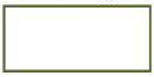
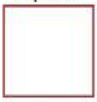
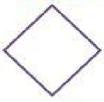
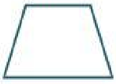
Quadrilateralsand Parallelograms

Exercise 9A

Type	Properties
Parallelogram 	• Opposite sides are equal and parallel • Opposite angles are equal
Rectangle 	• Opposite sides are equal and parallel • All angles are right angles (90°)
Square 	• Opposite sides are parallel • All sides are equal • All angles are right angles (90°)
Rhombus 	• Opposite sides are parallel • All sides are equal • Opposite angles are equal • Diagonals bisect each other at right angles (90°)
Trapezoid 	• One pair of opposite sides is parallel
Kite 	• Two pairs of adjacent sides are equal • One pair of opposite sides are equal • One diagonal bisects the other • Diagonals intersect at right angle (90°)

Question 1:

Let the fourth angle be x.
We know, that sum of the angles of a quadrilateral is $360°$

Then, $56° + 115° + 84° + x = 360°$

$\Rightarrow \quad 255° + x = 360°$

$\Rightarrow \quad x = 360° - 255° = 105°$

$\therefore$ The fourth angle is $105°$.

Question 2:

Let the angles of a quadrilateral be 2x, 4x, 5x and 7x.
We know, that sum of the angles of a quadrilateral is $360°$

Then, $2x + 4x + 5x + 7x = 360°$

$\Rightarrow \quad 18x = 360°$

$\Rightarrow \quad x = \frac{360}{18} = 20°$

$\therefore$ the angles of the quadrilateral are:

$2x = 2 \times 20 = 40°$

$4x = 4 \times 20 = 80°$

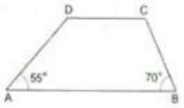
$5x = 5 \times 20 = 100°$

$7x = 7 \times 20 = 140°$

$\therefore$ the required angles are $40°, 80°, 100°$ and $140°$.

Question 3:

Since AB || DC



Since AB || DC, $\angle A$ and $\angle D$ are consecutive interior angles.
Consecutive interior angles sum upto 180°.

So, $\angle A + \angle D = 180°$

$\Rightarrow 55° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 55° = 125°$

Also, we know that, sum of the angles of a quadrilateral is 360°

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow 55° + 70° + \angle C + 125° = 360°$

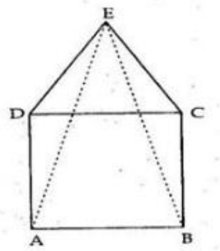
$\Rightarrow 250° + \angle C = 360°$

$\Rightarrow \angle C = 360° - 250° = 110°$

$\therefore \angle C = 110°$ and $\angle D = 125°$

Question 4:

Given: ΔEDC is an equilatateral triangle and ABCD is a square



To Prove: AE = BE

and $\angle DAE = 15^0$

(i) Proof: Since ΔEDC is an equilateral triangle,

$\angle EDC = 60^0$ and $\angle ECD = 60^0$

Since ABCD is a square,

$\angle CDA = 90^0$ and $\angle DCB = 90^0$

In ΔEDA

$\angle EDA = \angle EDC + \angle CDA$

$= 60^0 + 90^0$

$= 150^0$ ......(1)

In ΔECB

$\angle ECB = \angle ECD + \angle DCB$

$= 60^0 + 90^0 = 150^0$

$\Rightarrow \angle EDA = \angle ECB$ ......(2)

Thus, in ΔEDA and ΔECB

ED = EC [sides of equilateral triangle ΔEDC]

$\angle EDA = \angle ECB$ [from (2)]

DA = CB [sides of square ▫ABCD]

Thus, by Side-Angle-Side criterion of congruence, we have

$\therefore \Delta EDA \cong \Delta ECB$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ AE = BE [C.P.C.T]

(ii) Now in Δ EDA, we have

ED = DA

$\Rightarrow \angle DEA = \angle DAE$ [base angles are equal]

But $\angle EDA = 150^0$ [from (1)]

So, by angle sum property in ΔEDA

$\angle EDA + \angle DAE + \angle DEA = 180^0$

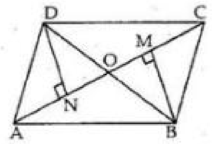
$\Rightarrow 150^0 + \angle DAE + \angle DAE = 180^0$

$\Rightarrow 2\angle DAE = 180^0 - 150^0$

$\Rightarrow 2\angle DAE = 30^0$

Question 5:

Given: BM ⊥ AC and DN ⊥ AC and BM = DN



To Prove : AC bisects BD.

We have,

$\angle DON = \angle MOB$ [Vertically opposite angles]

$\angle DNO = \angle BMO = 90°$

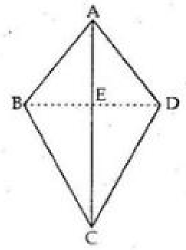
$BM = DN$ [Given]

$\therefore \Delta DNO \cong \Delta BMO$ [By AAS]

$\therefore OD = OB$ [C.P.C.T]

So, AC bisects BD.

Question 6:



Given: ABCD is quadrilateral in which AB = AD and BC = DC

To Prove: (i) AC bisects ∠A and ∠C

(ii) BE = DE

(iii) ∠ABC = ∠ADC

Proof: In ΔABC and ΔADC, we have

AB=AD [Given]

BC=DC [Given]

AC=AC [Common]

Thus by Side-Side-Side criterion of congruence,

$\Delta ABC \cong \Delta ADC$ ......(1)

The corresponding parts of the congruent triangles are equal.

