Angles, Lines and Triangles

Exercise 4A

Question 1:

(i) Angle: Two rays having a common end point form an angle.

(ii) Interior of an angle: The interior of ∠AOB is the set of all points in its plane, which lie on the same side of OA as B and also on same side of OB as A.

(iii) Obtuse angle: An angle whose measure is more than 90° but less than 180°, is called an obtuse angle.

(iv) Reflex angle: An angle whose measure is more than 180° but less than 360° is called a reflex angle.

(v) Complementary angles: Two angles are said to be complementary, if the sum of their measures is 90o.

(vi) Supplementary angles: Two angles are said to be supplementary, if the sum of their measures is 180°.

Question 2:

∠A = 36° 27′ 46″ and ∠B = 28° 43′ 39″

∴ Their sum = (36° 27′ 46″) + (28° 43′ 39″)

	Deg	Min	Sec
	36°	27′	46″
+	28°	43′	39″
	65°	11′	25″

[1° = 60′; 1′ = 60″]

Therefore, the sum ∠A + ∠B = 65° 11′ 25″

Question 3:

Let ∠A = 36° and ∠B = 24° 28′ 30″

Their difference = 36° – 24° 28′ 30″

	Deg	Min	Sec
	36°	0′	0″
–	24°	28′	30″
	11°	31′	30″

[1°=60′; 1′=60″]

Thus the difference between two angles is ∠A – ∠B = 11° 31′ 30″

Question 4:

(i) Complement of 58° = 90° – 58° = 32°

(ii) Complement of 16° = 90 – 16° = 74°

(iii) $\frac{2}{3}$ of a right angle = $\frac{2}{3}$ × 90° = 60°

Complement of 60° = 90° – 60° = 30°

(iv) 1° = 60′

⇒ 90° = 89° 60′

	Deg	Min
	89°	60′
	~~90°~~	~~0′~~
–	46°	30′
	43°	30′

Complement of 46° 30′ = 90° – 46° 30′ = 43° 30′

(v) 90° = 89° 59′ 60″

	Deg	Min	Sec
	89°	59′	60″
	~~90°~~	~~0′~~	~~0″~~
–	52°	43′	20″
	37°	16′	40″

Complement of 52° 43′ 20″ = 90° – 52° 43′ 20″

= 37° 16′ 40″

(vi) 90° = 89° 59′ 60″

	Deg	Min	Sec
	89°	59′	60″
	~~90°~~	~~0′~~	~~0″~~
–	68°	35′	45″
	21°	24′	15″

∴ Complement of (68° 35′ 45″)

= 90° – (68° 35′ 45″)

= 89° 59′ 60″ – (68° 35′ 45″)

= 21° 24′ 15″

Question 5:

(i) Supplement of 63° = 180° – 63° = 117°

(ii) Supplement of 138° = 180° – 138° = 42°

(iii) $\frac{3}{5}$ of a right angle = $\frac{3}{5}$ × 90° = 54°

∴ Supplement of 54° = 180° – 54° = 126°

(iv) 1° = 60′

⇒ 180° = 179° 60′

	Deg	Min
	179°	60′
	~~180°~~	~~0′~~
−	75°	36′
	104°	24′

Supplement of 75° 36′ = 180° – 75° 36′ = 104° 24′

(v) 1° = 60′, 1′ = 60″

⇒ 180° = 179° 59′ 60″

	Deg	Min	Sec
	179°	59′	60″
	~~180°~~	~~0′~~	~~0″~~
−	124°	20′	40″
	55°	39′	20″

Supplement of 124° 20′ 40″ = 180° – 124° 20′ 40″

= 55° 39′ 20″

(vi) 1° = 60′, 1′ = 60″

⇒ 180° = 179° 59′ 60″

	Deg	Min	Sec
	179°	59′	60″
	~~180°~~	~~0′~~	~~0″~~
−	108°	48′	32″
	71°	11′	28″

∴ Supplement of 108° 48′ 32″ = 180° – 108° 48′ 32″

= 71° 11′ 28″.

Question 6:

(i) Let the required angle be x^o

Then, its complement = $90^o - x^o$

$$\begin{aligned} \therefore \quad & x^o = 90^o - x^o \\ \Rightarrow \quad & x + x = 90 \\ \Rightarrow \quad & 2x = 90 \\ \Rightarrow \quad & x = \frac{90}{2} = 45 \end{aligned}$$

∴ The measure of an angle which is equal to its complement is 45°.

(ii) Let the required angle be x^o

Then, its supplement = $180^o - x^o$

$$\begin{aligned} \therefore \quad & x^o = 180^o - x^o \\ \Rightarrow \quad & x + x = 180 \\ \Rightarrow \quad & 2x = 180 \\ \Rightarrow \quad & x = \frac{180}{2} = 90 \end{aligned}$$

∴ The measure of an angle which is equal to its supplement is 90°.

Question 7:

Let the required angle be x^o

Then its complement is $90^o - x^o$

$$\begin{aligned} \Rightarrow \quad & x^o = \left(90^o - x^o\right) + 36^o \\ \Rightarrow \quad & x^o + x^o = 90^o + 36^o \\ \Rightarrow \quad & 2x^o = 126^o \\ \Rightarrow \quad & x = \frac{126}{2} = 63 \end{aligned}$$

∴ The measure of an angle which is 36° more than its complement is 63°.

Question 8:

Let the required angle be x^o

Then its supplement is $180^o - x^o$

$$\begin{aligned} \Rightarrow \quad & x^o = \left(180^o - x^o\right) - 25^o \\ \Rightarrow \quad & x^o + x^o = 180^o - 25^o \\ \Rightarrow \quad & 2x = 155 \\ \Rightarrow \quad & x = \frac{155}{2} = 77\frac{1}{2} \end{aligned}$$

∴ The measure of an angle which is 25° less than its supplement is $77\frac{1}{2}^{\circ} = 77.5^{\circ}$.

Question 9:

Let the required angle be x°

Then, its complement = 90° – x°

$$\Rightarrow \quad x^{\circ} = 4\left(90^{\circ} - x^{\circ}\right)$$
$$\Rightarrow \quad x^{\circ} = 360^{\circ} - 4x^{\circ}$$
$$\Rightarrow \quad 5x = 360$$
$$\Rightarrow \quad x = \frac{360}{5} = 72$$

∴ The required angle is 72°.

Question 10:

Let the required angle be x°

Then, its supplement is 180° – x°

$$\Rightarrow \quad x^{\circ} = 5\left(180^{\circ} - x^{\circ}\right)$$
$$\Rightarrow \quad x^{\circ} = 900^{\circ} - 5x^{\circ}$$
$$\Rightarrow \quad x + 5x = 900$$
$$\Rightarrow \quad 6x = 900$$
$$\Rightarrow \quad x = \frac{900}{6} = 150.$$

∴ The required angle is 150°.

Question 11:

Let the required angle be x°

Then, its complement is 90° – x° and its supplement is 180° – x°

That is we have,

$$180^{\circ} - x^{\circ} = 4(90^{\circ} - x^{\circ})$$
$$180^{\circ} - x^{\circ} = 360^{\circ} - 4x^{\circ}$$
$$4x^{\circ} - x^{\circ} = 360^{\circ} - 180^{\circ}$$
$$3x = 180$$
$$x = \frac{180}{3} = 60^{\circ}$$

∴ The required angle is 60°.

