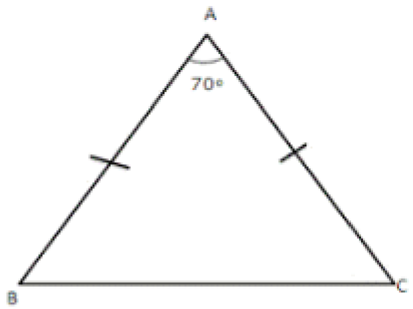
Class 9 Mathematics
RS Aggarwal Solutions
Chapter 5 Congruence of Triangles and Inequalities in a Triangle

Exercise 5A

Question 1:

AB=AC implies their opposite angle are equal



$\Rightarrow \quad \angle B = \angle C$ [angles opposite to equal sides]

But $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 70° + \angle B + \angle B = 180°$

$\Rightarrow 70° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 180° - 70°$

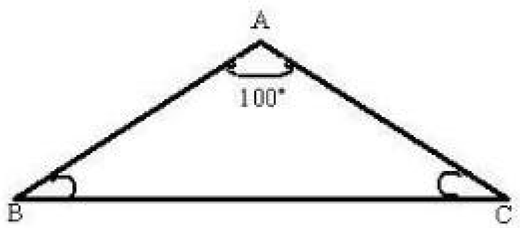
$\Rightarrow 2\angle B = 110°$

$\Rightarrow \angle B = \frac{110°}{2}$

$\Rightarrow \angle B = 55°$

$\Rightarrow \angle B = \angle C = 55°$

Question 2:



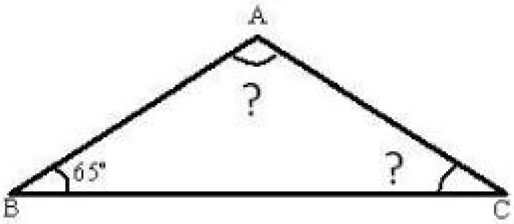
Consider the isosceles triangle ΔABC.

Since the vertical angle of ABC is 100° , we have, ∠A = 100°.

By angle sum property of a triangle, we have,

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\Rightarrow 100^\circ + \angle B + \angle C = 180^\circ$$
$$\Rightarrow 100^\circ + 2\angle B = 180^\circ \quad \text{[Since in an isosceles triangle base angles are equal, } \angle B = \angle C\text{]}$$
$$\Rightarrow 2\angle B = 180^\circ - 100^\circ = 80^\circ$$
$$\Rightarrow \angle B = \frac{80^\circ}{2}$$
$$\Rightarrow \angle B = 40^\circ$$
$$\Rightarrow \angle B = \angle C = 40^\circ$$

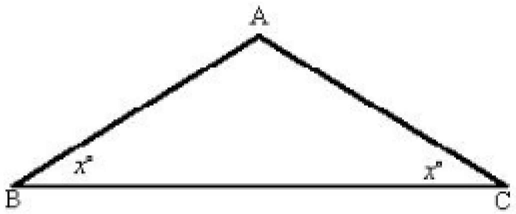
Question 3:



In $\triangle ABC$, if $AB = AC$
$\Rightarrow \triangle ABC$ is an isosceles triangle
$\Rightarrow$ Base angles are equal
$\Rightarrow \angle B = \angle C$
$\Rightarrow \angle C = 65^\circ$ [Since $\angle B = 65^\circ$]

Also by angle sum property, we have
$$\angle A + \angle B + \angle C = 180^\circ$$
$$\Rightarrow \angle A + 65^\circ + 65^\circ = 180^\circ \quad [\angle B = \angle C = 65^\circ]$$
$$\Rightarrow \angle A = 180^\circ - 130^\circ = 50^\circ$$

Question 4:



Let ABC be an isosceles triangle in which AB=AC.

Then we have $\angle B = \angle C$

Let $\angle B = \angle C = x$

Then vertex angle $A = 2(x+x) = 4x$

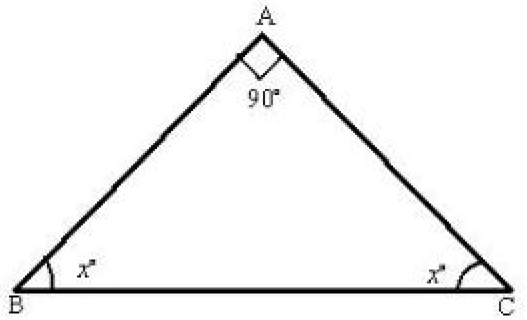
Now, $x + x + 4x = 180$

$$\Rightarrow 6x = 180$$
$$\Rightarrow x = \frac{180}{6} = 30$$

$\therefore$ Vertex $\angle A = 4 \times 30 = 120^0$

And, $\angle B = \angle C = 30^0$.

Question 5:



In a right angled isosceles triangle, the vertex angle is ∠A = 90° and the other two base angles are equal.

Let x° be the base angle and we have, ∠B = ∠C = 90°.

By angle sum property of a triangle, we have

$$\angle A + \angle B + \angle C = 180^\circ$$
$$\Rightarrow 90^\circ + x^\circ + x^\circ = 180^\circ$$
$$\Rightarrow 90^\circ + 2x^\circ = 180^\circ$$
$$\Rightarrow 2x^\circ = 180^\circ - 90^\circ$$
$$\Rightarrow 2x^\circ = 90^\circ$$
$$\Rightarrow x^\circ = \frac{90^\circ}{2}$$
$$\Rightarrow x^\circ = 45^\circ$$

Thus, we have, $\angle B = \angle C = 45^\circ$

Question 6:

Given: ABC is an isosceles triangle in which AB=AC and BC

Is produced both ways,

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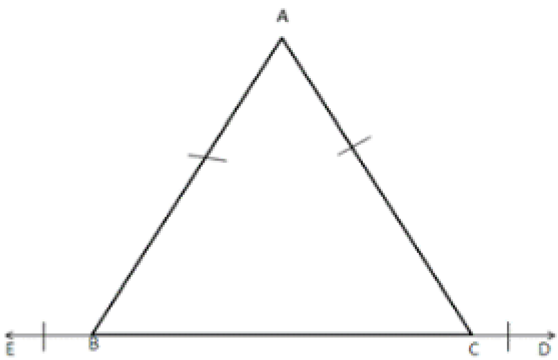


To Prove: ∠EBA=∠DCA

Proof : In ΔABC we have,

$$AB = AC$$
$$\Rightarrow \angle B = \angle C$$

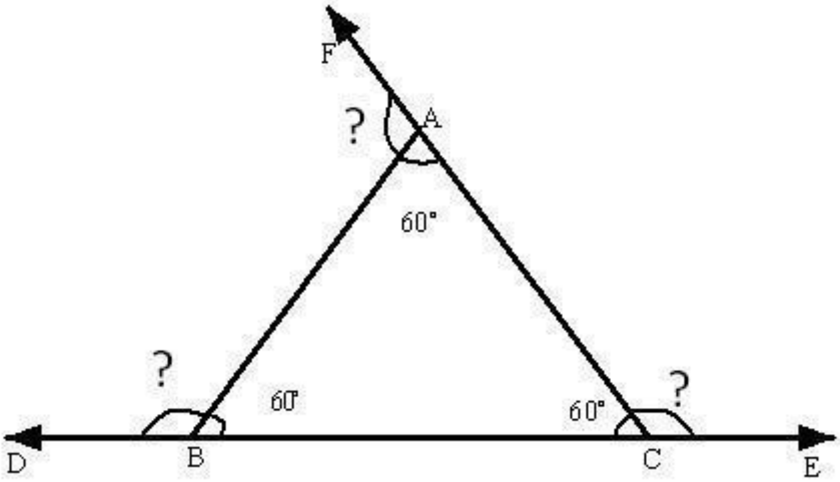
Now exterior ∠EBA=∠A+∠C = ∠A+∠B

[∴ ∠B =∠C]

and exterior ∠DCA =∠A+∠B

⇒ Exterior∠EBA =Exterior ∠DCA .

