1),

$$\begin{aligned} \text{ar}(\triangle ABD) &= \frac{1}{2} \times BD \times AE \\ &= \frac{1}{2} \times \frac{BC}{3} \times AE \quad [\text{from (3)}] \end{aligned}$$

$\therefore \text{ar}(\triangle ABD) = \frac{1}{3} \times \left(\frac{1}{2} \times BC \times AE \right)$

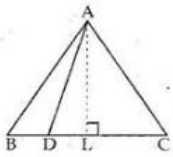
$= \frac{1}{3} \times \text{ar}(\triangle ABC)$ [from (2)]

$\therefore \text{ar}(\triangle ABD) = \frac{1}{3} \times \text{ar}(\triangle ABC)$

Question 26:

Given: ABC is a triangle in which D is a point on BC such that;

$$BD : DC = m : n$$



To Prove : $\text{ar}(\triangle ABD) : \text{ar}(\triangle ACD)$
 $= m : n$

Proof : $\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AL$

and, $\text{ar}(\triangle ADC) = \frac{1}{2} \times DC \times AL$

Now, $BD : DC = m : n$

$$\therefore BD = DC \times \frac{m}{n}$$

$$\begin{aligned}\therefore \text{ar}(\triangle ABD) &= \frac{1}{2} \times BD \times AL \\ &= \frac{1}{2} \times \left(DC \times \frac{m}{n} \right) \times AL \\ &= \frac{m}{n} \times \left(\frac{1}{2} \times DC \times AL \right) \\ &= \frac{m}{n} \times \text{ar}(\triangle ADC)\end{aligned}$$

$$\Rightarrow \frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle ADC)} = \frac{m}{n}$$

$$\Rightarrow \text{ar}(\triangle ABD) : \text{ar}(\triangle ADC) = m : n$$