Question 12:

Let the required angle be x°

Then, its complement is 90° – x° and its supplement is 180° – x°

$$\therefore \quad 90^{\circ} - x^{\circ} = \frac{1}{3}\left(180^{\circ} - x^{\circ}\right)$$
$$\Rightarrow \quad 90 - x = 60 - \frac{1}{3}x$$
$$\Rightarrow \quad x - \frac{1}{3}x = 90 - 60$$
$$\Rightarrow \quad \frac{2}{3}x = 30$$
$$\Rightarrow \quad x = \frac{30 \times 3}{2} = 45$$

∴ The required angle is 45°.

Question 13:

Let the two required angles be x° and 180° – x°.

Then,

$$\frac{x^{\circ}}{180^{\circ} - x^{\circ}} = \frac{3}{2}$$

⇒ 2x = 3(180 – x)

⇒ 2x = 540 – 3x

⇒ 3x + 2x = 540

⇒ 5x = 540

⇒ x = 108

Thus, the required angles are 108° and 180° – x° = 180° – 108° = 72°.

Question 14:

Let the two required angles be x° and 90° – x°.

Then

$$\frac{x^\circ}{90^\circ - x^\circ} = \frac{4}{5}$$

⇒ 5x = 4(90 – x)

⇒ 5x = 360 – 4x

⇒ 5x + 4x = 360

⇒ 9x = 360

⇒ x = $\frac{360}{9}$ = 40

Thus, the required angles are 40° and 90° – x° = 90° – 40° = 50°.

Question 15:

Let the required angle be x°.

Then, its complementary and supplementary angles are (90° – x) and (180° – x) respectively.

Then, 7(90° – x) = 3 (180° – x) – 10°

⇒ 630° – 7x = 540° – 3x – 10°

⇒ 7x – 3x = 630° – 530°

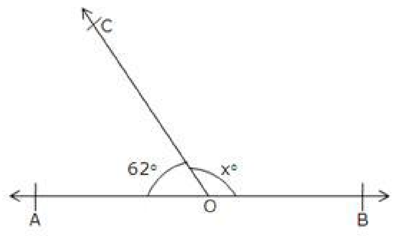
⇒ 4x = 100°

⇒ x = 25°

Thus, the required angle is 25°.

Exercise 4B

Question 1:



Since ∠BOC and ∠COA form a linear pair of angles, we have

∠BOC + ∠COA = 180°

⇒ x° + 62° = 180°

⇒ x = 180 – 62

∴ x = 118°

Question 2:

Since, ∠BOD and ∠DOA form a linear pair.

∠BOD + ∠DOA = 180°

∴ ∠BOD + ∠DOC + ∠COA = 180°

⇒ (x + 20)° + 55° + (3x – 5)° = 180°

⇒ x + 20 + 55 + 3x – 5 = 180

⇒ 4x + 70 = 180

⇒ 4x = 180 – 70 = 110

⇒ x = $\frac{110}{4}$ = 27.5

∴ ∠AOC = (3 × 27.5 – 5)° = 82.5-5 = 77.5°

And, ∠BOD = (x + 20)° = 27.5° + 20° = 47.5°.

Question 3:

Since ∠BOD and ∠DOA from a linear pair of angles.

⇒ ∠BOD + ∠DOA = 180^o

⇒ ∠BOD + ∠DOC + ∠COA = 180^o

⇒ $x^o + (2x - 19)^o + (3x + 7)^o = 180^o$

⇒ 6x – 12 = 180

⇒ 6x = 180 + 12 = 192

⇒ x = $\frac{192}{6}$ = 32

⇒ x = 32

⇒ ∠AOC = $(3x + 7)^o = (3\ 32 + 7)^o = 103^o$

⇒ ∠COD = $(2x - 19)^o = (2\ 32 - 19)^o = 45^o$

and ∠BOD = $x^o = 32^o$

Question 4:

x: y: z = 5: 4: 6

The sum of their ratios = 5 + 4 + 6 = 15

But x + y + z = 180^o

[Since, XOY is a straight line]

So, if the total sum of the measures is 15, then the measure of x is 5.

If the sum of angles is 180^o, then, measure of x = $\frac{5}{15} \times 180 = 60$

And, if the total sum of the measures is 15, then the measure of y is 4.

If the sum of the angles is 180^o, then, measure of y = $\frac{4}{15} \times 180 = 48$

And ∠z = 180^o – ∠x – ∠y

= $180^o - 60^o - 48^o$

= $180^o - 108^o = 72^o$

∴ x = 60, y = 48 and z = 72.

Question 5:

AOB will be a straight line, if two adjacent angles form a linear pair.

∴ ∠BOC + ∠AOC = 180^o

⇒ $(4x - 36)^o + (3x + 20)^o = 180^o$

⇒ 4x – 36 + 3x + 20 = 180

⇒ 7x – 16 = 180^o

⇒ 7x = 180 + 16 = 196

⇒ x = $\frac{196}{7}$ = 28

∴ The value of x = 28.

Question 6:

Since ∠AOC and ∠AOD form a linear pair.

∴ ∠AOC + ∠AOD = 180^o

⇒ 50^o + ∠AOD = 180^o

⇒ ∠AOD = $180^o - 50^o = 130^o$

∠AOD and ∠BOC are vertically opposite angles.

∠AOD = ∠BOC

⇒ ∠BOC = 130^o

∠BOD and ∠AOC are vertically opposite angles.

∴ ∠BOD = ∠AOC

⇒ ∠BOD = 50^o

Question 7:

Since ∠COE and ∠DOF are vertically opposite angles, we have,

∠COE = ∠DOF

⇒ ∠z = 50°

Also ∠BOD and ∠COA are vertically opposite angles.

So, ∠BOD = ∠COA

⇒ ∠t = 90°

As ∠COA and ∠AOD form a linear pair,

∠COA + ∠AOD = 180°

⇒ ∠COA + ∠AOF + ∠FOD = 180° [∠t = 90°]

⇒ t + x + 50° = 180°

⇒ 90° + x° + 50° = 180°

⇒ x + 140 = 180

⇒ x = 180 – 140 = 40

Since ∠EOB and ∠AOF are vertically opposite angles

So, ∠EOB = ∠AOF

⇒ y = x = 40

Thus, x = 40 = y = 40, z = 50 and t = 90

Question 8:

Since ∠COE and ∠EOD form a linear pair of angles.

⇒ ∠COE + ∠EOD = 180°

⇒ ∠COE + ∠EOA + ∠AOD = 180°

⇒ 5x + ∠EOA + 2x = 180

⇒ 5x + ∠BOF + 2x = 180

[∴ ∠EOA and BOF are vertically opposite angles so, ∠EOA = ∠BOF]

⇒ 5x + 3x + 2x = 180

⇒ 10x = 180

⇒ x = 18

Now ∠AOD = 2x° = 2 × 18° = 36°

∠COE = 5x° = 5 × 18° = 90°

and, ∠EOA = ∠BOF = 3x° = 3 × 18° = 54°

Question 9:

Let the two adjacent angles be 5x and 4x.

Now, since these angles form a linear pair.

So, 5x + 4x = 180°

⇒ 9x = 180°

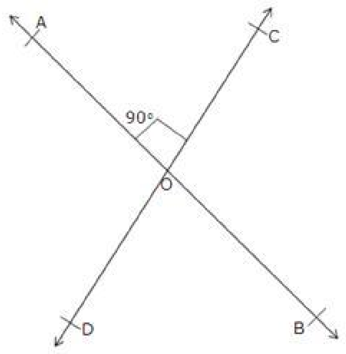
⇒ $x = \frac{180}{9} = 20$

∴ The required angles are 5x = 5x = 5 20° = 100°

and 4x = 4 × 20° = 80°

Question 10:

Let two straight lines AB and CD intersect at O and let ∠AOC = 90°.



Now, ∠AOC = ∠BOD [Vertically opposite angles]

⇒ ∠BOD = 90^o

Also, as ∠AOC and ∠AOD form a linear pair.