Question 7:



Let be an equilateral triangle.

Since it is an equilateral triangle, all the angles are equiangular and the measure of each angle is 60°

The exterior angle of ∠A is ∠BAF

The exterior angle of ∠B is ∠ABD

The exterior angle of ∠C is ∠ACE

We can observe that the angles ∠A and ∠BAF, ∠B and ∠ABD, ∠C and ∠ACE and form linear pairs.

Therefore, we have

$$\angle A + \angle BAF = 180^\circ$$
$$\Rightarrow 60^\circ + \angle BAF = 180^\circ$$
$$\Rightarrow \angle BAF = 180^\circ - 60^\circ$$
$$\Rightarrow \angle BAF = 120^\circ$$

Similarly, we have

$$\angle B + \angle ABD = 180^\circ$$
$$\Rightarrow 60^\circ + \angle ABD = 180^\circ$$
$$\Rightarrow \angle ABD = 180^\circ - 60^\circ$$
$$\Rightarrow \angle ABD = 120^\circ$$

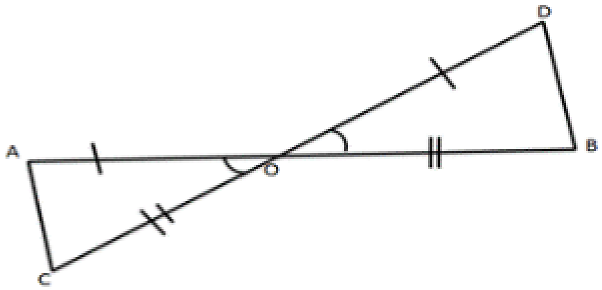
Also, we have

$$\angle C + \angle ACE = 180^\circ$$
$$\Rightarrow 60^\circ + \angle ACE = 180^\circ$$
$$\Rightarrow \angle ACE = 180^\circ - 60^\circ$$
$$\Rightarrow \angle ACE = 120^\circ$$

Thus, we have, ∠BAF = 120°, ∠ABD = 120°, ∠ACE = 120°

So, the measure of each exterior angle of an equilateral triangle is 120°.

Question 8:



Given: Two lines AB and CD intersect at O and O is the midpoint of AB and CD.

⇒ AO = OB and CO = OD

To prove: AC = BD and AC || BD

Proof: In △AOC and △BOD, we have,

AO = OB [Given: O is the midpoint of AB]

∠AOC = ∠BOD [Vertically opposite angles]

CO = OD [Given: O is the mipoint of CD]

So, by Side−Angle−Side congruence, we have, △AOC ≅ △BOD

The corresponding parts of the congruent triangles are equal.

Therefore, we have, AC = BD.

Similarly, by c.p.c.t, we have,

∠ACO = ∠BDO and

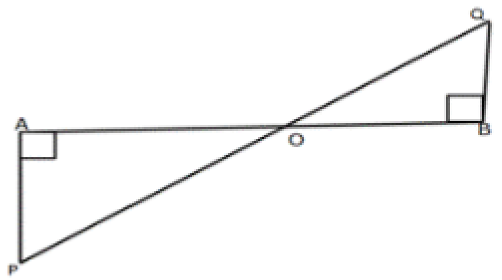
∠CAO = ∠DBO

This implies that alternate angles formed by AC and BD with transversal CD are equal. This means that, AC || BD.

Thus, AC = BD and AC || BD.

Question 9:

Given: $PA \perp AB$, $QB \perp AB$, and $PA = QB$
To Prove: $AO = OB$ and $PO = OQ$



Proof: In $\triangle APO$ and $\triangle BPO$,

$\angle PAO = \angle QBO = 90°$ [Given]

$PA = QB$ [Given]

$\angle PAO = \angle QBO$ [Since $PA \perp AB$, and $QB \perp AB$, $PA \parallel QB$, and thus PQ is a transversal, therefore, alternate angles are equal]

So, by Angle−Side−Angle criterion of congruence, we have

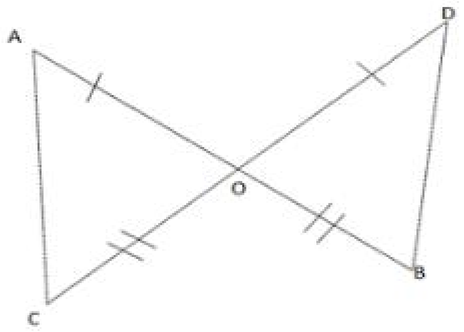
$\triangle APO \cong \triangle BPO$

$\Rightarrow$ $AO = OB$ and $PO = OQ$ [Since corresponding parts of congruent triangles are equal]

Thus, we have
O is the midpoint of AB and PQ.

Question 10:

Given: Line segments AB and CD intersect at O such that OA = OD and OB = OC.



To prove: AC = BD

Proof: In $\triangle AOC$ and $\triangle BOD$, we have

AO = OD [Given]

$\angle AOC = \angle BOD$ [Vertically opposite angles are equal]

OC = OB [Given]

So, by Side−Angle−Side criterion of congruence, we have,

$\Rightarrow \triangle AOC \cong \triangle BOD$

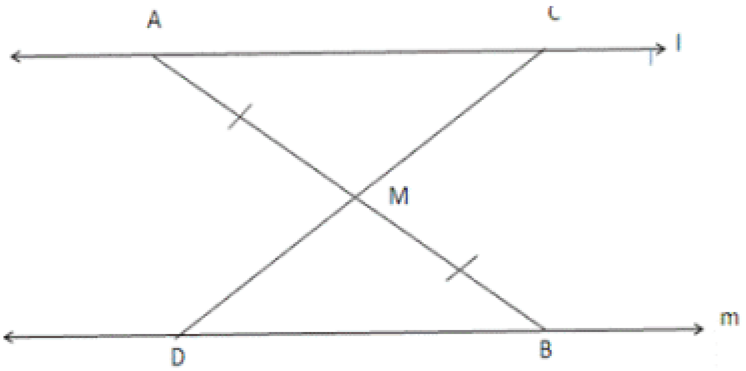
$\Rightarrow$ AC = BD [Since the corresponding parts of the congruent triangles are equal]

$\Rightarrow \angle CAO = \angle BDO$ [by c.p.c.t]

Thus, we have, AC = BD

In case $\angle ODB = \angle OBD$, then $\angle CAO = \angle OBD$ which means alternate angles made by lines AC and BD with transversal AB are equal and then lines AC and BD will be parallel.

Question 11:



Given: Two lines l and m are parallel to each other. M is the midpoint of segment AB. The line segment CD meets AB at M.