So, $\angle BAC = \angle DAC$ [C.P.C.T]

$\Rightarrow \angle BAE = \angle DAE$

It means that AC bisects ∠BAD, that is ∠A

Also, $\angle BCA = \angle DCA$ [C.P.C.T]

$\Rightarrow \angle BCE = \angle DCE$

It means that AC bisects ∠BCD, that is ∠C

(ii) In ΔABE and ΔADE, we have

AB = AD [given]

∠BAE = ∠DAE [from (i)]

AE = AE [Common]

Thus by Side-Angle-Side criterion of congruence, we have

$\therefore \Delta ABE \cong \angle ADE$ [∵ By SAS]

So, BE = DE [By c.p.c.t]

(iii) Since from equation (1) in subpart (i), we have

$\Delta ABC \cong \Delta ADC$,

Thus, by c.p.c.t, $\angle ABC = \angle ADC$

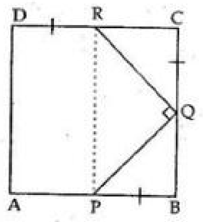
Question 7:

Given : A square ABCD in which $\angle PQR = 90°$ and $PB = QC = DR$

To Prove : (i) $QB = RC$

(ii) $PQ = QR$

(iii) $\angle QPR = 45°$



Proof :

(i) Consider the line segement QB:

$QB = BC - QC$

$= CD - DR$ [$\because$ ABCD is a square, so $BC = DC, QC = DR$(given)]

$QB = RC$ ......(1)

(ii) In ΔPBQ and ΔQCR, we have

$PB = QC$ [Given]

$\angle PBQ = \angle QCR = 90°$ [$\because$ ABCD is a square]

and $QB = RC$ [from (1)]

Thus by Side-Angle-Side criterion of congruence, we have

$\Delta PBQ \cong \Delta QCR$ [By SAS]

$\Rightarrow$ $PQ = QR$ [By c.p.c.t]

(iii) Given that, $PQ = QR$

So, in ΔPQR

$\angle QPR = \angle QRP$ [isosceles triangle, so base angles are equal]

By the angle sum property, in ΔPQR

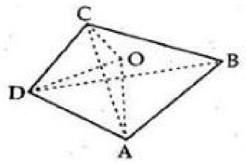
$\angle QPR + \angle QRP + 90° = 180°$

$\Rightarrow \angle QPR + \angle QPR = 180° - 90° = 90°$

$\therefore$ $\angle QPR = \frac{90}{2} = 45°$.

Question 8:

Given: O is a point within a quadrilateral ABCD



To Prove : $OA + OB + OC + OD > AC + BD$

Construction : Join AC and BD

Proof : In ΔACO,

$OA + OC > AC$ ...(i)

[$\because$ in a tringle, sum of any two sides is greater than the third side]

Similarly, In ΔBOD,

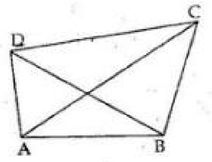
$OB + OD > BD$ ...(ii)

Adding both sides of (i) and (ii), we get;

$OA + OC + OB + OD > AC + BD$ (Proved)

Question 9:

Given: ABCD is a quadrilateral and AC is one of its disgonals.



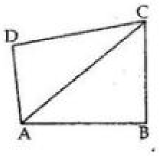
To Prove:
(i) $AB + BC + CD + DA > 2AC$
(ii) $AB + BC + CD > DA$
(iii) $AB + BC + CD + DA > AC + BD$
Construction: Join BD.
Proof: (i) In ΔABC,
$AB + BC > AC$ ...(1)
and, in ΔACD
$AD + CD > AC$ ...(2)
Adding both sides of (1) and (2), we get:
$AB + BC + CD + DA > 2AC$ ...(3)

(ii) In ΔABC,
$AB + BC > AC$
On adding CD to both sides of this inequality, we have,
$AB + BC + CD > AC + CD$ ...(4)
Now, in ΔACD, we have,
$AC + CD > DA$ ...(5)
From (4) and (5) we get
$AB + BC + CD > DA$ ...(6)

(iii) In ΔABD and ΔBDC, we have
$AB + DA > BD$ ...(7)
and $BC + CD > BD$ ...(8)
On adding (7) and (8), we get
$AB + BC + CD + DA > 2BD$ ...(9)
Adding (9) and (3), we have,
$2(AB + BC + CD + DA) > 2BD + 2AC$
i.e. $AB + BC + CD + DA > BD + AC$
[Dividing both sides by 2]

Question 10:

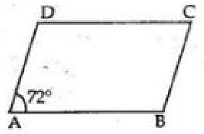
Given: ABCD is a quadrilateral.



To Prove: $\angle A + \angle B + \angle C + \angle D = 360°$
Construction: Join AC
Proof: In ΔABC
$\angle CAB + \angle B + \angle BCA = 180°$ ...(i)
In ΔACD,
$\angle DAC + \angle ACD + \angle D = 180°$ ...(ii)
Adding both sides of (i) and (ii) we get
$\angle CAB + \angle B + \angle BCA + \angle DAC + \angle ACD + \angle D = 180° + 180°$
$\Rightarrow \quad \angle CAB + \angle DAC + \angle B + \angle BCA + \angle ACD + \angle D = 360°$
$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360°$

Exercise 9B

Question 1:



In a parallelogram, opposite angles are equal.

$\therefore \quad \angle A = \angle C = 72°$

The sum of all the four angles of a parallelogram is 360°

So, $\quad \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad 72° + \angle B + 72° + \angle D = 360° \quad [\because \angle A = \angle C]$

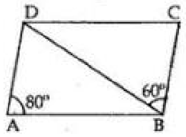
$\Rightarrow \quad 2\angle B + 144° = 360° \quad [\because \angle B = \angle D]$

$\Rightarrow \quad 2\angle B = 360° - 144° = 216°$

$\Rightarrow \quad \angle B = \frac{216}{2} = 108°$

$\therefore \quad \angle B = 108°, \angle C = 72° \text{ and } \angle D = 108°.$

Question 2:



ABCD is a parallelogram,
so opposite angles are equal.

$\therefore \quad \angle C = \angle A = 80°$

As $AD \parallel BC$ and BD is a transversal.

So, $\quad \angle ADB = \angle DBC = 60°$

[Alternate angles]

In $\triangle ABD$

$\angle A + \angle ADB + \angle ABD = 180°$

$\Rightarrow \quad 80° + 60° + \angle ABD = 180°$

$\Rightarrow \quad 140° + \angle ABD = 180°$

$\Rightarrow \quad \angle ABD = 180° - 140° = 40°$

$\therefore \quad \angle ABC = \angle ABD + \angle DBC = 40° + 60° = 100°$

In a parallelogram, opposite angles are equal.

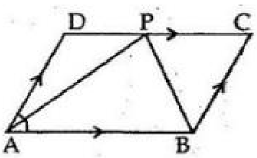
So, $\quad \angle ADC = \angle ABC = \angle 100°$

$\therefore \quad \angle CDB = \angle ADC - \angle ADB = 100° - 60° = 40°$

and $\quad \angle ADB = 60°.$

Question 3:

ABCD is a parallelogram in which DA=60° and bisectors of A and B meets DC at P.



(i) In a parallelogram, opposite angles are equal.

So, $\angle C = \angle A = 60^\circ$

In a parallelogram the sum of all the four angles is 360°.