⇒ 90^o + ∠AOD = 180^o

⇒ ∠AOD = 180^o – 90^o = 90^o

Since, ∠BOC = ∠AOD [Verticallty opposite angles]

⇒ ∠BOC = 90^o

Thus, each of the remaining angles is 90^o.

Question 11:

Since, ∠AOD and ∠BOC are vertically opposite angles.

∴ ∠AOD = ∠BOC

Now, ∠AOD + ∠BOC = 280^o [Given]

⇒ ∠AOD + ∠AOD = 280^o

⇒ 2∠AOD = 280^o

⇒ ∠AOD = $\frac{280}{2}$ = 140^o

⇒ ∠BOC = ∠AOD = 140^o

As, ∠AOC and ∠AOD form a linear pair.

So, ∠AOC + ∠AOD = 180^o

⇒ ∠AOC + 140^o = 180^o

⇒ ∠AOC = 180^o – 140^o = 40^o

Since, ∠AOC and ∠BOD are vertically opposite angles.

∴ ∠AOC = ∠BOD

⇒ ∠BOD = 40^o

∴ ∠BOC = 140^o, ∠AOC = 40^o , ∠AOD = 140^o and ∠BOD = 40^o.

Question 12:

Since ∠COB and ∠BOD form a linear pair

So, ∠COB + ∠BOD = 180^o

⇒ ∠BOD = 180^o – ∠COB .... (1)

Also, as ∠COA and ∠AOD form a linear pair.

So, ∠COA + ∠AOD = 180^o

⇒ ∠AOD = 180^o – ∠COA

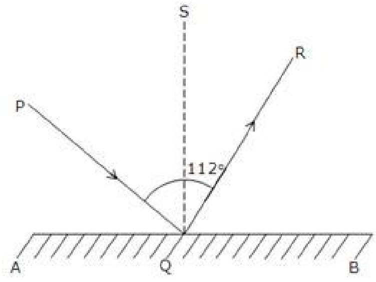
⇒ ∠AOD = 180^o – ∠COB .... (2)

[Since, OC is the bisector of ∠AOB, ∠BOC = ∠AOC]

From (1) and (2), we get,

∠AOD = ∠BOD (Proved)

Question 13:



Let QS be a perpendicular to AB.

Now, ∠PQS = ∠SQR

Because angle of incident = angle of reflection

⇒ ∠PQS = ∠SQR = $\frac{112}{2}$ = 56^o

Since QS is perpendicular to AB, ∠PQA and ∠PQS are complementary angles.

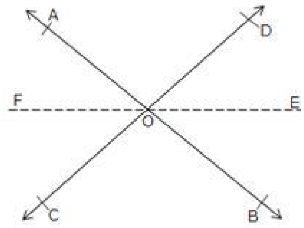
Thus, ∠PQA + ∠PQS = 90^o

⇒ ∠PQA + 56° = 90°

⇒ ∠PQA = 90° – 56° = 34°

Question 14:

Given : AB and CD are two lines which are intersecting at O. OE is a ray bisecting the ∠BOD. OF is a ray opposite to ray OE.



To Prove: ∠AOF = ∠COF

Proof : Since $\overrightarrow{OE}$ and $\overrightarrow{OF}$ are two opposite rays, $\overleftrightarrow{EF}$ is a straight line passing through O.

∴ ∠AOF = ∠BOE

and ∠COF = ∠DOE

[Vertically opposite angles]

But ∠BOE = ∠DOE (Given)

∴ ∠AOF = ∠COF

Hence, proved.

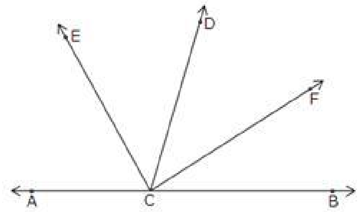
Question 15:

Given: $\overrightarrow{CF}$ is the bisector of ∠BCD and $\overrightarrow{CE}$ is the bisector of ∠ACD.

To Prove: ∠ECF = 90°

Proof: Since ∠ACD and ∠BCD forms a linear pair.

∠ACD + ∠BCD = 180°



∠ACE + ∠ECD + ∠DCF + ∠FCB = 180°

∠ECD + ∠ECD + ∠DCF + ∠DCF = 180°

because ∠ACE = ∠ECD

and ∠DCF = ∠FCB

2(∠ECD) + 2 (∠CDF) = 180°

2(∠ECD + ∠DCF) = 180°

∠ECD + ∠DCF = $\frac{180}{2}$ = 90°

∠ECF = 90° (Proved)

Exercise 4C

Question 1:

Since AB and CD are given to be parallel lines and t is a transversal.

So, ∠5 = ∠1 = 70° [Corresponding angles are equal]

∠3 = ∠1 = 70° [Vertically opp. Angles]

∠3 + ∠6 = 180° [Co-interior angles on same side]

∴ ∠6 = 180° – ∠3

= 180° – 70° = 110°

∠6 = ∠8 [Vertically opp. Angles]

⇒ ∠8 = 110°

⇒ ∠4 + ∠5 = 180° [Co-interior angles on same side]

∠4 = 180° – 70° = 110°

∠2 = ∠4 = 110° [Vertically opposite angles]

∠5 = ∠7 [Vertically opposite angles]

So, ∠7 = 70°

∴ ∠2 = 110°, ∠3 = 70° , ∠4 = 110°, ∠5 = 70°, ∠6 = 110°, ∠7 = 70° and ∠8 = 110°.

Question 2:

Since ∠2 : ∠1 = 5 : 4.

Let ∠2 and ∠1 be 5x and 4x respectively.

Now, ∠2 + ∠1 = 180° , because ∠2 and ∠1 form a linear pair.

So, 5x + 4x = 180°

⇒ 9x = 180°

⇒ x = 20°

∴ ∠1 = 4x = 4 × 20° = 80°

And ∠2 = 5x = 5 × 20° = 100°

∠3 = ∠1 = 80°[Vertically opposite angles]

And ∠4 = ∠2 = 100° [Vertically opposite angles]

∠1 = ∠5 and ∠2 = ∠6 [Corresponding angles]

So, ∠5 = 80° and ∠6 = 100°

∠8 = ∠6 = 100°[Vertically opposite angles]

And ∠7 = ∠5 = 80° [Vertically opposite angles]

Thus, ∠1 = 80°, ∠2 = 100°, ∠3 = ∠80°, ∠4 = 100°, ∠5 = 80°, ∠6 = 100°, ∠7 = 80° and ∠8 = 100°.

Question 3:

Given: AB || CD and AD || BC

To Prove: ∠ADC = ∠ABC

Proof: Since AB || CD and AD is a transversal. So sum of consecutive interior angles is 180°.

⇒ ∠BAD + ∠ADC = 180° ....(i)

Also, AD || BC and AB is transversal.

So, ∠BAD + ∠ABC = 180° ....(ii)

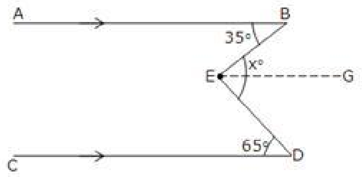
From (i) and (ii) we get:

∠BAD + ∠ADC = ∠BAD + ∠ABC

⇒ ∠ADC = ∠ABC (Proved)

Question 4:

(i) Through E draw EG || CD. Now since EG||CD and ED is a transversal.



So, ∠GED = ∠EDC = 65° [Alternate interior angles]

Since EG || CD and AB || CD,

EG||AB and EB is transversal.