To prove: M is the midpoint of CD, that is CM = MD

Proof: In △AMC and △BMD, we have

∠MAC = ∠MBD [Since l and m are parallel, AB is the transversal, and thus, alternate angles are equal]

AM = MB [given]

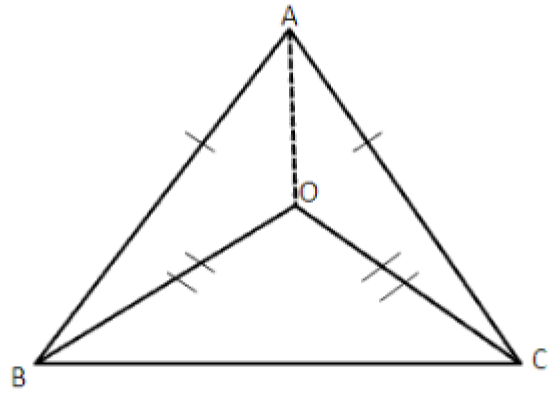
∠AMC = ∠BMD [vertically opposite angles are equal]

So, by Angle−Side−Angle criterion of congruence, we have

△AMC ≅ △BMD

Therefore, by corresponding parts of the congruent triangles are equal, we have, CM = MD

Question 12:



Given: AB = AC and O is an interior point of the triangle such that OB = OC

To prove: ∠ABO = ∠ACO

Construction: Join AO

Proof: In △AOB and △AOC, we have

AB = AC [Given]

AO = AO [Common]

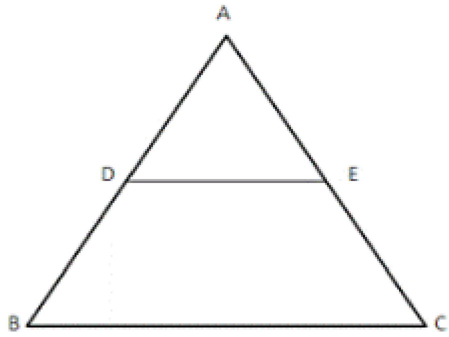
OB = OC [Given]

So, by Side−Side−Side criterion of congruence, we have,

△ABO ≅ △ACO

⇒∠ABO = ∠ACO [by corresponding parts of congruent triangles are equal]

Question 13:



Given: A ΔABC in which;

AB = AC

and, DE || BC

ToProve : AD = AE

Proof: Since DE || BC and AB is a transversal.

So, $\angle ADE = \angle ABC$ ...(i)

[∵ These are corresponding angles]

Also DE|| BC and AC is a transversal

So, $\angle AED = \angle ACB$ ...(ii)

[∵ these are corresponding angles]

But, AB = AC [Given]

So, $\angle ABC = \angle ACB$ ...(iii)

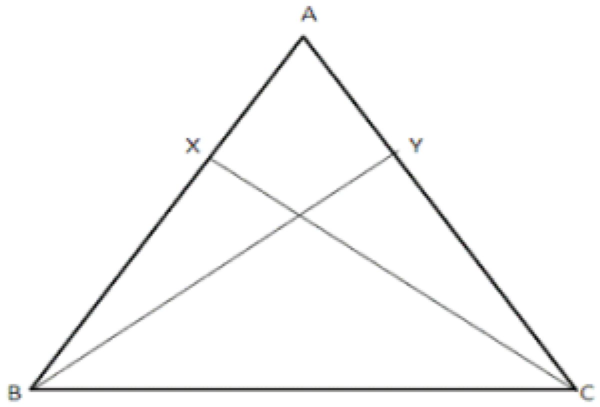
as oppsite angles are also equal in case sides are equal

So from (i), (ii) and (iii) we have

$\angle ADE = \angle AED$

and in ΔADE, this implies that AD = AE.

Question 14:



Given: AX = AY

To prove: CX = BY

Proof: In $\triangle AXC$ and $\triangle AYB$, we have

AX = AY [Given]

$\angle A = \angle A$ [Common angle]

AC = AB [Two sides are equal]

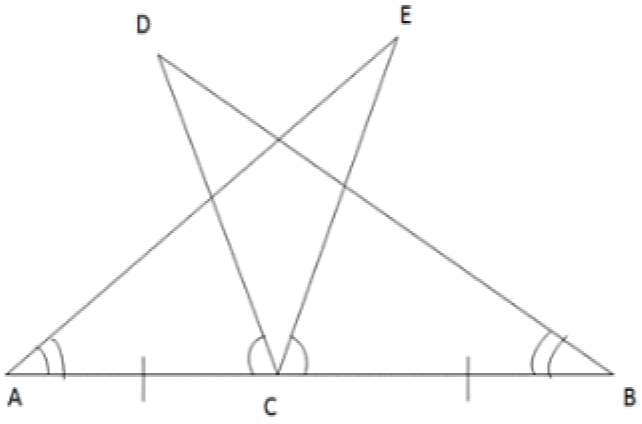
So, by Side−Angle−Side cirterion of congruence, we have

$\triangle AXC \cong \triangle AYB$

$\Rightarrow$ XC = YB [Since corresponding parts of congruent triangles are equal]

Question 15:

Given: C is the mid point of a line segment AB, and D is point such that,



$\angle DCA = \angle ECB$

and $\angle DBC = \angle EAC$

Toprove : $DC = EC$

Proof: In ΔACE and ΔDCB we have;

$AC = BC$ [Given]

$\angle EAC = \angle DBC$ [Given]

Also, $\angle DCA = \angle CDB + \angle DBA$ because exterior $\angle DCA$ in ΔDCB is equal to sum of interior opposite angles.

Again in $\angle ACE$, we have

ext.$\angle BCE = \angle CAE + \angle AEC$

But, $\angle DCA = \angle BCE$ [Given]

$\Rightarrow \angle CDB + \angle DBA = \angle CAE + \angle AEC$

$\Rightarrow \angle CDB = \angle AEC$ [$\because \angle DBA = \angle CAE$ (given)

Thus in ΔACE and ΔDCB,

$\angle EAC = \angle DBC$

$AC = BC$

and, $\angle AEC = \angle CDB$

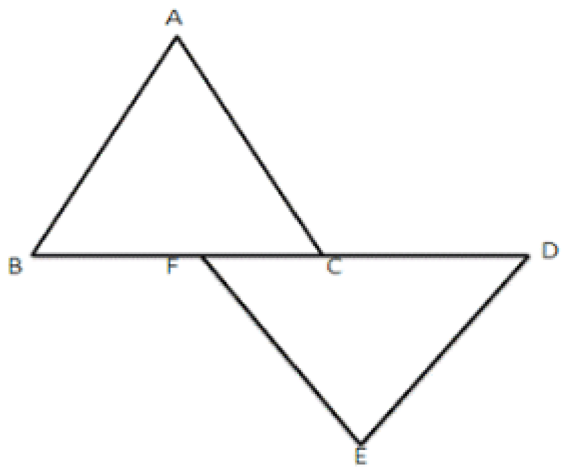
Thus by Angle-Side-Angle criterion of congruence, we have

$\Delta ACE \cong \Delta DCB$ (By ASA)

The corresponding parts of the congruent triangles are equal.