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$

Now, $\angle B + \angle D = 360^\circ - (\angle A + \angle C)$

$= 360^\circ - (60^\circ + 60^\circ) = 240^\circ$

$\therefore$ $2\angle B = 240^\circ$ $[\because \angle B = \angle D]$

So, $\angle B = \angle D = \frac{240^\circ}{2} = 120^\circ$

Since AB || DP and AP is a transversal

So, $\angle APD = \angle PAB = \frac{60^\circ}{2} = 30^\circ$ ....(1)

[$\therefore$ alternate angles]

Also, AB || PC and BP is a transversal.

So, $\angle ABP = \angle CPB$

But, $\angle ABP = \frac{\angle B}{2} = \frac{120^\circ}{2} = 60^\circ$

$\therefore$ $\angle CPB = 60^\circ$ ....(2)

Now, $\angle APD + \angle APB + \angle CPB = 180^\circ$

[As DPC is a straightline]

$30^\circ + \angle APB + 60^\circ = 180^\circ$

$\Rightarrow$ $\angle APB = 180^\circ - 30^\circ - 60^\circ = 90^\circ$

(ii) Since $\angle APD = 30^\circ$ [from (1)]

and $\angle DAP = \frac{60^\circ}{2} = 30^\circ$

So, $\angle APD = \angle DAP$

Now in ΔAPD,

$\angle APD = \angle DAP$......(3)

$\therefore$ DP = AD [isosceles triangle, sides are equal]

As $\angle CPB = 60^\circ$ [from (2)]

and $\angle C = 60^\circ$

So, $\angle PBC = 180^\circ - 60^\circ - 60^\circ = 60^\circ$

Since all angles in the ΔPCB are equal, it is an equilateral triangle.

$\therefore$ PB = PC = BC....(4)

(iii) $\angle DPA = \angle PAD$, [from(3)]

$\therefore$ DP = AD [isoscele striangle, sides are equal]

= BC [opposite sides are equal]

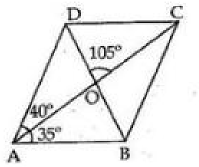
= PC [from (4)]

$= \frac{1}{2}DC$ [$\because$ DP = PC $\Rightarrow$ P is the midpoint of DC]

$\therefore$ DC = 2AD.

Question 4:

ABCD is a parallelogram



(i) $\angle AOB = \angle COD = 105^0$

[vertical opposite angle]

Now in ΔAOB, we have

$\angle OAB + \angle AOB + \angle ABO = 180^0$

$\Rightarrow 35^0 + 105^0 + \angle ABO = 180^0$

$\Rightarrow 140^0 + \angle ABO = 180^0$

$\Rightarrow \angle ABO = 180^0 - 140^0 = 40^0.$

(ii) Since $AB \parallel DC$ and BD is a transversal

So, $\angle ABD = \angle CDB$ [alternate angles]

$\Rightarrow \angle CDO = \angle CDB = \angle ABD = \angle ABO = 40^0$

$\therefore \angle ODC = 40^0$

(iii) As $AB \parallel CD$ and AC is a transversal

So, $\angle ACB = \angle DAC = 40^0$

[alternate opposite angles]

(iv) $\angle CBD = \angle B - \angle ABO$

But, $\angle A + \angle B + \angle C + \angle D = 360^0$

[$\because$ ABCD is a parrellogram]

$\Rightarrow 2\angle A + 2\angle B = 360^0$

$\Rightarrow 2 \times (40^0 + 35^0) + 2\angle B = 360^0$

$\Rightarrow 150^0 + 2\angle B = 360^0$

$\Rightarrow 2\angle B = 360^0 - 150^0 = 210^0$

$\Rightarrow \angle B = \frac{210^0}{2} = 105^0$

and $\angle CBD = \angle B - \angle ABO$

$= 105^0 - 40^0 = 65^0$

$\angle CBD = 65^0$

Question 5:

In a parallelogram, the opposite angles are equal.
So, in the parallelogram ABCD,

$$\angle A = \angle C$$

and $\angle B = \angle D$

Since $\angle A = (2x + 25)^0$

$\therefore$ $\angle C = (2x + 25)^0$

and $\angle B = (3x - 5)^0$

$\therefore$ $\angle D = (3x - 5)^0$

In a parallelogram, the sum of all the four angles is 360^0

$\therefore$ $\angle A + \angle B + \angle C + \angle D = 360^0$

$\Rightarrow (2x + 25) + (3x - 5) + (2x + 25) + (3x - 5) = 360^0$

$\Rightarrow$ $10x + 40 = 360^0$

$\Rightarrow$ $10x = 360^0 - 40^0 = 320^0$

$\Rightarrow$ $x = \frac{320}{10} = 32^0$

$\therefore \angle A = (2x + 25) = (2 \times 32 + 25) = 89^0$

$\angle B = (3x - 5) = (3 \times 32 - 5) = 91^0$

$\angle C = (2x + 25) = (2 \times 32 + 25) = 89^0$

$\angle D = (3x - 5) = (3 \times 32 - 5) = 91^0$

$\therefore \angle A = \angle C = 89^0$ and $\angle B = \angle D = 91^0$

Question 6:

Lets ABCD be a parallelogram.

Suppose, $\angle A = x^0$

Then, $\angle B$, which is adjacent angle of A is $\frac{4}{5}x^0$.

In a parallelogram, the opposite angles are equal

$\Rightarrow$ $\angle A = \angle C = x^0$ and $\angle B = \angle D = \frac{4}{5}x^0$

The sum of all the four angles of a parallelogram is 360^0.

$\Rightarrow$ $\angle A + \angle B + \angle C + \angle D = 360^0$

$\Rightarrow$ $x + \frac{4}{5}x + x + \frac{4}{5}x = 360^0$

$\Rightarrow$ $2x + \frac{8}{5}x = 360^0$

$\Rightarrow$ $\frac{18}{5}x = 360^0$

$\Rightarrow$ $x = \frac{360 \times 5}{18} = 100^0$

$\therefore$ $\angle A = x = 100^0$

$\angle B = \frac{4}{5}x = \frac{4}{5} \times 100 = 80^0$

$\angle C = x = 100^0$

$\angle D = \frac{4}{5}x = \frac{4}{5} \times 100 = 80^0$

$\therefore \angle A = \angle C = 100^0$ and $\angle B = \angle D = 80^0$.

Question 7:

Lets ABCD be the given parallelogram.