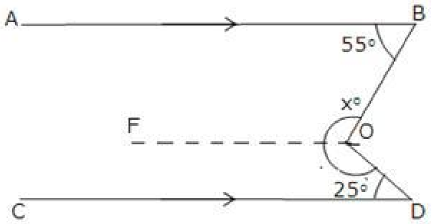
So, ∠BEG = ∠ABE = 35° [Alternate interior angles]

So, ∠DEB = x°

⇒ ∠BEG + ∠GED = 35° + 65° = 100°.

Hence, x = 100.

(ii) Through O draw OF||CD.



Now since OF || CD and OD is transversal.

∠CDO + ∠FOD = 180°

[sum of consecutive interior angles is 180°]

⇒ 25° + ∠FOD = 180°

⇒ ∠FOD = 180° – 25° = 155°

As OF || CD and AB || CD [Given]

Thus, OF || AB and OB is a transversal.

So, ∠ABO + ∠FOB = 180° [sum of consecutive interior angles is 180°]

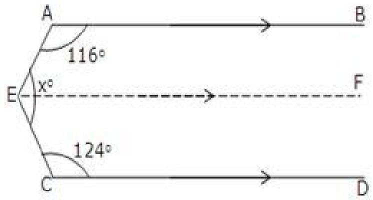
⇒ 55° + ∠FOB = 180°

⇒ ∠FOB = 180° – 55° = 125°

Now, x° = ∠FOB + ∠FOD = 125° + 155° = 280°.

Hence, x = 280.

(iii) Through E, draw EF || CD.



Now since EF || CD and EC is transversal.

∠FEC + ∠ECD = 180°

[sum of consecutive interior angles is 180°]

⇒ ∠FEC + 124° = 180°

⇒ ∠FEC = 180° – 124° = 56°

Since EF || CD and AB ||CD

So, EF || AB and AE is a trasveral.

So, ∠BAE + ∠FEA = 180°

[sum of consecutive interior angles is 180°]

∴ 116° + ∠FEA = 180°

⇒ ∠FEA = 180° – 116° = 64°

Thus, x° = ∠FEA + ∠FEC

= 64° + 56° = 120°.

Hence, x = 120.

Question 5:

Since AB || CD and BC is a transversal.

So, ∠ABC = ∠BCD [alternate interior angles]

⇒ 70° = x° + ∠ECD ....(i)

Now, CD || EF and CE is transversal.

So, ∠ECD + ∠CEF = 180° [sum of consecutive interior angles is 180°]

∴ ∠ECD + 130° = 180°

⇒ ∠ECD = 180° – 130° = 50°

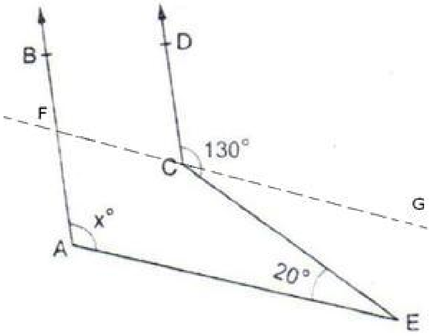
Putting ∠ECD = 50° in (i) we get,

$70^\circ = x^\circ + 50^\circ$

⇒ x = 70 – 50 = 20

Question 6:

Through C draw FG || AE



Now, since CG || BE and CE is a transversal.

So, ∠GCE = ∠CEA = 20° [Alternate angles]

∴ ∠DCG = 130° – ∠GCE

= $130^\circ - 20^\circ = 110^\circ$

Also, we have AB || CD and FG is a transversal.

So, ∠BFC = ∠DCG = 110° [Corresponding angles]

As, FG || AE, AF is a transversal.

∠BFG = ∠FAE [Corresponding angles]

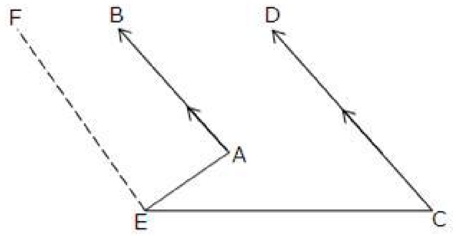
∴ x° = ∠FAE = 110°.

Hence, x = 110

Question 7:

Given: AB || CD

To Prove: ∠BAE – ∠DCE = ∠AEC



Construction : Through E draw EF || AB

Proof : Since EF || AB, AE is a transversal.

So, ∠BAE + ∠AEF = 180° ....(i)

[sum of consecutive interior angles is 180°]

As EF || AB and AB || CD [Given]

So, EF || CD and EC is a transversal.

So, ∠FEC + ∠DCE = 180° ....(ii)

[sum of consecutive interior angles is 180°]

From (i) and (ii) we get,

∠BAE + ∠AEF = ∠FEC + ∠DCE

⇒ ∠BAE – ∠DCE = ∠FEC – ∠AEF = ∠AEC [Proved]

Question 8:

Since AB || CD and BC is a transversal.

So, ∠BCD = ∠ABC = x° [Alternate angles]

As BC || ED and CD is a transversal.

$\angle BCD + \angle EDC = 180^o$

$\Rightarrow \angle BCD + 75^o = 180^o$

$\Rightarrow \angle BCD = 180^o - 75^o = 105^o$

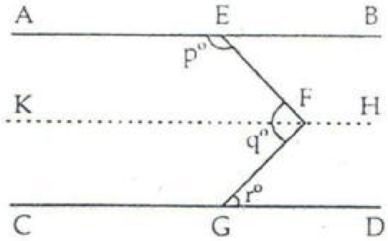
$\angle ABC = 105^o$ [since $\angle BCD = \angle ABC$]

$\therefore x^o = \angle ABC = 105^o$

Hence, x = 105.

Question 9:

Through F, draw KH || AB || CD



Now, KF || CD and FG is a transversal.

$\Rightarrow \angle KFG = \angle FGD = r^o$ .... (i)

[alternate angles]

Again AE || KF, and EF is a transversal.

So, $\angle AEF + \angle KFE = 180^o$

$\angle KFE = 180^o - p^o$ .... (ii)

Adding (i) and (ii) we get,

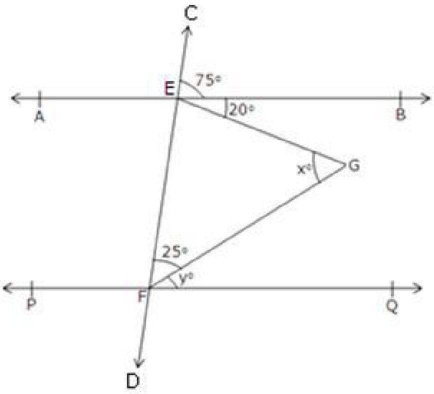
$\angle KFG + \angle KFE = 180 - p + r$

$\Rightarrow \angle EFG = 180 - p + r$

$\Rightarrow q = 180 - p + r$

i.e., $p + q - r = 180$

Question 10:



Since AB || PQ and EF is a transversal.

So, $\angle CEB = \angle EFQ$ [Corresponding angles]

$\Rightarrow \angle EFQ = 75^o$

$\Rightarrow \angle EFG + \angle GFQ = 75^o$

$\Rightarrow 25^o + y^o = 75^o$

$\Rightarrow y = 75 - 25 = 50$

Also, $\angle BEF + \angle EFQ = 180^o$ [sum of consecutive interior angles is 180^o]

$\angle BEF = 180^o - \angle EFQ$

$= 180^o - 75^o$

$\angle BEF = 105^o$

$\therefore \angle FEG + \angle GEB = \angle BEF = 105^o$

$\Rightarrow \angle FEG = 105^o - \angle GEB = 105^o - 20^o = 85^o$

In ΔEFG we have,

$x^o + 25^o + \angle FEG = 180^o$

$\Rightarrow x^\circ + 25^\circ + 85^\circ = 180^\circ$
$\Rightarrow x^\circ + 110^\circ = 180^\circ$
$\Rightarrow x^\circ = 180^\circ - 110^\circ$
$\Rightarrow x^\circ = 70^\circ$

Hence, x = 70.