So, $DC = CE$ [by c.p.c.t]

Question 16:



Given: AB ⊥ AC and DE ⊥ FE such that ,

AB =DE and BF =CD

To prove : AC=EF

Proof: In ΔABC, we have,

BC = BF +FC

and , in ΔDEF

FD =FC +CD

But, BF =CD [Given]

So, BC = BF+ FC

and, FD = FC+BF

⇒ BC =FD

So, in ΔABC and ΔDEF, we have,

∠BAC= ∠DEF =90° [Given]

BC = FD [Proved above]

AB = DE [Given]

Thus, by Right angle-Hypotenuse-Side criterion of congruence, we have

ΔABC ≅ ΔDEF [By RHS]

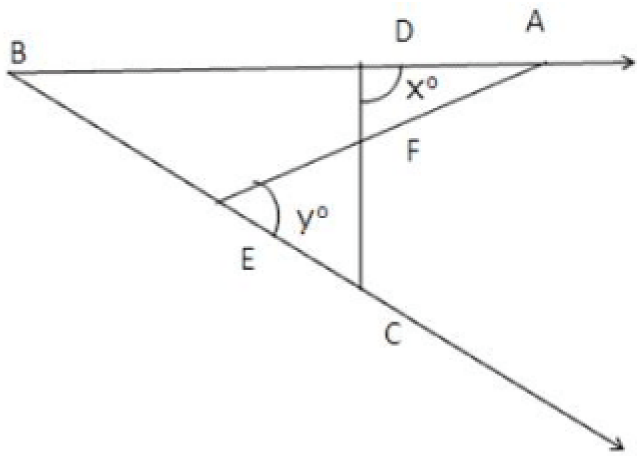
The corresponding parts of the congruent triangles are equal.

So, AC = EF [C.P.C.T]

Question 17:

Given: AB = BC

and, $x^\circ = y^\circ$



To prove: AE = CD

Proof: In ΔABE, we have,

Exterior∠AEB = ∠EBA + ∠BAE

⇒ y° = ∠EBA + ∠BAE

Again, in ΔBCD we have

x° = ∠CBA + ∠BCD

Since, x = y [Given]

So, ∠EBA + ∠BAE = ∠CBA + ∠BCD

⇒ ∠BAE = ∠BCD

Thus in ΔBCD and ΔBAE, we have

∠B = ∠B [Common]

BC = AB [Given]

and, ∠BCD = ∠BAE [Proved above]

Thus by Angle-Side-Angle criterion of congruence, we have

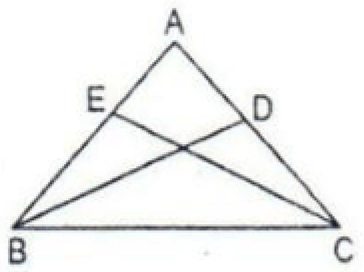
ΔBCD ≅ ΔBAE

The corresponding parts of the congruent triangles are equal.

So, CD = AE [Proved]

Question 18:

Given: A ΔABC in which $AB = AC$ and
BD and CE are the bisectors of $\angle B$ and $\angle C$ respectively.



To prove: $BD = CE$

Proof: In ΔABD and ΔACE

$$\angle ABD = \frac{1}{2}\ \angle B$$

and $$\angle ACE = \frac{1}{2}\ \angle C$$

But $\angle B = \angle C$ as $AB = AC$ [In Isosceles triangle, base angles are equal]

$\Rightarrow$ $\angle ABD = \angle ACE$

$AB = AC$ [Given]

$\angle A = \angle A$ [Common]

Thus by Angle-Side-Angle criterion of congruence, we have

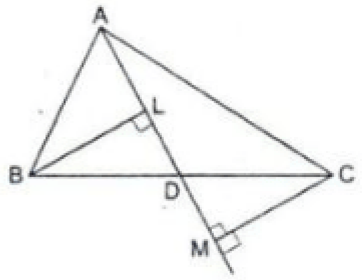
$\Delta ABD \cong \Delta ACE$ [By ASA]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $BD = CE$ [C.P.C.T]

Question 19:

Given: A Δ in which D is the mid point of BC and $BL \perp AD$ and $CM \perp AD$.



To Prove: BL = CM

Proof: In ΔBLD and ΔCMD

$\angle BLD = \angle CMD = 90^0$ [Given]

$\angle BDL = \angle MDC$ [Vertically opposite angles]

? BD = DC [Given]

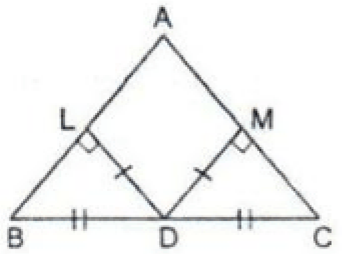
Thus by Angle-Angle-Side criterion of congruence, we have

$\Delta BLD = \Delta CMD$ [By AAS]

The corresponding parts of the congruent triangles are equal

So, BL = CM [C.P.C.T]

Question 20:



Given: In a ΔABC, D is the mid point of BC and $DL \perp AB$ and $DM \perp AC$. Also, $DL = DM$

To prove: $AB = AC$

Proof: In right angled triangles ΔBLD and ΔCMD

$\angle BLD = \angle CMD = 90^0$

Hypt.BD = Hypt.CD [Given]

DL = DM [Given]

Thus, by Right Angle-Hypotenuse-Side criterion of congruence, we have

$\Delta BLD = \Delta CMD$ [By RHS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $\angle ABD = \angle ACD$ [C.P.C.T]

In ΔABC, we have

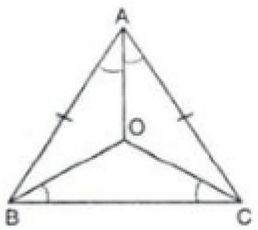
$\angle ABD = \angle ACD$

$\Rightarrow$ AB = AC

[$\therefore$ sides opposite to equal angles are equal]

Question 21:

Given : A ΔABC in which $AB = AC$, BO and CO are bisectors of $\angle B$ and $\angle C$



To Prove : In ΔBOC, we have,

$$\angle OBC = \frac{1}{2}\angle B$$

and , $\angle OCB = \frac{1}{2}\angle C$

But, $\angle B = \angle C$ [$\therefore$ AB= AC (given)]

So, $\angle OBC = \angle OCB$

Since base angles are equal, sides are equal

$\Rightarrow$ OB = OC ....(1)

Since OB and OC are the bisectors of angles,

$\angle B$ and $\angle C$ respectively, we have

$$\angle ABO = \frac{1}{2}\angle B$$

$$\angle ACO = \frac{1}{2}\angle C$$

$\Rightarrow \angle ABO = \angle ACO$ .....(2)

Now, in ΔABO and ΔACO

AB = AC [Given]

$\angle ABO = \angle ACO$ [from (2)]

BO = OC [from (1)]

Thus, by Side-Angle-Side criterion of congruence, we have

$\Delta ABO \cong \Delta ACO$ [By SAS]

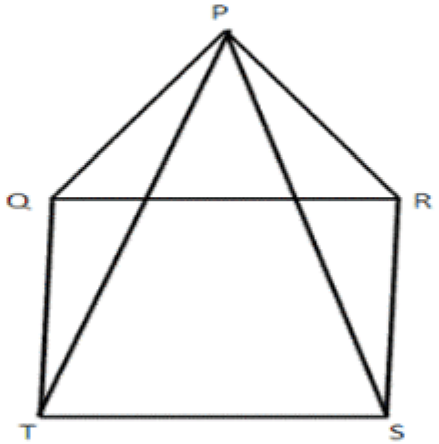
The corresponding parts of the congruent triangles are equal

$\therefore$ $\angle BAO = \angle CAO$ [By cpct]

i.e. AO bisects $\angle A$.

Question 22:

Given: PQR is an equilateral triangle and QRST is a square.