If $\angle A$ is smallest angle, then the greater angle

$\Rightarrow \quad \angle B = 2\angle A - 30^0$

In a parallelogram, the opposite angles are equal

$\Rightarrow \quad \angle A = \angle C$ and $\angle B = \angle D = 2\angle A - 30^0$

The sum of all the four angles of a parallelogram is 360^0.

$$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360^0$$
$$\Rightarrow \quad \angle A + (2\angle A - 30^0) + \angle A + (2\angle A - 30^0) = 360^0$$
$$\Rightarrow \quad \angle A + 2\angle A - 30^0 + \angle A + 2\angle A - 30^0 = 360^0$$
$$\Rightarrow \quad 6\angle A - 60^0 = 360^0$$
$$\Rightarrow \quad 6\angle A = 360^0 + 60^0 = 420^0$$
$$\Rightarrow \quad \angle A = \frac{420^0}{6} = 70^0$$

$\therefore \angle A = 70^0 \Rightarrow \angle C = 70^0$

$\angle B = (2\angle A - 30^0) = (2 \times 70^0 - 30^0) = 110^0$

$\angle D = \angle B = 110^0$

$\therefore \angle A = \angle C = 70^0$ and $\angle B = \angle D = 110^0$.

Question 8:

Perimeter of a parrallelogram ABCD

$$= AB + BC + CD + DA$$
$$= 9.5 + BC + 9.5 + BC$$

[$\therefore$ ABCD is a parrallelogram and its opposite sides are equal i.e. AB = CD and BC = DA]

$$30 = 19 + 2BC$$

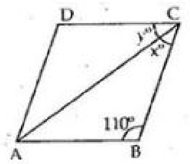
[Perimeter = 30 cm(given)]

$$\Rightarrow \quad 2\,BC = 30 - 19 = 11$$
$$\Rightarrow \quad BC = \frac{11}{2} = 5.5\text{cm}$$

$\therefore \quad$ AB = 9.5cm, BC = 5.5cm, CD = 9.5cm, DA = 5.5cm.

Question 9:

(i) ABCD is a rhombus, so its all sides are equal.



In ΔABC, we have

$AB = BC$

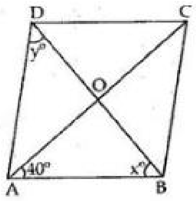
$\Rightarrow \quad \angle CAB = \angle ACB = x^0$

As, $\quad \angle CAB + \angle ABC + \angle ACB = 180^0$

$$\Rightarrow \quad x + 110^0 + x = 180^0$$
$$\Rightarrow \quad 2x = 180^0 - 110^0 = 70^0$$
$$\Rightarrow \quad x = \frac{70^0}{2} = 35^0$$

$\therefore x = 35^0$ and $y = 35^0$

(ii) Since in a rhombus, all sides are equal



So in ΔABD, $AB = AD$

$\Rightarrow \angle ABD = \angle ADB$

$\Rightarrow x = y$ ......(1)

Now in ΔABC, $AB = BC$

$\Rightarrow \angle CAB = \angle ACB$

$\Rightarrow \angle ACB = 40^0$

$\therefore \angle B = 180^0 - \angle CAB - \angle ACB$

$= 180^0 - 40^0 - 40^0 = 100^0$

$\Rightarrow \angle DBC = \angle B - x^0 = 100 - x^0$

But $\angle DBC = \angle ADB = y^0$ [alternate angle]

$\Rightarrow 100 - x^0 = y^0$

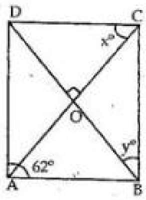
$\Rightarrow 100^0 - x^0 = x^0$ [from (1)]

$\Rightarrow 2x^0 = 100$

$\Rightarrow x^0 = \frac{100}{2} = 50^0$

So, $x = 50^0$ and $y = 50^0$.

(iii) Since ABCD is a rhombus



So, $\angle A = \angle C$, i.e. $\angle C = 62^0$

Now in ΔBCD,

$BC = DC$

$\Rightarrow \angle CDB = \angle DBC = y^0$

As, $\angle BDC + \angle DBC + \angle BCD = 180^0$

$\Rightarrow y + y + 62^0 = 180^0$

$\Rightarrow 2y = 180^0 - 62^0 = 118^0$

$\Rightarrow y = \frac{118}{2} = 59^0$

As diagonals of a rhombus are perpendicular to each other,

ΔCOD is a right triangle and $\angle DOC = 90^0, \angle ODC = y = 59^0$

$\Rightarrow \angle DCO = 90^0 - \angle ODC$

$= 90^0 - 59^0 = 31^0$

$\therefore \angle DCO = x = 31^0$

$\therefore$ $x = 31^0$ and $y = 59^0$

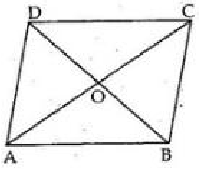
Question 10:

ABCD is a rhombus in which diagonal AC = 24 cm and BD = 18 cm.
We know that in a rhombus, diagonals bisect each other at right angles.
So in ΔAOB

$\angle AOB = 90^0$

$AO = \frac{1}{2}AC = \frac{1}{2} \times 24 = 12$ cm

and, $BO = \frac{1}{2}BD = \frac{1}{2} \times 18 = 9$ cm



Now, by Pythagoras Theorem, we have

$AB^2 = AO^2 + OB^2$

$\Rightarrow AB^2 = 12^2 + 9^2$

$= 144 + 81 = 225$

$\Rightarrow AB = \sqrt{225} = 15$ cm

So the length of each side of the rhombus is 15 cm.

Question 11:

Since diagonals of a rhombus bisect each other at right angles.

So, $AO = OC = \frac{1}{2}AC = \frac{1}{2} \times 16 = 8$ cm.

$\therefore$ In right ΔAOB,

$AB^2 = AO^2 + OB^2$

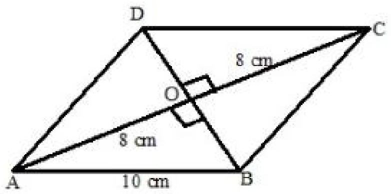
$\Rightarrow 10^2 = 8^2 + OB^2$

$\Rightarrow OB^2 = 100 - 64 = 36$

$\Rightarrow OB = \sqrt{36} = 6$ cm.

$\therefore$ Length of the other diagonal $BD = 2 \times OB$

$= 2 \times 6 = 12$ cm.



Area of $\Delta ABC = \frac{1}{2} \times AC \times OB$

$= \frac{1}{2} \times 16 \times 6 = 48$ cm^2.