Question 11:

Since AB || CD and AC is a transversal.

So, $\angle BAC + \angle ACD = 180^o$ [sum of consecutive interior angles is 180^o]

$\Rightarrow \angle ACD = 180^o - \angle BAC$

$= 180^o - 75^o = 105^o$

$\Rightarrow \angle ECF = \angle ACD$ [Vertically opposite angles]

$\angle ECF = 105^o$

Now in ΔCEF,

$\angle ECF + \angle CEF + \angle EFC = 180^o$

$\Rightarrow 105^o + x^o + 30^o = 180^o$

$\Rightarrow x = 180 - 30 - 105 = 45$

Hence, x = 45.

Question 12:

Since AB || CD and PQ a transversal.

So, $\angle PEF = \angle EGH$ [Corresponding angles]

$\Rightarrow \angle EGH = 85^o$

$\angle EGH$ and $\angle QGH$ form a linear pair.

So, $\angle EGH + \angle QGH = 180^o$

$\Rightarrow \angle QGH = 180^o - 85^o = 95^o$

Similarly, $\angle GHQ + 115^o = 180^o$

$\Rightarrow \angle GHQ = 180^o - 115^o = 65^o$

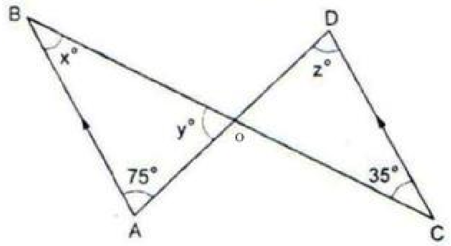
In ΔGHQ, we have,

$x^o + 65^o + 95^o = 180^o$

$\Rightarrow x = 180 - 65 - 95 = 180 - 160$

$\therefore x = 20$

Question 13:



Since AB || CD and BC is a transversal.

So, $\angle ABC = \angle BCD$

$\Rightarrow x = 35$

Also, AB || CD and AD is a transversal.

So, $\angle BAD = \angle ADC$

$\Rightarrow z = 75$

In ΔABO, we have,

$\angle AOB + \angle BAO + \angle BOA = 180^o$

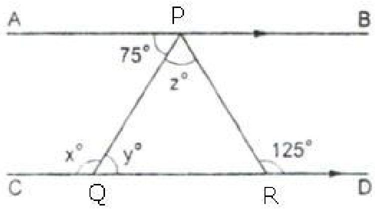
$\Rightarrow x^o + 75^o + y^o = 180^o$

$\Rightarrow 35 + 75 + y = 180$

$\Rightarrow y = 180 - 110 = 70$

∴ x = 35, y = 70 and z = 75.

Question 14:



Since AB || CD and PQ is a transversal.

So, y = 75 [Alternate angle]

Since PQ is a transversal and AB || CD, so x + APQ = 180^o

[Sum of consecutive interior angles]

⇒ $x^o = 180^o$ – APQ

⇒ x = 180 – 75 = 105

Also, AB || CD and PR is a transversal.

So, ∠APR = ∠PRD [Alternate angle]

⇒ ∠APQ + ∠QPR = ∠PRD [Since ∠APR = ∠APQ + ∠QPR]

⇒ $75^o + z^o = 125^o$

⇒ z = 125 – 75 = 50

∴ x = 105, y = 75 and z = 50.

Question 15:

∠PRQ = $x^o = 60^o$ [vertically opposite angles]

Since EF || GH, and RQ is a transversal.

So, ∠x = ∠y [Alternate angles]

⇒ y = 60

AB || CD and PR is a transversal.

So, ∠PRD = ∠APR [Alternate angles]

⇒ ∠PRQ + ∠QRD = ∠APR [since ∠PRD = ∠PRQ + ∠QRD]

⇒ x + ∠QRD = 110^o

⇒ ∠QRD = $110^o - 60^o = 50^o$

In ΔQRS, we have,

∠QRD + $t^o + y^o = 180^o$

⇒ 50 + t + 60 = 180

⇒ t = 180 – 110 = 70

Since, AB || CD and GH is a transversal

So, $z^o = t^o = 70^o$ [Alternate angles]

∴ x = 60 , y = 60, z = 70 and t = 70

Question 16:

(i) Lines l and m will be parallel if 3x – 20 = 2x + 10

[Since, if corresponding angles are equal, lines are parallel]

⇒ 3x – 2x = 10 + 20

⇒ x = 30

(ii) Lines will be parallel if $(3x + 5)^o + 4x^o = 180^o$

[if sum of pairs of consecutive interior angles is 180^o, the lines are parallel]

So, (3x + 5) + 4x = 180

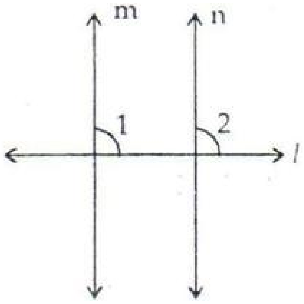
⇒ 3x + 5 + 4x = 180

⇒ 7x = 180 – 5 = 175

⇒ x = $\frac{175}{7}$ = 25

Question 17:

Given: Two lines m and n are perpendicular to a given line l.



To Prove: m || n

Proof : Since m ⊥ l

So, ∠1 = 90°

Again, since n ⊥ l

∠2 = 90°

∴ ∠1 = ∠2 = 90°

But ∠1 and ∠2 are the corresponding angles made by the transversal l with lines m and n and they are proved to be equal.

Thus, m || n.

Exercise 4D

Question 1:

Since, sum of the angles of a triangle is 180°

∠A + ∠B + ∠C = 180°

⇒ ∠A + 76° + 48° = 180°

⇒ ∠A = 180° – 124° = 56°

∴ ∠A = 56°

Question 2:

Let the measures of the angles of a triangle are (2x)°, (3x)° and (4x)°.

Then, 2x + 3x + 4x = 180 [sum of the angles of a triangle is 180°]

⇒ 9x = 180

⇒ x = $\frac{180}{9}$ = 20

∴ The measures of the required angles are:

2x = (2 × 20)° = 40°

3x = (3 × 20)° = 60°

4x = (4 × 20)° = 80°

Question 3:

Let 3∠A = 4∠B = 6∠C = x (say)

Then, 3∠A = x

⇒ ∠A = $\frac{x}{3}$

4∠B = x

⇒ ∠B = $\frac{x}{4}$

and 6∠C = x

⇒ ∠C = $\frac{x}{6}$

As ∠A + ∠B + ∠C = 180°

$$\Rightarrow \frac{x}{3} + \frac{x}{4} + \frac{x}{6} = 180$$
$$\Rightarrow \frac{4x + 3x + 2x}{12} = 180$$
$$\Rightarrow 9x = 180 \times 12$$
$$\Rightarrow x = \frac{180 \times 12}{9} = 240$$

$$\therefore \angle A = \frac{x}{3} = \frac{240}{3} = 80^\circ$$

$$\angle B = \frac{x}{4} = \frac{240}{4} = 60^\circ$$

$$\angle C = \frac{x}{6} = \frac{240}{6} = 40^\circ$$

Question 4:

$\angle A + \angle B = 108^\circ$ [Given]
But as $\angle A$, $\angle B$ and $\angle C$ are the angles of a triangle,
$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow 108^\circ + \angle C = 180^\circ$
$\Rightarrow C = 180^\circ - 108^\circ = 72^\circ$
Also, $\angle B + \angle C = 130^\circ$ [Given]
$\Rightarrow \angle B + 72^\circ = 130^\circ$
$\Rightarrow \angle B = 130^\circ - 72^\circ = 58^\circ$
Now as, $\angle A + \angle B = 108^\circ$
$\Rightarrow \angle A + 58^\circ = 108^\circ$
$\Rightarrow \angle A = 108^\circ - 58^\circ = 50^\circ$
$\therefore \angle A = 50^\circ$, $\angle B = 58^\circ$ and $\angle C = 72^\circ$.