To Prove: PT = PS

and $\angle PSR = 15^0$

Proof: Since ΔPQR is an equilateral triangle,

$\angle PQR = 60^0$ and $\angle PRQ = 60^0$

Since QRTS is a square,

$\angle RQT = 90^0$ and $\angle QRS = 90^0$

In ΔPQT

$$\angle PQT = \angle PQR + \angle RQT$$
$$= 60^0 + 90^0$$
$$= 150^0$$

In ΔPRS

$\angle PRS = \angle PRQ + \angle QRS$

$= 60^0 + 90^0 = 150^0$......(1)

$\Rightarrow \quad \angle PQT = \angle PRS$ ......(2)

Thus, in ΔPQT and ΔPRS

$PQ = PR$ [sides of equilateral triangle ΔPQR]

$\angle PQT = \angle PRS$ [from (2)]

$QT = RS$ [sides of square ▭QRST]

Thus, by Side-Angle-Side criterion of congruence, we have

$\therefore \quad \Delta PQT \cong \Delta PRS$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore \quad PT = PS$ [C.P.C.T]

Now in Δ PRS , we have

$PR = RS$

$\Rightarrow \quad \angle RPS = \angle PSR$

But $\angle PRS = 150^0$ [from (1)]

So, by angle sum property in ΔPRS

$\angle PRS + \angle SPR + \angle PSR = 180^0$

$\Rightarrow \quad 150^0 + \angle PSR + \angle PSR = 180^0$

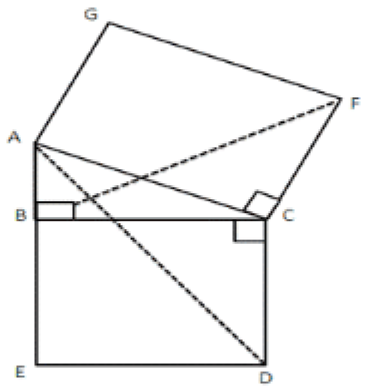
$\Rightarrow \quad 2\angle PSR = 180^0 - 150^0$

$\Rightarrow \quad 2\angle PSR = 30^0$

$\Rightarrow \quad \angle PSR = \frac{30}{2} = 15^0$

Question 23:

Given : ABC is atriangle , right angled at B. ACFG is a a square and BCDE is a square.



To prove: AD= EF

Proof: Since BCDE is a square,

$\angle BCD = 90^0$.....(1)

In ΔACD,

$\angle ACD = \angle ACB + \angle BCD$

$= \angle ACB + 90^0$ .....(2)

In ΔBCF,

$\angle BCF = \angle BCA + \angle ACF$

Since ACFG is a square,

$\angle ACF = 90^0$

Thus, we have

$\angle BCF = \angle BCA + 90^0$ ......(3)

From (2) and (3), we have

$\angle ACD = \angle BCF$ ......(4)

Thus in ΔACD and ΔBCF, we have

AC = CF [sides of a square]

$\angle ACD = \angle BCF$ [from (4)]

CD=BC [sides of a square]

Thus, by Side-Angle-Side criterion of congruence, we have

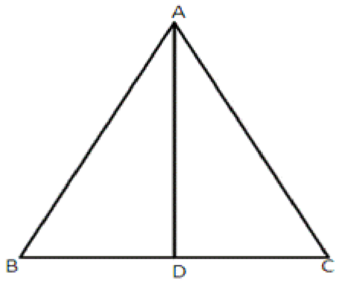
$\therefore \Delta ACD \cong \Delta BCF$ [By SAS]

The corresponding parts of congruent triangles are equal.

So, AD= BF (C.P.C.T)

Question 24:

Given : ABC is an isosceles triangle in which AB = AC and AD is the median through A.



To prove: ∠BAD = ∠DAC

Proof: In ΔABD and ΔADC

AB = AC [Given]

BD = DC [Given]

AD = AD [Common]

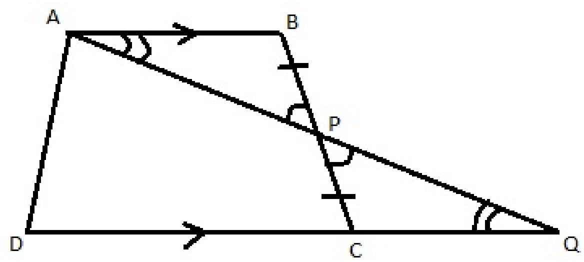
Thus by Side-Side-Side criterion of congruence, we have

ΔABD ≅ ΔADC [By SSS]

The corresponding parts of the congruent triangles are equal.

∴ ∠BAD = ∠DAC (Proved)

Question 25:



Given ABCD is a quadrilateral in which AB || DC

To Prove: (i) AB = CQ

(ii) DQ = DC+AB

Proof: In ΔABP and ΔPCQ we have

∠PAB = ∠PQC [alternate angles]

∠APB = ∠CPQ [Vertically opposite angles]

BP = PC [Given]

Thus by Angle-Angle-Side criterion of congruence, we have

ΔABP ≅ ΔPCQ

The corresponding parts of the congruent triangles are equal

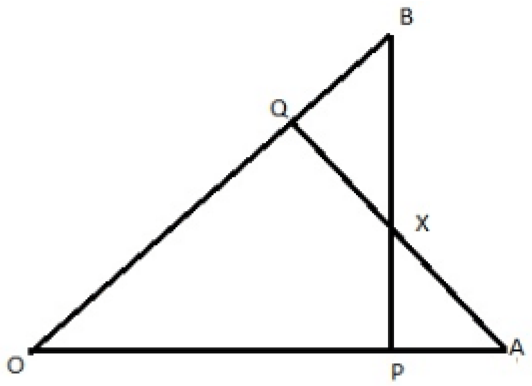
∴ AB = CQ .....(1)

Now, DQ = DC + CQ

= DC + AB [from (1)]

Question 26:

Given : OA = OB and OP = OQ



To Prove: (i) PX = QX

(ii) AX = BX

Proof: In ΔOAQ and ΔOPB, we have,

$OA = OB$ [Given]

$\angle O = \angle O$ [Common]

$OQ = OP$ [Given]

Thus by Side-Angle-Side criterion of congruence, we have

$\Delta OAQ = \Delta OPB$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore \angle OBP = \angle OAQ$ ....(1)

Thus , in ΔBXQ and ΔPXA, we have

$BQ = OB - OQ$

and, $PA = OA - OP$

But, $OP = OQ$

and $OA = OB$ [Given]

Therefore, we have, $BQ = PA$ ....(2)

Now consider triangles ΔBXQ and ΔPXA.

$\angle BXQ = \angle PXA$ [Vertical opposite angles]

$\angle OBP = \angle OAQ$ [from (1)]

$BQ = PA$ [from (2)]

Thus by Angle-Angle-Side criterion of congruence, we have,

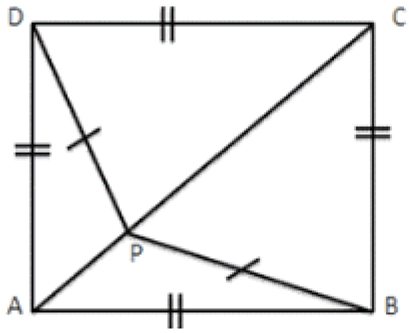
$\therefore \Delta BXQ \cong \Delta PXA$

$PX = QX$ [C.P.C.T]

$AX = BX$ [C.P.C.T]

Question 27:

Given: ABCD is a sqaure and P is a point inside it such that PB =PD



To Prove: CPA is a straight line.

Proof : In ΔAPD and ΔAPB

DA= AB [∴ ABCD is a square]

AP =AP [Common]

and, PB =PD [Given]

Thus by Side-Side-Side criterion of congruence, we have

$\Delta APD \cong \Delta APB$

The corresponding parts of the congruent triangles are equal.