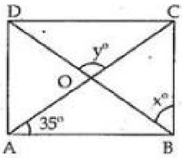
Area of $\Delta ACD = \frac{1}{2} \times AC \times OD$

$= \frac{1}{2} \times 16 \times 6 = 48$ cm^2.

$\therefore$ Area of rhombus ABCD = (Area of ΔABC + Area of ΔACD)

$= (48 + 48)$cm^2 = 96 cm^2.

Question 12:



We know that diagonals of a rectangle are equal and bisect each other.

So, in ΔAOB

$AO = OB$

$\Rightarrow \angle OAB = \angle OBA$ [base angles are equal]

i.e. $\angle OBA = 35^0$ [$\because \angle OAB = 35^0$, given]

$\angle AOB = 180^0 - 35^0 - 35^0 = 110^0$

and, $\angle DOC = y^0 = \angle AOB = 110^0$

[Vertically opp. angles]

Consider the right triangle, ΔABC, right angled at B.

So, $\angle ABC = 90^0$ [$\because$ ABCD is a rectangle]

Now, consider the ΔOBC

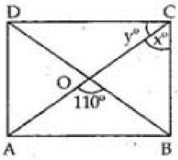
So, $\angle OBC = x^0 = \angle ABC - \angle OBA$

$= 90^0 - 35^0$

$= 55^0$

$\therefore x = 55^0$ and $y = 110^0$.

(ii) We know that diagonals of a rectangle are equal and bisect each other.



So, in ΔAOB, $OA = OB$

$\Rightarrow \angle OAB = \angle OBA$

Again in ΔAOB,

$\angle AOB + \angle OAB + \angle OBA = 180^0$

$\Rightarrow 110^0 + \angle OAB + \angle OBA = 180^0$

$\Rightarrow 2\angle OAB = 180^0 - 110^0 = 70^0$

$\Rightarrow \angle OAB + \angle OBA = \frac{70}{2} = 35^0$

Since $AB \parallel CD$ and AC is a transversal, $\angle DCA$ and $\angle CAB$ are alternate angles, and thus they are equal.

So, $\angle DCA = y^0 = \angle CAB$ and $\angle CAB = 35^0$ .....(1)

$\Rightarrow y^0 = 35^0$

Now consider the right triangle, ΔABC

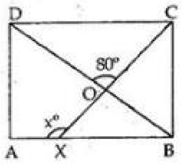
$\angle ACB = x^0 = 90^0 - \angle CAB$

$= 90^0 - 35^0$ [from (1)]

$= 55^0$

$\therefore x = 55^0$ and $y = 35^0$.

Question 13:



Consider the triangle ΔABD

$AB = AD$ [$\therefore$ ABCD is a square]

So, $\angle ADB = \angle ABD$ [base angles are equal]

$\therefore \angle ADB + \angle ABD = 90^0$ [$\because \angle A = 90^0$ as ABCD is a square]

$2\angle ADB = 90^0$

$\Rightarrow \angle ADB = \frac{90}{2} = 45^0$

Now in ΔOXB,

$\angle XOB = \angle DOC = 80^0$ [vertically opposite angle]

and $\angle ABD = 45^0 \Rightarrow \angle XBD = 45^0$.....(1)

So, exterior $\angle AXO = \angle XOB + \angle XBD$

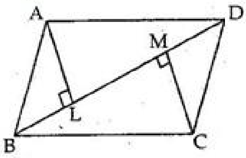
$x^0 = 80^0 + 45^0$ [from (1)]

$= 125^0$

$\therefore x^0 = 125^0$

Question 14:

A parallelogram ABCD in which AL and CM are perpendiculars to its diagonal BD



To Prove : (i) $\Delta ALD \cong \Delta CMB$

(ii) $AL = CM$

Proof : (i) In ΔALD and ΔCMB, we have

$\angle ALD = \angle CMB = 90^0$ [Given]

$\angle ADL = \angle CBM$ [AD || BC, BD is a transversal, so alternate angles are equal]

$AD = BC$ [Opposite sides of a parallelogram]

Thus by Angle-Angle-Side criterion of congruence, we have

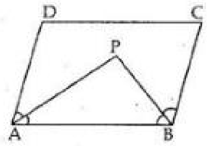
$\therefore \Delta ALD \cong \Delta CMB$ [By AAS]

(ii) Since $\Delta ALD \cong \Delta CMB$, the corresponding parts of the congruent triangles are equal.

$\therefore AL = CM$ [C.P.C.T.]

Question 15:

Given: A parallelogram ABCD in which angle bisectors of $\angle A$ and $\angle B$ intersect at P.



To Prove : $\angle APB = 90^0$

Proof : $\angle PAB = \frac{1}{2}\angle A$

and $\angle PBA = \frac{1}{2}\angle B$ [Given]

$\therefore$ AD and BC are parallel and AB is a transversal.

So sum of consecutive angles is 180^0.

$\Rightarrow \quad \angle A + \angle B = 180^0 \quad .....(1)$

$$\angle PAB + \angle PBA = \frac{1}{2}\angle A + \frac{1}{2}\angle B$$

$$= \frac{1}{2}(\angle A + \angle B)$$

$$= \frac{1}{2}\times 180^0 \quad \text{[from (1)]}$$

$$\angle PAB + \angle PBA = 90^0 \quad ......(2)$$

Now in ΔPAB,

$\angle PAB + \angle PBA + \angle APB = 180^0$

$\Rightarrow \quad 90^0 + \angle APB = 180^0$ [from (2)]

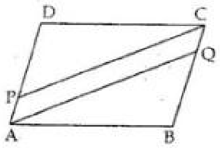
$\Rightarrow \quad \angle APB = 180^0 - 90^0 = 90^\circ$

$\therefore \quad \angle APB = 90^0$

Question 16:

Given: A parallelogram ABCD in which $AP = \frac{1}{3}AD$ and

$CQ = \frac{1}{3}BC$



To Prove: PAQC is a parallelogram.

Proof: In ΔABQ and ΔCDP

$AB = CD$

[$\because$ opposite sides of a parallelogram]

$\angle B = \angle D$

and $DP = AD - PA = \frac{2}{3}AD$

and, $BQ = BC - CQ = BC - \frac{1}{3}BC$

$= \frac{2}{3}BC = \frac{2}{3}AD$ [$\because AD = BC$]

$\therefore$ $BQ = DP$

Thus, by Side-Angle-Side criterion of congruence, we have,

So, $\Delta ABQ \cong \Delta CDP$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$AQ = CP$ [By cpct]

and $PA = \frac{1}{3}AD$

and $CQ = \frac{1}{3}BC = \frac{1}{3}AD$

$PA = CQ$ [$\because AD = BC$]

Also, by c.p.c.t, $\angle QAB = \angle PCD$.....(1)

Therefore,

$\angle QAP = \angle A - \angle QAB$

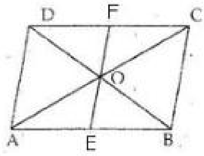
$= \angle C - \angle PCD$ [since $\angle A = \angle C$ and from (1)]

$= \angle PCQ$ [alternate interior angles are equal]

Therefore, AQ and CP are two parallel lines.