Question 5:

Since. $\angle A$, $\angle B$ and $\angle C$ are the angles of a triangle .
So, $\angle A + \angle B + \angle C = 180^\circ$
Now, $\angle A + \angle B = 125^\circ$ [Given]
$\therefore 125^\circ + \angle C = 180^\circ$
$\Rightarrow \angle C = 180^\circ - 125^\circ = 55^\circ$
Also, $\angle A + \angle C = 113^\circ$ [Given]
$\Rightarrow \angle A + 55^\circ = 113^\circ$
$\Rightarrow \angle A = 113^\circ - 55^\circ = 58^\circ$
Now as $\angle A + \angle B = 125^\circ$
$\Rightarrow 58^\circ + \angle B = 125^\circ$
$\Rightarrow \angle B = 125^\circ - 58^\circ = 67^\circ$
$\therefore \angle A = 58^\circ$, $\angle B = 67^\circ$ and $\angle C = 55^\circ$.

Question 6:

Since, $\angle P$, $\angle Q$ and $\angle R$ are the angles of a triangle.
So, $\angle P + \angle Q + \angle R = 180^\circ$ ....(i)
Now, $\angle P - \angle Q = 42^\circ$ [Given]
$\Rightarrow \angle P = 42^\circ + \angle Q$ ....(ii)
and $\angle Q - \angle R = 21^\circ$ [Given]
$\Rightarrow \angle R = \angle Q - 21^\circ$ ....(iii)
Substituting the value of $\angle P$ and $\angle R$ from (ii) and (iii) in (i), we get,
$\Rightarrow 42^\circ + \angle Q + \angle Q + \angle Q - 21^\circ = 180^\circ$
$\Rightarrow 3\angle Q + 21^\circ = 180^\circ$
$\Rightarrow 3\angle Q = 180^\circ - 21^\circ = 159^\circ$
$\angle Q = \frac{159}{3} = 53^\circ$

∴ ∠P = 42° + ∠Q

= 42° + 53° = 95°

∠R = ∠Q – 21°

= 53° – 21° = 32°

∴ ∠P = 95°, ∠Q = 53° and ∠R = 32°.

Question 7:

Given that the sum of the angles A and B of a ABC is 116°, i.e., ∠A + ∠B = 116°.

Since, ∠A + ∠B + ∠C = 180°

So, 116° + ∠C = 180°

⇒ ∠C = 180° – 116° = 64°

Also, it is given that:

∠A – ∠B = 24°

⇒ ∠A = 24° + ∠B

Putting, ∠A = 24° + ∠B in ∠A + ∠B = 116°, we get,

⇒ 24° + ∠B + ∠B = 116°

⇒ 2∠B + 24° = 116°

⇒ 2∠B = 116° – 24° = 92°

∠B = $\frac{92}{2}$ = 46°

Therefore, ∠A = 24° + 46° = 70°

∴ ∠A = 70°, ∠B = 46° and ∠C = 64°.

Question 8:

Let the two equal angles, A and B, of the triangle be x° each.

We know,

∠A + ∠B + ∠C = 180°

⇒ x° + x° + ∠C = 180°

⇒ 2x° + ∠C = 180° ….(i)

Also, it is given that,

∠C = x° + 18°….(ii)

Substituting ∠C from (ii) in (i), we get,

⇒ 2x° + x° + 18° = 180°

⇒ 3x° = 180° – 18° = 162°

x = $\frac{162}{3}$ = 54°

Thus, the required angles of the triangle are 54°, 54° and x° + 18° = 54° + 18° = 72°.

Question 9:

Let ∠C be the smallest angle of ABC.

Then, ∠A = 2∠C and B = 3∠C

Also, ∠A + ∠B + ∠C = 180°

⇒ 2∠C + 3∠C + ∠C = 180°

⇒ 6∠C = 180°

⇒ ∠C = 30°

So, ∠A = 2∠C = 2 (30°) = 60°

∠B = 3∠C = 3 (30°) = 90°

∴ The required angles of the triangle are 60°, 90°, 30°.

Question 10:

Let ABC be a right angled triangle and ∠C = 90°

Since, ∠A + ∠B + ∠C = 180°

⇒ ∠A + ∠B = 180° – ∠C = 180° – 90° = 90°

Suppose ∠A = 53°

Then, $53^o + \angle B = 90^o$

$\Rightarrow \angle B = 90^o - 53^o = 37^o$

$\therefore$ The required angles are 53^o, 37^o and 90^o.

Question 11:

Let ABC be a triangle.

Given, $\angle A + \angle B = \angle C$

We know, $\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle C + \angle C = 180^o$

$\Rightarrow 2\angle C = 180^o$

$\Rightarrow \angle C = \frac{180}{2} = 90^o$

So, we find that ABC is a right triangle, right angled at C.

Question 12:

Given : ΔABC in which $\angle A = 90^o$, AL $\perp$ BC

To Prove: $\angle BAL = \angle ACB$

Proof :

In right triangle ΔABC,

$\Rightarrow \angle ABC + \angle BAC + \angle ACB = 180^o$

$\Rightarrow \angle ABC + 90^o + \angle ACB = 180^o$

$\Rightarrow \angle ABC + \angle ACB = 180^o - 90^o$

$\therefore \angle ABC + \angle ACB = 90^o$

$\Rightarrow \angle ACB = 90^o - \angle ABC$ ....(1)

Similarly since ΔABL is a right triangle, we find that,

$\angle BAL = 90^o - \angle ABC$ ...(2)

Thus from (1) and (2), we have

$\therefore \angle BAL = \angle ACB$ (Proved)

Question 13:

Let ABC be a triangle.

So, $\angle A < \angle B + \angle C$

Adding A to both sides of the inequality,

$\Rightarrow 2\angle A < \angle A + \angle B + \angle C$

$\Rightarrow 2\angle A < 180^o$ [Since $\angle A + \angle B + \angle C = 180^o$]

$\Rightarrow \angle A < \frac{180}{2} = 90^o$

Similarly, $\angle B < \angle A + \angle C$

$\Rightarrow \angle B < 90^o$

and $\angle C < \angle A + \angle B$

$\Rightarrow \angle C < 90^o$

ΔABC is an acute angled triangle.

Question 14:

Let ABC be a triangle and $\angle B > \angle A + \angle C$

Since, $\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle A + \angle C = 180^o - \angle B$

Therefore, we get

$\angle B > 180^o - \angle B$

Adding $\angle B$ on both sides of the inequality, we get,

$\Rightarrow \angle B + \angle B > 180^o - \angle B + \angle B$

$\Rightarrow 2\angle B > 180^o$

$\Rightarrow \angle B > \frac{180}{2} = 90^o$

i.e., $\angle B > 90^o$ which means $\angle B$ is an obtuse angle.

ΔABC is an obtuse angled triangle.

Question 15:

Since ∠ACB and ∠ACD form a linear pair.