∴ $\angle APD = \angle APB$ ....(i)

Now consider the triangles, ΔCPD and ΔCPB.

CD = CB [∴ ABCD is a square]

CP = CP [Common]

and, PB = PD [Given]

Thus by Side-Side-Side criterion of congruence, we have

$\Delta CPD \cong \Delta CPB$

The corresponding parts of the congruent triangles are equal.

Hence we have

$\angle CPD = \angle CPB$ ......(ii)

Adding both sides of (i) and (ii) we get

$\angle APD + \angle CPD = \angle APB + \angle CPB$ ......(iii)

Angles around the point P add upto 360°,

$\Rightarrow \angle APD + \angle CPD + \angle APB + \angle CPB = 360^\circ$

$\Rightarrow \angle APB + \angle CPB = 360^\circ - (\angle APD + \angle CPD)$ ...(iv)

Substituting (iv) in (iii) we get,

$\angle APD + \angle CPD = 360^\circ - (\angle APD + \angle CPD)$

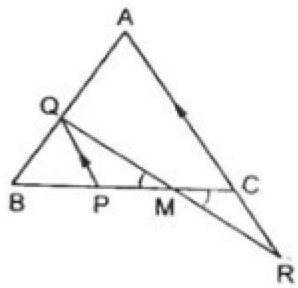
i.e $2(\angle APD + \angle CPD) = 360^\circ$

$\angle APD + \angle CPD = \frac{360}{2} = 180^\circ$

This proves that CPA is a straight line.

Question 28:

A ΔABC which is an equilateral triangle and PQ||AC.
AC is produced to R such that CR=BP



To Prove: PM=MC

Proof: Let QR intersects PC at M.

Since ΔABC is an equilateral triangle,

$\Rightarrow \angle A = \angle ACB = 60°$

Since PQ $\parallel$ AC and corresponding angles are equal.

$\Rightarrow \angle BPQ = \angle ACB = 60°$

In ΔBPQ, $\angle B = \angle ACB = 60°$

$\Rightarrow \angle BQP = 60°$

$\Rightarrow \Delta BPQ$ is an equilateral triangle

$\Rightarrow PQ = BP = BQ$

Since BP = CR, we have,

$PQ = CR$ ......(1)

Consider the triangles ΔPMQ and ΔCMR.

Since PQ || AC and QR is a transversal

So, $\angle PQM = \angle CRM$ [alternate angles]

$\angle PMQ = \angle CMR$ [vertically opposite angles]

$PQ = CR$ [from (1)]

Thus by Angle-Angle-Side criterion of congruence, we have

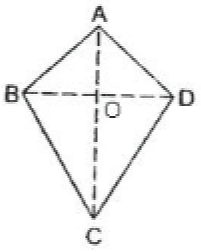
$\Delta PMQ \cong \Delta CMR$ [By AAS]

The corresponding parts of the congruent triangles are equal.

So, PM = MC [C.P.C.T](proved)

Question 29:

Given: a quarilateral ABCD in which AB=AD and BC=DC



To Prove: (i) AC bisects ∠A and ∠C

(ii) AC ⊥ BD and AC bisects BD

Proof: In ΔABC and ΔADC,we have

AB=AD [Given]

BC=DC [Given]

AC=AC [Common]

Thus by Side-Side-Side criterion of congruence,

ΔABC ≅ ΔADC [By SSS]

The corresponding parts of the congruent triangles are equal.

So, ∠BAC =∠DAC [C.P.C.T]

⇒ ∠BAO =∠DAO .....(1)

It means that AC bisects ∠BAD, that is ∠A

Also, ∠BCA=∠DCA [C.P.C.T]

It means that AC bisects ∠BCD, that is ∠C

(ii)

Now in ΔABO and ΔADO

AB =AD [Given]

∠BAO =∠DAO [from (1)]

AO =AO [Common]

Thus, by Side-Angle-Side criterion of congruence, we have

ΔABO ≅ ΔADO [By SAS]

The corresponding parts of the congruent triangles are equal.

∴ ∠BOA =∠DOA

But ∠BOA+∠DOA = 180°

Or 2∠BOA = 180°

$$\Rightarrow \quad \angle BOA = \frac{180^\circ}{2} = 90^\circ$$

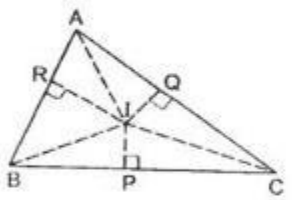
Also, as ΔABO ≅ ΔADO

So, BO = OD

which means that AC bisects BD.

Question 30:

Given: A triangle ABC in which bisectors of ∠B and ∠C meet at I.



Also, we have IP ⊥ BC, IQ ⊥ CA and IR ⊥ AB

To Prove: (i) IP=IQ=IR

(ii) ∠IAR=∠IAQ

Proof: (i) In ΔBIP and ΔBIR we have,

∠PBI = ∠RBI [Given]

∠IRB = ∠IPB= 90° [Given]

and, IB = IB [Common]

Thus by Angle-Angle-Side criterion of congruence, we have

∴ ΔBIP ≅ ΔBIR [By AAS]

The corresponding parts of the congruent triangles are equal.

So, IP = IR

Similarly IP = IQ

∴ IP = IQ = IR

(ii) Now in ΔAIR and ΔAIQ we have

IR = IQ [Proved above]

IA = IA [Common]

and, ∠IRA = ∠IQA = 90°

Thus by Side-Angle-Side criterion of congruence, we have

∴ ΔAIR ≅ ΔAIQ [By SAS]

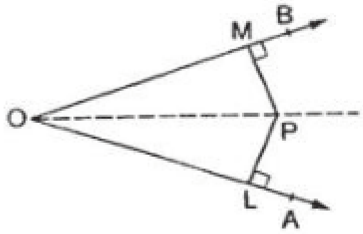
The corresponding parts of the congruent triangles are equal.

So, ∠IAR = ∠IAQ [by c.p.c.t]

⇒ IA bisects ∠A

Question 31:

Given: An angle AOB and P is a point in the interior of ∠AOB such that PL=PM. Also PL ⊥ OA and PM ⊥ OB



To Prove: ∠POL = ∠POM

Proof: In ΔOPL and ΔOPM, we have

∠OMP = ∠OLP = 90° [Given]

OP = OP [Common

PL = PM [Given]

Thus, by Right angle-Hypotenuse-Side criterion of congruence, we have

ΔOPL ≅ ΔOPM [By R.H.S]

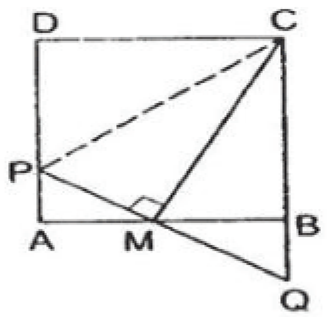
The corresponding parts of the congruent triangles are equal.

∴ ∠POL = ∠POM [C.P.C.T]

⇒ OP is the bisector of ∠LOM=∠AOB

Question 32:

Given M is the mid-point of side AB of a square ABCD and CM ⊥ PQ



To Prove : (i) PA = BQ

(ii) CP = AB+PA

Proof: (i) In ΔAMP and ΔBMQ

∠AMP = ∠BMQ [Vertically opposite angle]

∠PAM = ∠MBQ= 90° [∴ ABCD is a square]

and AM = MB [Given]

Thus by Angle-Angle-Side criterion of congruence, we have

ΔAMP ≅ ΔBMQ [By AAS]

The corresponding parts of the congruent triangles are equal.