So, PAQC is a parallelogram.

Question 17:



Given: A parallelogram ABCD, in which diagonals intersect at O. E and F are the points on AB and CD

To Prove: $OE = OF$

Proof: In ΔAOE and ΔCOF, we have

$\angle CAE = \angle DCA$ [Alternate angles]

$AO = CO$ [diagonals are equal and bisect each other]

and, $\angle AOE = \angle COF$ [Vertically opposite angles]

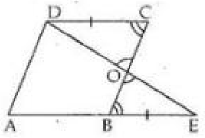
Thus by Angle-Side-Angle criterion of congruence, we have,

$\therefore$ $\Delta AOE \cong \Delta COF$ [By ASA]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $OE = OF$ [By cpct]

Question 18:

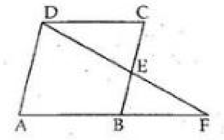


Given : ABCD is a parrallelogram in which AB is produced to E such that BE = AB. DE is joined which cuts BC at O.
To Prove : OB = OC
Proof : In ΔOCD and ΔOBE, we have,

$\angle DOC = \angle EOB$ [vertically opposite angles are equal]
$\angle OCD = \angle OBE$ [AB ∥ CD, BC is a transversal thus, alternate angles are equal]
DC = BE [AB = CD and BE = AB]

Thus, by Angle-Angle-Side criterion of congruence, we have
∴ $\Delta OCD \cong \Delta OBE$ [by AAS]
The corresponding parts of the congruent triangles are equal.
∴ OC = OB
Hence, ED bisect BC.

Question 19:



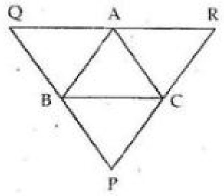
Given : A parrallelogram ABCD in which E is the mid point of side BC. DE and AB when produced meet at F.
To Prove : AF = 2AB
Proof : In ΔDEC and ΔFEB

$\angle DEC = \angle FEB$ [Vertically opposite angles]
$\angle DCE = \angle FBE$ [alternate angles]
CE = EB [Given]

Thus by Angle-Angle-Side criterion of congruence, we have
$\Delta DEC \cong \Delta FEB$ [By AAS]
The corresponding parts of the congruent triangles are equal.
∴ DC = FB [By cpct]
So, AF = AB + BF
= AB + DC
= AB + AB
= 2AB
∴ AF = 2AB

Question 20:

Given : A ΔABC in which through points A, B and C, lines QR, QP and RP are drawn parallel to BC, CA and AB.



To prove : $BC = \frac{1}{2} QR$

Proof : Since $AR \parallel BC$ and $AB \parallel RC$ [Given]

So, ABCR is a parallelogram. Therefore

$AR = BC$ ....(i)

Also, $AQ \parallel BC$ and $QB \parallel AC$

So, AQBC is a parallelogram. Therefore

$QA = BC$ ....(ii)

Adding both side of (i) and (ii), we get

$AR + QA = BC + BC$

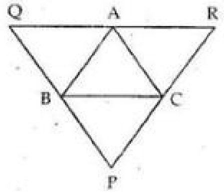
$\Rightarrow QR = 2BC$

$\Rightarrow BC = \frac{QR}{2}$

$\therefore BC = \frac{1}{2} QR$

Question 21:

Given : A ΔABC, in which through points A, B and C, lines QR, QP and RP have been drawn parrallel to BC, AC and AB of ΔABC respectively.



To Prove : Perimeter of $\Delta PQR = 2$(Perimeter of ΔABC)

Proof :

Since $AR \parallel BC$ and $AB \parallel RC$ [Given]

So, ABCR is a parallelogram. Therefore

$AR = BC$ ....(i)

Also, $AQ \parallel BC$ and $QB \parallel AC$

So, AQBC is a parallelogram. Therefore

$QA = BC$ ....(ii)

Adding both side of (i) and (ii), we get

$AR + QA = BC + BC$

$\Rightarrow QR = 2BC$

$\Rightarrow BC = \frac{QR}{2}$

$\therefore BC = \frac{1}{2} QR$

Similarly, we can prove $AB = \frac{1}{2} RP$ and $AC = \frac{1}{2} PQ$

So, Perimeter of $\Delta PQR = PQ + QR + RP$

$= 2AC + 2BC + 2AB$

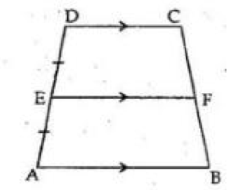
$= 2(AC + BC + AB)$

$= 2(\text{Perimeter of } \Delta ABC)$

Exercise 9C

Question 1:

Given : ABCD is trapezium in which AB || DC and through the mid – point E of AD a line drawn parallel to AB which cuts BC at F.



To prove : F is the mid – point of BC
Proof : Since AB || DC and EF || AB
So, AB || EF || DC

Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.

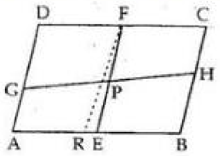
Now AD is a transversal and therefore,
Let us apply Intercepts Theorem.
Thus, the intercepts made by AB, EF and DC on transversal BC are also equal

$\therefore$ CF = FB

$\therefore$ F is mid – Point of BC.

Question 2:

Given : A parallelogram ABCD in which E and F are the mid points of AB and CD. A line segment GH cuts EF at P.



To prove : GP = PH
Proof : AD, EF and BC are three line segments and DC and AB are two transversal.
The intercepts made by the line on transversal AB and CD are equal because,

AE = EB

and DF = FC

We need to prove that FE is parallel to AD.
Let us prove by the method of contradiction.
Let us assume that FE is not parallel to AD.
Now, draw FR parallel to AD.
Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.

Thus, by Intercept Theorem, AR = RB because

DF = FC

But AE = EB [Given]

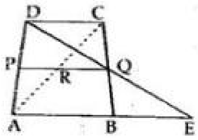
There cannot be two mid points R and E of AB.
Hence our assumption is wrong.
So, AD || EF || BC
Now, again by Intercept Theorem, we have

GP = PH

because GH is transversal and intecept made by AD, EF and BC on GH are equal as DF = FC.