So, $\angle ACB + \angle ACD = 180^\circ$

$\Rightarrow \angle ACB + 128^\circ = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 128 = 52^\circ$

Also, $\angle ABC + \angle ACB + \angle BAC = 180^\circ$

$\Rightarrow 43^\circ + 52^\circ + \angle BAC = 180^\circ$

$\Rightarrow 95^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 95^\circ = 85^\circ$

$\therefore \angle ACB = 52^\circ$ and $\angle BAC = 85^\circ$.

Question 16:

As ∠DBA and ∠ABC form a linear pair.

So, $\angle DBA + \angle ABC = 180^\circ$

$\Rightarrow 106^\circ + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - 106^\circ = 74^\circ$

Also, ∠ACB and ∠ACE form a linear pair.

So, $\angle ACB + \angle ACE = 180^\circ$

$\Rightarrow \angle ACB + 118^\circ = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 118^\circ = 62^\circ$

In ∠ABC, we have,

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$

$74^\circ + 62^\circ + \angle BAC = 180^\circ$

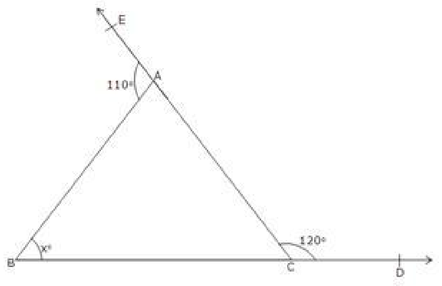
$\Rightarrow 136^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 136^\circ = 44^\circ$

$\therefore$ In triangle ABC, $\angle A = 44^\circ$, $\angle B = 74^\circ$ and $\angle C = 62^\circ$

Question 17:

(i) $\angle EAB + \angle BAC = 180^\circ$ [Linear pair angles]



$110^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 110^\circ = 70^\circ$

Again, $\angle BCA + \angle ACD = 180^\circ$ [Linear pair angles]

$\Rightarrow \angle BCA + 120^\circ = 180^\circ$

$\Rightarrow \angle BCA = 180^\circ - 120^\circ = 60^\circ$

Now, in ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

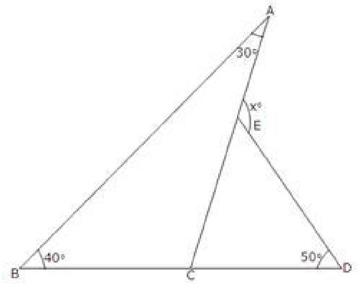
$x^\circ + 70^\circ + 60^\circ = 180^\circ$

$\Rightarrow x + 130^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 130^\circ = 50^\circ$

$\therefore x = 50$

(ii)



In ΔABC,

$\angle A + \angle B + \angle C = 180^o$

$\Rightarrow 30^o + 40^o + \angle C = 180^o$

$\Rightarrow 70^o + \angle C = 180^o$

$\Rightarrow \angle C = 180^o - 70^o = 110^o$

Now $\angle BCA + \angle ACD = 180^o$ [Linear pair]

$\Rightarrow 110^o + \angle ACD = 180^o$

$\Rightarrow \angle ACD = 180^o - 110^o = 70^o$

In ΔECD,

$\Rightarrow \angle ECD + \angle CDE + \angle CED = 180^o$

$\Rightarrow 70^o + 50^o + \angle CED = 180^o$

$\Rightarrow 120^o + \angle CED = 180^o$

$\angle CED = 180^o - 120^o = 60^o$

Since $\angle AED$ and $\angle CED$ from a linear pair

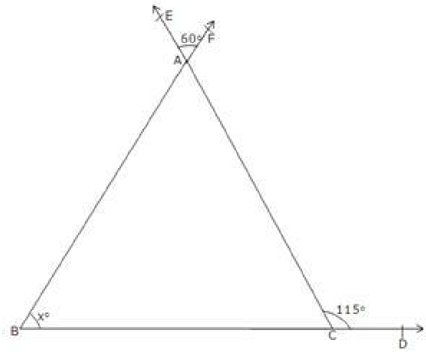
So, $\angle AED + \angle CED = 180^o$

$\Rightarrow x^o + 60^o = 180^o$

$\Rightarrow x^o = 180^o - 60^o = 120^o$

$\therefore x = 120$

(iii)



$\angle EAF = \angle BAC$ [Vertically opposite angles]

$\Rightarrow \angle BAC = 60^o$

In ΔABC, exterior $\angle ACD$ is equal to the sum of two opposite interior angles.

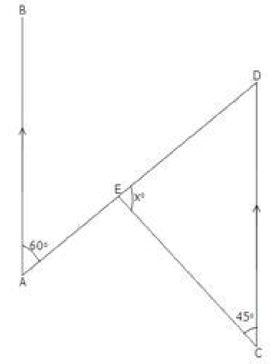
So, $\angle ACD = \angle BAC + \angle ABC$

$\Rightarrow 115^o = 60^o + x^o$

$\Rightarrow x^o = 115^o - 60^o = 55^o$

$\therefore x = 55$

(iv)



Since AB || CD and AD is a transversal.

So, ∠BAD = ∠ADC

⇒ ∠ADC = 60^{o}

In ∠ECD, we have,

∠E + ∠C + ∠D = 180^{o}

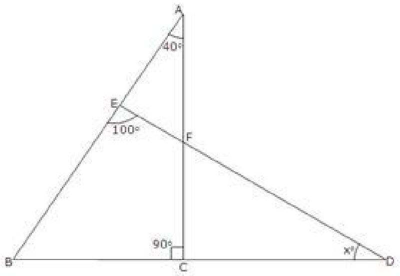
⇒ $x^{o} + 45^{o} + 60^{o} = 180^{o}$

⇒ $x^{o} + 105^{o} = 180^{o}$

⇒ $x^{o} = 180^{o} – 105^{o} = 75^{o}$

∴ x = 75

(v)



In ΔAEF,

Exterior ∠BED = ∠EAF + ∠EFA

⇒ $100^{o} = 40^{o}$ + ∠EFA

⇒ ∠EFA = $100^{o} – 40^{o} = 60^{o}$

Also, ∠CFD = ∠EFA [Vertically Opposite angles]

⇒ ∠CFD = 60^{o}

Now in ΔFCD,

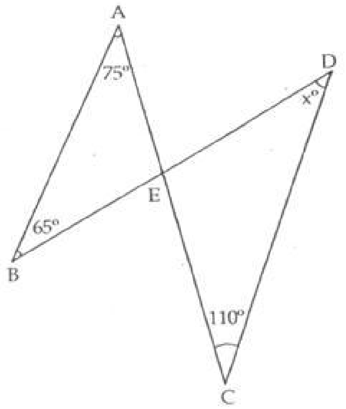
Exterior ∠BCF = ∠CFD + ∠CDF

⇒ $90^{o} = 60^{o} + x^{o}$

⇒ $x^{o} = 90^{o} – 60^{o} = 30^{o}$

∴ x = 30

(vi)



In ΔABE, we have,

∠A + ∠B + ∠E = 180°

⇒ 75° + 65° + ∠E = 180°

⇒ 140° + ∠E = 180°

⇒ ∠E = 180° – 140° = 40°

Now, ∠CED = ∠AEB [Vertically opposite angles]

⇒ ∠CED = 40°

Now, in ΔCED, we have,

∠C + ∠E + ∠D = 180°

⇒ 110° + 40° + x° = 180°

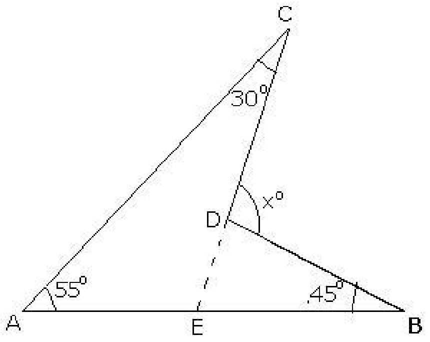
⇒ 150° + x° = 180°

⇒ x° = 180° – 150° = 30°

∴ x = 30

Question 18:

Produce CD to cut AB at E.