∴ PA= BQ and MP =MQ .....(1)

(ii) Now ΔPCM and ΔQCM, we have

PM=QM [from (1)]

∠PMC = ∠QMC= 90° [Given]

CM=CM [Common]

Thus by Side-Angle-Side criterion of congruence we have

∴ ΔPCM ≅ ΔQCM [By SAS]

The corresponding parts of the congruent triangles are equal.

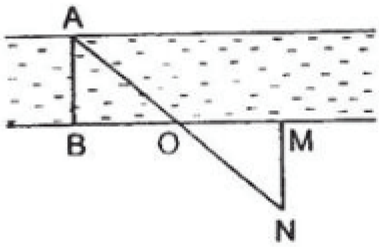
So, PC = QC [C.P.C.T]

⇒ PC = QB + CB

⇒ PC = AB + PA [∵ AB = CB and PA = QB]

Question 33:

Let AB be the breadth of a river. Now take a point M on that bank of the river where point B is situated. Through M draw a perpendicular and take point N on it such that point, A, O and N lie on a straight line where point O is the mid point of BM.



Now in ΔABO and ΔNMO we have,

$\angle OBA = \angle OMN = 90^0$

$OB = OM$ [$\therefore$ O is mid point of BM]

and $\angle BOA = \angle MON$ [Vertically opposite angles]

Thus, by Angle - Side - Angle criterion of congruence, we have,

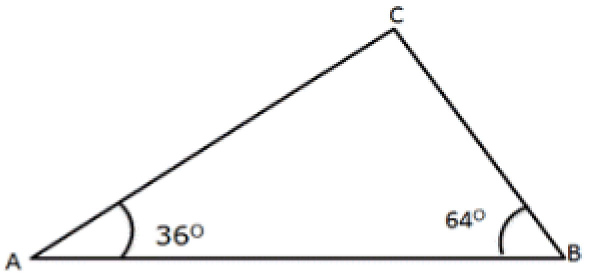
$\Delta ABO \cong \Delta NMO$ [By ASA]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $AB = NM$ [C.P.C.T]

Thus, we find that MN is the width of the river.

Question 34



We have $\angle A = 36°$ and $\angle B = 64°$

By the angle sum property in ΔABC, we have

$\angle A+\angle B+\angle C = 180^\circ$

$\Rightarrow 36^\circ + 64^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 100^\circ = 80^\circ$

Therefore, we have

$\angle A = 36°, \angle B = 64°$ and $\angle C = 80°$

$\therefore \angle C$ is largest and $\angle A$ is shortest.

Side opposite to $\angle C$ is longest and hence

AB is longest side.

Side opposite to $\angle A$ is shortest and hence

BC is shortest side.

Question 35:

In a right angle triangle, greatest angle is $\angle A = 90°$.

And hence other angles are less than $90°$ because sum

of the angles of a triangle is $180°$.

So, $\angle A$ is the greatest angle.

Therefore, side BC which is opposite to $\angle A$ is longest.

Question 36:

In ΔABC,

$$\angle A = \angle B = 45^\circ$$

So, $$\angle C = 180^\circ - \angle A - \angle B$$

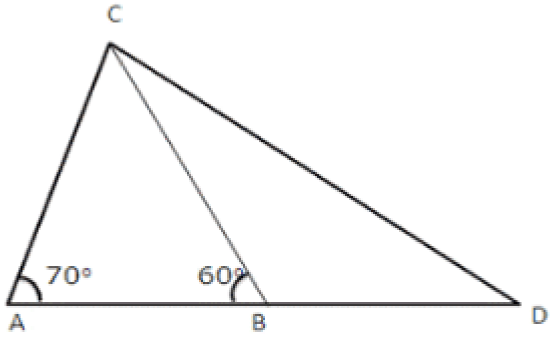
$$= 180^\circ - 45^\circ - 45^\circ$$

$$= 180^\circ - 90^\circ = 90^\circ$$

Thus we find that $\angle C$ is the greatest angle of ΔABC.

So, AB is the longest side which is opposite to $\angle C$.

Question 37:



In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad 70^\circ + 60^\circ + \angle C = 180^\circ$

$\Rightarrow \quad 130^\circ + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 180^\circ - 130^\circ = 50^\circ$

Now in ΔBCD we have,

$\angle CBD = \angle DAC + \angle ACB$ [$\because$ $\angle CBD$ is the exterior angle of $\angle ABC$]

$= 70^\circ + 50^\circ = 120^\circ$

Since $BC = BD$ [Given]

So, $\angle BCD = \angle BDC$

$\therefore \quad \angle BCD + \angle BDC = 180^\circ - \angle CBD$

$= 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow \quad 2\angle BCD = 60^\circ$

$\Rightarrow \quad \angle BCD = \angle BDC = 30^\circ$

Now in ΔACD we have

$\angle A = 70^\circ, \angle D = 30^\circ$

and $\angle ACD = \angle ACB + \angle BCD$

$= 50^\circ + 30^\circ = 80^\circ$

$\therefore \quad \angle ACD$ is the greatest angle.

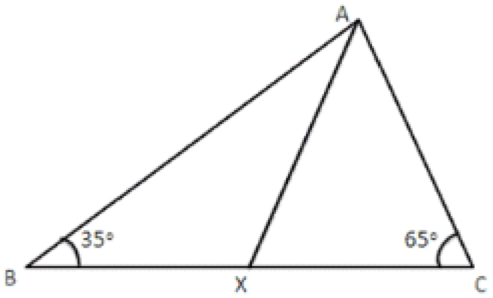
So the side opposite to $\angle ACD$, that is AD, is the longest side of ΔACD

$\therefore \quad AD > CD$

(ii) Since $\angle BDC$ is the smallest angle, the side opposite to $\angle BDC$, that is AC, is the shortest side of ΔACD

$\therefore \quad AD > AC.$

Question 38:



In ΔABC,

$$\angle A = 180^\circ - \angle B - \angle C$$
$$= 180^\circ - 35^\circ - 65^\circ$$
$$= 180^\circ - 100^\circ = 80^\circ$$

$$\therefore \quad \angle BAX = \frac{1}{2}\angle A$$
$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

Now in ΔABX,

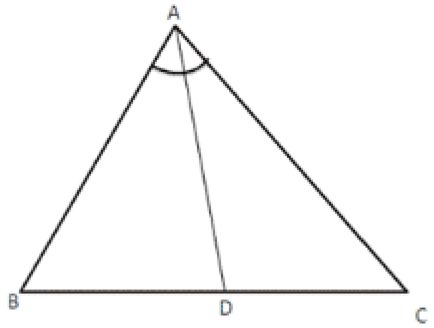
$$\angle B = 35^\circ$$
$$\angle BAX = 40^\circ$$
and $$\angle BXA = 180^\circ - 35^\circ - 40^\circ$$
$$= 180^\circ - 75^\circ = 105^\circ$$

So, in ΔABX,
$\angle B$ is smallest,so the side opposite to $\angle B$,
that is AX, is smallest
So $AX < BX$ ....(i)
Now consider ΔAXC

$$\angle CAX = \frac{1}{2} \times \angle A$$
$$= \frac{1}{2} \times 80^\circ = 40^\circ$$
$$\angle AXC = 180^\circ - 40^\circ - 65^\circ$$
$$= 180^\circ - 105^\circ = 75^\circ$$

Therefore, in ΔAXC, we have,
$\angle CAX = 40^\circ, \angle C = 65^\circ$ and $\angle AXC = 75^\circ$
$\therefore$ $\angle CAX$ is smallest in ΔAXC
So the side opposite to $\angle CAX$ is shortest.
$\Rightarrow$ CX is shortest
$\Rightarrow CX < AX$ ....(ii)
From (i) and (ii) ,we get
$BX > AX > CX$
This is the required descending order.