Question 3:

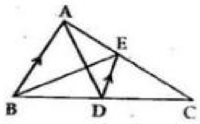
Given : ABCD is trapezium in which AB || DC
P and Q are the mid – points of AD and BC. DQ is joined and produced and AB is also produced and so that they meet at E.
AC cuts PQ at R.



To prove :
(i) DQ = QE
(ii) PR || AB
(iii) AR = RC
Proof :
(i) Consider the triangles ΔQCD and ΔQBE
$\angle DQC = \angle BQE$ [vertically opposite angles]
$CQ = BQ$ [$\because$ Q is the midpoint of BC]
$\angle QDC = \angle QEB$ [AE || DC, BC is a transversal, and thus alternate angles are equal]
Thus, by Angle-Side-Angle criterion of congruence, we have
$\Delta QCD \cong \Delta QEB$ [by ASA]
The corresponding parts of the congruent triangles are equal.
Thus, DQ = QE [by c.p.c.t]
(ii) Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
Thus by the midpoint Theorem, PQ || AE.
AB is a part of AE and hence, we have PQ || AB
Since the intercepts made by the lines AB, PQ and DC on AD
Since PQ || AB || DC
So, PR which is part of PQ is also parallel to AB
$\therefore$ PR || AB || DC
(iii) Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.
The three lines PR, AB and DC are cut by AC and AD.
So, by intercept Theorem, AR=RC

Question 4:

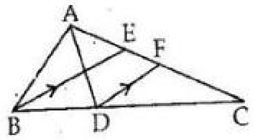
Given: A ΔABC in which AD is its median and DE || AB



To Prove : BE is a median of ΔABC.
Proof : In ΔABC,
DE || AB [Given]
D is the mid – point of BC.
The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.
So, by Mid point Theorem , E is the mid – point of AC.
$\therefore$ BE is the median of ΔABC drawn through B.

Question 5:

Given : A ΔABC in which AD and BE are the medians. DF is drawn parallel to BE.



To prove : $CF = \frac{1}{4}AC$

Proof : In ΔCBE,
D is the mid point of BC and DF is parallel to BE.
The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.

So, by Mid point Theorem F is the mid point of EC.

$\therefore CF = \frac{1}{2}EC$

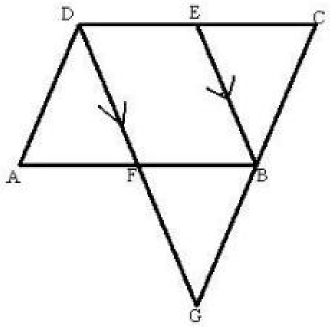
$= \frac{1}{2}\left(\frac{1}{2}AC\right)$ [BE is the median through B]

$= \frac{1}{4}AC.$

Thus, $CF = \frac{1}{4}AC.$

Question 6:

Given : ABCD is a paralleogram in which E is the mid point of DC.



Through D, a line is drwan parallel to EB meeting AB at F and BC produced at G.

To Prove : (i) $AD = \frac{1}{2}GC$

(ii) $DG = 2EB$

Proof : (i) In ΔCDG,
$EB \parallel DG$ and E is the mid – point of CD.
The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.
So, by Mid – point Theorem , B is the mid – point of CG.

$\therefore \quad CB = BG$

As, ABCD is a parallelogram,

So, $\quad AD = BC$

$\Rightarrow \quad BG = CB$

$\Rightarrow \quad AD = BG = \frac{1}{2}CG$

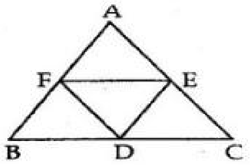
(ii) Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
Since E is the mid point of DC and B is the mid point Of CG
$\therefore$ By Mid point Theorem, in ΔCDG

$EB = \frac{1}{2}DG$

$\Rightarrow \quad DG = 2EB$

Question 7:

Given : A ΔABC in which D, E and F are the mid points of BC, AC and AB respectively.
DE, EF and FD are joined to get four triangles.



To Prove : Four triangle AFE, BFD, FDE and EDC are Congruent.
Proof : Since F, E are mid point of AB and AC

So, $EF = \frac{1}{2} BC$ [By Mid point Theorem]

Similarly $FD = \frac{1}{2} AC$

and $ED = \frac{1}{2} AB$

Now, in ΔAFE and ΔBFD, we have

$AF = FB$

$FE = \frac{1}{2} BC = BD$

$FD = \frac{1}{2} AC = AE$

Thus by Side-Side-Side criterion of congruence, we have

$\therefore \quad \Delta AFE \cong \Delta BFD$ [By SSS]

Again, in ΔBFD and ΔFED, we have

$FE \parallel BC$

i.e. $FE \parallel BD$ and $AB \parallel ED$

i.e. $FB \parallel ED$, by Mid point Theorem.

So, BDEF is a parallelogram.

$\therefore$ FD being a diagonal divides the parallelogram into two congruent triangles

$\therefore \quad \Delta BFD \cong \Delta FDE$

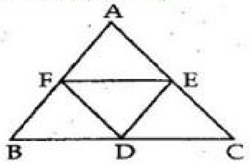
Similarly we can prove FECD is a parallelogram.

So, $\Delta FED \cong \Delta EDC$

Thus, all the four triangles $\Delta BFD, \Delta FDE, \Delta FED$ and ΔEDC are congruent to each other.

Question 8:

Given : A triangle ABC in which D,E and F are the mid points of BC, AC and AB respectively.



To prove : $\angle EDF = \angle A$
$\angle DEF = \angle B$
and $\angle DFE = \angle C$

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In ΔAFE and ΔDFE

$AF = \frac{1}{2}AB = ED$ [By Mid point Theorem]

$AE = \frac{1}{2}AC = FD$ [By Mid point Theorem]

$FE = EF$ [Common]

Thus by Side-Side-Side criterion of congruence, we have

$\Delta AFE \cong \Delta DFE$ [By SSS]

The corresponding parts of the congruent triangles are equal.

$\therefore \angle A = \angle FDE$ [C.P.C.T.]

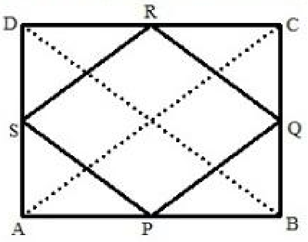
Similarly we can prove that

$\angle B = \angle DEF$

and $\angle C = \angle DFE.$

Question 9:

Given : ABCD is a rectangle and P, Q, R and S are the mid points of AB, BC, CD and DA respectively.