Now, in ΔBDE, we have,

Exterior ∠CDB = ∠CEB + ∠DBE

⇒ x° = ∠CEB + 45° .....(i)

In ΔAEC, we have,

Exterior ∠CEB = ∠CAB + ∠ACE

= 55° + 30° = 85°

Putting ∠CEB = 85° in (i), we get,

x° = 85° + 45° = 130°

∴ x = 130

Question 19:

The angle ∠BAC is divided by AD in the ratio 1 : 3.

Let ∠BAD and ∠DAC be y and 3y, respectively.

As BAE is a straight line,

∠BAC + ∠CAE = 180° [linear pair]

⇒ ∠BAD + ∠DAC + ∠CAE = 180°

⇒ y + 3y + 108° = 180°

⇒ 4y = 180° – 108° = 72°

⇒ y = $\frac{72}{4}$ = 18°

Now, in ΔABC,

∠ABC + ∠BCA + ∠BAC = 180°

y + x + 4y = 180°

[Since, ∠ABC = ∠BAD (given AD = DB) and ∠BAC = y + 3y = 4y]

⇒ 5y + x = 180

⇒ 5 × 18 + x = 180

⇒ 90 + x = 180

∴ x = 180 – 90 = 90

Question 20:

Given : A ΔABC in which BC, CA and AB are produced to D, E and F respectively.

To prove : Exterior ∠DCA + Exterior ∠BAE + Exterior ∠FBD = 360°

Proof : Exterior ∠DCA = ∠A + ∠B ....(i)

Exterior ∠FAE = ∠B + ∠C ....(ii)

Exterior ∠FBD = ∠A + ∠C ....(iii)

Adding (i), (ii) and (iii), we get,

Ext. ∠DCA + Ext. ∠FAE + Ext. ∠FBD

= ∠A + ∠B + ∠B + ∠C + ∠A + ∠C

= 2∠A + 2∠B + 2∠C

= 2 (∠A + ∠B + ∠C)

= 2 × 180°

[Since, in triangle the sum of all three angle is 180°]

= 360°

Hence, proved.

Question 21:

In ΔACE, we have,

∠A + ∠C + ∠E = 180° ....(i)

In ΔBDF, we have,

∠B + ∠D + ∠F = 180° ....(ii)

Adding both sides of (i) and (ii), we get,

∠A + ∠C +∠E + ∠B + ∠D + ∠F = 180° + 180°

⇒ ∠A + ∠B + ∠C + ∠D + ∠E + ∠F = 360°.

Question 22:

Given : In ΔABC, bisectors of ∠B and ∠C meet at O and ∠A = 70°

In ΔBOC, we have,

$\Rightarrow \angle BOC + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^\circ$

$\Rightarrow \angle BOC = 180^\circ - \frac{1}{2}\angle B - \frac{1}{2}\angle C$

$= 180^\circ - \frac{1}{2}(\angle B + \angle C)$

$= 180^\circ - \frac{1}{2}[180^\circ - \angle A]$

$[\because \angle A + \angle B + \angle C = 180^\circ]$

$= 180^\circ - \frac{1}{2}[180^\circ - 70^\circ]$
$= 180^\circ - \frac{1}{2} \times 110^\circ$

$\angle BOC + \angle OBC + \angle OCB = 180^o$
$= 180^o – 55^o = 125^o$
$\therefore \angle BOC = 125^o$.

Question 23:
We have a ΔABC whose sides AB and AC have been procued to D and E. A = 40^o and bisectors of $\angle CBD$ and $\angle BCE$ meet at O.
In ΔABC, we have,
Exterior $\angle CBD = C + 40^o$

$$\Rightarrow \angle CBO = \frac{1}{2} \text{ Ext. } \angle CBD = \frac{1}{2}(\angle C + 40^\circ) = \frac{1}{2}\angle C + 20^\circ$$

And exterior $\angle BCE = B + 40^o$

$$\Rightarrow \angle BCO = \frac{1}{2} \text{ Ext. } \angle BCE = \frac{1}{2}(\angle B + 40^\circ) = \frac{1}{2}\angle B + 20^\circ.$$

Now, in ΔBCO, we have,
$\angle BOC = 180^\circ - \angle CBO - \angle BCO$
$= 180^\circ - \frac{1}{2}\angle C - 20^\circ - \frac{1}{2}\angle B - 20^\circ$
$= 180^\circ - \frac{1}{2}\angle C - \frac{1}{2}\angle B - 20^\circ - 20^\circ$
$= 180^\circ - \frac{1}{2}(\angle B + \angle C) - 40^\circ$
$= 140^\circ - \frac{1}{2}(\angle B + \angle C)$
$= 140^\circ - \frac{1}{2}[180^\circ - \angle A]$

$= 140^\circ - 90^\circ + \frac{1}{2}\angle A$

$= 50^\circ + \frac{1}{2}\angle A$

$= 50^\circ + \frac{1}{2} \times 40^\circ$

$= 50^o + 20^o$
$= 70^o$
Thus, $\angle BOC = 70^o$

Question 24:
In the given ΔABC, we have,
$\angle A : \angle B : \angle C = 3 : 2 : 1$
Let $\angle A = 3x$, $\angle B = 2x$, $\angle C = x$. Then,
$\angle A + \angle B + \angle C = 180^o$
$\Rightarrow 3x + 2x + x = 180^o$
$\Rightarrow 6x = 180^o$

$\Rightarrow x = 30^o$

$\angle A = 3x = 3\ 30^o = 90^o$

$\angle B = 2x = 2\ 30^o = 60^o$

and, $\angle C = x = 30^o$

Now, in ΔABC, we have,

Ext $\angle ACE = \angle A + \angle B = 90^o + 60^o = 150^o$

$\angle ACD + \angle ECD = 150^o$

$\Rightarrow \angle ECD = 150^o - \angle ACD$

$\Rightarrow \angle ECD = 150^o - 90^o$ [since , AD $\perp$ CD, $\angle ACD = 90^o$]

$\Rightarrow \angle ECD = 60^o$

Question 25:

In ΔABC, AN is the bisector of $\angle A$ and AM $\perp$ BC.

Now in ΔABC we have;

$\angle A = 180^o - \angle B - \angle C$

$\Rightarrow \angle A = 180^o - 65^o - 30^o$

$= 180^o - 95^o$

$= 85^o$

Now, in ΔANC we have;

Ext. $\angle MNA = \angle NAC + 30^o$

$$= \frac{1}{2}\angle A + 30^\circ$$
$$= \frac{85^\circ}{2} + 30^\circ$$
$$= \frac{85^\circ + 60^\circ}{2}$$
$$= \frac{145^\circ}{2}$$

Therefore, $\angle MNA = \frac{145^\circ}{2}$

In Δ MAN. we have;

$\angle MAN = 180^o - \angle AMN - \angle MNA$

$= 180^o - 90^o - \angle MNA$ [since AM $\perp$ BC, $\angle AMN = 90^\circ$]

$= 90^\circ - \frac{145^\circ}{2}$ [since $\angle MNA = \frac{145^\circ}{2}$]

$= \frac{180^\circ - 145^\circ}{2}$

$= \frac{35^\circ}{2}$

$= 17.5^\circ$

Thus, $\angle MAN =$

Question 26:

(i) False (ii) True (iii) False (iv) False (v) True (vi) True.