Question 39:



Given: ABC is a triangle in which AD is the bisector of ∠A.

Proof: (i) In ΔACD

Exterior ∠ADB =∠ DAC+∠ACD

=∠BAD +∠ACD

[∴ ∠DAC= ∠BAD(given)]

∴ ∠ADB > ∠BAD

The side opposite to angle ∠ADB is the longest side in ΔADB

So, AB > BD

(ii) Again in ΔABD

Exterior ∠ADC = ∠ABD +∠BAD

=∠ABD+ ∠CAD

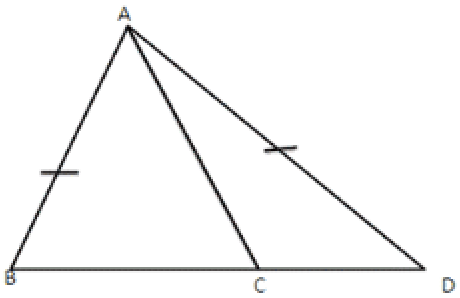
∴ ∠ADC > ∠CAD

The side opposite to angle ∠ADC is the longest side in ΔACD

So, AC > DC

Question 40:

Given : A ΔABC is which AB=AC side BC of ΔABC is produced to D.



To prove: AD > AC

Proof: In ΔABC

Ext.$\angle ACD = \angle B + \angle BAC$

$= \angle ACB + \angle BAC$ [$\because \angle B = \angle C$ as AB=AC]

$= \angle CAD + \angle CDA + \angle BAC$

[$\because$ Ext.$\angle ACB = \angle CAD + \angle CDA$]

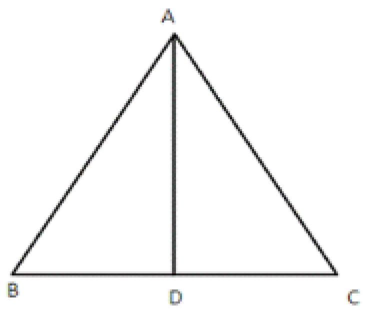
$\Rightarrow \angle ACD > \angle CDA$

So the side opposite to $\angle ACD$, is the longest.

$\therefore$ AD > AC

Question 41:

Given: A ΔABC in which AC> AB and AD is a bisector of $\angle A$



To prove: $\angle ADC > \angle ADB$

Proof : Since AC > AB

$\Rightarrow \angle ABC > \angle ACB$

Adding $\frac{1}{2}\angle A$ on both sides of inequality.

$\angle ABC + \frac{1}{2}\angle A > \angle ACB + \frac{1}{2}\angle A$

$\Rightarrow \angle ABC + \angle BAD > \angle ACB + \angle DAC$

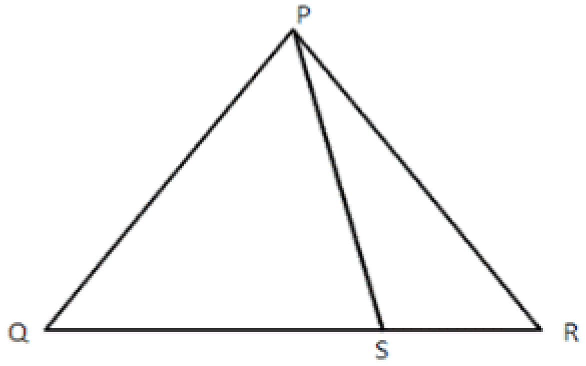
[$\because$ AD is a bisector of $\angle A$]

$\Rightarrow$ Exterior $\angle ADC >$ Exterior $\angle ADB$

$\therefore \angle ADC > \angle ADB$.

Question 42:

Given : A triangle PQR and S is a point on QR.



To prove: PQ + QR + RP >2PS

Proof: Since in a triangle, sum of any two sides is always greater than the third side.

So in ΔPQS, we have

PQ+ QS> PS ....(i)

Similarly, in ΔPSR, we have

PR + SR> PS ...(ii)

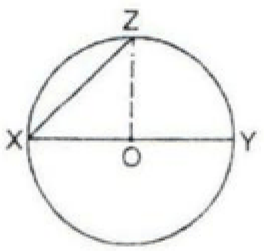
Adding both sides of (i) and (ii) , we get.

PQ + QS + PR + SR > 2PS

⇒ PQ + PR + QS + SR > 2PS

⇒ PQ + PR + QR > 2PS

Question 43:



Given : A circle with centre O is drawn in which XY is a diameter and XZ is a chord.

To prove : XY> XZ

Proof : In ΔXOZ, we have,

OX+OZ > XZ

[∴ sum of any two sides in a triangle is a greater than its third side]

⇒ OX +OY > XZ

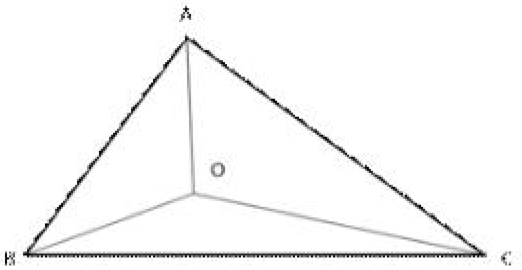
[∵ OZ = OY, radius of the circle]

∴ XY > XZ

[∵ OX+OY=XY]

Question 44:

Given : ABC is a triangle and O is appoint inside it.



To Prove : (i) AB+AC > OB +OC

(ii) AB+BC+CA > OA+OB+OC

(iii) OA+OB+OC > $\frac{1}{2}$ (AB+BC+CA)

Proof:

(i) In ΔABC,

AB+AC > BC ….(i)

And in , ΔOBC,

OB+OC > BC ….(ii)

Subtracting (i) from (i) we get

(AB+AC) – (OB+OC) > (BC-BC)

i.e. AB+AC>OB+OC

(ii) AB+AC > OB+OC [proved in (i)]

Similarly, AB+BC > OA+OC

And AC+BC > OA +OB

Adding both sides of these three inequalities, we get

(AB+AC) + (AC+BC) + (AB+BC) > OB+OC+OA+OB+OA+OC

i.e. 2(AB+BC+AC) > 2(OA+OB+OC)

Therefore, we have

AB+BC+AC > OA+OB+OC

(iii) In ΔOAB

OA+OB > AB ….(i)

In ΔOBC,

OB+OC > BC ….(ii)

And, in ΔOCA,

OC+OA > CA

Adding (i), (ii) and (iii) we get

(OA+OB) + (OB+OC) + (OC+OA) > AB+BC+CA

i.e 2(OA+OB+OC) > AB+BC+CA

$\Rightarrow$ OA+OB+OC > $\frac{1}{2}$ (AB+BC+CA)

Question 45:

Since AB=3cm and BC=3.5 cm

∴ AB+BC=(3+3.5) cm =6.5 m

And CA=6.5 cm

So AB+BC=CA

A triangle can be drawn only when the sum of two sides is greater than the third side.

So, with the given lengths a triangle cannot be drawn.