To prove : PQRS is a rhombus.
Construction : Join AC and BD
Proof : In ΔABC,
P and Q are the mid – points of AB and BC.
Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
So by Mid – point Theorem,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

Similarly, from ΔADC,

$RS \parallel AC$ and $RS = \frac{1}{2}AC$

$\Rightarrow PQ \parallel RS$ and $PQ = RS = \frac{1}{2}AC \ldots\ldots(1)$

Now, in ΔBAD,
P and S are the mid – points of AB and AD.
So by Mid – point Theorem, we have

$PS \parallel BD$ and $PS = \frac{1}{2}DB$

Similarly, from ΔBCD,

$RQ \parallel BD$ and $RQ = \frac{1}{2}DB$

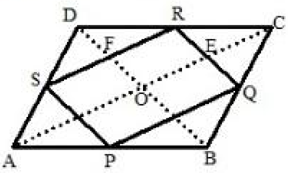
$\Rightarrow PS \parallel RQ$ and $PS = RQ = \frac{1}{2}DB \ldots\ldots(2)$

The diagonals of a rectangle are equal
$\therefore AC = BD$ ....(3)
From (1), (2) and (3) we have
$PQ \parallel RS$ and $PS \parallel RQ$ and
$\therefore$ $PQ = QR = RS = SP$

$\therefore$ PQRS is a rhombus.

Question 10:

Given : ABCD is a rhombus in which P,Q,R and S are the mid – points of AB, BC, CD and DA respectively.



To Prove :PQRS is a rectangle.
Construction : Join AC and BD.
Proof :
Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
In ΔABC
P and Q are the mid points of AB and BC.
So by Mid point Theorem,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

Similarly, from ΔADC,

$RS \parallel AC$ and $RS = \frac{1}{2}AC$

$\Rightarrow PQ \parallel RS$ and $PQ = RS = \frac{1}{2}AC$......(1)

Now, in ΔBAD,
P and S are the mid – points of AB and AD.

So by Mid – point Theorem, we have

$PS \parallel BD$ and $PS = \frac{1}{2}DB$

Similarly, from ΔBCD,

$RQ \parallel BD$ and $RQ = \frac{1}{2}DB$

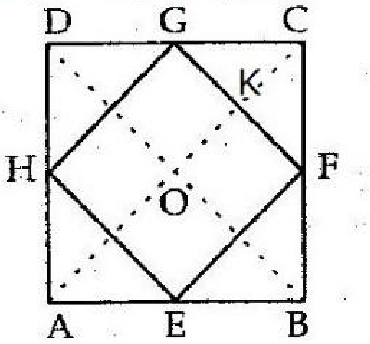
$\Rightarrow PS \parallel RQ$ and $PS = RQ = \frac{1}{2}DB$......(2)

From (1) and (2), we have
PQRS is a parallelogram as its opposite sides are parallel.
We know , that in a rhombus, diagonals intersects at right angles.

$\therefore$ $\angle EOF = 90^0$
Now, $RQ \parallel DB$
$\Rightarrow$ $RE \parallel FO$
Also, $SR \parallel AC$
$\Rightarrow$ $FR \parallel OE$
$\therefore$ OERF is a parallelogram.
In a parallelogram, opposite angles are equal.
So, $\angle FRE = \angle EOF = 90^0$
Thus, PQRS is a parallelogram with $\angle R = 90^0$
Hence, PQRS is a rectangle.

Question 11:

Given : ABCD is a square in which E, F, G and H are the mid points of AB, BC, CD and AD, respectively.
The mid points are joined together.



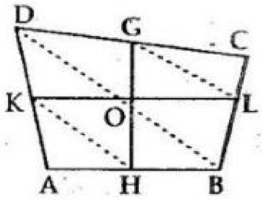
To prove : EFGH is a square.
Construction : Join AC and BD
Proof :
Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
In ΔABC
E and F are the mid – points and by the Mid points Theorem, we have
$EF \parallel AC$ and $EF = \frac{1}{2}AC$
Similarly, in ΔADC,
H and G are the midpoints and by the Mid points Theorem, we have
$HG \parallel AC$ and $HG = \frac{1}{2}AC$
Thus, we have,
$EF \parallel HG$ and $EF = HG = \frac{1}{2}AC$.....(1)

In ΔBAD,
H and E are the midpoints and by the Mid points Theorem, we have,
$HE \parallel BD$ and $HE = \frac{1}{2}BD$
In ΔBCD,
G and F are the midpoints and by the Mid points Theorem, we have,
$GF \parallel BD$ and $GF = \frac{1}{2}BD$
Thus, we have,
$HE \parallel GF$ and $HE = GF = \frac{1}{2}BD$.....(2)
The diagonals of a square are equal.
$\Rightarrow AC = BD$ ......(3)

From (1), (2) and (3), we have
$GF \parallel BD$ and $HE \parallel GF$.
Also, we have $EF = GF = GH = HE$
So, EFGH is a rhombus
Now, as diagonals of a square are equal and intersect at right angles.
So, $\angle DOC = 90^0$
In a parallelogram the sum of adjacent angles is 180^0.
So, $\angle DOC + \angle GKO = 180^0$
$\Rightarrow \angle GKO = 180^0 - 90^0 = 90^0$
But $\angle GKO = \angle EFG$ [Corresponding angles]
$\therefore \angle EFG = 90^0$
$\therefore$ EFGH is a square.

Question 12:

Given: A quadrilateral ABCD in which H, L, G and K are the mid points of AB, BC, CD and AD.
Points G and H are joined and k and L are joined.



To prove: GH and KL bisect each other.
Construction: Join KH, BD and GL.
Proof: Since K and H are the mid points of AD and AB.
So in ΔABD, by mid point theorem,

$\Rightarrow \quad KH = \frac{1}{2} BD$

Similarly, in ΔCBD,

$GL = \frac{1}{2} BD$

$\Rightarrow \quad KH = GL$

Now in ΔKOH and ΔGOL, we have

$KH = GL$

$\angle OKH = \angle GLO$ [Alternate angles]

$\angle OHK = \angle OGL$ [Alternate angles]

$\therefore \quad \Delta KOH = \Delta GOL$ [SAS]

$\Rightarrow \quad OK = OL$ and $OG = OH$ [C.P.C.T.]

$\therefore$ GH and KL bisect each other.